A frequency decomposition time domain model of broadband frequency-dependent absorption: Model II

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1. Summary

This report is the second in series about time-domain modelling of broadband frequency-dependent attenuation via a frequency decomposition approach. The other two reports are concerned with the **modified mode superposition model** [1] and the **fractional derivative model** [2].

The medical ultrasound transducers as well as other real-world acoustic sources mostly produce multiple-frequencies acoustic wave. This study aims to extend the frequency-dependent attenuation time-domain model of single frequency excitations [3] to broadband excitations via finite frequency decomposition. The basic idea behind this strategy is to divide a broadband excitation into a bank of narrowband excitations and then regard each narrowband source as a single frequency source. Finally, the responses of all narrowband sources are summed up as the system response of the corresponding broadband source. In essence, the frequency decomposition is based on the superposition principle. A frequency decomposition experimental simulation [4] has verified that the methodology is accurate, reliable and efficient.

It is expected that the strategy will also work with the FEM time-domain numerical simulation. Compared with the previous mode superposition model [1], the present broadband model also satisfies the principle of causality and is as simple to implement, more computationally affordable since it circumvents computationally expansive fractional power of a matrix. The additional computing effort of this model is to the forward and inverse bandpass filter transforms of broadband excitations, which is trivial among the whole simulation. However, the computing effort of broadband excitations is expected about ten times higher than that of a single excitation. Blessing is that the model is inherently very suitable for parallel computing. The direct extension of the present model to nonlinear media, however, is blocked since the superposition principle ceases to hold. A linearization iteration method [5] may revitalize the model to nonlinear problems, which we will investigate in the future.

The rest of the report is grouped into the four sections. In section 2, the time domain model of single frequency excitation is briefly discussed for the completeness of this self-contained report, and then, in section 3, we outline the frequency decomposition procedure to model the ultrasound propagation under a broadband excitation in time domain. Section 4 is dedicated to the bandpass filter which decomposes a broadband excitation into a bank of narrowband excitations. Finally, section 5 analyzes the dispersion properties of the present broadband time domain model.

2. Time-domain model for single frequency excitation

The frequency-power attenuation is given by

$$p(x+\Delta x)=p(x)e^{-\alpha(f)\Delta x}, \tag{1}$$

where p represents the amplitude pressure, and f is the frequency,

$$\alpha(f) = \alpha_0 f^y, \qquad y \in [0,2]. \tag{2}$$

Here α_0 is the attenuation constant and y the frequency-power exponent, both of which are dependent on tissues.

Assuming the viscous absorption linearly depends on the velocity, the standard time-domain damped ultrasound wave model can be expressed as

$$\frac{1}{c^2}\ddot{p} + \gamma\dot{p} - \nabla^2 p = 0, \tag{3}$$

where γ is the viscous coefficient, and upper dot denotes the temporal derivative. The FEM analogization of spatial Laplacian yields the semi-discretization equations

$$\ddot{p} + c^2\gamma\dot{p} + c^2 Kp = g(t), \tag{4}$$

where K is the symmetric FEM interpolation matrix of Laplace operator, and vector $g(t)$ is due to the external excitation source (transducers on boundary surface of our problems). In the case of a singular frequency excitation $g(t)$, [1] derived

$$\ddot{p} + 2\alpha_0 c f^y \dot{p} + c^2 Kp = g(t) \tag{5}$$

corresponding to the empirical frequency-dependent absorption (1). In the preceding superposition model report [1], it is shown that (5) satisfies the causality principle.

3. Frequency decomposition time domain scheme

If the external source is a broadband excitation $s(t)$ instead of a single frequency $g(t)$ in (4), the models (4) and (5) can not hold with the frequency-dependent attenuation (2) except when y=0. [3] suggested using the central frequency of $s(t)$ as a single frequency in determining the viscous coefficient, and then the formulation (5) is applied. In some cases, this procedure may introduce unpredictable significant model error. Following the frequency decomposition idea presented in [4], a more accurate alternative scheme should be to decompose the broadband excitation $s(t)$ into a sum of narrowband excitations $s_i(t)$, i.e.

$$s(t) = \sum_{i=1}^{m} s_i(t) \ . \tag{6}$$

When the bandwidth of each component is narrow enough, the attenuation effect can be simulated at the center frequency of each narrowband component [4]. Then, the model (5) for single frequency excitation is applied to each narrowband component source $s_i(t)$ with their respective central frequency, and we get m sets of semi-discrete FEM system equation

$$\ddot{p}_i + 2\alpha_0 c f_i^{\,y} \dot{p}_i + c^2 K p_i = s_i(t), \tag{7}$$

which f_i is the central frequency of $s_i(t)$. Although the FEM discretization for matrix K is only done once for all, the repeated solution of m sets of ordinary differential equation systems (7) is quite time-consuming given that each (7) is a semi-discrete FEM equation of large ODEs. The parallel solution of all temporal ordinary equation sets (7) could be crucial to improve the efficiency.

In terms of the superposition principle under the linear system, the response of a broadband excitation $s(t)$ is a sum of those of all of its narrowband component excitations

$$p(t) = \sum_{i=1}^{m} p_i(t) \ . \tag{8}$$

(6), (7) and (8) outline the basic methodology of the present scheme. Fig. 1 illustrates the schematic solution procedure via the frequency decomposition of a broadband pulse.

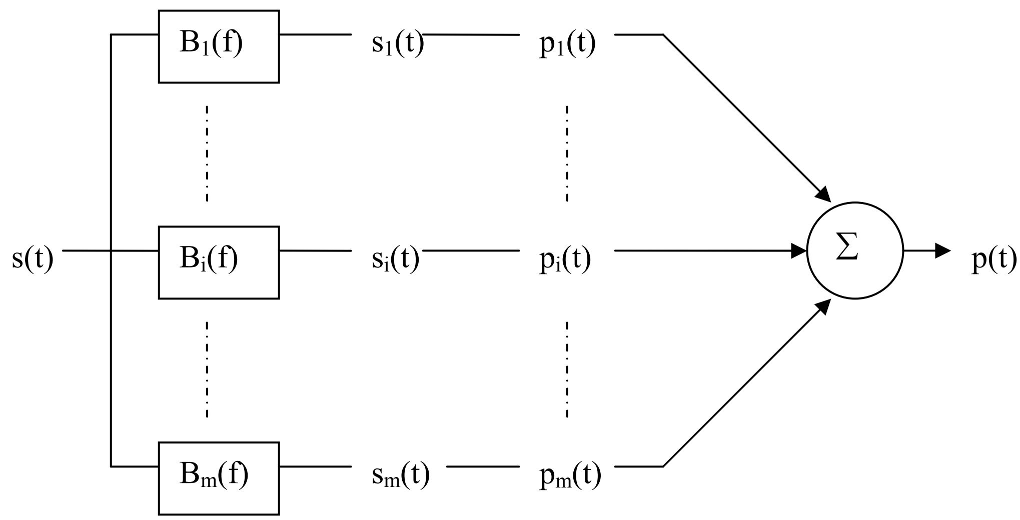


Fig. 1. Schematic procedure of frequency decomposition time domain model

The major sources of error of the present mathematical model are

1) the central frequency of narrowband component source in (7) is approximately used as in the single frequency case;
2) the formulation (5) for single frequency excitation itself is also an approximate analog of the damping effect which ignores the transient solution.

Since there is no need to calculate the fractional power of matrix *K*, the present model is computationally more efficient compared with the mode superposition model given in [1]. However, the present model also faces a new computing challenge: the repeated (about ten times) solution of ODEs (7) for all narrowband responses.

4. Bandpass filters for frequency decomposition

This section addresses the bandpass filter issue, a key part of the present broadband time domain model. The filter is to perform frequency-dependent alterations of a signal, i.e. filtering. As shown in Fig. 1, a set of the bandpass filter $B_i(f)$ divide the broadband $s(t)$ into a sum of narrowband $s_i(t)$. One of the simplest, fastest and convenient approaches to alter the frequency properties of a signal by filtering is to apply the linear frequency-domain filter

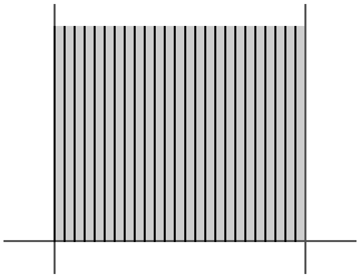
$$S_i(f) = B_i(f)S(f), \tag{9}$$

where f denotes the frequency parameter, B_i is the transfer function as shown in Fig. 1, S the Fourier transform of the input broadband signal $s(t)$, S_i the Fourier transform of the filtered narrowband signal $s_i(t)$. (9) can be equally expressed as a convolution operation

$$s_i(t) = \int_{-\infty}^{\infty} b_i(\tau) s(t-\tau) d\tau, \tag{10}$$

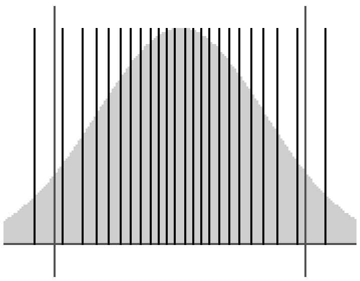
where b_i is the inverse Fourier transform of the frequency domain filter B_i. Filtering in frequency domain gives superior performance compared with the other filters design techniques. Its main drawback is to require complete signal accessible. This is not an issue in our medical ultrasound case. Some typical bandpass filters are illustrated and briefly commented in Table 1 [6].

Table 1. Typical bandpass filters



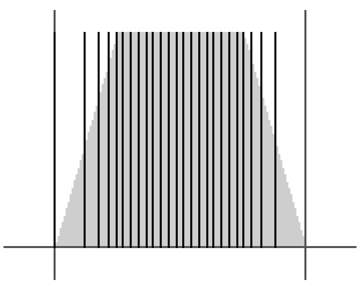
Box filter

This filter is an ideal frame aperture. A frame using this filter distributes the time samples regularly over the specified amount of time.



Gauss filter

The gauss filter simulates the behavior of a tube camera. The time behavior looks like a bell where the samples are concentrated in the middle of the frame.



Shutter filter

The shutter filter simulates the shutter of a mirror reflex camera. In the first quarter of the frame the shutter opens. Then the shutter remains open for the half time and then the shutter closes. Under **Blizzard II** the proportion is fixed to 1:2:1.

He [4] recommended the use of the Gauss filter to minimize the reconstruction error. This is owing to that if the system transfer function is Gaussian, the Gaussian solution for an impulse input is an excellent approximation of the exact causal solution for most application problems [7]. Each bandpass filter used in [4] has a Gaussian magnitude function

$$B_i(f) = \frac{1}{\sqrt{\pi}} e^{-\left(\frac{f - f_L - (i-1)B}{B}\right)^2}, \quad i=1,2,\ldots,n, \tag{11}$$

where

$$B = \frac{f_H - f_L}{m-1}, \tag{12}$$

f_L and f_H respectively denote the lower and upper limits frequencies. Since the energy at higher frequency components are dissipated much more rapidly that those of lower frequencies, it may be more efficient to divide the frequency in a non-uniform fashion unlike (12). We will explore this issue in the later research. [4] pointed out that the division number m is not necessary big since the experiments show that m=10 or so produces very accurate reconstructions. As an example, the energy spectrum of a broadband source is illustrated in Fig. 2, where W_f is the bandwidth and f_C central frequency.

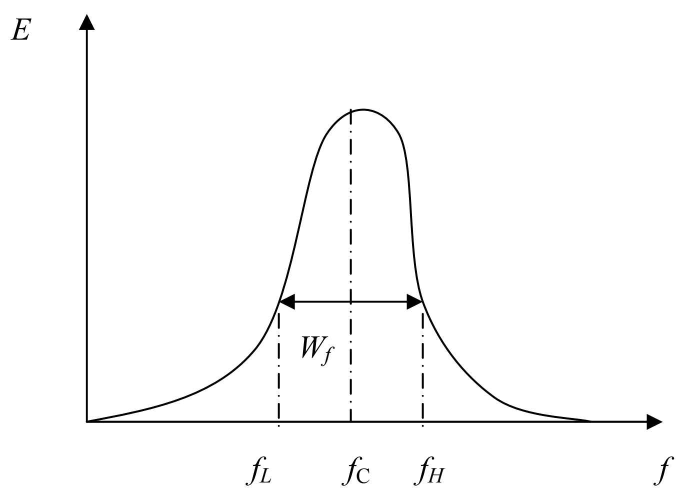


Fig. 2. A broadband pulse

5. Dispersion analysis

With the Duhamel integral and when $\alpha_0 f_i^y < \omega_j$, where ω_j are the eigenvalues of matrix K in (7), we can have the solution of each modal equation (7)

$$p_i(t) = \sum_j^m \frac{1}{\hat{c}_{ij}} \int_0^t s_i(\tau) e^{-\alpha_0 c f_i^y (t-\tau)} \sin \hat{c}_{ij}(t-\tau) d\tau + e^{-\alpha_0 c f_i^y t} \left\{ \alpha_{ij} \sin \hat{c}_{ij} t + \beta_{ij} \cos \hat{c}_{ij} t \right\} \qquad (13)$$

where α_{ij} and β_{ij} are calculated using the initial conditions, and the distorted phase velocity $\hat{c}_{ij}$ dependent on the attenuation is calculated by

$$\hat{c}_{ij} = c\sqrt{\omega_j^2 - \alpha_0^2 f_i^{2y}} \,. \qquad (14)$$

(14) shows that the viscous effect causes the dispersion. When $\alpha_0 f_i^y > \omega_j$, the solution of (7) is

$$p_i(t) = \sum_j^m \int_0^t s_i(\tau) h(t-\tau, f_i, \omega_j) d\tau + \alpha_{ij} e^{-\alpha_0 c f_i^y t - ct\sqrt{\alpha_0^2 \omega_j^2 - f_i^{2y}}} + \beta_{ij} e^{-\alpha_0 c f_i^y t + c\omega_i t \sqrt{\alpha_0^2 \omega_j^2 - f_i^{2y}}} \,, \qquad (15)$$

and under critically damped ($\alpha_0 f_i^y = \omega_j$), the solution is

$$p_i(t) = \sum_j^m \int_0^t s_i(\tau) r(t-\tau, f_i, \omega_j) d\tau + e^{-\alpha_0 c f_i^y t} (\alpha_{ij} + \beta_{ij} t), \qquad (16)$$

where h and r are the impulse response function of systems. It is well known that the ultrasound wave is heavily attenuated while traveling through soft tissues. The transient solutions, which are the second right terms of (13), (15) and (16), are rapidly evanescent and the frequency spectrum of the propagating ultrasound wave is close to that of its transducer excitation source. Thus, it is reasonable to split the broadband ultrasound wave in terms of the excitation frequency spectrum.

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