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Why the hydrodynamics is valid at early stage of heavy-ion collisions?

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Abstract. Evolution of hot and dense nuclear matter produced in central Au+Au collisions at energies of NICA, FAIR, and SPS is studied within two transport models. Two interesting features in the matter behaviour are observed almost from the very beginning of the collisions, at $t \geq 2$ fm/c, for all studied reactions. (i) Expansion of the system proceeds with constant entropy-per-baryon ratio. (ii) Effective equation of state has a linear form, $P = a\varepsilon$, $a \simeq const$. Both observations support the formal application of hydrodynamics at the early stages of heavy-ion collisions, when the system is very far from the equilibrium.

1. Introduction

Hydrodynamic model for the description of hadronic, or rather nuclear, collisions at ultrarelativistic energies was formulated by Landau more than 60 years ago [1, 2]. It states that

$$\partial_\mu T^{\mu\nu} = 0, \quad (1)$$

where the energy-momentum tensor $T^{\mu\nu}$ reads

$$T^{\mu\nu} = (\varepsilon + P)u^\mu u^\nu + Pg^{\mu\nu} + \eta^{\mu\nu} \quad (2)$$

Here ε is the energy density, P is the pressure, u^μ is the local four-velocity, and $\eta^{\mu\nu}$ represents the dissipative tensor. In the first approximation one may neglect the dissipative processes in the system, and equations (1)-(2) are reduced to well-known Euler equations for ideal fluid. To complete this system of equations one needs to know the equation of state (EOS), which links pressure to energy density. Landau employed the EOS of ultrarelativistic gas of particles, $P = \varepsilon/3$. Later on, when the influence of resonances was taken into account, this estimate was reduced to $P \approx \varepsilon/5$ [3] or even to $P \approx \varepsilon/7$ [4]. However, the basic features of the model persist. Modern hydrodynamic models usually include hadronic cascade as afterburner after the chemical freeze-out in the system, when the inelastic collisions have ceased. They nicely describe the basic features of heavy-ion collisions in a broad range of bombarding energies (for review, see e.g. [5] and references therein). Nevertheless, the thermalization time needed to match the experimental data is very short, e.g. $\tau \equiv \sqrt{t^2 - z^2} \approx 0.5$ fm/c or even $\tau \approx 0.1$ fm/c. The extremely fast thermalization mechanism needs convincing explanations. Two interesting solutions were mentioned not far ago among others. One approach formulated in [6] concludes that hydrodynamic description with a large coarse-graining scale is sufficient for observables



which do not require a precise space-time resolution of the system. In this case, the local thermal equilibrium is not a necessary prerequisite of the hydrodynamic description. According to other explanation, the low-order hydrodynamics can well approximate the hydrodynamic attractor solution describing out-of-equilibrium systems, thus justifying its applicability [7, 8].

What about the microscopic transport models, which do not rely on hypothesis of (fast) local thermal equilibrium? Relaxation of hot and dense nuclear matter, produced in heavy-ion collisions at relativistic and ultrarelativistic energies, to equilibrium have been studied in these models in e.g. [9, 10, 11, 12, 13, 14, 15]. The hadron yields and energy spectra extracted from the central area of heavy-ion collisions were compared to those obtained (i) within the statistical model of ideal hadron gas and (ii) in infinite nuclear matter simulated by box with periodic boundary conditions within the same microscopic model. It was found that the system reached the vicinity of local chemical and thermal equilibrium at $t \approx 6 - 10$ fm/ c after beginning of the collision. However, in line with the conclusions of [6, 7, 8], hydrodynamic description may be valid already at a far-from-equilibrium stage. To verify this scenario we apply two transport string models, ultra-relativistic quantum molecular dynamics (UrQMD) [16, 17] and quark-gluon string model (QGSM) [18, 19]. Our goal is to investigate the evolution of the fireball in heavy-ion collisions at the energies from NICA and FAIR to top SPS energy. Basic features of both models are sketched in section 2, whereas section 3 presents the results of model calculations. Conclusions are drawn in section 4.

2. Basic features of UrQMD and QGSM

Both UrQMD [16, 17] and QGSM [18, 19] are Monte Carlo event generators designed for description of relativistic hh , hA and $A + A$ interactions. The multiparticle production takes place via formation and fragmentation of specific colored objects, strings, stretching uniformly between the quarks, diquarks, and their antistates. The string tension is about $\kappa \approx 1$ fm/ c , and strings break into hadrons via the Schwinger-like mechanism of (di)quark-anti(di)quark formation. However, both mechanisms of the string formation and string fragmentation in the models are different.

There are two possible methods of string excitation. UrQMD employs the longitudinal excitation of strings which is characteristic for all Lund-based string models [20]. Here the mass of the string arises from the momentum transfer, and the strings are stretching between the constituents belonging to the same hadron. QGSM utilizes the color exchange mechanism [21], in which the constituents at the string ends belong to different hadrons. The variety of subprocesses in the latter case is much richer compared to the longitudinal excitation. For the string fragmentation process the string models utilize three possible schemes. The first scenario, suggested by Lund [20], implies that the string always splits on a sub-string and a particle on a mass shell at the end of the fragmenting string. This option is realized in UrQMD. The second scheme splits the string into two sub-strings according to the area law [22]. The third option is the Field-Feynman mechanism [23], where the fragmentation takes place independently from both ends of the string. This scenario is employed in QGSM.

Both models utilize available experimental information, such as hadron cross sections, resonance widths and decay modes. For the description of hadron-nucleus and nucleus-nucleus collisions hadronic cascade is used. Particle propagation between the collisions is governed by Hamilton equations of motion. To obey the uncertainty principle, newly produced particles can interact only after the certain formation time. Pauli principle is implemented by blocking the final state if the outgoing phase space is already occupied.

3. Effective EOS of matter in the central cell

When studying the thermalization problem, one has to check first the isotropy of pressure gradients [9, 11, 15]. After that, three characteristics dealing with the conserved quantities,

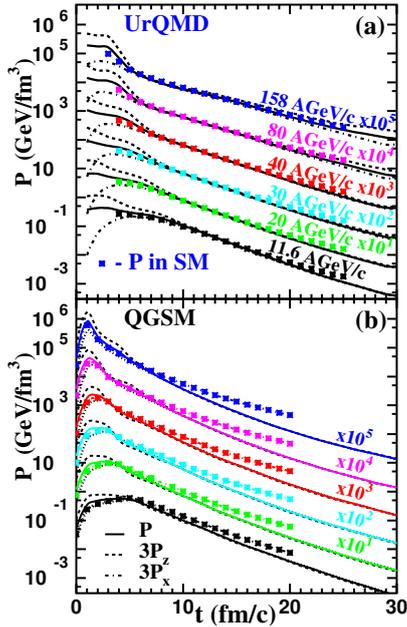


Figure 1. Triple longitudinal (dashed curves) and triple transverse (dash-dotted curves) diagonal components of the microscopic pressure tensor in the central 125 fm³ cell in (a) UrQMD and (b) QGSM calculations of central Au+Au collisions at energies from 11.6 AGeV to 158 AGeV. Solid lines indicate the total microscopic pressure and asterisks denote the pressure given by the statistical model.

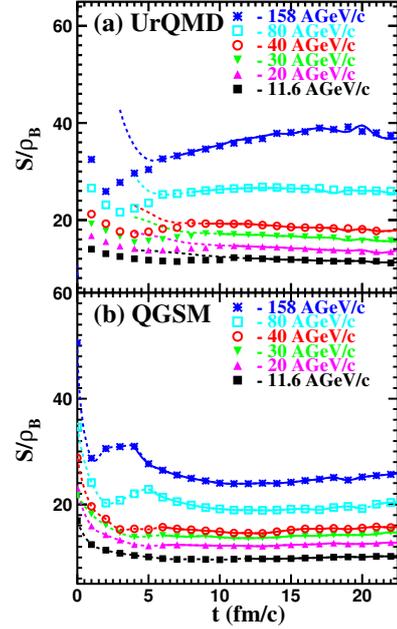


Figure 2. Time evolution of entropy per baryon S/ρ_B in the central 125 fm³ cell in (a) UrQMD and (b) QGSM calculations of central Au+Au collisions at energies from 11.6 AGeV to 158 AGeV. Dashed lines indicate the nonequilibrium stage whereas solid lines are related to the phase of local equilibrium.

i.e. energy density, net-baryon and net-strangeness densities, should be extracted from the tested volume. One inserts these parameters to the system of nonlinear equations provided by the statistical model of ideal hadron gas with the same number of baryon and meson states as independent degrees of freedom in order to get temperature, baryon chemical potential, and strangeness chemical potential. If the hadron abundances and energy spectra in microscopic and macroscopic calculations would match each other, the system can be considered as being in the vicinity of local equilibrium. For formal applicability of hydrodynamic description it would be enough to show that (i) expansion of the fireball proceeds isentropically [1, 24] and (ii) pressure as a function of energy density in the tested volume remains approximately constant.

For our study we choose central Au+Au collisions at six different energies: $E_{lab} = 11.6, 20, 30, 40, 80, 158$ AGeV. The tested volume is defined as central cubic cell with volume $V = 5 \times 5 \times 5 = 125$ fm³. To calculate diagonal elements of the pressure tensor $P_{\{x,y,z\}}$ one has to employ virial theorem

$$P_{\{x,y,z\}} = \frac{1}{3V} \sum_{i=h} \frac{p_i^2_{\{x,y,z\}}}{(m_i^2 + p_i^2)^{1/2}} \quad (3)$$

Here V , p_i and m_i are the volume of the cell, the momentum of the i th hadron, and its mass, respectively. Figure 1 displays evolution of pressure gradients in longitudinal and transverse directions, respectively, compared to the total pressure and the pressure, given by the statistical

model. The convergence of the longitudinal and the transverse pressure components takes place at $t \approx 6 - 8$ fm/c in both models. The total microscopic pressure, however, appears to be very close to the "equilibrium" pressure already at $t \leq 3$ fm/c (UrQMD) or even earlier (QGSM) for all bombarding energies. Recall, that at such early times the hadron-string matter in the cell is very far from both thermal and chemical equilibrium. Then, the expansion of ideal fluid proceeds isentropically. But our central cell is an open system, particles can leave it freely, and none of the three key parameters is conserved. Nevertheless, one is able to verify the behaviour of the entropy-per-baryon ratio. Evolution of this ratio in model calculations is shown in in figure 2. We see that the entropy per baryon weakly depends on time at $t \geq 2$ fm/c. The matter seems to expand isentropically from quite early times.

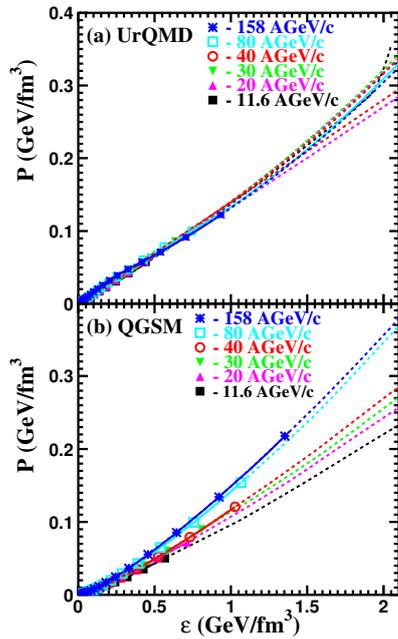


Figure 3. Time evolution of the microscopic pressure P versus the energy density ε in the central 125 fm^3 cell in (a) UrQMD and (b) QGSM calculations of central Au+Au collisions at six energies in question. Dashed lines correspond to the nonequilibrium stage of the reaction; solid lines represent the equilibrium phase.

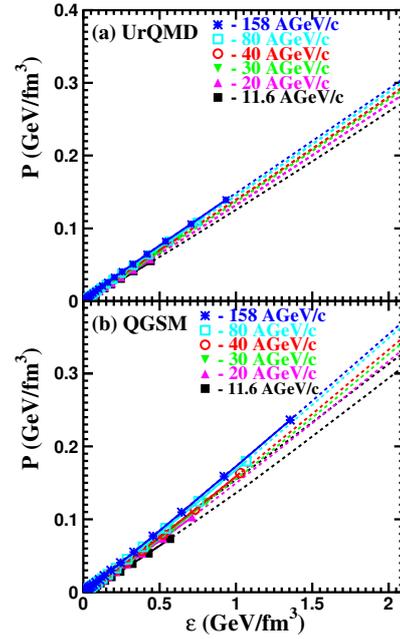


Figure 4. The same as in figure 3 but for macroscopic pressure P extracted from the SM fit to microscopic model generated spectra of hadrons.

Now we can trace the time evolution of the pressure and the energy density. The ratio of these quantities is presented in figures 3 and 4. Figure 3 shows microscopically calculated pressure, P^{mic} , whereas figure 4 uses the macroscopic pressure, P^{mac} , determined from the fit to the statistical model. Symbols and solid lines in both figures indicate the stages of chemical and thermal equilibrium. Dashed lines are related to non-equilibrium phase. But even here the pressure depends nearly linearly on the energy density, $P = a\varepsilon$, where $a = c_s^2$, and c_s is sonic velocity in the system. It increases slightly from $c_s^2 = 0.12 \pm 0.03$ at $E_{lab} = 11.6$ AGeV to $c_s^2 = 0.14 \pm 0.02$ at $E_{lab} = 158$ AGeV, in full accord with the statistical model estimates [4]. The most important observation for us is that the formal ratio P/ε remains nearly constant almost from the beginning of the collision.

4. Conclusions

Relaxation of hot and dense nuclear matter to equilibrium in relativistic heavy-ion collisions is a very interesting topic. More and more studies nowadays are investigating the applicability of hydrodynamic description to the systems far from equilibrium. It looks that the local thermal equilibrium is not a necessary condition for application of hydrodynamics anymore. We have reconsidered our studies of thermalization problem within the microscopic model simulations and found two interesting features. The matter expands with constant entropy-per-baryon ratio, and the effective EOS, $P = a\varepsilon$, remains remarkably linear, starting already at $t \approx 2$ fm/c, when the system is quite far from the equilibrium. Both findings support the application of hydrodynamic description to early stages of relativistic heavy-ion collisions.

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