The role of research in common pool problems

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A R T I C L E   I N F O

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Significant amounts of public spending are allocated towards research on climate change, but considerable uncertainties remain. We analyze the strategic role of information acquisition and the determinants of investments in information in a common pool game. In the first stage, countries can acquire a signal about their own environmental damages caused by total emissions. Because signals are public, there are information spillovers between countries. In the second stage, the countries decide how much pollution to emit. We show that there can be an inefficiently high amount of investments in information in the non-cooperative equilibrium compared to the cooperative solution if the countries are risk averse and the expected emissions are sufficiently large. In addition, we analyze what happens if the countries cooperate in one of the stages but not in the other. We show numerically that if the emissions are decided non-cooperatively, countries might agree not to acquire any information at all. But if the emissions levels are decided cooperatively, investments in the non-cooperative equilibrium are always too low.

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1. Introduction

Considerable uncertainties surround the extent of environmental damages caused by common pool emissions. The consequences of anthropocentric greenhouse gas emissions are one of the most important examples, with one central gauge being the equilibrium climate sensitivity. The likely temperature effect from doubling of carbon dioxide levels ranges from 1.5 to 4.5 °C (Stocker, 2014). Investments in more and better information can help alleviate these and other climate related uncertainties. For example, the United States federal government spent roughly 2.7 billion dollars on climate change related research in 2014. Likewise, the European Union funds several climate change related research projects through its Horizon 2020 program. Much of this research is public and disseminated by organizations such as the Intergovernmental Panel on Climate Change (IPCC), which means that there are information spillovers to other countries from the investments made by individual countries. While

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information is thus a public good, national priorities remain an important driver for determining research needs.\footnote{For example, the Australian Government’s Science and Research Priorities state: “Research will build Australia capacity to respond to environmental change.” The figure for the U.S. spending is from Federal Climate Change Expenditures Report to Congress, August 2013. EU Horizon 2020 website (https://ec.europa.eu/programmes/horizon2020/, accessed Nov 1, 2019) provides information on the program’s research funding.}

Because findings from research can affect common pool emission decisions, information acquisition is a strategic choice. This interaction raises the question whether there is too little or too much investment in information, and what the socially optimal level of investment would be. To answer these questions, we analyze a two-stage game where countries first choose how much information they acquire and then decide how much pollution to emit.

An important part of the uncertainty surrounding the damages from common pool emissions is that damages can differ significantly between countries (see e.g. Ricke et al., 2018). One way to conceptualize the asymmetry of damages from a changing climate is that there are two different components that jointly determine the negative impact. First, there is the component that is common to all countries: the rise in global temperature. Second, there is an idiosyncratic component that is specific to each country: how well countries can adapt to higher temperatures. To give an extreme example, the Maldives will be more adversely affected than the United States in relative terms and will have considerably fewer opportunities for adaptation. On the other hand, the damage realizations in two neighboring countries can be very similar. Due to the public nature of the research efforts, such correlation (or non-correlation) between damages is potentially an important driver for research incentives, but it remains a neglected element in the existing literature on information acquisition.

In our model, two risk averse countries face \textit{ex-ante} unknown and asymmetric damages from emissions but have symmetric prior beliefs. In the first stage, each country can invest in research effort which determines the probability of receiving a public signal that reveals the country’s own true damages. Because signals are public, countries can also update their beliefs based on the signal of the other country. In the second stage, the countries decide how much pollution they emit after observing the information revealed in the first stage. The expected payoffs in the pollution stage depend on how uncertain the countries are about the true damages. Hence the common pool problem in the pollution stage becomes dependent on the first stage information acquisition game, and vice versa. Risk aversion is an important element for information acquisition because it means that being uninformed entails a direct cost. Risk aversion can be justified by the non-diversifiable nature of the risk that each country faces.

The combination of learning, risk aversion and strategic interaction makes the analysis of the information acquisition decision challenging. We use a model that we think is especially well-suited to the task: constant absolute risk-aversion (CARA) preferences together with Gaussian beliefs over the true damages. This allows us to model the players’ information acquisition decisions using certainty equivalents. In addition, Gaussian beliefs mean that the correlation between damages captures how much countries learn from each other’s signals, i.e. how much information spills over to the other country. This is a key factor for the countries’ investment decisions. Due to information spillovers, a natural hypothesis is that when the correlation is high, non-cooperative countries acquire little information because of free-riding.

To evaluate our hypothesis, we compare the non-cooperative (Bayes-Nash) equilibrium to the welfare maximizing cooperative solution. Our main result is to show analytically that there can be more investment in the non-cooperative equilibrium than in the cooperative solution even when the correlation between the countries damages is perfect. This happens when the pollution stage is sufficiently inefficient in terms of expected emissions (i.e. private benefits from polluting activities are high) and when countries are sufficiently risk averse. Thus, the free-riding hypothesis we mentioned above is not true in terms of relative investment levels when countries are risk averse.

Why would non-cooperative countries invest more in information than cooperative countries? The value of information in our model consists of the direct effect of reducing the risk and the strategic effect on the equilibrium emissions in the pollution stage. When the correlation between damages equals one, the latter effect disappears as countries hold the same information irrespective of learning, and thus the value of information is determined only by the amount of risk the countries face. Non-cooperative countries then invest more than cooperative countries only if there is additional risk in the non-cooperative equilibrium. The latter is true in our model due to the common pool externality in the pollution stage, which causes the non-cooperative countries to emit more than the efficient level. Larger emissions translate to larger risks because damages are uncertain. Thus, acquiring information is a way for the non-cooperative countries to insure themselves against the risk brought by higher emissions. We confirm this intuition by showing that absent risk aversion, there is always less investment in the non-cooperative equilibrium than in the cooperative solution, irrespective of the correlation.

Given the complexity of the model, we use numerical methods to further explore what happens to investments in information when correlation in damages can take any value between zero and unity. Consistent with the analytical results, we are able to demonstrate that the non-cooperative countries invest more than the cooperative countries when countries are risk averse. Our numerical results also suggest that, given high enough risk aversion, low correlation in damages can lead to relatively larger investments in the non-cooperative equilibrium than in the cooperative solution. This is explained by the non-cooperative countries’ aversion to being the country with less information: the country with higher degree of uncertainty emits less than the more certain country.

After analyzing the incentives to invest in information, a natural question to ask from our model is what happens if the countries could agree on an efficient policy in the information stage given that the pollution stage is non-cooperative. We first show analytically that when the correlation between damages equals one, the countries would agree to invest even more in
information than in the non-cooperative equilibrium. This implies that even though the non-cooperative countries invest more than the fully cooperative countries, it would be welfare improving to invest even more. We also analyze investment levels numerically and show that when the correlation is small enough, the efficient policy in the information stage might be to not invest in information at all, which effectively means a ban on research. In this case, the information stage cooperative countries prefer to be mutually ignorant rather than risk the possibility of being the country with larger damages or the country not observing its true damages.

We also analyze what happens when emissions are controlled by a first-best policy but countries act non-cooperatively in the information stage. Because a first-best policy internalizes the effect of uncertainty, there can be no private gains from information, and the non-cooperative countries prefer to free-ride. Thus, countries acting non-cooperatively in the information stage always invest less than fully cooperative countries. This result highlights the difficulty of solving the common pool problem and of achieving efficient information acquisition at the same time. Emissions policies necessarily affect not only emissions but also the incentives to acquire information.

Our research is closely related to several other studies on the role of information in pollution problems (Gollier et al., 2000; Bramoullé and Treich, 2009; Boucher and Bramoullé, 2010; Morath, 2010). Bramoullé and Treich (2009) analyze how exogenous uncertainty and risk aversion affect common pool problems. They show that, in their specification, emissions are always lower under uncertainty, and that the presence of uncertainty can in fact improve welfare. We continue their work by examining how much countries would invest in information if such investments were possible. Boucher and Bramoullé (2010) look at the effect of uncertainty in a game with public good provision and treaty formation. They show that uncertainty can help in terms of participation, but it decreases the effort in the case of a public good. In a single decision maker model, Gollier et al., (2000) examine how risk aversion and learning might explain precautionary motives when setting an emissions policy. They give general conditions under which precautionary motives exist.

The paper closest to ours in terms of research questions is Morath (2010). He analyzes a quasilinear public good game with information acquisition and finds that countries can choose to remain uninformed even when information is costless. Our paper is different in the sense that we focus on comparing the cooperative and the non-cooperative outcomes. Additionally, the presence of risk aversion in our model means that uncertainty has a direct effect on emissions and on information acquisition.

The question of how information affects games with externalities has been analyzed in other contexts. Ulph and Maddison (1997) and others have pointed out that information can make players worse-off in games with negative externalities. Similar conclusions can arise also in games with emission decisions and coalition formation such as Kolstad and Ulph (2008) and Kolstad and Ulph (2011), although the generality of these conclusions has been questioned by Finus and Pintassilgo (2013). The novelty in our paper to these papers is twofold: first, we endogenize learning and, second, we analyze how information spillovers affect learning. We, however, do not analyze how information affects coalition formation as we focus on the two player case.

In the literature outside environmental economics, Hellwig and Veldkamp (2009) and Myatt and Wallace (2012) analyze information acquisition in a beauty contest game where players can observe a set of signals. In Hellwig and Veldkamp (2009), players can choose what signals they observe and hence the information choice only affects other players through its effect on the subsequent actions. In Myatt and Wallace (2012), both the signal precision and the correlation with other players’ information are endogenously determined by how much attention players pay to each signal. Because information processing is always costly in both of these papers, there are no information spillovers unlike in our model.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 analyzes the information acquisition stage and Section 4 analyzes the pollution stage. Section 5 first compares information acquisition between the non-cooperative and cooperative countries and then considers what happens if countries can cooperate in either stage of the game. Section 6 shows the numerical results. Section 7 discusses what happens if we change some of the main assumptions of the model and then concludes. All proofs can be found in the Appendix.

2. Model

2.1. Actions and payoffs

We analyze a game between two countries, \( i \in \{1, 2\} \), who can invest in information before deciding how much pollution to emit. Before countries can acquire information, nature chooses the true marginal damages from pollution, \( \theta = (\theta_1, \theta_2) \), for each country. The countries do not know the true marginal damages but can acquire a signal about the damages in the first stage. After acquiring information, the countries observe signal realizations, \( s = (s_1, s_2) \), based on their decisions and update their prior beliefs on the marginal damages. Signal realizations and information acquisition decisions are public. After observing the signals, countries move to the pollution stage and decide how much pollution to emit, \( g = (g_1, g_2) \). Fig. 1 shows the timeline of the game.
The payoffs of the game are as follows. Countries' own emissions generate known private benefits $B(g_i)$ but total emissions, $G = g_1 + g_2$, cause ex-ante unknown damages $D(G, \theta)$. We let the benefits from emissions be $B(g_i) = -0.5(g_i - g)^2$, where $g > 0$ (saturation point) denotes the optimal amount of emissions when there are no damages. We let the damages from pollution be $D(G, \theta_j) = \theta G$. The ex-ante unknown variable $\theta$ measures country $i$'s sensitivity to emissions. The following payoff function determines country $i$'s ex-post net benefits from emissions $g_i$

$$y_i = B(g_i) - D(G, \theta) = -0.5(g - g_i)^2 - \theta G. \quad (1)$$

To introduce risk preferences in our model, we define a concave utility function, $u(y_i)$, over the net benefits from emissions. We let $u(y_i) = -e^{-\rho y_i}$ so that countries preferences are of the constant absolute risk aversion (CARA) form. The parameter $\rho > 0$ measures countries' risk aversion. As a benchmark, we also analyze the risk neutral case ($\rho = 0$) in which we let utility be linear in the ex-post payoff, $u(y_i) = y_i$.

We assume that the marginal damages $\theta_1$ and $\theta_2$ have a prior joint normal distribution $N(\mu, \Sigma)$ with a mean vector $\mu$ and a covariance matrix $\Sigma$. We let the symmetric prior mean be $\mu_0 > 0$, the symmetric prior variance be $\nu_0 > 0$ and the correlation between the marginal damages be $\gamma \in [0, 1]$. Together with the CARA utility function, this allows us to express an individual country's expected utility in the pollution stage, $E(u(y_i) \mid s)$, using the following certainty equivalent:

$$\phi(g, \mu, \nu) := -\frac{1}{2}(g - g_i)^2 - \mu G - \frac{1}{2} \rho \nu G^2, \quad (2)$$

where $\mu_i$ is the mean of country $i$'s posterior belief of $\theta_i$ and $\nu_i > 0$ its variance, which measures how well informed a country is of its damages. We let $g = (g_1, g_2)$ denote countries' emissions, $\mu = (\mu_1, \mu_2)$ denote mean damages and $\nu = (\nu_1, \nu_2)$ denote variances for both countries. Notice that the last term in the certainty equivalent (2) is the effect risk aversion has on the expected utility—the greater the uncertainty the smaller the certainty equivalent if emissions are positive.

A key parameter in the model is the saturation point, $\bar{G}$, which measures the importance of polluting activities to the countries. It essentially determines the marginal benefit from emissions. To guarantee positive expected emissions, we assume that $\bar{G} > 2 \mu_0 > 0$.

### 2.2. Information acquisition and learning

We model the countries' information acquisition problem as choosing the probability of observing the true marginal damages, $\theta_i$, and let $p_i \in [0, 1]$ denote this probability. The cost of research investment is given by a twice continuously differentiable convex cost function $c(.)$. We normalize the cost to be zero when investment is zero: $c(0) = 0$ and $c'(0) = 0$.

After acquiring information, the countries either learn true damages or not. This means that we have four possible signal realization pairs: $s_{1,1} = (\theta_1, \theta_2)$, $s_{1,0} = (\theta_1, \theta)$, $s_{0,1} = (\theta, \theta_2)$ and $s_{0,0} = (\emptyset, \emptyset)$ where $\emptyset$ denotes the null signal when a country does not learn the true damages. After observing the signals, countries update their prior belief using the Bayes' rule. Because the signals are public, the countries learn from each others' signals and hold identical beliefs. If both countries learn their true damages, their posterior is the degenerate distribution around the true damages $\mu = (\theta_1, \theta_2, \nu = (\nu_1, \nu_2)$. If, for example, country $i$ does not learn but country $j$ does learn, then the posterior for $\theta_i$'s damages is the conditional distribution given $\theta_j$. That is, the posterior for $\theta_i$ is the normal distribution with mean $\mu^*$ and variance $\nu^*$:

$$\mu^* := \gamma \theta_j + (1 - \gamma) \mu_0,$$

$$\nu^* := (1 - \gamma^2) \nu_0. \quad (3)$$

Notice that the posterior variance for country $i$'s damages is $(1 - \gamma^2) \nu_0$ so that the higher the correlation, the more certain will country $i$ also be of its damages. The correlation between the countries' damages thus measures the information spillover from country $j$ to country $i$ and vice versa. The larger the correlation, the larger the spillover.

### 3. Information stage

In this section, we characterize the non-cooperative (Bayes–Nash) equilibrium and the cooperative solution to the information stage. In the non-cooperative equilibrium, the countries maximize their own expected payoff and disregard the effects of information and pollution to the other country. The cooperative solution, conversely, maximizes the sum of the countries' expected payoffs and therefore serves as a natural benchmark for the non-cooperative equilibrium.

#### 3.1. Non-cooperative equilibrium

We start by characterizing the non-cooperative equilibrium. Let $V_i(\mu, \nu)$ denote country $i$'s expected payoff (in utility terms) in the non-cooperative pollution stage for a given posterior $(\mu, \nu)$ (which depends on the signals countries observe). Given the other country's behavior in the equilibrium, we can define it as

$$V_i(\mu, \nu) := \max_{g_i} \mathbb{E} \left[ u \left( B(g_i) - D(G, \theta_i) \right) \mid s \right], \quad (4)$$

where the signal $s$ depends whether the countries learn the true damages or not: $s \in \{s_{1,1}, s_{1,0}, s_{0,1}, s_{0,0}\}$. With CARA preferences we can express the conditional expectation in (4) more simply with the certainty equivalent as $u(\phi_1(\mu, \nu)) = -e^{-\rho \phi_1(\mu, \nu)}$.

We have four possible expected payoffs $V_i(\mu, \nu)$ (four different posteriors) in the pollution stage. We can define the expected payoffs in the information stage, before the countries observe the signals, in all four cases in the following way:

$$U^NC_{1,1} := \mathbb{E}_0[V_i(\mu_1, \theta_2), (0, 0))], \quad U^NC_{1,0} := \mathbb{E}_0[V_i((\mu_1, \mu^*), (0, \nu^*))].$$

$$U^NC_{0,1} := \mathbb{E}_0[V_i((\mu^*, \theta_2), (\nu^*, 0))], \quad U^NC_{0,0} := \mathbb{E}_0[V_i((\mu_0, \mu_0), (\nu_0, \nu_0))].$$

The subindices in $U_{ij}$ refer to whether the countries have learned the true damages (1) or not (0), with the first subindex referring to country 1 and the second to country 2. The expectation is taken with respect to $\theta_1$ and $\theta_2$ given the prior belief $(\mu_0, \nu_0)$. The structure of the expected payoffs underline the fact that the only information the countries have in the information stage is at which probability they will observe the true damages.

Given the non-cooperative pollution stage payoffs, country 1’s information acquisition problem can be formulated as

$$\max_{p_1} \left\{ p_1 p_2 U^NC_{1,1} + p_1 (1 - p_2) U^NC_{1,0} + (1 - p_1) p_2 U^NC_{0,1} + (1 - p_1)(1 - p_2) U^NC_{0,0} - c(p_1) \right\}.$$  

(6) Similar to Gollier et al. (2000), we assume in (6) that the utilities are separable between the two stages and we also assume that the cost function $c(\cdot)$ is separable from the utility derived from the emissions outcome.

The problem in (6) can be written in a more useful form:

$$\max_{p_1} \left\{ \Delta U^NC_0 + p_2 \left( \Delta U^NC_1 - \Delta U^NC_0 \right) \right\}. $$

(7) where function $A(p_2)$ depends only on $p_2$ and hence does not factor into the choice of optimal $p_1$. The coefficients $\Delta U^NC_0$ and $\Delta U^NC_1$ determine the (marginal) value of information. We can define them as:

$$\Delta U^NC_0 := U^NC_{1,0} - U^NC_{0,0},$$

$$\Delta U^NC_1 := U^NC_{1,1} - U^NC_{0,1}.$$  

Coefficient $\Delta U^NC_0$ measures country 1’s gain from learning when country 2 has not learned its own damages. Coefficient $\Delta U^NC_1$ measures country 1’s gain from learning when country 2 has learned its own damages. The difference $\Delta U^NC_1 - \Delta U^NC_0$ measures the strategic complementarity of information. If the difference is positive, then country 1’s gain from learning is larger when country 2 has also learned its damages. This implies that information is a strategic complement. If the difference is negative, it means that country 1’s gain from learning is greater when country 2 is uninformed. Information is then a strategic substitute.

We now proceed to characterize the Bayes-Nash equilibrium of the game in which both countries maximize their own expected payoff given the other country’s behavior. The first order condition to the optimization problem in (7) determines the best response of country 1:

$$\Delta U^NC_0 + \left( \Delta U^NC_1 - \Delta U^NC_0 \right) p_2 = \phi'(p_1).$$

(8)

Because we assume that the countries are symmetric, the equilibrium is defined by a pair of first order conditions identical to (8). Analyzing them yields the following proposition for the equilibrium of the information stage.

**Proposition 1.** Suppose that $\Delta U^NC_0 > 0$ and $\Delta U^NC_1 < \phi'(1)$. Then there exists a unique symmetric interior Bayes-Nash equilibrium to the information stage and the equilibrium investment level $p^NC$ is determined by

$$\Delta U^NC_0 + \left( \Delta U^NC_1 - \Delta U^NC_0 \right) p^NC = \phi'(p^NC).$$

An interior equilibrium exists as long as the value of information at zero investment is greater than the cost ($\Delta U^NC_0 > 0$) and below the cost when investment equals one ($\Delta U^NC_1 < \phi'(1)$). Notice that when information is a strategic substitute ($\Delta U^NC_1 < \Delta U^NC_0$) asymmetric equilibria might exist along with the symmetric one, but we can rule them out by having a convex enough cost function (details in the Appendix).

### 3.2. Cooperative solution

Next, we characterize the cooperative solution to the countries’ information acquisition problem. Let $W(\mu, \nu)$ denote the expected cooperative payoff in the pollution stage. We can define it as the maximized sum of the two countries’ utilities:

$$W(\mu, \nu) := \max_{\mu_1, \mu_2} \mathbb{E} \left[ u(B(g_1) - D(G, \theta_1)) + u(B(g_2) - D(G, \theta_2)) \mid s \right].$$

(9)
where $s \in \{s_{1,1}, s_{1,0}, s_{0,1}, s_{0,0}\}$. The objective function in (9) contains both countries’ payoffs, and hence the emissions set by the cooperative are different than the emissions in the non-cooperative equilibrium. Similar to the non-cooperative case, we can define the expected values of the pollution stage payoffs for all possible posteriors:

$$
\begin{align*}
U_{1,1}^{C} := & E_0[W((\theta_1, \theta_2), (0, 0))], \\
U_{1,0}^{C} := & E_0[W((\theta_1, \mu^*), (0, v^*))], \\
U_{0,1}^{C} := & E_0[W((\mu^*, \theta_2), (v^*, 0))], \\
U_{0,0}^{C} := & E_0[W((\mu_0, \mu_0), (v_0, v_0))].
\end{align*}
$$

The subindices again denote whether countries have learned the true damages or not and the expectation is taken with regards to $\theta_1$ and $\theta_2$ given the prior belief.

Using the definitions in (10), the cooperative information acquisition problem can be formulated as

$$
\max_{p_1, p_2} \left\{ p_1 p_2 U_{1,1}^{C} + p_1 (1 - p_2) U_{1,0}^{C} + (1 - p_1) p_2 U_{0,1}^{C} + (1 - p_1)(1 - p_2) U_{0,0}^{C} - c(p_1) - c(p_2) \right\}.
$$

Reorganizing the objective function in a similar fashion as in the non-cooperative case gives

$$
\Delta U_{0}^{C} + (\Delta U_{1}^{C} - \Delta U_{0}^{C}) p_2 + \Delta U_{0}^{C} p_2 + B - c(p_1) - c(p_2),
$$

where $B$ is a constant independent of $p_1$ and $p_2$ and we can define $\Delta U_{0}^{C}$ and $\Delta U_{1}^{C}$ as

$$
\Delta U_{0}^{C} := U_{1,0}^{C} - U_{0,0}^{C}, \\
\Delta U_{1}^{C} := U_{1,1}^{C} - U_{0,1}^{C}.
$$

We thus see that the cooperative’s information acquisition problem has a structure similar to the non-cooperative problem except that the coefficients $\Delta U_{0}^{C}$ and $\Delta U_{1}^{C}$ are determined by the cooperative pollution stage payoffs. Also notice that the problem is fully symmetric from the perspective of each country. Taking the first order condition with respect to country 1’s investment level $p_1$ then gives

$$
\Delta U_{0}^{C} + \Delta U_{1}^{C} p_2 = c'(p_1).
$$

In the symmetric solution $p_1 = p_2 = p^C$ and the above first order condition has to hold for both countries. We thus have the following result for the cooperative solution.

**Proposition 2.** Suppose that $\Delta U_{0}^{C} > 0$ and $\Delta U_{1}^{C} < c'(1)$. Then the (symmetric) cooperative investment level $p^C$ is determined by

$$
\Delta U_{0}^{C} + (\Delta U_{1}^{C} - \Delta U_{0}^{C}) p^C = c'(p^C).
$$

Similar to the non-cooperative case, asymmetric solutions might exist when $\Delta U_{0}^{NC} < \Delta U_{1}^{NC}$ but a convex enough cost function rules them out. In sum, we can characterize the investment level in a similar fashion as we did for the non-cooperative equilibrium. Because the cooperative emissions in general differ from the non-cooperative emissions, the investment levels are also different in the two cases.

### 3.3. Perfect correlation

So far we have assumed that the marginal damages are imperfectly correlated between the two countries. Suppose instead that the marginal damages are identical, $\theta_1 = \theta_2$, so that the correlation between damages is perfect ($\gamma = 1$). Because now both countries either know their damages or not, we can write country 1’s problem in the non-cooperative equilibrium as

$$
\max_{p_1} \{ (p_1 + p_2 - p_1 p_2) U_{1,1}^{NC} + (1 - p_1)(1 - p_2) U_{0,0}^{NC} - c(p_1) \}.
$$

Any of the signal realizations $s_{1,1}, s_{1,0}$ or $s_{0,1}$ reveals the true $\theta$; hence the first term in the objective function (13). The problem can be rewritten as

$$
\max_{p_1} (\Delta \tilde{U}_{0}^{NC} - \Delta \tilde{U}_{0,0}^{NC}) p_1 + D(p_2) - c(p_1),
$$

where $\Delta \tilde{U}_{0}^{NC} := U_{1,1}^{NC} - U_{0,0}^{NC}$. Compared to (7), we see that perfect correlation implies $\Delta \tilde{U}_{1}^{NC} = 0$, which means that the investment levels are strategic substitutes, as long as information is valuable ($\Delta \tilde{U}_{0}^{NC} > 0$). Hence, investments from country 2 decrease country 1’s marginal value of information. This is our first result and we formalize it with the following lemma.

**Lemma 1.** When the correlation between the marginal damages is perfect and $\Delta \tilde{U}_{0}^{NC} > 0$, information is a strategic substitute.

The result in Lemma 1 is that with perfect correlation countries do not care who acquires information, because signals acquired by either country carry the same information. The information stage is therefore purely a public good game, which
implies that the non-cooperative equilibrium suffers from free-riding. A natural hypothesis therefore is that investments in information are lower in the non-cooperative equilibrium than in the cooperative solution when the correlation is perfect.

Given the analogous structure, exactly the same condition as in Lemma 1 holds in the cooperative case. However, unlike in the non-cooperative equilibrium, there is no free-riding because the cooperative solution internalizes the effect of information for the other country through the coefficient $\Delta U^C_0$. In fact, because higher correlation means the signals carry more information, we should expect that the cooperative investments are increasing in the correlation. To say something more precise about the relative investment levels in the two solutions, however, we have to analyze the pollution stage in more detail. We turn to this next.

4. Pollution stage

4.1. CARA preferences

4.1.1. Non-cooperative equilibrium

A non-cooperative country’s problem in the pollution stage is to maximize her expected utility, $\mathbb{E}[u(y_i) \mid s]$, by choosing emissions $g_i$. Assuming CARA preferences, we can use the certainty equivalent to express the expected utility as $u(\phi(g_i, \mu, \nu))$. It then follows that a non-cooperative country’s problem is equivalent to maximizing the certainty equivalent $\phi(g_i, \mu, \nu)$:

$$-\frac{1}{2}(\mathbb{E} - g_i)^2 - \mu G - \frac{1}{2} \rho\nu G^2.$$  

Taking the first order condition with respect to country $i$’s own emissions, $g_i$, we can derive the best response of country $i$ as

$$g_i(g_i) = \frac{\mathbb{E} - \mu_i}{1 + \rho\nu_i} - \frac{\rho\nu_i g_i}{1 + \rho\nu_i}.$$  

(14)

This tells us that country $i$’s emissions are a function of its information, $\mu_i$ and $\nu_i$. In particular, the more uncertain the country is the more willing it is to sacrifice its own emissions vis-à-vis the other country’s emissions, $g_j$. Analyzing the best responses of both countries yields the following proposition.

**Proposition 3.** The unique Bayes-Nash equilibrium to the pollution stage game is such that country $i$ emits according to

$$g_i^{NC}(\mu, \nu) := \frac{(1 + \rho\nu_i)(\mathbb{E} - \mu_i) - \rho\nu_i(\mathbb{E} - \mu_i)}{1 + \rho(\nu_i + \nu_j)}$$  

(15)

and thus the total emissions are

$$G^{NC}(\mu, \nu) := \frac{2\mathbb{E} - \mu_1 - \mu_2 + \rho(\nu_1 - \nu_2)(\mu_2 - \mu_1)}{1 + \rho(\nu_1 + \nu_2)}.$$  

The equilibrium emissions of each country are decreasing in their own uncertainty ($\nu_i$) but increasing in the other country’s uncertainty ($\nu_j$).

An important property of the non-cooperative equilibrium is highlighted in the last part of the proposition: relative uncertainty matters for the equilibrium emissions. More informed countries emit more than less informed countries. Furthermore, informational asymmetries can increase total emissions. It can be also easily shown that both the individual equilibrium emissions, $g_i^{NC}$, and the total equilibrium emissions, $G^{NC}$, are decreasing in risk aversion, $\rho$.

The effect of the saturation point $\mathbb{E}$ on an individual country’s emissions depends on the sign of the expression $1 + \rho(\nu_j - \nu_i)^2$. If the sign is positive, country $i$’s emissions are increasing in the saturation point. This is true, when country $i$ observes its damage realization and country $j$ does not. When the sign is negative, country $i$’s emissions are decreasing in $\mathbb{E}$. This is true, when country $i$ does not learn its true damages whereas country $j$ does, and furthermore assuming that $\rho\nu_j > 1$ holds.

Using the non-cooperative equilibrium emissions given by Proposition 3, we can define the non-cooperative pollution stage payoff as

$$V_i(\mu, \nu) := -e^{-\rho\phi_i^{NC}},$$  

(16)

where $\phi_i^{NC} = \phi_i(g_i^{NC}, \mu, \nu)$ and $g_i^{NC} = (g_i^{NC}, g_j^{NC})$. The expression in (16) then determines the information stage payoffs in (5).

4.1.2. Cooperative solution

The cooperative solution to the pollution stage maximizes the expected sum of both countries’ utilities: $\mathbb{E}[u(y_1) + u(y_2) \mid s]$. We again use the certainty equivalents and formulate the cooperative problem as

$$W(\mu, \nu) := \max_{g_1, g_2} \{(-e^{-\rho\phi_1(g, \mu, \nu)} - e^{-\rho\phi_2(g, \mu, \nu)})\},$$  

(17)

$^2$ Take the derivative of the equilibrium emissions with respect to $\mathbb{E}$. 
Taking the first order conditions with regards to both countries’ emissions level yields the following necessary conditions for optimality:

\[
\frac{\partial W}{\partial g_1} = \rho \frac{\partial \phi_1(g, \mu, v)}{\partial g_1} e^{-\rho \phi_1(g, \mu, v)} + \rho \frac{\partial \phi_2(g, \mu, v)}{\partial g_1} e^{-\rho \phi_2(g, \mu, v)} = 0,
\]

\[
\frac{\partial W}{\partial g_2} = \rho \frac{\partial \phi_1(g, \mu, v)}{\partial g_2} e^{-\rho \phi_1(g, \mu, v)} + \rho \frac{\partial \phi_2(g, \mu, v)}{\partial g_2} e^{-\rho \phi_2(g, \mu, v)} = 0.
\]

(18)

In general, we cannot solve the optimal emission levels \( g^C_1 \) and \( g^C_2 \) in closed form from the necessary conditions. Because of informational asymmetries (different posteriors), the cooperative problem does not reduce to maximizing the sum of the certainty equivalents. Looking at the first order conditions in (18), we can see that the exponential terms weigh the derivatives of informational asymmetries (different posteriors), the cooperative problem does not reduce to maximizing the sum of the signals.

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The following proposition gives the cooperative solution to the pollution stage.

**Proposition 4.** The cooperative emissions levels \( g^C = (g^C_1, g^C_2) \) are determined by a (unique) solution to (18).

If the posteriors of the countries are fully symmetric, \( \mu_1 = \mu_2 = \hat{\mu} \) and \( v_1 = v_2 = \hat{v} \), the cooperative solution to the pollution stage specifies the following emission levels for each country

\[
g^C_1(\hat{\mu}, \hat{v}) = g^C_2(\hat{\mu}, \hat{v}) = \frac{\overline{g} - 2\hat{\mu}}{1 + 4\rho \hat{v}}
\]

and thus the total emissions are

\[
G^C(\hat{\mu}, \hat{v}) = \frac{2(\overline{g} - 2\hat{\mu})}{1 + 4\rho \hat{v}}.
\]

The cooperative emissions are given by a solution to the system of equation (18), which we can solve in closed form only in the case of fully symmetric posteriors. When the posteriors are symmetric, both higher risk aversion and higher uncertainty reduce cooperative emissions. In the case of symmetric posteriors, the individual and total emissions are always increasing in the saturation point, \( \overline{g} \).

Using Proposition 4, we can define the cooperative pollution stage payoff as

\[
W(\mu, v) := -e^{-\rho \phi^C_i} - e^{-\rho \phi^C_i},
\]

(19)

where \( \phi^C_i(g^C, \mu, v) \) for \( i = 1, 2 \). The expression in (19) then determines the expected information stage payoffs in (10).

The underlying reason we generally cannot solve the cooperative emissions in closed form is that the cooperative solution tends to allocate more emissions to a country with a bad damage realization or with a higher posterior variance. This comes from the concavity of the utility function: the marginal increase in total utility is greater if the cooperative allocates more emissions to the worse-off country. The cooperative solution in effect tries to equalize countries’ expected payoffs. In the special case when posteriors are identical (e.g. damages are identical), this is clear because also the emissions—and not only the payoffs—are identical.

### 4.1.3. Pollution stage inefficiencies

In this section, we analyze the inefficiencies that arise in the non-cooperative equilibrium. We focus on excess emissions relative to the cooperative solution, \( G^{NC} - G^C \). We can define the expected excess emissions as

\[
\Delta G := \mathbb{E}_0(G^{NC}(\mu, v) - G^C(\mu, v)).
\]

The expectation is taken with respect to \( \theta_1 \) and \( \theta_2 \) given the prior belief. We have the following result for the expected excess emissions.

**Lemma 2.** Excess total emissions are always positive, \( \Delta G > 0 \). Furthermore, when posteriors are symmetric (e.g. identical damages) \( \Delta G \) is increasing in the saturation point \( \overline{g} \).

The expected excess emissions are always positive as we would expect because the pollution stage suffers from the (standard) common pool externality: countries do not internalize damages to other countries. This implies that the non-cooperative equilibrium suffers from excess damages and therefore from excess risk taking.

Lemma 2 tells us something interesting about the role the saturation point \( \overline{g} \) plays for our model (in the case of perfect correlation): excess emissions are increasing in the saturation point. Hence, a larger saturation point implies that the common...
4.2. Risk neutral preferences

The risk neutral preferences ($\rho = 0$) provide an interesting benchmark for the CARA preferences. Then the countries’ utility is linear in the ex-post payoff $y_i$. Thus, in the pollution stage, the non-cooperative countries are maximizing their expected ex-post payoff $\mathbb{E}(y_i \mid s)$:

$$-\frac{1}{2}(\bar{g} - g_i)^2 - \mu_i G.$$

The above non-cooperative objective function is the same as the certainty equivalent with $\rho = 0$. Hence (15) can be used to determine the equilibrium emissions: $g_i^{\text{NC}} = \bar{g} - \mu_i$. Consequently, the emission levels are determined by the posterior means and the saturation parameter $\bar{g}$ so that the posterior variances do not play any role. In fact, the strategic interaction in the pollution stage disappears with risk neutral preferences because the countries’ best responses are independent of the other country’s emissions.

For the cooperative solution, the objective function simplifies to $\mathbb{E}(y_1 + y_2 \mid s)$ when countries are risk neutral. Thus, we can write the cooperative problem as

$$\max_{g_1, g_2} \left[-\frac{1}{2}(\bar{g} - g_1)^2 - \frac{1}{2}(\bar{g} - g_2)^2 - (\mu_1 + \mu_2)G \right].$$

Taking the first order conditions with regards to $g_1$ and $g_2$ allows us to solve the cooperative emissions. This yields $g_i^{\text{C}} = g_i^{\text{NC}} = \bar{g} - \mu_i - \mu_2$. Hence, if countries are risk neutral the emissions are identical for both countries irrespective of the posterior beliefs. The symmetry follows because it is optimal to equalize the marginal value of emissions between the countries.

It is easy to see that the total emissions are lower in the cooperative solution than in the non-cooperative equilibrium also when countries are risk neutral. The non-cooperative equilibrium thus suffers from excess emissions, as expected.

5. Comparison of information acquisition

5.1. Non-cooperative equilibrium versus cooperative solution

5.1.1. Comparison with risk neutral preferences

When do non-cooperative countries have strong incentives to invest in information? Next, we compare information acquisition in the non-cooperative equilibrium to the cooperative solution. We start by analyzing the case when countries are risk neutral, which allows us to derive closed-form solutions for all key variables.

While the countries’ emissions are independent of the posterior variance with risk neutral preferences, information still matters because in the information stage countries do not know the realized posterior: they are taking expectation over the posterior means so that the posterior variances do not play any role. In fact, the strategic interaction in the pollution stage disappears with risk neutral preferences because the countries’ best responses are independent of the other country’s emissions.

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$$\max_{g_1, g_2} \left[-\frac{1}{2}(\bar{g} - g_1)^2 - \frac{1}{2}(\bar{g} - g_2)^2 - (\mu_1 + \mu_2)G \right].$$

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It is easy to see that the total emissions are lower in the cooperative solution than in the non-cooperative equilibrium also when countries are risk neutral. The non-cooperative equilibrium thus suffers from excess emissions, as expected.
and the cooperative marginal value of information is determined by
\[ \Delta U_0^C = (1 + \gamma)^2 \nu_0, \]
\[ \Delta U_1^C = (1 - \gamma^2) \nu_0. \]

One interesting observation from Lemma 3 is how the correlation affects the marginal value of information. We can see that for both non-cooperative and cooperative countries the correlation increases the coefficient \( \Delta U_0 \) but diminishes the strategic value of information, \( \Delta U_1 \).

Given Lemma 3, we can compare the investments in research. Recall from Proposition 1 that investment in information is pinned down by the first order condition
\[ \Delta U_0 + (\Delta U_1 - \Delta U_0)p^* = c'(p^*), \]

where \( p^* \) is the equilibrium (or the cooperative) investment level. The comparison of investment levels between the two solutions reduces to comparing the left-hand sides (marginal value of information) of this equation, because the marginal cost of information, \( c'(p^*) \), is increasing. Because the left-hand side is linear in investment \( p^* \), a sufficient condition for the non-cooperative countries to invest more than the cooperative countries is that the marginal value of information is greater at both \( p^* = 0 \) and \( p^* = 1 \). This leads to the following two conditions
\[ \Delta U_0^{NC} > \Delta U_0^C, \]
\[ \Delta U_1^{NC} > \Delta U_1^C, \]

where the superscript NC denotes the non-cooperative equilibrium and C the cooperative solution. The first condition says that the marginal value of information is greater for the non-cooperative countries at \( p^* = 0 \) and the second condition says that it is greater at \( p^* = 1 \). The next proposition summarizes our results for the risk neutral case.

**Proposition 5.** Suppose countries are risk neutral and that the conditions in Proposition 1 and in Proposition 2 hold. Then

(i) The cooperative countries always invest more in information than the non-cooperative countries.

(ii) Given \( p^C \leq 3/4 \), the difference in the marginal value of information between the cooperative solution and the non-cooperative equilibrium is increasing in the correlation \( \gamma \) and is at its largest when \( \gamma = 1 \).

Proposition 5 tells us that there is always less investment in the non-cooperative equilibrium than in the cooperative solution when countries are risk neutral. This is true because the non-cooperative countries do not internalize the other country’s gains from learning and prefer to instead free-ride on the other country’s investment. Additionally, the difference in the value of information is increasing in correlation (given that investment levels are not too close to one), because correlation strengthens the free-riding incentives in the non-cooperative equilibrium. In fact, the difference in the value of information is at its greatest when correlation is perfect (\( \gamma = 1 \)). This is the precisely the case we analyze next when we let the countries be risk averse.

### 5.1.2. Comparison with CARA preferences

Next, we extend the comparison between non-cooperative and cooperative investment in research to the case when risk aversion is present (\( \rho > 0 \)). The key simplifying assumption we make is that damages are identical, \( \theta_1 = \theta_2 \), which means that the correlation between the marginal damages is perfect (\( \gamma = 1 \)). In Section 6, we analyze numerically the more general model with imperfect correlation, \( \gamma \in [0, 1] \). While restrictive, the assumption of identical damages reveals much of the central forces at work because it shuts down any private gains from learning. Because of this, the free-riding incentives in the non-cooperative equilibrium are at their strongest.

We can evaluate the expected payoffs in the information stage by using the emissions given by Proposition 3 (non-cooperative) and by Proposition 4 (cooperative) and plugging them into the certainty equivalent \( \phi(g, \mu, \nu) \). Integrating over the prior belief then yields the expected payoffs in the information stage. We can characterize the expected payoffs in a closed form in the case of identical damages (in the Appendix), but the intractability of the cooperative solution in the pollution stage prevents a closed form characterization for the more general case when correlation is imperfect (\( \gamma < 1 \)).

After calculating the expected information stage payoffs, we can compare the investment levels between the non-cooperative equilibrium and the cooperative solution. Recall from Proposition 1 and Lemma 1 that investment in information in the perfect correlation case is determined by
\[ (1 - p^*) \Delta U_0 = c'(p^*), \]

where \( p^* \) is the equilibrium (or the cooperative) investment level. The comparison of the investment levels thus reduces to:
\[ p^{NC} > p^C \iff \Delta U_0^{NC} > \Delta U_0^C. \]

---

3 If one of the conditions fails, it does not follow that the non-cooperative countries invest more than the cooperative countries. If both conditions fail, then there must be more investment in the cooperative solution.
Our findings are summarized by the next proposition.

**Proposition 6.** Suppose that the marginal damages are identical (perfect correlation) and that the conditions in Proposition 1 and in Proposition 2 hold. Then non-cooperative countries invest more in information than cooperative countries if risk aversion is sufficiently high and the saturation point is sufficiently large: \( \rho > \hat{\rho} \) and \( \Gamma \geq \hat{\Gamma} \).

We prove in Proposition 6 that there can be more investment in information in the non-cooperative equilibrium than in the cooperative solution. This is true even when correlation is perfect. The non-cooperative countries invest more when the coefficient of risk aversion and the saturation point are sufficiently large. In other words, when expected excess emissions are large (recall Lemma 2), risk averse non-cooperative countries respond by investing more in information. To understand where the result comes from, let us write the inequality in (21) in terms of the underlying expected payoffs:

\[
U_{1,1}^{NC} - U_{0,0}^{NC} > U_{1,1}^{C} - U_{0,0}^{C}.
\]

What happens to these payoffs when we increase the saturation point \( \hat{\Gamma} \)? Each of the payoffs is decreasing in the saturation point, but given large enough risk aversion the fastest decrease happens in the non-cooperative payoff when countries do not know the true damages, \( U_{0,0}^{NC} \). This is because it contains the quadratic risk aversion term in the certainty equivalent and because the excess emissions are increasing in the saturation point.\(^4\) Hence, when \( \hat{\Gamma} \) increases, the value of information is increasing faster for the non-cooperative countries given high enough risk aversion. This then gives the result in Proposition 6: there are cutoffs for the risk aversion and the saturation point above which the non-cooperative countries invest more than the cooperative countries.

In terms of the underlying incentives, the result in Proposition 6 tells us that the countries’ need to insure themselves against the excessive risk taking in the non-cooperative equilibrium can dominate the free-riding incentives present in the equilibrium. Thus, another way of interpreting the effect of the saturation point is that it increases the risk taking in the non-cooperative equilibrium (by increasing excess emissions). The importance of risk aversion is highlighted by the fact that when the countries are risk neutral, the cooperative countries always invest more, as shown in Proposition 5. The non-cooperative countries invest more than the cooperative countries only when they are risk averse and when there is sufficient amount of risk in the pollution stage equilibrium.

### 5.2. Cooperative information stage, non-cooperative pollution stage

Previous section showed that non-cooperative behavior in the information stage might result in more investment in information relative to the cooperative solution. What would happen if the countries could cooperate in either stage?

We first analyze what happens if countries cannot cooperate on emissions but can agree on information investments. That is, countries play cooperatively in the information stage and then play non-cooperatively in the pollution stage. We call this solution the information stage cooperative solution. The value of information for this solution is determined by the sum of the non-cooperative information stage payoffs:

\[
\Delta U_0^{C1} = U_{0,1}^{NC} + U_{0,0}^{NC} - 2U_{0,0}^{NC} = \Delta U_0^{NC} + U_{0,1}^{NC} - U_{0,0}^{NC},
\]

\[
\Delta U_1^{C1} = 2U_{1,1}^{NC} - U_{0,1}^{NC} - U_{1,0}^{NC} = \Delta U_1^{NC} + U_{1,1}^{NC} - U_{1,0}^{NC}.
\]

The difference from the non-cooperative equilibrium is that the information stage cooperative solution internalizes the effect of information on the other country. We can see this from the coefficients \( \Delta U_0^{C1} \) and \( \Delta U_1^{C1} \), which include the payoffs of both countries. The effect on the other country consists of the positive effect of the other country learning and the negative effect of the equilibrium outcome changing (possibly) adversely with more information. Both of these effects are captured by the difference \( U_{0,1}^{NC} - U_{0,0}^{NC} \) in \( \Delta U_0^{C1} \) and \( U_{1,1}^{NC} - U_{1,0}^{NC} \) in \( \Delta U_1^{C1} \). Whether these differences are positive or negative determines if information is more valuable for the information stage cooperative than for the non-cooperative countries.

By comparing the investment levels between the non-cooperative equilibrium and the information stage cooperative solution, we are in effect holding the emissions stage behavior constant. Such a comparison provides an alternative way of assessing whether the non-cooperative equilibrium is characterized by higher or by lower levels of investments in information than what would be efficient given the assumed non-cooperative behavior in the pollution stage. For the case with perfect correlation, the following proposition compares the investments in information between the information stage cooperative solution and the non-cooperative equilibrium.

**Proposition 7.** Suppose that the marginal damages are identical (perfect correlation) and that the conditions in Proposition 1 hold. Then, the information stage cooperative countries always invest more in information than the non-cooperative countries.

The result in Proposition 7 follows from observing that, when the damages are identical, we have \( \Delta U_0^{C1} = 2\Delta U_0^{NC} \) and \( \Delta U_1^{C1} = \Delta U_1^{NC} = 0 \). Hence, the value of information must be greater for the information stage cooperative countries under the conditions laid out in Proposition 1. Proposition 7 says that the investments in the non-cooperative equilibrium are still lower than what

\(^4\) The risk aversion term is \(-0.5\rho v_{G}\), which is decreasing rapidly in total emissions \( G \).
would be efficient (given the non-cooperative pollution stage), even though the equilibrium investment levels might still be higher than in the fully cooperative solution (Proposition 6). This is because in the non-cooperative equilibrium countries want to free-ride on each other’s investments. More generally, when the correlation is imperfect, relative investments in information between the non-cooperative equilibrium and the information stage cooperative solution can go either way. We analyze this numerically in Section 6.

5.3. Non-cooperative information stage, cooperative pollution stage

We next analyze what happens when the pollution stage is cooperative but the information stage is non-cooperative. This would be true, for example, if the pollution stage externality is internalized by an effective policy instrument (e.g. a Pigouvian tax) but countries still decide their own research budgets. We call this solution the pollution stage cooperative solution. The value of information for the pollution stage cooperative countries is determined by

\[ \Delta U^C_{i0} = U^C_{i0} - U^C_{i1} = \frac{1}{2} \Delta U^C_{i1}, \]

\[ \Delta U^C_{i1} = U^C_{i1} - U^C_{i0} = \frac{1}{2} \Delta U^C_{i0}, \]

where the payoffs are country i’s expected payoff in the cooperative solution as defined in Section 3. The second equality for both \( \Delta U^C_{i0} \) and \( \Delta U^C_{i1} \) follows from the symmetry of the prior: each country is identical in expectation. This immediately yields the following proposition.

**Proposition 8.** Suppose the conditions in Proposition 2 hold. Then the pollution stage cooperative countries always invest less in information than the fully cooperative countries.

It is clear from (23) that the value of information is always greater in the fully cooperative solution than in the pollution stage cooperative solution. This is because the pollution stage cooperative solution does not internalize the value of information to the other country, while the fully cooperative solution does. Hence, the non-cooperative investments are at a lower than the efficient level (given the cooperative behavior in the pollution stage).

An important lesson from the result in Proposition 8 is that an efficient emissions policy is not enough to achieve the ex-ante optimal outcome if there is no collaboration in research investments. Imposing e.g. a carbon tax necessarily implies that non-cooperative countries have less incentives to invest in information. This is true because countries want to free-ride on each other’s investments in the absence of any private gains from information.

6. Numerical analysis

This section presents numerical results for our model. In particular, we analyze the case when the correlation between damages, \( \gamma \in (0,1) \). To do this, we first need to evaluate the information stage expected payoffs \( (U_{1,1}, U_{1,0}, U_{0,1}, U_{0,0}) \) for both the cooperative solution and the non-cooperative equilibrium. We compute them numerically using Monte Carlo integration. This entails drawing a large sample of realizations of the marginal damages, \( \theta = (\theta_1, \theta_2) \), from the prior distribution and computing the cooperative and non-cooperative emissions for each realization. These emissions then determine the expected non-cooperative and the expected cooperative pollution stage payoffs \( V(\mu, \nu) \) and \( W(\mu, \nu) \) as defined in Section 4. The sample averages of those payoff realizations yield approximations for the expected information stage payoffs \( (U_{1,1}, U_{1,0}, U_{0,1}, U_{0,0}) \).

We use the following parameters for the numerical solutions: the prior beliefs are \( \mu_0 = 0.3 \) and \( v_0 = 0.11 \) and the benefit function parameter is \( \gamma = 1 \). We assume that the cost function for research effort takes the following form

\[ c(p) = \frac{c_0}{1-p} - c_0, \]

where we set \( c_0 = 0.05 \). Given the cost function and the expected payoffs, the non-cooperative and the cooperative investments in information, \( p^{NC} \) and \( p^C \), can be solved using the expressions in Propositions 1 and 2, respectively.

Fig. 2 shows investments in research as a function of the correlation between the countries’ damages \( (\gamma) \) in all four solution concepts defined earlier (in Section 3 and in Section 5). The (fully) cooperative solution is depicted using a solid black line, whereas the (fully) non-cooperative equilibrium is depicted using a dashed black line. The investments in the information stage cooperative solution are depicted using broken dashed line, whereas the dotted line shows the investments in the pollution stage cooperative solution. The effect of risk aversion \( (\rho) \) on investments can be seen by comparing the outcomes between the three panels. The first panel shows investment levels when the countries are risk neutral (linear utility, denoted by \( (\rho = 0) \)). The second panel shows investment levels when the countries are moderately risk averse (\( \rho = 2 \)), and the third panel show investments levels when the countries exhibit high risk aversion (\( \rho = 4 \)).

Several observations can be made from Fig. 2. We first focus on comparing the fully cooperative solution to the non-cooperative equilibrium. First, investments in research are increasing in risk aversion in both the cooperative solution and in the non-cooperative equilibrium. When countries are risk neutral (\( \rho = 0 \), first panel), investments are also increasing in correlation.
However, higher risk aversion can change this. For example, from the third panel it can be seen that the non-cooperative investments are actually decreasing in correlation. In the first and the second panel, the cooperative investments are at a higher level than the non-cooperative investments. However, high enough risk aversion (third panel) can change this as well. With high risk aversion (the third panel), the non-cooperative countries actually invest more than the cooperative countries when the correlation is low enough. The main reason the correlation matters so much is that in the non-cooperative equilibrium emissions react to information—more informed countries emit more—but this effect naturally becomes smaller as the correlation increases.

An interesting observation from Fig. 2 is that when the countries cooperate in the information stage but act non-cooperatively in the pollution stage (broken dashed line), they might not invest in research at all when the correlation is small. This effect is enhanced by higher risk aversion. Effectively, countries issue a ban on research to ensure that the pollution stage externality problem is minimized through “mutual ignorance”. This result is in line with Bramoullé and Treich, 2009, who also find that the presence of uncertainty can improve welfare in a common pool problem. Such action only works when the countries are risk averse, because risk aversion leads to lower levels of emissions in the pollution stage. Recall as well that informational asymmetries increase total emissions in the non-cooperative equilibrium (Proposition 3) and this effect is strengthened by higher risk aversion. However, when correlation is higher, cooperation in the information stage leads to increased investments in information because now both countries benefit from observing either $\theta_1$ or $\theta_2$ (the value of information becomes greater).

Fig. 3 shows the expected total welfare, the sum of the countries’ utilities, in the four solution concepts. The relevant comparison here is how far the other solutions are from the fully cooperative solution (solid line) as it gives the highest expected welfare. The difference tells us how valuable better cooperation is. The pollution stage cooperative solution (dotted black line) has the smallest difference to the cooperative solution. This tells us that most of the inefficiency (with these parameters) stems from the pollution stage behavior. The expected welfare in the information stage cooperative solution (broken dashed line) is higher than in the non-cooperative equilibrium but lower than in the pollution stage cooperative. The difference to the non-cooperative equilibrium effectively disappears when the correlation is higher. This reflects Fig. 2, which shows that the investment behavior is very similar in the two solutions with higher correlation. These results suggest that the value of cooperation is high in the pollution stage and low in the information stage. However, we should keep in mind that the prior variance used for the numerical analysis is fairly small ($\nu_0 = 0.11$).

Fig. 4 shows the expected emissions in the four different solution concepts. The first observation is that risk aversion generally decreases the expected emissions, because larger emissions also mean higher risk. However, with high enough risk aversion, the non-cooperative equilibrium might actually “bounce back” in expected emissions compared to lower risk aversion especially with higher correlation (left panel vs. center panel). The effect of risk aversion is explained by increased research investments (Fig. 2). The effect of the correlation is explained by the fact that a higher correlation between the damages more than offsets the decrease in the investment level (Fig. 2) so that the probability of both countries learning is larger with higher correlation. In general, the expected emissions are increasing in the investments to information, and this is indicated by the fact the pollution

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5 For example, in the non-cooperative case the expected total emissions, $\overline{E}$, are defined as $\overline{E} = p_{NC}p_{NC}E_0(G_{11}) + 2p_{NC}(1 - p_{NC})E_0(G_{10}) + (1 - p_{NC})(1 - p_{NC})E_0(G_{00})$, where $G_{11}$, $G_{10}$, $G_{00}$ denote the pollution stage total emissions for each of the possible signal realization. Note that $G_{10} = G_{01}$. 

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Fig. 2. Research investments as a function of the correlation $\gamma$ in the non-cooperative equilibrium (dashed line) and in the cooperative (solid line), the cooperative information stage (dash-dot line) and the cooperative pollution stage (dotted line) solutions. The first panel shows the risk neutral ($\rho = 0$) case; the second panel moderate risk aversion ($\rho = 2$); and the third panel high risk aversion ($\rho = 4$).
Fig. 3. Expected total welfare as a function of the correlation $\gamma$ in the non-cooperative equilibrium (dashed line) and in the cooperative (solid line) solutions. The first panel shows the risk neutral ($\rho = 0$) case; the second panel moderate risk aversion ($\rho = 2$); and the third panel high risk aversion ($\rho = 4$).

Fig. 4. Expected total emissions as a function of the correlation $\gamma$ in the non-cooperative equilibrium (dashed line) and in the cooperative (solid line) solutions. The first panel shows the risk neutral ($\rho = 0$) case; the second panel moderate risk aversion ($\rho = 2$); and the third panel high risk aversion ($\rho = 4$).

Finally, Fig. 5 shows that there can be more investment in the non-cooperative equilibrium than in the cooperative solution even when the correlation equals one, $\gamma = 1$. For the same parameters, the information stage cooperative countries invest even more than the non-cooperative countries. Note that in Fig. 5 we have plotted investments as a function of risk aversion $\rho$. The other parameter values are $g = 10, c_0 = 10, \mu = 0.3, v_0 = 0.11$. These results are consistent with the analytical findings in Proposition 6 and Proposition 7.

7. Discussion and conclusions

7.1. Discussion

Next, we briefly discuss what happens if we change some of the central assumptions of the model. Multiple countries. We assume that there are only two countries playing the game. While there is nothing in our model that
would prevent analyzing the game with more than two countries, this would complicate analysis considerably. With already three countries, there are eight possible realizations for the signals, so the dimensionality of the problem expands rapidly as the number of countries increases. Generally speaking, having more (symmetric) countries in the pollution stage exacerbates the common pool problem as each country internalizes a smaller part of the total damages. Thus under the conditions in Proposition 6, we would expect non-cooperative countries to invest even more in information when there are multiple countries. Such a scenario is especially relevant in a global common pool problem, such as carbon emissions. An interesting topic for future research would be to examine the formation of coalitions in the presence of multiple countries, and how such coalitions might invest in information.

Private signals. A natural extension of the model is to analyze private signals, that is, what happens if countries do not observe each other’s signals. How likely such a situation is depends on the type of common pool problem and available mechanisms for sharing research. In the interest of space, we do not analyze the case of private signals here. However, the model presented in this paper still provides a prediction for how private signals might influence investments in information. Not observing each other’s signals is similar to having zero correlation in damages, with the addition that countries do not know whether the other country has learned it or not. Thus, for non-cooperative countries the value of information should be at least as large as with public signals.

Sequential moves. We assume a simultaneous move game in both stages. Alternatively, we could have a situation where one country moves first in information acquisition, e.g. commits to a research infrastructure. As with simultaneous moves, the outcome in the non-cooperative equilibrium depends on whether information is a strategic complement or a substitute (i.e. on the level correlation in damages). When information is a strategic complement, the first mover suffers because the second mover finds it optimal to invest more than the first mover. When information is a strategic substitute, the first mover can commit not investing and will gain as a result, because the second mover will then have to bear the cost of providing the public good. Thus, the correlation between damages again plays a key role. We leave a more detailed analysis of such a game for future research.

Normal distribution. We assume that the countries’ damages from pollution have a joint normal distribution. This implies that there is always a positive probability of damages being in fact ex-post benefits. While some countries might indeed benefit from, for example, increased CO2 emissions, we emphasize here that in expectation damages are always negative. For our analysis, this is crucial as we focus on information acquisition i.e. when countries use expectations to evaluate the consequences of their decisions. Furthermore, we can always make the probability of positive damages (i.e. benefits) arbitrarily small by moving the prior mean and this has no qualitative effect on our results. The main benefit of normally distributed damages is to simplify the analysis while still allowing us to model risk aversion and information spillovers in a tractable way.

Cooperative objective function. We assume that the cooperative solution assigns equal weights to both countries. Alternatively, the cooperative could assign unequal weights to countries because of bargaining power or equity concerns, for example. In the pollution stage, this would make emissions more sensitive to information regarding the damages of the country with a larger weight. This effect, by itself, already makes the cooperative solution more willing to invest in learning the damages of the country with the larger weight. Thus, a larger weight might have a non-linear effect on research investments. However, we leave the full analysis of this possibility for future research.

7.2. Conclusions

This paper examined countries’ incentives to invest in research in a two-stage game in which they can first acquire information about uncertain damages and then decide how much pollution to emit. We identified the degree of inefficiency in the
pollution stage and the countries’ risk aversion as important determinants of investments in research. In particular, we compared the non-cooperative equilibrium to the cooperative solution. We showed analytically that non-cooperative countries may invest more in research even when the correlation between countries’ damages is perfect, as long as the benefits from polluting activities are large enough. This is because the non-cooperative countries want to insure themselves against the higher risk that is present in the non-cooperative pollution stage. When the correlation can take any value between zero and unity, our numerical results confirm that non-cooperative countries may invest more than cooperative countries. When the correlation is low, the non-cooperative countries have an additional reason to invest in research: to increase their share of the total emissions because countries with higher uncertainty emit less in the non-cooperative equilibrium.

When countries can cooperate in the information stage but act non-cooperatively in the pollution stage, the solution might entail a total ban on research when the correlation between damages is low enough. Such a ban ensures mutual ignorance in the pollution stage game, thus mitigating inefficiencies arising from the non-cooperative pollution game. When countries cooperate in the pollution stage, our results suggest that a non-cooperative information stage leads to lower investments in research. This is explained by the non-cooperative countries’ desire to free-ride on each others’ research.

Judging the efficiency of research investments in a real-world setting requires empirical knowledge about the beliefs and risk aversion of the decision makers. We are careful not to make such judgements given our highly stylized model. However, one could argue that the establishment of the EU Emissions Trading System, for example, in effect signals a cooperative pollution stage for the countries in the European Union. Thus, collaboration in research and more research funding at the EU level could suggest that the non-cooperative research investments are viewed to be too low. Another possible interpretation for our results is that the type of the research matters: countries should invest in research projects that inform all countries (high correlation) rather than on those that provide information more narrowly (low correlation).

While we have focused on research investments related to climate change as our motivation, our model can also apply to analyze other information acquisition problems that involve externalities and public information. These include, for example, pollution in oceans and in other major bodies of water. The model is also relevant for applications outside environmental economics. One such example is firms engaged in quantity competition and who also invest in publicly disseminated research.

### Appendix A. Information acquisition

**Proof of Proposition 1**

First part of the proposition: the best response for country $i$ is found from the first order condition:

$$\Delta U_0^{NC} + (\Delta U_1^{NC} - \Delta U_0^{NC})p_j = c'(p_i).$$

The symmetric equilibrium is found by setting $p_i = p_j = p^{NC}$, which leads to

$$\Delta U_0^{NC} + (\Delta U_1^{NC} - \Delta U_0^{NC})p^{NC} = c'(p^{NC}).$$

From the conditions of the proposition we have that at $p^{NC} = 0 \Delta U_0^{NC} > c'(0)$ and at $p^{NC} = 1 \Delta U_0^{NC} < c'(1)$, and thus by continuity there must exist a $p^{NC} \in (0, 1)$ such that the symmetric first order condition holds. Furthermore, as the left-hand side is linear and the right-hand side is convex in $p^{NC}$, the solution is unique.

**Asymmetric equilibria.** Assume that the conditions in Proposition 1 hold. When $\Delta U_0^{NC} > \Delta U_1^{NC}$, i.e. information is a strategic substitute, asymmetric equilibria might exist along with the symmetric one. However, we can rule these out by having a convex enough cost of information. To see this first observe that under the conditions of Proposition 1 we have interior investment levels: $1 > p^*_2 > p^*_1 > 0$. In an equilibrium we must then have that the first order condition (24) holds for both country 1 and 2. Subtracting the country 2’s first order condition from country 1’s first order condition then yields (note that both conditions have to hold):

$$(\Delta U_1^{NC} - \Delta U_0^{NC})(p^*_2 - p^*_1) = c'(p^*_1) - c'(p^*_2).$$

This can only be true if $\Delta U_1^{NC} > \Delta U_0^{NC}$ because $c'(p^*_2) > c'(p^*_1)$. Then notice that writing $p^*_2 = p^*_1 + h$ with $h = p^*_2 - p^*_1$ we have that $c'(p^*_2) = c'(p^*_1) + c''(p^*_1)h + 0.5c'''(p^*_1)h^2 + \ldots$. Supposing that $c'''(p^*_1) \geq 0$ we then have from Taylor’s theorem (positive remainder) that $c'(p^*_2) - c'(p^*_1) \geq c''(p^*_1)h$. Using this and reorganizing the condition gives

$$(\Delta U_0^{NC} - \Delta U_1^{NC})(p^*_2 - p^*_1) \geq c''(p^*_1)(p^*_2 - p^*_1).$$

Thus, if we have that $c''(p^*_1) > \Delta U_0^{NC} - \Delta U_1^{NC}$ the asymmetric equilibrium cannot exist. More generally we need $c''(p) > \Delta U_0^{NC} - \Delta U_1^{NC}$ and $c'''(p) \geq 0$ for all $p \in (0, 1)$. Intuitively, when the cost is convex enough asymmetric equilibria cannot exist because the cost of increasing the investment level increases too fast.
Proof of Proposition 2

The cooperative investment level is determined by the first order condition with regards to \( p_1 \)
\[
\Delta U_0^C + (\Delta U_1^C - \Delta U_1^E) p_1 = c'(p_1).
\]

In the symmetric solution we have \( p_1 = p_2 = p^C \) so that the investment level is determined by
\[
\Delta U_0^C + (\Delta U_1^C - \Delta U_1^E) p^C = c'(p^C).
\]

Again, to have and interior solution we must have \( \Delta U_0^C > 0 \) and \( \Delta U_1^C < c'(1) \). These conditions then give existence and uniqueness of the solution with similar arguments to the non-cooperative case. \( \square \)

Asymmetric solutions. If we have \( \Delta U_0^C > \Delta U_1^C \), we can have asymmetric solutions to the cooperative’s problem in a similar fashion to the asymmetric non-cooperative equilibria. Then the cooperative solution picks whatever gives the largest expected payoff. Similarly to the equilibria we can rule out these asymmetric solutions by having a convex enough cost: the conditions are that \( c^\mu(p) > \Delta U_0^C - \Delta U_1^C \), \( c^\mu(p) \geq 0 \) for all \( p \in (0, 1) \).

Appendix B. Pollution stage

Proof of Proposition 3

The best response for country \( i \) is found from the first order condition (\( \phi_i \) is concave in \( g_i \)):
\[
(g_i - g^C) - \mu_i - \rho \nu_j G = 0.
\] (25)

Solving the best response (in the text) and substituting in \( g_j \) to \( G = g_i + g_j \) then yields the expression in the proposition.

Uniqueness follows from construction: the best responses are linear in \( g_i \) so they either cross once, always or never. Since \( \rho \nu_j/(1 + \rho \nu_i) < 1 \) they must cross once.

Taking derivatives with regards to \( \nu_i \) and \( \nu_j \) we see that \( \partial g^NC_i/\partial \nu_j > 0 \) and \( \partial g^NC_i/\partial \nu_i < 0. \) \( \square \)

Proof of Proposition 4

The cooperative problem is to maximize the sum of the countries’ payoffs. The objective function and the first order (necessary) conditions are in the text. First, to see why the necessary conditions are also sufficient observe that the objective function is concave in \((g_1^C,g_2^C)\). This follows from the fact that the certainty equivalents \( \phi_1(g, \mu, \nu) \) and \( \phi_2(g, \mu, \nu) \) are concave and from the fact that \( -\exp(-\rho \phi_1(g, \mu, \nu)) \) is a concave and monotonic transformation of a concave function and thus concavity of the certainty equivalent is preserved (the sum of two concave functions is concave).

When the posteriors are identical, the first order conditions are fully symmetric and so we must have \( g_1 = g_2 \) and \( \phi_1(g, \mu, \nu) = \phi_2(g, \mu, \nu) \). We can then cancel the exponential terms out, and the resulting first order condition becomes:
\[
(g^C - g_1) - \mu_1 - \mu_2 - \rho(\nu_1 + \nu_2) G = 0.
\] (26)

Using \( g_1 = g_2 \) and letting \( \mu_1 = \mu_2 = \hat{\mu} \) and \( \nu_1 = \nu_2 = \hat{\nu} \) yields the expression in the proposition. \( \square \)

Proof of Lemma 2

Proof. The emissions, \( g^NC_i \) and \( g^C_i \), are determined by the following pair of first order conditions:
\[
e^{-\rho \phi_1} \frac{\partial \phi_1}{\partial g^NC_i} = 0,
\]
\[
e^{-\rho \phi_2} \frac{\partial \phi_2}{\partial g^C_i} = -e^{-\rho \phi_2} \frac{\partial \phi_2}{\partial g^NC_i},
\]
where \( \phi_1 \) and \( \phi_2 \) are the certainty equivalents for country 1 and country 2 respectively.

We first prove that excess emissions are positive for any posterior belief. Due to concavity in \( g_1 \), the LHS of both first order conditions is decreasing in \( g_1 \). The difference lies in the RHS, which is always strictly positive (for any \( g_1, g_2 \) because we have \( \partial \phi_2/\partial g^C_i < 0 \) for the cooperative and increasing in \( g_1 \). This implies that for any solution to these equations, \((g^NC_i, g^C_i)\), we must have \( g^NC_1 > g^C_1 \), because otherwise the non-cooperative condition cannot hold. Furthermore, because this reasoning applies for both countries we must have \( g^NC_2 > g^C_2 \) as well and thus \( G^NC > G^C \). This then implies that \( \Delta G > 0 \), because the result holds for any posterior and thus has to hold in expectation as well.

We then prove that excess emissions are increasing in \( g \) when posteriors are symmetric. First observe that we can then write the inequality \( E(G^NC) > E(G^C) \) as (from Proposition 3 and Proposition 4)
\[
E(G^NC) = \frac{2(g - \mu)}{1 + 2\rho \nu^*} > E(G^C) = \frac{2(g - 2\mu)}{1 + 4\rho \nu^*}.
\]
The derivative of both sides with regards to $\overline{g}$ is

$$\frac{2\overline{g}}{1 + 2\rho^*} > \frac{2\overline{g}}{1 + 4\rho^*}.$$ 

This is true since the left-hand side denominator is smaller so we have that the difference $E(G^{NC}) - E(G^C)$ is increasing in $\overline{g}$. □

Appendix C. Comparison of information acquisition

Risk neutral information stage payoffs

First recall that using the rules for variance and covariance we can write the expectations of $\theta_i^2$ and $\theta_1\theta_2$ as

$$E(\theta_i^2) = \mu_i^2 + \nu_0,$$

$$E(\theta_1\theta_2) = \mu_0^2 + \gamma \nu_0.$$ 

We use these two results several times. Recall as well that the conditional distribution for $\theta_1$ given $\theta_2$ is given by

$$E(\theta_1 | \theta_2) = \gamma \theta_2 + (1 - \gamma)\mu_0,$$

$$\text{Var}(\theta_1 | \theta_2) = (1 - \gamma^2)\nu_0.$$ 

Non-cooperative information stage payoffs

The non-cooperative pollution stage payoff (from $g_i = \overline{g} - \mu_i$) is

$$V_i(\mu, \nu) = -0.5\mu_i^2 - \mu_i (2\overline{g} - \mu_i - \mu_j).$$

We need to take expectations of this in all four different cases: when both countries learn, when country 1 learns and country 2 does not and so on. This yields the expected information stage payoffs $U^{NC}_{1,1}, U^{NC}_{1,0}, U^{NC}_{0,1}$ and $U^{NC}_{0,0}$ as defined in the text. Doing this first in the case when both countries learn, we find that

$$U^{NC}_{1,1} = E_0[-0.5\theta_i^2 - \theta_i(2\overline{g} - \mu_i - \theta_i)]$$

$$= E_0(-0.5\theta_i^2 - (2\overline{g}\theta_i - \theta_i^2 - \theta_j\theta_i))$$

$$= - (2\overline{g} - 1.5\mu_0)\mu_0 + (0.5 + \gamma)\nu_0.$$ 

Then doing the same steps for $U^{NC}_{1,0}$ we find that

$$U^{NC}_{1,0} = E_0[-0.5\theta_i^2 - \theta_i(2\overline{g} - \theta_i - \gamma \theta_i - (1 - \gamma)\mu_0)]$$

$$= E_0(-0.5\theta_i^2 - (2\overline{g}\theta_i - (1 + \gamma)\theta_i^2 - (1 - \gamma)\theta_i\mu_0))$$

$$= - (2\overline{g} - 1.5\mu_0)\mu_0 + (0.5 + \gamma)\nu_0.$$ 

For $U^{NC}_{0,1}$ we have

$$U^{NC}_{0,1} = E_0[-0.5(\gamma \theta_j + (1 - \gamma)\mu_0)^2 - (\gamma \theta_j + (1 - \gamma)\mu_0)(2\overline{g} - \gamma \theta_j + (1 - \gamma)\mu_0 - \theta_j)]$$

$$= - (2\overline{g} - 1.5\mu_0)\mu_0 + (1 + 0.5\gamma)\gamma \nu_0.$$ 

And finally for $U^{NC}_{0,0}$ we have

$$U^{NC}_{0,0} = E_0[-0.5\mu_0^2 - \mu_0(2\overline{g} - \mu_0 - \mu_0)] = -(2\overline{g} - 1.5\mu_0)\mu_0.$$ 

Cooperative information stage payoffs

The cooperative pollution stage payoff (from $g_1 = g_2 = \overline{g} - \mu_1 - \mu_2$) is

$$W(\mu, \nu) = - (\mu_i + \mu_j)^2 - 2(\mu_i + \mu_j)(\overline{g} - \mu_i - \mu_j).$$
Doing the same steps as in the non-cooperative case we can then calculate the expected information stage payoffs $U_{1,1}$, $U_{1,0}$, $U_{0,1}$ and $U_{0,0}$:

\[
U_{1,1} = \mathbb{E}[\{(\theta_i + \theta_j)^2 - 2(\theta_i + \theta_j)(\bar{\theta}_i - \bar{\theta}_j)] \\
= -4(\bar{\theta} - \bar{\mu})\mu_0 + 2(1 + \gamma)\nu_0, \\
U_{1,0} = -4(\bar{\theta} - \bar{\mu})\mu_0 + (1 + \gamma)^2\nu_0 = U_{0,1}^C, \\
U_{0,0} = -4(\bar{\theta} - \bar{\mu})\mu_0.
\]

**Proof of Proposition 5**

**Proof.** We start with the comparison result. We show that the conditions for the non-cooperative to invest more are never satisfied which then directly implies that they always hold for the cooperative (opposite signs). First, the condition for the $\Delta U_0$ terms is

\[
\Delta U_{NC}^C - \Delta U_0^C > 0.
\]

Writing out this difference gives

\[
(-0.5 - \gamma - \gamma^2)\nu_0 > 0,
\]

which is never true. Moving on to the $\Delta U_1$ terms, the condition is

\[
\Delta U_{NC}^C - \Delta U_1^C > 0.
\]

Or

\[
-0.5(1 - \gamma^2)\nu_0 > 0,
\]

which again is never true. Hence, the cooperative countries always invest (at least weakly) more than the non-cooperative countries.

We then prove the result regarding correlation $\gamma$. First we can write the difference in the marginal value of information between the solutions as

\[
\Delta U_0^C + (\Delta U_1^C - \Delta U_0^C)p^C - \Delta U_{NC}^C - (\Delta U_{NC}^C - \Delta U_0^C)p^NC.
\]

Taking the derivative of this difference with regards to $\gamma$ yields

\[
\nu_0(2(1 + \gamma) - (4\gamma + 2)p^C) - \nu_0(1 - (1 + \gamma)p^NC).
\]

We want to prove that the derivative is positive for all $\gamma > 0$ so that our claim follows. From the first part we have that $p^C \geq p^NC$. Letting $p^C = p^NC$ we thus get a lower bound for the derivative:

\[
\nu_0(1 + 2\gamma - (1 + 3\gamma)p^C) > 0.
\]

This is true if

\[
1 + 2\gamma - (1 + 3\gamma)p^C > 0 \iff p^C \leq \frac{1 + 2\gamma}{1 + 3\gamma}.
\]

At $\gamma = 1$ the RHS is at the largest and thus we get an upper bound $p^C \leq 3/4$, which is true from our assumption in the claim.

**CARA information stage payoffs with perfect correlation**

**Non-cooperative information stage payoffs:** In this section we assume that countries are risk averse ($\rho > 0$) throughout. First, we need to evaluate the information stage payoffs by using the maximized certainty equivalents:

\[
\mathbb{E}_0(V_i(\mu, \nu)) = \mathbb{E}_0\left(\max_{\tilde{g}} - e^{-\rho\phi_i(\tilde{g}, \mu, \nu)}\right) = \mathbb{E}_0\left(-e^{-\rho\phi_{NC}(\mu, \nu)}\right),
\]

where $\phi_{NC}(\mu, \nu)$ is the non-cooperative certainty equivalent using the emission from Proposition 3. Recall that we can write the certainty equivalent as

\[
\phi_i(\tilde{g}, \mu, \nu) = -\frac{1}{2}(\bar{g} - \tilde{g})^2 - \mu \tilde{G} - \frac{1}{2} \rho \nu \tilde{G}^2.
\]
We first need to solve $\phi^{NC}(\theta, 0)$ and $\phi^{NC}(\mu_0, v_0)$ by plugging in $g^{NC} = (1 + \rho v_i)\bar{g} - \mu_i - \rho v_i(\bar{g} - \mu_i)/(1 + \rho (v_i + v_j))$ to $\phi_i(g, \mu, \nu)$ from Proposition 3. Because we assume identical damages $\theta_1 = \theta_2$ (perfect correlation) this yields (simplifying the best as we can):

$$\phi^{NC}(\theta, 0) = -2\overline{g}\theta_1 + 1.5\theta_1^2,$$

$$\phi^{NC}(\mu_0, v_0) = -\frac{4\overline{g}^2\rho v_0(1 + \rho v_0) + (4\overline{g} - 3\mu_0)\mu_0 + 4(\overline{g} - \mu_0)\mu_0\rho v_0}{2(1 + 2\rho v_0)^2}.$$

Because $\phi^{NC}(\mu_0, v_0)$ does not depend on the random variable $\theta_1$, the expected information stage payoff is simply $\exp(-\rho \phi^{NC}(\mu_0, v_0))$. When the countries learn the true damages, $\phi^{NC}(\theta, 0)$, we need to evaluate the following expectation:

$$E_0\left(-e^{-\rho \phi^{NC}(\theta, 0)}\right) =$$

$$-\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi} v_0} e^{-\rho \phi^{NC}(\theta, 0)} e^{-\frac{(\theta_1 - \mu_1)^2}{2v_0}} d\theta_1.$$

It is worthwhile to solve this integral in the general case when $\exp(-\rho \phi^{NC}(\theta, 0))$ is written as $\exp(-a\theta^2 + b\theta)$ with $a, b > 0$. The resulting integral is a standard Gaussian integral that can be evaluated with the usual methods.\footnote{E.g. “completing the square”. Note as well that if we let $a = 0$, we get the certainty equivalent in the pollution stage.} Thus, we get

$$-\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi} v_0} e^{-\theta^2 + b\theta} e^{-\frac{(\theta_1 - \mu_1)^2}{2v_0}} d\theta_1 = -\exp\left(\frac{b\mu_0 - a\mu_0^2 + 0.5b^2v_0}{1 + 2av_0}\right) \frac{1}{\sqrt{2\pi}v_0} \frac{\sqrt{\pi}}{\sqrt{a + (2v_0)^{-1}}}$$

$$= -\exp\left(\frac{b\mu_0 - a\mu_0^2 + 0.5b^2v_0}{1 + 2av_0}\right) \frac{1}{\sqrt{1 + 2v_0a}}.$$

(28)

When evaluating the integral over $-\rho \phi^{NC}(\theta, 0)$ we have $a = 1.5\rho$ and $b = 2\overline{g}\rho$ (recall that we have $-\rho$ in front of the certainty equivalent). We thus get

$$E_0\left(-e^{-\rho \phi^{NC}(\theta, 0)}\right) = -\kappa_{NC} e^{-\rho \phi^{NC}_{1,1}},$$

where

$$\phi^{NC}_{1,1} := -\frac{2\mu_0\overline{g} - 2\rho v_0\overline{g}^2 + 1.5\mu_0^2}{1 + 3\rho v_0},$$

$$\kappa_{NC} := \frac{1}{\sqrt{1 + 3\rho v_0}}.$$

Thus, we can write the marginal value of information as

$$\Delta I^{NC}_0 = -\kappa_{NC} e^{-\rho \phi^{NC}_{0,0}} + e^{-\rho \phi^{NC}_{1,1}},$$

where $\phi^{NC}_{0,0} := \phi^{NC}(\mu_0, v_0)$, which we defined earlier.

**Cooperative information stage payoffs:** Again we use the fact that damages are identical, $\theta_1 = \theta_2$. The cooperative emissions levels are then given by Proposition 4. Using the cooperative emissions, we repeat here the same steps we already did for the non-cooperative case. We again need to find the expected payoffs in the information stage:

$$E_0(W(\mu, v)) = E_0\left(\max_{\delta_1, \delta_2} - e^{-\rho \phi^{C}(\xi, \mu, v)} - e^{-\rho \phi^{C}(\bar{g}, \mu, v)}\right) = 2E_0\left(-e^{-\rho \phi^{C}(\mu, v)}\right),$$

where $\phi^{C}(\mu, v)$ is the pollution stage certainty equivalent of each individual country and the equality follows from symmetry. We first evaluate $\phi^{C}(\theta, 0)$ and $\phi^{C}(\mu_0, v_0)$ using $g^{C}(\mu, v)$ from Proposition 4:

$$\phi^{C}(\theta, 0) = -2\overline{g}\theta_1 + 2\theta_1^2,$$

$$\phi^{C}(\mu_0, v_0) = -\frac{2(\overline{g} - \mu_0)\mu_0 + \overline{g}^2\rho v_0}{1 + 4\rho v_0}.$$
Then we need to evaluate the information stage payoffs by taking the expectation over the pollution stage payoffs. Again, \( \Phi^C(\mu, v_0) \) does not depend on the random variable \( \theta_1 \), so we have that the information stage payoff when countries do not learn equals \(-2\exp(-\rho \Phi^C(\mu, v_0))\). When countries learn the true damages, we need to evaluate the following integral:

\[
\mathbb{E}_0 \left( -2e^{-\rho \Phi^C(\theta, 0)} \right) = -2 \int_{-\infty}^\infty \frac{1}{\sqrt{2\pi}v_0} e^{-\rho \Phi^C(\theta, 0)} e^{-\frac{(\theta_1 - \mu)^2}{2v_0}} d\theta_1.
\]

From the same steps as for the non-cooperative case we get that to the Gaussian integral in (28). Letting \( a = 2\rho \) and \( b = 2\bar{\theta}\), then yields:

\[
2\mathbb{E}_0 \left( -e^{-\rho \Phi^C(\theta, 0)} \right) = -2\kappa_C e^{-\rho \Phi^C_1,1},
\]

where

\[
\Phi^C_1,1 := -\frac{2(\mu_0 + \frac{\alpha \bar{\theta}}{v_0} - \mu_0^2)}{1 + 4\rho v_0} = -2\left( \frac{\delta_0 - \mu_0 + \frac{\alpha \bar{\theta}}{v_0} - \mu_0}{1 + 4\rho v_0} \right) = \Phi^C(\mu_0, v_0),
\]

\[
\kappa_C := \frac{1}{\sqrt{1 + 4\rho v_0}}.
\]

Remarkably, we have that \( \Phi^C_{1,1} = \Phi^C(\mu_0, v_0) \). The marginal value of information for the cooperative is thus

\[
\Delta U^C_0 = -2\kappa_C e^{-\rho \Phi^C_{1,1}} + 2e^{-\rho \Phi^C_{0,0}} = 2(1 - \kappa_C)e^{-\rho \Phi^C_{0,0}},
\]

where \( \Phi^C_{0,0} := \Phi^C(\mu_0, v_0) \).

**Proof of Proposition 6**

To prove the claim in the proposition, we need to show that \( \Delta U^C_{0NC} > \Delta U^C_0 \) (recall that \( \theta_1 = \theta_2 \)) or

\[
-\kappa_{NC}e^{-\rho \Phi^C_{1,1}} + e^{-\rho \Phi^C_{0,0}} > 2(1 - \kappa_C)e^{-\rho \Phi^C_{0,0}}.
\]

where we have used the fact that \( \Phi^C_{1,1} = \Phi^C_{0,0} \). This leads to the two following conditions which are sufficient for the claim:

\[
-\kappa_{NC}e^{-\rho \Phi^C_{1,1}} + 0.5e^{-\rho \Phi^C_{0,0}} > 0
\]

\[
0.5e^{-\rho \Phi^C_{0,0}} > 2(1 - \kappa_C)e^{-\rho \Phi^C_{0,0}}.
\]

The first condition ensures that the left-hand side in original condition (29) is positive and the second condition ensures that it is then in fact greater than the right-hand side.

We first show that the first condition in (30) is true. To begin, note that we can write an exponential function using the Taylor series expansion as \( e^x = 1 + x + \cdots + (1/n!)x^n + \cdots \). In our case we have two variables \( x = -\rho \Phi^C_{1,1} \) and \( y = -\rho \Phi^C_{0,0} \) and for large enough \( \bar{x} \), say \( \bar{x} > \bar{g} \), both \(-\rho \Phi^C_{1,1} \) and \(-\rho \Phi^C_{0,0} \) are strictly positive (from the definitions above). It then follows that we can write the first condition in (30) as

\[
1 - \kappa_{NC} + (0.5y - \kappa_{NC}x) + \cdots + \frac{1}{n!} (0.5y^n - \kappa_{NC}x^n) + \cdots > 0.
\]

To argue that the first order expansion is positive, first observe that \( 1 - \kappa_{NC} > 0 \) always. To argue that \( (0.5y - \kappa_{NC}x) = \kappa_{NC} \Phi^C_{1,1} - 0.5\Phi^C_{0,0} > 0 \), we write it as a quadratic equation:

\[
A_1(\rho, v_0) \bar{\theta}^2 + B_1(\mu_0, \rho, v_0)\bar{\theta} + C_1(\mu_0, \rho, v_0),
\]

where

\[
A_1(\rho, v_0) = \frac{\rho v_0 (1 + \rho v_0)}{(1 + 2\rho v_0)^2} - \frac{2\rho v_0 \kappa_{NC}}{1 + 3\rho v_0}.
\]

\[
B_1(\mu_0, \rho, v_0) = \frac{(1 + \rho v_0) \mu_0}{(1 + 2\rho v_0)^2} - \frac{2\mu_0 \kappa_{NC}}{1 + 3\rho v_0},
\]

\[
C_1(\mu_0, \rho, v_0) = -\frac{(0.75 + \rho \mu_0) \mu_0^2}{(1 + 2\rho v_0)^2} + \frac{3\mu_0^2 \kappa_{NC}}{2 + 6\rho v_0}.
\]
Now, $A_1(\rho, \nu_0)$ is positive if and only if
\[
(1 + \rho \nu_0)(1 + 3 \rho \nu_0)^{1/3} - 2(1 + 2 \rho \nu_0)^2 > 0,
\]
where we have substituted $\kappa_{\text{NC}} = 1/(1 + 3 \rho \nu_0)^{0.5}$ and simplified the resulting inequality. Because the highest order term is positive, there must exist $\hat{\rho}_1$ such that $A_1(\rho, \nu_0) > 0$ for $\rho > \hat{\rho}_1$. This then implies that there must also exist a cutoff $g_1$ such that for $g > g_1$ we have $\kappa_{\text{NC}} \Phi_{1,1}^{\text{NC}}(0.5) \nu_0 > 0$ (the highest order term in $\Phi$ is positive). We still need to argue that the remaining Taylor series terms are positive (positive remainder), to do that observe that we can write $0.5 \kappa_{\text{NC}}^{-1} g > 0$ as $0.5^{1/n} g > \kappa_{\text{NC}}^{-1} x$. Writing this out gives:

\[
\kappa_{\text{NC}}^{-1} \Phi_{1,1}^{\text{NC}} - 0.5 \Phi_{0,0}^{0,0} > 0.
\]

This condition is almost identical to the one we had for the first order expansion. We can write it as a quadratic equation:

\[
A_n(\rho, \nu_0) + B_n(\mu_0, \rho, \nu_0) + C_n(\mu_0, \rho, \nu_0),
\]

where

\[
A_n(\rho, \nu_0) = \frac{2 \rho \nu_0 (1 + \rho \nu_0) 0.5^{1/n}}{(1 + 2 \rho \nu_0)^2} - \frac{2 \rho \nu_0 \kappa_{\text{NC}}^{1/n}}{1 + 3 \rho \nu_0},
\]

\[
B_n(\mu_0, \rho, \nu_0) = \frac{2 \mu_0 (1 + \rho \nu_0) \mu_0 0.5^{1/n}}{(1 + 2 \rho \nu_0)^2} - \frac{2 \mu_0 \kappa_{\text{NC}}^{1/n}}{1 + 3 \rho \nu_0},
\]

\[
C_n(\mu_0, \rho, \nu_0) = - \frac{(1.5 + 4 \rho \nu_0) \mu_0 0.5^{1/n}}{(1 + 2 \rho \nu_0)^2} + \frac{3 \mu_0^2 \kappa_{\text{NC}}^{1/n}}{1 + 3 \rho \nu_0}.
\]

Now, $A_n(\rho, \nu_0)$ is positive if

\[
0.5^{1/n} (1 + \rho \nu_0)(1 + 3 \rho \nu_0)^{1/n} - (1 + 2 \rho \nu_0)^2 > 0.
\]

Because the highest order term is positive, for any $n \in \mathbb{N}$, we can find $\hat{\rho}_n$ such that for $\rho > \hat{\rho}_n$ the inequality holds. This then implies that there must exist a cutoff $\hat{g}_n$ such that for $g > \hat{g}_n$ we have $\kappa_{\text{NC}}(\Phi_{1,1}^{\text{NC}}(0.5) \nu_0 > 0).$ To have that all $n$th order Taylor expansions are positive we can pick $\rho_n = \max(\hat{\rho}_1, \ldots, \hat{\rho}_n)$ and $g_n = \max(\hat{g}_1, \ldots, \hat{g}_n)$. To conclude the proof for the first inequality in (30), note that the remainder for the Taylor series is at least $\epsilon_n = (y_{n+1} - x_{n+1})/n!$ (from both exponential functions). Given $\rho > \rho_n$ and $\Phi > \Phi_n$, the $n$th order Taylor series expansion sums to, say, $\Delta_n > 0$. We are done when $\epsilon_n > 0$ or we find $n$ such that $\Delta_n > \epsilon_n$. Now note that for us $n = 1$ is enough because letting $\rho > \rho_2$ and $\Phi > \Phi_2 = \max(\Phi_2, \Phi_3, \ldots) \implies \Delta_1 > 0$ and $\epsilon_1 > 0$ from the arguments above.

We then show that the second inequality in (30) is true. This is equivalent to showing that $\Phi_{0,0}^{C} - \Phi_{0,0}^{NC}$ (multiply both sides with $\exp(-\rho_0 \Phi_{0,0}^{C})$) is increasing in $\Phi$ so that the there exists a cutoff $g_d$ such that the inequality is true for $g > g_d$. Writing the difference $\Phi_{0,0}^{C} - \Phi_{0,0}^{NC}$ as a quadratic equation in $g$ gives

\[
H(\rho, \nu_0) + I(\mu_0, \rho, \nu_0) + J(\mu_0, \rho, \nu_0),
\]

where

\[
H(\rho, \nu_0) = \frac{0.125 \rho^2 \nu_0^2}{(\rho \nu_0 + 0.25)^2},
\]

\[
I(\mu_0, \rho, \nu_0) = \frac{0.125 \mu_0 \nu_0}{(\rho \nu_0 + 0.25)^2}.
\]

\[
J(\mu_0, \rho, \nu_0) = \frac{0.03125 \mu_0^2 \nu_0}{(\rho \nu_0 + 0.25)^2}.
\]

Because $H(\rho, \nu_0) > 0$ the difference $\Phi_{0,0}^{C} - \Phi_{0,0}^{NC}$ must ultimately be increasing in $\Phi$ so that the cutoff $g_d$ exists. This completes the proof for $\Delta U_{0,0}^{NC} > \Delta U_{0,0}^{C}$ and thus that there is more investment in the non-cooperative equilibrium when $\rho > \hat{\rho} = \rho_2$ and $\Phi > \Phi = \max(g_0, g_2, g_3)$. □
References