

DIFFERENT FACETS OF PRE-SERVICE TEACHERS' BELIEFS ON THE HISTORY OF MATHEMATICS

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ABSTRACT

The study ÜberLeGMa examined the beliefs of 141 mathematics pre-service teachers on the structure of mathematics, the history of mathematics, and the teaching and learning of the history of mathematics. The paper presents the key findings of the study, including statements on the structure and distribution of these beliefs and their relationships. The aim of the paper is to provide empirical foundations for studying higher education learning opportunities in this area and to formulate recommendations for incorporating appropriate learning arrangements into teacher education.

1 Introduction

At many universities, mathematics pre-service teachers study the history of mathematics in addition to subject-related, didactic and pedagogical learning content during their study. Although benefits to the so-called HPM perspective have been raised (Clark, Kjeldsen, Schorcht & Tzanakis, 2018), there is widespread consent among mathematicians and mathematics educators that the inclusion of the historical, philosophical, and developmental context of mathematics in the university curriculum provides prospective teachers with learning opportunities that can enrich the teachers' future pedagogical practice (Mosvold, Jakobsen, & Jankvist, 2014, Burns, 2010, Smestad, 2011). For example, the AMTE recommend in its standards for teacher education in mathematics (2017, p. 38): "Well-prepared beginning teachers of mathematics realize that the social, historical, and institutional contexts of mathematics affect teaching and learning and know about and are committed to their critical roles as advocates for each and every student."

Universities offer different ways of dealing with mathematical-historical content during teacher education. Study content can be anchored in seminars or lectures in the mathematics teacher education program for primary and secondary level as well as for upper secondary level.

From the perspective of evidence-based teacher education, however, the question of impact remains open. It has not yet been sufficiently clarified how pre-service teachers perceive these study contents and to what extent the university learning opportunities contribute to professionalization processes of the teachers. Specifically, the question arises as to whether pre-service teachers later refer to mathematics historical references in their pedagogical practice and are prepared to address them explicit-

ly in the classroom, or whether the study content as a whole has only little sustainability.

Previous research into the integration of historical references in mathematics education and in mathematics teacher education is mainly concerned normatively with the questions of why historical references are meaningful and how these references can be addressed in teaching (Clark, Kjeldsen, Schorcht & Tzanakis, 2018, Clark, Kjeldsen, Schorcht, Tzanakis & Wang, 2016, Fauvel & van Maanen, 2002). With regard to empirical work, empirical task analyzes by Schulte (2016) or Schorcht (2018) at the level of textbooks used in practice have been able to identify various types of tasks that allow for the integration of historical references into mathematics education. A wide range of international empirical studies also address teachers' perceptions of mathematics history in the classroom (Alpaslan, Işıksal, & Çiğdem, 2014, Bütüner, 2018, Burns, 2010, Charalambous, Panaoura, & Philippou, 2009, Furinghetti, 2007, Goodwin, 2007, Ho, 2008, Jankvist, 2010, Philippou & Christou 1998, Smestad, 2011). Although the empirical research on the history of mathematics in the classroom suggests that the embedding of historical references has advantages (Furinghetti, 2007, Glaubitz, 2011), the meaning and significance of the content of history of mathematics is an issue of discussions in mathematics teacher education (Nickel, 2013).

Against the backdrop of the issue of impact, the aim of our article is to create empirical foundations for a discussion of study content on the history of mathematics and to create opportunities for empirically analyzing the impact of university learning arrangements in this area. As part of our study *ÜberLeGMa* (“Überzeugungen von Lehramtsstudierenden zur Geschichte der Mathematik” – “Beliefs of pre-service teachers on the history of mathematics”, Schorcht & Buchholtz, 2015), we have developed an instrument to analyze pre-service teachers' beliefs on the history of mathematics and the teaching and learning of history of mathematics. With the help of this instrument, we have collected the beliefs of 141 pre-service teachers and examined their connection to further beliefs.

We hope that colleagues working in teacher education in the history of mathematics will use this tool to examine the impact of seminars or learning activities in history of mathematics. Ideally, we hope that whatever university learning opportunity in the history of mathematics will result in pre-service teachers becoming open to this study content and willing to address it later in the classroom. In the ongoing didactic discussion, normative ideas and demands can thus be systematically substantiated with empirical results – at least on the level of beliefs of pre-service teachers.

2 Theoretical Framework

2.1 Strategies for addressing mathematics history in the classroom

In terms of dealing with history of mathematics, teachers use two perspectives on mathematics: on the one hand, developments of mathematics can be focused as a product and, on the other hand, as a process of changes. In this section, we describe these perspectives and their impact on integrating history of mathematics in education. Therefore, we take an insight view on studies and thoughts about different perspectives on history of mathematics and its integration in courses.

Furinghetti (2007) states in her study that prospective teachers (15) used two different modes during a course of 42 hours in a teacher program to integrate historical sources into mathematics teaching. In the first mode, prospective teachers pursued the goal of clarifying mathematical concepts through their genesis over time. Hereby, the product of the concepts at the end of this process are used as a starting point to create a path of development. Furinghetti (2007, p. 137) calls this mode “evolutionary”. In the second mode, the prospective teachers used selected original sources of history of mathematics from a writer or author to situate the foundations of a concept in its particular historical situation. Thus, the cognitive origins are traced back to historical roots and thus prevent adherence to the product of a completed genesis of a concept. This mode is called “situated” (Furinghetti, 2007, p. 137). Prospective teachers therefore use the history of mathematics to either present a current process in its historical genesis or to introduce students to mathematical thinking based on a mathematical-historical example.

Similar conclusions states Lakoma (2002, p. 28 f.) for Polish textbooks, but calls these two modes “discursive style” or “dogmatic style”. She argues that the choice of mode depends on authors’ beliefs about teaching and learning mathematics. Depending on which goals are pursued in lessons containing the history of mathematics, tasks can contribute to the mediation of a mathematical discourse culture or emerge as a “set of curious details” for the purpose of motivating the learners.

Summarizing this might lead us to the assumption that pre-service teachers therefore might use history of mathematics to either present a current process in its historical genesis or to introduce students to mathematical thinking based on a mathematical-historical example. From this and other empirical studies on the beliefs of teachers and pre-service teachers of mathematics, it is known that mathematics teachers either understand mathematics as a static, rather un-changeable product or understand mathematics as a dynamic, continuous process of change initiated by human mathematical activities (Blömeke, Kaiser, & Lehmann, 2010, Voss, Kleickmann, Kunter, & Hachfeld, 2013). The exciting question that emerges from this finding is how views on mathematics are related to the use of history of mathematics in education.

Studies by Buehl, Alexander, and Murphy (2002) or Hofer and Pintrich (1997) show this dependency between epistemological beliefs and teacher-student interaction in the classroom. Previous studies on the history of mathematics in teaching do not fully address this notion and focus, e.g. rather the methodical use of mathematics history. However, relations between the methodical use and the mathematical worldview of teachers could be identified (Alpaslan et al., 2014, Furinghetti 2007, Goodwin, 2007).

2.2 Reasons for and against mathematics history in the classroom

There are various reasons for the use of history of mathematics in the education. Bütüner (2018, p. 9) presents a list of these justifications as a synopsis of his literature research, which is briefly described below. This is to demonstrate the possible intentions of teachers to use history of mathematics in the classroom (see Fried, 2001, Liu, 2006, Tzanakis & Arcavi, 2002). Mathematics history should accordingly

- (1) make students clear that mathematics is a human activity and a human product,

- (2) increase motivation and positive attitude towards mathematics,
- (3) open up perspectives on the nature of mathematics to students and broaden the subject-didactic repertoire of teachers,
- (4) provide a deeper understanding of mathematical concepts, problems and solutions.

This list includes *cognitive*, *affective*, and *evolutionary* justifications. The students should be cognitively demanded with regard to mathematical or mathematical-historical content in order to expand their mathematical discourse skills. Likewise, they should be motivated by the history of mathematics to deal with a specific subject area, and thus to understand the fundamental change processes of mathematics influenced by protagonists, culture, or social environment.

On the other side, studies by Ho (2008) or Panasuk and Horton (2012) revealed barriers of teachers to using history of mathematics. The teachers interviewed in both studies most often cited insufficient training in dealing with the history of mathematics as an obstacle to the integration of mathematical-historical topics. Likewise, teachers do not use mathematics history because they try to use the time spend in class for other content. Teachers also report that mathematics history can confuse students. In the end, mathematics history can be considered history and therefore is not content of mathematics education. In addition to these critical arguments, some teachers also mentioned inadequate opportunities for assessment or a lack of teaching materials.

In their articles, Tzanakis and Arcavi (2002) and Siu (2006) put together lists of obstacles that teachers encounter in the classroom if they want to integrate the history of mathematics. Tzanakis and Arcavi distinguish *philosophical* and *practical* objections. They add to philosophical objections the ontological distinction between mathematics and history, which leads to a prioritization of mathematical learning content by teachers, insufficient historical prior knowledge of students and their lack of motivation, as well as the danger of having history can help cultivate cultural chauvinism and narrow-minded nationalism. As a practical objection, Tzanakis and Arcavi, add the inadequate expertise of teachers, which is related to a lesser self-concept with regard to mathematical-historical content (Tzanakis & Arcavi, 2002, p. 203).

Siu (2006) describes a similar list, but differentiates it further. He additionally refers to teachers' doubts about the value of mathematics history for mathematics education. Overall, the discussion on the use of mathematics history involves affirmative and negative beliefs. Affirmative beliefs are based on affective, cognitive, or evolutionary benefits, while negative beliefs include practical or philosophical objections. Following the findings of Buehl et al. (2002), Furinghetti (2007), Goodwin (2007), Hofer and Pintrich (1997) or Lakoma (2002), these reasons show this two-fold nature, depending on the epistemological beliefs teachers have in their field.

2.3 Research on Teachers Beliefs

The prospect that study content on the history of mathematics is used in the classroom ties in with the hope that university learning opportunities can change the beliefs of pre-service teachers in such a way that they thematize mathematics history in teaching. This hope is supported by the existing assumption for the field of research on teacher actions that the application of professional knowledge in context situations is

only successful if there are corresponding subjective beliefs among the teachers. Beliefs are given an orienting and action-guiding function for the application of learned content (Ernest, 1989, Schmotz, Felbrich, & Kaiser, 2010, Schoenfeld, 1998, 2010, Thompson, 1992).

Despite intensive research into beliefs of teachers, especially in the context of pedagogical-psychological research, there is yet no clear and precise definition of the concept of beliefs (see, for example, Pajares, 1992). Richardson (1996) therefore proposes an area-unspecific definition of beliefs, based on a broader understanding. He understands beliefs as “psychologically held understandings, premises, or propositions about the world that are felt to be true” (Richardson, 1996, p. 103). Following Richardson, we also understand beliefs as subjective opinions and attitudes of a person to an object, which also include affective stances and the willingness to act (see Grigutsch, Raatz, & Törner, 1998). With regard to the long-term development of beliefs, it can be assumed that they are relatively stable to restructuring and, to a certain extent, can act as psychological “filters” and / or “barriers” (Reusser & Pauli, 2011). On the other hand, however, justifications for beliefs may change in the professional development of teachers (Eichler & Erens, 2015). For mathematics teachers, however, despite the blurring of the term, there is a broad consensus on the differentiation of professional beliefs (Ernest, 1989). It is assumed that beliefs can be domain-specific (Eichler & Erens, 2015, Törner, 2002) or even situation-specific (Kuntze, 2011, Schoenfeld, 2010). In addition to epistemological beliefs on the structure of mathematics (see, Grigutsch et al., 1998), beliefs on the acquisition of mathematical knowledge or on the teaching and learning of mathematics (Buchholtz & Kaiser, 2017, Handal, 2003, Kuntze, 2011, Staub & Stern, 2002) are other important dimensions of epistemological beliefs.

For the study of beliefs on the history of mathematics, it made sense to expand these existing dimensions by further specific beliefs. To this end, in the present study, based on the above-described theoretical framework on the history of mathematics and its use in the classroom, tools have been developed that specifically capture beliefs about the history of mathematics and the teaching and learning of history of mathematics. Similar to the beliefs on the structure of mathematics, the beliefs on the history of mathematics include both static and dynamic perspectives (Buehl et al., 2002, Furinghetti, 2007, Goodwin, 2007, Hofer & Pintrich, 1997, Lakoma, 2002).

Static perspectives include, for example, the assumption that mathematical findings are axiomatic and therefore have ideal or eternal existence. Mathematics is understood as a perfect logical and consistent system. This view can also suggest an anecdotal understanding of the history of mathematics, limited to the narrative of the work of eminent personalities or their biographies. However, the perspective frequently neglects that mathematical findings are often the subject of disputes, and without questioning mathematical theorems, it would hardly be possible to uncover contradictions and initiate further developments.

Dynamic perspectives on the history of mathematics emphasize this aspect in particular. They have a critical attitude to mathematical findings and do not exclude that today's mathematics can be questioned and further developed. On the other hand, they regard mathematics as an intellectual creation of humans in their respective historical

and cultural context and see the origins of mathematical thinking in a strong reference of the discipline to every day's life.

The beliefs on the teaching and learning of history of mathematics pick up the different justifications described above (see Bütüner, 2018; Siu, 2006; Tzanakis & Arca-vi, 2002).

Affirmative beliefs are fed by affective, cognitive or evolutionary reasons, emphasizing the motivational nature of mathematics history in the classroom, the cognitive added value of using the history of mathematics in class, or overall a processual image of mathematics. Teachers who share these benefits use mathematics history to engage students in a mathematical subject area and get to know the genesis of and relations between mathematical content. Here, especially historical references, which can motivate the students, play a role. Mathematics should be understood as a human product and provide a specific perspective on the nature of mathematics. Overall, these references should favor a positive attitude towards mathematics.

Negative beliefs refer to philosophical or practical objections. Teachers who share these reasons tend to reject mathematics history in the classroom. They see the high complexity, the low motivation and the lack of learning prerequisites of the students as well as the high time pressure as obstacles to thematize relevant content in the classroom. It can be assumed that dynamic beliefs about mathematics and constructivist teaching-learning beliefs are related to a more strongly emphasis of a process-based, iterative operation with mathematics in lesson design (Reusser et al., 2011). For this reason, we suspect that open-mindedness to incorporating historical aspects in mathematics education is most likely to be found among teachers with appropriate dynamic beliefs. Conversely, in the field of mathematics history, however, one could argue, that teachers with a more static belief of mathematics might also be open to historical references in mathematics teaching, because these put the spotlight on the universal and eternal validity of mathematical theorems.

Ultimately, this raises the empirical question of how teachers' different facets of beliefs are interrelated and whether convergent or more differentiated structures can be identified between beliefs on the structure of mathematics, the history of mathematics, and the teaching and learning of history of mathematics. In the present study, these structures could be examined empirically, at least at the level of pre-service teachers.

2.4 Research questions

Existing research on the beliefs of teachers refers to a differentiated image of beliefs in the field of mathematics and mathematics as a school subject. For instance, research describes beliefs on the origin of mathematical knowledge and the nature of mathematical problems (Grigutsch et al., 1998, Törner, 2002). Empirical studies highlight the importance of such beliefs in teaching (Buehl et al., 2002, Hofer & Pintrich, 1997, Staub & Stern, 2002). Despite this research background, the research discussion lacks of empirical findings in the area of teachers' beliefs on history of mathematics. Most of studies in this area are either normative studies on the epistemological foundations of teachers' beliefs on history of mathematics (e.g, Siu, 2006, Tzanakis & Arca-vi, 2002). Other, more qualitative case studies engage with students' learning processes in classroom contexts about history of mathematics (e.g, Chorlay, 2016, Glau-

bitz, 2011, Jankvist, 2009). A third kind of studies analyses historical documents, textbooks and teacher materials for didactical implications (e.g. Biegel, Reich & Sonnar, 2008, Clark, et al., 2018, Clark et al., 2016, Fauvel & van Maanen, 2002). The aim of our study, and thus of this article is therefore, to deepen the research findings on teachers' beliefs on history of mathematics in education and to support the discussion with empirical results. Our research questions are:

- 1) What kind of beliefs on mathematics, on history of mathematics and on teaching and learning of history of mathematics have pre-service teachers?
- 2) How are their beliefs on the history of mathematics related to their epistemological beliefs about mathematics, and what relations exist to their beliefs on teaching and learning of history of mathematics?

3. Methodology

The following section outlines the method of data collection. The instrument is further provided as a whole in the appendix.

3.1. Sample

By means of an online survey in the summer term 2015 and in the winter term 2015/2016, the study examined the beliefs of 159 German mathematics pre-service teachers studying both for primary and secondary level. For the administration of the survey, pre-service teachers at various universities received a link via email. As is the case with studies in tertiary education, it is not uncommon to gather different sized subsamples at individual universities due to limited access to the field. Participation in the study was voluntary and the data was collected anonymously. Eighteen pre-service teachers had to be excluded from our analysis due to missing values or unfinished surveys. Overall, the study is based on a sample of 141 pre-service teachers from nine universities: University of Hamburg (6), Justus Liebig University Giessen (12), University of Siegen (11), University of Wuppertal (37), Technical University Dresden (28), University of Kassel (6), University of Vechta (1), University of Bielefeld (1), Technical University of Dortmund (39). On average, the pre-service teachers were about 24 years old with a standard deviation of slightly more than 4 years, studying mostly in the 6th semester (with a relatively large span from the 1st to the 33rd semester) and predominantly female (111 pre-service teachers, 79%). The evaluation of the study degree of the pre-service teachers showed a differentiated picture of the sample. The vast majority of pre-service teachers (108, 77%) were studying for primary or lower secondary level. 23 pre-service teachers (16%) studied for upper secondary or vocational level and 10 students (7%) studied for special needs education.

3.2 Instrument

Pre-service teachers' beliefs about mathematics, the history of mathematics, and the teaching and learning of history of mathematics were collected in a survey using three scales. With reference to empirical research on attitudes and beliefs (Grigutsch et al., 1998), and empirical and theoretical work on beliefs on the history of mathematics (Alpaslan et al., 2014, Siu, 2006, Tzanakis & Arcavi, 2002), we developed scales on the history of mathematics (26 items) and the teaching and learning of the history of

mathematics (21 items) as part of the piloting of the study. An already existing component of the instrument consisting of 12 items was questions on beliefs about the structure of mathematics (Grigutsch et al., 1998). For all scales, pre-service teachers should indicate their agreement on a five-point Likert scale (1 = "strongly disagree" to 5 = "strongly agree"). Table 1 gives an overview of the descriptive statistics and reliabilities of the developed scales and illustrates them with example items. Appendix A and B shows the developed scales.

With the help of confirmatory factor analyzes (CFA), the assumed factor structure was then empirically examined individually for each dimension of the beliefs. The models were specified in the form of structural equation models.

4. Results

4.1 Beliefs on the structure of mathematics

In essence, we were able to replicate the four-factor solution of beliefs about the structure of mathematics based on the work of Grigutsch et al. (1998). We identified the factors *formalism*, *application*, *process*, and *scheme orientation* (see Table 1), where formalism and scheme orientation represent static perspectives and application and process dynamic perspectives. The model had an acceptable to good fit ($\chi^2/df = 1.85$, RMSEA = 0.07, CFI = 0.91, SRMR = 0.06). In addition, we identified significant correlations (Fig. 1) between the factors already revealed by Grigutsch et al. (1998). Since these are correlations at the latent level, the correlation coefficients were correspondingly a bit higher. Figure 1 shows how static beliefs, such as the formalism aspect and scheme orientation, are positively related and distinct from other related dynamic beliefs, such as the application and process aspects. Despite the fact that there were clear correlations between the factors, not all correlations between the individual factors were significant.

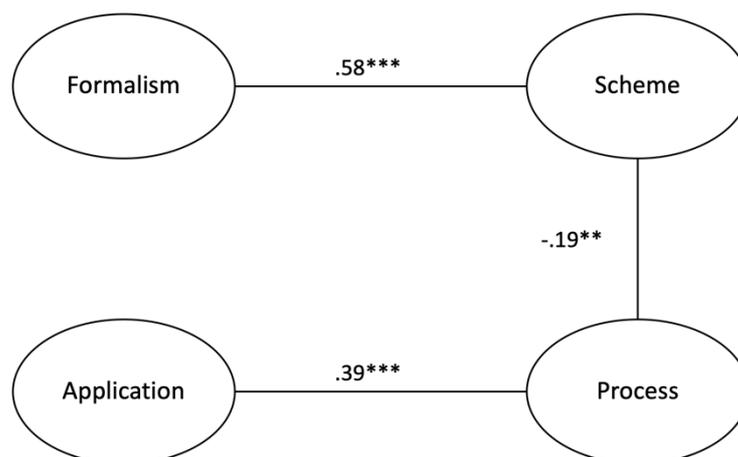


Fig. 1: Model for the beliefs on the nature of mathematics

From Table 1 it can be seen that the pre-service teachers on average agree slightly more with the dynamic beliefs than with the static beliefs, with the highest, average value (4.31) in the process-view, but the agreement with static beliefs is relatively high (> 3.38).

4.2 Beliefs on the history of mathematics

For the beliefs on the history of mathematics, the five-factor solution of different perspectives on the history of mathematics based on the assumptions could empirically be confirmed (see Fig. 2).

The *protagonist view* includes items that focus on the work of mathematicians or show how people have used mathematics in the past.

With the *perfectionist view*, pre-service teachers agreed with the statements that formulas have always played a significant role in mathematics, and that mathematics history describes a move toward perfect mathematics.

On the other hand, the *real-life view* focuses on the high everyday value of mathematics for humans. This includes the cultural significance of mathematics and application problems that arise within mathematical development.

With a *process-oriented view*, pre-service teachers see mathematics undergoing constant change. They would accept the refutation of today's valid mathematical knowledge, if it is proven wrong. Mathematics history shows accordingly that mathematical knowledge must constantly be questioned.

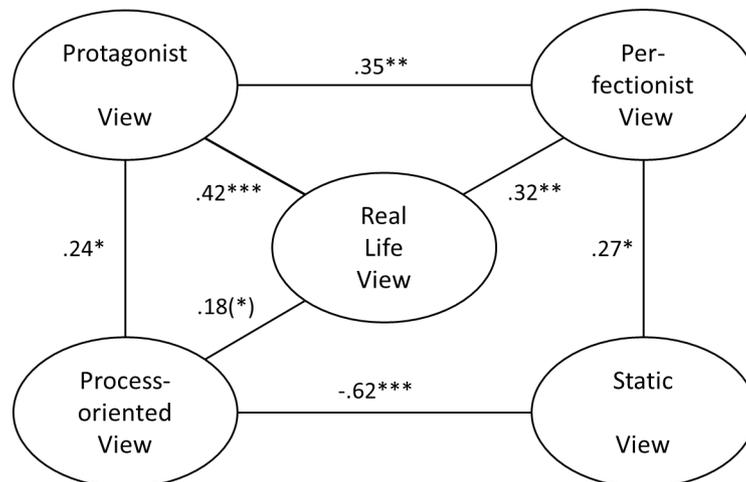


Fig. 2: Model for the beliefs on the history of mathematics

Within the *static view*, items are summed up that do not put possibilities to gain any significant insights past mathematics. The pre-service teachers agreed with the statements that there is only one “right” mathematics that has not changed over time. This view understands mathematics history essentially also as a collection of biographies.

The model in Fig. 2 fits well ($\chi^2/df = 1.33$, RMSEA = 0.05, CFI = 0.91, SRMR = 0.07). While the static view correlates negatively with the process-oriented view, this negative correlation does not apply to all dynamic beliefs – such as the real-life view.

There were still no significant relations between the protagonist view and the static view. Rather, the protagonist viewpoint is associated with both the beliefs of the perfectionist view and the real-life and process-oriented view of the history of mathematics. Interpreting these correlations, this could mean that the work of important personalities in mathematics is more strongly associated with the dynamic development of the discipline than with the creation of eternally valid theorems. The perfectionist view, on the other hand, correlates both with the protagonist view and the real-life

| Scale | Items | M | SD | Cronbach's α | Example items |
|---|-------|------|-----|---------------------|--|
| Beliefs on the structure of mathematics (Grigutsch et al. 1998) | | | | | |
| Formalism | 4 | 3,67 | ,62 | .77 | Fundamental to mathematics is its logical rigor and precision. |
| Application | 2 | 3,88 | ,61 | .63 | Many aspects of mathematics have practical relevance. |
| Process | 4 | 4,31 | ,49 | .70 | Mathematical problems can be solved correctly in many ways. |
| Scheme | 2 | 3,38 | ,80 | .68 | When solving mathematical tasks, you need to know the correct procedure, else you would be lost. |
| Beliefs on the history of mathematics (Schorcht & Buchholtz, 2015) | | | | | |
| Process-oriented View | 5 | 3,63 | ,62 | .78 | The history of mathematics shows us that mathematical knowledge must constantly be scrutinized. |
| Real-Life View | 4 | 3,96 | ,53 | .67 | The history of mathematics shows how people solved everyday problems with mathematics |
| Protagonist View | 4 | 3,64 | ,48 | .54 | The history of mathematics shows us the work of outstanding personalities. |
| Static View | 5 | 2,02 | ,58 | .71 | The history of mathematics shows that there is nothing new to explore in mathematics. |
| Perfectionist View | 2 | 2,99 | ,76 | .54 | The history of mathematics describes the path of mathematics towards a consistent system without contradictions. |
| Beliefs on the teaching and learning of history of mathematics | | | | | |
| Application reasons | 4 | 3,65 | ,68 | .76 | Historical references in mathematics lessons teach students the practical applicability of mathematical concepts. |
| Deepening reasons | 4 | 3,50 | ,73 | .81 | Historical references in mathematics lessons help students recognize interconnections between mathematical concepts. |
| Motivation reasons | 3 | 3,59 | ,77 | .75 | Students should learn historical references in mathematics education, because it also allows mathematically less interested students to learn mathematics. |
| Critical stance reasons | 2 | 3,59 | ,66 | .37 | In dealing with the erroneous paths of mathematics, students can develop a critical attitude towards mathematical insights. |
| Time reasons | 3 | 2,74 | ,80 | .78 | Historical references depend on many interpretations and require too much time in mathematics lessons. |
| Relevance reasons | 3 | 2,50 | ,76 | .76 | Students do not need to learn historical references in mathematics lessons because they usually are not subject to exams. |
| Complexity reasons | 2 | 2,44 | ,80 | .70 | Historical references to earlier errors and fallacies of mathematics only confuse students. |

Tab. 1: Scales and descriptive statistics

view, as well as with the static view. All in all, it was thus possible to identify a much more differentiated structure of the beliefs on the history of mathematics, which cannot simply be attributed to the distinction between static and dynamic beliefs. However, the relationships are only very cautiously interpretable, since some scales showed poor reliability (see Tab. 1). We attribute this to the difficulty of developing scales of very heterogeneous and nuanced content. Table 1 shows that the pre-service teachers agreed least on the history of mathematics in the static (2.02) and the perfectionist (2.99) view-points, while the agreement on the real-life view (3.96) is much higher.

4.3 Beliefs on the teaching and learning of history of mathematics

For the beliefs on the teaching and learning of history of mathematics, we here present a seven-factor solution that showed satisfying model fit ($\chi^2/df = 1.53$, RMSEA = 0.06, CFI = 0.93, SRMR = 0.05). Although the model had a good fit, very high correlations could be identified between the factors (multicollinearities), which makes it difficult to distinguish between individual belief facets. There was a clear correlation pattern between the three objections (time, relevance and complexity) and the four affirmative belief facets (application, deepening, motivation and critical stance). This suggests an underlying structure of higher order (Byrne, 2012), which we elaborated on elsewhere (Buchholtz & Schorcht, submitted). Note that the factor critical stance also showed unsatisfactorily reliability (see Tab. 1). However, we decided to take up this factor here and to present the whole seven-factor model in order to present our findings to a higher degree of detail and to better map the theoretical anchoring of the instrument (see Fig. 3).

Affirmative beliefs therefore contain four justification patterns: deepening reasons, motivation reasons, application reasons and critical stance reasons.

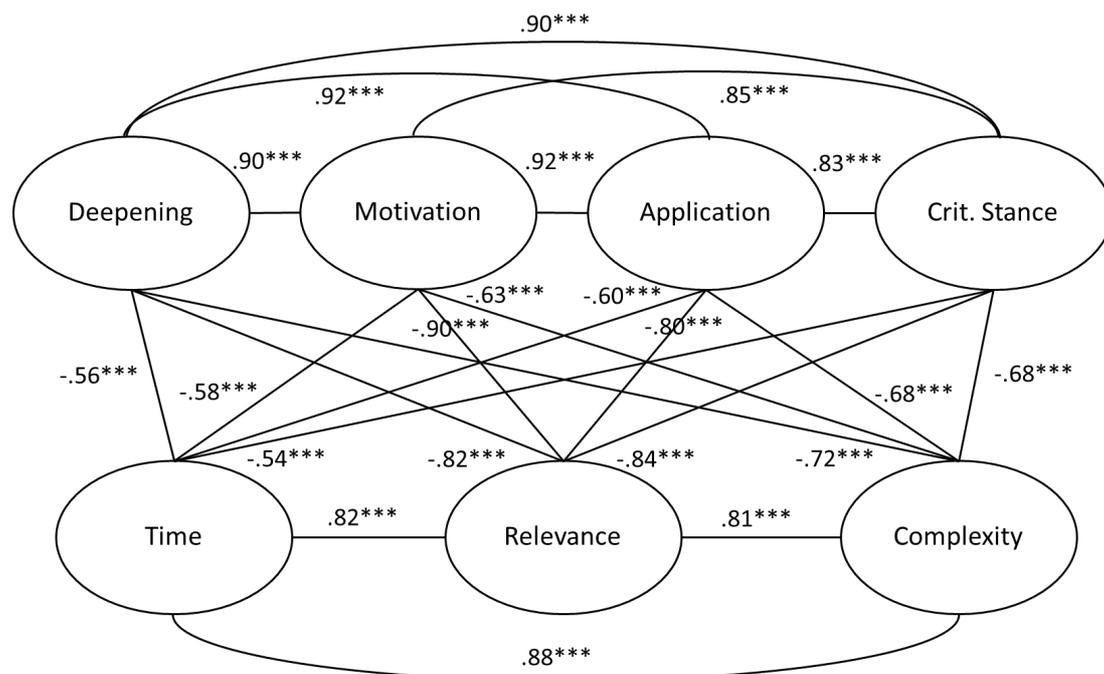


Fig. 3: Model for the beliefs on the teaching and learning of history of mathematics

Deepening reasons are a collection of beliefs that aim to help students understand the genesis and inter-connectedness of mathematical terminology in the classroom.

Reasons for justification, which are more likely to be attributed to *motivation*, point to the functional role of mathematics history in mathematics teaching like when used in problem-posing for a lesson or the motivation of less interested students.

The *application* reasons focus on the training of problem-solving skills. Pre-service teachers who agreed with these statements would claim to use mathematics history to clarify the applicability of mathematical concepts. The students would thereby better recognize a sense of their own learning.

Other reasons point to the evolution of a *critical stance* towards mathematics and the development of an inquiry-oriented mind-set.

Beliefs about objections also have three justification patterns: time reasons, relevance reasons, and complexity reasons.

Under *time* reasons, we find beliefs, which attach too much time to the treatment of mathematical historical references, which is therefore not available when teaching. At best, mathematics history is seen as a digression in the lesson and is considered time-consuming due to its complexity.

The *relevance* factor includes statements that assume that the history of mathematics is boring students and that the content of mathematical-historical references is not relevant to examinations. The knowledge of the historical development of a mathematical concept is therefore of little relevance as long as one knows the definition of the concept.

Under the *complexity* factor, pre-service teachers also regard mathematics history as too complex to handle in class. Especially, they agree on the statement that mathematical errors and fallacies, often mentioned in mathematics historical content, could rather confuse the learners than help to build up mathematical understanding.

In the seven-factor model, the very high significant correlations between all affirmative factors and between all factors of objections and the very high negative correlations between the four affirmative and the three factors of objections are striking. This belief pattern indicates that affirmative reasoning as well as objections patterns are closely related, but pre-service teachers tend to emphasize one or the other aspect of their beliefs in a complementary manner. Interestingly, the correlations between the time factor and the affirmative factors in the range of $-.54$ to $-.60$ are somewhat smaller than between the other two negative factors and the affirmative factors ($-.68$ to $-.90$). This may suggest that even though pre-service teachers see a benefit in the thematization of mathematics-historical content in the classroom, concerns about lack of time cannot be completely dispelled. However, particularly in the case of objections, the interpretation of a rather dichotomy between the reasons is supported by the descriptive statistics (see Table 1). For all facets without the exception of the critical stance factor we achieved good reliability, which could be because the factor only consists of two items. Interestingly, the factors of objections consistently averaged less agreement and a higher standard deviation, suggesting a larger divergence of pre-service teachers' answers. The lowest agreement could be identified in the complexity (2.44) and relevance aspect (2.50), and the application aspect (3.65) received the strongest agreement.

4.4 Relations between different dimensions of beliefs

In analyzing the relations of beliefs on the history of mathematics and beliefs on the structure of mathematics, we find significant correlations between the different facets, but with the exception of a mid-high correlation between real life view and application orientation (.49) all other significant correlations are relatively low (see Table 2).

| | Process-oriented View | Real-life View | Protagonist View | Static View | Perfectionist View |
|--------------------|-----------------------|----------------|------------------|-------------|--------------------|
| Formalism | n.s. | n.s. | .23** | n.s. | .32** |
| Application | .18* | .49** | .23** | -.22** | n.s. |
| Process | .22** | .41** | .27** | -.25** | .18* |
| Scheme | n.s. | n.s. | .23** | n.s. | n.s. |

Tab. 2: Relations between beliefs about the history of mathematics and the structure of mathematics

Between the dynamic beliefs on mathematics - application and process - and the process-oriented view and the real-life view on the history of mathematics we find relationships as expected. This also applies to the negative relations between the static view on the history of mathematics and the dynamic beliefs on mathematics. Interestingly, the protagonist view correlates significantly with all beliefs on the structure of mathematics, which can be taken as an indication that (anecdotal) beliefs about outstanding personalities and their work in the development of mathematics may be overarching beliefs that are independent from whether mathematics is perceived as a more logical-deductively ordered structure or as applied science. However, these relationships are only carefully interpretable here due to the poor reliability of the scale. Another interpretable result is that the belief that the history of mathematics witnesses the evolution of mathematics toward a perfect system is related to structural beliefs about formalism. Epistemological similarities such as the orientation towards (the development of) universal formulas and logical statements, which contain both facets of beliefs, are likely to be decisive here. Interestingly, however, formalistic beliefs and static beliefs on the history of mathematics (including the belief that mathematics does not change over time) are not related.

| | Process-oriented view | Real-life view | Protagonist view | Static View | Perfectionist View |
|------------------------|-----------------------|----------------|------------------|-------------|--------------------|
| Application | .23** | .45** | .29** | -.24** | .18* |
| Deepening | .27** | .41** | .29** | -.26** | n.s. |
| Motivation | .21* | .41** | .29** | -.22** | .17* |
| Critical Stance | .25** | .35** | .26** | -.25** | n.s. |
| Time | n.s. | -.24** | n.s. | .26** | n.s. |
| Relevance | n.s. | -.29** | -.16(*) | .26** | n.s. |
| Complexity | n.s. | -.28** | -.15(*) | .29** | n.s. |

Tab. 3: Relations between the beliefs on history of mathematics and the teaching and learning of history of mathematics

The analysis of the relations between beliefs on the history of mathematics and the beliefs on the teaching and learning of history of mathematics initially revealed a clear pattern (see Table 3).

The process-oriented view, the real-life view and the protagonist view (and at 10% significance level also the perfectionist view for some instances) are weakly to medium highly positively correlated to affirmative beliefs on the teaching and learning of history of mathematics. (see Table 3).

Partly - as in the real-life view and in the protagonist view - the beliefs on the history of mathematics are also negatively correlated with the negative beliefs about teaching and learning. On the other hand, the static perspective is positively related to the rather negative beliefs, although the size of the relationship does not differentiate between the aspects of justification. Overall, however, the correlations are only low to medium high here.

5. Discussion

Using confirmatory factor analyzes, our study has elucidated pre-service teachers' various beliefs about the structure of mathematics, as well as various views and justifications in the beliefs of history of mathematics and in the teaching and learning of the history of mathematics.

The results on the beliefs on the structure of mathematics (see 4.1) have already replicated well-known structural results with regard to static and dynamic beliefs of (pre-)service teachers on mathematics (Blömeke et al., 2010, Voss et al., 2013). However, static or dynamic beliefs on the structure mathematics do not translate clearly into beliefs on the history of mathematics. With regard to the beliefs on the history of mathematics, a more differentiated picture of different static and dynamic points of view was found, which are however interrelated. A process-oriented view of the history of mathematics, a real-life view, a protagonist view, a perfectionist view and a static view could be distinguished (see 4.2). The majority of the pre-service teachers in the sample clearly agree with the process-oriented and the real-life view on the history of mathematics, the static view of the history of mathematics receives the lowest approval. Among the reasons in favour or against the use of mathematics history in the classroom were affirmative as well as negative beliefs among the pre-service teachers. The affirmative beliefs for the teaching and learning of history of mathematics (see 4.3) included various justification patterns, such as application, motivation, deepening and critical stance reasons. The beliefs of objections were captured in three typical justification patterns: timing, relevance and complexity reasons.

Correlation analyzes of the different beliefs provided interesting insights into the structural relationships of the various facets. Overall, the majority of pre-service teachers support the use of the history of mathematics in the classroom. Our convergent findings show that the affirmative justifications for this advocacy are also related to dynamic views on the history of mathematics, which in turn are linked to dynamic views of the structure of mathematics. For the inclusion of the history of mathematics in the classroom, this means that in this case, pre-service teachers will use the history of mathematics in the classroom to emphasize the

dynamic aspects of mathematics and its history, and to provide a deeper understanding of mathematics. On the other hand, if pre-service teachers increasingly take a static view on the history of mathematics, this relates to their greater rejection of the use of mathematics history in teaching. However, this relationship seems to be independent of static beliefs about the structure of mathematics, such as the formalism aspect or schema orientation. Pre-service teachers with a static view reject mathematics history in the classroom because they think it is too complex for the students. In addition, they see the available time in the classroom as too tight, as that mathematics history could be integrated in addition. Also, the lack of relevance for exams is a justification for not use mathematics history.

Interestingly, we were able to identify differentiated relationships between static beliefs about the structure of mathematics and dynamic beliefs on the history of mathematics. Thus, beliefs on the formalism aspect and scheme orientation are related, albeit only weakly, to the protagonist view and the perfectionist view of the history of mathematics, but not to the static view of the history of mathematics. All in all, we infer from our results that prospective teachers specifically locate the reflection about people and outstanding figures in the history of mathematics within two different views on the history of mathematics, which are related differently to beliefs on the teaching and learning of history of mathematics and the structure of mathematics. On the one hand, people and their influence on mathematics are reflected within the protagonist view, with the focus then being on the mathematical work of the persons in their time. Appropriate beliefs are agreed on by pre-service teachers with all sorts of structural beliefs and are positively associated with affirmative beliefs about teaching and learning about the history of mathematics. On the other hand, pre-service teachers with a static view perceive mathematics history also as a pure collection of biographies (which may have anecdotal value for teaching at best). These pre-service teachers may not regard the knowledge of human achievements in the development of mathematics as very relevant, and accordingly, they might assess the importance of this knowledge as less important for teaching and learning of mathematics.

Overall, however, the results of the study ÜberLeGMa can only be interpreted with caution. Some scales showed poor reliability, so it seems appropriate to replicate the findings in further studies. Moreover, the small sample of pre-service teachers does not allow a generalization of the results. Since we did not forge any direct comparisons between universities, we refrained from displaying a site-specific presentation of the individual sub-samples, but it cannot be ruled out that results are distorted by the influence of locations with high numbers of pre-service teachers. In further follow-up studies our quantitative findings could be extended by additional qualitative studies. For example, new research questions arise about the relationships of beliefs or the impact of courses in history of mathematics on respective beliefs.

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APPENDIX A

26 Items: „Beliefs on the history of mathematics“

The history of mathematics shows that the basic mathematical findings are centuries old and have survived the time.

The history of mathematics shows us the work of outstanding personalities.

The history of mathematics for me is a collection of interesting anecdotes.

The history of mathematics testifies that formulas have always played a significant role in mathematics.

The history of mathematics shows us that mathematical knowledge must constantly be scrutinized.

The history of mathematics shows that mathematics has its origins in application problems.

How people used mathematics at their own time is shown by the history of mathematics.

Mathematics has also gone astray in the course of its development.

Mathematics history shows that mathematics is undergoing constant change.

The history of mathematics testifies to the development of mathematical ideas towards a perfect mathematics.

In the future, no fundamentally new mathematical findings will be discovered.

The history of mathematics shows us that you have to deal critically with mathematical findings.

History of mathematics is essentially a collection of biographies.

The history of mathematics shows the constant elimination of mathematical inconsistencies.

The history of mathematics shows that there is nothing new to explore in mathematics.

The history of mathematics shows that mathematics does not change over time.

The history of mathematics testifies that there is only one "correct" mathematics.

The history of mathematics shows how people solved everyday problems with mathematics.

Mathematics history shows that mathematical discoveries are eternally valid and unchangeable.

The history of mathematics illustrates the high everyday benefits that mathematics has for people.

The history of mathematics shows the high cultural significance of mathematics.

In the future, today's accepted mathematical findings could be discarded again.

History of Mathematics describes how people practiced math in their time.

The history of mathematics describes the path of mathematics towards a consistent system without contradictions.

The history of mathematics documents the constant progress in mathematics.

The history of mathematics describes mathematics as the spiritual creation of humanity.

APPENDIX B

21 Items: „Beliefs on the teaching and learning of history of mathematics“

Historical references in mathematics education contribute to an application-oriented image of mathematics.

In mathematics it is not only important to know a concept, but also its historical development.

At best, historical references can be used for a digression in the classroom.

Historical references in mathematics lessons teach students the practical applicability of mathematical concepts.

Historical references in mathematics classes can take away students' fear of the "scientific" mathematics.

Students should learn historical references in mathematics education, because it also allows mathematically less interested students to learn mathematics.

Dealing with the struggle for solutions to mathematical problems make students understand the meaning of their own learning.

Historical references are ideal as an introduction to a substantive mathematical topic.

Dealing with the history of mathematics trains problem solving abilities of students.

Dealing with historical references in mathematics teaching motivates students.

Historical references in mathematics lessons help students recognize interconnections between mathematical concepts.

Students will gain a deeper understanding of mathematical procedures as they see how they have changed over time.

By dealing with the historical genesis of mathematical concepts, they can be better memorized and understood.

The historical development of mathematics is too complex to handle in class.

Historical references depend on many interpretations and require too much time in mathematics lessons.

The knowledge of the historical development of a mathematical concept is of little relevance as long as one knows the definition of the concept.

In dealing with the erroneous paths of mathematics, students can develop a critical attitude towards mathematical insights.

Historical references in mathematics education are too time consuming.

Historical references in mathematics lessons bore students.

Students do not need to learn historical references in mathematics lessons because they usually are not subject to exams.

Historical references to earlier errors and fallacies of mathematics only confuse students.
