Heterogeneous Growth rates in Norwegian Firms

Determinants of Norwegian firm growth rates in the different Quantiles of its distribution

Trym Langballe

Thesis submitted for the degree of
Master in Economics
30 credits

Department of Economics
Faculty of Social Sciences

UNIVERSITY OF OSLO

Spring 2021

Heterogeneous Growth rates in Norwegian Firms

Determinants of Norwegian firm growth rates in the different Quantiles of its distribution

Trym Langballe

Heterogeneous Growth rates in Norwegian Firms

http://www.duo.uio.no/

Printed: Reprosentralen, University of Oslo

Abstract

In this thesis I analyse a wide panel data set from the Central Coordinating Register of Legal Entities (VoF) of Norwegian firms between 2007 and 2016. The main goal is to present an analysis of the distribution of Norwegian firm growth-rates, and to identify important firm- and industry specific determinants of these growth rates. Further, a main goal of the thesis is to distinguish between how the determinants affect the different parts of the firm growth-rate distribution, and to identify whether any of these determinants are particularly important for achieving high growth rates. The main way to identify these differences will be the Quantile Regression method, which allows me to identify how the different determinants are affecting different parts of the distribution.

Contents

List of Figures

List of Tables

Preface

I would like to thank my supervisor Karen Helene Ulltveit-moe. This thesis was originally intended as a contribution to the INNOPROD project at the Department of Economics - UiO, and I hope it still could be useful to the project. I am thankful to the project for providing financial support and for providing the data used in this thesis. This thesis would not have been possible without generous help and support from friends and family, of which I am truly thankful. Among these a special thank you is due to Maja Olderskog Albertsen, Ingrid Engebretsen and Kristen Vamsæter for the generous use of their time reading the thesis, and for providing valuable insights. Last, a thank you to the Institute of Economics for being patient with me, and for the years I was able to spend there.

Chapter 1

Introduction

In 2017, the Norwegian government issued a commission to study Norwegian businesses' access to capital and Norwegian capital markets (NOU2018:5, 2018). In this study, the commission remarked that Norway has a small number of high-growth firms compared to other OECD countries (NOU2018:5, 2018, p.102). Why are Norwegian firms unable to achieve the same levels of firm growth as firms in other OECD countries? The purpose of this thesis is to answer this question. In particular, the thesis aims to identify the determinants of Norwegian firm growth and what distinguishes the different parts of the growth distribution, particularly how the top quantiles containing the high-growth firms are affected by the determinants. Such traits include firm-specific determinants like firm age, number of employees, and some sector-specific determinants like industry size, industry growth, and level of competitiveness within each industry. There is a large body of empirical research on firm growth and its determinants. In general, researchers have found that the productivity and growth of firms is heterogeneous. Thus, some firms perform better and some firms perform worse than others due to the heterogeneous nature of firm productivity. The empirical method I use in this thesis aims to identify what makes some Norwegian firms perform better than others while respecting the heterogeneous nature of firm growth. The analysis I present is threefold. First, I examine the growth rate distribution of firms. I do this to determine the shape of the statistical distribution of the growth rates and to investigate whether it remains constant over time or not. The second part of the analysis will identify determinants of firm growth and whether the effects of the determinants differ for different parts of the distribution. I do this by estimating a quantile regression model for firm growth rates, which allows me to account for the firms' heterogeneous nature. In the QR analysis, I estimate two models. The first model only considers firm-specific variables like age and size, while the second model also includes industry-specific variables. In the third part of the analysis, I look into the persistence of the growth rates over time. For this, I use a transitional probability matrix, which looks at firms in different quantiles of the distribution and estimates the transitional probabilities that a firm is in any quantile, conditional on the quantile where it resided the previous period. This way, it is possible to determine whether the firms achieving high growth will repeat their success.

Chapter 2

Theory and Previous Empirical research

2.1 Theory

There is a substantial body of literature concerning the growth of firms. Some of the main theories, and the ones providing the theoretical basis for this thesis, are summarized in the section below. Certainly the most mentioned paper in the literature on firm growth is the paper by Gibrat (1931) concerning firm growth rates. However, his theory is not concerned with the heterogeneous nature of growth rates, and it is not too instructive for the purpose of this thesis. Ideally there would be some theoretical work regarding the firms that outperform others (i.e. the high-growth firms), but I have not been able to find any such papers. However the heterogeneity of firm growth rates has been considered in some theoretical work, and two of these are summarized below with Jovanovic (1982), and Hopenhayn (1992b)

2.1.1 Gibrat's Law

A lot of the literature I have reviewed is concerned with Gibrat (1931). In particular, researchers have tried to examine whether or not Gibrat's "law" holds, and what it says about firm growth. The "law" suggests that the logarithm of firm size develop in such a way that the expected firm size in the next period is proportional to the current size of the firm, and that small additive independent increments in firm size would generate a normally distributed variable. In each period, a new set of opportunities arise, and the probability of exploiting them is proportional to the size of the firm. Following Sutton's (1997, p. 40-41) way of presenting this problem: In a model where the growth is defined by the change of size between two periods, and where the random variable ε_t denotes the proportional rate of growth between period $(t-1)$ and period t, such that $x_t - x_{t-1} = \varepsilon_t x_{t-1}$ we have that

$$x_t = (1+\varepsilon_t)x_{t-1} = x_0(1+\varepsilon_1)(1+\varepsilon_2)...(1+\varepsilon_t)$$

For a short" time period ε_t can be regarded as being "small," and we can approximate $log(1+\varepsilon_t) \simeq \varepsilon_t$. Taking logs, we obtain

$$log(x_t) = log(x_0) + \varepsilon_1 + \varepsilon_2 + ... + \varepsilon_t$$

If we assume the increments ε_t to be independent with mean m and variance σ^2, as $t \to \infty$, the term $log(x_0)$ will be small compared to the sum of the ε_t's and thus $log(x_t)$. This means that the distribution of $log(x_t)$ is approximated by a normal distribution with mean mt and variance $\sigma^2 t$. In other words, the limit distribution of x_t is log-normal. The log transformed size will follow some process:

$$log\, x_t = \alpha + \beta\, log\, x_{t-1} + v_t$$

Were β is the effect of previous size on current size. Gibrat's law imply that $\beta = 1$. In general, empirical research suggests that the "law" does not hold. The empirical literature concerning the relationship between growth and firm size is discussed in chapter 2.2.1 below.

2.1.2 Jovanovic

Jovanovic (1982) presents a theory of firm growth with passive learning that is frequently mentioned in the literature I have reviewed. He proposes a model of industry evolution, where an industry consists of a continuum of firms. The firms in the industry are all unaware of their true productivity (he labels the true productivity "type") when they enter the industry, but they do know the distribution of the efficiency parameter that they draw their type from. When they enter the market, they randomly draw their productivity θ from a $N(\bar{\vartheta}, \varsigma^2_{\vartheta})$ distribution, and then they start to produce. To gain information about their type, they observe the costs they get from production, and gain imperfect information about their true type through these costs. Over time, the firms with lower productivity, i.e. the types that are less efficient, will realize that they are less efficient, produce less, and eventually exit. The firms that do not exit will continue to learn about their productivity through repeated observation of their costs, and as they gain more information over time, their uncertainty decreases, and production stabilizes . The profit function of every firm maximizes with respect to output (q) is:

$$q_t \, p_t - c\,(q_t)\, E_{t-1}\, \zeta\,(\theta + \epsilon_t)$$

Output q is decreasing in the expected productivity θ, so firms believing they are less efficient will have less output. The firms face a dynamic problem that has to satisfy the value function;

$$V\,(x, n, t; p) = \pi\,(p_t, x) + \beta \int max\,[\,W,\ V(z, n+1, t+1; p)]\, P\,(dz|x, n)$$

Here W is the value the firm get from selling their assets when exiting. The function V is strictly increasing in x (the beliefs about productivity), and z the observed productivity. The model predicts that, as new firms are uncertain about their type when they are young, they might be under- or overestimating their productivity. The effect of revealing their true productivity through the encountered costs then results in higher or lower growth rates for the surviving firms. As time moves on the firm learn their types, and production stabilizes. Thus, a firm sets its output (and employment) based on its guess about its type. If, at the end of the period profits are larger than expected, the firm infers that it is more efficient than it guessed in the previous period. If this is the case, firms update their guess and increase their output (and employment). Since younger firms experience more uncertainty about their type then their older counterparts, they are more likely to make mistakes and set their size at a lower or higher level than their level of efficiency would require. Thus, the update is much stronger and the growth rates are both larger and more volatile. Smaller firms are more likely to fail, and so the model implies that those who fail are the ones who would have grown more slowly, leading to a selection bias towards the young firms with high growth rates.

2.1.3 Hopenhayn

Hopenhayn (1992b) introduces a model with heterogeneous firms for steady state industry dynamics. The steady state in the model predict that firms enter, grow, decline, and exit, but that overall distribution of firms is unchanging. Denoting the a firm's productivity with $\varphi, \in [0, 1]$, Hopenhayn assumes that productivity of each firm follows a Markov process with a density function such that $\varphi_{t+1} \sim F(\,\cdot \mid \varphi_t)$ where the CDF is strictly decreasing in φ. Higher φ_t means higher F by first order stochastic dominance, and the higher the productivity shock is in period t, the more likely they are to draw higher shocks in period $t+1$. Firms have perfect foresight of output and input prices (p_t, w_t).

Entrants to the draw their initial productivity from a distribution $G(\cdot)$ and pay an initial cost to do so. Within each period, incumbents decide to remain in the industry or exit; entrants decide if they want to enter or not. The incumbent who chose stays pays fixed cost c_f and gets to draw a realization of its productivity from the distribution, then produces. Entrant who chose to enter pays a different cost c_e as entry cost, then draws its productivity realization, and produces. The state of the industry in a period t is denoted by μ_t , which is the measure over all productivity levels mass of firms that are currently active in the industry.

They solve the following dynamic programming problem:

$$V_t(\varphi, z) = \pi(\varphi, z_t) + \beta max\left\{0, \int V_{t+1}(\varphi', z)\, F(d\varphi'|\varphi)\right\}$$

The value function is increasing in φ under regularity assumptions. And there exists a cutoff pointx_t, such that the firms below this threshold below exit. Entrants will keep entering the industry until the expected value from production, is zero, so that there is a positive mass M_t of entrants when

$$V_t^e = \int V_t(\varphi', z)\, G(d\,\varphi') = c_e$$

Saying that the expected value (value of producing times the initial productivity distribution) is equal to entry cost. This implies an evolutionary process for the state of the industry following:

$$\mu_{t+1}([0, \varphi']) = \int_{\varphi \geq x_t} F(\varphi'|\varphi)\mu_t(d\varphi) + M_{t+1}G(\varphi')$$

The sum of all firms with productivity draws over the cutoff x_t + [the expected mass of entrants the following period M_{t+1}]. For a $c* > 0$ such that for any $c_e < c*$ there exists a competitive stationary equilibrium with positive entry and exit. If certain assumptions are satisfied, we get a unique solution. The implications of the model is as follows: The size, profits and value distribution of firms increases with age. An increase in entry cost reduces entry number of entrants (M) and the turnover $(M/\mu(s))$.

2.1.4 The Tent-shaped Distribution

Bottazzi and Secchi's (2003)(2006) papers is often cited for their work on growth rate distribution. Their model describe a mechanism where the assignment procedure of business opportunities to a firm has a self-reinforcing effect. The probability that a firm gets a new business opportunity depends on the number already caught. And a positive feedback is generated that can be caused by, among others: Increasing returns in the growth process of firms, economies of scale, and knowledge accumulation. This leads to the presence of a fat tail in the distribution. And this indicates that a large numbers of opportunities is assigned to a single firm. A successful firm is the outcome of this firm's ability to build on its success based on previous successful behavior in a volatile environment. Bottazzi and Secchi found a simple generalization that is able to describe the empirically identified tent shape of the empirical growth rate density. The model they present converges to what is known as a Laplace distribution:

$$f_L(g; \mu, a) = \frac{1}{2a} e^{\frac{-|g-\mu|}{a}}$$

2.2 Empirical work

I have found much empirical research on the growth rates of firms. The papers that are discussed in the following section all try to identify one or several determinants of firm growth using different research designs. Among these, the ones that have received most attention in the literature is the size of firms and the age of firms, industry affiliation and Persistence of growth rates have not been as widely examined, but still it has been considered in some of the papers I have reviewed here. I have not found many papers on high-growth firms in particular, but it is sometimes considered in the papers concerning firm growth in general. The papers that influenced me the most when I choose the method applied in the thesis are summarized in section 2.2.5.

2.2.1 Firm size

When examining the relationship between firm growth and firm size, Gibrat's law has attracted most attention from researchers. Sutton (1997) and Caves (1998) survey a number of empirical research papers concerning the validity of Gibrat's law. They find some proof that Gibrat's law applies to some subsamples of firms. Particularly, they find that firms who are initially large grow more randomly,but for other subsamples, small firms tend to grow more than larger firms. This means that firm growth might be influenced by the size of firms, so the law does not always hold. More recently, Daunfeldt and Elert (2013) tests whether firm growth is independent of firm size or not. They look at a sample of Swedish limited liability firms between 1998 and 2004. They found (when looking at the pooled data from all industries) that growth rates are higher for smaller firms compared to larger firms. When they look at sub-samples by individual industries, the results become more diffuse, however. Three insights about the relevance of industries is especially highlighted. First, when an industry is larger, there is a smaller the probability that Gibrat's law will hold. Second, Gibrat's law is more likely to be rejected in industries characterized by high Minimum Efficient Scale, and a high share of firms in metropolitan areas. Third, Gibrat's law was more likely to hold in mature industries characterized by a high degree of group ownership, and in industries with high market concentration. With these insights in mind, they conclude that industry affiliation seems to matter a great deal for the relationship between firm size and growth rates. Industry affiliation will be further discussed in section 2.3 below. Some concerns are raised about the validity of their analysis. For example, high market concentration might occur in natural oligopolies, or small firms might migrate to larger markets to realize growth (selection effect). They also did not dis-aggregate on the geographical level, which might be a problem, as not all firms compete on a national level.

Delmar et al. (2003) differentiate between different types of growth and discover that high-growth firms of different sizes tend to grow in different ways. Firms characterized by steady overall growth were dominated by large firms (especially for steady growth in sales). These firms also had the largest absolute growth in employment. Coad et al. (2013) find that the effect of firm size has a negative effect on firm growth for employee-, sales-, and profit growth, but positive for effect on growth in productivity. Segarra and Teruel Carrizosa (2014) perform a QR, and find that large firm size impacts firm growth rates negatively, and the impact decreases as one move towards the top quantiles. In other words, the effect becomes more negative among the firms with higher growth rates. The impact is positive in the lower quantiles, but here they are not able to get significant results. They explain this by hypothesizing that a number of firms in the higher growth quantiles is small, which means that they are further away from their minimum efficient scale, and thus needs to grow more.

Henrekson and Johansson (2010) present results regarding firm size and observe that Gazelles (firms that grow with more than 20% over three years) are found in all sizes, but smaller firms are over-represented. They find that larger Gazelle firms are more important for job creation in absolute terms. Finally, they comment that age (being a young firm) is a more important determinant for Gazelles than their size.

2.2.2 Firm age

Much research is concerned with the relationship between age and growth, but again not so much about high-growth firms specifically. **coad** analyze the growth rates of Spanish manufacturing firms between 1998 and 2006. In particular, they look at the growth rate distributions for firms of different ages and find that young firms are more likely to grow faster than older. Still, young firms have roughly the same chances of accelerated decline in size as older firms. For young firms, growth is positively correlated with financial performance, and profitable young firms have a higher expected growth rate than less profitable firms. Younger firms are also more successful at converting employment growth into growth in sales, profits, and productivity, whereas older firms convert sales growth into growth in profits and productivity. Moreover, they find that older firms tend to have lower expected growth rates in sales, profits, and productivity compared to younger firms. Growth in employees is more common for young firms, while a focus on sales growth seems more appropriate when the firm gets older. Delmar et al. (2003) find that age has a significant effect on different growth patterns. In particular, 71% of the firms with high relative growth rates was created during the measurement period, which means that they were 10 years old or younger. This is compared to firms growing mainly through the acquisition of other firms, and in this group only about 15% of the firms were created during the measurement period. Barba Navaretti et al. (2014) look at a group of French, Italian, and Spanish manufacturing firms. Using a QR approach, they find that young firms grow faster than older firms, especially in the highest growth quantiles. They find that young firms face the same probability of declining as older firms and that the results are robust to other firm characteristics, such as labor productivity, capital intensity, and the firm's financial structure. The QR suggests that the effects of age on growth rates are not constant, and the effect is larger for firms with higher growth rates (being young is especially important for the fastest-growing firms). Productivity and access to credit have a strong positive effect on the firms experiencing the highest growth rates. Their results are consistent across the three countries, despite non-negligible cross-country differences in the age structure of firms. The papers that Henrekson and Johansson (2010) looked at in their literature review, all suggest that being a young firm rather than an old firm is associated with higher growth. All the studies that concern the age of firms rapport that gazelles tend to be younger firms.

2.2.3 Industry Affiliation

Further the literature review of Henrekson and Johansson (2010), is concerned with the representation of Gazelles in various industries. They find that Gazelles are represented in all industries but appear to be over-represented in service industries. Most of the studies examined in the review are concerned with total growth, and some consider differences in organic and acquired growth. These studies find that small firm growth tends to be more organic, while large firm growth tends to be characterized by acquired growth. Thus, young firms could be responsible for significant portion of new jobs created. Delmar et al. (2003) find that both firms with high relative growth and high absolute growth are found in knowledge-intensive industries. The growers in absolute terms were over-represented in manufacturing industries (high-tech. and technology-oriented manufacturing). The relative growers were mostly in professional service industries (business, information technology, consultants, advertising, education, and health care). The steady overall growers were primarily found in manufacturing industries. Levratto et al. (2010) examine French firms, with more than 10 and less than 250 employees between 1997 and 2007. Using structural and strategic determinants of firm growth, they carry out an anlysis using a multinomial logit model. They find that there is a higher probability of being a high-growth firm when the firm belongs to either the computer manufacturing industry or the electronic manufacturing industry. Daunfeldt et al. (2016) analyse a data set of limited liability firms in Sweden during the period 1997 to 2008. They find that high-growth firms are over-represented in knowledge-intensive service industries (i.e. service industries with a high share of human capital). They use a fractional logit regression. Both sales and employment are used as a measure of growth and they define high-growth firms as firms with the top 1% growth rates in sales or employment over 3 years. They separate between four groups of high-growth firms: Absolute employment- , relative employment- , absolute sales- , and relative sales high-growth firms. In all four groups, high growth is defined the same way. Knowledge-intensive service industries were more likely to have a higher share of high-growth firms. They suggest that human capital rather than R&D is crucial in explaining fat tails in the growth distribution. A higher share of high-growth firms is found in industries with larger firms. The share of high growth firms also seems to be determined by firm age within the industry, but the direction of the results depends on the choice of growth measure. Industries characterized by older firms have a higher share of high growth firms when firm growth is measured in terms of absolute changes but a lower share when growth is measured in relative terms. Segarra and Teruel (2014) find a difference between firms in industrial manufacturing and service industries when looking at the effect of innovation on high-growth firms. They find that manufacturing firms are significantly affected by R&D investment per employee, while growth in service firms appear not to be affected as much by R&D investment.

2.2.4 Persistence of Growth

Although not a main focus of this thesis, the possibility of auto-correlation in growth rates should not be entirely neglected, as it seems to be an important determinant of firm growth. Daunfeldt and Halvarsson (2015) test the persistence of growth in Swedish high-growth firms. Their main focus is the firms' number of employees and total sales growth. They define high-growth at different cut-off levels: the top 10% and 1% of the growth distribution. They find that fast high-growth firms are more likely to show declining growth in the next period, regardless of the growth indicator, and growth rate cut off-level. One possible explanation for this is, in their view, that firms recover in the three years following the high growth, possibly due to adjustments in costs. Further, they find that small firms do not tend to repeat their growth, while it is more probable for larger firms.

Hölzl (2014) use data from Austrian firms, and study the survival, persistence, and growth of fast-growing firms 3 and 9 years after their fast-growth period. He employs two definitions of fast growth: First, high-growth firms are defined as the firms that achieve a growth rate of at least 20% three years in a row (similar to gazelles), and have a size of at least 10 employees at the start of the period. And second, he uses the Birch index, which is a combination of absolute change with relative growth rates. For firms defined as high-growth, he finds that the probability of survival after both 3 and 9 years increases if a firm is defined as a high-growth at the beginning of the period. However, if a firm is defined as high-growth at the end of the period the chance of survival is not affected.

He does not find that high-growth in itself affects chances of survival, as the increased survival probability in beginning period, is explained by the increase in size during the fast-growth period. For the high Birch-firms, the effect of achieving a high growth at either the start or the end of the period is associated with an increase in the probability of survival of firms after 9 years. He finds that the Birch index primarily captures absolute employment changes.

Haltiwanger et al. (2010) Use matching to construct control groups of firms that are similar in age, size, and industry affiliation to the fastest-growing firms. High-Birch firms have a much higher probability of repeating their growth compared to high-growth firms. Both high-growth firms and high-Birch firms have better growth performance than the firms selected for the control groups. Being a high-growth firm does not increase the likelihood of survival in future periods compared to the control firms. The high-growth firms have higher probability of being a high-growth firm 3 or 9 years after the growth compared to the control firms. However, this effect is small for high-growth firms. Thus, most high-growth firms are "one-hit wonders".

2.2.5 Empirical strategies

The papers reviewed above produce pretty consistent results, and they suggest which determinants that could be relevant to understand Norwegian firm growth. However, the papers apply a range different methods, which are not all suitable when we want to identify heterogeneous firm growth rates. Still, there are some papers that use empirical methods that accounts for heterogeneity in growth rates, i.e. quantile-regression (QR). This thesis' research design is inspired by the following papers. The empirical distribution of firm growth rates is studied in the papers by Bottazzi and Secchi (2003, 2006), talked about in the first section. In their (2006) paper they look at firms in the Italian manufacturing industry, and conclude that a tent shape (double-exponential) distribution is a very robust feature of their data. Coad and Rao (2008) relate innovation to growth in sales for incumbent firms in high-tech sectors. They observe that growth rate distributions are heavy-tailed, use a quantile regression approach, and observe that innovation is important for a handful of fast-growth firms. Reichstein et al. (2010) examine the growth rate distribution through QR. They look at the size of firms and industry dynamics. Their results suggest that for firms in the upper quantiles of the distribution, firm size has a positive effect on growth when a firm is in an industry with high growth rates or in a growing market. Capasso et al. (2013) use QR to examine the effect of pure auto-correlation in firm growth for several sub-samples of firm sizes. The QR suggests that extreme growth events are likely negatively correlated over time, and it is difficult for a firm to repeat an extreme growth event. Daunfeldt and Halvarsson (2015) also run a QR on lagged firm growth, but over a longer period (four to six years instead of a one year lag). They find it unlikely that a firm will repeat high growth, and they characterize these high-growth firms as “one hit wonders”. Some papers also examine the dependency of growth rates on previous growth rates through probability transition matrices. This has been done by bothCapasso et al. (2013) and Daunfeldt and Halvarsson (2015) who construct probability transition matrices in order to examine the probabilities of moving between quantiles between periods. Both find similar results: that the firms with highest losses in employees in one period is the most likely to be the highest growing firms in the next period.

Chapter 3

Method

The empirical approach is divided into three different parts. First, we look at the distribution of the growth rates for the entire sample, and examine whether or not it remains consistent over time. In this part, we also run a simple test to find out whether firm growth rates in the data follow an auto regressive process. Second, we estimate simple model of firm growth using a QR and OLS model on a pooled data set. Then we try to run a QR accounting for the panel structure of the data-set, including demeaned variables. In this part, we also estimate a more complete model, with determinants based on industry dynamics. Third, we compute a transitional probability matrix in order to examine how growth rates influence each other time. The three elements of the method is based on the papers reviewed previously, and will hopefully yield insight into different aspects of firm growth.

3.1 Quantile regression

Ordinary Least Squares (OLS) regression focuses on the expected value of a variable Y conditional on a set of variables X. This type of regression restricts our attention to a specific part of the location of the conditional distribution of Y with respect to X. Quantile Regression however, extends our attention to different locations of the conditional distribution. Thus, it allows us to examine how the relation between X and Y changes with the distribution of Y. As the main focus of this thesis is identifying what differentiates firms that are located at different parts of the growth rate distribution, QR is a suitable analytical tool for a number of reasons. First, OLS regression might fail to correctly identify the growth determinants of the firms located at the tails of the distribution. Due to the heterogeneous nature of firm growth rates and our interest in the top one percent growth firms in particular, we do not want to dismiss the firms in the tails of the distribution as outliers. Rather we want to analyze these firms by estimating the coefficients in their respective quantiles. When an OLS-model is used, we restrict our attention to the mean of the distribution, meaning, that the OLS-model can obtain summary estimates that calculate the average effect of the independent variables on the average firm, but it will not say anything about the effect the non-average firms (e.g. the high growth firms). QR is not restricted to analyze the means of the variables, so it is able to provide a more complete picture of the relationships between the dependent and independent variables at different quantiles(Koenker, 2005). The QR is robust to outliers and heavy-tailed distributions, which suits our distribution of the firm growth-rates, as it is definitely more on the heavy-tailed side (section 6.1). Coad and Rao (2008) and textciteNavaretti2014 also suggest that just analyzing the average effect on average growth could be misleading. The QR produces more robust coefficient estimates than the OLS, this is especially true in the presence of outliers and for distributions of error terms that deviate from normality, (Buchinsky, 1998; Koenker and Hallock, 2001). Second, the shape of the growth rate distribution (section 6.1) suggests that a traditional OLS regression model might be a poor fit with the data at hand. A QR model however does not rely on any assumption about the shape of the distribution of the dependent variable, so it presents a compelling alternative to the standard OLS regression. As discussed in section 6.1, the growth rate distributions of firms in this data set displays a distributon not well estimated by a normal-distribution. As a result of the data being exponentially rather than normally distributed, the assumption of normally distributed error terms does no longer hold. When using the QR approach, we can avoid the assumption of identically distributed error terms at all points of the conditional distribution. When this assumption is relaxed, we are able to account for firm heterogeneity and the possibility that the estimated coefficients vary at different quantiles of the conditional growth rate distribution. These are important reasons for considering an alternative to OLS when studying a dependent variable that does not have a normal distribution.

3.1.1 Regular Quantile Regression

Consider the linear model

$$y_{it} = \boldsymbol{\beta}' \boldsymbol{x}_{it} + \varepsilon_{it}$$

where $\boldsymbol{x}$ is a set of predictor variables, $\boldsymbol{\beta}$ is a vector of estimators, ε is an unknown error, and y is a random sample from a random variable, Y. In particular, assume that Y is pinned down by the cumulative distribution function (CDF)

$$F_Y(y) = F(y) = P(Y \leq y).$$

Given the model above, the ordinary least squares estimate is found as the estimator that minimize the sum of squared error terms. In particular:

$$\boldsymbol{\beta} = \min_{\boldsymbol{\beta}} E[(y_{it} - \boldsymbol{\beta}' \mathbf{x}_{it})^2]$$

In a similar fashion, the median regression estimates the median of a dependent variable, conditional on the values of the independent variable. Median regression is computed the same way as the QR estimates when the quantile $\theta = 0,5$. The minimization problem becomes takes the absolute loss function as a starting point. In particular, one minimize the absolute sum of deviations:

$$\hat{\mathrm{Me}} = \min_{\beta} E|y_{it} - \boldsymbol{x}_{it}\boldsymbol{\beta}| = \frac{1}{NT} \sum_i \sum_t |y_{it} - \boldsymbol{x}_{it}\boldsymbol{\beta}|$$

Using the sample observations, one can obtain the sample estimator for $\hat{\mathrm{Me}}$ for the median. In other words, median regression finds the regression plane that minimizes the sum of the absolute residuals rather than the sum of the squared residuals.

Estimators of the QR function is found through minimizing a weighted sum of the absolute sum of positive and negative residuals. In the case of the discrete variable Y with probability density function $f(y) = P(Y = y)$ any quantile will be

$$Q_Y(\theta, \mathbf{X}) = \mathbf{X}\beta_\theta$$

The quantile function is defined as the inverse of the CDF inverse :

$$Q_Y(\theta) = Q(\theta) = F_Y^{-1}(\theta) = \inf y : F(y) > \theta,\ \theta \in [0, 1].$$

If the CDF $F(\cdot)$ is strictly increasing and continuous, then $F^{-1}(\theta)$ is a real number y such that $F(y) = \theta$. In particular

$$Q_Y(\theta, \mathbf{X}) = Q_\theta[Y|\mathbf{X} = \mathbf{x}]$$

denotes the generic conditional quantile function ande θ denotes the parameters and the corresponding estimators for a specific quantile θ.

One can present the quantiles as particular centers of the distribution, minimizing the weighted absolute sum of deviations . The θ-th quantile will then be

$$\hat{q}_Y(\theta, \mathbf{X}) = \min_{Q_Y(\theta, \mathbf{X})} E|\rho_\theta(Y - Q_Y(\theta, \mathbf{X}))|$$

and the minimization problem for the estimators become

$$\hat{\beta}(\theta) = \min_{\beta} E|\rho_\theta(Y - \boldsymbol{X\beta_\theta})|,$$

where $\rho_\theta(.)$ denotes the following check- or loss function:

$$\rho_\theta(y) = [\theta - I(y < 0)]y = [(1-\theta)I(y \leq 0) + \theta I(y > 0)]|y|.$$

Here I is an indicator function. This loss function is an asymmetric absolute loss function; that is a weighted sum of absolute deviations, where a $(1-\theta)$ weight is assigned to the negative deviations and a θ weight is used for the positive deviations. Univariate quantiles are defined as particular locations of the distribution, that is the θ-th quantile is the value y such that $P(Y \leq y) = \theta$.

In order to obtain the quantile estimators the sum of both positive and negative absolute deviations are minimized. We get the following minimization problem

$$\min_{\beta} \left\{ \sum_{i,t \epsilon i,t: y_{it} \geq \vec{x}_{it} \vec{\beta}} \theta |y_{it} - \boldsymbol{x_{it}\beta_\theta}| + \sum_{i,t \epsilon \{i,t: y_{it} < \vec{x}_{it} \vec{\beta}\}} (1-\theta)|y_{it} - \boldsymbol{x_{it}\beta_\theta}| \right\}$$

$$= \min_{\beta} \sum_{i=1}^{n} \rho_\theta(u_{\theta it})$$

Here, θ may vary within its bounded interval $(0 < \theta < 1)$ representing different quantiles. The equation is solved by linear programming methods. As one increases θ continuously from 0 to 1, one traces the entire conditional distribution of y, conditional on x. The coefficient estimate for the exogenous variable is interpreted in much the same fashion as the OLS regression coefficients. Specifically, the quantile coefficients may be interpreted as the marginal change in the dependent variable due to a marginal change in the exogenous variable, conditional on being in the θ-th quantile of the distribution. If the coefficients change with the quantiles, it indicates heteroskedasticity issues (Koenker, 2005). As mentioned above, quantile regression makes no assumptions about the variance of the error-terms or of the distribution of the variables, but assumes that the expected value of the error term for every quantile equals 0 Koenker and Bassett Jr, 1978[1]. If the growth rates are heteroskedastic, the standard errors produced by the default formula-based on Koenker and Bassett might not be satisfactory, however. For this reason, we obtain standard errors through bootstrap replications, considering each year as a cluster when drawing the samples. Bootstrapping allows us to obtain standard errors for any statistic even when an analytical formula is not available. The method is simple, but turns out to be quite computationally expensive.

3.1.2 Quantile regression and fixed effects

The panel structure of the data might be an issue. In brief, the problem is that of incidental parameters, and there is no transformation that I can easily use to eliminate the problem. There are some different methods I have come across trying to amend this problem. Kato et al. (2012) use a model similar to the one I present bellow, and study the properties of QR when the fixed effects are included as dummies. They find that the estimator is consistent and asymptotically normal with a large number of observations and a long time span: $n \to \infty$ and $T \to \infty$ and with $n^2[ln(n)]^3/T \to 0$. There is an issue however, because in our data set the number of entities n is much larger than the number of periods. This wull likely result in biased stimates. This is not something I have been able to amend easily, and it should be kept in mind when looking at the results in the estimated coefficients in the regression bellow. However, I do believe that, even if the estimates are not completely unbiased, they should provide a decent estimate. different approaches to this problem that was considered, but discarded are as follows: Abrevaya and Dahl (2008) for example. Who argue that the only realistic option for fixed effects QR is a "correlated random effects" (Mundlak) estimator when there is a fixed number of periods T. This seems like it might have been the good method for estimating the model, but I have not been able to apply it to my data. Machado and Silva (2019) have developed, and use a quantiles-via-

[1]Standard errors for a Koenker and Bassett model are obtained through the STATA command "qreg"

moments estimation technique. They have made a STATA module "xtqreg" in order to estimate models based on their method. It might be useful in estimating panel data models with individual effects. They consider a location-scale model for panel data:

$$y_{it} = \alpha_i + x'_{it}\beta + (\delta_i + x'_{it}\gamma)u_{it}$$

$$\eta(\tau)_i = \alpha_i + \delta_i Q_u(\tau),$$

$$\beta(\tau) = \beta + \gamma Q_u(\tau)$$

where x_i and u_i are independent and $Pr((\delta_i + x'_{it}\gamma) > 0) = 1$. The estimation is performed using two fixed effects regressions (xtreg). The model requires that $(n, T) \to \infty$ with $n = (T)$ in order to achieve cosistency. And for a fixed number of periods T the model will not be consistent.

3.2 Transition probability matrix

As the last part of the analysis, a transition probability matrix is estimated for some quantiles of the growth rate distribution. After the regression models in the previous part, regression of the lagged residuals(period t-1) on the current period residuals (period t) from my growth model. This is done in order to determine if growth rates follow an Autoregressive Process. If this is the case, then I will estimate a transition probability matrix. This simple matrix is computed in order to examine the intro-distributional mobility of the firms between the different quantiles of the growth rate distribution. When using a transition probability matrix and focusing on the firm growth rates, and on the relative positions of firms in the distribution, we can identify the probability of a firm being present in the the top quantile between two periods where the actual growth rates in the top quantile is different between the two are different. As a result one can identify the probability of persistently being a high-growth firm, but not what the growth rates of these firms are. (Capasso et al., 2013)

The transitional probability matrix is calculated through a discrete-time Markov chain, where the state changes at certain time instants (every year in our case). The time instant is indexed by t. At each time t, the state of the chain is denoted by X_t and belongs to a finite set S of possible states called the state space. In this case, the set of states is the following quantiles: $S \equiv [1, 10, 25, 50, 75, 90, 99]$. The Markov chain is described in terms of its transition probabilities p_{ij}: whenever the state is i, there is a probability

p_{ij}that the next state is equal to j. Mathematically[2] :

$$p_{ij} = \mathbf{P}(X_{t+1} = j \mid X_t = i).\, i, j \in S$$

The main underlying assumption for Markov chains is that the transition probabilities p_{ij} apply whenever state i is visited, no matter what happened in the past, and no matter how state i was reached. Mathematically the assumed Markov property requires that

$$\mathbf{P}(X_{t+1} = j \mid X_t = i, X_{t-1} = i_{t-1}.....X_0 = i_0) = \mathbf{P}(X_{t+1} = j \mid X_t = i) = p_{ij}$$

for all times t, and all states $i, j \in S$ for all states and all possible sequences $i_0, ..., i_{n-1}$ of earlier states. Thus, the probability law of the next state X_{t+1}depends on the past only through the value of the present state X_t.

The transition probabilities p_{ij} is non-negative, and sum to one:

$$\sum_{j=1}^{100} p_{ij} = 1 \,\forall\, i.$$

The probabilities p_{ii} is allowed to be positive, so it will be possible for the next state to be the same as the current state. Even though the state does not change, the “non-changed” state will still be viewed as a state transition of a special type (a "self-transition"). All of the elements of a Markov chain model can be encoded in a transition probability matrix, whose element at the ith row and jth column is p_{ij}. Formally:

$$\begin{matrix} p_{1,1} & p_{1,5} & \cdots & p_{1,100} \\ p_{5,1} & p_{5,5} & \cdots & p_{5,100} \\ \vdots & \vdots & \ddots & \vdots \\ p_{100,1} & p_{100,5} & \cdots & p_{100,100} \end{matrix}$$

[2]Regarding the code used to calculate the transitional probability matrix in a cross-sectional time-series data, StataCorp (2016) states that one “can estimate the probability that $X_{i,t+1} = v_2$ given that $X_{i,t} = v_1$ by counting transitions. The rows reflect the initial values, and the columns reflect the final values. The transition probabilities reported by "xttrans" are not necessarily the Markov transition probabilities. "Xttrans" counts transitions from each observation to the next once the observations have been put in t order within i. It does not normalize for missing periods. "xttrans" does pay attention to missing values of the variable being tabulated, however, and does not count transitions from non-missing to missing or from missing to non-missing.” In order to amend for this the advice from the manual, the data is fully rectangularized using the "fillin" command. "fillin" adds observations with missing data so that all interactions of growth rates exist, thus making a complete rectangularization of growth rates. The "xttrans" command will now produce estimates of the Markov transition matrix.

Chapter 4

Model

4.1 Growth rate

The main variables of interest in the analysis is the growth rate, the size and age of each firm. I have chosen to define a firm's size in any period t as the total number of employees in the firm during that period. The size in period t is denoted by $S_i(t)$. To obtain an estimate of firm growth rates $g_i(t)$, the natural logarithm of firm size $S_i(t)$ is taken, and subtracted by the growth rate in the previous period $S_i(t-1)$. This means that the growth rate in period t is:

$$g_i(t) = lnS_i(t) - lnS_i(t-1) \tag{4.1}$$

In period $t-1$ it is:

$$g_i(t-1) = S_i(t-1) - S_i(t-2)$$

This way growth rate of firm i's size in a period t is defined as the difference in the re-scaled logarithmic size between two consecutive years. Following Levratto et al. (2010), a high growth firm is considered as a firm bellonging to the top 1% of growers (the $99^t h$ quantile)

4.2 Panel structure

Ideally I would like to estimate a model including industry dummies to control for fixed effects. However, this has turned out to be computationally heavy, and it was not possible due to limited available computational power. As discussed above, there is some issues with bias in the estimation of standard errors when considering panel-data using QR, but hopefully the standard errors will be sufficiently accurate when I use bootstrapping. In order to control for fixed effects I estimate a within estimation model for firm growth instead. A within estimation model will be estimated in the following fashion:

$$g_{ijt} = a_i + \beta x_{ijt} + \varepsilon_{ijt}$$

Here g_{ijt} is the growth rate of firm i operating in industry j in period t. With a total of N firms and J industries over T periods. In the model, I assume that the outcome of g_{ijt} is a function only of the industry fixed effect a_j and a set of observed characteristics x_{ijk} that could vary across individual firms, industries, and periods. We let ε_{ijt} be a homoskedastic error term that has a mean of zero and is uncorrelated with firm effects a_i and x_{ijt}. I assume that all variables g_{ijt} and x_{ijt} are measured as deviations from their overall sample means. Formally this can be written as:

$$E(\varepsilon_{ijt}|x_{ijt}) = 0, E_i(\varepsilon_{ijt}) = 0,$$

and

$$corr(\varepsilon_{ijt}, a_i) = corr(\varepsilon_{ijt}, x_{ijt}) = 0$$

I can fit this model directly, without estimating the actual individual fixed effects, but by subtracting the within-industry mean from all variables in the model:

$$E(y_{ijt}|i = i) = \bar{g}_i = a_i + \bar{x}_i \beta$$

Here, $\bar{g}_i(\bar{x}_i)$ is the within-industry j average across all firms and time periods of $g(x)$. Subtracting $E(g_{ijt}|i = i)$ from g_{ijt}, we obtain the following transformed data:

$$g_{ijt} - \bar{g}_i = (x_{ijt} - \bar{x}_i)\beta + \varepsilon_{ijt}$$

$$\ddot{g}_{ijt} = \ddot{x}_{ijt}\beta + \varepsilon_{ijt}$$

So in this case we for example have that the geometric mean of employment of all firms belonging to a 2-digit industry $\bar{S}_i(t)$ is subtracted from each observation within the industry j. $\ddot{S}_i(t)$ is then the demeaned logarithm of employment in period t. Given by:

$$\ddot{S}_i(t) = lnS_i(t) - \frac{1}{N_I} \sum_{j \epsilon I} ln \left(S_j\right)$$

That the log size variable now represent deviations from the industry average and the same is done with growth rates and the age of firms. In other words, the growth rates are normalized with respect to the average growth rate of each industry. The firm age variable is demeaned in the same manner as the growth rates and the firm sizes.

These equations will be estimated using the quantile regression procedures discussed above. Looking at the normalized logarithmic size has some advantages compared to just using the absolute firm size. First, it allows us to use new variable controls for differences in size across industries at the 2-digit level. Second, it controls for differences in sector growth rates. Third, common shocks and general common factors in the economy such as business cycles and inflation are removed. Fourth, the re-scaled variable can be used to characterize the distribution of firm growth if the number of firms changes over time, and will make it easier to compare distributions with different numbers of firms Bottazzi et al., 2001Bottazzi et al., 2011 Capasso et al., 2013)

4.3 Linear Regression Models and Variables

4.4 Models

I will estimate two models. First, I estimate a simple linear model of growth as a function of log transformation of firm age and firm size. Second, I estimate a more complex model trying to identify certain industry dynamics that might be important in determining firm growth in line with the predictions by the models of Hopenhayn (1992) and Jovanovic (1982). The following equation makes up the simple growth model:

$$\ddot{g}_{i,t} = \beta_1 ln(\ddot{S}_{i,t-1}) + \beta_2 ln(\ddot{A}_{i,t-1}) + \varepsilon_{i,t} \tag{4.2}$$

This model only considers firm specific characteristics. In particular, the only independent variables are firm age and size where $\ddot{A}_{i,t}$ denotes the demeaned logarithm of firm age, which is included in almost all of the firm growth literature,for example Jovanovic (1982), and $\varepsilon_{i,t}$ is the error term. If the Norwegian firms follow a process similar to the one described in Hopenhayn (1992) or Jovanovic (1982) then the growth rates should be negatively dependent on firm size and age, as these work as a proxy for the learning process of the firms entering the industry for the first time. This classical firm-specific approach is den extended by considering industry-specific variables. The industry dynamics model can be represented by the following equation:

$$\begin{aligned}\ddot{g}_{i,t} &= \beta_1 ln(\ddot{S}_{i,t-1}) + \beta_2 ln(\ddot{A}_{i,t-1}) + \beta_3 CITY + \beta_4 \ddot{g}_{i,t-1} \\ &\quad + \beta_5 I\_SIZE_j + \beta_6 I\_CON_j + \beta_7 MES_j + \beta_8 I\_GR_j + \beta_9 I\_TUR_j + \varepsilon_{i,t}\end{aligned}$$

A number of industry variables is included in the second model: I_SIZE is the log transformed size of the industry the firm belongs to; I_CON is the industry concentration (measured as squared sum of firms industry shares across the industry); MES is a measure of the minimum efficient scale and (measured as the median number of employees divided by the total number of employees in industry j at time t Daunfeldt and Elert, 2013; I_Gr. are industry growth rates, estimated by the size variable above. (Reichstein et al., 2010); I_TUR: Industry turbulence, measured as the sum of absolute changes in market shares within an industry, which could also be interpreted as a proxy for entry costs, as Hopenhayn (1992) predicts industry turbulence is affected by entry costs (a decrease in entry costs reduces industry turnover); CITY is the share of firms located in urban areas, which might be relevant since firms located in areas where there is higher demand for final goods may exhibit higher performance than other firms (Krugman, 1991, Levratto et al., 2010, Daunfeldt and Elert, 2013). Three non-industry variables are included. Size, age, and as discussed in the literature review above and examined bellow in section 6, the growth rates of each firm is likely affected by the growth rates in the previous period (at least an AR1 process) so a lagged growth variable is included to control for this. Summary statistics are provided in *Table*2 and *Table*3

Chapter 5

Data and Descriptive Statistics

5.1 Data source and standards

The data used in this thesis are taken from the Norwegian Central Bureau of Statistics' (SSB) Central Coordinating Register of Legal Entities (VoF). The data is collected and provided by SSB. It contains a registry of all establishments and enterprises in the Norwegian public and private sectors for the years considered. The definition of firms used in the thesis follows the Standard for Industry Classification (SIC), meaning that a firm is defined as the smallest combination of legal entities that produce goods or services with a degree of independent decision-making autonomy. Examples of such legal units are limited companies, general partnership, sole proprietorship, etc. (Hansson, 2009). A firm is is registered in an industry according to their most important activities. If a firm consists of several smaller entities, the mother-firm will be registered in the industry that contains most of its employees. VoF contains firm data from the years 1966 to 2016, with yearly rapports between 1966 and 1997 and monthly thereafter. The database contains information on the sector of each firm, at the 5-digit SN (Norwegian standard industry classification) level, number of employees and dates of registry. Date of exit is assumed to be the last date it is observed. When a firm changes its sector of activity, the firms is still regarded as continuing.

5.2 Data selection

The firms included in the final data-set are the firms registered with an activity code verifying them as active in the respective year (Hansson, 2009). The data also identify whether or not a firm is registered in an enterprise group. When dealing with enterprise groups only the parent corporations of enterprise groups are kept in the data. This is so because the parent corporation has to keep account of the total activity in the enterprise group, and the links between the different enterprises within the enterprise group is recorded in the VoF. An enterprise group consists of one parent corporation and at least one subsidiary corporation where the parent corporation during its ownership controls the subsidiary corporation in accordance to legal provisions (Hansson, 2009).

Each month, a data set is provided with observations. However, employment is not necessarily updated for each month, so the employment-observation is carried over to the next month unless they are updated. To be consistent, and to follow the custom of other research, only one yearly observation of each firm is considered. The data-set was limited to the years 2007 - 2016 (9 years). The reason for limiting the observations to these years is due to the fact that the VoF-registry changed the industry code (NACE-codes) definitions in 2009 (the SN-2007 standard) (implemented from the 2006 data). I was unable to track which firms changed to what new NACE-code without making assumptions on where some of the firms are registered in the new standard. The data set is limited further by the way growth rates are calculated. In particular, I only consider firms with more than zero employees. The reason for excluding the firms with zero employees is due to the fact that it is easier to handle log-transformations when not considering observations of zero (i.e. not having to transform the data to account for the to-be missing values), as well as the computational cost of using bootstrap re-sampling methods to obtain estimates. Further, as a requirement for the calculating the growth rates, only the non-subsidiary firms with two consecutive observations of non-missing and non-zero employment numbers is considered. This means that only short-term growth rates are considered. Longer growth rates could be an option, and could possibly better balance the data, but very large parts of the data would be excluded as many firms have a short lifespan. Firms that are registered as sole proprietors are also excluded from the data. This is in part because the majority of such firms are excluded anyway because they are registered with zero employees, and also to follow common procedure in previous empirical research. Of the full data-set of firms registered as active there are about 3 500 000 observations over all 9 years. Of these about 1 700 000 are registered as sole proprietors. Of those not registered as sole proprietors, but with zero observed employees there is about 550 000 observations (about 16% of all observations). The remaining sample still has a couple of observations with missing growth rates (i.e. singular observations of employment), when these are removed, we are left with a data-set of about 1 050 000 observations, with observations per year summarized in the tables bellow:

5.3 Descriptives

Table 5.1: Number of firms per year

Year	N Firms	Percent
2007	97 864	9,35
2008	98 476	9,40)
2009	99 773	9,53
2010	101 347	9,68
2011	102 773	9,81
2012	105 770	10,10
2013	106 806	10,20
2014	107 945	10,31
2015	110 221	10,52
2016	116 260	11,10
Total	1 047 235	100

Table 5.2: Descriptive statistics

Variable	1^{st} qrtl.	Mean	Median	3^{rd} qrtl.	SD
Growth	-.0660	1.40e-10	-.0364	.0296	.3459
$ln(\ddot{S})$	-.6689	.1000	.0041	.7384	1.1493
$ln(\ddot{A})$	-.4613	.0842	.2477	.7311	.8564
CITY	.4011	.4683	.4542	.5609	.1463
I_SIZE	21557	71976	41520	97776	90032.51
I_CON	.0025	.0141	.0050	.0122	.0342
MES	.00004	.0023	.0001	.0002	.0037
I_GR	17	1884.199	821	2558	4283.49
I_TUR	.1232	.1759	.1560	.1976	.1031

N = 1 047 235

Descriptive statistics for all variables in the model

Table 5.3: Correlation matrix, all variables

	Growth	$ln(\ddot{S})$	$ln(\ddot{A})$	CITY	I_SIZE	I_CON	MES	I_GR	I_TUR
Growth	1.0000								
$ln(\ddot{S})$	-0.0845	1.0000							
$ln(\ddot{A})$	-0.1230	0.2215	1.0000						
CITY	-0.0018	0.0005	0.0074	1.0000					
I_SIZE	-0.0022	0.0039	-0.0056	-0.1005	1.0000				
I_CON	-0.0015	0.0025	0.0092	0.2299	-0.1181	1.0000			
MES	0.0011	0.0001	-0.0000	-0.0265	-0.0367	0.1485	1.0000		
I_GR	-0.0032	-0.0034	-0.0004	-0.0460	0.5312	-0.0507	-0.0211	1.0000	
I_TUR	-0.0096	-0.0060	-0.0003	-0.0551	-0.1024	-0.1518	-0.0025	0.0182	1.0000

Chapter 6

Results

6.1 Growth rate distribution

Figure 8.1 in the appendix shows the pooled growth rates for the entire data set. In the figure, the growth rates g_{it} appear along the horizontal axis, and the relative frequencies are on the vertical axis. It seems clear that the distribution of growth rates for Norwegian firms carries a close resemblance to empirical distributions of firm growth rates found in other countries (Coad, 2009, Capasso et al., 2013). In particular, the growth rate distribution seems to follow the mirrored exponential distribution, also known as a Laplace type distribution, as described by Bottazzi and Secchi (2006). When looking at the four different periods, it is apparent that the growth rate distribution remains similar, and consistent over all the years considered in the data. The probability density function of the Laplace distribution has the following form:

$$f_G(g; \mu, a) = \frac{1}{2a} e^{\left(-\frac{[g-\mu]}{a}\right)}$$

Here, g is the growth rate of a firm, μ is the location parameter, and a is the scale parameter. The distribution is mirrored exponentially distributed on both the left and right side of zero when the parameter μ is close to 0 and a is close to 0,5, which seems to be a good fit for the data [1].

[1] The distribution in figure 6.1 is estimated with 10 000 random draws from a Laplace distribution

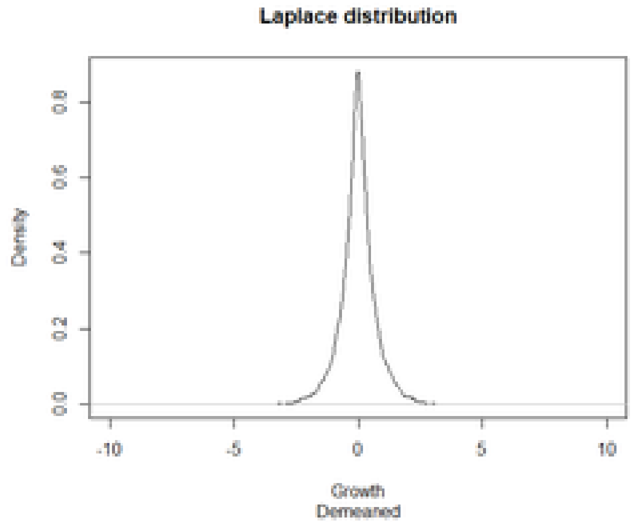

Figure 6.1: Laplace distribution

A visual examination of the growth rate distribution over the four years indicate that the distributions remain a stable double exponential with b close to 1 over the whole period examined. I perform a Kolmogorov–Smirnov test to test whether the distribution of growth rates is similar in the first and last year of the data set (the year 2007 and year 2016 respectively). Table 8.1 in the appendix reports the result. In general, the compared distributions do not seem to show any significant difference, indicating that the distribution of growth rates remains stable over time. When plotting the growth rates at the 10th-, 25th-, 50th-, 75th-, and 90th-percentile in Figure 2, we can see that the same trend for aggregated growth rates remains quite similar over the entire period.[2] The fact that the distributions are stable over time may be due to different mechanisms. One possible explanation is that firms in the tail of the distribution are replaced each period by firms in other parts of the distribution, meaning that there is some intra-distributional mobility of the firms. Another possible explanation is that firms in the tails of the distribution remain there in the following periods.

6.2 Regression

I want to start this section by comparing a simple OLS regression with the simple growth model to highlight the differences between OLS and the QR

[2]With a conspicuous bump in the year 2015, which I cannot explain. It might be an artefact of the data, or some shock (e.g. the Norwegian oil crisis in 2014-2015

method. A simple OLS regression on the pooled data, with robust standard errors, gives the following estimates:

Table 6.1: Simple OLS-model, robust SE

Growth	$\ddot{S}_{i,t-1}$	$\ddot{A}_{i,t-1}$	_cons
Coef.	-.030658***	-.0181172***	.0251355***
	(.0003)	(.0004)	(.0009)

Standard errors in parentheses: *** p<0.01, ** p<0.05, * p<0.1

We see that both the size of firms and their age negatively impact the growth rates of firms. The negative and significant coefficients both indicate that the growth rate of firms tend to diminish when they become older and larger. This phenomenon is well established in the literature (as discussed in section 2.2). Now consider the estimates for a simple QR model:

Table 6.2: Demeaned model, on 2-digit NACE code level, and resampled by bootstrapping year clusters

Growth Quantiles	1	10	25	50	75	90	99
ln(Employees)	0.032*	-0.014***	-0.025***	-0.028***	-0.036***	-0.050***	-0.092***
	(0.016)	(0.003)	(0.002)	(0.003)	(0.006)	(0.002)	(0.024)
ln(Age)	0.158***	0.023***	-0.009***	-0.016***	-0.039***	-0.081***	-0.206***
	(0.023)	(0.002)	(0.001)	(0.002)	(0.002)	(0.007)	(0.025)

Standard errors in parentheses: *** p<0.01, ** p<0.05, * p<0.1
N = 1 047 235

Here, our QR model is estimated for the 1^{st}, 10^{th}, 25^{th}, 50^{th}, 75^{th}, 90^{th}, and 99^{th} quantiles. This estimation provides a more nuanced picture of how firm growth behaves over different parts of the distribution. The results from the median ($50^{th} quantile$ resemble the results from OLS regression above, indicating that the median and average firms are quite similar. In the tails of the distribution, things are different, however. Consider first the firms in the lower quantile. In the 1^{st} quantile, we find that the impact of size on growth is positive. As we move up the distribution, we see that firm size in period $t-1$ has an increasingly negative impact on firm growth. The effect of size on growth becomes more and more negative the further to the right of the distribution we move, and the growth is most negatively impacted by size for those in the top 1% of the growth distribution (99^{th} quantile), i.e. the high growth firms. These results align with what researchers find in other countries and reject Gibrat's (1931) law and confirm Jovanoic's (1982) law. A possible explanation for this, following Jovanovic (1982), is that small and young firms are not fully aware of their productivity, such that updating information causes a high relative growth rate. Alternatively, the smallest firms are not yet at the minimum efficient scale and need high growth to achieve this scale as soon as possible.

Before moving to the model containing industry-specific determinants, we test whether the model in equation 3 follows an autoregressive process. I will only do a simple regression of the residuals from the second model on these residuals' lags. This provides the following results:

Table 6.3: AR(1) process of model 2

$\hat{u}$	Coef.	$P > \|t\|$
$\hat{u}_{t-1}$	-0.0526***	0.000
_cons	-0.0048***	0.000

*** p<0.01, ** p<0.05, * p<0.1

Thus, there is a highly significant correlation between the residuals, and there is a probability that the growth process of firms follow at least an AR(1) process. Note that the test carried out might not be very thorough, but it gives an indication. We should also bear in mind that correlation has been found in other countries (as mentioned in the literature review). I have included the same regression for firms above the 90^{th} quantile, and the effect seems to be more prominent towards the tail of the distribution. I examine this further in Table 6.5 in section 6.3.

Table 6.4 displays the results from the second model. These results are not entirely reliable as the estimates had to be simplified to make the deadline for the thesis. In particular, the estimates are only done on 10% of the entire data set and only with 50 bootstrap replications. Hopefully, they will still provide some indication of what the well-estimated parameters should be.

Table 6.4: Estimation of Model 2

Quantiles	01	10	25	50	75	90	99
ln(Employees)	-0.120***	-0.037***	-0.015***	0.001	0.020***	-0.058***	-0.133***
	(0.031)	(0.009)	(0.002)	(0.001)	(0.003)	(0.012)	(0.018)
ln(Age)	0.017	0.019*	-0.001**	-0.001	-0.024***	-0.072***	-0.078**
	(0.016)	(0.007)	(0.001)	(0.000)	(0.006)	(0.005)	(0.030)
$Growth_{t-1}$	-0.014	-0.097***	-0.006	-0.000	0.013***	-0.056***	-0.125***
	(0.018)	(0.020)	(0.003)	(0.001)	(0.005)	(0.016)	(0.029)
CITY	-0.009	0.035*	0.006	-0.006	-0.036*	-0.027	0.129
	(0.133)	(0.016)	(0.017)	(0.006)	(0.017)	(0.059)	(0.233)
Ind. Size	0.010	-0.004	-0.005*	-0.004***	0.005	-0.021***	-0.042***
	(0.017)	(0.008)	(0.003)	(0.001)	(0.003)	(0.005)	(0.014)
Ind. Con.	-0.962	-0.110	-0.219***	0.061	0.056	0.126	1.110**
	(0.680)	(0.136)	(0.060)	(0.046)	(0.033)	(0.108)	(0.413)
MES	4.305	0.430	-2.824	-0.541	7.126*	-3.300***	-9.303
	(3.000)	(1.560)	(3.784)	(0.896)	(3.499)	(0.436)	(1.717)
Ind. Gr	0.007	0.001	-0.004	-0.001	0.001	0.001	-0.028
	(0.013)	(0.006)	(0.002)	(0.001)	(0.003)	(0.010)	(0.027)
Ind. Tur	-0.480*	-0.180	0.018	0.027	-0.023	0.039	0.150
	(0.240)	(0.106)	(0.035)	(0.014)	(0.028)	(0.119)	(0.309)
_cons	-0.962***	-0.202**	-0.001	-0.038***	0.006	-0.004	1.514***
	(0.135)	(0.075)	(0.034)	(0.011)	(0.019)	(0.133)	(0.415)

N = 104 724
Standard errors in parentheses: *** p<0.01, ** p<0.05, * p<0.1
Resampled: bootstrapping by year clusters, 50 repetitions

When we study the results of the second model presented above, we see that the estimation from this smaller subsample provides mostly insignificant estimates. Some of the results also look pretty suspicious. Given the simplification assumptions we used to obtain them, we should be careful when interpreting. The most general observation about the estimates is that they are all relatively small for the median firms. A possible explanation could be that median firms have achieved a stable size and adjusted to the industry dynamics. In this way, they will remain close to the median growth rates. The coefficient on firm age seems to follow a similar pattern as in the first model, where the coefficients become more and more negative as we move towards the highest quantiles of the distribution. Size, however, does not follow the same procedure, at least for the firms with the lowest growth rates. The conclusion from earlier remains the same for our high-growth firms: high growth is still negatively dependent on firm size. The growth rate of the previous period has surprisingly a negative effect over almost the entire distribution; the effect seems to be more negative as we move towards the top growers. The coefficient indicating the effect of the share of firms per industry operating in an urban area (CITY) does not seem to indicate how it influences firm growth. If we only look at the high-growth firms (99^{th} quantile) the effect is positive. A possible explanation could be that industries with higher shares of firms located in urban areas face greater demand for final goods, higher manufacturing shares, economies of scale or lower transportation costs (Krugman, 1991, p. 429). But the effect is not present in the 75^{th} quantile or the 90^{th} quantile. The log transformation of industry size (Ind. Size) gives a more consistent picture. As we move up to the higher quantiles of the distribution, the effect of an increase in industry size has a more significant negative effect on growth rates. Industry concentration (Ind. Con.) has an increasingly positive impact over the growth rates and a substantial positive effect on high-growth rates. We might interpret this in light of Schumpeter (1942) where each firm

"*...has a small and precarious market of his own which he tries—must try—to build up*"(Schumpeter, 1942, p.79).

A high level of concentration could induce a higher level of innovative activity and increased investments in research and development, leading to higher growth rates. Minimum efficient scale (MES) does not provide a consistent picture either, but in general, it seems to become more negative as we move along to the upper quantiles. This is surprising as one could expect firms to close the efficiency gap when entering an industry and thus grow more in industries with high MES. The negative coefficients could be caused by entrant firms not joining industries characterized by high MES. Perhaps due to large incumbent firms driving out entrants and only taking on short-run losses(Daunfeldt and Elert, 2013). The coefficients for Industry growth (Ind. Gr.) and industry turbulence (Ind. Tur.) look like they are working in opposite directions. This indicates that industry growth negatively affects the top quantile and positively the lower and middle ones, while the opposite is true for industry turbulence. Higher cost of entry could lead to lower turbulence (young firms might be more volatile as suggested by Jovanovic (1982)). So when looking at the high-growth firms, industries with higher turbulence has a positive effect on growth as high turbulence indicates that entry costs are lower, more young firms can enter, and then grow to achieve their MES. Reducing the mass of entrants and the rate of entry in the industry could cause higher entry costs (lower turbulence), and imply higher growth for large firms, but not necessarily for small ones (Hopenhayn, 1992a, p. 1142). The relative growth (which is how growth is measured here) would not be as high for a firm already large, and the lower turbulence industries display lower relative growth for firms. For the industry growth, the effect is negative for the top quantiles. This might be due to most growth being created by a larger mass of firms with lower growth rates, and high-growth firms existing in relatively large industries characterized by few high-growth firms, where their total contribution to the industry growth rates does not amount to much. It does, however, not line up with other findings (e.g. Johansson (2005)) that find industry growth and industry turbulence to be positively correlated, but the estimated effects of industry turbulence and industry growth are at odds with each other. Again, I want to emphasize that the regression only use 10% of the complete set of firms, with only 50 bootstrap replications. Drawbacks has largely been discussed earlier in the thesis, but a further drawbacks to the model might be omitted variable bias, apparent from the section above is probably the inclusion of interaction terms between the industry determinants, and between industry and firm determinants. Squared variables of age and size could probably also be useful in examining a more complex relationship between growth, age, and size. Not considered at all are factors such as financial variables and innovation data that the literature on firm growth have found to be quite important as determinants of firm growth.

6.3 Persistance of Growth

Table 6.5: TPM between quantiles

Quantile	q1	q10	q25	q50	q75	q90	q99
q1	2.62	5.60	1.18	56.16	3.53	26.31	4.60
q10	3.15	13.13	7.08	46.45	15.76	13.67	0.76
q25	1.13	9.94	26.84	34.77	24.35	3.33	0.09
q50	1.37	6.57	6.75	68.11	9.61	7.07	0.51
q75	1.05	10.71	19.32	36.91	27.55	4.38	0.08
q90	3.60	15.47	6.76	43.10	18.32	12.29	0.45
q99	5.24	12.80	10.61	27.45	26.23	16.85	0.83
Total	1.72	8.87	10.46	55.64	15.03	7.78	0.50

Table 6.5 shows the estimated transition probabilities of firms located in each of the different quantiles. q1 is firms with growth rates smaller than the first quantile, q10 contains the firm situated between the first and the

25^{th} quantile, and so on. q99 includes the firms above the 99^{th} quantile (i.e. the high growth firms). The reported estimates show the transition probabilities of moving to a quantile in the next period, given the quantile it currently is in (with current quantile displayed horizontally. When looking at the matrix for the total distribution, we can determine some patterns of consecutive firm growth rates. Most obviously, there is a reversion to the mean for all growth rates, i.e. no matter what part of the distribution a firm is a part of, the firm has the highest probability of ending up in the middle of the distribution. Note however, that the size of the quantile intervals are much larger in the middle, so numbers are relatively higher then at the tales due to this.

More relevant to the thesis, however, are the events unfolding at the ends of the distribution. When looking at both the 1st and 10th quantiles (firms in the top left of the tables), i.e. the firm with large negative growth rates over both periods, it is estimated that the probability seems quite of being one of these firms. The same pattern repeats itself in the lower right corner for the persistent high-growth firms, indicating that the firms with top 1% growth (high growth firms) in one period have a very low probability of repeating that growth in the following period. So we have some undeniable winners and losers.

When looking at the lower left and upper right corner values, the transition probability increases substantially, suggesting that a firm with low growth in one period has a relatively high likelihood of achieving high growth in the next period and vice versa. This indicates a sort of rebound or balancing effect in firm growth rates. Also worth noticing is that firms in the extremes are more likely to experience (opposite) extreme growth the following period.

Generally, we see that the firms most likely to be a future high-growth firm will be found among the firms currently experiencing a high amount of job losses regardless of which size group the firm is in. There also seems to be a strong "balancing" or "rebound" effect at work, indicated by the higher values of the main diagonal (from left to right). This means that firms are more likely to experience a proportional opposite growth in the following period. Finally, the estimates indicate that the probability of repeating high growth is small. One caveat to that analysis is that it does not take acquisitions into account. This probably leads to an upwards bias in the growth rates of larger firms, as it seems reasonable to assume that the firms acquiring other firms are large rather than small.

Chapter 7

Conclusion

The main goal of this thesis was to provide insight into what determines the growth rates of Norwegian firms and what distinguishes the Norwegian high-growth firms from the rest. To this end, I carried out a substantial literature review. In the review, I found some papers that consider high-growth firms. Still, most of the empirical research I found regarding firm growth was concerned with averages and not outliers. I nevertheless found some possible determinants of growth and a suitable method for the analysis. In the first part of the analysis, I examined the distribution of the firm growth rates. I found the distribution to be approximately following the theoretical Laplace distribution. Then, I estimated a quantile regression model to find the determinants of firm growth in different parts of the growth distribution. The estimation results showed a distinguishable difference between the determinants of growth when looking at the different parts of the growth distribution. Moreover, I have found that some results that hold for other countries also hold for Norwegian firms: There is a negative relationship between size and growth, especially for firms with high growth rates. Moreover, it seems that small and young firms are most likely to be high-growth firms. The mechanism proposed in Jovanovic's (1982) "theory of passive learning" is a possible explanation for this. It implies that the young firms update their knowledge about their productivity, resulting in significant growth for the firms underestimating their productivity and exit for the firms overestimating their productivity. After that, the firm growth rates decrease as the firm's gain information about their productivity and get older and larger, in the end stabilizing. I could not fully realize the analysis that was supposed to make up the main part of this thesis. This limited the novelty of this thesis, as little research have looked at the relationship between industry dynamics and firm growth. I believe I could have provided valuable insights into Norwegian firm growth characteristics, particularly high-growth firms. The model that I estimated was largely unreliable and insignificant. But, if one insisted on interpreting the estimates, they would indicate that the Norwegian high-growth firms are estimated to be in smaller industries, industries with higher concentration, industries with a low minimum efficient scale of production, and industries characterized by higher degrees of turbulence. In the last part of the analysis, I considered the persistence of growth rates. The literature I reviewed generally found that there probably is a correlation between current and previous growth rates.When performing a simple test for autocorrelation by regressing current residuals on the residuals of the prior period, I found that the growth rates of the Norwegian firms probably follow an autoregressive process. In particular, the growth rates are at least following an AR(1) process. I did not test if the autocorrelation extended to earlier periods. Suspecting that the growth rates were autocorrelated, I further developed the analysis by computing a simple transitional probability matrix of growth rates. The resulting matrix indicated that the firm's probability for repeating high growth is very low. The firms most likely to achieve high growth rates are those in the lower quantiles with significant negative growth in the previous period. This indicates that the process suggested by Jovanovic (1982) could be a suitable approximation of the growth process for Norwegian firms. In general, it is hard to draw any very definitive conclusions for high-growth firms in Norway other than for the first period of growth in this thesis. The firms achieving high growth are the ones who are small, young, and belonged to the bottom part of the growth distribution in the previous period. With the methods used in this thesis, it is not easy to distinguish between the firms that will continue to achieve high growth rates and those that will not. Any further research on this subject could probably gain insight by estimating a similar model with more time and computational resources. Extensions to the analysis should probably examine the persistence of high growth rates. The model including industry dynamic probably suffers from omitted variables and could benefit from the inclusion of interaction terms between industries. The transitional probabilities estimation could be extended to look at smaller subsamples of the firm population, such as different industries, sizes and ages.

Chapter 8

Appendix

Table 8.1: Kolmogorov–Smirnov test for 2007 and 2016

Year	D	P-value
2007:	0.0007	0.956
2016:	-0.0457	0.000
Combined:	0.0457	0.000

The statistics reported is a result of testing two hypothesis. First, that one group contains smaller values than the other and second, that one group contains larger values than the other. The statistics reported in table table 5 contains the largest absolute value of the two tests The two hypotheses are evaluated with the two following statistics:

$$D^{+} = \max\left\{F_t(g) - F_{t-1}(g)\right\}$$

$$D^{-} = \min\left\{F_t(g) - F_{t-1}(g)\right\}$$

where $F_t(g)$ and $F_{t-1}(g)$ are the empirical distributions at year t and t-1 being compared. The combined statistic reported in (StataCorp, 2019):

$$D = \max\left(|D^{+}|, |D^{-}|\right)$$

Table 8.2: AR(1) process of model 2. 90^{th}q

$\hat{u}$	Coef.	$P > |t|$
$\hat{u}_{t-1}$	-.1680***	0.000
_cons	.3238***	0.000

*** p<0.01, ** p<0.05, * p<0.1

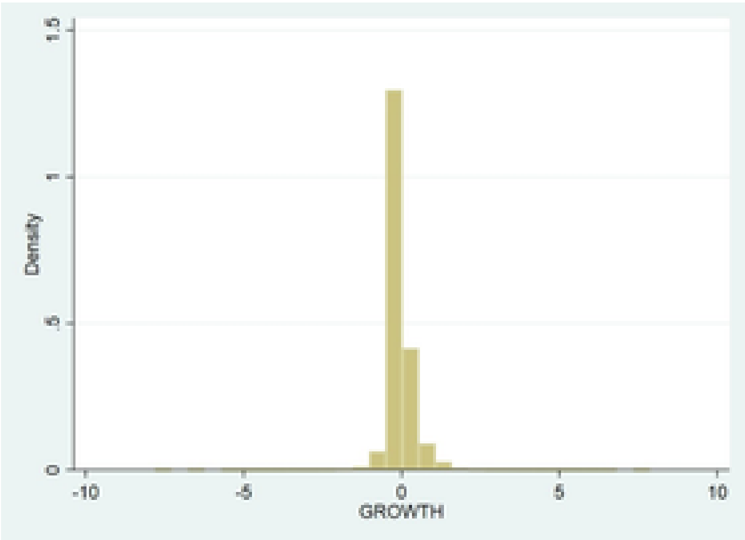

Figure 8.1: Pooled growth rates all years (2007 - 2016)

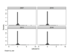

Figure 8.2: Growth rate distribution in years 2007, 2010, 2013 and 2016

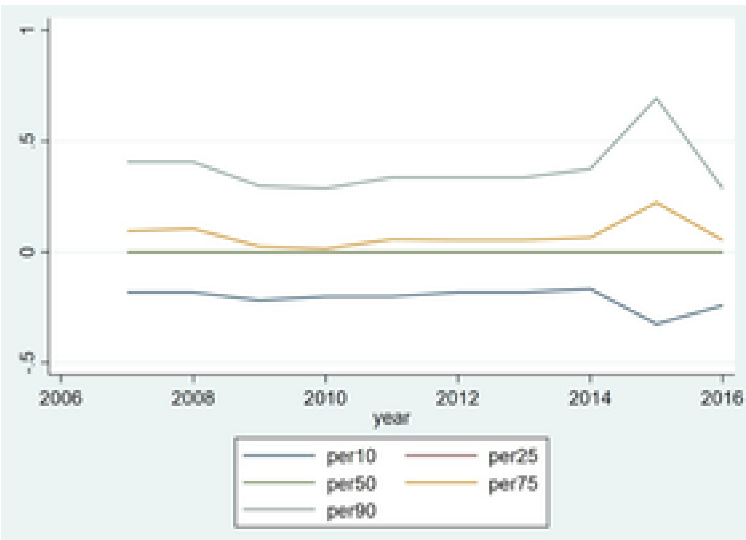

Figure 8.3: Evolution of the growth rate distribution (2007-2016)

The distribution in figure 1 is computed using the "histogram" function in STATA16

Bibliography

Abrevaya, J., & Dahl, C. M. (2008). The effects of birth inputs on birthweight: Evidence from quantile estimation on panel data. *Journal of Business & Economic Statistics*, *26*(4), 379–397.

Barba Navaretti, G., Castellani, D., & Pieri, F. (2014). Age and firm growth: Evidence from three european countries. *Small Business Economics*, *43*, 823–837. https://doi.org/10.1007/s11187-014-9564-6

Bottazzi, G., Coad, A., Jacoby, N., & Secchi, A. (2011). Corporate growth and industrial dynamics: Evidence from french manufacturing. *Applied Economics*, *43*(1), 103–116.

Bottazzi, G., Dosi, G., Lippi, M., Pammolli, F., & Riccaboni, M. (2001). Innovation and corporate growth in the evolution of the drug industry. *International journal of industrial organization*, *19*(7), 1161–1187.

Bottazzi, G., & Secchi, A. (2003). Why are distributions of firm growth rates tent-shaped? *Economics Letters*, *80*(3), 415–420.

Bottazzi, G., & Secchi, A. (2006). Explaining the distribution of firm growth rates. *The RAND Journal of Economics*, *37*(2), 235–256.

Buchinsky, M. (1998). Recent advances in quantile regression models: A practical guideline for empirical research. *Journal of human resources*, 88–126.

Capasso, M., Cefis, E., & Sapio, A. (2013). Reconciling quantile autoregressions of firm size and variance–size scaling. *Small Business Economics*, *41*. https://doi.org/10.1007/s11187-012-9445-9

Caves, R. E. (1998). Industrial organization and new findings on the turnover and mobility of firms. *Journal of Economic Literature*, *36*(4), 1947–1982. https://EconPapers.repec.org/RePEc:aea:jeclit:v:36:y:1998:i:4:p:1947-1982

Coad, A. (2009). *The growth of firms: A survey of theories and empirical evidence*. Edward Elgar Publishing.

Coad, A., & Rao, R. (2008). Innovation and firm growth in high-tech sectors: A quantile regression approach. *Research policy*, *37*(4), 633–648.

Coad, A., Segarra, A., & Teruel, M. (2013). Like milk or wine: Does firm performance improve with age? *Structural Change and Economic Dynamics*, *24*, 173–189. https://doi.org/https://doi.org/10.1016/j.strueco.2012.07.002

Daunfeldt, S.-O., & Elert, N. (2013). When is gibrat's law a law? *Small Business Economics*, *41*(1), 133–147.

Daunfeldt, S.-O., Elert, N., & Johansson, D. (2016). Are high-growth firms overrepresented in high-tech industries? *Industrial and Corporate Change*, *25*(1), 1–21.

Daunfeldt, S.-O., & Halvarsson, D. (2015). Are high-growth firms one-hit wonders? evidence from sweden. *Small Business Economics*, *44*(2), 361–383.

Delmar, F., Davidsson, P., & Gartner, W. (2003). Arriving at the high-growth firm. *Journal of Business Venturing*, *18*, 189–216. https://doi.org/10.1016/S0883-9026(02)00080-0

Gibrat, R. (1931). Les inégalits économiques. *Sirey*.

Haltiwanger, J., Jarmin, R. S., & Miranda, J. (2010). Who creates jobs? small vs. large vs. young. *US Census Bureau Center for Economic Studies*, (CES-WP-10-17). https://papers.ssrn.com/sol3/papers.cfm?abstract_id=1666157

Hansson, A.-K. (2009). *Bedrifts- og foretaksregisteret*. Statistisk Sentralbyrå. https://www.ssb.no/a/publikasjoner/pdf/notat_200702/notat_200702.pdf

Henrekson, M., & Johansson, D. (2010). Gazelles as job creators: A survey and interpretation of the evidence. *Small business economics*, *35*(2), 227–244.

Hölzl, W. (2014). Persistence, survival, and growth: A closer look at 20 years of fast-growing firms in austria. *Industrial and corporate change*, *23*(1), 199–231.

Hopenhayn, H. A. (1992a). Entry, Exit, and Firm Dynamics in Long Run Equilibrium. *Econometrica*, *60*(5), 1127–1150. https://ideas.repec.org/a/ecm/emetrp/v60y1992i5p1127-50.html

Hopenhayn, H. A. (1992b). Entry, exit, and firm dynamics in long run equilibrium. *Econometrica: Journal of the Econometric Society*, 1127–1150.

Johansson, D. (2005). The turnover of firms and industry growth. *Small Business Economics*, *24*(5), 487–495.

Jovanovic, B. (1982). Selection and the evolution of industry. *Econometrica*, *50*(3), 649–70. https://doi.org/https://doi.org/10.2307/1912606

Kato, K., Galvao Jr, A. F., & Montes-Rojas, G. V. (2012). Asymptotics for panel quantile regression models with individual effects. *Journal of Econometrics*, *170*(1), 76–91.

Koenker, R. (2005). *Quantile regression*. Cambridge University Press. https://doi.org/10.1017/CBO9780511754098

Koenker, R., & Bassett Jr, G. (1978). Regression quantiles. *Econometrica: journal of the Econometric Society*, 33–50.

Koenker, R., & Hallock, K. F. (2001). Quantile regression. *Journal of economic perspectives*, *15*(4), 143–156.

Krugman, P. (1991). Increasing returns and economic geography. *Journal of political economy*, *99*(3), 483–499.

Levratto, N., Zouikri, M., & Tessier, L. (2010). The determinants of growth for smes-a longitudinal study from french manufacturing firms. *Available at SSRN 1780466*.

Machado, J. A., & Silva, J. S. (2019). Quantiles via moments. *Journal of Econometrics*, *213*(1), 145–173.

NOU2018:5. (2018). *Kapital i omstillingens tid - næringslivets tilgang til kapital*. Nærings- og fiskeridepartementet. https://www.regjeringen.no/no/dokumenter/nou-2018-5/id2590735/

Reichstein, T., Dahl, M., Ebersberger, B., & Jensen, M. (2010). The devil dwells in the tails a quantile regression approach to firm growth. *Journal of Evolutionary Economics, 20 (2), 219-231*, *20*. https://doi.org/10.1007/s00191-009-0152-x

Schumpeter, J. A. (1942). *Capitalism* (Vol. 3). New York.

Segarra, A., & Teruel, M. (2014). High-growth firms and innovation: An empirical analysis for spanish firms. *Small Business Economics*, *43*(4), 805–821.

Segarra, A., & Teruel Carrizosa, M. (2014). High-growth firms and innovation: An empirical analysis for spanish firms. *Small Business Economics*, *43*(4), 805–821. https://EconPapers.repec.org/RePEc:kap:sbusec:v:43:y:2014:i:4:p:805-821

StataCorp. (2016). *Stata statistical software : Release 16*. College Station, TX: StataCorp LLC.

Sutton, J. (1997). Gibrat's legacy. *Journal of Economic Literature*, *35*(1), 40–59. http://www.jstor.org/stable/2729692