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Magdalena Kersting

# **General Relativity in Secondary School**

**Research-Based Development of Learning Resources and Analyses of Students'  
Conceptual Understanding Using the Model of Educational Reconstruction**

**Dissertation submitted for the degree of Philosophiae Doctor**

Department of Physics

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Alice: “Would you tell me, please, which way I ought to go from here?”

The Cheshire Cat: “That depends a good deal on where you want to get to.”

Alice: “I don’t much care where – ”

The Cheshire Cat: “Then it doesn’t much matter which way you go.”

— *Lewis Carroll, Alice in Wonderland*







# Preface

This thesis is submitted in partial fulfillment of the requirements for the degree of *Philosophiae Doctor* at the University of Oslo. The research presented here is conducted under the supervision of Ellen K. Henriksen, Maria Vetleseter Bøe, and Carl Angell. The thesis builds on a collection of three journal papers and two conference papers, presented in chronological order. The papers are preceded by chapters that provide background information and motivation for the work. These chapters introduce the research goals, present the theoretical and methodological approaches, and discuss and contextualize the findings to link the individual papers. One paper is joint work with Ellen K. Henriksen, Maria Vetleseter Bøe, and Carl Angell. One paper is joint work with Rolf Steier. I am the sole author of the remaining conference papers.

## Acknowledgements

In every field of creative activity, it is one's taste, together with ability, temperament, and opportunity, that determines one's style and through it one's contribution. That taste and style have so much to do with one's contribution in physics may sound strange at first, since physics is supposed to deal objectively with the physical universe. But the physical universe has structure, and one's perceptions of this structure, one's partiality to some of its characteristics and aversion to others, are precisely the elements that make up one's taste. Thus it is not surprising that taste and style are so important in scientific research, as they are in literature, art, and music.

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*Chen-Ning Yang*

As much as this thesis reflects my taste, ability, and temperament it is only because of opportunities I have been given and support I have received that I was able to develop my style and find my voice as a physics educator. I feel fortunate and privileged that I got to spend the last four years pursuing creative activities at the crossroads of science and education.

I am grateful to Ellen K. Henriksen, Maria Vetleseter Bøe<sup>1</sup>, and Carl Angell for having given me the opportunity to pursue this PhD-project as part of project ReleQuant. ReleQuant has offered me to learn and grow in numerous ways and it is hard to think of another project that would have allowed me to combine

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<sup>1</sup>Thanks for joining me for weekly Pilates classes, sharing nerdy content on Twitter, and getting excited about quantum field theory!

my interests in the same way. I would like to thank you for providing structures that have allowed me to thrive, for sharing your expertise and valuable feedback when I needed it, and for giving me the freedom to be creative and to explore many strands of research while growing as a physics educator.

I would like to thank the Norwegian Centre for Science Education for their support and for hosting *General Relativity* on the learning platform Viten. Specifically, I would like to thank Øystein Sørborg who has turned so many of our ideas into reality. You responded quickly, worked hard, and were just amazing at making *General Relativity* look nice and work flawlessly. Thanks to Tor-Martin Austad and Visual Lab who skillfully implemented our ideas and helped us visualize warped time. Thanks to the UniMedia team and Henning Vinjusveen Myhreagen for producing a fun and instructive slow-motion video of a falling bottle of water. Finding myself in a film studio was a welcome change from my office space and I thoroughly enjoyed the whole production process. Thanks to Ester Robstad who created wonderful and colorful graphics for our learning environment.

My work has been fueled by coffee and pastries and I would like to thank three colleagues with whom I have consumed copious amounts of both. First, I couldn't have asked for a better colleague than Rolf Steier. Thanks for showing me possibilities beyond the Department of Physics, thanks for providing me with plenty of awesome reading material, and thanks for always making me excited about our present and future projects. I've enjoyed every coffee and every slice of banana bread we have shared along the way. Second, I'd like to thank Henrik Galligani Ræder who bribed me with brioche and whisky and who has become both a colleague and dear friend. Thanks for having strong opinions and for not having dead cow eyes. Third, I wish to thank Marjan Zadnik for so many good conversations about our work and research but more importantly about science, art, literature, film, philosophy, and everything else that makes life worth living. Our weekly coffee chats at UWA were something I was looking forward to every Monday.

Sometimes, like-minded colleagues happen to work and live at the other end of the world. In Einstein-First and the EPER-collaboration I have found colleagues from Australia and around the world who have showed me how much you can achieve if you choose to collaborate across institutions and countries. First of all, I'd like to thank David Blair for his passion, his childlike curiosity, and his extraordinary ability to make things happen. The formation and development of the EPER-collaboration and not least the trajectory of my PhD-project owe much to your enthusiasm and relentless efforts. Thanks to Ruby Chan for doing her magic behind the scenes, for always being there when I needed help, and for having the most wonderful energy. Einstein-First and OzGrav are lucky to have you as their node administrator! Thanks to John Moschilla for providing some much needed inspiration during the early days of the EPER-collaboration and the early days of my work. Thanks to Warren Stannard for his enthusiasm,



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his stellar symposium contributions and for sharing food and drinks in Perth, Dublin, and Oslo. Thanks to Ron Burman for many insightful discussions and his patience in sharing his knowledge with the Einstein-First team. It's been a pleasure to work with Grady Venville who gave me the opportunity to pursue new projects in Australia. Your encouragement and positive outlook on our projects meant a lot. Meeting the OzGrav education and public outreach team was a most fortuitous encounter. Thanks to Jackie Bondell, Mark Myers, Lisa Horsley, and Carl Knox for being such an energetic, talented, and positive team of science educators, science communicators, content developers, and digital artists. I truly appreciated your hospitality and your willingness to share your perspectives and help me grow and learn during my visit in Melbourne.

There were many moments when I questioned this work and life in academia. I am grateful to Lars Hasvoll Bakke for having listened to my doubts, for having shared his valuable perspectives, and for having helped me understand Norwegian culture a bit better. Ich danke Ihnen. Pursuing a PhD presents one with many challenges, not only of the scientific kind. I would like to thank Rike Bostelmann for having been with me on this journey. Oceans, continents, and time zones have never stood in our way to support each other.

Finally, I would like to thank David for being there, for keeping me grounded, for making me laugh, and for having taught me to always start with why. Thanks for being the best, dummdidumm.

• **Magdalena Kersting**

Oslo, November 2019



# List of Papers

This thesis builds on the following original publications, which I refer to by their Roman numerals.

## Paper I

Kersting, M., Henriksen, E. K., Bøe, M. V., & Angell, C. (2018). General relativity in upper secondary school: design and evaluation of an online learning environment using the model of educational reconstruction. *Physical Review Physics Education Research*, 14(1), 010130-1-010130-18

## Paper II

Kersting, M., & Steier, R. (2018). Understanding curved spacetime - the role of the rubber sheet analogy in learning general relativity. *Science & Education*, 27(7), 593–623.

## Paper III

Kersting, M. (2019). Free fall in curved spacetime - how to visualise gravity in general relativity. *Physics Education*, 54(3), 035008.

## Paper IV

Kersting, M. (2019). Navigating four dimensions – upper secondary students' understanding of movement in spacetime, *Journal of Physics: Conference Series* 1287 012007

## Paper V

Kersting, M. (in press). How History and Philosophy of Science Can Inform Teaching and Learning of General Relativity in Upper Secondary School, *Proceedings of the Fifteenth Marcel Grossman Meeting on General Relativity*, World Scientific, Singapore

The following articles do not lie within the scope of this PhD-thesis because they either employ different theoretical and methodological perspectives or present popular scientific accounts of my work. Readers interested in teaching and learning of general relativity more broadly might find the following articles interesting nonetheless.

### **Paper A**

Steier, R., & Kersting, M. (2019). Metaimagining and embodied conceptions of spacetime. *Cognition and Instruction*, 37(2), 145–168.

### **Paper B**

Steier, R., Kersting, M., Silseth, K. (2019). Imagining with improvised representations in CSCL environments. *International Journal of Computer-Supported Collaborative Learning*, 14:109.

### **Paper C**

Kersting, M. (2018). General Relativity – Why high school students should learn about Einstein’s most revolutionary idea. Titan, University of Oslo. <https://titan.uio.no/node/2671>

### **Paper D**

Kersting, M. (2018). Navigating four dimensions. *Lateral Magazine*. <http://www.lateralmag.com/articles/issue-27/navigating-four-dimensions>

### **Paper E**

Kersting, M., Toellner, R., Blair, D., Burman, R. (in press) Gravity and warped time – clarifying conceptual confusions in general relativity. *Physics Education*.

# Abstract

The field of physics education aims to accomplish both research and practical goals: physics educators move between the world of research and scholarship and the world of school and teacher practice. This thesis takes the general theory of relativity as a learning domain to present physics education research that has contributed to both worlds. The first contribution of this PhD-project is a research-based digital learning environment about general relativity (GR) that was developed for physics students in upper secondary schools in line with the Norwegian curriculum. Second, by developing and evaluating the digital learning resources, the project has produced new research findings on student learning processes and their conceptual understanding in the learning domain of GR.

GR is Albert Einstein's theory of gravity that describes the deep link between space, time, and matter. GR is a major scientific theory of the 20th century that has shaped our modern physical and philosophical understanding of the Universe. Yet, teaching and learning of physics in schools remains mostly dominated by classical physics. In many countries, GR is not part of the physics curriculum; only few secondary school students worldwide learn about our current best description of gravity and gravitational astronomy. In countries where GR is taught in the school curriculum, such as in Norway, challenges remain to find suitable instructional approaches that make the abstract scientific theory teachable and understandable at the secondary school level. This PhD-project responds to the challenges of GR education through physics education research.

Even though physics educators and teachers have started to bring topics of GR to secondary schools, many of the investigations so far have only had an exploratory character. Field-tested and research-based material is rare and there is a scarcity of educational studies that report on the efficacy of these learning resources. Moreover, physics educators still do not know much about student learning processes and the ways students develop conceptual understanding in GR. To make teaching and instruction successful, there remains the need to approach GR education systematically and within a sound educational framework.

The objective of this thesis is to address this need in form of an educational reconstruction of GR in the context of the Norwegian educational system. More specifically, by employing the Model of Educational Reconstruction as its framework, this thesis reports, first, on the research-based development of a digital learning environment and, second, on investigations of student learning processes in GR. An educational reconstruction of a new learning domain requires grounding in a learning theory and specification of suitable methods. Sociocultural

theory is the learning theory that has informed this research project. Methods of design-based research have guided the research design. The findings of this thesis are two-fold and touch upon the world of scholarship and the world of school practice. First, I report on the design and development of a digital learning environment about GR that includes a content-structure of instruction and design principles to facilitate teaching and learning of GR. Second, I present findings on students' learning processes and conceptual understanding of key concepts in GR. Doing so, I demonstrate how a sociocultural perspective and a qualitative research approach can fruitfully inform modern science education research.

The design principles summarize 1) how GR relates to and sometimes breaks with classical physics; 2) how to counteract students' lack of experience with relativistic phenomena by linking students' lifeworlds to relativistic concepts; 3) how to draw on students' prevailing motivation and interest to introduce key concepts in GR; 4) how to use written and oral language to facilitate understanding of abstract concepts in GR.

Learning processes in GR are often characterized by a conflict between students' experiential understanding of gravity and the abstract description of GR. In order to conceptualize the physical mechanism of gravity in the domain of GR, students need to develop awareness of the tension between the physical force of gravity in the everyday experiential sense and the curved spacetime explanation. Awareness of the strengths and weaknesses of instructional analogies such as the popular rubber sheet analogy for curved spacetime can help students address their conceptual difficulties. More generally, my findings suggest that GR is a learning domain in which students can benefit from a teaching approach with a greater emphasis on the nature of science and scientific models, and in particular on the scope and limitations of such models.

All in all, findings of this research show that secondary school students were able to obtain a qualitative understanding of GR when provided with appropriately designed learning resources and sufficient scaffolding of learning through interaction with teachers and peers. Indeed, I argue that the time has come for educators and teachers to embrace GR as an important topic of modern physics education.

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# Chapter 1

## Prologue

After several unsuccessful attempts to weld my results together into such a whole, I realized that I should never succeed. (...) my thoughts were soon crippled if I tried to force them on in any single direction against their natural inclination. And this was, of course, connected with the very nature of the investigation. For this compels us to travel over a wide field of thought criss-cross in every direction.

---

*Ludwig Wittgenstein*

The main task of this thesis is to construct a convincing narrative that links my work and research projects of the past four years. To do so I draw on Aikenhead (1996) who likens students to tourists or travellers who traverse new landscapes when learning science. Yet, teachers and science educators in their roles as travel guides often find themselves in unknown territory as well. “Reading” new landscapes requires the ability to “see” the landscape in a special and disciplinary way that facilitates the generation of insightful understanding (Eriksson, 2019; Wylie, 2007). For the past four years, I have been both a guide and a tourist myself traveling over a wide field of thoughts. This thesis is an album that collects the landscape sketches I have made along the way.

The thesis starts with a general introduction and presents the two overarching research aims that set the stage for my PhD-project. I briefly summarize the key ideas of general relativity both from a scientific and an educational perspective. Contextualising my work, I present project ReleQuant as the greater educational project in which my PhD-thesis was situated. I frame my research by presenting the research questions that have guided the individual studies. In the theoretical framework, I introduce the learning theory and educational models that I chose to approach the research questions. In the method section, I describe the research design and give an account of the data collection and data analysis before presenting the learning environment *General Relativity*<sup>1</sup> that the Norwegian Centre for Science Education launched in January 2018. I then summarize the main findings of my research on student learning processes in the result section and discuss the results before drawing a final conclusion that casts my work into a greater educational context of the Einsteinian Physics Education Research (EPER) collaboration.

The field of science education is at the crossroads of different disciplines. Traditionally, science educators are at home in the sciences, the humanities, and

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<sup>1</sup>[www.viten.no/relativity](http://www.viten.no/relativity)

## 1. Prologue

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the social sciences. Naturally, these vast landscapes are too big to just stay in one place. I have been very fortunate to hold a PhD-position that has allowed me to explore many of these landscapes by reading and learning and writing and traveling. As a result, many of my investigations have concerned issues not directly related to the heart of my PhD-project, namely the educational reconstruction of GR. Thus, I have produced more sketches of landscapes than those that have made it into this thesis. I turned some of those additional sketches into research articles that employ different methods than those that I present in this thesis. Others ended up as popular science articles that present my investigations in a less formal way. Readers and travellers who are keen to explore the vast landscape of general relativity are invited to venture off the beaten path that this thesis presents.







## Chapter 2

# Research Aims

It should be possible to make Einstein's view of the Universe as much a part of the intellectual equipment of ordinary people as is that of Newton.

---

*Clement Durell*

The objective of this PhD-project was two-fold: First, to develop a research-based digital learning environment in GR in line with the Norwegian physics curriculum. Second, to study students' learning processes in order to gain a better understanding of how GR can be taught and learned in secondary schools. Guided by these general objectives, I formulated two overarching research aims for my research project:

**First, to propose a way of turning general relativity, a physical theory of gravity, into a subject area that can be taught at the secondary school level.**

**Second, to characterize physics students' understanding of key concepts in GR and to investigate how students develop their understanding through work within a digital learning environment.**

Naturally, these aims are broad. To find satisfactory answers, I unpacked the two overarching aims by formulating more specific research questions that I approached in my individual studies. In the PhD research frame, I describe the general nature of educational research before presenting an overview of the different studies and the more specific research questions. In the theoretical framework, I rephrase and specify these two research aims based on the Model of Educational Reconstruction (Duit et al., 2012).







# Chapter 3

## Introduction

Everybody knows that Einstein did something astonishing, but very few people know exactly what it was. It is generally recognised that he revolutionised our conception of the physical world, but the new conceptions are wrapped up in mathematical technicalities. (...) What is demanded is a change in our imaginative picture of the world - a picture which has been handed down from remote, perhaps pre-human, ancestors, and has been learned by each one of us in early childhood. A change in our imagination is always difficult, especially when we are no longer young. The same sort of change was demanded by Copernicus, who taught that the earth is not stationary and the heavens do not revolve about it once a day. To us now there is no difficulty in this idea, because we learned it before our mental habits had become fixed. Einstein's ideas, similarly, will seem easier to generations which grow up with them; but for us a certain effort of imaginative reconstruction is unavoidable.

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*Bertrand Russell*

Arguably, GR is one of the most beautiful physical theories ever invented (Carroll, 2003). The theory proposes a radically new way of describing space and time. Doing so, GR propelled 20th century physics into a new and modern arena of scientific inquiry. From the big bang to black holes, from cosmology to gravitational wave astronomy, relativistic ideas present a fantastic vision of physics for the future and continue to inspire scientists and the next generation of scientists alike.

In February 2016, the LIGO Scientific Collaboration announced the first direct observation of gravitational waves (Abbott et al., 2016). Shortly after, the Nobel Prize committee awarded the 2017 physics prize “for decisive contributions to the LIGO detector and the observation of gravitational waves” positioning the theory of general relativity at the forefront of research. In 2019, the first image showing the shadow cast by a black hole confirmed yet again GR and opened a new era of astrophysics (Akiyama et al., 2019). Large research collaborations such as the LIGO Scientific Collaboration in the USA or the ARC Centre of Excellence for Gravitational Wave Discovery OzGrav in Australia attract physics and engineering students, pooling and channeling the expertise of thousands of young minds. Extensive education and outreach campaigns such as the Perimeter Institute series<sup>1</sup> or the OzGrav education and public outreach campaigns<sup>2</sup> aim at increasing public understanding of relativity. Physicists such as Brian Greene or Brian Cox fill halls of passionate science lovers. The public is attracted by

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<sup>1</sup><http://www.perimeterinstitute.ca/outreach>

<sup>2</sup><https://www.ozgrav.org/education>

### 3. Introduction

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scientists who ponder the nature of our universe and who publicly share their understanding and enthusiasm for science.

Yet, despite the relevance and public popularity, there are surprisingly few attempts of bringing GR to classrooms. In many countries, physics students hardly ever come across modern concepts of space and time in their classrooms. Rather, it is left to science popularizers and outreach campaigns to teach students our current best understanding of the universe. To most people nowadays, Einstein's ideas still seem as challenging as they did when Bertrand Russell (1925) wrote his "ABC of Relativity". Young learners continue to grow up with notions of classical physics, probably making it harder for them to wrap their heads around ideas of curved space and warped time later in life. An increased focus on GR and gravitational astronomy presents enormous opportunities and substantial responsibilities for scientists and educators.

This PhD-project responds to the challenges of GR education and addresses the scarcity of research-based approaches to teaching and learning of GR. In this section, I give an introduction to GR both from physics and education perspectives. These accounts contextualize my research in two ways: First, I lay the groundwork for an educational reconstruction of GR by introducing the key concepts of GR. Second, I relate my research to previous educational work on relativity and integrate my studies within an ongoing series of educational explorations that aim at teaching students our current best understanding of the universe.

#### 3.1 General relativity from a physics perspective

From the 17th century onward, physicists started to treat gravity as an attractive force between massive objects. Despite the experimental success of this description, no one was able to explain the true source of gravitational phenomena — a problem that led Isaac Newton, father of the gravitational force model, to call his own theory a "great absurdity" (Ohanian, 1976, pp 75).

In 1905, Einstein published his theory of special relativity, which explained the relationship between space and time. Even though special relativity and Newton's theory of gravity described the world quite well, Einstein realised that both theories could not be combined: According to Newton, gravity acts instantaneously over great distances. Special relativity is in conflict with this model; the theory puts a universal speed limit on every possible physical effect – and in particular, on gravity.

It took ten more years before Einstein could present his general theory of relativity. GR is a mathematical theory of gravity, time, and space. The theory generalizes both Newton's classical theory of gravity and special relativity. Avoiding the "conceptual absurdities" of Newton's gravitational force, GR interprets gravity as a manifestation of the curvature of our universe. In Newtonian physics, time and space are absolute. They furnish the background against which distance and time are measured, and ultimately, against which the laws of nature unfold. Einstein's take on the workings of our universe was astoundingly different: Not



only did Einstein merge time and space into a dynamic four-dimensional fabric called spacetime but he allowed time and space to take active roles in the laws of physics. Spacetime reacts to the presence of massive objects, it bends and ripples, and it deflects the path of light and the orbits of planets.

### 3.1.1 Einstein's happiest thought: Gravity is a geometrical phenomenon

Einstein was a master of thought experiments. He called one of his thought experiments the happiest thought of his life – the idea that a person in free fall would feel weightless (Einstein, 1920). This physical principle led to the formulation of the equivalence principle. The principle of equivalence, in turn, suggests the mathematical strategy of describing gravity as the geometry of curved spacetime.

At its heart, the principle of equivalence encapsulates the universal nature of gravity. The behavior of freely falling objects is universal because it is independent of their masses (or any other feature of their composition). Whereas other forces can be detected by acting only on certain particles (such as the electromagnetic force acting on charged particles), there is no “gravitationally neutral” particle. A force is something that causes acceleration. Saying that there is no gravitationally neutral object amounts to saying that there is no reliable way to define acceleration due to gravity. There is no particle with respect to which one could measure such gravitational acceleration. In other words, gravity is inescapable. This observation of universality suggests that gravity is not a force but an intrinsic feature of the universe. The relativistic notion of zero acceleration is "moving freely in the presence of whatever gravitational field happens to be around." (Carroll, 2003)

The principle of equivalence formalizes the universality of gravity by stating that, locally, the motion of freely falling objects is the same in a gravitational field and in a uniformly accelerated frame. In larger regions of spacetime, however, one cannot disregard inhomogeneities in the gravitational field (Schutz, 2003). Therefore, the principle of equivalence is a local statement about the nature of gravity. A slight generalization of the principle of equivalence states that all laws of physics – not only gravitational phenomena - reduce to those of special relativity in small enough regions of spacetime. This formulation allows linking physics to geometry: The physics of special relativity takes place in flat spacetime. If, locally, the laws of physics reduce to those of special relativity, an appropriate mathematical framework to model spacetime is a structure that looks locally flat but can have a non-flat geometry globally. Such a structure is called a differentiable manifold. Differential geometry, the study of such manifolds, is the mathematical backbone of general relativity. Once Einstein challenged the traditional conception of gravity being a force, he was elegantly led to the idea that gravity was geometry.

#### 3.1.2 Matter tells spacetime how to curve. Curvature tells matter how to move.

There are two parts to describe the physics of gravitation: first, how the gravitational field influences the movement of matter and second, how matter determines the gravitational field. Classically, these two parts are encapsulated by one equation that describes the acceleration  $a$  of an object in a gravitational potential  $\phi$

$$a = -\nabla\phi,$$

and by Poisson's equation that expresses the potential in terms of the matter density  $\rho$  and Newton's gravitational constant  $G$

$$\nabla^2\phi = 4\pi G\rho.$$

It took Einstein ten years to describe how the gravitational field influences the behavior of matter and how matter determines the gravitational field in his relativistic framework. The curvature of spacetime replaces the gravitational potential in its role to act on matter; in turn, energy and momentum influence spacetime and create curvature. In short, matter tells spacetime how to curve; curvature tells matter how to move (Wheeler, 1998). This description is encapsulated in a tensor equation called Einstein's field equation:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$$

Here, the left-hand side of the equation corresponds to the curvature of spacetime where  $R_{\mu\nu}$  is the curvature tensor,  $g_{\mu\nu}$  the metric tensor, and  $R$  the Ricci scalar. The right-hand side of the equation describes the energy content of the spacetime region.  $T_{\mu\nu}$  is the stress-energy tensor and  $G$  is the gravitational constant that also appears in Newton's equation of gravity.

### 3.2 General relativity from an educational perspective

GR is a new theory of gravity. While it seems straightforward to summarise the theory from a physics perspective by stating Einstein's equation, it is a challenge to convey key ideas of GR to students that have not learned the sophisticated language of differential geometry yet. In this section, I briefly sketch educational challenges that arise when one tries to teach and learn GR at the secondary school level and trace the historic development of educational research in GR.

There is general agreement about the challenges of teaching GR to secondary and undergraduate students.<sup>3</sup> The abstract nature of the theory contributes to its perceived difficulty: GR builds on an advanced level of mathematics that draws

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<sup>3</sup>In study (I), I elaborate on these educational and conceptual challenges in detail (Kersting et al., 2018a).

on linear algebra, multidimensional analysis, and topology. Moreover, the theory requires students to have sound background knowledge in Newtonian gravity, classical dynamics including Lagrange and Hamiltonian mechanics, classical field theory, and special relativity.

Aside from these technical difficulties, there are conceptual obstacles as well. GR describes the physics of gravitation in a four-dimensional setting. Research has shown that astronomy students often struggle to move between two and three-dimensional representations of the Universe (Eriksson et al., 2014). Students need to master discipline-specific ways of moving between dimensions in order to appreciate a three-dimensional reality (Eriksson et al., 2014). Adding yet another dimension in form of the time-dimension is likely to make this task harder in GR and astronomy education. Indeed, acquiring a three- and four-dimensional understanding of the Universe is part of a more general competence in astronomy that Eriksson (2019) has termed “reading the sky”. Moreover, there are conceptual challenges other than the challenge of dimensionality in the learning domain of GR. Students have hardly any direct experience of relativistic phenomena. Life on earth unfolds in seemingly classical ways, thus rendering the nature of relativistic phenomena often as counter-intuitive to learners.

Approaches to help students overcome these technical and conceptual challenges vary. By and large, the main educational efforts have focused on developing appropriate teaching approaches that rely on qualitative and conceptual understanding (Henriksen et al., 2014) historic and cultural reconstructions (Levrini, 2002b,a), geometrical approaches (Zahn and Kraus, 2018, 2014), or simplified mathematical treatments (diSessa, 1981; Stannard et al., 2017). However, to make teaching and learning successful, there remains the need to study students’ knowledge and conceptual understanding of key features of GR which then, in turn, can inform the refinement of learning resources and instructional approaches.

When I started my PhD-project in 2015, detailed accounts of secondary students’ conceptual understanding in GR were missing in the literature. Possibly due to the conceptual challenges, it seemed that the physics education community had mostly overlooked GR as a learning domain for middle and secondary school students. For example, a bibliography of research on teaching and learning science comprising 2000 articles on conceptual understanding in physics and published within a period of 30 years lists only eight instances of special relativity and no account of GR at all (Duit, 2009). A few exploratory studies on middle and upper secondary school students’ understanding of GR did exist though (Baldy, 2007; Pitts et al., 2014; Velentzas and Halkia, 2013).

Addressing the concept of gravity at the middle school level, a French project introduced the so-called pillow-model to study French students’ (15 years old) ideas of attraction between objects (Baldy, 2007). The pillow-model is similar to the popular rubber sheet model and compares curved spacetime to a flexible fabric. Baldy compared two teaching methods, one based on Newtonian physics and one based on the pillow-model, and studied student’s conceptions of falling bodies. She found that the Newtonian approach was less effective, even though she admitted that her findings did not mean that the students built a correct

### 3. Introduction

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Einsteinian understanding of gravity and the universe.

Velentzas and Halkia (2013) looked at the role of thought experiments to teach the principle of equivalence to secondary school students. Their findings suggest that thought experiments are effective tools to convey abstract relativistic ideas. A key insight relates to the difference between everyday and scientific thinking. While students were able to conduct the relativistic thought experiments, many struggled to accept the implications because relativistic phenomena often were in conflict with everyday experiences. While Velentzas and Halkia's study addressed relativistic concepts, the main focus of the study was the use of thought experiments as an instructional tool at the secondary school level. The findings did not give in-depth insights into students' understanding of GR.

One of the pioneering studies in "Einsteinian Physics Education" was an exploratory case study that probed whether it was possible and beneficial to teach general relativity to students of age 11 through an enrichment program in Western Australia (Pitts et al., 2014). Using pre-post questionnaires, Pitts et al. looked at the measurable improvements in students' understanding of aspects of GR. Students were able to demonstrate knowledge but were not able to provide their own explanations of relativistic concepts. In particular, the concept of gravity remained elusive to many students.

The scarcity of literature on GR education was a motivation for me to fill this gap with my own research on learning processes in GR. Remarkably, since 2015, quite a bit has changed in the science education research community. Partly due to the birth of gravitational wave astronomy (Abbott et al., 2016), partly due to the formation of the Einsteinian Physics Education Research (EPER) collaboration (Blair et al., 2016; Venville et al., 2017; Kersting et al., 2018b) interest in teaching and learning of GR has steadily increased. This development has even led physics educators to call for "Einsteinian Physics Education" as a new way to improve physics education (Kim and Lee, 2018).

Indeed, just within the last four years, a new body of research on teaching and learning of Einsteinian Physics has emerged. This research is two-fold, presenting new instructional approaches to teaching and learning relativity on one side (Gould, 2016; Kim and Lee, 2018; Overduin et al., 2018; Kraus and Zahn, 2018; Pereira, 2016; Stannard, 2018; Stannard et al., 2017; Ryston, 2019b,a; Kamphorst et al., 2019) and research on middle and secondary students' learning and understanding of GR on the other side (Choudhary et al., 2018; Kaur et al., 2017a; Ruggiero, 2019). In the discussion, I comment on these recent efforts and contextualise my work in relation to other educational projects that address GR education around the world.

### **3.3 Project ReleQuant and the Norwegian upper secondary curriculum**

It is important to present this PhD-project as part of the greater educational project ReleQuant (Henriksen et al., 2014). The ReleQuant agenda has informed much of the basic assumptions and approaches to learning modern physics that

I have employed in my research. ReleQuant is an interdisciplinary educational project that pools the expertise of physicists, science educators, physics teachers, learning scientists, and designers to develop digital research-based learning resources in modern physics and to conduct research on students' learning processes and motivation.

The physics education group at the Department of Physics, University of Oslo, manages the ReleQuant project, which is conducted in collaboration with the Norwegian University of Science and Technology, the Norwegian Center for Science Education, and four partner schools in the greater Oslo area. Practicing physics teachers as well as physics teacher students are included through the sub-project ReleQuant Competence. Within the scope of ReleQuant Competence, a community of practice is created in which physics teachers, researchers, and teacher students work actively together in order to develop varied and engaging teaching in physics.

An integral part of ReleQuant centers on utilizing the online learning platform *Viten*<sup>4</sup> to offer students easy access to the learning resources and to empower teachers to use these resources flexibly and effectively in their practice. A developer of the *Viten* development team is part of the ReleQuant project as well.

With the school reform introduced in autumn 2006, GR was included in the Norwegian curriculum for upper secondary physics. In addition to the more traditional topics in the curriculum, such as Newtonian mechanics and electromagnetism, Norwegian physics students are expected to explore different interpretations and philosophical aspects of modern physics as well. Such a focus on qualitative and philosophical as well as modern aspects of physics is unusual compared to science curricula in many other countries (Bungum et al., 2018, 2015; Henriksen et al., 2018, 2014).<sup>5</sup>

The specific curriculum goal for GR remains vague: "*The aims of the studies are to enable pupils to give an account of the postulations that form the basis for the special theory of relativity, discuss qualitatively some of the consequences of this theory for time, momentum and energy, and give a qualitative description of the general theory of relativity*" (The Norwegian Directorate for Education and Training, 2006). This broad description leaves scope for various interpretations of what constitutes GR at a qualitative level.

While it seems innovative and exciting to introduce topics of modern physics to the physics curriculum, research suggests that demands on both students and teachers are high: Students struggle with relativity and quantum physics since these topics describe phenomena that cannot be visualized or experienced directly (Bungum et al., 2015; Velentzas and Halkia, 2013; Kersting et al., 2018a). Often, teachers lack sufficient background in modern physics, thus finding the topics conceptually demanding as well. Moreover, topics of modern physics often require novel ways of teaching and learning that differ from teacher-centered

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<sup>4</sup>[www.viten.no](http://www.viten.no)

<sup>5</sup>In the discussion, I elaborate on the development of the Norwegian physics curriculum and show how my work contributes to shaping modern physics education.

### 3. Introduction

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traditional physics lessons that focus on content knowledge and mathematical problem solving. These novel learning approaches, in turn, challenge both teachers and students (Bøe et al., 2018).

In response to these challenges, ReleQuant aims to investigate how modern physics can be taught in ways that are scientifically sound and that motivate young learners. The development of learning resources in ReleQuant is based on a sociocultural perspective on learning which emphasizes language as an integral part of the learning process (Lemke, 1990; Vygotsky, 1962). This approach entails that students make physics concepts their own through use of language and interaction with others. Another feature of the development of ReleQuant resources is the inclusion of history, philosophy, and nature of science aspects because these aspects have long been advocated as important elements in modern science education (Höttecke et al., 2012). In the theoretical framework of this thesis, I describe these theoretical approaches in more detail in relation to my own research.







# Chapter 4

## PhD Project Frame

All science is either physics or stamp collecting.

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*Ernest Rutherford*

Just looking at my CV, the content of this thesis might surprise some readers. I am a mathematical physicist by training and have worked in the Department of Physics at the University of Oslo, the School of Physics at the University of Western Australia, and the Centre for Astrophysics & Supercomputing at Swinburne University of Technology as part of my PhD-project. Yet, my PhD-research is not physics research in the narrow sense of the word. Obtaining a degree in physics education, or science education more generally, entails conducting educational research which draws on methods from the social rather than the natural sciences. In this section, I position physics and science education research within the greater landscape of the educational sciences. Moreover, I frame my approach to study conceptual understanding as one example of how physics educators study people instead of natural phenomena.

### 4.1 Physics education and science education research

Does physics education research belong to the realms of physics or the realms of education research? It might be natural to think of physics education research as a branch of science education research. But this placement begs the next question: What is science education research? What distinguishes the discipline-based fields of science education from educational research more generally?

In answering these questions, it is important to acknowledge that science educators comprise a diverse group of people: educational researchers, science teachers, science communicators, public outreach staff, and even educational policy makers often work in the service of science education. As diverse as its practitioners is the field of science education itself: science education research lies at the crossroads of different disciplines. There are diverging opinions on whether physics and science education research should be conceptualized as a branch of the social sciences (Davis and Russ, 2015) or as a relevant and authentic part of the discipline-based research communities (Sydney University Physics Education Research Group, 2001; Eriksson, 2014). Both sides have convincing arguments.

In this thesis, I wish to put particular focus on the methods and theoretical frameworks that have informed my research. To me, these methods and frameworks from the social sciences have been more instructive and more crucial in conducting my PhD-research than the actual physics content of GR. Starting my PhD with a degree in physics, I had already sound knowledge of

GR. However, I did hardly know anything about learning theories, educational models, and qualitative and quantitative research methods. I therefore choose to position physics and science education research as a branch of educational research, which, in turn, can be viewed as a branch of the social sciences.

Framing this thesis as a research project within the social sciences reflects my personal PhD-trajectory. Throughout the past four years, I have been trained in tools and methods of the educational sciences rather than having gained new knowledge in physics.<sup>1</sup> Indeed, “developing comprehensive understanding of research epistemologies should be a core goal of graduate research education” according to Peter Charles Taylor, Professor of STEAM Education (Taylor, 2014, pp 40). In the method section, I further elaborate on the nature of educational research and different approaches to conducting such research.

### 4.2 Studying conceptual understanding

Science educators try to obtain a complete picture of how people come to know science (Davis and Russ, 2015). The attempt to channel this interest along specific strands of investigations, thus forming a distinct field of research, can be traced back to mid-nineteenth century efforts that tied education to new scientific advancements (Davis and Russ, 2015). Over its history, the field has remained a domain-specific sub-field of the larger field of educational research. As such, the development of science education mirrors the larger evolution of educational research. Today, there are several important areas of science education research. For example, the biannual conference of the European Science Education Research Association (ESERA)<sup>2</sup> lists 18 strands in science education in 2019; the National Association for Research in Science Teaching (NARST)<sup>3</sup> a worldwide organization for improving science teaching and learning through research, lists 15 strands.

In this section, I elaborate on one strand that has been of particular interest for my research. ESERA calls this strand “Learning Science: Conceptual Understanding” and NARST calls it “Science Learning, Understanding and Conceptual Change”. When trying to understand how students come to learn and know science, one inevitably comes across the notion of conceptual understanding. While there seems to be agreement among science educators that successful instruction should foster students’ conceptual understanding, different science educators have interpreted the notion of conceptual understanding in different ways. Some physics educators even argue that there is no satisfactory definition either of the notion of a concept nor of conceptual understanding (Sands, 2014). It is therefore important to specify how I frame this vague notion in my PhD-thesis before I present my research on students’ conceptual understanding in GR.

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<sup>1</sup>However, I very much appreciate the new physical insights I have gained through working as a teaching assistant in the GR course FYS4160 and through taking a graduate course in quantum field theory FYS4170. Thanks to Torsten Bringman for being a wonderful lecturer!

<sup>2</sup><https://www.esera2019.org>

<sup>3</sup><https://www.narst.org>

In a very broad sense, the term conceptual understanding encompasses the growth of students' understanding in various learning domains (Greeno et al., 1992). In a more narrow sense, conceptual understanding encapsulates students' knowledge of specific scientific concepts and their ability to reason qualitatively based on this knowledge (Sands, 2014). Irrespective of how narrow or broad one defines the notion of conceptual understanding, though, there remains the challenge of investigating students' actual understanding of and their reasoning with scientific concepts.

One approach to this challenge, and the one that I have followed in my research, is to make use of the close connection between language and thought (Vygotsky, 1962, 1978). According to psychologist Lev Vygotsky (1962, p. 218), "Thought is not merely expressed in words; it comes into existence with them." Thus, for me, conceptual understanding in GR is linked to an active level of word knowledge in the learning domain of GR. According to this view, students are able to understand and employ scientific vocabulary as it is situated within a network of other scientific words and ideas (Lemke, 1990; Vygotsky, 1962; Wittgenstein, 1997). For example, looking at the transitions from a passive to a more active control of science vocabulary is thus a way to study conceptual understanding (Haug and Ødegaard, 2014).

The active control of physics vocabulary is not the only way to study conceptual understanding from a linguistic perspective though. Findings from metaphor theory suggest that certain figures of speech, such as metaphors and analogies, shape conceptual structures significantly (Lakoff and Johnson, 2003). Metaphor theory thus provides a method for investigating how people understand abstract concepts: metaphorical language helps us to articulate abstract ideas in terms that are intelligible to others (Lancor, 2018). For the remainder of this thesis, I assume that conceptual knowledge develops alongside an increased level of word knowledge and that metaphors are pervasive in students' thoughts and actions.







## Chapter 5

# Research Questions

Anyone who has ever tried to present a rather abstract scientific subject in a popular manner knows the great difficulties of such an attempt.

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*Albert Einstein*

In the beginning of this thesis, I formulated two overarching research aims. First, to propose a way of turning GR, a physical theory of gravity, into a subject area that can be taught at secondary school level. Second, to characterize physics students' understanding of key concepts in GR and to investigate how students develop their understanding through work within a digital learning environment. I unpacked these overarching aims by formulating more specific research questions that I addressed in the individual studies. In the following, I present an overview of these studies with their more specific research questions that guided my inquiries. Table 5.1 summarises this overview.

**Study (I) - Kersting, M., Henriksen, E. K., Bøe, M. V., & Angell, C. (2018). General relativity in upper secondary school: design and evaluation of an online learning environment using the model of educational reconstruction. *Physical Review Physics Education Research*, 14(1), 010130-1-010130-18 .**

This first study presents the core of my thesis and set the ground for the subsequent studies. Indeed, all other studies build on the first publication insofar as they address issues that we identified as being crucial or challenging for students when trialling our learning environment for the first time. Study (I) reports on the design and structure of the learning environment and gives a broad account of students' understanding of key concepts in GR. In a sense, this study is a miniature version of my thesis. It introduces the theoretical and methodological frameworks that informed my research and it touches upon students' understanding of the most important concepts in GR - albeit somewhat briefly. Specifically, I formulated the following research questions:

- I-RQ1 What characterizes the understanding of key features of GR that participating students express while engaging with the learning environment?
- I-RQ2 In what ways do the participating students' experiences support or challenge the design hypotheses that guided the development of our online learning environment?

## 5. Research Questions

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- I-RQ3 Based on findings from RQ1 and RQ2, what design principles can be formulated for the development of learning resources in GR at the upper secondary level?

The dual interest of designing learning resources and investigating students' conceptual understanding becomes apparent. Formulating the second research question allowed us to emphasise the actual process of designing learning resources specifically.

**Study (II) - Kersting, M., & Steier, R. (2018). Understanding curved spacetime - the role of the rubber sheet analogy in learning general relativity. *Science & Education*, 27(7), 593–623 .**

Arguably, one of the most important insights in GR is that gravity is a manifestation of the curved geometry of spacetime. As a subsequent step towards gaining a better idea of students' conceptual understanding in GR, I focused on the concept of spacetime specifically. One of the most ubiquitous representations of spacetime in physics education is the so-called rubber sheet analogy (RSA). The RSA served as a convenient starting point to investigate students' conceptualizations of spacetime. Since every analogy is limited in its explanatory power, I was interested to see which conceptual features of spacetime the analogy highlights or hides. This interest resulted in the formulation of the first research question in study (II). The second question then addressed students' understanding of the RSA specifically. Moreover, I was interested to see to what extent students showed awareness of the limitations of the RSA as an instructional analogy:

- II-RQ1 What features of gravity as they were explained by Einstein does the rubber sheet analogy hide and highlight?
- II-RQ2 What characterizes students' understanding of the rubber sheet analogy?
- II-RQ3 In what ways do students show awareness of the analogical nature of the rubber sheet analogy when conceptualizing gravity and curved spacetime?

**Study (III) - Kersting, M. (2019). Free fall in curved spacetime - how to visualise gravity in general relativity. *Physics Education*, 54(3), 035008 .**

Guided by the question of how to visualise gravity in GR, this paper presents a new instructional model that we developed in line with the findings of the first two studies. Thus, the paper presents a specific example of how a key idea of GR – the geodesic equation – was transformed into an interactive model that illustrates this equation qualitatively.

- III-RQ1 How can one visualise gravity in general relativity?



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**Study (IV) - Kersting, M. (2019). Navigating four dimensions – upper secondary students’ understanding of movement in spacetime, GIREP 2018 conference proceeding (manuscript in press) .**

The starting point of this study was the scarcity of comprehensive accounts of secondary school students’ conceptual understanding of movement in spacetime. The research questions in this study built on findings from study (II) that illustrated the conceptual struggles students faced when conceptualizing movement in space and time:

- IV-RQ1 What characterizes upper secondary school students’ understanding of movement in four-dimensional spacetime?
- IV-RQ2 What are the difficulties and challenges that upper secondary school students face when conceptualizing movement along geodesic curves?

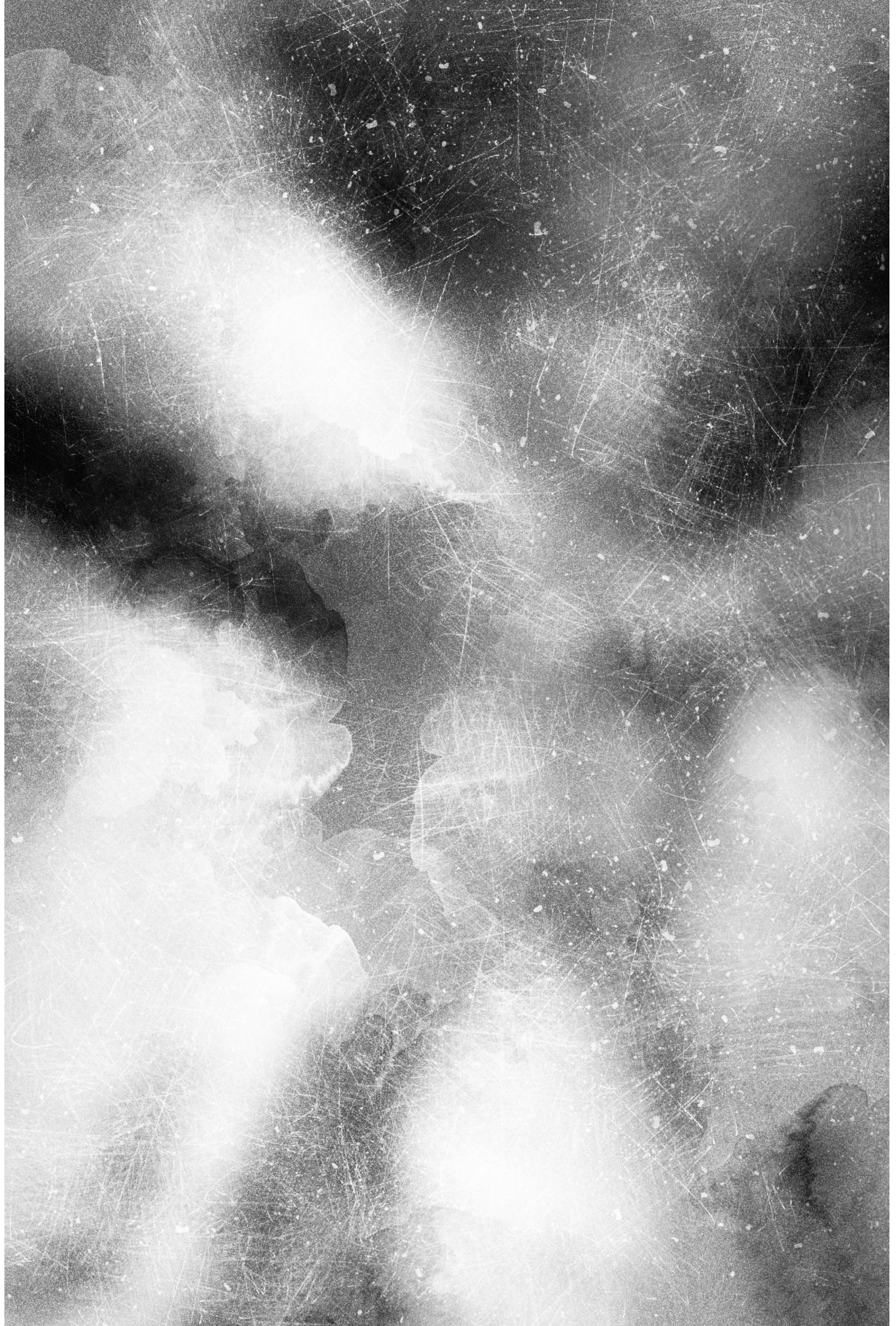
**Study (V) - Kersting, M. (2019). How History and Philosophy of Science Can Inform Teaching and Learning of General Relativity in Upper Secondary School, Marcel Grossmann 2018 conference proceeding (manuscript in press) .**

Research suggests that history and philosophy of science can be fruitful in teaching and learning of physics. Building on these findings, I was interested to see in which ways history and philosophy of science could promote understanding and motivation in the domain of GR.

- V-RQ1 How can a historical and philosophical approach to teaching and learning Einsteinian physics foster understanding of and motivation for general relativity?

Table 5.1: Overview of the research aims, research questions, and studies that are part of this PhD-project

<b>Research aims</b>	<b>Specific research questions</b>	<b>Publication</b>
<p>To propose a way of turning general relativity, a physical theory of gravity, into a subject area that can be taught at secondary school level.</p>	<p>In what ways do students' experiences support or challenge the design hypotheses that guided the development of our online learning environment?</p> <p>What design principles can be formulated for the development of learning resources in GR at the upper secondary level?</p> <p>What features of gravity as they were explained by Einstein does the rubber sheet analogy hide and highlight?</p> <p>How can one visualise gravity in general relativity?</p> <p>How can a historical and philosophical approach to teaching and learning Einsteinian physics foster understanding of and motivation for general relativity?</p>	<p>Study (I)</p> <p>Study (I)</p> <p>Study (II)</p> <p>Study (III)</p> <p>Study (V)</p>
<p>To characterize physics students' understanding of key concepts in GR and to investigate how students develop their understanding through work within a digital learning environment.</p>	<p>What characterizes the understanding of key features of GR that participating students express while engaging with the learning environment?</p> <p>What characterizes students' understanding of the rubber sheet analogy?</p> <p>In what ways do students show awareness of the analogical nature of the rubber sheet analogy when conceptualizing gravity and curved spacetime?</p> <p>What characterizes upper secondary school students' understanding of movement in four-dimensional spacetime?</p> <p>What are the difficulties and challenges that upper secondary school students face when conceptualizing movement along geodesic curves?</p>	<p>Study (I)</p> <p>Study (II)</p> <p>Study (II)</p> <p>Study (IV)</p> <p>Study (IV)</p>





## Chapter 6

# Theoretical Framework

Frameworks have a tendency to disappear when they are intuitive and carefully planned, because our attention is on the wonderful fruits of the process.

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*Frank Chimero*

The following sections explore the landscapes of my PhD-research from a theoretical perspective. Each section specifies my research approach on a finer grain-size. Starting from the overarching learning theory of sociocultural theory, I zoom in onto the educational and methodological frameworks that have guided the development of my overarching research design, before I present History and Philosophy of Science (HPS) as a specific approach to GR education.

### 6.1 Sociocultural Theory

To find better methods of teaching, one has to gain better understanding of learning. To gain better understanding of learning, one has to study the nature of knowledge. There are distinct traditions in educational research and practice that stem from different perspectives on knowing and learning. These perspectives in turn can be traced back to different philosophical schools of thought (Greeno et al., 1992). While the scope of this thesis does not allow for a thorough discussion of my epistemological assumptions, it is important to acknowledge that philosophy and learning theories shape educational practices to a tangible extent. These theories are not only of academic interest but do indeed affect the design of learning resources, teaching approaches, the way research is carried out, and even curriculum design. There is great value in adopting a theory on knowing and learning as a first step towards conducting science education research (Redish, 2014).

The overarching learning theory that informed the research of this thesis is sociocultural theory or socioculturalism in short. There are certain themes that are characteristic of socioculturalism. Here, I briefly summarize these basic tenets in relation to science education research and in relation to my own work. In line with my interest to study conceptual understanding, I elaborate on the relation between knowledge and language that is a characteristic feature of socioculturalism. Finally, I attempt to clarify some confusion between sociocultural theory and socioconstructivism that can obscure the underlying assumptions of sociocultural research projects.

At its heart, sociocultural theory posits that one cannot separate learning from its social context. Learners actively construct knowledge through interactions within the social contexts of their communities. Learning is embedded within

## 6. Theoretical Framework

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activity, context and culture – a position that is broadly known as situated learning (Lave and Wenger, 1991). Learning is a process of enculturation; students learn science by learning the culture of science, including how to speak, write, and interact in ways that are acceptable to the scientific community (Lancor, 2018). Describing learning as a situated process of becoming integrated into a knowledge community entails a close interdependence between social and individual processes in the construction of knowledge. Thus, knowledge is contextual and a product of social patterns. In particular, knowledge is not confined to the learner’s mind but is collaboratively created and can be distributed over mind, body, and the environment (Hakkarainen et al., 2013). Therefore, student learning must be studied in context (Lancor, 2018).

Sociocultural theory puts particular focus on language as the lens through which humans experience, communicate, and understand reality (Vygotsky, 1962, 1978). Sociocultural teaching approaches therefore reflect the collaborative nature of learning by fostering collaboration and joint activities that are often mediated through language (Greeno et al., 1992; Packer and Goicoechea, 2000). Students learn by using language as a tool to process and understand new information. The tools students develop are based on the (classroom) cultures they find themselves in.

The perspective of sociocultural theory becomes apparent in the ReleQuant-approach – both in the design of the learning environment and in the way we have conducted our research. First, the design of the learning environment builds on a sociocultural stance: The physics classroom serves as the community in which students make physics concepts their own through use of language and interaction with each other. Within the scope of this thesis, I assume that knowledge of GR is co-constructed in settings of joint activity. These patterns of knowledge creation unfold as students collaboratively work with our digital learning resources. To this end, the ReleQuant resources foster discussions both in small groups as well as in the classroom. Moreover, the instructional tasks encourage students to use oral and written language to practice talking physics (Bungum et al., 2018; Henriksen and Angell, 2010; Lemke, 1990) - and thus, to practice participation in the community of their physics classroom (Bøe et al., 2018).

Second, the ReleQuant research design reflects the sociocultural stance as well: We collected data of focus group interviews, audio records of discussion tasks, and video data of group work. In all cases, we aimed to capture accounts of students’ knowledge productions in authentic classroom settings.<sup>1</sup> This way of collecting data contrasts with methods of educational measurement which often assess students’ knowledge gain through individual pre-post questionnaires. In the methods section, I describe our qualitative research approach in line with sociocultural perspectives in more detail. Finally, also the way I report on the findings has a clear focus on processes - both the learning processes and the intermediate steps of the design process.

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<sup>1</sup>A more thorough treatment of nontraditional ways of teaching and learning physics and how students react to those approaches can be found in (Bøe et al., 2018).

### 6.1.1 Talking science, writing science

Employing a sociocultural view entails a commitment to studying the role of language in science education more closely. This emphasis on language traces back to Vygotsky (1962, 1978) who proposed that the development of concepts and the development of word meanings are the same process: Thought is not merely expressed in words; it comes into existence with them. Language is a socially provided tool that mediates the development of concepts (Wertsch, 1993).

Jay Lemke (1990) was one of the first to apply ideas of language mediation to science classrooms. Lemke coined the phrase “talking science” by noticing that “the mastery of science is mainly a matter of learning how to talk science” (Lemke, 1990, pp 153). The work of science is a linguistic act and doing and teaching science are complex and language-laden activities (Bratkovich, 2018). Science teaching is a social process to bring students into the community of people who talk science. Learning the languages of science is not only learning the words, but the world in which students are expected to use these words (Lemke, 1990; Wittgenstein, 1997). Here, learning the language of science entails both the fluent use of oral and written language.

More specifically, Lemke studied the interplay between language as social action and language as thematic resources. Students use language to build relationships among each other and to create relationships among ideas. Interestingly, this observation leads to the insight that scientific concepts and meanings do not exist in the abstract; rather, they are constructed through the use of words and other signs. Scientific concepts are bits of thematic patterns: they do not exist as ideas in their own separate reality; they are thematic items that make up a semantic pattern of relationships of meaning. Building understanding of a scientific concept means identifying relationships between words and phrases, that is, learning them in the context of a thematic pattern. For example, to understand the concept of spacetime students need to identify and correctly make use of the semantic links between spacetime, gravity, and curvature. In study (II), I presented examples of how students produced incorrect semantic patterns when talking about spacetime.

In the setting of physics classrooms, the idea of talking science gets naturally adapted to “talking physics” (Henriksen and Angell, 2010). Facilitating physics talk has been a driving force within ReleQuant (Bungum et al., 2018; Bøe et al., 2018; Kersting et al., 2018a). Throughout my PhD-work, talking science and talking physics played a prominent and recurrent role both in the research design and the design of the learning resources. Science cannot be easily separated from language, and science content cannot fully be understood without language (Bratkovich, 2018). Already Einstein chose a dialogical approach to physics by presenting many of the relativistic key ideas in dialogue-form (Einstein and Infeld, 1938).

One particular interesting form of talking science relates to the use of metaphors and analogies in scientific discourse. Science often deals with the abstract and unobservable. In response, analogies and metaphors can be powerful

tools in scientific knowledge construction. Kapon and diSessa noted that the generation of analogies and the reasoning stemming from these analogies play a central role in scientific practice, thought, and creativity (Kapon and DiSessa, 2012). Indeed, Stinner (2003) observed that the big theories in science, including Einstein's theory of relativity, are often the product of imaginative thinking which, according to Stinner, includes to see analogies between disparate events. It seems that Einstein was particularly apt at finding fruitful analogies. He introduced analogies similar to the rubber sheet analogy to reason about abstract mathematical concepts (Hentschel, 1998) and he used the analogy of riding on a ray of light to work out his theory of relativity (Kind and Kind, 2007).

Metaphors are one particular example of talking physics and instructional analogies and metaphors have become a popular tool in science education (Aubusson et al., 2006; Harrison and Treagust, 2006). Systematic metaphor analysis (Schmitt, 2005) is a recent fruitful approach that draws on findings from cognitive science and linguistics to understand the use of analogies and metaphors in science education (Amin et al., 2015; Lancor, 2014; Niebert et al., 2012). This approach goes back to Lakoff and Johnson (2003) who argued that metaphors are not only a linguistic phenomenon, but also a fundamental feature of thought and mind. Metaphors serve as a principal tool for processes of understanding because learners systematically use inference patterns from one conceptual domain to reason about another conceptual domain.

### 6.1.2 Socioculturalism is not socioconstructivism

There are four distinct learning theories that have shaped the discourse in educational research to a great extent: behaviorism, cognitivism, constructivism, and socioculturalism (Lancor, 2018). None of these theories is all encompassing or exclusive. Just as we have physics, chemistry, and biology to account for the different levels at which nature can be studied, there are different approaches to describe the complex phenomenon that is human learning (Greeno et al., 1992; Treagust and Duit, 2008).

It is outside the scope of this thesis to give an account of each learning theory. It is, however, important to distinguish between two learning theories that often get confounded in educational research and practice – namely socioculturalism and socioconstructivism. There seems to be a lack of clarity in delineating socioculturalism from socioconstructivism. To get a better understanding of the sociocultural contributions of this thesis, it is helpful to contrast socioculturalism to socioconstructivism. Specifically, I try to flesh out some key differences between socioconstructivism and the sociocultural approach in this PhD-work.

Broadly, constructivism focuses on mental processes and how learners organize their experience of the world. The basic idea behind constructivism is that students learn by integrating new ideas into existing conceptual frameworks (Lancor, 2018). Socioconstructivism extends constructivism to social settings. Learners add to and reshape their mental models of reality through shared language practices and social collaboration. The cognitive tools individuals are given from the community allow them to construct new knowledge on the basis



of knowledge previously acquired. Importantly, though, the focus remains on the mental models within individual students. While social constructivists do engage in an analysis of cultural norms, they maintain a conceptual dichotomy between the individual's constructive activities on one hand and social processes on the other. It is this dichotomy that is the key to understanding the difference between the two learning theories.

In contrast to socioconstructivist views, sociocultural theory does not separate individual processes of knowledge construction from social processes of joint understanding. The sociocultural perspective regards the individual as being an inseparable part of the environment – in contrast to socioconstructivist perspectives that make a distinction between individual cognitive activities and the environment (de Laat and Simons, 2002). Since learning often entails both a change in knowing and in being, some researchers even argue that the difference between the two learning theories is not only at the level of epistemology but that there are indeed quite different ontological assumptions underlying the two learning theories (Packer and Goicoechea, 2000).

Within the scope of this thesis, I take the sociocultural stance: Learning GR with the ReleQuant resources cannot be separated from the environment of the physics classroom and the interactions with peers and the teacher. In my research, I explore how students adopt learning strategies and tools that are specific to the culture of teaching and learning physics in secondary schools (Bøe et al., 2018). These explorations entail a dialectic perspective: In the process of "doing physics" with their peers and our learning resources, students construct the social environment of the physics lesson and are at the same time constructed by this very environment (John-Steiner and Mahn, 1996; Vygotsky, 1978). This is why the units of analysis in this thesis are rarely the individual students but rather students interacting with each other. My focus lies on the conceptual understanding that groups of physics students build in collaboration and in encounter with our digital learning resources.

## 6.2 Model of Educational Reconstruction

The first research aim of my PhD-project can be condensed into the following task: to find a way of making GR, a physical theory of gravity formulated in the language of differential geometry, teachable at the secondary school level without using advanced mathematics. To approach this task, I chose the Model of Educational Reconstruction (MER) (Duit et al., 2012). The basic goal of MER is to provide "a theoretical framework for studies as to whether it is worthwhile and possible to teach particular content areas of science." (Duit et al., 2012, pp 19)

### 6.2.1 Reconstructing novel learning domains

The Model of Educational Reconstruction is a well-established model that offers a framework for science education researchers who want to assess novel learning

## 6. Theoretical Framework

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domains. To that end, MER provides guidance for integrating empirical research on teaching and learning in the development of learning resources. The framework comprises the basic idea that “science subject matter issues as well as student learning needs and capabilities have to be given equal attention in attempts to improve the quality of teaching and learning” (Duit et al., 2012, pp 13). This idea is mirrored in the structure of MER that links three strands of educational research (Figure 6.1): First, science educators analyze the particular science content of a topic and identify its key concepts. Second, science educators take student and teacher perspectives into account. Third, science educators design and evaluate learning environments and suitable learning activities.

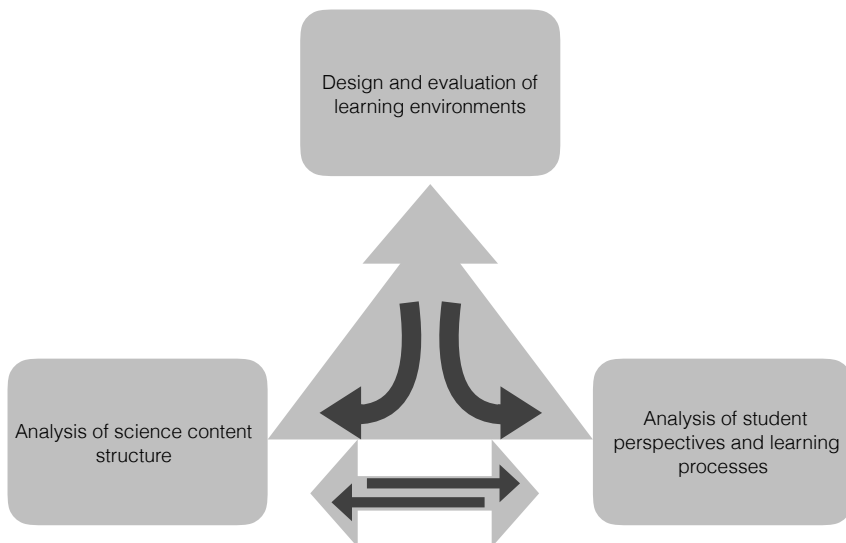


Figure 6.1: The Model of Educational Reconstruction links three components of educational research.

MER aligns well with the objectives of my PhD-project. The holistic approach of MER serves as a useful and flexible tool to scrutinize the educational relevance of GR, a learning domain that has not entered mainstream physics education yet. MER has previously been employed to reconstruct novel learning domains such as chaos theory (Duit and Komorek, 1997), nonlinear systems (Stavrou, 2015), nanoscience (Laherto, 2010, 2012), climate change (Niebert and Gropengießer, 2013), physical geography (Felzmann, 2017; Reinfried et al., 2015), and recently also special relativity (Kamphorst et al., 2019).

To attempt an educational reconstruction of a new learning domain, educators do not view the science content as given but acknowledge that the scientific content has to undergo certain reconstruction processes. In short, the science

content structure of GR has to be transformed into a content structure for instruction. Naturally, the two structures are different. In particular, the content structure for instruction is not just a simplified version of the science content (Duit et al., 2012). The educational goal of the transformation is to present a content structure that introduces students to the key ideas while keeping students' needs in mind.

The three different components of MER are intertwined: Findings in one component inform development and research in the other two. Consequently, the nature of an educational reconstruction is an iterative one. In Figure 6.2, I give an overview of my research cast into the MER-framework. Having introduced MER as the educational framework that guides my research, I am able to rephrase my research aims. **The aim of my PhD-research is to propose an educational reconstruction of GR.**

## 6.3 Design-based Research

While MER serves as an overarching framework to reconstruct GR from an educational perspective, it does not explicitly specify the research methods to do so. However, seeing that MER is a design-oriented research approach (Laherto, 2013), it seems natural to complement an MER-research design with methods of design-based research (DBR). DBR provides an iterative frame for developing and testing learning resources effectively; indeed, DBR shares several key features with MER. To bridge the gap between my overarching research approach on teaching and learning GR and the actual classroom practices, I employed the methodology of DBR to develop the digital learning environment *General Relativity*.

### 6.3.1 Design-based research - bridging the gap from research to classrooms

The main goal of design-based research, also called Design Experiments (Brown, 1992), Design Research (Sandoval, 2014), or Educational Research Design (Bungum et al., 2015; McKenney and Reeves, 2012), is to develop and implement systematic solutions to educational problems (Anderson and Shattuck, 2012). Through the development of learning resources for classroom settings, the DBR framework builds a bridge between research and school practice.

The approach of DBR relies on repeated rounds of development and testing in close collaboration with practitioners. Those iterative phases comprise analysis and reflection of the problem, development and design of solutions, and evaluation using classroom research (McKenney and Reeves, 2012). Specifically in the process of analysis and reflection, the model emphasizes reflections of teachers as well as the involvement of researchers. Typically, DBR researchers report on successful design processes in form of descriptive models since results consist both of widely usable artifacts such as teaching and learning sequences and knowledge of how to use these artifacts in praxis (Juuti and Lavonen, 2006).

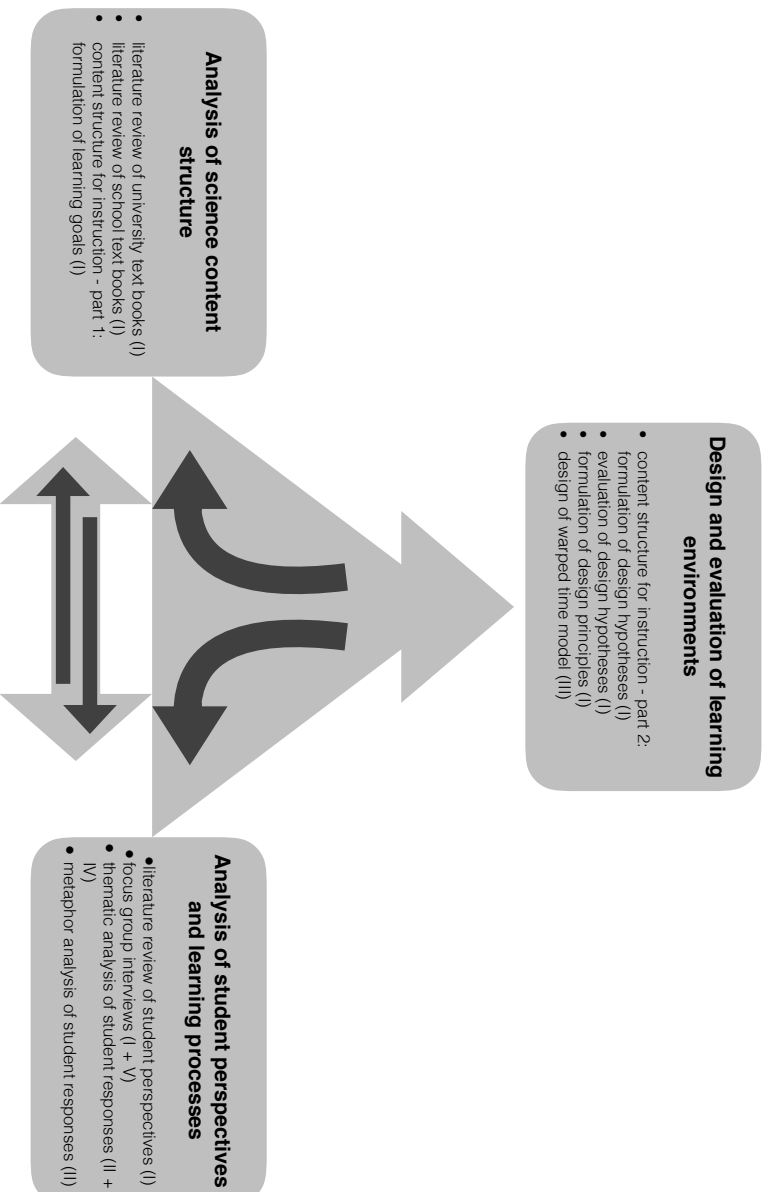


Figure 6.2: Overview of the research design in line with the Model of Educational Reconstruction. The Roman numerals in brackets denote the individual studies.

In particular, the formulation of design principles is a common result of DBR research praxis (Edelson, 2002; Bungum et al., 2015).

Originally, DBR arose in response to the need of understanding the nature of learning in context, that is to say learning in the real world as opposed to sterile laboratory experiments (Brown, 1992). Wishing to go beyond narrow measures of learning and to emphasize the role of social interaction, DBR combines both educational research and theory-driven design of learning environments. The dual goal of refining theory and refining practice is reflected in a shared commitment to the production of innovative learning environments, knowledge about how learning environments work in context, and fundamental knowledge about teaching and learning (Sandoval, 2014).

DBR methods lend themselves well to the domain of GR, because they have been used successfully in physics classrooms both in the domains of classical physics and quantum physics. Tiberghien et al. (2009) used the DBR-framework to develop teaching sequences on topics in classical mechanics targeted at students ranging from grade 7-12. Bungum et al. (2015) reported on the development of a teaching module in quantum physics that was guided by DBR methods within project ReleQuant. Both research groups highlighted the close collaboration with teachers during the iterative phases of development.

The gradual process of interlacing perspectives of scientists, practitioners, and students is a distinguishing feature of the development of research-based instructional activities in science education that aim at improving student understanding of scientific knowledge (Méheut, 2004). In fact, both DBR and MER draw on the tradition of design and evaluation of teaching-learning sequences. In a special issue of the *International Journal of Science Education* on teaching-learning sequences, Méheut (2004, pp. 516) noted in the editorial: “Yet whereas there is often extensive communication of learning results, the various explicit and implicit assumptions and decisions concerning the design of a teaching-learning-sequence, its teaching features or the interlacing of teaching with learning are less widely discussed, and may not even be made clear and comprehensible.”

This observation is interesting in light of my research that emphasizes processes rather than outcomes. Addressing the difficulty of making explicit assumptions and decisions in the design of my GR learning environment, MER and DBR offer frameworks for elaborating and validating the design by dividing the design process into distinct phases. Juuti and Lavonen (2006) emphasized the benefit of such a division: The researchers even suggested to report on the various phases in separate articles as to foster communication between DBR scholars and designers during the process. Following this advice, I reported on the development of my learning environment in study (I): Design hypotheses guided the evaluation of the first prototype and it was only after a discussion of the success (or failure) of these hypotheses that we synthesized the results in form of design principles. In the result section, I describe this process in detail.

### 6.4 History and Philosophy of Science

Even though not an educational framework in the strict sense, this PhD project takes inspiration from two research perspectives that have shaped much of the design of our digital learning resources and have channeled my research interests along certain lines: History and Philosophy of Science (HPS) and Nature of Science (NoS) (Abd-El-Khalick, 2013; Galili, 2018; Hodson, 2014; Monk and Osborne, 1997). In recent years, researchers have called for physics education to move beyond traditional content-focused instruction by including historical, epistemological, and sociocultural aspects (Bøe et al., 2018; Duit et al., 2014).

HPS and NoS are two approaches that address these calls by raising awareness of the nature of science as a human enterprise and by emphasising the historic development of scientific ideas as well as their philosophical implications. There is a vast literature on the merits of using such an approach in physics and science education to foster understanding of science as a process, to promote deeper understanding of scientific ideas, to create awareness of the public role of science in our society, and to increase student interest and motivation (Höttecke and Silva, 2011; Kim and Lee, 2018; Galili, 2018; Matthews, 2014; Irzik and Nola, 2011).

In this thesis, I focus on HPS and NoS in relation to GR specifically. This context aligns with a broader discourse in physics education research that shows that the use of HPS and NoS can foster understanding in modern physics and in particular in special relativity and quantum physics (Henriksen et al., 2018; Levrini, 2014). Thus, drawing from HPS and NoS serves as a suitable approach to an educational reconstruction of GR that focuses on qualitative understanding.

To understand why HPS and NoS can be successful entryways to teaching and learning of GR, it is important to understand the actual historic context of the development of GR. For Einstein, the history of physics had special meaning for understanding physics (Kim and Lee, 2018). Indeed, GR can be thought of as a culminating response in a long-standing philosophical dispute on the nature of space and time. GR did not only herald a new scientific age, but fostered also a new interest in the philosophy of space and time as I explain in more detail in study (II) (Kersting and Steier, 2018). Indeed, the significance of the theory extended beyond the mere contents of scientific laws and theories - adopting a relativistic perspective entailed a change in the worldview of many scientists at the time of Einstein (Chandler, 1994).

Seeing that GR challenges students' views on space and time, physics educators have drawn on HPS and NoS to develop teaching strategies that soften the impact of a counter-intuitive theory (Holton, 1973; Levrini, 2014). This approach aligns with Einstein's perspective on teaching relativity as a way to understand the nature of science as an inquiry that is not completed but continuing (Kim and Lee, 2018). While these developments reflect the historic and philosophical context of GR in the early 20th century, it is important to acknowledge that the practice of GR continues to be "in its making". Physicists continue to articulate, test, and debate implications of and alternatives to GR.







# Chapter 7

## Methods

To use a magnifying glass is to pay attention, but isn't paying attention already having a magnifying glass? Attention by itself is an enlarging glass.

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*Gaston Bachelard*

In this section, I first explain the nature of qualitative research before presenting the educational context and research design of my PhD-project. I summarize the data collection that laid the basis for my empirical studies and describe the data analysis methods that I employed to evaluate the learning environment and to study student learning processes. I motivate the choice of these analytical methods through the sociocultural focus on language. At the end of the section, I address quality standards of my methodological choices.

### 7.1 The nature of educational research

Arguably, there are different ways of conducting educational research. Educational researchers have to take a stance towards the phenomena they want to describe and the approaches that inform their inquiries. Doing so, most researchers acknowledge a basic distinction between qualitative and quantitative approaches even though new perspectives have started to shape the debate towards more holistic research perspectives (Taylor, 2014). Traditionally, science educators have approached their studies based on methods inherited from the natural sciences (Heron and Meltzer, 2005). This approach has focused on quantitative research methods such as educational measurements, experimental validation, and statistical methods. Quantitative methods belong to a paradigm of empiricism or recurrence-oriented research that seeks to find reproducible and representative patterns in human behaviour (Robertson et al., 2018).<sup>1</sup> Assuming that certain laws govern human behaviour is close to a perspective held by many scientists that try to describe natural phenomena by formulating laws of nature.

However, humans differ from the world around them in their ability to make meaning (Gallagher, 1991). Human action is to a great extent shaped by the meanings that participants make of their local environments (Robertson et al., 2018). Researchers aiming to gain insightful understanding of the “meaning-perspectives” – that is ideas, beliefs, values, or worldviews that underpin classroom interactions – have to draw on qualitative methods (Taylor, 2014). To a greater extent than quantitative educational researchers, qualitative

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<sup>1</sup>In fact, knowledge production in the natural sciences has almost become synonymous with the paradigm of positivism or empiricism (Taylor, 2014).

## 7. Methods

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researchers report on the physical, social, and cultural contexts that shape and are shaped by students' and teachers' interactions. This focus on the social and cultural context resonates with the sociocultural perspective that I have described in the theoretical framework.

Quantitative and qualitative research approaches provide different perspectives that can complement each other fruitfully. Since quantitative approaches often run danger of overlooking local contexts and individual students' subjective experiences, qualitative methods (or case-oriented research) serve as a convenient entryway to unpack the complexity, context, and dynamics of teaching and learning in the classroom (Robertson et al., 2018; Taylor, 2014). Moreover, quantitative approaches sometimes reinforce the gap between research and practice because they seldom focus on individual participants and often require a team of specifically trained academic researchers (Taylor, 2014). To counterbalance this tendency of dividing research from practice, qualitative methods have the potential to highlight the local contexts in the classrooms. In these contexts, qualitative researchers acknowledge that the researcher as an observer cannot be divided from the observed. This aspect of bringing researchers and practitioners into dialogue aligns well with the DBR-perspective that I have adopted in my research.

Irrespective of the specific methodological perspective, educational researchers should display awareness of the theoretical assumptions that underpin research processes and critically bring into questions these assumptions (Taylor, 2014). This critical awareness even extends to the language and nomenclature of educational research. For example, the very word "research" is readily taken to imply empiricism, the discovery of new facts, and the analysis of data - in short, quantitative approaches (Conroy and Smith, 2017). Yet, in the social sciences, non-empirical or mixed disciplines such as philosophy or sociology count as research as well (Smeyers and Smith, 2014). Conroy and Smith (2017) point out that fresh thinking and the clarification of ideas count as research, too. One should therefore keep in mind that educational research serves as an umbrella term for diverse paradigms in knowledge production - none of which is "better" or "truer" than another. Both quantitative and qualitative methods allow valuable insights into educational practices.

In this thesis, I ascribe to a qualitative research approach in line with the sociocultural perspective of ReleQuant. My approach has a particular focus on the processes of learning rather than the learning outcomes. Since GR is a challenging learning domain, I want to be able to give rich accounts of students' multiple and perhaps even contradictory views. How do students conceptualize that gravity might be both a force and a geometric phenomenon? With such questions in mind, qualitative research methods allow me to probe deeply into student learning processes that unfold in the classroom context while the students work with the digital learning resources. My research focus on processes instead of outcomes gets mirrored in the structure of the thesis as well: Instead of only reporting on the form of the final learning environment, I also elaborate on the design processes and the various iterations during the different stages of development.

## 7.2 Research design

Broadly, the research design of this PhD-work was informed by the MER and DBR frameworks. A key feature of the research design was its iterative nature and the close collaboration between the teachers and researchers throughout the whole research process. Since my PhD-project was situated within the greater educational project ReleQuant, I was able to seize on the existing ReleQuant network of partner schools and teachers to conduct my research. In total, we worked with eight teachers in five secondary schools in the greater Oslo area. All schools had middle to high-achieving students in national comparison. All teachers had participated in at least one ReleQuant workshop or seminar, which provided the opportunity to familiarise teachers with the learning resources, discuss different learning approaches, get feedback from previous classroom trials, and pool efforts to improve teaching and instruction more generally.

Over the course of three consecutive years, from spring 2016 to spring 2018, we trialled the learning resources in twelve physics classes with 239 students in total. The students were in their final year (age 18–19) and enrolled in the most advanced physics course available in Norwegian upper secondary school. The teachers led the physics lessons and teaching comprised two units of 90 minutes each. All students and teachers had consented to take part in the research project.<sup>2</sup> Figure 7.1 shows an overview of the research design consisting of three consecutive rounds of development and classroom testing.

## 7.3 Data collection

In line with DBR-methods and the previously outlined research design, classroom trials and data collection took place over a three-year period. As often is the case in DBR-research projects, data collection was rich. This choice to collect rich data was a purposeful one and aligns with the exploratory nature of much of my research. The learning domain of GR is abstract. Because of the scarcity of previous research on how students deal with the abstract concepts in GR, it seemed appropriate to collect data of how students make meaning of relativistic concepts during the actual lessons so that we would be able to identify crucial and challenging issues.

To be able to document and unpack students' learning processes with the digital learning resources, one to two field workers (one of them always me) observed each lesson. In six classes, video recordings supplemented the observations. To investigate how students talked physics, we asked them to take audio recordings of small-group discussions that were built into the learning environment. Students recorded the discussions with their smartphones and sent the files to the teacher who then forwarded these audio-files to the ReleQuant-team. We collected written responses that we retrieved directly from the learning platform; often these written responses summarised a student group discussion instead of presenting an individual student's ideas. To gain additional insight into students'

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<sup>2</sup>The consent form that students signed is attached in the appendix.

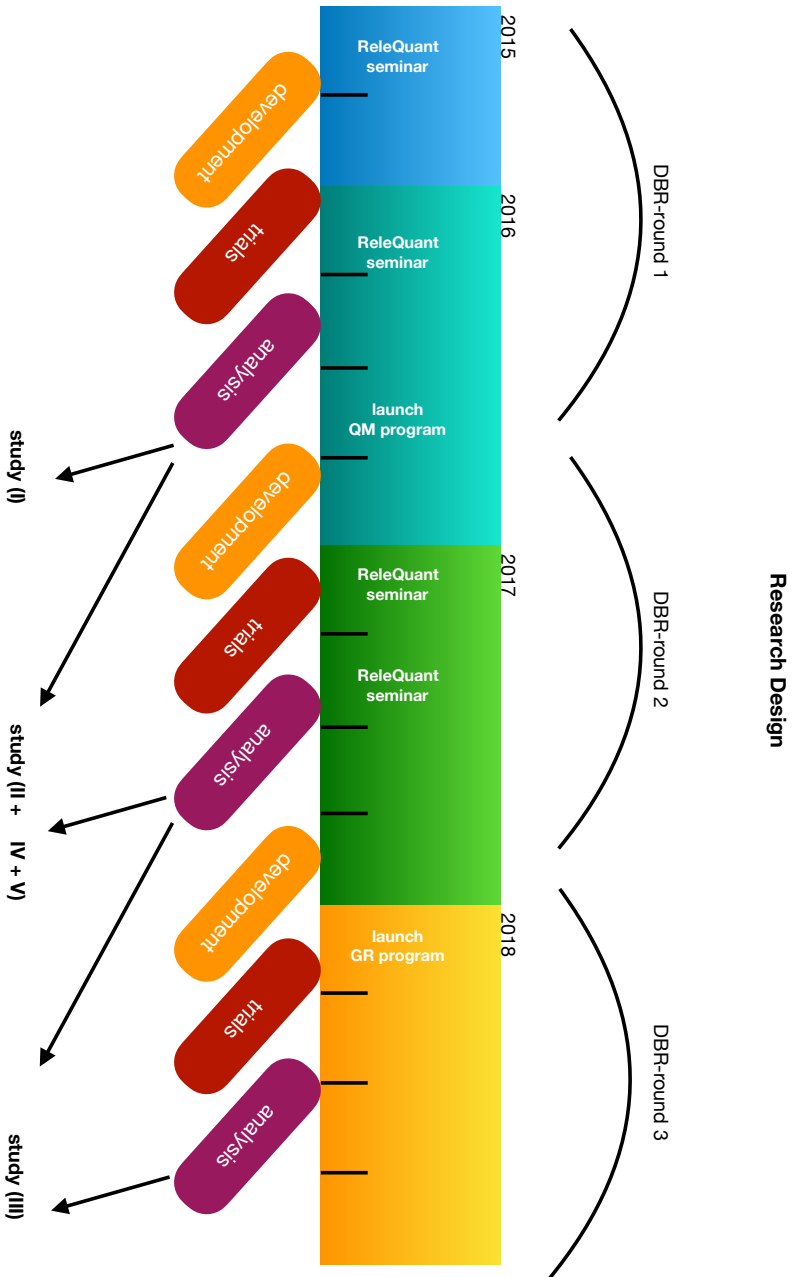


Figure 7.1: Overview of the research design consisting of three consecutive DBR-rounds.

experiences with the learning environment, we conducted semi-structured focus group interviews with five to nine participants in each school during the first two trials. This led to seven focus group interviews in total. The interview guide can be found in the appendix. During the first trial, we also conducted individual interviews with the teachers to get immediate feedback on how to improve the learning resources. Figures 7.2 to 7.4 give an overview of the collected data.

## 7.4 Data analysis

The two main methods of analysis I used across my empirical studies are thematic analysis and metaphor analysis. In the following section, I explain these methods and why I chose them to address my research questions. Reporting on the assumptions that informed my analyses allows other educators to contextualise and to compare my findings to other studies on teaching and learning of GR. It is worthwhile pointing out that language serves as a magnifying glass to focus attention to the way learning processes unfold. Thus, in my work, language runs as a common thread from the theoretical framework through the data collection to the methods of analysis.

### 7.4.1 Thematic analysis

Thematic analysis is a general method for identifying, analysing, and reporting patterns within data sets (Braun and Clarke, 2006). While qualitative approaches in social sciences can be very diverse and nuanced, thematic analysis is a foundational method for qualitative researchers. Identifying patterns is a basic feature of the ways humans understand the world around them. Promoting this search for patterns to a rigorous analysis technique makes qualitative research methods more transparent. Strengths of the method are its flexibility and theoretical freedom: irrespective of the chosen theoretical approach, thematic analysis offers an accessible tool to provide a rich and detailed, yet complex account of data. Braun and Clarke (2006) divide the thematic analysis into six distinctive steps:

1. First, a researcher familiarizes himself with the corpus of data. In my case, this familiarization meant transcribing and translating the audio and interview files, synthesizing my own field notes and those of other observers into more structured accounts, and closely reading through the written responses of students.
2. Based on the first familiarization with the data set, the researcher generates initial codes. I chose the qualitative data analysis software Atlas.ti to code our collected data because the software allows for a flexible and intuitive handling of large bodies of textual, audio, and video data. The flexibility of Atlas.ti to arrange, reassemble, and manage the material was particularly important during the second step of thematic analysis since this phase entails coding for as many potential codes and patterns as possible. In

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DBR-round 1		number of students	field observation	video observation	audio recordings of discussions	written responses	student focus group interview	individual teacher interview
<b>School A</b>								
class A1		20	X		X (3 discussion tasks)	X (6 written tasks)	X (6 students)	X
<b>School B</b>								
class B1		18	X	X	X (3 discussion tasks)	X (6 written tasks)		X
class B2		22	X	X	X (3 discussion tasks)	X (6 written tasks)	X (6 students)	X
<b>School C</b>								
class C1		21	X	X	X (3 discussion tasks)	X (6 written tasks)	X (5 students)	X
<b>School D</b>								
class D1		22	X		X (3 discussion tasks)	X (6 written tasks)		X
class D2		22	X		X (3 discussion tasks)	X (6 written tasks)	X (6 students)	X
<b>In total</b>		<b>125 students</b>			<b>97 audio recordings</b>	<b>418 written responses</b>	<b>25 students</b>	<b>6 teachers</b>

Figure 7.2: Overview of the data collection in DBR-round 1.

DBR-round 2	number of students	field observation	video observation	audio recordings of discussions	written responses	student focus group interview
<b>School A</b>						
class A1	19	X		X (4 discussion tasks)	X (4 written tasks)	X (6 students)
<b>School B</b>						
class B1	20	X	X	X (4 discussion tasks)	X (4 written tasks)	
class B2	20	X	X	X (4 discussion tasks)	X (4 written tasks)	X (6 students)
<b>School C</b>						
class C1	19	X	X	X (4 discussion tasks)	X (4 written tasks)	
class C2	19	X	X	X (4 discussion tasks)	X (4 written tasks)	X (9 students)
<b>in total</b>	<b>96 students</b>			<b>92 audio recordings</b>	<b>246 written responses</b>	<b>21 students</b>

Figure 7.3: Overview of the data collection in DBR-round 2.

DBR-round 3	number of students	field observation	video observation	audio recordings of discussions	written responses	student focus group interview
School A						
class A1	19	X			X (4 written tasks)	
In total	19 students				48 written responses	

Figure 7.4: Overview of the data collection in DBR-round 3.



generating initial codes, I was careful to code meaningful chunks of the data such as whole sentences instead of single words to maintain the context of the code.

3. As a next step, the researcher searches for themes based on the initial coding. This step is crucial because it redirects the analysis from the level of codes to the broader level of themes and larger patterns. In study (I), I coded almost all of the data (interviews, written responses, audio discussions) that we had collected in the first DBR-round of trials. Because of the sheer amount of data and the multitude of initial codes, it was difficult to structure and communicate patterns in the data in a meaningful way. Therefore, we discussed this particular step of the thematic analysis during several meetings with all authors of study (I). In study (II) and (IV), in contrast, the identification of themes was more straightforward because the data sets were smaller and the research questions more focused.
4. Step four entails reviewing and refining the tentative codes and themes. Figure 7.5 shows a network that I used to structure the codes and themes of study (I). The visual map of the codes helped me to review the codes and discuss them with my co-authors in a meaningful way.
5. Fifth, the researcher defines and names the final codes and themes. Based on the previous steps, I found this process quite easy. I made sure to choose names that would help readers link the analysis to the research questions. In study (I), for example, themes relate to the key concepts in GR.
6. The last step of the thematic analysis involves the production of the report. These reports have become the method and result sections in studies (I), (II), (IV), (V).

The process of thematic analysis involves a constant moving back and forward between the entire data set, the coded extracts, and the tentative findings. In particular, writing and reporting on the findings is an integral part of the analysis, not something that takes place at the end of it (Smith, 2018). Importantly, codes do not “emerge” from the data set and researchers do not “discover” them either. There is no “hidden pattern” in the data set other than what the researcher reads into it. Thus, researchers who naturally have theoretical and epistemological commitments play an active role in arriving at themes and in selecting which of the findings are of interest (Braun and Clarke, 2006). In the section on quality standards, I address my role as a researcher in the research process in more detail.

## **7.4.2 Metaphor analysis**

Thematic analysis is a broad and flexible method to identify patterns in data. Yet, to probe deeper into the conceptual understanding of abstract relativistic concepts that students develop when interacting with their peers and with our

## 7. Methods

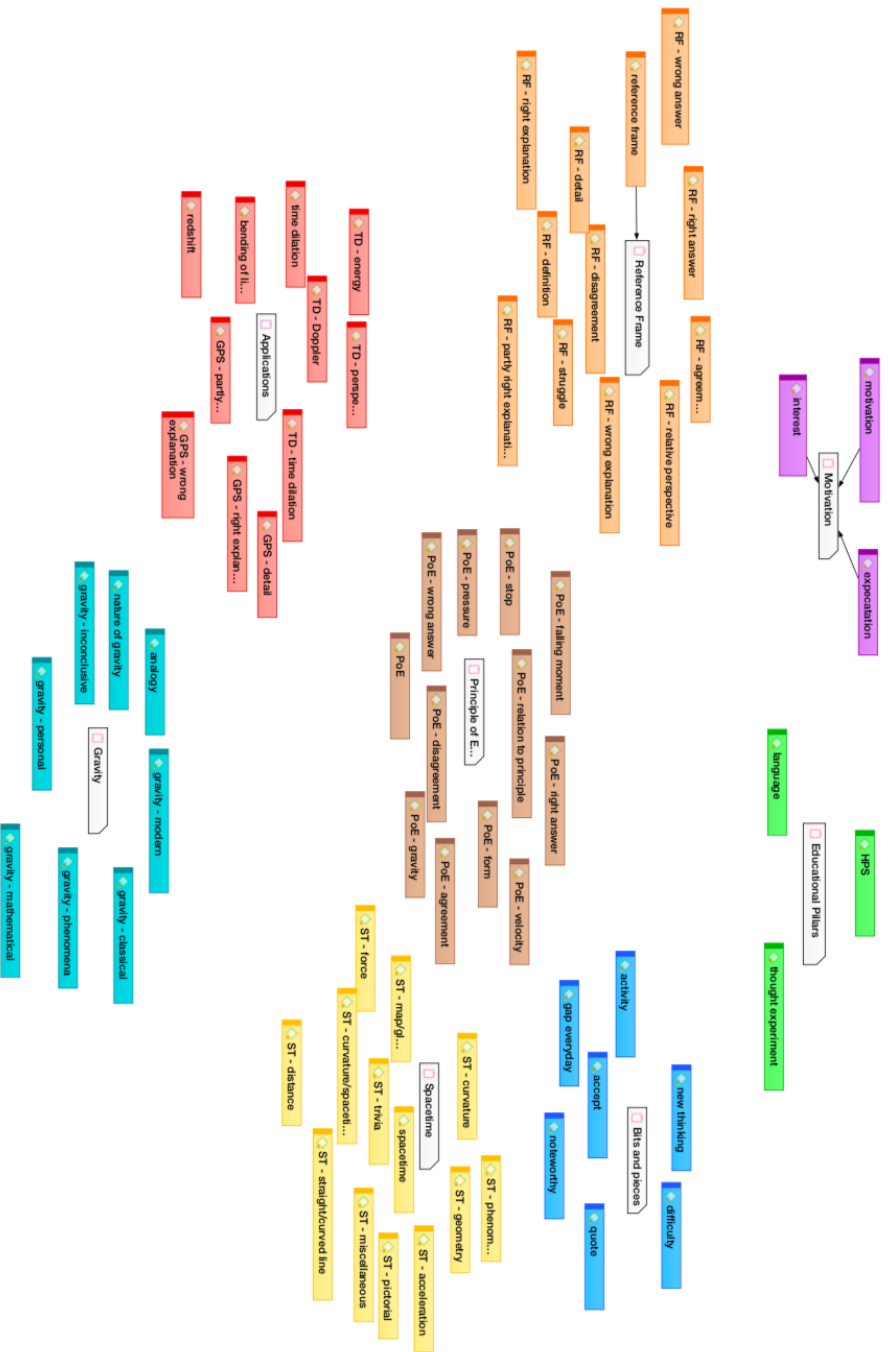


Figure 7.5: As part of the thematic analysis, codes get gathered into themes. This network of themes and codes stems from study (I).

digital resources, I needed a more refined analytical tool. I chose systematic metaphor analysis (Schmitt, 2005) as a means to obtain deeper insights into students' understanding of relativistic key concepts for two reasons. First, our inability to visualise many of the abstract concepts in GR makes metaphors and analogies an appealing tool to make meaning of key ideas of GR. Second, instructional analogies and metaphors have become a popular tool in science education. This trend in turn has prompted science educators to use metaphor analysis to study students' conceptual understanding from a new perspective, namely the perspective of embodied cognition (Amin et al., 2015; Lancor, 2014, 2018; Niebert et al., 2012; Treagust and Duit, 2015).

Systematic metaphor analysis allows educators to study students' use of scientific language on a fine-grain scale by drawing from findings from cognitive science and linguistics. The approach goes back to Lakoff and Johnson (2003) who argued that metaphors are not only a linguistic phenomenon but also a fundamental feature of thought and mind. Forming the basis of our conceptual systems, metaphors serve as a principal vehicle for understanding. We systematically use inference patterns from one conceptual domain to reason about another conceptual domain. Metaphors are thus not merely comparisons between two different things or concepts. Metaphors are frames through which we perceive and make meaning of the world (Schön, 1979).

Generally, a metaphor is a statement that characterises one thing in terms of another. This broad definition of metaphors encompasses analogies as well (Lakoff and Johnson, 2003). Indeed, most science educators treat analogies and metaphors synonymously (Niebert et al., 2012). Systematic metaphor analysis promotes the analysis of metaphors and analogies to a rigorous qualitative research procedure that allows reconstructing metaphorical concepts based on written or oral accounts (Schmitt, 2005). The two crucial steps in a systematic metaphor consist in

- (1) identifying a metaphor and
- (2) reconstructing metaphorical models.

First, to identify metaphors, one looks for phrases that can be understood beyond their literal meaning (source area) and that are transferred to a new and often abstract area (target area). Niebert et al. (2012) present the example "An Atom Is Like The Solar System". Here, the abstract properties of an atom that are difficult to visualize are related to a familiar model of the solar system.

Second, to reconstruct metaphorical models, a process that Niebert et al. (2012) called "categorizing the level of conceptual metaphor", one groups the metaphorical phrases that have the same source and the same target area. This categorization can be condensed in form of the so-called metaphor equation:

$$\mathbf{target\ area = source\ area}$$

For the above example, the metaphor equation would read "atoms = solar systems". We may imagine an atom (including its difficult to perceive properties)

through metaphor by drawing on our previous experiences with physical models of the solar system. The metaphor equation captures the underlying logic of metaphors in language.

In study (II), I was interested to unpack the affordances that the rubber sheet analogy provides when students attempt to make meaning of curved spacetime (Kersting and Steier, 2018). Applying systematic metaphor analysis to textbook accounts and our own empirical data of student discussions allowed us to study the underlying logic of the rubber sheet from a linguistic perspective. This linguistic exploration provided deeper insights into students' conceptualisation of curved spacetime than the thematic analyses of study (I) (Kersting et al., 2018a).

### 7.5 Quality standards and context of the research project

The objective of my PhD-project was a broad one targeting an educational reconstruction of GR for upper secondary school students with a specific interest in understanding student learning processes. I have chosen a qualitative research approach to address my research aims. In this section, I discuss issues of transparency, context, and research ethics that are related to my methodological choices.

I introduced the nature of qualitative research methods by contrasting these methods to traditional quantitative approaches. Likewise, to discuss quality standards for qualitative research, it is useful to look at the standards for quantitative research first. Quantitative research methods are regulated by standards of objectivity embodied in various forms of validity and reliability (Taylor, 2014). The notion of validity describes the truthfulness of a study in the sense that the study tries to give an authentic portrait of what the researcher is looking at (Miles and Huberman, 1994). The notion of reliability captures the consistency of a study in the sense that the research process should be consistent and methods and findings stable over time and stable across researchers (Maxwell, 2013).

While the standards of validity and reliability seem appropriate for quantitative approaches that are characterised by recurrence-oriented research and a “principle of verificationism” (Taylor, 2014, pp 41), many qualitative researchers argue that these standards are epistemologically irrelevant for their work (Schwandt, 2001). Quantitative researchers try to obtain descriptions of reality based on statistic methods that are related to concepts of probability. The underlying goal of quantitative educational research is to generalize from a certain sample of students to a greater population. Qualitative researchers, in contrast, acknowledge and emphasize that they discover interpretations of reality married to a certain social context without the ultimate goal to obtain generalizable findings.

There is no final agreement on quality standards for qualitative research yet. However, there are epistemic and ethical criteria that support qualitative researchers to account for the production of their results and to give reasonable

interpretations in line with the observational data (Maxwell, 2013; Taylor, 2014). One such quality criterion is trustworthiness which addresses methodological issues that are related to the concepts of validity and reliability. Researchers aim to produce credible findings by constructing deep and rich insights of the meaning-perspectives of participants based on prolonged immersion in the participants' social environment and context. Another criterion is authenticity that is unique to methods of qualitative research. Qualitative researchers should aim to establish and maintain relationships of mutual understanding and mutual benefit with their participants always keeping in mind that they can never truly be objective observers or "outside" of the social context of their participants (Taylor, 2014).

### 7.5.1 Trustworthiness and authenticity

Since MER and DBR are two frameworks that allow for a variety of choices in research methods, it is important to look at the interplay of the individual components of the research design to critically examine the trustworthiness and authenticity of my research. The iterative nature of both MER and DBR facilitate trustworthiness of the research. By conducting iterative rounds of trial and (re-)design, I was able to gain deep insights into the context of upper secondary physics learning in Norwegian high schools over the course of three years and within twelve physics classrooms. Our observations were credible to a high degree because all of the data collection took place during the physics lesson or, in the case of focus group interviews, during lunch breaks in the familiar classroom setting. Moreover, data collection was guided by a particular focus on collaboration, never separating the students from the learning environment or from their peers.

By collecting a diverse set of data – thus often capturing the same learning processes through various means (a student discussion might have been observed by a field worker, captured on camera, audio-taped by the students on their phone, and recalled by students during a subsequent focus group interview) – we were able to gather rich descriptions of the learning processes that allowed us to bring together diverse perspectives. In study (I), for example, we strengthened the trustworthiness of our findings through analysis of rich data coming from field notes, written responses, audio recordings, and interviews. In study (II), in which we based our inquiry only on one source of data (written summaries of student discussions of one particular discussion task), we combined two different methods of analysis to gain different perspectives on students' meaning making processes using the rubber sheet analogy. Study (V), on the other hand, combined interview data from two consecutive rounds of classroom trials to account for a greater diversity when looking at students' motivation and engagement in GR.

To achieve a greater authenticity of the research design, we involved teachers in every step of the process: teachers helped to design learning resources, they carried out the lessons, they provided feedback, and shared their own experiences of challenges in teaching and learning of GR. Moreover, having the same group of researchers and teachers involved over the course of the three

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iterations strengthened authentic relationships of mutual understanding and mutual benefit.<sup>3</sup> This collaboration between researchers and practitioners is a methodological strength of DBR: “In design-based research, the process of forcing the same people to engage the theory, the implementation of interventions, and the measurement of outcomes encourages a greater degree of methodological alignment” (Hoadley, 2004, pp 205).

Of course, when addressing issues of authenticity, my own involvement and my continued role across the research design and execution need to be critically examined as well. I brought to my PhD-project a broad variety of professional experience as well as certain theoretical commitments. Therefore, I have to give a clear account of my background and my assumptions that informed the processes of arriving at findings and drawing conclusions. Reflecting over my own situatedness in the research and showing awareness of how we obtain data is one way of raising quality standards in qualitative research projects (Johansson, 2018; Maxwell, 2013). Being trained as a theoretical physicist and mathematician, I had a sound knowledge in topics of GR and differential geometry when I started my PhD-project. I was therefore familiar with the relativistic concepts of GR, which allowed me to get a grasp of the physics context easily and to focus on educational aspects of the project right from the beginning.

However, my familiarity with GR was also a potential risk since many relativistic concepts seemed a lot easier or more “natural” to me than to my colleagues, teachers, and students. I had worked as a teaching assistant in undergraduate maths and physics courses and had therefore already some teaching experience. Still, it was a learning curve for me to adapt my explanations and instructional approaches to audiences other than university students and in particular to the audience of upper secondary school students. In the next section, I sketch the trajectory of the development of one particular learning activity that reflects the gradual simplification and unpacking of the idea that “gravity is geometry”. This trajectory also reflects my growing experience with teaching GR at various levels of difficulty.

My experience of having studied GR also helped me establish authentic relationships with the participating students in the classrooms. During the trials of the learning environment, I was not only a passive field observer but I introduced the ReleQuant-project and my research to the classes at the start of the unit as well. Often, I would help answer questions related to topics of GR but also questions that related to my experience of having studied physics and mathematics. My background gave me credibility and allowed me to share my experiences with students, which in turn allowed us to co-construct meaning of GR by shared and authentic attempts of talking physics. This joint construction of meaning within a shared social context became particularly prominent in the focus group interviews. From a sociocultural perspective, students did not

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<sup>3</sup>While the group of researchers stayed the same during the whole project, there was variation within the group of participating teachers. Since not all teachers have a „Fysikk 2” course every year, some teachers of the first trial did not participate in the second trial while new teachers joined the project. Nonetheless, we invited all teachers, old and new, to ReleQuant seminars and workshops to pool everyone’s experiences.

give objective insights into their true experiences of learning with the digital learning resources; rather, they shared certain views and interpretations of their experiences that were contingent on the interaction with their peers and me as a physicist and researcher facilitating the interview situation (Kvale and Brinkmann, 2015).

As described earlier in this section, qualitative research does not necessarily sit easily within the traditions of science education because many science education researchers ascribe to the objectivity of the scientific method in the process of knowledge production (Taylor, 2014). In a similar vein, having been trained in applying the scientific method and in thinking along natural scientific lines of reasoning, I grappled with many of the educational and social science perspectives that were not based on the familiar (post-)positivist paradigm. To conduct this research in an authentic and sound way, I had to learn to “talk like an educational researcher” and become part of the community of social scientists. I had to learn the languages of pedagogy and philosophy, the vocabulary of post-positivism and postmodernism, hoping that with a more active level of word knowledge my understanding and appreciation of educational methods would increase.

The resulting clash of perspectives stemming from the natural and social sciences and my struggle to understand different paradigms of knowledge production have helped me to generate a more nuanced view on science education and science education research. I have exercised scepticism towards many of the underlying assumptions in science education research; in fact, often I have been frustrated by a lack of clarity in the methods and foundations of the field. Therefore, much of my inquiries have been motivated by the wish to get to the bottom of educational theories and concepts. This approach is for example reflected in the presentation of the theoretical framework of this thesis in which I carefully situate my research within a sociocultural research tradition.

Finally, it is important to mention that I conducted my PhD-project as a non-native Norwegian speaker in a Norwegian educational setting. The focus on language in my PhD-project and my interest in the ways that language allows us to understand and express concepts in physics were therefore motivated both by the sociocultural stance of ReleQuant but also very much by my personal experience of navigating familiar landscapes in an unfamiliar language. Being a non-native speaker clearly influenced my role as a researcher.

As opposed to most of my colleagues who had been educated in Norway, I was not familiar with the Norwegian school system. While a lack of closeness to the research context can be a problem when it comes to interpreting observations, the novelty of the experience can be an asset as well. Familiarity and boredom are common problems of observation in educational settings (Johansson, 2018). I did hardly suffer from boredom or familiarity during my field work because much of the Norwegian school context remained interesting enough for me to stay focused and alert during observations. Moreover, my perspective was a fresh one allowing me to identify patterns that might have slipped the attention of observers too familiar with the classroom scenes.

While a lack of familiarity with a research setting can help focus the attention of a researcher, there are obvious disadvantages of not being a native speaker

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when conducting research in authentic classroom settings. Since my Norwegian vocabulary was limited and since I did not have the same cultural background as our students and teachers, there was always the danger of me misunderstanding utterances, missing subtleties, or not getting cultural references in student conversations. Consequently, the transcription of audio data took me longer than it would have taken a native Norwegian speaker and was probably more prone to errors. Also the translation of excerpts from Norwegian to English might have been affected by my limited vocabulary.

To counteract these problems during the transcription process, I asked our research assistants for help when I was struggling to understand certain words or phrases in the audio files. Moreover, many of the audio files were transcribed by our research assistants who were native Norwegian speakers and who had worked as field observers during classroom trials with the ReleQuant resources. To ensure that the English translations of student quotes were accurate, my co-authors double-checked several of my translations in study (I). In study (II), in which we put particular emphasis on the language of students through the systematic metaphor analysis, we discussed the translation of ambiguous words within the research group. For example, the literal translation of the Norwegian verb “*å se for seg*” is “*to see in front of you*”. However, this expression is better translated as “*to visualise*”, “*to envision*”, or “*to see in one’s mind’s eye*”. Through continual discussions on how to best translate words and expressions, we tried to ensure that we would stay as close to the original statements as possible.

My limited vocabulary and my growing mastery of the Norwegian language led to interesting discussions during group meetings when we addressed the design of the learning resources. A particular memorable discussion centered on the question of how to translate “spacetime” to Norwegian because there were two competing translations in common use. In fact, it was only two years after I started my PhD-project that a team of astrophysicists agreed on the official Norwegian translation of spacetime (Hammerstrøm, 2017).<sup>4</sup>

A similar discussion addressed the question of how to express the “warpedness” of time. Gravitational fields influence the passing of time which is often explained by saying that massive objects warp time. But what is time and can time even be warped or curved? What is a good translation of “warped” when the Norwegian “*krumning*” describes both “curvature” and “warpedness”? The struggle to find suitable Norwegian words for relativistic concepts might seem

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<sup>4</sup>“Egentlig burde det ikke være så vanskelig å oversette «spacetime» – det kan gjøres helt direkte: romtid. Så kan man lure på hvordan oversettelsen tidrom har dukket opp og blitt en tilsynelatende hyppigere brukt term for å beskrive nøyaktig den samme tingen. Disse to variantene brukes om hverandre, og det er vel egentlig ikke så farlig. Astrofysikere vet at det er snakk om det samme, men det er kanskje ikke like tydelig for alle andre. Og hvilken av de to variantene er det egentlig best å bruke? (...) Den norske oversettelsen må være tidrom. Det er vanskelig å sette fingeren på akkurat hvorfor det må være sånn, men tidrom høres rett og slett mest riktig ut for det vi snakker om. Det høres ut som om romtid betyr noe litt annet, bare ved at de norske ordene kommer i den rekkefølgen: Tidrom høres ut som en forening av tid og rom, mens romtid høres ut som tiden til et rom – en liten nyanseskjell.” (Hammerstrøm, 2017)



like a linguistic banality but it cuts right through to the heart of the matter: conceptual knowledge is intimately linked to an active level of word knowledge. Since Norwegian was a third language for me, this relation between knowledge and language was a particular prominent lens through which I approached my research.

### 7.5.2 Research ethics

Besides aspects of trustworthiness and authenticity, quality standards for well-designed research must also encompass an ethical dimension. Since educational research is concerned with humans and their behaviour, questions of ethics inevitably enter into the research process. These questions bear on more than traditional guidelines for good scientific practice (All European Academies (ALLEA), 2017). A big concern when conducting research with human participants is the potential of causing harm because it can be hard to predict the effects that observing and interacting with participants can have on them (Johansson, 2018). In the case of educational research, this potential harm relates mostly to social or psychological factors. As educational researchers, we do not want to upset or disturb students.

To ensure that this PhD-project complied with the respective laws in Norway, the Norwegian Centre for Research Data was notified of the data collection and approved the ReleQuant studies. Moreover, all teachers and students participated voluntarily. We obtained permission from the participants through written forms that informed about the background and the aims of the research project and the use and storage of the participants' data. To avoid a potential risk of harm through disclosure of personal information, we avoided naming the schools where research had been conducted. Moreover, quotes of students were always presented in a way that did not reveal student identities.

### 7.5.3 The Norwegian context

Qualitative research is not set up to generalize from the sample of a specific study. Instead, qualitative research is for the purpose of developing critical, analytical, in-depth insights. According to Steier (2014), there is a common hesitancy to generalize highly contextualized DBR-research studies beyond the scope of the particular intervention: "The expectation is not that a particular study is perfectly reproducible, but that the findings have some applicability in other contexts. Generalizability, in the context of DBR, is understood as the development of usable knowledge which can be applied to future iterations of a design." (Steier, 2014, pp 64)

The online learning environment developed in this project was built to help Norwegian upper secondary school students achieve the specific aims in the national physics curriculum. Thus, the design principles may need to be adapted to other groups of learners and different curricular aims. I think, however, that my findings demonstrate the feasibility of communicating central aspects of GR

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qualitatively and that our design principles represent insights that may be quite broadly applicable.

Since my studies of upper secondary school students' conceptual understanding of key concepts in GR had an exploratory character, the main focus was to use different methods and research perspectives as ways to explore a variety of phenomena observed in student learning processes in the learning domain of GR. The ReleQuant partner schools and our partner teachers were all situated in the greater Oslo area and had middle to high-achieving students, which made it easy for me to collect rich data for my explorations. However, these schools and the context of the research were not necessarily representative for all Norwegian upper secondary physics classrooms or more international contexts. In studies (I), (II), and (IV), I discuss questions of how to translate findings from the Norwegian to other contexts in more detail. I return to this question in the conclusion as well when giving an outlook on international efforts in Einsteinian Physics education research.





## Chapter 8

# Results: Design of the Learning Environment

Historically, some of the best minds in the world have addressed themselves to education; for example, Plato, Rousseau, Dewey, Bruner and Illich. But they have addressed education essentially as theorists, even where they have tried to design schools or curricula to implement their ideas. What is different today is that some of the best minds in the world are addressing themselves to education as experimentalists: their goal is to compare different designs to see what affects what. Technology provides us with powerful tools to try out different designs, so that instead of theories of education, we may begin to develop a science of education. But it cannot be an analytic science like physics or psychology; rather it must be a design science more like aeronautics or artificial intelligence. For example, in aeronautics the goal is to elucidate how different designs contribute to lift, drag, manoeuvrability, etc.. Similarly, a design science of education must determine how different designs of learning environments contribute to learning, cooperation, motivation, etc.

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*Allan Collins*

The objective of my PhD-project was two-fold addressing both the development of a digital learning environment and students' learning processes in GR. Therefore, the presentation of my results follows a two-fold structure as well. In this section, I focus on the design and evaluation of the learning environment. In the next section, I present my findings on students' learning processes and conceptual understanding. Both of these objectives are integrated in the MER-framework: The design of the learning environment (including the content structure for instruction and the design principles) is an important part of my proposed educational reconstruction of GR. Yet, the reconstruction does not end just there. New findings on learning processes and student perspectives feed into the constant re-development and refinement of learning resources and/or content structures for instruction. This iterative process is particularly interesting when educators try to adapt the material to different contexts and different age groups.

Even though the final learning environment can be viewed as a result in its own right, the interesting challenges and insights that led to its final form arose during the development. In line with DBR-methods, the presentation in this section thus encompasses a detailed discussion of the design hypotheses and design principles that guided the development of the learning environment. Moreover, I present the development of one specific learning activity in more detail. In the first round of our trials, this activity failed to engage students productively. In

describing the trajectory of the development process of this activity, I want to bring greater clarity to the process of designing learning resources in modern physics – an aspect that is often neglected in published research articles (Méheut, 2004). Again, this focus on iterations and processes aligns with the choice of the frameworks of MER and DBR and with my qualitative research approach.

### 8.1 Content structure for instruction

In line with the MER-framework, the first steps towards an educational reconstruction of GR consist in identifying key concepts of the theory as well as understanding student perspectives. Both investigations then lead to the development of a content structure for instruction that is suitable at the secondary school level. In this section, I present my proposed content structure for instruction. To link the DBR-methodology to the MER-framework, I integrated design hypotheses, and eventually design principles, into the content structure of instruction. My proposed content structure for instruction is therefore twofold: The content structure consists of specific learning goals that are accompanied by design principles on how to present the key ideas of GR in a qualitative way. I present both the learning goals and the design hypotheses/principles in this section.

In the introduction of this thesis, I briefly describe key ideas and educational challenges of GR. In study (I), I presented both of these topics as two components of the MER-framework (Kersting et al., 2018a). First, based on a literature review of university and high school textbooks, I summarised key concepts of GR from a physics perspective.<sup>1</sup> I divided these concepts into two parts, the conceptual foundation on one side and relativistic phenomena on the other side. Interestingly, Einstein did a similar divide in his own accounts of SR and GR (Kim and Lee, 2018). Second, I reviewed the science education literature to obtain a tentative list of challenges that secondary and undergraduate students face when learning about relativistic concepts. It is important to note that I conducted this literature review in 2015/16. Since then, the science education community has seen a rise in published papers on students' conceptual understanding in SR and GR. I comment on this development and the latest research findings in the conclusion.

#### 8.1.1 Content structure – part 1: Learning goals

The Norwegian physics curriculum asks secondary school students to give a qualitative description of GR. To develop a learning environment about GR in line with this curriculum goal, I unpacked the key concepts of GR that would correspond to a qualitative understanding of GR, thereby reconstructing the

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<sup>1</sup>In a sense, the presentation of GR in high school textbooks presupposes a certain form of “educational reconstruction” that the textbook authors undertook. However, in contrast to many such implicit “educational reconstruction”, my work made the reconstruction of GR for educational purposes explicit by adhering to the structure of an educational framework.

scientific content structure. I structured this reconstruction in form of specific learning goals that I present in Table 8.1. In the following, I explain how I arrived at these learning goals and discuss how they laid the basis for the development of the digital learning resources. These explanations serve as a supplement to study (I) and draw particular attention to the development process of the educational reconstruction. Moreover, being clear about the assumptions and decisions that informed my suggested learning goals will help educators and teachers to adapt these learning goals to different contexts more easily.

In the introduction, students should become aware of the need for a new theory of gravity. Levrini (2014) argued that students would only be willing to accept novel ideas of space and time if they became dissatisfied with current models. Thus, our first learning goals introduce GR as a new theory of gravity and ask students to explain why Newton's theory of gravity is not compatible with Einstein's theory of special relativity.

Following the historical development of GR, we suggest introducing GR based on the principle of equivalence which is in line with the presentation in school and university textbooks that I analyzed. Several famous thought experiments can illustrate the idea that locally, one cannot distinguish between accelerated systems and gravitational fields. We chose to omit a learning goal about the equality of inertial and gravitational mass because both a pilot study in Norwegian classrooms and previous research suggested that students often fail to grasp the implication of this equality (Bandyopadhyay and Kumar, 2010a,b; Ytterhaug, 2015). Since students often struggle to apply the principle of equivalence to new situations (Velentzas and Halkia, 2013), our learning goal incorporates two different perspectives of looking at the principle. Being accelerated is equivalent to being in a gravitational field and feeling weightless is equivalent to being far away from any gravitational source.

To enable students to draw on their previous knowledge and to show how GR extends special relativity, the principle of equivalence is used to formulate a new definition of an inertial frame as a reference frame in free fall. With this definition, the special principle of relativity can be extended to the general principle of relativity: the laws of physics take the same form in all reference frames.

The principle of equivalence and the general principle of relativity can seem very abstract. Therefore, we suggest presenting three relativistic phenomena as direct consequences of those principles: bending of light, gravitational time dilation, and gravitational frequency shift. All three phenomena can be deduced from the principle of equivalence and allow making links to students' everyday lives. Gravitational time dilation plays a crucial role in modern GPS technology, the bending of star light around the sun was the first observation that confirmed GR in 1919, and gravitational redshift draws a link to the classical Doppler effect.

The principle of equivalence suggests that gravity is not a force. We formulated several learning goals that describe how gravity can be viewed as a manifestation of curved spacetime to offer new explanations of gravity. We were careful to include explicit learning goals that describe the universe as having three

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Table 8.1: The first part of the content structure for instruction consists of learning goals. These learning goals are presented in study (I). A simplified version of the learning goals can be found in the Viten-programme *General Relativity*.

<b>Content</b>	<b>Learning goals</b>
Introduction	Describe general relativity as a new theory for gravity. Explain how the fact that Newton's force of gravity acts instantaneously contradicts Einstein's claim that nothing can move faster than the speed of light.
The principle of equivalence	Use the fact that locally it is impossible to distinguish between a gravitational field and a uniform acceleration between free fall and the absence of a gravitational field to explain how acceleration and gravity are equivalent phenomena.
Reference frames and principle of relativity	Describe an inertial reference frame as a <u>reference frame in free fall</u> . Explain that the laws of physics take the same form in all reference frames.
Relativistic phenomena (bending of light, gravitational redshift, time dilation)	Give examples of phenomena that are predicted by GR but not by Newton's theory of gravity. Describe how light travelling through the gravitational field of the sun is deflected and use the principle of equivalence to explain why this is <u>predicted by GR</u> . Describe how time goes slower close to massive objects and use the principle of equivalence to <u>explain why this is predicted by GR</u> . Explain how light that moves away from a gravitational source is redshifted and light that moves towards a gravitational source is blueshifted.
Spacetime and curvature	Explain how general relativity is a theory describing the relationship between space, time, and gravity. Describe our universe as having three special and one temporal dimension. Explain that gravity is not a force, but a geometric phenomenon. Describe how mass curves spacetime and how curvature influences the movement of mass.



spatial dimensions and one time dimension and that describe general relativity as a theory linking those space and time dimensions to gravity. Moreover, our formulations emphasize the geometric nature of gravity and focus on conceptual ideas that underpin GR.

## **8.1.2 Content structure – part 2: Design hypotheses and design principles**

Characteristic features of the DBR-framework are the iterative rounds of design revisions and the subsequent reporting on these different phases of design research (Collins et al., 2004). To account for this focus methodologically, I made the design of the GR learning environment a crucial part of study (I). Doing so, I formulated tentative design hypotheses that guided the development of the learning resources. Specifically, I analysed students' experiences during the first trial in line with these design hypotheses. Testing the design hypotheses in the classroom setting allowed for iterative revisions in the design process and enabled us to arrive at research-based design principles.

### **8.1.2.1 Design hypotheses**

The hypotheses that guided the development and evaluation of the learning environment stemmed from the review of students' difficulties in GR as well as theoretical perspectives on how language and HPS and NoS can support motivation and learning. In this section, I explain how we arrived at our design hypotheses. Study (I) presents more details and screenshots of the online learning environment that illustrate the implementation of the design hypotheses (Kersting et al., 2018a).

**Design hypothesis 1: Students can grasp central ideas in GR qualitatively without advanced mathematics by relying on geometric ideas.** In order to avoid the advanced mathematical framework of GR, we proposed an instructional approach based on elementary geometry. This approach has the potential to work well because it is possible to separate the geometric ideas of GR from its sophisticated mathematical descriptions: "The conceptual and the technical are often inseparable in advanced domains of physics. Yet Einstein's relativity is one domain where this separation is meaningfully possible." (Bandyopadhyay and Kumar, 2010b, pp 020104–1)

In her attempt to clarify basic concepts of GR from an educational and cultural perspective, Levrini (2002b,a) discusses conceptions of space as one central idea in GR. These conceptions align with geometric lines of reasoning. Indeed, using geometric ideas to foster understanding of GR is a strategy that has been promoted by several education researchers and physicists (diSessa, 1981; Hartle, 2005; Zahn and Kraus, 2018).

**Design hypothesis 2: Thought experiments, analogies and visualizations of relativistic phenomena foster understanding in GR.** Students lack direct

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experiences of relativistic phenomena because everyday life on earth meshes well with classical models of physics. One can use several strategies to deal with that lack of experience. First, we suggested a presentation of GR relying on thought experiments. Not only was Einstein himself a master of thought experiments but research suggests that thought experiments can be a powerful instructional tool as well: Thought experiments can bridge the gap between students' everyday lives and relativistic phenomena (Velentzas and Halkia, 2013).

Second, instructional analogies and metaphors have become a popular tool in science education because they can help to communicate abstract scientific concepts (Aubusson et al., 2006). In study (II), I explore this aspect of teaching and learning GR in more detail (Kersting and Steier, 2018).

Third, making use of new technologies in the form of videos and animations can open new ways of visualizing GR. Both Kraus (2007) and Weiskopf et al. (2006) argue for the need of new visualizations. According to Kraus, visualizations serve several important teaching purposes: they can introduce learners to the subject, they can counterbalance the development of misconceptions, they can be easily remembered, and they target a general audience.

**Design hypothesis 3: Emphasizing the break of relativistic with classical physics helps students overcome their classical preconceptions.** Since pre-existing ideas that stem from classical physics can hinder students' understanding of relativistic phenomena, one should try to present the break between classical physics and GR explicitly to make students become aware of their own preconceptions. Such a strategy has been used successfully in relativity (Velentzas and Halkia, 2017) and quantum physics (Bungum et al., 2015).

**Design hypothesis 4: Recalling background knowledge in special relativity allows students to align relativistic ideas from special relativity with general relativity.** Since GR extends special relativity to the realm of curved spacetime, explanations of special relativity alongside a presentation of GR can enable students to draw on previous knowledge. In particular, recalling the principle of special relativity and the definition of reference frames can help students integrate novel ideas such as the principle of equivalence. The integration of relativistic descriptions into the greater framework of physics aligns with Einstein's perspective on understanding relativity (Kim and Lee, 2018).

**Design hypothesis 5: Linking abstract topics to students' everyday life motivates and fosters understanding in GR.** Often, learners experience the nature of relativistic phenomena as counter-intuitive. Grounding the presentation of GR in students' everyday life, for instance by presenting technological applications, can facilitate understanding in GR. This approach has been successfully employed in quantum physics within project ReleQuant before (Bungum et al., 2015).

**Design hypothesis 6: Students are generally motivated by topics in relativistic physics such as black holes and spacetime.** Students are engaged by topics of modern physics (Angell et al., 2004). In particular, relativistic concepts such as black holes, gravitational waves, or curved spacetime have the potential to promote motivation among young learners (Pitts et al., 2014; Zahn and Kraus, 2014).

**Design hypothesis 7: Use of language and talking physics facilitates understanding of abstract concepts in GR.** Students at upper secondary level are not able to use the sophisticated language of mathematics to learn GR. However, they can use their everyday language when discussing with peers and in written exercises to deepen their understanding of physics concepts. Henriksen and Angell (2010) demonstrated that use of language was important for Norwegian physics students' understanding of classical physics; Bungum et al. (2015) showed that talking physics facilitated understanding in quantum physics among Norwegian upper secondary school students.

**Design hypothesis 8: Students are interested in the historical development of GR and its philosophical implications.** Teaching GR at a qualitative level requires different teaching approaches than traditional ways of teaching physics mainly for two reasons. First, without the mathematical tools at hand, students have to reason conceptually to arrive at conclusions. Second, one basic assumption of classical physics is that space and time are fixed. Yet, in GR this assumption does no longer hold because space and time evolve dynamically. Emphasizing historical and philosophical aspects of GR can be a fruitful approach to tackle these challenges: the "use of history is usually conceived as an effective teaching strategy for softening the impact of a theory that requires that space and time views be deeply revised" (Holton, 1973) as cited in (Levrini, 2014, pp 158). Such an approach prompts students to recognize the historical debate that has led to the development of modern physics. Acknowledging this debate might enable students to reconstruct fundamental issues of physics (Levrini, 2002b).

### 8.1.2.2 Evaluation of design hypotheses

Study (I) reports on the findings of the first round of trials in six Norwegian physics classrooms in 2016. We used the design hypotheses to structure our analysis of the experiences of the participating students. The detailed analysis can be found in study (I) (Kersting et al., 2018a). Here, I give a brief summary of the findings in relation to each design hypothesis to emphasize two features of this research project in particular:

1. I want to show that design hypotheses can serve as a useful tool to unpack the rich data sets that DBR-projects often collect. I used data from the focus group interviews and written and oral responses to compare the design hypotheses to the ways students experienced the lessons based

on our learning resources. The evaluation of student responses against a pre-defined set of hypotheses serves as a convenient link between the second and third component of MER.

2. I want to report on the actual development process of starting from an initial design task and eventually arriving at a final product. Through formulating and evaluating design hypotheses, researchers have a means of breaking down design processes systematically. In my case, the task was to develop a digital learning environment in GR that is suitable for teaching in upper secondary schools in Norway. The final product consists of the research-based learning environment and a set of design principles that can give guidance for similar tasks in the future.

**Evaluation of design hypothesis 1: Students can grasp central ideas in GR qualitatively without advanced mathematics by relying on geometric ideas.** I found that a geometric approach to GR was very successful in engaging students and facilitated learning of the geometric features of curved spaces. However, based on the collected data it was not clear whether such a geometric approach actually fostered a deeper understanding of curvature in four-dimensional spacetime or whether it only helped students to grasp ideas of two-dimensional and three-dimensional spatial curvature. While the first round of trials encouraged us to use geometric ways of reasoning in the learning resources, the open questions that remained inspired study (II).

**Evaluation of design hypothesis 2: Thought experiments, analogies and visualizations of relativistic phenomena foster understanding in GR.** Our findings suggest that thought experiments helped deepen students' engagement with key concepts in GR. When asked about the role of thought experiments in the learning environment, students answered that those were engaging and challenged their understanding, thus making it easier to probe their own knowledge of GR. Thought experiments were especially successful to explain the principle of equivalence. Many students confirmed that the thought experiments of Einstein stepping off a tall building and of a laboratory in an accelerated spaceship helped them develop deeper understanding of GR.

The use of analogies is a more ambiguous issue. We observed a heavy dependence on the rubber sheet analogy to visualize curved spacetime. The fact that this analogy was ubiquitous throughout the data set suggested that learners have a great need for visualizations. However, the analogy troubled some students and they criticized the comparison, because it relied on gravity. Study (II) therefore addressed the controversial nature of the rubber sheet analogy in more detail.

**Evaluation of design hypothesis 3: Emphasizing the break of relativistic with classical physics helps students overcome their classical preconceptions.** In the interviews, students repeatedly expressed the insight that GR greatly differs from classical physics. Often, they articulated the need for a

new way of thinking to understand GR. Students found this shift of perspective challenging but also exciting and fun. This finding supports our hypotheses that emphasizing the break between GR and classical physics can engage students and help them become aware of their classical understanding of physics. Yet, there is a difference between becoming aware of one's classical preconceptions and overcoming them. From the data we gathered it was hard to say if students truly overcame their classical preconceptions. Many students seemed to continue to reason along classical lines of thought or mixed Newtonian and Einsteinian concepts.

**Evaluation of design hypothesis 4: Recalling background knowledge in special relativity allows students to align relativistic ideas from special relativity with general relativity.** The interviews confirmed previous findings that students found special relativity and the new way of describing time and space difficult. Moreover, students tended to confuse principles of special and general relativity. These findings suggest that one way of helping students overcome their confusion might be to present more clearly the distinction between key concepts of special and general relativity and how GR aligns (or breaks) with special relativity and classical physics. Because of time and curriculum constraints in schools, drawing on special relativistic concepts could, in fact, make it more difficult for students to understand abstract concepts in GR. In particular, the concept of reference frames requires students to handle conflicting definitions in classical physics, special relativity, and general relativity.

**Evaluation of design hypothesis 5: Linking abstract topics to students' everyday life motivates and fosters understanding in GR.** A learning domain as abstract as GR runs into danger of seeming too far away from students' everyday lives. In the focus group interviews as well as in written responses on Newton and Einstein's descriptions of gravity, students repeatedly expressed this perceived gap between their everyday lives and relativistic ideas. At the same time, students confirmed that they appreciated everyday examples to illustrate the principle of equivalence, the geometry of curved surfaces, and relativistic phenomena. It seems that the presentation of both the conceptual foundations of GR as well as the implications of these key concepts benefit from links and references to everyday life.

**Evaluation of design hypothesis 6: Students are generally motivated by topics in relativistic physics such as black holes and spacetime.** Addressing their motivation and attitude towards learning GR, students confirmed our hypothesis that relativistic topics were engaging. Students wanted to learn more about the nature of space and time because they perceived the topic to be modern and relevant. The recent detection of gravitational waves has probably contributed to this perspective since many students mentioned this observation. Students' fascination might also be due to popular culture because Einstein has become a scientific icon in our society.

**Evaluation of design hypothesis 7: Use of language and talking physics facilitates understanding of abstract concepts in GR.** Generally, the increased focus on language was well received by students. The focus groups interviews supported previous findings on using language in science classrooms (Bungum et al., 2015; Chen et al., 2016; Henriksen and Angell, 2010). Talking physics fostered students' overall understanding in GR and discussions with peers and with the teacher were experienced as an engaging variation from regular teaching. In particular, students appreciated to think aloud and they liked that their understanding of GR was challenged by discussions which forced them to reason and to find arguments.

Nonetheless, students asked for more mathematical approaches and easy calculations to probe their understanding as well. This was a recurrent critique in the interviews that relates to the frustration that students experience when approaching GR only qualitatively. This finding shows that students are aware of and rely on the close relationship between mathematics and physics (Uhden et al., 2012). However, the challenge in GR is that we do not have easy calculations at hand and that students have to rely almost exclusively on non-mathematical explanations. To counteract the development of frustration among students, teachers have to raise awareness of different ways of learning physics: qualitative understanding and historical and philosophical perspectives should be seen as valuable learning goals in their own right (Bøe et al., 2018). Being able to contribute meaningfully in a physics discussion should be acknowledged as an accomplishment for a physics student in a similar way to the more familiar accomplishment of getting correct answers to physics problems.

**Design hypothesis 8: Students are interested in the historical development of GR and its philosophical implications.** Aside from aspects regarding the specific content of GR, students were particularly motivated by approaches relying on the history and philosophy of science. They approved of a historic perspective that gave them insight into the development of GR. Being able to follow Einstein's reasoning and his struggle to find a new theory for gravity seemed to enable them to draw connections to previous knowledge and to the theory as such. These findings support previous research highlighting how historical examples may support motivation and learning in physics (Chandler, 1994). In study (V), I used data from two rounds of classroom trials to look closer at the role of HPS and NoS.

### 8.1.2.3 Design principles

A key result of study (I) was the formulation of design principles that encapsulate the findings from the first round of development of the digital learning environment. In this section, I first comment on the final form of these principles and compare them to the initial hypotheses. In the next section, I present the learning environment in more detail. This presentation serves as an example of how to translate my proposed content structure for instruction to actual learning resources that can be used in classrooms.

Generally, many of the findings from our first trial confirmed the design hypotheses. This agreement is not surprising in light of the fact that previous research and literature findings informed the formulation of the hypotheses. Still, the final design principles offer more than the original hypotheses. Indeed, the principles extend and challenge several of the hypotheses by showing that I originally took a too simplistic perspective. The design principles thus take a more nuanced stance towards designing learning resources in GR.

As a result, there are more design principles than hypotheses. The principles cover a wider scope by encapsulating both general findings and more specific suggestions to guide the design of learning resources. Table 8.2 presents these empirically found principles that follow a two-level structure with four basic principles that are specified by more specific design principles.

1. The first design principle takes a nuanced perspective on relating GR to classical physics. Based on findings from the quantum physics trials in ReleQuant (Bungum et al., 2015), I originally assumed that it would be fruitful to emphasise in which ways GR breaks with classical physics. However, many students seemed confused to meet ideas that contradicted many of the previously learned physics concepts (Kersting et al., 2018a). Presenting GR as a topic that breaks with classical physics might actually make it harder for students to accept the theory. Indeed, such an approach is contrary to what Einstein envisioned when developing his ideas in relativity (Kim and Lee, 2018). Einstein tried to unify many ideas stemming from classical physics. At its heart, GR is still a classical theory even though the relativistic phenomena seem counter-intuitive. Consequently, the new design principle is more careful in emphasizing how GR generalizes and extends many ideas from classical physics. In fact, research from Australia suggests that it might be fruitful to introduce students to Einsteinian physics first before presenting classical physics as a valid approximation for everyday settings where speeds are low and the gravitational field is weak (Kaur et al., 2018).
2. The second design principle and its sub-principles encompass how GR can be taught successfully using various instructional tools. A key notion is students' lifeworld that replaces the originally used notion of students' "everyday life". After all, many of the relativistic phenomena do not overlap with students' everyday experience. Yet, many students felt that topics of GR were more relevant to their lives than classical topics of physics. Astronomical objects such as black holes or gravitational waves intrigued students to a great extent and, therefore, constitute part of their lifeworlds.
3. The observation that many topics of GR seemed relevant and of interest to students led naturally to the third design principle that links teaching and instruction of GR to students' interests and motivations. Students are motivated to learn about astronomical phenomena and they appreciate to take a broad perspective of history and philosophy to understand the development of GR from its origin to present day physics.

## 8. Results: Design of the Learning Environment

Table 8.2: Research-based design principles for learning resources in GR.

<b>Basic design principles and specific features</b>
<b>1) Emphasize how GR relates to and sometimes breaks with classical physics.</b>
a) Present the need for a new theory of gravity by showing that special relativity and classical mechanics are irreconcilable.
b) Present relativistic phenomena such as gravitational bending of light and time dilation to show that GR extends the scope of classical physics.
c) Point out how the definition of inertial frames in GR differs from similar notions in special relativity and classical mechanics. Ask students to apply the abstract definition of an inertial frame to specific problems.
d) Show that there exist concepts in GR that are NOT relative, such as the notion of inertial reference frames, to help students connect relativistic ideas to their classical understanding of physics.
<b>2) Link key concepts of GR to students' lifeworlds to counteract the lack of experience with relativistic phenomena.</b>
a) Use everyday examples to illustrate relativistic ideas and to enable students to connect GR to their everyday life. GPS technology can explain gravitational time dilation and the geometry of world maps can explain motion in curved spaces.
b) Use thought experiments as educational tools to help students understand abstract concepts in GR. Thought experiments that illustrate free fall and weightlessness are particularly successful when explaining the principle of equivalence.
c) Use analogies with caution. State shortcomings of analogies explicitly to prevent the formation of misconceptions. In particular, explain how the rubber sheet analogy oversimplifies the notion of curved spacetime.
d) Use visualizations in the form of digital simulations and animations to introduce students to relativistic concepts and to prevent the formation of misconceptions.
<b>3) Draw on students' prevailing motivation and interest to introduce key concepts in GR.</b>
a) Use astronomical phenomena to engage students. Gravitational lensing around black holes can illustrate gravitational bending of light and curvature of spacetime. Thought experiments involving spaceships can illustrate the principle of equivalence.
b) Present GR in light of its historical development. The solar eclipse in 1919 can serve as historical example for an experimental verification of GR. Relate Einstein's quest to find a new theory for gravity to abstract descriptions of GR.
c) Emphasize epistemological aspects of GR and explain how Einstein's new interpretation of space, time, and gravity has shaped our worldview.
d) Present GR as an active field of research by referring to the recent observation of gravitational waves.
<b>4) Invite students to use written and oral language to facilitate understanding of abstract concepts in GR.</b>
a) Give students the opportunity to 'talk physics' to their peers by using discussion tasks that probe conceptual understanding of key concepts in GR.
b) Ask students to summarize their understanding of key concepts in written exercises to let them practice the use of new physics vocabulary.
c) Use plenary discussions guided by the teacher to consolidate understanding of GR and resolve misconceptions.
d) Explain that our qualitative understanding of GR can be made rigorous by employing advanced mathematics.



4. The fourth design principle captures the focus on language when dealing with GR on a qualitative level. In trialing the resources, I found that students were repeatedly asking for equations, proofs, or other mathematical treatments of the theory. The wish to approach GR mathematically might reflect the education of students in traditional physics classrooms and their use of correct answers as a key source of their self-concept of ability (Bøe et al., 2018). We therefore included a principle that would make sure students learned that a mathematical treatment of GR was possible and less prone to confusion than a description relying on everyday language. After all, the limits of students' language often mark the limits of their understood world (Wittgenstein, 1921).

The formulation of design principles was, of course, not the final step of my DBR-project. I used these principles to inform the design of an improved version of the learning environment that was in turn tested in the second iteration of our data collection in 2017. Eventually, the design principles, learning goals, and empirical findings from several rounds of trialling led to the final form of the learning environment. The learning environment contains three units, which are designed for a total of 180 minutes of classroom time. Each unit features text and images, videos, animations and simulations, writing and discussion tasks, thought experiments, and contextualization using historical as well as everyday examples. Figures 8.1 and 8.2 show the main page of the final learning environment that contains three different units.

There are some general features of the structure of the learning environment that I want to emphasise in particular. The layout of the GR learning environment aligns with the presentation of the quantum physics learning environment that was developed within ReleQuant as well. Thus, the design of the activities is consistent throughout and across the units of different ReleQuant programs. The overall structure of the learning resources is simple and intuitive. Recurrent motives such as a cartoon figure of young Albert Einstein add to an intuitive navigation of the learning environment. Each unit starts with specific learning goals to orientate students and teachers towards key ideas of this unit. The learning goals in the learning environment use a simpler language than the ones I proposed in study (I). This simplification helps students approach the goals more easily.

Each unit displays various forms of engagement that offer variety and cater to the needs of as many students as possible. Frequent discussion-tasks are meant to foster collaborative explorations of key issues in GR. Classroom discussions at the end of the unit provide opportunity to consolidate the new ideas. Furthermore, key ideas are emphasised throughout each unit and summarised at the end of each unit in a concise way. For students who want to learn more there is an additional section with recommended extra material that is tailored to each unit.

## 8. Results: Design of the Learning Environment



Figure 8.1: The main page of the Viten-program *General Relativity* - part 1.

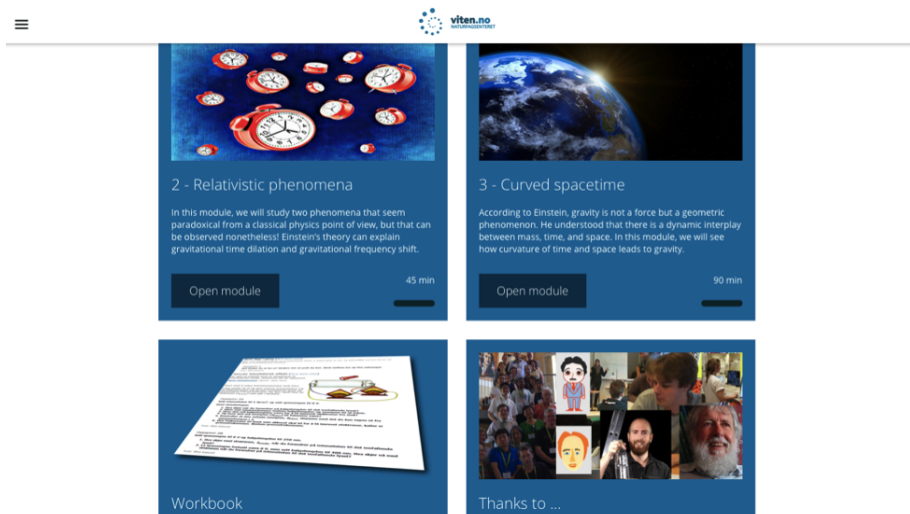


Figure 8.2: The main page of the Viten-program *General Relativity* - part 2.

## 8.2 Design trajectory - gravity is geometry

During three consecutive rounds of development, many design changes, both big and small, took place. In this section, I sketch the trajectory of the development of one specific learning activity. In showing how the design hypotheses have informed the design of the activity and how our trials, in turn, have probed the hypotheses, I hope to bring greater clarity to the interplay of DBR and MER methods in my PhD-project.

The learning activity addresses the relativistic key idea that gravity is geometry. In the first trial, this idea was introduced through a fictitious letter exchange between Albert Einstein and Marcel Grossman, a mathematician and friend of Einstein (Figures 8.3 - 8.6). The animation of the letter exchange presents Einstein's quest to find a mathematical description of GR. In the letter, Einstein asked Grossmann for help. Einstein explained that gravity was not a force but that he had not been able to find a better explanation yet. Grossmann replied that geometry might offer a solution to Einstein's problem.

Grossmann challenged Einstein's understanding of geometry by inquiring about the shortest path between Oslo and New York. After this animated introduction, students were presented with a world map that showed the curved flight route of a plane from Oslo to New York. Students were asked to discuss why the route traversed Iceland instead of taking a straight path over England. Afterwards, a second animation showed Einstein writing a second letter. He introduced the idea of geodesic curves and entertained the possibility that the earth might follow a geodesic curve around the sun.

The animated sequence of fictional letters illustrates Einstein's quest to find a geometric description of gravity and casts the development of GR into its historical context. Four design hypotheses informed the development of this activity:

1. The focus on the geometric nature of gravity echoes DH1. Throughout the letter exchange, both Einstein and Grossmann refer repeatedly to geometric ideas, such as the contrast of straight and curved paths and the explicit mentioning of differential geometry as a way to approach a physical problem.
2. Grossmann invites Einstein to engage in a thought experiment of a plane traveling between Oslo and New York. We invited students to explore this thought experiment to become familiar with the features of curved spaces. This emphasis on thought experiments reflects DH2.
3. In letting students explore a thought experiment, we asked them to discuss Grossmann's question in small groups. Thus, we fostered collaborative explorations of doing physics by talking physics (DH7).
4. Both Einstein and Grossmann were historic figures and indeed friends and collaborators. Even though our letter exchange was fictional, Einstein and

## 8. Results: Design of the Learning Environment

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Grossmann (1913) collaborated on topics of GR. Through the activity, we hoped to convey the historic context of GR and present Einstein as a real person who struggled to find a new theory of gravity in line with DH8.

The first trial of the learning environment in 2016 gave valuable feedback on the “gravity is geometry”-activity. Field observations suggested that the format of the animated letter exchange was not ideal. Students were not able to follow animations that relied heavily on that much text. The sound of the animation would get drowned in the classroom noise and the text was too small to read it off from a big screen (and many teachers showed the animation in front of the whole class). Moreover, there was a lot of information and many new concepts condensed into two short letters. This presentation assumed a rather optimistic and idealistic classroom setup. In a way, this first prototype reflected my inexperience with the reality of upper secondary physics classrooms. I had naively assumed that secondary students would be able to pay close attention and follow a long line of reasoning in the same way as undergraduate students with whom I had previously worked.

In the interviews, both teachers and students commented on the activity. Teachers criticised the small font size of the animations that made it difficult to read on a big screen. While students appreciated the historic context and liked the presentation of Einstein as a real person who struggled to find a new theory, the idea that gravity is geometry did not seem to have been communicated successfully. Even though students appreciated the letter format as a way to follow Einstein’s line of thought they struggled to recall the actual content of the letters.

The integrated discussion task of the world map and the flight route between Oslo and New York engaged students. Analysis of the audio records showed that almost all groups understood that the curvature of the earth leads to distortions of straight lines. However, most students were not able to connect the curved flight route to the more general concept of curved geometry and gravity.

Based on the findings from the first trial, we revised the “gravity is geometry”-activity. In the second version of the activity, we dropped the animation of the letter exchange completely. We replaced the letters with an extended world map task that now took center stage as a key activity in the unit on spacetime. Specifically, we drew on everyday examples and thought experiments to help students relate more easily to the activity. These adaptations contributed to the formulation of design principles 2a and 2b. In line with the idea of talking physics, we continued to put focus on collaborative explorations by letting students talk physics (what would later become design principle 4a).

Moreover, we introduced “Einstein’s law” (Figure 8.7) as a generalization of Newton’s first law: objects that are not influenced by forces move along geodesic curves in spacetime. Einstein’s law is a reformulation of the geodesic equation and connects the movement of freely falling particles to the geometric nature of gravity. By supplementing the world map task with Einstein’s law, we hoped to make the geometric nature of gravity more explicit.


Instructional steps of the learning activity "gravity is geometry"	English summary of the Norwegian instructions and explanations	Screenshots of the learning sequence
<p><b>An interactive letter exchange between Einstein and Grossmann presents Einstein's quest for a new theory of gravity. In his first letter, Einstein asks Grossmann for help: He has figured out that gravity is not a force but he cannot quite explain what gravity is.</b></p>	<p>Einstein was really good at physics and he knew a lot about mathematics as well. Still, he needed help with his theory of relativity. He worked closely together with Marcel Grossmann, an old university friend who had become a professor of mathematics. A letter exchange between the two could have looked like that:</p> <p><i>"Dear Marcel,</i></p> <p><i>I need your help. I have pondered one question for a while now but have not succeeded in finding a satisfactory answer: If gravity is not a force, why does the Earth circle around the Sun? (...) A particle that is not influenced by forces remains either at rest or moves at a constant velocity following the straightest path through space. This is a consequence of Newton's first law. But we know that the Earth moves around the Sun following an ellipse trajectory - which is not straight! Where is my mistake?!"</i></p>	

Figure 8.3: Prototype of an activity that illustrates the geometric nature of gravity – step 1.


Instructional steps of the learning activity "gravity is geometry"	English summary of the Norwegian instructions and explanations	Screenshots of the learning sequence
<p><b>In the next animation, Grossmann writes a letter in response. He suggests that a solution to Einstein's problem might be found in the geometry of space and time and challenges Einstein to find the shortest path between Oslo and New York.</b></p>	<p>Dear Albert,</p> <p>I have thought about your problem for a couple of days now and I think that your solution might be found within geometry.</p> <p>If I am not mistaken, you should brush up your knowledge on differential geometry. I have a question for you to get started: Take a look at a world map. What is the shortest path between Oslo and New York?</p> <p>(...)</p>	

Figure 8.4: Prototype of an activity that illustrates the geometric nature of gravity – step 2.

The second trial of the learning environment in 2017 revealed that students were highly engaged by the world map task (Steier et al., 2019). While discussing the shortest path between Oslo and New York, the groups would often introduce improvised representations such as globes or basketballs to explore the geometry of curved surfaces. This observation led us to develop an interactive version of the world map task in the final iteration of the learning environment. Moreover, even though students were able to explore the geometry of curved spaces, they continued to struggle to link the physics of free fall to the world map task.

These findings led to another design revision that made the link more explicit. The final version of the “gravity is geometry”- activity consists of a series of interactive simulations. In a first step, students explore the geometry of curved spaces by drawing flight routes on an interactive world map. The activity allows

## 8. Results: Design of the Learning Environment


Instructional steps of the learning activity "gravity is geometry"	English summary of the Norwegian instructions and explanations	Screenshots of the learning sequence
<p><b>In a last step, students get to read one final letter written by Einstein before being presented with the solution to the problem of straight and curved lines: Gravity is a geometric phenomenon.</b></p>	<p>"Marcel,</p> <p>(...) I am no expert in geometry but I do remember the concept of a geodesic curve: I want to take a closer look at a particle that is not influenced by any forces. The trajectory of such a particle is a geodesic curve. I can think of this curve as the shortest path between two points in space. If I draw such a curve I get a straight line. But this is only true on a flat map!</p> <p>Is it possible that the Earth follows a geodesic curve around the Sun? And that the universe is actually curved...?"</p> <p>Einstein was captivated by Grossmann's ideas. If the universe is curved, the elliptical orbit of the Earth is related to this curved geometry. Then one does not need a force to explain gravity. Massive objects curve spacetime. Phenomena that are related to spacetime curvature are gravitational phenomena.</p>	

Figure 8.5: Prototype of an activity that illustrates the geometric nature of gravity – step 3.


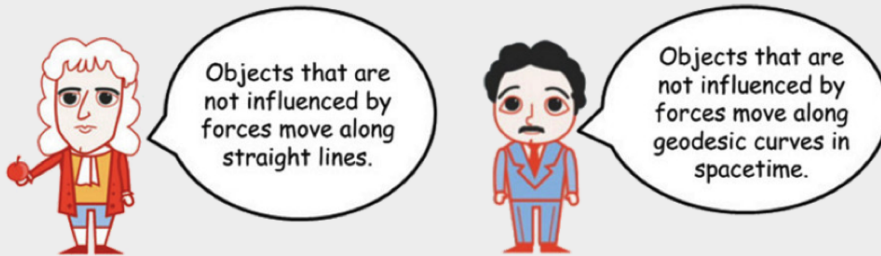
Instructional steps of the learning activity "gravity is geometry"	English summary of the Norwegian instructions and explanations	Screenshots of the learning sequence
<p><b>After the animated letter exchange, students were invited to discuss the question of finding the shortest path between Oslo and New York. The students took audio records of their discussions using mobile phones.</b></p>	<p>Why do we fly over Iceland when heading to New York? On a map it looks as if the shortest path is a straight line from Oslo over England to New York.</p>	

Figure 8.6: Prototype of an activity that illustrates the geometric nature of gravity – step 4.

## “Einstein’s law”

With general relativity, Einstein presented a completely new view on gravity as geometry in four-dimensional spacetime. Einstein realized that geometry created the illusion of gravity being a force. Nonetheless, he described movement similarly to Newton. He just had to reformulate Newton’s first law by taking curvature into account.



Einstein and Newton agree on objects following a straight line/ a geodesic curve in spacetime if they are not influenced by forces.

Figure 8.7: Einstein’s law is a generalization of Newton’s first law to the realms of curved spacetime.

In curved spaces, geometry works differently from what we are used to. An example is straight lines on a curved surface.

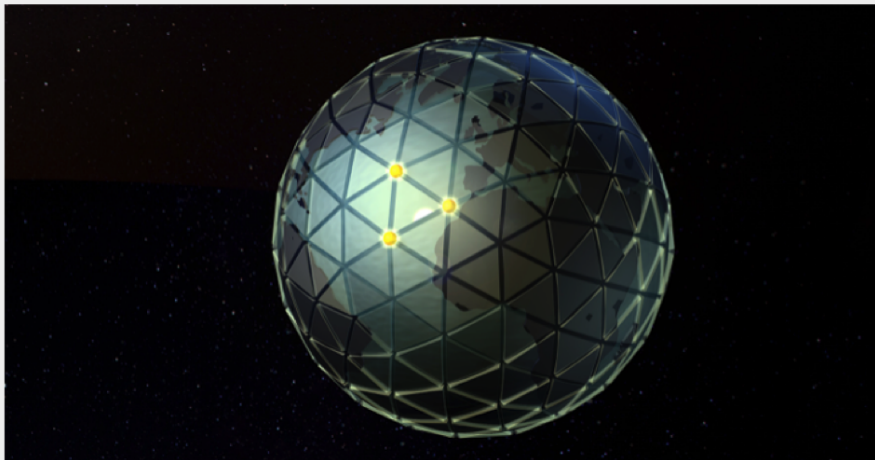



Figure 8.8: A sphere provides an example of how geometry works differently in curved spaces.

## 8. Results: Design of the Learning Environment

Draw and discuss in pairs:

1. Draw the shortest path that a plane can take from Oslo to New York.
2. Explain to each other the choice of path and compare to the correct answer. Explain the shape of the path.




By Strebbe (Own work) [CC-BY-SA 3.0], via Wikimedia Commons

reset show answer

Figure 8.9: Students are asked to find the shortest path between Oslo and New York.

The Earth is a sphere. Therefore, it is difficult to represent the Earth on a flat map. Below you can see a different way of representing the Earth. Try to draw the shortest path between Oslo and New York on this map.



By Strebbe (Own work) [CC-BY-SA 3.0], via Wikimedia Commons

reset show answer

Figure 8.10: Whether or not a line between two points looks straight on a map depends on the specific map projection.



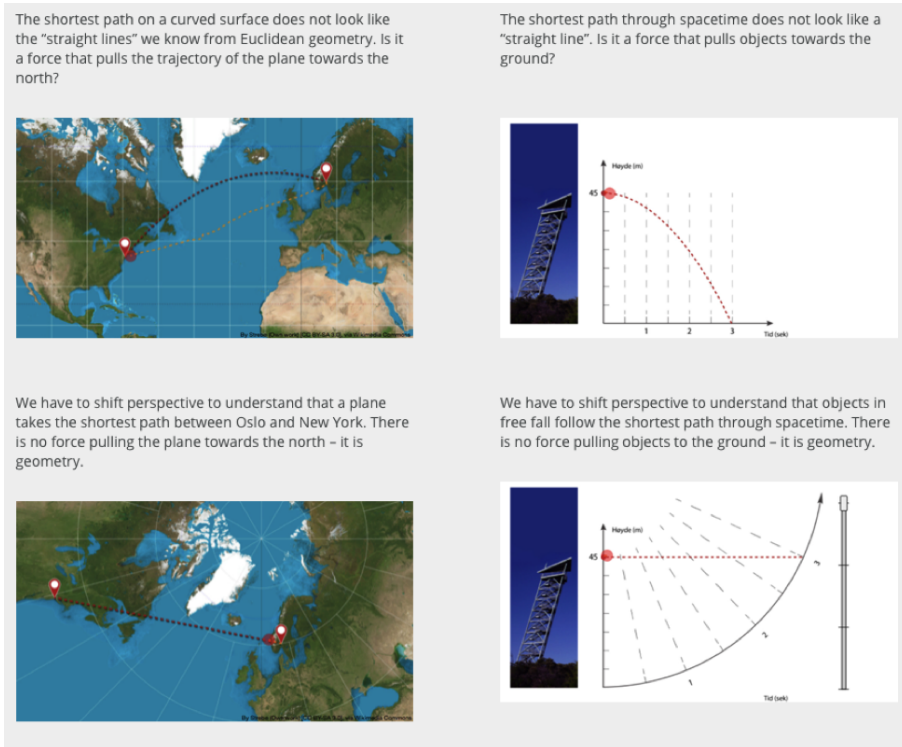


Figure 8.11: A summary at the end of the interactive warped-time activity helps students link the map-activity to the warped-time-activity thus linking gravity and geometry.

students to compare their suggested routes to the correct one. In addition, two world map projections illustrate the idea that shortest paths do not necessarily look straight (Figures 8.8-8.10).

In a next step, students are introduced to the concept of geodesic curves and Einstein’s law. Finally, a second series of interactive simulations connects the concept of geodesic curves to the phenomenon of gravity by introducing warped height-time diagrams (Figure 8.11). This activity illustrates the geodesic equation and is presented in detail in study (III).

In summary, the idea that gravity is geometry has guided the development and iteration of a series of activities in the unit on curved spacetime. Originally condensed into an animated exchange of letters between Einstein and Grossmann, this idea was unpacked through several interactive tasks in the final learning environment. Interactive elements as well as collaborative tasks now allow students to explore the concept of gravity by looking separately at the concepts of curved spaces, warped time, and geodesic curves.







## Chapter 9

# Results: Student learning processes

Knowledge (...) is an ocean of alternatives channelled and subdivided by an ocean of standards. It forces our mind to make imaginative choices and thus makes it grow. It makes our mind capable of choosing, imagining, criticising.

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*Paul Feyerabend*

I set out my PhD-project with two overarching research aims. First, to transform GR from a scientific theory to a theory that can be taught at secondary school level. Second, to study learning processes and in particular how students develop conceptual understanding of key concepts in GR. Developing a learning environment as explained in the previous section was one important part of my PhD-project to address the first research aim. Based on this development, I was able to pursue the second aim, namely to investigate student learning processes in GR in more detail. Since I originally based my educational reconstruction on literature accounts of student learning processes in relativity, the natural next step within the MER-framework was to use the digital learning resources to conduct studies on student perspectives on my own.

In this section, I broadly present my findings that characterize learning processes in GR and that shed light onto students' understanding of key concepts in GR. The presentation of the findings follows the individual publications. In the next section, I discuss and contextualise the findings. Study (I) lies at the heart of this thesis and stands out insofar as it presents my proposed educational reconstruction of GR within the combined frameworks of MER and DBR. Therefore, findings from study (I) appear both in the result section on the design of the learning environment and in this result section on learning processes.

Studies (II) and (IV) are follow-up studies that build on study (I) by adding to the component of MER that addresses student perspectives. These two studies address different relativistic concepts and show how students developed understanding of them. Being follow-up studies, they are in line with the iterative nature of MER and DBR: several iterations of developing and trialling learning resources enabled me to probe deeper into students' conceptual understanding. Even though study (III) mainly focuses on the design of the learning resources by presenting a new model to visualise gravity, the study addresses student perspectives that have led to the development of this model as well. Study (V) investigates how history and philosophy of science can foster students' understanding and motivation in GR.

### **9.1 Study I - General relativity in upper secondary school: design and evaluation of an online learning environment using the model of educational reconstruction**

The starting point of this study was the scarcity of research-based approaches to teaching and learning GR at the upper secondary school level. I employed the framework of MER to study whether it was worthwhile and possible to teach GR qualitatively. In the previous section, I have outlined how the DBR framework and specifically the formulation and testing of design hypotheses guided the development of the learning environment. In this section, I focus on students' conceptual understanding of key concepts in GR. I was interested to investigate the understanding of key features of GR that participating students expressed while engaging with the learning environment. To characterize students' conceptual understanding after the first round of classroom trials, I analyzed students' oral and written responses as well as focus group data. I used the design hypotheses and relativistic key concepts as a guide to structure the analysis.

Overall, the findings indicate that the ReleQuant students were able to obtain a qualitative understanding of GR when provided with appropriately designed learning resources and sufficient scaffolding through interaction with their teachers and peers. Since the aim was to broadly map out student perspectives on all key concepts and since the thematic analysis took into account a large set of data from our first round of trials, the findings remained at a rather general level. Naturally, these general findings then informed more specific research questions and more specific methods of analysis that allowed me to explore student learning processes in more detail in the follow-up studies.

One important insight that I gained from this first study relates to the gap that students perceive between the abstract description of spacetime curvature and their own experience of gravity. Even though most students were able to reproduce the explanation that mass curves spacetime and the curvature of spacetime influences the movement of mass, few students could explain why gravity pulls them down to the floor. In the focus group interviews, students confirmed that they perceived Einstein's explanation of gravity as being far away from their own lives.

This observation relates to the abstract nature of spacetime. Even though the concept of spacetime lies at the heart of GR, there is not much research on how students understand this concept. Our first study suggested that spacetime was an engaging, yet challenging concept that students felt very uncertain about. Even though curved spacetime was perceived as a difficult concept, students generally found the geometric way of looking at space and time very exciting. This interest supports an observation made by Levrini and Fantini (2013) who argued that challenges in modern physics can be productive and engaging for upper secondary students. Studies (II), (IV), and (V) explore different conceptual aspects of spacetime and gravity as a geometric phenomenon in more detail.

Another interesting finding from study (I) relates to the definition of reference

frames. The analysis of the audio discussions revealed that there seemed to be a disconnect between knowing the definition of an inertial system and being able to apply this definition to a given situation. For example, one group of students came to the conclusion that an inertial frame was a relative concept; other students formulated similar thoughts but would not go so far as to say that one cannot give a definite answer to the question of whether or not a reference frame was inertial. Many studies have reported on the challenging nature of reference frames in physics education (Arlego and Otero, 2017; Aslanides and Savage, 2013; Bandyopadhyay, 2009; De Hosson et al., 2010). Findings from our study corroborate this observation and show how a poor understanding of concepts in classical physics makes it more difficult to understand concepts of modern physics. Emphasizing that there exist several concepts in relativity that are not relative – or in other words, helping students see the continuity of core assumptions in physics (Kim and Lee, 2018) – is one important finding of paper (I) that can guide future work in GR education.

## **9.2 Study II - Understanding curved spacetime - the role of the rubber sheet analogy in learning general relativity**

After having obtained a first understanding of students' learning processes based on the first round of trials, I addressed students' conceptions of specific key concepts in GR in my subsequent studies. In study (II), we looked closer at the concept of spacetime and how learners conceptualized their experience of gravity in the setting of GR.

A popular analogy compares spacetime to a two-dimensional rubber sheet distorted by massive objects. This analogy is ubiquitous in teaching GR to upper secondary and undergraduate physics students. However, physicists and physics educators criticize the analogy for being inaccurate and for introducing conceptual conflicts. Addressing these criticisms, we analyzed the rubber sheet analogy through systematic metaphor analysis of textbooks and research literature, and presented an empirical analysis of upper secondary school students' use and understanding of the analogy.

We took a theoretical perspective of embodied cognition to account for the relationship between students' experiences in relation to the abstract nature of spacetime. This perspective drew directly from the findings of study (I) that suggested that students struggled to relate their experiential understanding of gravity to the relativistic explanation. Our goal was to understand the rubber sheet analogy and the affordances it provides for students to conceptualize gravity as curved spacetime.

In the second version of the learning environment, one task asked students to use their knowledge of GR to discuss the cartoon in Figure 9.1. The second round of trials in 2017 allowed me to collect data of these discussions in form of written responses that the groups of students produced after their discussion. To characterize students' understanding of the analogy, we first looked at the metaphorical ways in which students talked about the rubber sheet. Analyzing

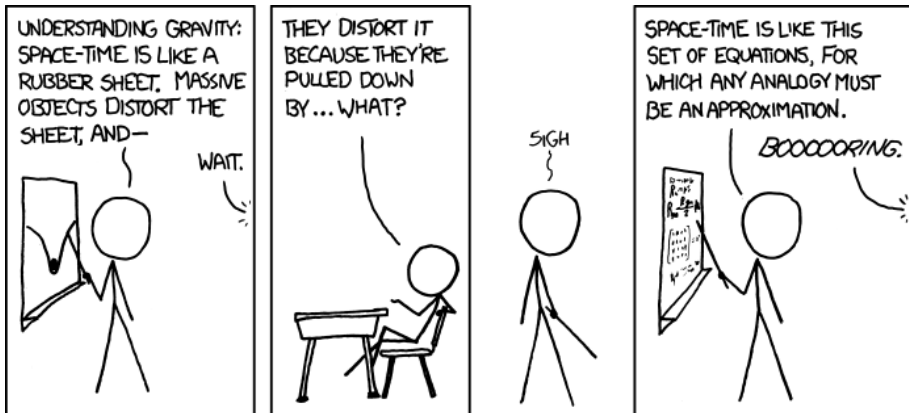


Figure 9.1: A cartoon invited students to discuss the rubber sheet analogy and the concept of spacetime. (<https://xkcd.com/895>)

students' written accounts through the lens of metaphor analysis enabled us to gain insights into their conceptual understanding. This close-up view on language was interesting because the use of the right terminology can in some cases mask students' lack of conceptual understanding. A metaphor analysis allowed us to explore whether students used the rubber sheet analogy in productive ways.

One key finding suggested that the link between gravity and the force concept was very robust and prevalent among the ReleQuant students. Whereas GR posits that gravitational effects are a consequence of the curvature of spacetime, students turned this reasoning upside down; they described a force of gravity that curved spacetime. Thus, the findings of our metaphor analysis suggested that some students confused cause and effect when working with the rubber sheet model.

Moreover, students' everyday experience of gravity often got in the way of inferring the right analogical mappings. Even though the source domain of the rubber sheet analogy draws on students' embodied experience, it seems that exactly this embodied experience might have interfered with a more abstract understanding of the disembodied concept of spacetime. Students were asked to explain gravity in the Einsteinian sense by drawing on their everyday understanding of gravity as a force that warps the rubber sheet. This conflict might add to the common perception of GR being counter-intuitive. In order to conceptualize the physical mechanism of gravity in the domain of GR, students need to develop awareness of the tension between the physical force of gravity in the everyday experiential sense and the curved spacetime explanation.

While students displayed a good understanding of the need for analogies and visualizations in the domain of GR, few groups addressed shortcomings of the rubber sheet analogy specifically. The most common flaw that students identified was the dimensional reduction of four to two dimensions. There were only few responses that addressed the time dimension. Thus, our findings suggested that



many students were not aware that the time dimension played a crucial role in the mechanism of gravity; those who showed such awareness admitted that this part of the theory was difficult to understand. This observation suggests that the role of time might have posed a conceptual challenge for students when dealing with gravity in the setting of GR. This finding was the starting point for the development of the warped-time activity as presented in study (III).

### **9.3 Study III - Free fall in curved spacetime - how to visualize gravity in general relativity**

In presenting a new instructional model of warped time, study (III) illustrates how we adapted one key idea of GR, the geodesic equation, to the secondary school setting. Doing so, the study also synthesizes findings on student learning processes in form of instructional strategies to use the warped-time model successfully:

1. During the trials, some students criticized the warped-time model for not being representative of the relativistic phenomenon of warped time. This criticism reveals an understanding of the limitations of the warped-time model. Successful instruction should therefore complement the warped-time activity with other examples from cosmology and astrophysics where relativistic phenomena indeed have a more significant effect.
2. The warped-time model allows for a direct comparison between Newton and Einstein's models of gravity by showing how two different models can describe the same physical phenomenon. Teachers could use this opportunity to help students build awareness of the nature of scientific models.
3. It is important to emphasize that every object in the universe always moves in time. This insight can help students link geometric descriptions more easily to their everyday experiences of gravity. Study (IV) elaborates on this idea in more detail.
4. In everyday life, many relativistic phenomena cannot be observed directly. To help students make sense of warped time, the warped-time model can be used to discuss gravitational time dilation as well.

These instructional strategies present examples of how research-based efforts to understand student learning processes can in turn inform and improve teaching and instruction in physics classrooms.

### 9.4 Study IV - Navigating four dimensions – upper secondary students' understanding of movement in spacetime

In study (II), we investigated how students conceptualized the abstract concept of spacetime. With the aim of gaining deeper insight into students' understanding of movement in spacetime, I characterized challenges that students faced when conceptualizing movement along geodesic curves in relation to gravity. Study (IV) reports from the second classroom trial of the learning environment in 2017. In a built-in activity, students were asked to discuss in pairs or small groups whether they were moving in spacetime and whether or not they were following a geodesic curve.

Based on the presentation of the concepts of spacetime and geodesic curves in the learning environment, a correct response from students could have confirmed that they were moving in spacetime because they were moving in time (they were aging) and in space (the Earth circles around the Sun). Moreover, students would ideally have come to the conclusion that they were not following a geodesic curve through spacetime: a geodesic curve is followed by objects in free fall but students in the classroom were, of course, not in free fall.

The findings of the thematic analysis showed that students were able to describe their movement in spacetime by drawing on movement in space and movement in time. Common examples of students' spatial movement included the Earth circling around the Sun or the movement of our galaxy in an expanding universe. Even though students realized that they moved in time because they grew older, they admitted that temporal movement was hard to visualize.

Movement in spacetime did not pose significant challenges to students; however, the concept of movement along geodesic curves did challenge students. The group discussions featured a broad variety of different ways to characterize geodesic curves such as the shortest, straightest, or fastest path between two points or as a curve followed by an object on which no forces act. This variety in responses suggests that students were able to use different characterizations of geodesics, which, in turn, suggests that they had obtained a reasonable understanding of geodesic curves.

Yet, movement along geodesic curves in spacetime challenged almost all groups. Only few groups were able to connect the geometric description of a geodesic curve as the straightest path in a curved space to the physical state of being in free fall or alternatively, to the state of not being affected by external forces. There seemed to be a gap in students' understanding between the geometric framework of GR and the physics of gravitation. Interestingly, students often stated that they were following a geodesic curve because Einstein said so. There was a common (mis)conception that movement in spacetime was the same as movement along geodesic curves.

## **9.5 Study V - How History and Philosophy of Science Can Inform Teaching and Learning of General Relativity in Upper Secondary School**

Study (V) looked at the potential of history and philosophy of science to motivate and engage students in GR. Many of the instructional approaches in our learning environment invited students to explore the historical development and philosophical aspects of GR. I used the focus group interviews from the first two rounds of classroom trials in 2016 and 2017 to explore if and how HPS was a motivating factor for students. I analyzed seven focus group interviews with 46 students in total using methods of thematic analysis.

The findings showed that students generally approved of an HPS approach in the learning domain of GR. Students mentioned four factors as being particularly motivating or interesting to learn about: First, the learning environment presented Einstein as a real person who struggled to find a new theory of gravity. Second, physics was presented as important part of our cultural heritage with social relevance for our daily lives. Third, students learned that physics was a modern field that has more to offer than just out-dated textbook knowledge. Finally, the historic contextualization helped students link new ideas in GR to their knowledge of classical physics.

Overall, the findings of study (V) suggested that employing history and philosophy of science in the service of physics education can serve as a successful approach to motivate and engage students in GR.







## Chapter 10

# Discussion

Words and chronological time create all these total misunderstandings of what's really going on at the most basic level.

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*David Foster Wallace*

According to Einstein, common sense is nothing more than a deposit of prejudices laid down in the mind prior to the age of eighteen (Barnett, 2005). It is therefore a noble and necessary cause to expose young minds to our current best theory of gravity. So far, all observations continue to confirm Einstein's theory as physicists and astronomers open new windows to the Universe (Grimberg et al., 2019). The first direct observation of gravitational waves of two merging black holes has been called the discovery of the century, akin to Galileo's first turning of his telescope to the sky (Grimberg et al., 2019). This exciting time in astronomy represents a critical opportunity to engage students and the general public if they are to understand such a landmark result (Key and Hendry, 2016). However, with great opportunities come great educational responsibilities Kersting (2019). As physics educators we face the challenge of introducing novel topics to the physics curriculum, of translating abstract scientific theories into appropriate learning resources, and of investigating students' learning processes and conceptual challenges to improve instructional practices in the physics classroom.

This thesis contributes to the emerging field of general relativity education by having suggested a research-based educational reconstruction of GR. I pursued my PhD-project based on two overarching research aims:

1. To propose a way of turning GR, a physical theory of gravity, into a subject area that can be taught at the secondary school level.
2. To characterize students' understanding of key concepts in GR and to investigate how students develop their understanding through work within a digital learning environment.

To address these research aims, I employed the MER-framework to suggest a content structure for instruction that is suitable to communicate key ideas of GR qualitatively at the secondary school level. Naturally, the components of MER are interrelated. In line with the content structure, I developed a digital learning environment that we designed and trialed over a 3-year period according to the DBR-framework. Using this digital learning environment, I studied students' learning processes in the domain of GR in more depth. By employing the MER framework, I was able to link both research aims: while the development of the

content structure for instruction and the design of the learning resources were the starting point for my inquiries, the findings of student learning processes fed directly back into a refinement of the digital learning environment.

In the following, I discuss my approach to pursuing my research aims and critically examine the findings I obtained along the way. The first part of this discussion is structured along the design principles that I formulated in paper (I) with additional insights added from the follow-up studies. I put particular emphasis on students' conceptual understanding of curved spacetime because one important contribution of my PhD-research is empirical research on spacetime as conceptualized by students at the upper secondary school level. Since design aspects have featured so prominently in my work, I then discuss the fruitful interplay of design and educational research. Finally, I contextualize my research and situate my work within a broader tradition of physics education in Norway to reflect on the impact of physics education in the 21st century.

### 10.1 Research aim 1: An educational reconstruction of GR

MER provides an educational framework to study whether it is possible to teach new content areas of science (Duit et al., 2012). The framework takes a broad and holistic perspective by including considerations of content knowledge, student and teacher perspectives, and design processes. As such, I have found the framework to be very useful to guide my research. Indeed, MER has provided the very language with which I have described my research. Instead of remaining rather vague by saying that I was aiming to develop a digital learning environment about GR in line with the Norwegian curriculum, MER has allowed me to specify this research aim: I have proposed an educational reconstruction of GR and this reconstruction entails the development of a content structure for instruction, the development of a digital learning environment that is freely available in English and Norwegian, and in-depths studies of students' conceptual understanding of key ideas in GR.

In article (I), I broadly presented my attempt of an educational reconstruction of GR for secondary school students. This educational reconstruction took into account all three components of the MER-model. In particular, I demonstrated the applicability of the MER-model to yet another topic area in science education by providing empirical evidence of its workings. This evidence comprises research-based design-principles that can give guidance for other educators and teachers that aim to introduce students to relativistic concepts.

The only other reconstruction of GR from an educational perspective that I am aware of was proposed by Levrini (2002b). Levrini reconstructed the historic debate about the nature of space and spacetime to identify primitive ideas that make up the basis of modern physics. Specifically, Levrini used the concept of space as an educational criterion to unpack the debate on the foundations of GR. Even though this reconstruction is carried out in a masterful and knowledgeable way and therefore provides valuable epistemological insights, the work remains theoretical without explicit links to school practices.



Levrini (2002b) comments on the increasingly difficult formal language of modern physics. She concludes that many relativistic concepts are not accessible without the proper mathematical formalism but that the basic ideas of GR can be enough to give students an understanding of the importance of GR nonetheless: “As far as teaching the basic ideas of GR is concerned, the main problem is that only few concepts of GR can be taught avoiding the use of the tensorial formalism. Hence teaching at secondary school level does not usually go beyond the presentation of the EP [equivalence principle], the gravitational red-shift and the lift thought experiment. My opinion is that these topics can be enough to give students ideas about the physical and cultural value of GR, provided that their discussion is anchored to the controversy about space and if the classical roots of the controversy are made explicit.” (Levrini, 2002b, pp 276)

While I agree with Levrini’s assessment that the basic principle of GR can help students acknowledge the physical and cultural value of relativity, I think there is value in trying to convey more advanced concepts as well. Therefore, I have put particular focus on reconstructing the more abstract concepts such as spacetime without using the advanced mathematical formalism. The results in studies (I), (II), and (IV) indicate that upper secondary students can indeed obtain a qualitative understanding of the more abstract ideas of GR while also building awareness of the physical and cultural value of GR. These findings are in line with recent research conducted in Australia that showed that key concepts of Einsteinian Physics such as the relativity of time, the warping of time, and the curvature of spacetime were conceptually intelligible to middle school students (Kaur et al., 2018). However, there is a need for more research on different contexts and larger student populations. Physics educators have only started to realise the potential of Einsteinian Physics education research.

## 10.2 Research aim 2: Learning processes in GR

By studying student learning processes and students’ conceptual understanding, we can determine how different designs of learning environments contribute to learning, cooperation, and motivation in the classroom (Collins, 1992). Specifically, the framework of DBR allows answering this question by providing first design hypotheses and eventually design principles that encapsulate successful ways of teaching relativistic concepts based on empirical evidence. The design principles encompass overarching themes that characterise appropriately designed learning resources and specific recommendations that promote successful instruction in GR. In the following, I discuss key insights on student learning processes that provide more nuanced views on teaching and learning of GR than what has been reported in the literature so far. I supplement this discussion with examples from our digital learning resources to give more weight to the empirical nature of my research.

**Classical versus modern physics** Often, GR is presented as a revolutionary new way to describe gravity, space, and time. Yet, our original hypothesis that

## 10. Discussion

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emphasizing the break of relativistic and classical physics would help students understand relativistic concepts did not turn out to be completely accurate. Such an approach has shown to be useful in quantum mechanics (Bungum et al., 2015) and continues to guide much research in Einsteinian Physics education research where educators assume that students must go through ontological conceptual changes to cope with the “radical shift to the Einsteinian paradigm” (Kaur et al., 2018).

However, the findings of our classroom trials suggested that students felt baffled or bewildered that much of what they previously had learned about gravity, space, and time was outdated knowledge. Instead of just focusing on the differences between classical physics and relativity, it might be more fruitful to show how Newton and Einstein’s theories of gravity are two different models to describe gravitational phenomena (Kim and Lee, 2018). The findings of this research give also more weight to the question of whether an early exposure to relativistic concepts might help students to accept relativity more easily (Pitts et al., 2014). Such a question is in line with Russell’s assumption that relativity seems difficult because we did not learn about relativistic concepts before our mental habits had become fixed (Russell, 1925). Educators argue for a re-education of our intuitions to make them compatible with what seems to be the best science around (Chandler, 1994). Helping students build awareness of the scope and limitations of physical theories as well as showing how GR extends classical physics can be one step towards such a re-education in line with the presentation of GR in our digital resources (Figures 10.1-10.3).

**Gravity as seen from two points of view**

If the rocket standing on Earth is accelerating up, does this mean that the ground is accelerating up as well? That's weird.

A mysterious, invisible force that acts momentarily over great distances? Who is weird now?

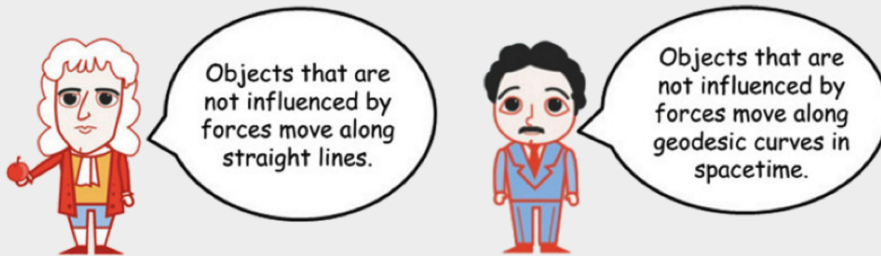
**Writing task 2**

Use your knowledge in physics to discuss the two statements and write down a short summary of what you have discussed. Which perspective do you prefer?

Figure 10.1: In a discussion task, students are invited to discuss gravity from two perspectives to help them build awareness of how gravity is described from an Einsteinian and a Newtonian point of view.

## “Einstein’s law”

With general relativity, Einstein presented a completely new view on gravity as geometry in four-dimensional spacetime. Einstein realized that geometry created the illusion of gravity being a force. Nonetheless, he described movement similarly to Newton. He just had to reformulate Newton’s first law by taking curvature into account.



Einstein and Newton agree on objects following a straight line/ a geodesic curve in spacetime if they are not influenced by forces.

Figure 10.2: GR does not completely break with classical physics but extends many familiar ideas to a four-dimensional spacetime setting. In this panel, students learn about “Einstein’s law”, a relativistic generalization of Newton’s first law.

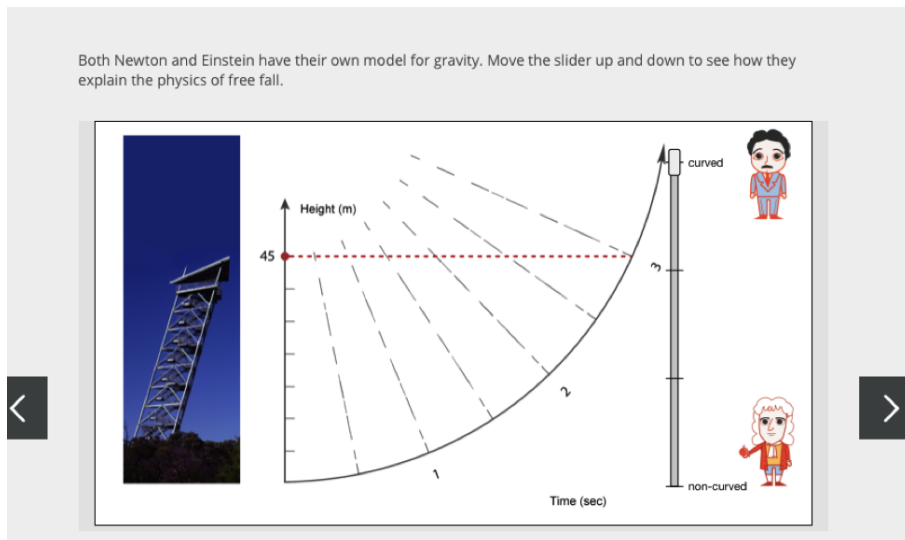


Figure 10.3: In this activity, students can move between Newton’s and Einstein’s model of gravity to understand how both physicists explained the physics of free fall through a force and a spacetime model.

**The role of experience** Students (as well as most teachers and physicists!) lack experiential knowledge of relativistic phenomena because classical physics is a very good approximation of the world we observe in everyday life. To counteract this lack of experience, our learning resources make use of instructional tools such as thought experiments, analogies, visualizations, and simulations that have been found to be beneficial for learning (Harrison and Treagust, 2006; Kluge and Bakken, 2010; Niebert et al., 2012; Velentzas and Halkia, 2013; Weiskopf et al., 2006). By and large, the findings of my research corroborate the usefulness of these tools in teaching and learning of GR. Despite the aid of these instructional tools, there still remains the challenge to close the gap between abstract descriptions of GR and the everyday experience of relativistic phenomena.

In study (II), we demonstrated that most students showed awareness of some limitations of the rubber sheet analogy that illustrates spacetime. Yet, students' experiential knowledge of gravity often got in the way of inferring analogical mappings that would allow them to reason with the analogy productively. At an abstract level, many students were able to explain planetary motion as result of curved spacetime. However, the same students struggled to explain how curved spacetime kept them pulled down to the floor (Kersting and Steier, 2018; Steier and Kersting, 2019). For example, students often confused cause and effect when conceptualizing curved spacetime through the rubber sheet analogy.

In study (IV), I found a similar disconnect between abstract descriptions of GR and students' experience of gravity. Even though students displayed a good understanding of the concept of a geodesic curve, many found the idea of movement along geodesic curves in spacetime very challenging. During their discussions, only few groups were able to connect the geometric description of a geodesic curve as the straightest path in a curved space to the physical state of being in free fall. This observed gap in students' understanding between the geometric framework of GR and the physics of gravity challenges an observation by Bandyopadhyay and Kumar (Bandyopadhyay and Kumar, 2010b) who stated that the separation between the conceptual and technical aspects of GR is possible in a meaningful way.

While it seems that one can separate the technical from the conceptual features in GR, there remains the danger that students do not build a holistic understanding of GR. Already in 1980, Sexl (1980) identified the problem that students might not be able to stitch together pieces of their understanding in GR. One contribution of my research is to offer research-based suggestions on how teachers can help students link their everyday experiences of gravity to more abstract description, thus helping them obtain more holistic perspectives during their collaborative learning processes.

**Student motivation** Science education research has repeatedly found that secondary school students are motivated by topics of astrophysics and astronomy (Angell et al., 2004; Conlon et al., 2018; Eriksson et al., 2014; Kaur et al., 2018, 2017a). It is thus not surprising to see that ReleQuant students felt motivated

and engaged to learn about Einstein’s theory of gravity, space, and time. The direct discovery of gravitational waves in 2016 and the subsequent public interest in gravitational wave astronomy has probably added to students’ motivation. Physicists announced the discovery shortly before we trialed the first prototype of the digital learning environment in 2016 and students showed a particular interest in gravitational waves. In response to this interest, the final design of the learning environment now presents gravitational waves not only as a hook to catch students’ interest but also as an example of physics in the making 10.4.

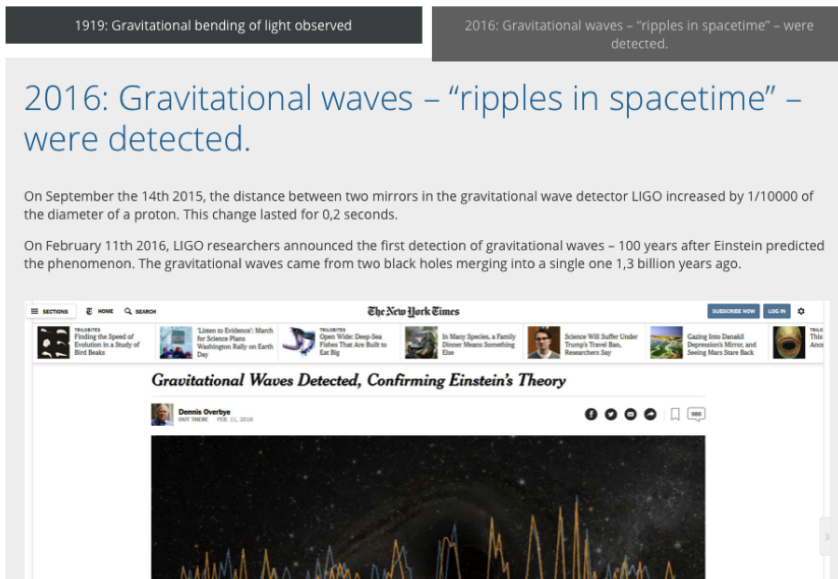


Figure 10.4: The direct observation of gravitational waves serves as an example of physics in the making and hooks students’ interest.

Aside from topics in astrophysics, students were particularly motivated and engaged by approaches that relied on the history and philosophy of science. The importance of HPS and NoS in teaching special relativity has been acknowledged before (Levrini, 2014). My PhD-work extends these findings from the special to the general theory of relativity: historical contextualization and philosophical implications of GR foster students’ motivation and engagement which in turn can facilitate productive learning processes. GR is an abstract theory that challenges students’ ideas of space and time. Yet, such challenges can in fact be productive. In quantum physics, Levrini and Fantini (2013) showed that students often felt motivated when presented with complex learning environments. My research suggests that the same can hold true in GR. Students felt particularly motivated by historic perspectives that gave them insight into the development of GR. Moreover, being able to follow Einstein’s reasoning and his struggle to find a new theory of gravity seemed to enable students to draw connections to previous

knowledge of physics. Thus, HPS perspective in GR open for opportunities to engage students in productive ways.

**Talking physics** Using language to facilitate learning of GR was a central tenet of the ReleQuant approach. Inviting students to talk physics and to learn about relativistic concepts through discussions and written tasks present one way to deal with the inaccessibility of the advanced mathematical framework at the secondary school level. Indeed, working physicists routinely use language to construct representations and to explore scientific ideas (Aubusson et al., 2006; Ochs et al., 1994). The focus on qualitative understanding emphasizes the thematic patterns that students learn to identify when building conceptual understanding (Lemke, 1990). The digital learning resources invited students to practice these semantic relationships between words and phrases which has been shown to be important in learning physics (Tang and Tan, 2017).

Even though research suggests that discursive ways of learning can be productive (Chi and Menekse, 2015; Mortimer and Scott, 2003; Tang et al., 2011), our ReleQuant students held divided opinions on learning physics through small group discussions. While some students appreciated the opportunity to discuss with their peers and probe their understanding in oral and written form, other students felt that they were lacking a correct answer or a resolution that would help them make sense of their discussion. This criticism is a valid one in light of the analysis of audio discussions. Study (I) presented a discussion task on gravitational redshift and study (IV) reported on a discussion task on movement in spacetime both of which are examples of tasks that students struggled to solve. The analyses revealed that most groups of students either did come to a wrong conclusion or gave up the discussion remaining undecided on their final conclusions.

Even though each unit in the digital learning environment includes a final classroom discussion to let students and teachers consolidate ideas and discuss unclear concepts, field observations showed that those discussions were often omitted because of a lack of time. Teachers should therefore be careful to pencil in enough time for consolidation phases because these phases are crucial for students to check whether or not their discussions have led them to right conclusions. In another ReleQuant publication, Bungum et al. (2018) found that a large proportion of small-group discussions had potential for enhancing students' understanding in quantum physics. But this potential was likely to only be realized when teachers could place student discussions in a broader frame in the classroom and thereby involve all students, also the ones who did not have productive discussions.

When discussing the role of language in the focus group interviews, another recurrent theme was the wish for more quantitative and mathematical treatments of GR. Students asked specifically for easy mathematical examples to support their reasoning. This wish to employ the mathematical language to approach relativistic concepts suggests that upper secondary school students have already been encultured in a traditional way of doing physics (Bøe et al.,

2018). Traditionally, many physics students like to solve physics problems by using calculations. To make the expectations that are built into the ReleQuant resources explicit to students, the digital learning environment has an introductory module: here, students learn that they are expected to “think like a physicist” by arguing like physicists (Figure 10.5). In addition, the learning resources emphasize that GR is a mathematical theory (Figure 10.6) and they present students with Einstein’s field equation to give them a flavour of the mathematical description of GR (Figure 10.7).

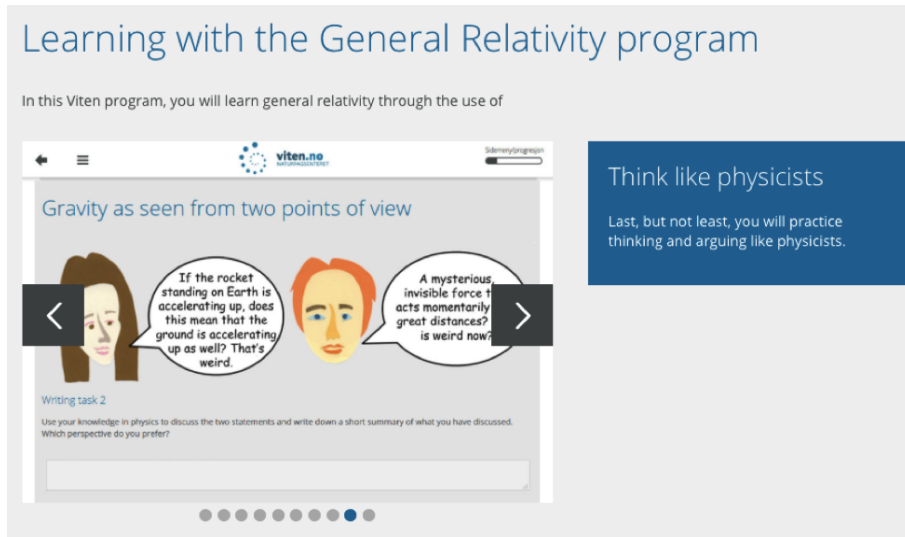


Figure 10.5: Students learn that they are expected to think and argue like a physicist.

### **Spacetime - conceptual understanding of an abstract scientific concept**

Four-dimensional spacetime serves as the dynamic stage on which the physics of GR unfolds. The concept of spacetime and the dynamic interplay between mass and spacetime that give rise to gravitational phenomena are therefore key ideas that students encounter in GR. Surprisingly enough, empirical accounts of students’ understanding of spacetime are scarce and have only started to emerge very recently (Kaur et al., 2018). One main contribution of this PhD-project is the systematic study of upper secondary students’ conceptual understanding of spacetime as a particularly abstract concept and the presentation of a new instructional model of curved spacetime (Kersting, 2019). Thus, the concept of spacetime presents a recurring motive in my work.

Study (I) served as a first and exploratory step towards investigations of students’ conceptual understanding of spacetime. The focus group interviews revealed that spacetime was an engaging, yet challenging concept that many students felt very uncertain about. Moreover, students found it difficult to relate

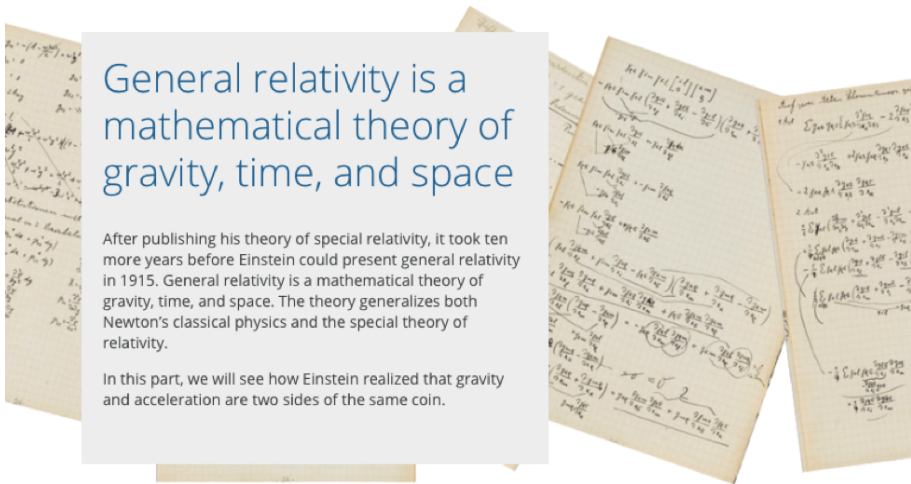


Figure 10.6: Even though the digital learning environment focuses on qualitative understanding of GR, it acknowledges the role of mathematics.

### Mass curves spacetime

Gravity is a phenomenon connected with mass. Massive objects distort the geometry of spacetime by curving both space and time. It is difficult to visualize curvature in four dimensions, but we can use analogies in two and three dimensions.

Einstein's field equation

Even though it is difficult to visualize curvature in four dimensions, it is possible to calculate it. Einstein described gravity with an equation showing that the curvature of spacetime is proportional to the mass and energy present in spacetime. The constant of proportionality is Newton's classical gravitational constant.

Figure 10.7: Einstein's field equation is explained in easy terms by illustrating the proportionality between spacetime curvature and mass and energy.



the abstract description of curvature of spacetime to their own experience of gravity. While they could state that mass curves spacetime and the curvature of spacetime leads to gravitational effects, they struggled to explain why gravity kept them grounded on the floor.

From this first and rather broad exploration of the spacetime concept, study (II) zoomed in on students' conceptual understanding of spacetime by analysing how students used the rubber sheet analogy in their explanations. Study (III) presented a new instructional model of warped time that teachers can use as a supplement to the rubber sheet analogy. Study (IV) looked at the ways students conceptualized movement in spacetime.

In study (II), we employed methods of metaphor analysis to gain insights into students' shared explorations of the rubber sheet analogy and the spacetime concept. Perspectives of psychology and cognitive linguistics informed this analysis because these fields explain how the abstract domain of time gets its relational structure from the more concrete domain of space (Boroditsky, 2000; Lakoff and Johnson, 2003; Matlock et al., 2005). It seems that the domains of space and time share conceptual structures which becomes relevant if we ask students to conceptualize spacetime as an entity in which space and time are merged into a higher-dimensional object. Research suggests that learners tap into their experiential knowledge, for example their understanding of physical space and motion, to make sense of abstract domains such as time (Matlock et al., 2005). It is exciting that recent findings from psychology and cognitive linguistic have the potential to inform GR education. My research has given first examples of how embodied cognition can be fruitfully employed in the service of GR education.

The source domain of the rubber sheet analogy draws on students' embodied experience. Yet, contrary to previous research that suggests that understanding needs embodiment (Niebert et al., 2012), we found that the embodied experience of gravity often got in the way of inferring the right analogical mappings. It seems that students' sensory experience of gravity may actually have contradicted the concept of spacetime. Despite the fact that embodied mechanisms usually help learners make sense of abstract concept, in the learning domain of spacetime embodiment can run into conflicts. In order to conceptualize the physical mechanism of gravity in the domain of GR, students need to develop awareness of the tension between the physical force of gravity in the everyday experiential sense and the curved spacetime explanation. More generally, the question of how students understand the abstract concept of spacetime through spatial metaphors and their experiential knowledge of space and forces should be investigated further in future work.

### **10.3 Science education as a design science**

Throughout this thesis, I have used the notion of design principles quite uncritically as a means to structure and report on the development process of the digital learning environment *General Relativity*. In this section, I take a critical step

back – first, to contextualize my interpretation of design principles against the different DBR-practices; second, to discuss how design practices and educational practices share common features that can fruitfully inform each other.

Seeing that researchers are still struggling to agree on clear standards for DBR as a methodology in its own right (Sandoval, 2014), it is interesting to discuss the variety of DBR practices in light of my own research in physics and science education. The way DBR practices informed my research clearly reflects the educational agenda of ReleQuant, namely to design a digital learning environment that works in classroom settings and thus, to improve teaching and learning in GR. It is insightful, however, to contrast the DBR practices in my PhD-work with the ones in the greater educational community. Different traditions, goals, and motivations among scholars of different but related fields naturally lead to different DBR practices. There is value in spelling out how DBR practices can be of service in science education research and practice.

In the theoretical framework, I have briefly traced the origins of DBR to sketch its emergence as a new paradigm for educational research within the learning sciences (Design-based Research Collective, 2003). Now I want to compare this paradigm to the interpretation of DBR practices within ReleQuant and my own research. DBR is characterized by multiple design iterations and a collaborative partnership between researchers and practitioners (such as teachers). Accordingly, the development of the ReleQuant quantum physics and GR learning resources followed standard DBR practice. The design of the learning environments was carried out over four and three rounds respectively and each round of testing was accompanied by seminars and workshops where teachers from six schools in the Oslo area provided feedback and shared their experiences.

Key to these iterations was the collaborative momentum that we tried to keep up by having feedback loops that allowed us to quickly incorporate feedback and shared experiences. As such, the DBR development process was self-correcting; we could keep track of the influence of the learning resources and make changes as needed. Educational assumptions can easily fall out of step with the context of design work (Chimero, 2012). Reflecting on (implicit) assumptions and discussing them with practitioners made our design choices observable so that they could be improved upon. By formulating design hypotheses and testing our interventions in line with these hypotheses, we aimed to stay on track and stay true to our assumptions and the context of the development.

However, the motivation of theory building and viewing (science) education as a design science (Collins, 1992; Collins et al., 2004) went somewhat short in my work and in the greater ReleQuant agenda. According to Chimero (2012), all design work has three common traits: there is a message to the work, the tone of that message, and the format that the work takes. The message relates to the content and the information that is communicated. The tone is the domain of design; the format is the artifact that is being produced. Guided by the wish to find workable solutions to teaching and learning GR, we emphasized the characteristics of the message (the content of GR) and the artifact (the website of *General Relativity*). However, we did not put more efforts into determining how the tone and the arrangement of the message contributed to the learning

processes and how the relationship between message, tone, and style influenced learning and collaboration. Thus, we did not fully realise the potential of viewing (science) education as a design science.

Thus, our emphasis on the message and the physics content of the learning resources (as opposed to more overarching design perspectives) has led to slightly different interpretations of what design principles are. While design principles, at the most general level, are commonly seen as an intermediate step between scientific findings, which must be generalizable and replicable, ReleQuant interprets design principles more pragmatically. Here, these principles are mostly content-related and do not add to the theory building of science education as a design science. Few of our design principles concern the actual design of learning resources; rather, they relate to the presentation of content-specific issues (Kersting et al., 2018a; Bungum et al., 2015).

In summary, my work and the ReleQuant research have used the DBR framework quite pragmatically as a means to develop research-based learning resources that work in the classroom setting. This use makes sense in light of our goal to improve teaching and learning in modern physics. This practice has, however, neglected more general purposes of DBR as emphasized by educational researchers - for example to contribute to theory building by elucidating how different designs contribute to different features of learning processes (Collins et al., 2004). While both science educators and educational researchers apply similar frameworks and methods, the motivations for employing DBR might diverge between those two communities of practice. Nonetheless, the underlying goal of DBR, to study the link between educational processes and outcomes, remains visible in both communities of practice.

Design-based research is one example of how science education research can benefit from design methods. I now want to discuss the nature of design more broadly because I think there is an interesting commonality between the disciplines of design and education. This commonality suggests that design practices have the potential to inform science education research in fruitful ways. There are different ways of characterizing design. First, good design moves, that is its marker (Chimero, 2012): design functions as the road we follow to reach what is possible. Second, design gains value as it moves from hand to hand: good design work turns into a shared experience (Chimero, 2012). The same can be said about education because education has transformational power that stimulates and moves students to reach their full potential in collaboration with their peers.

If science education researchers become aware of this transformational and collaborative power, they might look more closely at design practices to learn how to successfully enact changes. Design has a tendency to live between things in order to connect them. Bridging two things means a bond with both of them (Chimero, 2012). Designers have established a vocabulary and developed methods to negotiate problem spaces between themselves, their clients, and their audiences. Science education researchers can learn from these experiences because science education, in a similar vein, sits at the crossroads of disciplines and tries to connect diverse group of people.

A design framework is an example of how design practices can be transferred to educational practices. A design framework acts as the bridge that connects the designer to his audience because the framework allows a systematic gathering of thoughts, opinions, and proposed solutions; in other words, frameworks reap the contributions of an intelligent and experienced audience and give designer and audience shared ownership of the products of design (Chimero, 2012). The way I have employed the MER framework in my research shows that the same holds true in education if we specify the generic design audience to be students and teachers and if we replace the designer by the ReleQuant team of educational researchers.

### 10.4 Curriculum and classroom perspectives

In this section, I contextualize my PhD-work by showing how my research relates to physics education practice in Norway and around the world. Just as physics is a dynamic scientific discipline that has grown and developed over time, so has physics education undergone changes that were often propelled by changing values in our society at large.

Arguably, physics is one of the most fundamental scientific disciplines because physicists try to describe how the universe works at the most fundamental level. Physics education research should therefore aim at finding successful strategies to teach students about space, time, matter, and the dynamics of the fundamental forces. Yet, there is more to ask and expect from successful physics education than just conveying these key concepts. Such expectations get often expressed in the physics curriculum. Since this PhD-research was conducted within the Norwegian educational context, I use the Norwegian physics curriculum to exemplify how perceptions of physics and expectations of what constitutes good physics teaching have changed over time. This presentation serves as the backdrop to contextualize the contributions of my work within a greater tradition of teaching physics in Norway and worldwide<sup>1</sup>.

Good physics education is equally grounded in physics and pedagogy. Pedagogical perspectives relate both to different modes of instruction and to skill sets that students require in addition to learning about the content specific aspects of physics. I argue that my research promotes physics education that is modern both from the content and the pedagogical perspectives. To support that argument it is useful to sketch the historic development of physics education in Norway first.

During the first half of the 20th century and up to the 1970s, teaching physics in Norway was closely linked to the practical and technical applications of physics (Angell et al., 2019). This emphasis aligned with a society that was, by and large, characterized by industrial transformations and technical advancements.

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<sup>1</sup>Interestingly, it seems that physics curricula across different countries do not differ significantly from each other. As part of the TIMSS Advanced project, physics educators found that national curricula have great overlap in their presentation of content and competencies (Mullis and Martin, 2014).

The physics curriculum asked students to study technical applications such as steam engines, hydraulic turbines, and radio valves (Angell et al., 2019). In a similar vein, students were expected to create detailed construction drawings of these apparatuses and conduct experiments to understand the workings of technical and everyday phenomena.

During the 1970s and 1980s, the presentation of physics in Norwegian schools got increasingly abstract and more scientific. The emphasis shifted from teaching explicit technical applications of physics to conveying more general physical principles. This shift brought also forth an increased use of mathematics to describe physical phenomena.

Subsequently, cultural, historical, and philosophical perspectives were integrated into the physics curriculum. Physics became part of the general knowledge and education (“allmenndannelse”); students were expected to understand how physics affects society and how ethical considerations go along with scientific progress (Angell et al., 2019). For example, the topic of energy and the energy consumption of our society entered the Norwegian physics curriculum in the 1980s to illustrate the interdependence of physics and society. In line with such cultural and ethical perspectives, the curriculum asked students to build qualitative understanding in addition to solving problems quantitatively.

In 2006, a new physics curriculum was launched in Norway (The Norwegian Directorate for Education and Training, 2006). It is this curriculum reform that initiated project ReleQuant and that provided the starting point for my PhD-research.<sup>2</sup>

However, the introduction of GR is not surprising in light of the historic development of the Norwegian physics curriculum. Even though relativistic concepts only have few immediate practical applications, GR serves as a good example to illustrate the nature of science as well as historical and philosophical features of physics. Introducing GR as a topic in upper secondary physics education thus reflects the changing values of a society that has moved from an age of industrial growth towards a modern age of services, knowledge, and information technology. The emphasis on cultural perspectives in the 2006 physics curriculum goes hand in hand with an increased focus on skills that allow students to communicate key ideas of physics successfully. These skills entail talking, writing, and reading physics in addition to doing calculations and using digital tools.

It is against the backdrop of this curriculum development, that my work contributes to modern physics education in a double-sense. First, the content of my research is modern because GR is an emerging learning domain that has traditionally not been taught in schools. Teaching students GR reflects the belief that knowledge of our current best understanding of the universe will set students up for a successful future as informed citizens of our modern society. Second,

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<sup>2</sup>At the time of writing this thesis, a new curriculum is currently developed in Norway.<sup>3</sup> Even though special relativity had been part of the Norwegian physics curriculum since the 1960s, it was only in 2006 that GR was introduced as a mandatory topic in the final year of upper secondary physics education in Norway. Such a choice of topic is uncommon internationally (Henriksen et al., 2014; Kersting et al., 2018a).

the pedagogical perspectives of my research are modern because the digital learning resources foster modern forms of instruction that differ from practices in traditional physics classroom. Research suggests that students encounter a very specific discipline culture in the physics classroom. Traditionally, this culture has been characterized by blackboard-centered instruction and a strong focus on solving problems quantitatively using mathematics (Bøe, 2012; Bøe et al., 2018; Duit et al., 2014; Höttecke et al., 2012). In contrast, the instructional approaches that we have implemented in our digital learning environment differ from traditional instructional practices in three significant ways:

1. **History and Philosophy of Science** Increasingly, science educators call for inclusion of history, philosophy and the nature of science to make the culture of the physics classroom more inclusive (Abd-El-Khalick, 2013; Johansson, 2018; Monk and Osborne, 1997). With my research, I have shown that HPS and NoS provide promising opportunities to introduce secondary school students to GR.
2. **Qualitative Understanding** Often, the notion of conceptual understanding implies qualitative reasoning (Sands, 2014). Contrary to common opinions, this kind of reasoning is far from simple. Concepts are dynamic and they emerge from unique circumstances in the classroom. In fact, physics educators still discuss the nature of concepts and call for a reframing of concepts and a discussion of the meaning of conceptual understanding in science education (Hardman, 2019; Kampourakis et al., 2018). In line with this view, my research presents successful examples of how to implement instructional approaches that invite students to reason qualitatively and to build awareness of the scope and limits of conceptual understanding in abstract learning domains.
3. **Talking Physics** Science cannot easily be separated from language; science content cannot fully be understood without language (Bratkovich, 2018). Related to my focus on qualitative understanding is the active use of language that the ReleQuant learning resources have promoted in the physics classroom. In line with the sociocultural perspective, the learning resources provided students with the tools to build conceptual understanding through language (Wertsch, 1993). This approach is directly linked to the Norwegian physics curriculum that encourages students to talk, write, and read physics.

By providing research-based evidence of the efficacy of these three instructional approaches, my research contributes to the ongoing debate on what good physics education should look like both in Norway and internationally. Even though physics education research cannot give universal recipes for success, my findings suggest how to facilitate successful learning in the context of GR education. In this context, non-traditional approaches to physics education have the potential to ignite a passion for physics among many students which, in turn, can contribute to educating science-literate citizens. Likewise, findings of my research can

contribute to an improved education of physics and science teacher students. Thus, in addition to directly supporting in-service teachers and students in Norwegian physics classrooms, my research is likely to influence the ways modern physics is taught in teacher education and will be taught in Norway in the future as well (Angell et al., 2019).









## Chapter 11

# Conclusion and Outlook

When the ideas involved in Einstein's work have become familiar, as they will do when they are taught in schools, certain changes in our habits of thought are likely to result, and to have great importance in the long run.

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*Bertrand Russell*

The field of physics education aims to accomplish both research and practical goals. Physics educators move between the world of research and scholarship and the world of school and teacher practice. With this PhD-research, I have contributed to both worlds:

1. I have developed a digital learning environment about GR to make the ideas of Einstein's work teachable in schools. The learning environment targets upper secondary physics students in Norway and aligns with the Norwegian physics curriculum. The design of the learning resources builds on a content structure for instruction that aims to support teachers to make informed choices about instructional practices in the learning domain of GR.
2. I have conducted qualitative research that investigates how students build conceptual understanding of abstract scientific concepts in the domain of GR. Moreover, I have demonstrated how the MER and DBR frameworks can be fruitfully combined to develop learning resources and to study student learning processes.

The practical implications of my work contribute to an ongoing debate on how to make "Einsteinian Physics" more accessible at the primary and secondary school level. In the following, I summarize the most important contributions and implications of my research along the three dimensions of why, what, and how. Doing so, I cast my work into a greater perspective of international efforts in Einsteinian Physics education research. Concluding, I formulate directions for future research.

### 11.1 Why should students learn GR?

Time in schools is limited and school curricula cannot cover all topics in science. It is therefore legitimate to ask why students should learn GR. Asking why unearths a purpose and helps the physics education community define students' needs. This identification of needs allows formulating objectives which, in turn, help us improve teaching and instruction.

## 11. Conclusion and Outlook

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The most obvious response to why students should learn about Einstein's theory of gravity pertains to the immediate context of my PhD-project. My research responded to a curriculum in Norway that had introduced GR as a mandatory part for final year physics students in secondary schools. Thus, my project addressed educational challenges of an existing physics curriculum.

However, referring to an existing curriculum is of course a rather lazy response that does not provide motivation beyond the narrow constraints of a national educational system. Indeed, there are more interesting and more universally applicable reasons for why we should introduce young minds to relativistic ideas.

One important purpose of education is to introduce children to the society into which they are born and to the physical universe that surrounds this society (Feyerabend, 1975). To achieve this goal, educators often draw on basic myths with which they can explain the world around us. (Feyerabend, 1975). GR is our current best theory of gravity, space, and time. By describing gravity as a geometric phenomenon and spacetime as a dynamic fabric, the theory provides a myth and through this myth a "language of reality" which helps us conceptualize the world around us. Indeed, research in psychology, cognitive science, and linguistics suggests that language is more than a mere system of communication (Boroditsky, 2000; Lakoff and Johnson, 2003; Chomsky, 1972). Language might contribute to shape our understanding of the very world we inhabit.

However, learning new languages gets harder the older one becomes. The same holds true for learning languages of reality. Once certain mental habits have formed, new ideas that are contrary to students' pictures of the world are more difficult to accept (Russell, 1925). We owe it to future generations to teach them a language of reality that will allow them facing a quickly changing world and an uncertain future.

According to Asimov (1981), "it is change, continuing change, inevitable change, that is the dominant factor in society today. No sensible decision can be made any longer without taking into account not only the world as it is, but the world as it will be." Our wish to teach students our best understanding of the Universe is underpinned by the belief that exposure to Einsteinian Physics will make young minds grow. We wish to educate students so that they become capable of choosing, imagining, and criticizing different myths of the world. Ideally, students will accommodate Einsteinian perspectives as their language of reality while at the same being able to view Newtonian gravity as a useful approximation of this Einsteinian reality.

Moreover, GR serves as a great example to illustrate the nature of science as an evolving human endeavour that involves creativity, mathematical expression, thought experiments, controversies, scholarly discussions, and experimental and observational tests of the theory. Scientific laws are not set in stone and modern physics is not completed. Scientists improve existing knowledge; they work creatively and propose ideas that can push and transform current scientific understanding. The recent birth of gravitational wave astronomy and the first-ever taken picture of the shadow of a supermassive black hole show the relevance of GR as an important field of scientific inquiry in the future.

## 11.2 What should students learn in GR?

Thirty years after attempting to explain relativity to a general audience (Einstein, 1917), Einstein had adopted a somewhat sobering perspective towards communicating relativistic ideas to non-experts:

“Anyone who has ever tried to present a rather abstract scientific subject in a popular manner knows the great difficulties of such an attempt. Either he succeeds in being intelligible by concealing the core of the problem and by offering the reader only superficial aspects or vague allusions, thus deceiving the reader by arousing in him the deceptive illusion of comprehension; or else he gives an expert account of the problem, but in such a fashion that the untrained reader is unable to follow the exposition and becomes discouraged from reading any further. If these two categories are omitted from today’s popular scientific literature, surprisingly little remains. But the little that is left is very valuable indeed.” (Barnett, 2005)

Despite those difficulties, I agree with Einstein that the little that is left and the little that conveys key ideas of relativity to a non-expert audience is very valuable indeed. As part of my educational reconstruction, I have specified the vague Norwegian learning goal of “giving a qualitative description of GR” in form of a content structure for instruction. By identifying key concepts of GR and translating these concepts into explicit learning goals that are suitable for secondary school students I hope I have avoided to arouse the deceptive illusion of comprehension and to discourage students from wanting to learn more about relativistic concepts.

Thus, an important contribution of my research presents an answer to the question of what students should learn in the learning domain of GR. This contribution is significant since comparable educational studies only look at a few relativistic concepts without attempting to capture a more holistic picture of GR (Bandyopadhyay and Kumar, 2010b; Baldy, 2007; Pitts et al., 2014; Kaur et al., 2017b). Moreover, by presenting specific learning goals as presented in Table 8.1, my educational reconstruction of GR is easily transferable to school practices in other contexts.

## 11.3 How can physics education research support students in learning GR?

While asking why unearths a purpose and develops a point of view, questions about how enable actions, improve craft, and elevate form (Chimero, 2012). By developing our digital learning environment, I have made assumptions about how students can effectively learn relativistic topics and have tested these assumptions against a set of design hypotheses. My work thus demonstrates the feasibility of teaching GR at the secondary school level based on a qualitative approach facilitated by digital learning resources. Sharing my learning findings and assumptions, I hope to offer valuable impetus to the physics education community.

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However, my research is not the only attempt of introducing GR at the secondary school level. Physicists and physics educators around the world have made various efforts to develop instructional strategies and instructional models to teach GR (Pössel, 2018; Jonsson, 2004). For example, the Australian project Einstein-First relies on hands-on activities and active learning to engage students (Kaur et al., 2017b). The Australian Centre of Excellence for Gravitational Wave Discovery OzGrav develops immersive Virtual Reality experiences to inspire and educate students<sup>1</sup>. German physicists have developed sector models as a geometrical approach to teach GR (Kraus and Zahn, 2018; Zahn and Kraus, 2018). Latest efforts focus on turning sector models into digital applications (Weissenborn, 2018), thus aligning German and Norwegian instructional approaches to a greater extent. In Korea, educators look at historic and cultural approaches towards teaching Einsteinian Physics and call for Einsteinian Physics education as a new way to improve physics education (Kim and Lee, 2018). In Scotland, the curriculum for excellence<sup>2</sup> has been a forerunner in innovative curriculum design.

The diversity of teaching approaches that are being developed around the world certifies to the relevance of GR and Einsteinian Physics as an emerging learning domain in physics education at the primary and secondary school level. While the above research groups and educational projects each have pioneered different approaches to teaching and learning Einsteinian Physics, they have encountered similar obstacles in implementing these approaches in schools and curricula (Moschilla et al., 2017). This observation led to the formation of the Einsteinian Physics Education Research (EPER) collaboration that wants to teach students our most contemporary scientific understanding of the Universe while finding new ways of conducting physics education research on a global scale (Blair et al., 2016; Choudhary et al.).

The EPER collaboration is an example of how much impact physics education research can have if physicists and educators collaborate at a large scale to introduce students to Einsteinian Physics. The main objectives of EPER have addressed four specific areas:

- **Resources:** to share existing Einsteinian Physics learning resources and develop novel learning resources for all levels of education
- **Teachers:** to develop resources for teachers and to run teacher professional development workshops
- **Students:** to investigate and promote students' conceptual understanding in the learning domain of Einsteinian Physics
- **Dissemination:** to disseminate best-practice examples and research results by building an international research network

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<sup>1</sup><https://www.ozgrav.org/education.html>

<sup>2</sup>[https://education.gov.scot/scottish-education-system/policy-for-scottish-education/policy-drivers/cfe-\(building-from-the-statement-appendix-incl-btc1-5\)/What%20is%20Curriculum%20for%20Excellence](https://education.gov.scot/scottish-education-system/policy-for-scottish-education/policy-drivers/cfe-(building-from-the-statement-appendix-incl-btc1-5)/What%20is%20Curriculum%20for%20Excellence)

Within the EPER collaboration, we have worked with teachers and students around the world to find instructional approaches that work in practice. Of course, paradigmatic changes do not happen over night. We have taken the first steps towards a future of modern physics education, though, and will continue to push boundaries of modern physics education research. My PhD-research has contributed to this ongoing effort by providing research-based evidence of how students can learn GR qualitatively at the upper secondary school level.

## 11.4 Outlook on future research

Finally, to conclude this section, I give an outlook on future areas of research that seem promising to pursue based on the findings of my research.

1. The role of the teacher is an important component in teaching and learning of GR that I have not studied in my PhD-project. By adopting a sociocultural perspective, the ReleQuant resources have facilitated student discussions that reflect dialogic approaches. Traditional physics instruction often lacks these dialogic approaches (Mortimer and Scott, 2003). However, balancing dialogic and authoritative modes of instruction might be better suited to foster conceptual understanding of abstract scientific concepts because teachers can better realize the potential of small-group discussions by integrating the discussions within a broader pedagogical frame (Bungum et al., 2018; Tang and Tan, 2017). Future research could look more specifically at interactions between teachers and students to realize the potential of sociocultural approaches to GR education.
2. Already in the 1970s, Stenhouse (1975) observed that “there can be no educational development without teacher development.” Thus, studies of teacher perspectives and their professional development will be an important contribution to firmly establish Einsteinian Physics in secondary school curricula around the world.
3. My PhD-research has been based on qualitative methods to unpack learning processes in GR. Quantitative approaches have the potential to complement my research on students’ conceptual understanding in fruitful ways. In particular, based on the exploratory character of much existing research in Einsteinian Physics education (Pitts et al., 2014; Steier and Kersting, 2019), promising future projects could involve greater student populations and different national contexts to obtain more generalizable findings. The creation of a concept inventory in GR would be worthwhile contribution to the field similar to the concept inventory in special relativity (Aslanides and Savage, 2013).
4. My work has centered on utilizing modern technology in form of digital learning resources to support students in their learning of GR. With the rise of virtual and augmented reality technology, it seems that there is even greater potential to develop learning resources that help students

## 11. Conclusion and Outlook

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counterbalance their lack of experience of relativistic phenomena. Studying the specific processes through which students come to build understanding within digital and virtual reality seems a great vision for future physics education research in GR. It is only through a better understanding of how students interact with each other and across digital and virtual modes of representations that educators can fully capitalize on the potential of new technologies.







## Chapter 12

# Epilogue

When faced with a totally new situation, we tend always to attach ourselves to the objects, to the flavor of the most recent past. We look at the present through a rear-view mirror. We march backwards into the future.

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*Marshall McLuhan*

The world and its landscapes shape us. To conduct the research presented in this thesis I took on the role of a traveler traversing and reading new landscapes both literally and figuratively. Much of my research is inspired by the roads I have travelled and the connections I have made. Many of my papers were written not in my office but in planes and trains, in coffee shops and hotel rooms abroad. Many of my ideas took shape while conversing with colleagues and friends from near and far. During the past four years my work has led me to ten different countries and has allowed me to attend almost twenty conferences. All these places asked questions of me just as searchingly as I questioned them. Random encounters fueled my imagination. I have fallen down countless rabbit holes - many of which I have climbed up again, bruised and confused but glad all the same. While writing these lines I sip on iced coffee watching life in Bologna pass by. Putting the finishing touches on this thesis after having attended ESERA 2019 seems like an apt way to wrap up this album of landscape sketches. The beauty and pleasure of understanding have guided my explorations; I have repeatedly fallen in love with beautiful ideas, words, and minds while traversing unknown fields of thoughts.

(...) marching is no way to go into the future. It is too methodical and restricted. The world often subverts our best laid plans, so our road calls for a way to move that is messier, bolder, more responsive. The lightness and joy afforded by creating suggests that we instead dance.

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*Frank Chimero*



# Papers



## General relativity in upper secondary school: Design and evaluation of an online learning environment using the model of educational reconstruction

Magdalena Kersting,\* Ellen Karoline Henriksen, Maria Vetleseter Bøe, and Carl Angell  
*Department of Physics, University of Oslo, P.O. Box 1048 Blindern, 0316 Oslo, Norway*

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Because of its abstract nature, Albert Einstein's theory of general relativity is rarely present in school physics curricula. Although the educational community has started to investigate ways of bringing general relativity to classrooms, field-tested educational material is rare. Employing the model of educational reconstruction, we present a collaborative online learning environment that was introduced to final year students (18–19 years old) in six Norwegian upper secondary physics classrooms. Design-based research methods guided the development of the learning resources, which were based on a sociocultural view of learning and a historical-philosophical approach to teaching general relativity. To characterize students' learning from and interaction with the learning environment we analyzed focus group interviews and students' oral and written responses to assigned problems and discussion tasks. Our findings show how design choices on different levels can support or hinder understanding of general relativity, leading to the formulation of design principles that help to foster qualitative understanding and encourage collaborative learning. The results indicate that upper secondary students can obtain a qualitative understanding of general relativity when provided with appropriately designed learning resources and sufficient scaffolding of learning through interaction with teacher and peers.

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### I. INTRODUCTION

Albert Einstein's theory of general relativity is an important pillar of modern physics. Classically, gravity is described as a force, but Einstein described gravitational effects as a manifestation of a deep connection between time, space, and massive objects. This revolutionary insight is rarely present in school physics curricula because general relativity (GR) is abstract in a double sense: First, learners lack experience with relativistic phenomena, because the realm of GR covers extreme situations not to be found in everyday life. Second, GR builds on a sophisticated mathematical framework that is not readily accessible to secondary school students.

Nonetheless, physicists and science educators argue for introducing GR in schools because of its far-reaching scientific, philosophical, and cultural importance [1–3]. Not only does the theory constitute our best understanding of the Universe, it continues to inspire scientific and technological progress and has become an issue of practical concern to mainstream physicists and engineers [4]. GPS

technology relies on GR as does the discovery of gravitational waves that caught public interest in 2016 [5].

Clearly, when introducing GR to school physics, we need to find ways of communicating important features of GR without relying on its advanced mathematical foundations [6]. However, it seems that the educational community has not properly laid the groundwork for bringing GR to schools [7]. Studies on secondary school students' conceptual development of key concepts in GR are scarce. Most research papers focus either on special instead of general relativity [8–10] or investigate learning among undergraduate university students [4,11–13]. Relying mostly on case studies and interviews, these studies agree in their findings that students struggle with the interpretation of relativistic phenomena and that they hold a large number of misconceptions.

Despite first steps towards teaching GR in schools and an increasing awareness to teach GR at the undergraduate level [4], there is a lack of research literature describing field-tested learning resources in GR that are tailored to the needs of secondary school students. The present paper aims to fill this gap by reporting on the development and evaluation of an online learning environment and upper secondary students' understanding of key concepts in GR.

### II. BACKGROUND OF THE STUDY

In this section we present the greater educational project that this work is part of and discuss the curriculum

\*[magdalena.kersting@fys.uio.no](mailto:magdalena.kersting@fys.uio.no)

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constraints of learning physics in Norway to contextualize our study and its results.

### A. Project ReleQuant

This study was conducted within the broader ReleQuant project that develops collaborative online learning environments in general relativity and quantum physics for the optional physics subject in the final year of upper secondary school in Norway [14]. ReleQuant was established to investigate novel ways of teaching modern physics and to study students' learning processes and motivation. In the domain of modern physics, demands on both students and teachers are high. Students struggle with relativity and quantum physics, since these topics describe phenomena that cannot be visualized or experienced directly [7,15,16]. Often, teachers lack sufficient background in modern physics, thus finding the topics conceptually demanding [17]. Moreover, by emphasizing qualitative understanding and philosophical aspects, the Norwegian curriculum requires novel ways of teaching and learning that differ from teacher-centered traditional physics lessons focused on content knowledge and mathematical problem solving.

The development of learning resources in ReleQuant is based on a sociocultural perspective on learning, which emphasizes language as an integral part of the learning process [18,19]. This approach entails that students make physics concepts their own through use of language and interaction with others [19,20]. In particular in the domain of GR, upper secondary students have to rely on language since the advanced mathematical framework of GR is not available.

Another feature of the development of ReleQuant resources is the inclusion of history, philosophy, and nature of science (NOS) aspects. History, philosophy, and NOS have long been advocated as important elements in science education [21,22]. Physics educators have made calls for physics education to move beyond traditional content-focused instruction and include historical, epistemological, and sociocultural aspects (see, e.g., Ref. [23]). Moreover, research has shown that use of history, philosophy, and NOS can foster understanding in relativity and quantum physics content [8,24], making this a suitable approach for an educational reconstruction of GR that focuses on qualitative understanding.

### B. General relativity in the physics curriculum

With the school reform introduced in autumn 2006, GR was included in the Norwegian curriculum for upper secondary physics. Such a physics curriculum is uncommon. Of five other European countries and Australia, which have an educational system comparable to Norway's, only Sweden includes GR [15]. In addition to the more formalistic and traditional topics on the curriculum, such as Newtonian mechanics and electromagnetism, Norwegian physics students are expected to

explore different interpretations and philosophical aspects of modern physics. However, the specific curricular goal for GR remains vague:

*The aims of the studies are to enable pupils to give an account of the postulations that form the basis for the special theory of relativity, discuss qualitatively some of the consequences of this theory for time, momentum and energy, and give a qualitative description of the general theory of relativity [25].*

This broad description leaves scope for various interpretations of what constitutes GR qualitatively. Indeed, the physics curriculum in Norway gives some guidance for interpretation since it emphasizes historical and philosophical aspects of physics in particular:

*[The subject] also deals with how scientific knowledge is established and with conflicts and dilemmas that might arise during this process. (...) [the] subject shall help create an awareness that physics is part of our cultural heritage and that the subject must be viewed in a historical perspective [25].*

In addition, the curriculum emphasizes basic skills such as expressing oneself orally and in writing when learning physics. Thus, the approach chosen in the present work aligns both with the Norwegian curriculum goals and with research literature that describes the important role of language in a sociocultural setting [19,20], and that recommends to include historical and philosophical aspects in school science in order to foster understanding of the nature of science [21]. We believe that such an approach will be broadly applicable to preuniversity physics curricula in other countries as well, thus advancing modern physics education in general.

## III. THEORETICAL AND METHODOLOGICAL FRAMEWORK

In this study, we combine two powerful frameworks: The model of educational reconstruction (MER) [26] provides a theoretical frame for choices of physics content and learning resources, while design-based research (DBR) [27] serves as a methodological frame for the development and evaluation of the learning resources.

Both DBR and MER draw on the tradition of designing and evaluating teaching-learning sequences (TLS). In a special issue of the International Journal of Science Education on TLSs, Méheut and Psillos note in the editorial:

*Yet whereas there is often extensive communication of learning results, the various explicit and implicit assumptions and decisions concerning the design of a TLS, its teaching features or the interlacing of teaching with*



learning are less widely discussed, and may not even be made clear and comprehensible [28].

By dividing the design process into distinct phases, MER and DBR address this challenge and offer methods for validating and reporting on the design of learning environments.

### A. Model of educational reconstruction

The model of educational reconstruction [26] offers a methodological framework for science education research and provides guidance for integrating empirical research on teaching and learning in the development of content structures for instruction and learning resources [28,29]. The framework comprises the basic idea that three strands of educational research are closely connected (Fig. 1): First, to analyze the particular science content of a topic and to identify its key concepts. Second, to take student and teacher perspectives into account and to assess the crucial features of students' learning processes in this topic. Third, to design and evaluate learning environments and suitable learning activities.

MER aligns well with the objectives of this study since its broad and holistic approach can serve as a useful tool for scrutinizing the educational relevance of fields of science that have not entered mainstream education yet. In particular in the field of physics, the model has previously been employed to reconstruct novel learning domains such as chaos theory [30], nonlinear systems [31,32], nanoscience [33,34], climate change [35], and physical geography [36].

In this study, an analysis of the science content structure of GR and students' perspectives according to the literature served as a basis for the design, development, and evaluation of an online learning environment in GR. Presenting new empirical results, we add, moreover, to the understanding of upper secondary physics students' learning challenges and discuss the mutual relation between the three strands of MER.

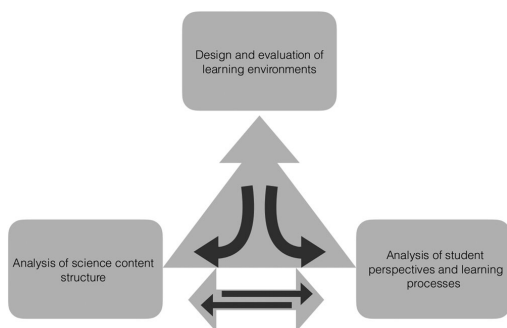


FIG. 1. The model of educational reconstruction combines three strands of educational research iteratively [26].

### B. Design-based research

The main goal of the DBR methodology, also referred to as educational design research [37], is to develop and implement systematic solutions to educational problems [27,38]. The framework offers suitable tools for developing field-tested learning resources, because it relies on repeated rounds of development in close collaboration with practitioners. The iterative phases comprise analysis and reflection of the problem, development and design of solutions, and evaluation using classroom research [37]. Bridging the gap from research to classroom settings, the formulation of design principles is a common result of DBR research praxis [17,39].

Focusing on design revisions and reporting more generally on the design research and its elements, goals, and phases are characteristic features of DBR methods [40]. Therefore, we have made the design of our learning environment a crucial part of our research agenda. In this study, tentative design hypotheses were derived from the content structure for instruction that guided the development of the learning environment. Testing these design hypotheses in the classroom setting allowed for iterative revisions in the design process and enabled us to formulate research-based design principles.

In line with the stance of project ReleQuant, the design of learning resources drew on the sociocultural tradition of viewing knowledge as constructed within and distributed among people and their environments [41]. In particular, we view learning and conceptual development in GR as a process of students' interaction with peers and the teacher and with their (physical and technological) environment. Students in the physics classroom construct knowledge through collaborative activities that are partly mediated through the online learning environment. In line with Vygotsky [42], who highlights the interrelationship of language and the development of abstract thoughts, our teaching approach uses discussion and writing tasks as cognitive tools. "Talking physics" [19,43] is one important way to reach understanding of abstract relativistic concepts.

### C. Aims and research questions

The aim of this study is to describe and propose an educational reconstruction of GR based on the science content structure and published literature on teaching and learning the topic supplemented with our own empirical results. Specifically, we have investigated Norwegian upper secondary physics students' work and interaction with an online learning environment to answer the following research questions:

- RQ1: What characterizes the understanding of key features of GR that participating students express while engaging with the learning environment?
- RQ2: In what ways do the participating students' experiences support or challenge the design hypotheses

that guided the development of our online learning environment?

- RQ3: Based on findings from RQ1 and RQ2, what design principles can be formulated for the development of learning resources in GR at the upper secondary level?

#### IV. AN EDUCATIONAL RECONSTRUCTION OF GENERAL RELATIVITY

Following the organization of MER, in this section we bring together all three strands that have guided our attempt to reconstruct GR from an educational perspective. The first steps towards an educational reconstruction of GR consisted of identifying key concepts of GR and summarizing published research concerning students' conceptions and challenges. Based on these insights, we created a content structure for instruction consisting of a sequence of specific learning goals and design hypotheses that provided guidance for the development of the learning environment. We then tested the learning environment in classrooms and evaluated and adjusted the design iteratively. In the subsequent sections, we will report on the 2016 class room trial of this learning environment and the resulting design principles.

##### A. MER-component 1: Key concepts of general relativity according to textbooks

To identify key concepts of GR and to gain insight into how educators treat those concepts, we started by analyzing the presentation of GR in the two Norwegian upper secondary physics textbooks that are on the market [44,45]. This analysis enabled an insight into how the book authors interpreted the vague curriculum goals explicitly. Since the Austrian physics curriculum treats GR similarly to the Norwegian one [46], an Austrian textbook [47] was scrutinized to obtain an additional and possibly contrasting educational perspective.

All three books present similar key features of GR and differ mostly in the degree of given details. The principle of equivalence and its implication to the equality of gravitational and inertial mass serves as a first step into the topic: *Locally, one cannot distinguish between a gravitational field and uniform acceleration.* This is followed by a discussion of reference frames and a generalized version of the principle of relativity: *An inertial frame is a reference frame in free fall and the laws of physics take the same form in all reference frames.* Gravitational redshift, gravitational bending of light, and gravitational time dilation are mentioned as three important phenomena. Finally, curvature of spacetime is presented: *Gravity is not a force, but a geometric phenomenon that is connected to the curvature of spacetime.*

Interestingly, when it comes to curvature of spacetime, the foci of the two Norwegian textbooks differ to a great

extent. While Callin *et al.* only mention the geometric nature of gravity on the final page of the chapter on GR without detailed explanations [44], Jerstad *et al.* choose this topic as a main focus and present Einstein's struggle to find a mathematical description of GR from a historical point of view [45]. More in line with this second presentation, the Austrian textbook discusses curved spacetime in detail, but highlights features of non-Euclidian geometry instead of a historical approach. This difference in presentation of curved spacetime suggests that physics educators have not yet reached consensus on how to teach this feature of GR successfully on a qualitative level and an educational reconstruction of GR should take this aspect into account in particular.

To complement upper secondary interpretations of GR with the understanding of experts in the field, we studied an acclaimed university textbook in addition [48]. Here, the main emphasis is clearly on the geometry of spacetime as stated already in the introduction:

*General relativity is Einstein's theory of space, time, and gravitation. (...) The essential idea is perfectly straightforward: while most forces of nature are represented by fields defined on spacetime, gravity is inherent in spacetime itself. In particular, what we experience as "gravitation" is a manifestation of the curvature of spacetime [48].*

On the same line, the renowned physicist John Archibald Wheeler summarized GR with the famous phrase "Spacetime tells matter how to move; matter tells spacetime how to curve" [49]. Once more, we see that the challenge of teaching the physics of curved spacetime is crucial, since the explanation of gravity as a manifestation of curved spacetime is an important feature of GR according to leading experts.

Summarizing the findings from our analysis of textbooks, we divide the main features of GR into two categories: Key ideas that make up the conceptual foundation of GR and key phenomena that can be derived from this conceptual foundation. Our classification of the science content structure is summarized in Table I.

##### B. MER-component 2: Student perspectives on general relativity according to research literature

Aiming for a comprehensive understanding of students' perspectives and challenges when faced with GR, we analyzed the science education literature on GR. Surprisingly, there are hardly any systematic reviews on teaching GR and field-tested educational material is rare. Moreover, also studies on conceptual understanding of learners are scarce, leading Velentzas and Halkia in their study on the use of relativistic thought experiments to conclude that

TABLE I. Science content structure: Key features of general relativity.

Conceptual foundation	<p>The principle of equivalence:</p> <ul style="list-style-type: none"> <li>- In small enough regions of spacetime, the laws of physics reduce to those of special relativity. In particular, this implies that locally there is no way to distinguish between the effect of a gravitational field and of being in a uniformly accelerated reference frame.</li> </ul> <p>Reference frames and the principle of relativity:</p> <ul style="list-style-type: none"> <li>- One can only define inertial frames locally.</li> <li>- An inertial system is defined as a system in free fall.</li> <li>- All reference frames are equivalent to formulate the laws of physics.</li> </ul> <p>Spacetime and curvature:</p> <ul style="list-style-type: none"> <li>- Gravity is not a force, but a manifestation of the curvature of spacetime.</li> <li>- Energy and momentum influence spacetime to create curvature.</li> <li>- Free particles move in straight paths in curved spacetime.</li> </ul>
Relativistic phenomena	<p>Gravitational bending of light:</p> <ul style="list-style-type: none"> <li>- Massive objects deflect light.</li> </ul> <p>Gravitational red shift:</p> <ul style="list-style-type: none"> <li>- The frequency of light is influenced by gravity.</li> <li>- Light that moves away from a massive body is redshifted.</li> </ul> <p>Gravitational time dilation:</p> <ul style="list-style-type: none"> <li>- A gravitational field influences the rate at which time passes.</li> <li>- The farther a clock is away from a source of gravitation, the faster the time passes.</li> </ul>

(...) *the way physics theories of the twentieth century are taught in the classrooms has not yet been fully investigated by the educational community. Looking in the recent literature, the relative works are very few and the proposals are even fewer* [7].

Indeed, out of more than 2000 articles on conceptual understanding in physics that were published within a period of 30 years, only eight focus on relativity while zero of them address GR explicitly [50]. Moreover, recent works on teaching relativity to preuniversity students focus usually on special as opposed to general relativity [8–10] or contribute to an important broader discussion on interpretations of GR [51] without suggesting actual learning goals that could guide the development of learning resources. Most of the publications that investigate learners' perspectives on GR study undergraduate instead of upper secondary students [11–13], an exception being Pitts *et al.* [1] and Kaur *et al.* [52] from the Einstein-First project [53]. The Einstein-First project aims to change the paradigm of school science teaching through the introduction of modern Einsteinian concepts of space and time, gravity and quanta at an early age. Through several so-called enrichment programs with Australian secondary school students, Einstein-First researchers found that already children of age 10–16 can understand basic principles of GR and are motivated by concepts of Einsteinian physics [1,52].

Teaching GR is challenging both technically and conceptually, because of the advanced math and the large amount of previous knowledge, notably special relativity, required [2,54,55]. Learners lack experience with relativistic phenomena and the counterintuitive

nature of these phenomena poses yet another difficulty [15,56]. The disagreement between relativistic concepts and preexisting ideas of space and time that stem from everyday knowledge and classical physics teaching can lead to a situation where secondary school students suffer a “cognitive conflict” [7]. This conflict often manifests itself in upper secondary students not accepting consequences of the theory. For example, preexisting ideas on the absolute nature of time proved to be strong and students struggled to accept that time is different for moving observers, even though their own reasoning had led them to this conclusion [7].

Also, undergraduate students struggle with deeply rooted classical views on time and space. They have problems distinguishing between different reference frames and observers [11] which can hinder a proper understanding of the principle of equivalence. Indeed, in their study on alternative conceptions of the principle of equivalence, Bandyopadhyay and Kumar [12] identified an altered view of reference frames as “the most important cognitive transition that needs to be affected in the context of the principle of equivalence.” [12]. Dimitriadi and Halkia [9] and Levirini and diSessa [10] confirmed that also upper secondary students have misleading conceptions of frames of reference and particularly the role of observers.

The most comprehensive study on conceptual difficulties in GR was conducted by Bandyopadhyay and Kumar [13]. They followed senior undergraduate students throughout a course on GR and investigated alternative conceptions of eight key issues that Einstein brought up in his own exposure of the subject [57]. In probing several aspects of GR, Bandyopadhyay and Kumar affirmed an

TABLE II. Students' learning challenges in GR according to the literature.

General challenges	Specific challenges
GR builds on an advanced level of mathematics.	Students struggle with the definition of reference and inertial frames.
GR requires a lot of background knowledge, in particular special relativity.	Students struggle with the role of observers in different reference frames.
Students have no direct experience of relativistic phenomena.	Students struggle to apply the principle of equivalence.
The nature of relativistic phenomena seems to be counterintuitive to learners.	Students cannot connect the equality of inertial and gravitational mass to the principle of equivalence and generally fail to see the difference between inertial and gravitational mass.
Preexisting ideas stemming from classical physics hinder understanding of GR.	Students take the Euclidean nature of our universe for granted.
Students struggle to accept the implications of GR even when they have understood the basic principles of the theory.	

assertion made by Sexl [58] that students often understand single ideas of GR but are not able to put these pieces together to understand the bigger picture. Moreover, they confirmed the above-mentioned difficulties and observed a lack of understanding for Einstein's achievement to explain the equality of inertial and gravitational mass. Students tended to think of Euclidean geometry as an obvious feature of space and often preferred an outside view on the universe as if it was embedded in a higher dimensional space. It is interesting to note that Bandyopadhyay and Kumar are the only authors who have reported on students' understanding of spacetime. However, their study only touches upon the non-Euclidean nature of spacetime and does not investigate student understanding of curved spacetime in detail which

### Einstein's happiest thought

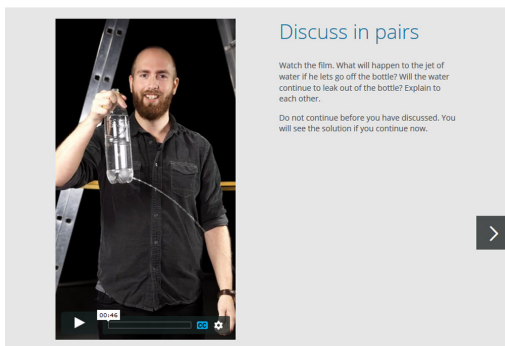


FIG. 2. A video exercise introduces students to the principle of equivalence and makes a connection to students' everyday life by asking them to discuss the physics of a falling water bottle. (DH2, DH5, DH7).

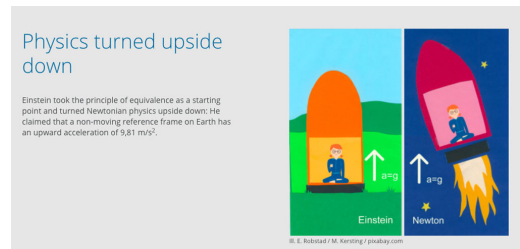


FIG. 3. A series of illustrations and thought experiments contrasts Newton's and Einstein's definition of an accelerated system to illustrate the concept of reference frames. (DH2, DH3).

### Video task: GPS

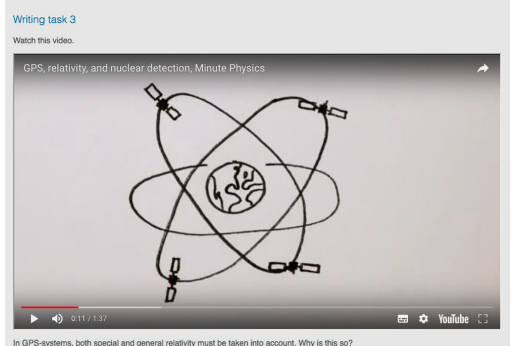


FIG. 4. A video exercise on GPS technology links the relativistic concept of time dilation to students' everyday life and allows them to make connections between special and general relativity. (DH4, DH5).

### Gravity as seen from two points of view

**Writing task 2**

Use your knowledge in physics to discuss the two statements and write down a short summary of what you have discussed. Which perspective do you prefer?

You need to be signed in as a student to write and save answers.

FIG. 5. We invite students to “talk physics” by discussing key concepts of GR. In this exercise, students are asked to discuss the two different viewpoints on gravity as held by Newton and Einstein. (DH7).

### The light from distant stars *does* get bent

Einstein calculated how much the light from distant stars gets deflected in the curved spacetime around the Sun. The British researcher Sir Arthur Eddington conducted an expedition to Africa to test Einstein's calculations by taking pictures of the night during the eclipse. The theory passed the test! This was the turning point in Einstein's career. From now on, he was not only well known within the scientific community, he had become an international celebrity.

FIG. 6. An animated thought experiment presents gravitational bending of light as a phenomenon that GR predicts but that classical physics cannot explain. (DH2, DH3, DH5, DH8).

### Why a new theory?

Both Newton's theory of gravity and Einstein's theory of special relativity describe the world quite well, but still, they cannot be combined. Why?

According to Newton, gravity acts over great distances with infinite speed. Special relativity is in conflict with this model – after all, it puts a speed limit on every possible physical effect – in particular, on gravity.

**KEY POINTS**

Special relativity describes the relation between time and space. It builds on two postulates:

- 1) The laws of physics take the same form in all inertial frames.
- 2) The speed of light in vacuum has the same value in all inertial frames.

FIG. 7. The learning environment builds on students' previous knowledge of classical physics and special relativity to show the need for a new theory of gravity. (DH3, DH4).

### Gravitational bending of light observed

1919: Gravitational bending of light observed

2016: Gravitational waves – “ripples in spacetime” – were detected.

#### 1919: Gravitational bending of light observed during total solar eclipse

Here is what the English newspaper The Times wrote about this observation.

Articles in The Times from Nov. 10, 1919, 5th, Nov. 16, 1919, center and Dec. 3, 1919.

<https://nyti.ms/29L7jps>

FIG. 8. To show that GR is a topic with historic and present-day relevance, we contrast the first experimental confirmation of GR to the recent breakthrough in gravitational wave astronomy. Students can move between newspaper headlines from 1919 and 2016. (DH5, DH6, DH8).

### Geometry in curved spaces

Draw and discuss in pairs:

1. Draw the shortest path that a plane can take from Oslo to New York.
2. Explain to each other the choice of path and compare to the correct answer. Explain the shape of the path.

reset show answer

FIG. 9. Several exercises ask students to discuss in pairs to foster understanding for relativistic concepts. In this task, a flight route on the world map illustrates the geometric nature of gravity. (DH1, DH7).

therefore still seems to be a mostly unexplored issue in science education research.

Also, other parameters can make learning GR difficult, such as a lack of time for teaching [55] or a lack of suitable learning activities [54]. Yet, despite various challenges, relativity seems to engage students to a high degree [59] and “most or all students show much interest, independent of age, of gender and of their general interest in physics” [56]. In Table II, we present the findings on students' challenges according to the literature more concisely in summary.

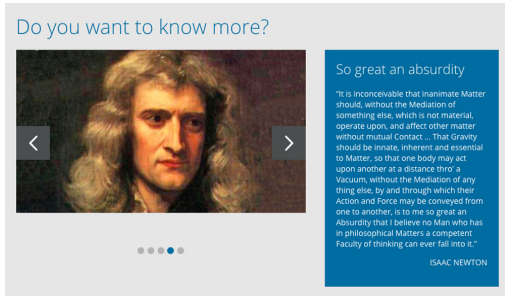


FIG. 10. By presenting direct quotes of Newton and Einstein, we introduce students to the historical development of GR and present physics as a human endeavor. (DH8).

**C. MER-component 3: Development of the online learning environment**

We have identified key features of GR as they appear in textbooks and established an understanding of students’ learning challenges through a literature search. We have argued for an instructional approach emphasizing students’ use of language in the learning process and for including historical and philosophical perspectives. In this

section, we present how these findings and their mutual relationship have guided our transformation of the science content structure of GR into a content structure for instruction in accordance with the MER framework. This content structure of instruction lies at the heart of the design of our online learning environment.

Since a successful educational reconstruction of general relativity must take both content and design features into account, we suggest a twofold content structure that includes specific learning goals for GR in secondary physics supplemented with design hypotheses. Focusing on a qualitative presentation, we propose an educational reconstruction of GR based on the learning goals presented in Table III. To arrive at those learning goals, we have taken the key features of GR (Table I) and reformulated and structured them in a way that is suitable for teaching at the upper secondary level. The learning goals serve as an interpretation of the vague Norwegian curriculum goal and as a possible approach for other contexts where GR is to be presented to learners with some physics background, but without advanced mathematics. Our design hypotheses (Table IV) complement the learning goals to facilitate teaching and learning of GR. Figures 2–10 exemplify the implementation of the design hypotheses through screenshots of the online

TABLE III. GR content structure for instruction specified by learning goals.

Content	Learning goals
Introduction	Describe general relativity as a new theory of gravity. Explain how the fact that Newton’s force of gravity acts instantaneously contradicts Einstein’s claim that nothing can move faster than the speed of light.
The principle of equivalence	Use the fact that locally it is impossible to distinguish between a gravitational field and a uniform acceleration and/or between free fall and the absence of a gravitational field to explain how acceleration and gravity are equivalent phenomena.
Reference frames and principle of relativity	Describe an inertial reference frame as a reference frame in free fall. Explain that the laws of physics take the same form in all reference frames.
Relativistic phenomena (bending of light, gravitational redshift, time dilation)	Give examples of phenomena that are predicted by GR but not by Newton’s theory of gravity. Describe how light travelling through the gravitational field of the sun is deflected and use the principle of equivalence to explain why this is predicted by GR. Describe how time goes slower close to massive objects and use the principle of equivalence to explain why this is predicted by GR. Explain how light that moves away from a gravitational source is redshifted and light that moves towards a gravitational source is blueshifted.
Spacetime and curvature	Explain how general relativity is a theory describing the relationship between space, time, and gravity. Describe our universe as having three spatial and one temporal dimension. Explain that gravity is not a force, but a geometric phenomenon. Describe how mass curves spacetime and how curvature influences the movement of mass.

TABLE IV. Design hypotheses for design of learning resources.

Design hypotheses	
DH1	Students can grasp central ideas in GR qualitatively without advanced mathematics by relying on geometric ideas.
DH2	Thought experiments, analogies and visualizations of relativistic phenomena foster understanding in GR.
DH3	Emphasizing the break of relativistic with classical physics helps students to overcome their classical preconceptions.
DH4	Recalling background knowledge in special relativity allows students to align relativistic ideas from special relativity with general relativity.
DH5	Linking abstract topics to students' everyday life motivates and fosters understanding in GR.
DH6	Students are generally motivated by topics in relativistic physics, such as black holes and spacetime.
DH7	Use of language and talking physics facilitates understanding of abstract concepts in GR.
DH8	Students are interested in the historical development of GR and its philosophical implications.

learning environment. In Sec. VI we will critically discuss the design hypotheses in light of student experiences to arrive at finalized design principles according to the DBR framework.

## V. CLASSROOM IMPLEMENTATION AND METHODS

### A. Implementation and data collection

The design hypotheses, together with the learning goals, led us to develop an online learning environment designed for 180 min of classroom time, containing text and images, videos, animations and simulations, writing and discussion tasks, thought experiments, and contextualization using historical as well as everyday examples. The learning environment was implemented in six physics classrooms at four generally high-performing schools in the Oslo area in spring 2016. The students were in their final year of upper secondary school and enrolled in the most advanced physics course available in Norwegian upper secondary school, namely, Level 2 physics. The students' teachers, who had participated in the DBR process of developing and testing previous versions of the resources, led the physics lessons. Teaching comprised two units of 90 min each. In total, 122 upper secondary students (age 18–19), 44 girls (36%) and 78 boys, and their six teachers participated. Informed consent to participate in the research was retrieved from all students and teachers.

To be able to follow students' learning paths, each lesson was observed by 1–2 field workers, one of whom was always the first author. In three classes, video recordings supplemented the observations. To investigate how students "talk physics," we asked students to make audio recordings of small-group discussions that are built into the learning environment. Students recorded the discussions with their smartphones and sent the files to the teacher, who forwarded them to researchers. Students' written responses were retrieved from the online learning platform. To gain insight into students' experiences with the learning environment and their own judgment of their learning in GR, we conducted four semistructured focus group interviews, one in each participating school with 5–8 participants per

group and 25 students in total. The students were chosen by the teachers to allow for a balance of gender and to include both stronger and weaker students. To ensure consistency, the first author conducted all interviews. The interviews were based on an interview guide focusing on the design of the learning activities, use of language, and students' challenges and motivation.

### B. Analysis of data

The first author and research assistants transcribed audio recordings from small-group discussions and focus group interviews, and these files, together with students' written responses, were imported into the Atlas.ti software for qualitative data analysis. Data sources for students' understanding of key concepts (RQ1) and for evaluation of the design hypotheses (RQ2) consist of focus groups and responses to written and audio exercises.

The analysis was based on thematic coding [60] and driven by our research questions. The first author performed the initial coding, and the coding process, formulation, and interpretation of findings, and the selection of representative quotes was discussed among ourselves over several rounds.

In the first stage of analysis, focus group interviews were coded for key concepts of GR that we chose as overarching themes (principle of equivalence, reference frames, spacetime, gravity, relativistic phenomena). In addition, focus group data were coded for features of our design hypotheses (language, history and philosophy, thought experiments, motivation, gap every-day, new way of thinking).

Written and oral responses were coded for correctness and for overarching themes (as above), which were again broken down into recurring motives that students drew on to support their arguments.

## VI. RESULTS AND DISCUSSION

To answer our research questions, we evaluated the participating students' understanding of key concepts in GR (RQ1) and investigated how students' experiences support or challenge the design hypotheses (RQ2). Concluding, we synthesized our empirical results to

formulate design principles for the development of learning environments in GR for upper secondary school (RQ3). Specifically, our interest lay in evaluating the design hypotheses and understanding the process of learning in the setting where the activity and collaboration with peers and the teacher took place [61]. This focus aligns both with the sociocultural approach of ReleQuant, with the MER model, and with recent calls that invite educational researchers to “examine more closely how the meaning and functions of CSCL [Computer Supported Collaborative Learning] applications are actually constituted in practice” [62]. The authors translated quotes of students and teachers from Norwegian to English. We denote quotes from focus group interviews by FG, written responses by WR, and audio records of the oral discussions by OD.

### A. Student understanding of key features of GR

Next, we present our findings on students’ understanding of key concepts in GR.

#### 1. The principle of equivalence

In the interviews, students could explain the principle of equivalence and were able to connect it to relativistic phenomena such as the deflection of light around the Sun. Nonetheless, some students found it difficult to believe that there is no experiment at all which could resolve whether one is accelerated or under the influence of gravity.

*FG—student: No, for me, so it is mostly that it is a bit difficult to, sort of, believe that you cannot do any experiment to test whether you’re not in an accelerated room instead of being in a pure gravitational field. This is something, which I think is a bit difficult to understand?*

We introduced the principle of equivalence through a discussion exercise and an experiment with a leaking bottle of water that can be explained by classical physics and that relates to Einstein’s own thought experiment (Fig. 2). This approach has been successful in connecting students’ everyday experience to the physics of relativity. Students remembered the activity well and found it easier to relate to it than to the example of a falling or floating elevator that is usually found in textbooks and that has been analyzed in the literature before [7].

*FG—student: I remember that there [in the online learning environment] were much easier examples to understand, because the book uses those examples that you yourself can’t relate to. But in the project we used examples which we could actually feel familiar with.*  
*FG—researcher: Do you have some examples for that?*  
*FG—student: For example, the one, the water bottle.*

However, many students would have liked a summarizing explanation at the end of the activity to connect the principle of equivalence explicitly to the physics of a falling water bottle. This criticism was partly confirmed by an analysis of the corresponding audio-recorded discussions. We found that the question “What will happen to the jet of water if the bottle falls?” was poorly formulated, because many students did not relate their discussion directly to the principle of equivalence but discussed related topics such as air resistance instead. Consequently, many groups did not come to the right conclusion, showing indeed the need for a clarifying summary as part of the activity. This finding suggests that learning activities in GR have to be very explicit when linking relativistic phenomena to students’ everyday experience. As a consequence, questions for pair and group discussions have become more specific in the final learning environment.

#### 2. Reference frames

In the interviews, none of the students mentioned the new definition of inertial frames explicitly, which could hint towards a lacking acknowledgement of the important reinterpretation of reference frames in GR [12]. The analysis of the audio discussions revealed that there seems to be a big gap between knowing the definition of an inertial system and being able to apply it to a given situation, suggesting that rote rather than deep learning had occurred. Students showed mixed understanding trying to find an answer to the question “According to GR, are you in an inertial reference frame while sitting in the classroom right now?” Even though many gave the correct answer, namely, that they are not in an inertial frame according to GR, the discussions revealed that students often did not feel confident in their reasoning. Students not trusting their own reasoning or sticking to classical views of acceleration—even though they could reproduce Einstein’s definition of being accelerated—supports findings from Veleztsas and Halkia [7].

*OD—student 1: I thought we agreed on being in an inertial system.*

*OD—student 2: We are not...*

*OD—student 1: Yes, we are surely not in free fall, so then we are in an inertial system.*

*OD—student 2: Yes, we are influenced by acceleration. Never mind.*

One group of students came to the conclusion that an inertial frame depends on the perspective of the observer and is therefore a relative concept; other students formulated similar thoughts but would not go so far as to say that one cannot give a definite answer to the question. Emphasizing that there exist several concepts in relativity that are not relative (the notion of inertial frames being one



of them) might help students to connect novel ideas in GR to their previous knowledge of physics.

### 3. Gravity and spacetime

In an introductory exercise, students were asked to answer the question “What do you know about gravity?” The written responses revealed that there was great variation in the understanding of gravity. Ideas ranged from phenomenological to mathematical descriptions as well as personal experiences of gravity. To contrast students’ initial knowledge with their ideas at the end of the unit, the nature of gravity was a topic in the focus group interviews. Most students answered in line with Einstein. However, they admitted that both Newton and Einstein seemed to have valid explanations, but Einstein’s explanation was perceived as far away from real life.

*FG—student 1: I think if you have to explain to an average person what gravity is, like just now, then it gets easier to say Newton’s, yes, everything that has mass attracts mass and so, yes, then there is opposing force and attracts each other equally and stuff. But it, yes.*

*FG—student 2: Yes, I could have never gone and said I talk to mum about, about what Einstein meant, because she wouldn’t have understood anything.*

Connected to the idea of gravity is the notion of curved spacetime. In an activity at the end of the chapter on curved spacetime, students were asked to summarize Newton’s and Einstein’s description of gravity. The interviews revealed that spacetime is an engaging, yet challenging concept that the students felt very uncertain about:

*FG—student: This is very exciting, because you can look at spacetime and then it curves if there is mass, around mass, but this is, you can see it in your mind, but then you think that you are in there, you are on the surface of the earth, you are sort of in a small sink. So then it gets sort of difficult to imagine it again, because if you take a look around you see the Universe, you don’t think of being down in a well somehow; this is very difficult.*

However, even though curved spacetime is perceived as a difficult concept, students generally found the geometric way of looking at space and time exciting. This interest supports a claim made by Levrini and Fantini [63], who argued that challenges in modern physics can be productive and engaging for upper secondary students.

This statement illustrates another finding that aligns with the perception of GR being far away from students’ life: Students found it difficult to relate the abstract description of curvature to their own experience of gravity. While they could reproduce the general description that mass curves spacetime and the curvature of

spacetime leads to gravitational effects, they struggled to explain why gravity keeps them grounded on the floor. This gap between abstract and experiential understanding of gravity is further elaborated on in a related publication from the ReleQuant project [64] and has also been identified among middle school students [1].

### 4. Relativistic phenomena: Redshift, time dilation, and gravitational bending of light

In the interviews, students could name gravitational bending of light and gravitational time dilation as relativistic phenomena that GR predicts. It seems that these phenomena have acted as a complement to the more abstract principles and have helped to bridge the gap between scientific concepts and students’ everyday understanding of time, space, and gravity. Students exemplified movement through curved spacetime with light rays that bend around the sun. Moreover, they appreciated the historic perspective of introducing deflection of light in the context of the solar eclipse from 1919.

While the bending of light was well received by students, gravitational time dilation and redshift seemed to have been more problematic. Even though well remembered in the interviews, students found these features complicated. This observation was confirmed by an analysis of a written task in which the gravitational redshift of a light signal detected in a spaceship was used to explain time dilation. In their explanations, students often mixed up classical and relativistic concepts, presented incoherent arguments, or tried to argue with the relativity of observation to explain time dilation:

*WR—student: The blueshift is relative to the rear, not to the space around the spaceship. This way, a detector in the spaceship will read different values for the number of wave crests than a detector in the surrounding space. This suggests the contraction of objects that accelerate.*

Thus, when presenting applications of GR it is important to take enough time to explain the relativistic phenomena in detail instead of just using them as an example of a more general principle of the theory.

## B. Evaluation of design hypotheses for the GR learning environment

The design of the learning environment was framed by eight design hypotheses. In this section, we will discuss these hypotheses in light of our empirical findings.

*DH1: Students can grasp central ideas in GR qualitatively without advanced mathematics by relying on geometric ideas.*

Using geometric ideas to foster understanding of the curvature of spacetime is a strategy that has been proposed

before by education researchers and physicists [2,53, 54,65]. Even though GR relies on an advanced mathematical foundation, the theory is geometric in its nature. We made use of this feature by developing learning activities that relied on geometry to explain curved spacetime. In the learning environment students encountered a simulation that presented Einstein's quest to find a geometric description of GR. Students were asked to discuss the form of a flight route on a world map to become familiar with the concept of shortest paths on curved surfaces (Fig. 9). The simulation continued to compare the geometry of the world map to the geometry of spacetime. We found that this geometric approach was very successful in engaging students and facilitated learning of the geometric features of curved spaces.

*OD—student 1: We believe that eh...you don't fly along a straight line because the Earth is round, and...*

*OD—student 2: And then the shortest path becomes such a curved line instead of a straight line because the Earth is not a plane, like on a sheet. This is a bit exciting because then you know that maybe not always a straight line is the shortest path between two points.*

However, it is not clear whether such a geometric approach actually did foster a deeper understanding for curvature in four-dimensional spacetime or whether it only helped students to grasp ideas of 2D and 3D spatial curvature. In another study, we looked closer at students' understanding of the space and time dimensions in GR exemplified by the rubber sheet analogy [66]. This analogy compares spacetime to a rubber sheet that gets distorted by massive objects. It is a popular, yet controversial way of illustrating GR [2,53,67]. We found that even though most students showed awareness of the limitations of an analogy that reduces the number of dimensions, many did not address the temporal dimension of spacetime explicitly [66].

*DH2: Thought experiments, analogies and visualizations of relativistic phenomena foster understanding in GR.*

Students lack direct experience of relativistic phenomena and one can use several strategies to deal with that problem. Generally, it seems that thought experiments have helped to deepen students' engagement with key concepts in GR. When asked about the role of thought experiments in the learning environment, students answered that those were engaging and challenged their understanding, thus making it easier to probe their own knowledge in GR.

*FG—student: I actually liked all the thought experiments, because you have to, sort of, think a bit, now we have learned something and then you have to think a bit*

*about why it is like this, how can you use this after all? Ehm, this was actually pretty nice, since then you are in a sort of different mode, you have to think and not only read something.*

These findings support Valentza's and Halkia's [7] conclusion that thought experiments are useful educational tools to help students deal with abstract concepts in relativity. Thought experiments were especially successful to explain the principle of equivalence. Many students confirmed that the thought experiments of Einstein stepping off a tall building and of a laboratory in an accelerated spaceship helped them in their understanding.

The use of analogies is a more ambiguous issue. We observed a heavy dependence on the rubber sheet analogy when students talked about curved spacetime. The fact that this analogy was ubiquitous throughout the data set suggests that learners have a great need for visualizations. In particular, the analogy of the rubber sheet seems to inform the way students think about spacetime:

*FG—researcher: What is gravity?*

*FG—student 1: Yes, I have this (laughs) warps in, eh, or according to Einstein warps, in, eh, curvature of space. Then, so like, this is sort of pretty difficult to explain that you need these visual, yes, you need it visualized to just understand it better.*

*FG—student 2: Like he said with this trampoline that, eh ...*

*FG—student 1: Yes, if you think of spacetime then it gets a lot easier to understand.*

Students' reliance on the rubber sheet analogy could be partly explained by a lack of other convenient and visual ways to grasp the abstract idea, especially because a mathematical description is not accessible. These findings confirm the need for suitable visualizations as expressed by Kraus [56] and Weiskopf *et al.* [68].

However, the analogy troubled some students and they criticized the comparison, because it relied on gravity to explain gravity, which is a common criticism among physicists as well [67].

*FG—student: Hm, the only problem I have with the rubber sheet analogy is that it requires a gravity (laughs) to make the model work as well. This is what I struggle with, because there won't be any curvature in space without a gravity that makes a well in it.*

The use of this particular analogy should therefore be reconsidered in light of analogies being "two-edged swords" when viewed as educational tools [69]. Insufficient discussion of the flaws of the rubber sheet analogy in the learning environment might have led to unnecessary misconceptions. Teachers should therefore point out explicitly in which ways the rubber sheet analogy

oversimplifies the geometric description of GR. This observation guided a revision of the learning environment that includes a new discussion task on the shortcomings of the rubber sheet analogy in its final version.

*DH3: Emphasizing the break of relativistic with classical physics helps students to overcome their classical preconceptions.*

Since preexisting ideas that stem from classical physics often hinder students' understanding, one can present the break between classical physics and GR explicitly to make students become aware of their own preconceptions. In the interviews, students repeatedly expressed the insight that GR greatly differs from classical physics. Often, they articulated the need for a new way of thinking to understand GR. Students found this shift of perspective challenging, but also exciting and fun:

*FG—student: Otherwise, it was fun to just see everything from a completely different angle all the time, a different perspective, like ohh, you can look at this also in this way.*

This finding supports our hypotheses that emphasizing the break between GR and classical physics can engage students and help them become aware of their classical understanding of physics. Moreover, such a strategy has already been successfully used in special relativity [7] and quantum physics [17].

*DH4: Recalling background knowledge in special relativity allows students to align relativistic ideas from special with general relativity.*

The interviews confirmed previous findings that students find special relativity with its new way of describing time and space difficult [9]. Moreover, we found that students tended to confuse principles of special and general relativity:

*FG—student 1: Mass curves time and space, this was a bit difficult; I didn't manage to (mumble). And it is very weird to think that centimeter and meter are kind of different in movement than if they stand still. This I think is difficult to think, difficult to accept, or how, kind of, that if one centimeter and one meter are like. Yes, I think this was difficult.*

*FG—student 2: This was special relativity.*

*FG—student 1: I have little overview on that. But I think this was difficult to understand.*

These findings suggest that one way of helping students overcome their confusion might be to present more clearly the distinction between key concepts of special and general relativity and how GR aligns with special relativity and classical physics. Because of time and curriculum constraints

in schools, drawing on special relativistic concepts could make it even more difficult for students to understand abstract concepts in GR. In particular, the concept of reference frames requires students to handle conflicting definitions in classical physics, special relativity, and general relativity.

*DH5: Linking abstract topics to students' everyday life motivates and fosters understanding in GR.*

Learners experience the nature of relativistic phenomena as counterintuitive. Overall, using everyday examples to illustrate relativistic ideas worked well to explain the principle of equivalence, the geometry of curved surfaces and relativistic phenomena. As explained in Sec. VI.A and in Sec. VI.B in DH1, both the discussion exercise about curved flight routes on flat maps (Fig. 9) and the physics of a falling bottle of water (Fig. 2) stimulated discussions among students and were mentioned in the interviews. Students understood the application of GR in GPS technology well. In an exercise in which students were asked to explain how special and general relativity affect GPS systems (Fig. 4), almost all students gave a correct or partly correct explanation and many supplemented their responses with detailed explanations showing that students were able to connect newly learned content in GR to their previous knowledge:

*WR—student: If you look at special relativity, then moving clocks will go a bit slower than clocks at rest. You have to take this into account considering that the satellites, and the Earth, follow their orbit with a certain velocity. But, if you look at general relativity, clocks that are in a gravitational field go slower than clocks that are outside of a gravitational field. This means that the clocks on Earth go slower than the clocks in the GPS satellites, so you have to take this into account as well.*

Thus, emphasizing links between everyday life and abstract phenomena seems to be fruitful when introducing students to GR. This observation aligns with a similar one in quantum physics [17].

*DH6: Students are generally motivated by topics in relativistic physics, such as black holes and curved spacetime.*

Concerning their motivation and attitude towards learning GR, students confirmed our hypothesis, in line with observations of Zahn and Kraus [2]. They wanted to learn more about the nature of space and time because they perceived the topic to be modern and relevant:

*FG—student 1: ...the more I think about it, the more fascinated do I get...*

*FG—student 2: ... what we have learned has been very old—old knowledge—so (...)*

TABLE V. Empirically based design principles for learning resources in GR.

Basic principles and specific features
<p>1. Emphasize how GR relates to and sometimes breaks with classical physics.</p> <p>(a) Present the need for a new theory of gravity by showing that special relativity and classical mechanics are irreconcilable.</p> <p>(b) Present relativistic phenomena such as gravitational bending of light and time dilation to show that GR extends the scope of classical physics.</p> <p>(c) Point out how the definition of inertial frames in GR differs from similar notions in special relativity and classical mechanics. Ask students to apply the abstract definition of an inertial frame to specific problems.</p> <p>(d) Show that there exist concepts in GR that are NOT relative, such as the notion of inertial reference frames, to help students connect relativistic ideas to their classical understanding of physics.</p> <p>2. Link key concepts of GR to students' life worlds to counteract the lack of experience with relativistic phenomena.</p> <p>(a) Use everyday examples to illustrate relativistic ideas and to enable students to connect GR to their everyday life. GPS technology can exemplify gravitational time dilation and the geometry of world maps can illustrate motion in curved spaces.</p> <p>(b) Use thought experiments as educational tools to help students understand abstract concepts in GR. Thought experiments that illustrate free fall and weightlessness are particularly successful when explaining the principle of equivalence.</p> <p>(c) Use analogies with caution. State shortcomings of analogies explicitly to prevent the formation of misconceptions. In particular, explain how the rubber sheet analogy oversimplifies the notion of curved spacetime.</p> <p>(d) Use visualizations in the form of digital simulations and animations to introduce students to relativistic concepts and to prevent the formation of misconceptions.</p> <p>3. Draw on students' prevailing motivation and interest to introduce key concepts in GR.</p> <p>(a) Use astronomical phenomena to engage students. Gravitational lensing around black holes can illustrate gravitational bending of light and curvature of spacetime. Thought experiments involving spaceships can illustrate the principle of equivalence.</p> <p>(b) Present GR in light of its historical development. The solar eclipse in 1919 can serve as historical example for an experimental verification of GR. Relate Einstein's quest to find a new theory of gravity to abstract descriptions of GR.</p> <p>(c) Emphasize epistemological aspects of GR and explain how Einstein's new interpretation of space, time, and gravity has shaped our worldview.</p> <p>(d) Present GR as an active field of research by referring to the recent observation of gravitational waves.</p> <p>4. Invite students to use written and oral language to facilitate understanding of abstract concepts in GR.</p> <p>(a) Give students the opportunity to "talk physic" with their peers by using discussion tasks that probe conceptual understanding of key concepts in GR.</p> <p>(b) Ask students to summarize their understanding of key concepts in written exercises to let them practice the use of new physics vocabulary.</p> <p>(c) Use plenary discussions guided by the teacher to consolidate understanding of GR and resolve misconceptions.</p> <p>(d) Explain that our qualitative understanding of GR can be made rigorous by employing advanced mathematics.</p>

*when we learned about those gravity waves (...) I found it very interesting.*

Students' fascination might be due to popular culture, because GR has become part of the 20th century cultural heritage [1,7] and Einstein has become a scientific icon. The recent detection of gravitational waves [5] can have contributed to this perspective, since many students mentioned the discovery.

*DH7: Use of language and talking physics facilitates understanding of abstract concepts in GR.*

Generally, the focus on using language to build and articulate physics understanding and insight was well received by students. The focus groups interviews supported previous findings on using language in science classrooms [17,43,70]: Talking physics fostered students' overall understanding in GR and discussions with peers and with the teacher were experienced as an engaging variation from regular teaching. In particular, students appreciated to think aloud, and they liked that their understanding of GR

was challenged by discussions, which forced them to reason and to find arguments:

*FG—student: I understand it a lot better if I have to explain it, so even though I haven't quite understood it myself, I start to try to explain it. So this, sort of, the pieces fit together while I work on that, so this helps. This way I get a much clearer picture of how this is.*

Nonetheless, students asked for more mathematical approaches and easy calculations to probe their understanding as well. This was a recurrent critique in the interviews that relates to the frustration that students experience when approaching GR only qualitatively. This finding shows that students are aware of the close relationship between mathematics and physics [71]. However, the challenge in GR is that we do not have easy calculations at hand and that students have to rely almost exclusively on nonmathematical explanations:

*FG—student: (...) it is difficult to understand like.*  
*FG—researcher: Yes, but could we have made it easier to understand?*

*FG—student: This is a bit difficult to say, but maybe like try to explain by doing a, a short math calculation (...) ehm, a bit more proof of it, it doesn't need to be a very complicated proof.*

To counteract the formation of frustration among students, teachers have to raise awareness for different ways of learning physics: qualitative understanding and historical and philosophical perspectives should be seen as valuable learning goals in their own right, enabling students on preuniversity level to grasp important areas of modern physics without the full mathematical-formalistic “toolbox” [72].

*DH8: Students are interested in the historical development of GR and its philosophical consequences.*

Aside from aspects regarding the specific content of GR, students were particularly motivated by approaches relying on the history and philosophy of science. They approved of a historic perspective that gave them insight into the development of GR. Being able to follow Einstein's reasoning and his struggle to find a new theory of gravity seemed to enable them to draw connections to previous knowledge and to the theory as such. In particular, students appreciated the famous experimental verification of GR during the solar eclipse in 1919 (Fig. 8).

*FG—student: I think it was very interesting that we kind of learned about, like, the history as well kind of. How this developed and this, (...) experiment with when they test bending of light with him, I don't remember his name, Sir...*

*FG—researcher: Sir Eddington.*

*FG—student: Yes, Eddington, that someone tested this kind of and verified it.*

This supports previous research [73] highlighting how historical examples may support motivation and learning in physics.

### C. Empirically based design principles for developing GR learning resources

After synthesizing our findings from the classroom trials with our design hypotheses and with previously published research, we now formulate design principles (RQ3) that we believe can guide the design of learning resources in GR and that foster qualitative understanding and encourage collaborative learning. The empirically based design principles follow a two-level structure with four basic principles that are specified by more specific features for each of them, as shown in Table V. These principles together with the insight on students' challenges identified in this study informed revisions of our learning environment. The revisions led to the final version that is now freely accessible on the Norwegian learning platform Viten [74].

## VII. SUMMARY AND CONCLUSION

The aim of this study was to describe and propose an educational reconstruction of GR for upper secondary physics students. Following the framework of MER, we identified key features of GR and studied learners' difficulties based on an analysis of physics textbooks and relevant literature. These results enabled us to turn the science content structure of GR into a content structure for instruction. DBR methods guided the development and evaluation of a collaborative online learning environment which were based on a sociocultural view of learning and a historical-philosophical approach to teaching GR. By extracting central principles for the design of an online learning environment in GR and by characterizing students' understanding and learning challenges, we added new empirical results to two of the three components in the MER framework. Seeing that field-tested educational material in GR is still rare, this study thus contributes to a growing body of research into teaching and learning GR at secondary school level.

Our findings corroborate earlier results reported in the literature [1,7,13,52] and have added to a deeper and more comprehensive understanding of students' challenges in learning GR. Specifically, we have presented first empirical results on students' understanding of curved spacetime, which is scarce in the literature. In summary, our findings indicate that upper secondary students can obtain a qualitative understanding of GR when provided with appropriately designed learning resources and sufficient scaffolding of learning through interaction with teachers and peers.

By synthesizing our empirical findings with the design hypotheses that we based on a sociocultural perspective and historical and philosophical approaches to learning physics, our final design principles are equally grounded in theory and practice. The principles arose from an iterative process of development and their formulation completes our proposed educational reconstruction of GR targeted at upper secondary students.

The online learning environment presented in this paper was developed to help Norwegian upper secondary students achieve the specific competence aims in the national curriculum for physics. Thus, design principles may need to be adapted to other groups of learners and different curricular aims. While our results are not generalizable *per se*, we claim, however, that our results demonstrate the feasibility of communicating central aspects of GR qualitatively, without mathematics, and that our design principles represent insights that may be quite broadly applicable.

We anticipate future research that will throw further light on how GR can be communicated to and understood by different groups of learners as well as research investigating students' learning processes in more detail. GR is an important pillar of modern physics. We believe that our take on an educational reconstruction of GR brings us one step closer to teaching

students our most contemporary scientific understanding of the Universe, thus turning GR into an important pillar of modern physics education as well.

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# Understanding Curved Spacetime

## The Role of the Rubber Sheet Analogy in Learning General Relativity

Magdalena Kersting<sup>1</sup>  · Rolf Steier<sup>2</sup>

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### Abstract

According to general relativity (GR), we live in a four-dimensional curved universe. Since the human mind cannot visualize those four dimensions, a popular analogy compares the universe to a two-dimensional rubber sheet distorted by massive objects. This analogy is often used when teaching GR to upper secondary and undergraduate physics students. However, physicists and physics educators criticize the analogy for being inaccurate and for introducing conceptual conflicts. Addressing these criticisms, we analyze the rubber sheet analogy through systematic metaphor analysis of textbooks and research literature, and present an empirical analysis of upper secondary school students' use and understanding of the analogy. Taking a theoretical perspective of embodied cognition allows us to account for the relationship between the experiential and sensory aspects of the metaphor in relation to the abstract nature of spacetime. We employ methods of metaphor and thematic analysis to study written accounts of small groups of 97 students (18–19 years old) who worked with a collaborative online learning environment as part of their regular physics lessons in five classes in Norway. Students generated conceptual metaphors found in the literature as well as novel ones that led to different conceptions of gravity than those held by experts in the field. Even though most students showed awareness of some limitations of the analogy, we observed a conflict between students' embodied understanding of gravity and the abstract description of GR. This conflict might add to the common perception of GR being counterintuitive. In making explicit strengths and weaknesses of the rubber sheet analogy and learners' conceptual difficulties, our results offer guidance for teaching GR. More generally, these findings contribute to the epistemological implications of employing specific scientific metaphors in classrooms.

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✉ Magdalena Kersting  
magdalena.kersting@fys.uio.no

<sup>1</sup> Department of Physics, University of Oslo, Postboks 1048 Blindern, 0316 Oslo, Norway

<sup>2</sup> Department of Education, University of Oslo, Postboks 1092 Blindern, 0317 Oslo, Norway

## 1 Introduction

The Earth circles around the Sun and we stay grounded on Earth because of gravity. Yet, the nature of gravity eluded human understanding for centuries. It was only with Albert Einstein's theory of general relativity (1915) that physicists found a fundamental description of gravity which set the stage for the development of modern physics in the twentieth century. General relativity (hereafter GR) is a modern theory of gravitation that extends classical mechanics to cosmic scales. Motion at the speed of light and physics close to extremely massive objects such as black holes require a more powerful framework than Newton's classical mechanics can offer. By describing gravity as geometry, GR offers such a framework with greater explanatory power than classical mechanics: the fabric of our universe can be modeled by four-dimensional spacetime and it is the curvature of spacetime that manifests itself in form of gravity.

Gravity as a manifestation of curved spacetime is an abstract concept that students, not being able to rely on the advanced mathematical formalism, must grasp in terms of other areas of experience. Thus, the description of this concept requires metaphoric language. In this study, conducted within the Norwegian design-based research project ReleQuant, we aimed to understand how upper secondary students reason with analogies and metaphors to conceptualize gravity and curved spacetime.

The formulation of GR not only provided impetus to the further development of physics, but it also inspired the emergence of new fields such as cosmology. In fact, the significance of the theory extended beyond the mere contents of scientific laws and theories. Indeed, adopting a relativistic perspective entailed a change in the worldview of many scientists at the time of Einstein (Chandler 1994). With its apparent metaphysical implications, GR brought about a new heyday of the philosophy of space and time as well (Reichenbach 1928). With Einstein's insight into the deep connection between gravity, space, and time, a century-long dispute on the nature of space and time found its culmination. The dispute reaches back to the beginning of the eighteenth century to Newton and Leibniz, who held opposing views on this topic (Vailati 1997). Whereas Leibniz argued that space and time are relational and can only be defined through orderings between objects, Newton described space and time as absolute entities that are as real as any object in the world. The Newtonian view of absolute space and time dominated academic discourse for almost 200 years. It was only at the end of the nineteenth century that philosophers and scientists started to question notions of absolute space and time (Mach 1893; Poincaré 1898). These considerations were predecessors to the revolutionary ideas of Einstein who eventually replaced absolute space and time with the notion of dynamical spacetime—a replacement whose philosophical impact still can be felt today (Chandler 1994).

Surprisingly, the great importance of GR in physics and philosophy has not corresponded to equivalent attention in education on how students understand such concepts. Even though current fields of physics research such as gravitational wave astronomy (Abbott et al. 2016) as well as the working of modern communication technologies rest greatly on our relativistic understanding of gravity, physics in high schools remains mostly dominated by classical theories of gravity (Henriksen et al. 2014; Velentzas and Halkia 2013). However, students are confronted with a growing number of representations in the media and popular culture, such as in recent discoveries about gravitational waves, which present gravity as a relativistic phenomenon. While other domains of modern physics such as quantum physics and special relativity have already entered high school and undergraduate education in many countries (Henriksen et al. 2014; Krijtenburg-Lewerissa et al. 2017; Levrini 2014; Stadermann and

Goedhart 2017), it was only very recently that physics educators made first attempts to introduce GR to school curricula and to investigate students' understandings of it (Kaur et al. 2017a; Kersting et al. 2018). In a society that is pushing knowledge and technological advancement ever further, it is important to teach students our best understanding of the universe, and this can only be done if we know how to communicate relativistic concepts effectively.

Studies on secondary school students' conceptual development of key concepts in GR are scarce (Kersting et al. 2018). Existing research either looks at special relativity instead of general relativity (Dimitriadi and Halkia 2012; Levrini 2014; Levrini and DiSessa 2008) or studies undergraduate physics learning (Bandyopadhyay and Kumar 2010a, b; Hartle 2005). Based mostly on case studies and interviews, the findings in these studies suggest that students often struggle with the interpretation of relativistic concepts and phenomena.

Recently, educational projects in Australia and Norway (Kaur et al. 2017a; Kaur et al. 2017b; Kersting et al. 2018) have started to investigate the learning of GR at the high school level in response to increased emphasis in national curricula. Efforts in Australia rely on so-called enrichment programs that introduce modern concepts of space and time to 10–16-year-old students. Work in Norway relies on digital learning resources that were trialled with 18–19-year-old students. In an attempt to achieve an educational reconstruction of GR, we reviewed the literature to identify the main challenges of teaching and learning relativity (Kersting et al. 2018). General challenges include the advanced level of mathematics, the lacking experience with relativistic phenomena, and the counterintuitive nature of these phenomena in light of classical physics. More specific challenges concern the role of observers in different reference frames and the Euclidean nature of our universe that students take for granted. Despite those challenges, the results from Australia and Norway are encouraging. Findings suggest that younger students are motivated by topics of Einsteinian Physics (Kaur et al. 2017b) and that students can gain a qualitative understanding of GR when provided with appropriately designed learning resources and support from peers (Kersting et al. 2018).

Moreover, the latter study is among the first to present empirical results on upper secondary students' understanding of curved spacetime. Focus group interviews revealed that spacetime is an engaging, yet challenging concept that students felt very uncertain about. The only other study that we are aware of to report on students' conceptual understanding of spacetime looked at senior undergraduate students taking a course on GR (Bandyopadhyay and Kumar 2010b). However, these researchers only touched upon non-Euclidean geometry and did not investigate students' understanding of the geometry of spacetime in detail. Therefore, the conceptual understanding of curved spacetime still seems to be a mostly unexplored topic in science education research.

Teaching GR on undergraduate and upper secondary school level requires teaching approaches that rely on qualitative explanations and elementary mathematics (Kersting et al. 2018). Such approaches entail the use of thought experiments (Velentzas and Halkia 2013), geometric models (diSessa 1981; Zahn and Kraus 2014), hands-on experiments (Pitts et al. 2014), and simple mathematical approximations (Stannard et al. 2017). Common to these teaching strategies is the shared understanding that the mathematical foundation of GR is very abstract and that many of its consequences are counterintuitive (Bandyopadhyay and Kumar 2010b; Kersting et al. 2018). These challenges affect high school and undergraduate students alike, because GR contradicts what most students have learned in previous physics classes, namely that gravity is a force.

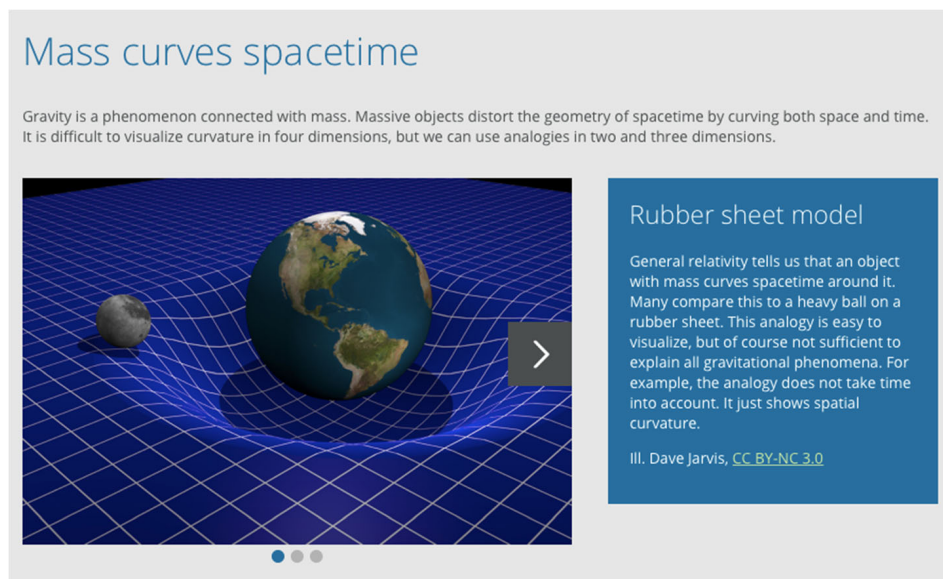
While there seems to be consensus about the educational challenges of GR, the most prevailing popular representation of the theory gives rise to a debate among physicists and physics educators. Both in teaching resources and in popular science culture, the so-called rubber sheet analogy (hereafter RSA) is a widely used tool to make sense of four-dimensional curved spacetime (Greene 2010). The analogy compares the fabric of the universe to a stretched rubber sheet (Fig. 1). Gravitation and the dynamic interplay between the movement of massive objects and the curvature of spacetime are illustrated by placing a bowling ball and marbles on the rubber sheet. The bowling ball produces a warp of the rubber, which results in an inward tug that will influence the movement of the marbles. The bowling ball represents for example the Earth and the marble is like the Moon circling around the massive ball. It is the warp of the rubber sheet that creates the gravitational tug. There is no need to introduce a force that, mysteriously, acts at a distance.

The ubiquity of the RSA in teaching resources and popular science literature nowadays stems from the challenge to visualize a theory whose geometry continues to confound. Einstein had admitted that our imaginative faculty cannot conceive of four dimensions:

No man can visualize four dimensions, except mathematically. We cannot even visualize three dimensions. I think in four dimensions, but only abstractly. The human mind can picture these dimensions no more than it can envisage electricity. Nevertheless, they are no less real than electro-magnetism, the force which controls our universe, within, and by which we have our being. (Einstein in Viereck 1929)

In response to this challenge, Einstein was presumably the first to employ the analogy that compares spacetime to a cloth. In a correspondence with his colleague Willem de Sitter, who would later publish joint work with Einstein on the curvature of the universe, Einstein explained: “Our problem can be illustrated with a nice analogy. I compare the space to a cloth floating (at rest) in the air, a certain part of which we can observe. This part is slightly curved similarly to a small section of a sphere’s surface.” (Hentschel 1998, p. 301).

Only shortly after the publication of GR, Einstein attempted to present the theory of relativity to a more general audience (Einstein 1917). Similar expositions by others followed



**Fig. 1** A screenshot of the Norwegian learning environment that introduces the rubber sheet analogy

shortly after that, and already in 1925 the eminent mathematician and philosopher Bertrand Russell used a “soft india-rubber” to illustrate the idea of curved spacetime (Russell 1925). Interestingly, at the same time Russell cautioned of the risks of simplifying scientific ideas too much: “Einstein revolutionized our conception of the physical world, but the innumerable popular accounts of his theory generally cease to be intelligible at the point where they begin to say something important”—a wise remark that foreshadowed the debate around the RSA that scientists and educators still lead today.

On the one hand, advocates of the RSA have praised it as an “excellent analogy” (Thorne 2009, p. 77) because of its visual power and its intuitive appeal both to students (Farr et al. 2012) and physicists in the field:

*The rubber membrane-bowling ball analogy is valuable because it gives us a visual image with which we can grasp tangibly what we mean by a warp in the spatial fabric of the universe. Physicists often use this and similar analogies to guide their own intuition regarding gravitation and curvature. (Greene 2010, p. 71)*

On the other hand, critics consider the RSA to be “misleading” (Price 2016, p. 588) and to pose a “considerable risk to the formation of misconceptions” among students (Zahn and Kraus 2014), because of oversimplification and incorrect presentation of the physics:

*Unfortunately, the illustration makes no sense. Students observe that space is not a rubber sheet, does not curve into an unseen dimension, and does not push objects into circular orbits. The rubber sheet does not even reflect the symmetry of the central mass—if you turn the illustration upside down the explanation fails. (Gould 2016, p. 396)*

Seeing that experts hold divided opinions on the educational value of the RSA when teaching GR, it is surprising that the ongoing debate is mostly based on opinions and claims without a proper evidential base. Gould, for example, claimed that “(...) students are often confused by literal illustrations of the concept [of curved spacetime]” (2016, p. 396), but he presented no empirical evidence to support this claim. Looking into the literature, the works that address the RSA explicitly can be grouped into two camps. On one side, physicists focus on the mathematics of the RSA to show why the analogy can be an instructive teaching tool (Middleton and Weller 2016), or to replace it with more appropriate mathematical models (Gould 2016; Price 2016). On the other side, science educators investigate how students understand the RSA (Baldy 2007; Steier and Kersting n.d.; Watkins 2014). However, these very few investigations have, so far, addressed the RSA rather as a way to explain gravitational phenomena in the framework of Newtonian physics rather than to shed light onto how the RSA might facilitate students’ understanding of curved spacetime in the context of GR.

Addressing the problem that secondary students display with a force of gravity that acts magically at distance, Baldy (2007) introduced the “pillow-model” to study French ninth-graders’ (15 years old) ideas of attraction between objects. The pillow-model replaces the rubber sheet by a soft pillow, but serves conceptually the same purpose as the RSA. Baldy compared two teaching methods, one based on Newtonian physics and one based on the pillow-model, and studied student’s conceptions of falling bodies. She found that the Newtonian approach is less effective, even though she admitted that her results “(...) are not intended to mean that the students built a representation of the universe that conformed to Einstein’s theory on all points, nor that they understood the theory.” (Baldy 2007, p. 1784).

In an exploratory study on the conceptual understanding of curved spacetime that was conducted within the same project as the present work, we analyzed a discussion between two Norwegian upper secondary physics students who showed deep engagement with gravity and

spacetime, but struggled to accept certain aspects of the new concepts. The results suggested that the RSA might be problematic for learners, because it makes use of two different concepts of gravity and relies on classical gravity to make the analogy of “Einsteinian” gravity work. (The fact that the analogy draws on classical gravity, like the force that creates a well in the rubber sheet, to explain a new interpretation of gravity lets the pair of students struggle conceptually.) However, it is not clear whether these results can be readily generalized to a broader sample of students (Steier and Kersting *n.d.*).

Addressing the controversy around the use of the RSA in the domain of GR, we want to bring the debate forward by offering actual empirical results on upper secondary school students’ ideas about curved spacetime in relation to the RSA. Insights into students’ understanding and their use of the most common representation of curved spacetime are critical in order to investigate learning processes and conceptualization of spacetime in GR and to develop efficient teaching approaches.

We aim to understand how upper secondary students reason with the RSA to conceptualize gravity and curved spacetime. To guide our examination, we ask the following research questions:

1. What features of gravity as they were explained by Einstein does the rubber sheet analogy hide and highlight?
2. What characterizes students’ understanding of the rubber sheet analogy?
3. In what ways do students show awareness of the analogical nature of the rubber sheet analogy when conceptualizing gravity and curved spacetime?

We hope that addressing these questions will serve as an impetus for the ongoing educational debate around the RSA and that it will add to the emerging body of knowledge concerning the teaching of GR. More generally, we hope that our findings will contribute to the epistemological implications of employing specific scientific metaphors and analogies in science classrooms.

## 2 Theoretical Background

In the following sections, we frame the challenge of analyzing the RSA and students’ ideas of curved spacetime in relation to research about the use of analogies and metaphors in science education.

### 2.1 Analogies, Metaphors, and Embodied Cognition in Science Education

Both the wish to approach GR from a qualitative perspective and our inability to visualize four dimensions make the RSA an appealing tool to communicate aspects of curved spacetime. Indeed, instructional analogies and metaphors have become a popular tool in science education, because they can help to communicate abstract scientific concepts (Aubusson et al. 2006). However, science educators have also recognized limitations to this approach due to the often-unpredictable ways that students interpret analogies and metaphors (Harrison and Treagust 2006).

Before we unpack further aspects of this criticism in relation to the RSA, let us define what we mean by an analogy or a metaphor in our context. Niebert et al. (2012) reviewed the use of both terms in the science education literature and came to the conclusion that most science educators treat analogies and metaphors synonymously as statements that characterize one

thing in terms of another. This characterization goes back to Lakoff and Johnson whose broad definition of metaphors encompasses analogies as well (Lakoff and Johnson 2003). Gentner et al. observed that the processes of understanding metaphors and analogies are the same (Gentner et al. 2001). On the basis of this observation, Niebert et al. concluded that the difference between analogies and metaphors is not theoretical but rather technical and basically depending on the number and quality of mappings between the target and source domain. Adopting this perspective, we understand analogies and metaphors as comparisons that construct a similarity between two objects and we do not distinguish between those two notions. This definition will allow us to treat the RSA in the broader framework of metaphor analysis. More generally, understanding the nature of analogy and metaphor is a process central to scientific models and modeling (Gilbert 2004). For the purpose of this study, we refer to models as artifacts which may be interacted with or visualized and we treat analogies and metaphors as one particular form of model in science education.

The increased interest in metaphors and analogies in science education stems partly from the fact that these models play an important role in scientific knowledge construction. There is a long tradition in the philosophy of science to argue for the epistemological importance of analogies (Hesse 1953). Kapon and diSessa noted that “the generation of analogies and the reasoning stemming from these analogies play a central role in scientific practice, thought, and creativity” (2012, p. 262). Stinner (2003, p. 340) observed that the big theories in science including Einstein’s theory of relativity or Maxwell’s theory of electromagnetism are often the product of imaginative thinking which, according to Stinner, includes “to see analogies between disparate events.” Thus, historical accounts of scientific discoveries abound with examples of how scientists used metaphors and analogies to build their theories (Chandler 1994; Hesse 1952; Kind and Kind 2007; Silva 2007). It seems that Einstein was particularly apt at finding fruitful analogies. He was presumably the one to introduce the RSA to reason about curved spacetime (Hentschel 1998), and he used the analogy of riding on a ray of light to work out his theory of relativity in the first place (Kind and Kind 2007).

Systematic metaphor analysis (Schmitt 2005) is a recent fruitful approach that draws on findings from cognitive science and linguistics to understand the use of analogies and metaphors in science education (Amin et al. 2015; Lancor 2014a; Niebert and Gropengießer 2014; Niebert et al. 2012). This approach goes back to Lakoff and Johnson who, in their seminal work (2003), argued that metaphors are not only a linguistic phenomenon, but a fundamental feature of thought and mind. Forming the basis of our conceptual systems, metaphors serve as a principal vehicle for understanding, because we systematically use inference patterns from one conceptual domain to reason about another conceptual domain. Since such metaphors are grounded in the everyday human experiences of “having a physical body in a physical world” (Roth and Lawless 2002, p. 336) Lakoff and Johnson suggested that cognition is ultimately embodied. Embodied cognition extends the boundaries of the mind from merely being inside the brain to including the body’s physical interactions with the world. Metaphors are thus the mediators that extend one physical experience to other conceptual domains. For example, the “leg” of a table is an extension of the leg of a body, and allows us to make sense of its function as a structure for support (Lakoff and Johnson 2003, p. 54). We think about table legs in terms of our bodily experiences of being supported by our own legs and feet. Metaphors are thus not merely comparisons between two different things or concepts, but are rather frames through which we perceive and make meaning of the world (Schön 1979). Applying systematic metaphor analysis through a perspective of embodied cognition highlights the bodily and experiential aspects of metaphor use.

The position that knowledge is embodied and that metaphors can reveal fundamental conceptions allows science educators to study learning processes through the lens of embodied cognition. Amin et al. (2015) acknowledged the emergence of a critical mass of studies that apply ideas from the perspective of embodied cognition in science education. These applications entail investigations into how the use of language and gestures can support conceptualization of abstract scientific ideas. We want to draw on those findings and employ similar methods to investigate the metaphorical patterns of the RSA in order to figure out in which ways students map basic features of the rubber sheet metaphorically onto the abstract scientific concept of spacetime. Embodied cognition does not imply that bodily understanding in some way supersedes the role of language in cognition, but rather suggests that language use and bodily understanding are intertwined.

Exploring the conceptual domain of GR from a linguistic perspective resonates with a broader movement in science education that emphasizes “talking science” in the classroom (Lemke 1990). Reaching ultimately back to Vygotsky (1962), the assumption that language and the development of abstract thoughts are interrelated has brought about fruitful approaches to scaffold learners’ development of scientific knowledge (Chen et al. 2016). Viewing language as a “window in the conceptions of students” (Niebert and Gropengießer 2014, p. 281) aligns particularly well with the objective of our study: students are not familiar with the mathematical language of GR and have to reason by using the everyday language available to them to talk about abstract relativistic concepts. Metaphors are one particular example of talking physics. By choosing metaphors as our unit of analysis, we are able to employ a powerful linguistic tool to explore students’ conception in GR.

## 2.2 Metaphor Analysis as an Analytic Framework in Science Education

An important study to employ embodied cognition as a framework in science education investigated students’ struggles to understand analogies and metaphors as intended by teachers and instructors (Niebert et al. 2012), by reanalyzing 199 instructional analogies and metaphors on the basis of a metaphor analysis. By recognizing metaphors as a useful part of the material that can be analyzed and integrated into a broader research strategy, Schmitt (2005) proposed a systematic procedure for the reconstruction of metaphors to uncover patterns of thought. Niebert et al. built on this procedure to identify and classify conceptual metaphors in science education by first grouping metaphorical terms with the same source and target area and then summarizing the metaphorical model on the level “target is source.” For instance, “the gene is a code” and “equilibrium is a dance” are popular metaphors in biology and chemistry textbooks (Niebert et al. 2012). Their findings suggest that good analogies and metaphors in science education need embodied sources. This conclusion is an interesting one in light of the observation that the embodied source of the RSA and students’ embodied understanding of gravity confront students with profound imaginative challenges; the analogy prompts students to transfer embodied understandings of gravity between three and four dimensions (Steier and Kersting n.d.).

Research studies applied the concept of conceptual metaphor in a variety of ways, and developing a specific and operationalized definition of conceptual metaphor is a challenging but necessary task (Treagust and Duit 2015). In the context of science education, Niebert et al. defined a conceptual metaphor as the “imaginative principles behind the analogy or metaphor” (2012, p. 855) that becomes apparent once metaphors and analogies have been arranged according to their target and source domain. That is, conceptual metaphors allow learners to



imagine one thing in terms of another. Likewise, Lancor (2014a, b) understood a conceptual metaphor as an overarching relationship between target and source domain that is supported by explicit metaphors/analogies that highlight or obscure characteristics of the scientific concept.

Metaphors and imagination are closely linked because metaphors mediate imaginative processes. Approaches to imagining depend on the notion of presence. As Nemirovsky et al. defined it (2012, p. 131), imagining is the “experience of bringing to presence something which is absent in the current surroundings of the participants (Casey 1979; Sartre 2004)”. Imaginers are interacting with objects, ideas, and situations that are not immediately *there* or perceivable. Metaphors, then, function as a way to give presence to these objects of imagination. Niebert et al. explained: “we employ conceptions from a source domain (...) and map them onto an abstract target domain (...) to understand abstract phenomena. Thus, the use of imagination requires a source–target mapping” (2012, p. 852). This imaginative mapping occurs through metaphor. One example used by Niebert et al. (2012) is the metaphor that *atoms are solar systems*. The abstract, difficult to visualize properties of an atom, may become present for learners by relating atoms to the more concrete or familiar models of the solar system. We may imagine an atom (including its difficult to perceive properties), through metaphor, by drawing on our previous experiences with physical models of the solar system. By analyzing the structural properties and relationships of metaphor use, we are thus able to gain insight into how learners conceptualize, imagine, and make present abstract ideas.

While Niebert et al. (2012) presented a broad picture of understanding instructional analogies in science education, other studies have used systematic metaphor analysis to focus on metaphors for individual scientific concepts such as the greenhouse effect or energy. Niebert and Gropengießer (2014) employed metaphor and qualitative content analysis to gain insight into students’ and climate scientists’ resources for understanding the greenhouse effect. Lancor (2014a) studied conceptions of energy in biology, chemistry, and physics and demonstrated that metaphor analysis can be a fruitful framework to analyze scientific discourse. She took a closer look at the substance metaphor for energy in textbooks and the science education literature and identified six conceptual metaphors within this broad metaphor: “Energy as a substance that can be accounted for, can flow, can be carried, can change forms, can be lost, and can be an ingredient, a product or stored in some way.” (Lancor 2014a, p. 1245) This analysis in turn helps to investigate how students understand science content, since each conceptual metaphor affords a different understanding of a scientific concept.

Since both the greenhouse effect and energy are particularly abstract concepts in science education, the above studies suggest that abstract scientific concepts might be too complex to be described by just one metaphor or analogy. Rather, they seem to be embedded in a metaphorical network that structures our understanding of a scientific concept (Lancor 2014a); this is an observation that mirrors Lemke’s (1990) suggestion that scientific concepts do not exist as ideas in their own separate reality, but that they are thematic items that make up a semantic pattern of relationships of meaning. This observation encourages us further to employ the framework of conceptual metaphors and embodied cognition in our study of the abstract concept of curved spacetime.

### 2.3 The Bad Use of Metaphors and the Use of Bad Metaphors

Ultimately, studying the role of metaphors in science education has the goal to improve instructional practices. In a recent editorial in this journal, Kampourakis (2016) pointed out that science educators have an important contribution to make: in communicating scientific

knowledge, they bridge the gap between experts and non-experts. The use of metaphors plays a crucial role in this translation process. Calling for an increased awareness for the inherent limitations of metaphorical language and for the pitfalls that come with communicating conceptual issues, Kampourakis invited us to study “the bad use of metaphors and the use of bad metaphors.” Genes are one example of the “bad use” of metaphors in biology education. According to Kampourakis, the popular metaphors of information encoded in DNA and the genome as a book of life can be misleading: those metaphors present genes as autonomous entities without taking the cellular context into account. One has to be explicit in communicating that encoding information is not an inherent property of genes.

Kampourakis’ call created a common interest in metaphorical practices to which we aim to contribute with this study. Investigating how the—possibly “bad”—RSA can be put to good use in teaching and learning of GR is very much in line with a recent exploration by Haglund (2017), who studied the scientific concept of entropy that is metaphorically conceptualized as disorder. Just like spacetime, entropy is “a genuinely challenging concept for students to grasp, due to its abstract, complex, and mathematical nature” (Haglund 2017, p. 208). Haglund argued that the disorder metaphor can give a first flavor of entropy that students in turn can use to develop and refine their understanding of entropy.

In contrast to entropy, the notion of curved spacetime, although abstract and mathematical in nature, is intimately linked to the embodied experience of being under the influence of gravity. Coming to full circle with the starting point of our investigation, we wish to understand how learners conceptualize their experience of gravity in the setting of GR.

### 3 Methods

Before we can explore the ways in which students conceptualize gravity and curved spacetime with the help of the RSA, it is important to have a sound understanding of the RSA. Therefore, our methodological approach entails the analysis of two different data sets: first, we use metaphor theory to analyze the rubber sheet analogy based on the general accounts of physicists and physics educators as found in the literature. These findings serve as basis for the second part: our empirical investigation of students’ use and understanding of the RSA in relation to gravity and curved spacetime.

#### 3.1 Metaphor Analysis in RSA-Relevant Literature

To study the presentation of the RSA in the relevant literature, we followed the systematic procedure for the reconstruction of metaphors as outlined in Schmitt (2005) and further refined in Niebert et al. (2012). This approach promotes the analysis of metaphors to a qualitative research procedure that allowed us to reconstruct metaphorical concepts based on written accounts.

The two crucial steps in a systematic metaphor analysis consist in (1) identifying a metaphor and (2) reconstructing metaphorical models (Schmitt 2005). First, to identify metaphors, one looks for phrases that can be understood beyond their literal meaning, which stems from physical or cultural experience (source area) and is transferred to a new, and often abstract, area (target area). Second, to reconstruct metaphorical models, a process that Niebert et al. (2012) called “categorizing the level of conceptual metaphor,” one groups the metaphorical phrases that have the same source and the same target area. Condensing this categorization

in the equation “target area = source area,” one thus reconstructs the complete metaphor by identifying its underlying logic.

To exemplify the process of the systematic metaphor analysis, we look at an exposition from Baldy (2007, p. 1772) that makes the mapping between target and source area in general relativity very specific:

Einstein’s theory is introduced to students via the so-called “pillow” model: the pillow represents space, and steel balls of different sizes and masses are used to represent celestial bodies. When a marble representing a body is placed next to a ball, it falls into the dip in the pillow created by the ball. And if the marble is rolled fast enough, it deviates from its normal trajectory in the vicinity of the ball.

Here, the identification of metaphors reveals a rich network of source and target areas that, furthermore, interact dynamically. We can identify several source areas rooted in everyday experience—namely a pillow, a dip in the pillow, a steel ball, and marbles. We find three abstract target areas—space, celestial bodies, and trajectories. To structure the analogy on the level “target-is-source,” we can formulate “space is pillow,” “steel balls are celestial bodies,” and “marbles are celestial bodies.” In addition to these mappings of objects, we have another dimension to the metaphor, namely the dynamic interplay between target and source objects: a ball creates a dip in the pillow, a marble falls into the dip, a marble deviates from its trajectory. We return to this example in our presentation of the results in the next section.

Since science educators are not only interested in identifying analogies and metaphors in scientific discourse, but are also concerned about communicating scientific ideas fruitfully, one can extend the systematic metaphor analysis in a way that encompasses educational concerns. Niebert et al. (2012) proposed two additional steps as part of an extended metaphor analysis that is valuable in the educational context: the identification of the metaphor’s deficiencies and resources, and the comparison and interpretation of students’ and teachers’ source domains. We incorporate these two steps in our analysis, noting that they allow us to make the transition from our literature review to the empirical interpretation of students’ conceptual understanding.

Since we conducted this study in the context of the Norwegian physics curriculum, our selection of relevant texts for a metaphor analysis of curved spacetime includes the two Norwegian physics textbooks on the market (Callin et al. 2012; Jerstad et al. 2014), two popular science books by renowned physicists in the field of general relativity (Greene 2010; Thorne 2009), six peer-reviewed research articles that address the RSA explicitly and that were published within the last 25 years (Baldy 2007; Chandler 1994; Gould 2016; Kaur et al. 2017a; Middleton and Weller 2016; Price 2016), as well as one master’s thesis in science education (Watkins 2014). Following the systematic procedure as outlined above, we identified 41 instances of metaphorical phrases that relate the scientific concept of curved spacetime to a rubber sheet-like object. To simplify the classification in terms of “target-is-source,” we further structured the metaphorical phrases with the help of three subcategories “spacetime is,” “objects are,” “dynamical action via.” This subdivision follows our observation in the previous example that there is a metaphorical mapping of target and source objects, as well as a dynamical interplay between the two. Based on this subdivision, we were able to identify four conceptual metaphors for curved spacetime that allow for a full reconstruction of the RSA. After having unpacked the presentation of the RSA in this way, we followed the extension of the metaphor analysis by Niebert et al. (2012) in order to identify deficiencies and resources of each conceptual metaphor: we took into account the strengths and weaknesses of the RSA that were mentioned explicitly in the analyzed literature and compared those to the individual conceptual metaphors that make up the RSA in order to identify features that the RSA possibly highlights or hides.

## 3.2 Metaphor and Thematic Analysis of Students' Responses

With the systematic metaphor analysis of the literature, we have laid the groundwork for investigating students' conceptualization of gravity and curved spacetime. Before we explain how the literature analysis has informed the way that we framed the empirical analysis, we outline the data collection procedure and the greater educational research project that this study is part of.

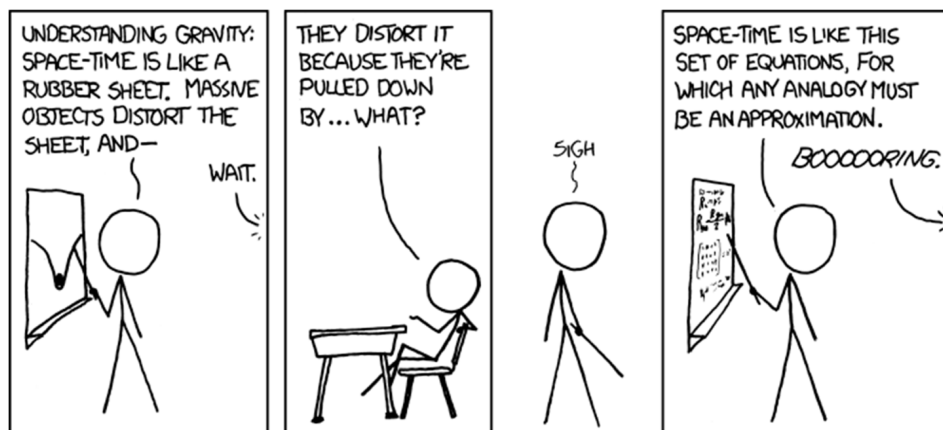
### 3.2.1 Data Collection

This work was conducted within the design-based research project ReleQuant that developed collaborative online learning environments in modern physics for upper secondary schools in Norway (Henriksen et al. 2014). Drawing on the tradition of Vygotsky (1962), project ReleQuant builds on a sociocultural approach to learning physics that emphasizes the use of language (Lemke 1990; Scott and Mortimer 2005) and the interdependence between the individual student and his or her surroundings in the learning process (Rasmussen and Ludvigsen 2010). Students were encouraged to work in pairs or small groups and discuss key concepts of GR and quantum physics while using the learning environments.

This study reports on findings from the second round of testing a GR learning environment in five upper secondary physics classes in three Norwegian schools that were considered to be high achieving in national comparison. The schools are partner schools of the ReleQuant project and the teachers were involved in the development of the learning resources that were jointly designed by physics educators and learning scientists from the project. In total, 97 students (70 boys, 27 girls, 18–19 years old) participated in a series of two 2-h lessons that were part of the regular physics curriculum for final year secondary school students in Norway. The curriculum states that students should be able to “give a qualitative description of general relativity” (The Norwegian Directorate for Education and Training 2006). The learning environment consists of three thematic units the last of which covers the topic of curved spacetime.

Our interest in understanding student ideas of the RSA led us to choose one particular discussion task, which addresses the RSA directly, for further analysis (Fig. 2). The task invited students to reflect on the RSA by discussing a cartoon that addresses the analogical nature of the rubber sheet representation. The open format of the question is well suited to investigate students' ideas of curved spacetime and prompts them to consider the role of analogies more generally. In a second step, students had to write a short summary of their group discussion. This summary provides insight into their use of scientific language, as well as what they felt were the most important conclusions in their discussions. Our data comes from 65 written responses to this task retrieved from the online learning platform. The reason that the total number of collected written responses (65) is smaller than the total number of participating students (97) is that several groups of students chose to submit a joint group response instead of writing individual summaries.

The discussion task is part of a longer learning sequence that introduces students to the concept of spacetime by presenting different models and interactive visualizations of curved spacetime. In the discussion, we relate the findings of this study to the broader context of investigating students' conceptual understanding of spacetime.



**Fig. 2** In the spacetime unit, students worked on the following task: “Use your knowledge in general relativity and discuss the cartoon. Write down a short summary of what you have discussed.” The comic is licensed under a Creative Commons Attribution-NonCommercial 2.5 License and can be accessed under <https://xkcd.com/895/>

### 3.2.2 Data Analysis

We conducted two independent analyses of students’ responses: a metaphor analysis to identify conceptual metaphors and a thematic analysis to characterize students’ awareness of the strengths and weaknesses of the RSA. This double approach resembles the one employed by Lancor (2014b), who characterized students’ conceptual understanding of the energy concept through the lens of metaphor analysis.

Employing methods of the systematic metaphor analysis and mirroring our procedure of the systematic metaphor analysis of the literature, we took students’ written responses and identified 39 instances of metaphorical language connected to curved spacetime. The metaphorical phrases were thus again divided into the three subcategories “spacetime is,” “objects are,” “dynamical action via.” Following the scheme “target-is-source,” we then continued to decompose each metaphorical phrase into its various mappings between target and source area.

We found students’ responses to be often somewhat muddled and not very clear about mappings between target and source areas. To deal with this ambiguity in the written responses, we were very careful in conducting the metaphor analysis. In particular, we found many phrases in which students used a kind of rubber sheet analogy without directly mapping from the target to the source area, i.e., they remained either in the target or in the source area. Therefore, we chose to generate an additional code for implicit metaphorical mapping and tagged 22 instances of those phrases in addition.

To illustrate our method, we present an example of analyzing two responses:

*Student response 1: If you put a mass on a sheet it will bend and create a deflected/curved spacetime around the mass like that we have seen before where the sheet was time.*

*Student response 2: It has to do with that the mass of an object went down in the paper as it was described.*

In response 1, we can identify the mappings “sheet is spacetime,” “sheet is time,” “spacetime bends,” “spacetime is deflected/curved.” Response 2 is an example of an implicit mapping, because the student remains in the source area of paper and mass without explicitly mentioning spacetime or celestial objects. Nonetheless, we can identify the conception “mass goes down in the paper.”

Since we were not only interested in the way students' conceptualize curved spacetime linguistically, but also wanted to gain insight into their awareness of the analogical nature of the RSA as well, we conducted an additional thematic analysis (Braun and Clarke 2006) of the data set to unpack students' understanding thereof. This analysis corresponds to the additional step of identifying strengths and weaknesses of metaphorical mappings as suggested by Niebert and Gropengießer (2014). However, the important difference to the corresponding analysis of the literature is that we aimed to bring to light students' *own* ideas of strengths and weaknesses of their metaphorical reasoning, instead of reconstructing general features that the RSA highlights and hides.

Following the five-step procedure of a thematic analysis (Braun and Clarke 2006), we got familiar with the data set by coding it for general occurrences of students' elaboration on analogies or scientific models. Based on our literature analysis, we used the identified strengths and weaknesses of the RSA as starting point to generate a set of initial codes. With this set we analyzed the identified analogical responses and started to create new codes that captured recurring patterns in the responses that we could not have anticipated solely from the metaphor analysis of the literature. For example, eight groups of students addressed the interaction between the student and the teacher in the cartoon. This observation gave rise to the code "teaching situation." With this enriched set of codes, it became evident that we could group student responses dealing with the RSA into three themes: responses that elaborate on the general nature of analogies in physics, those that address specific characteristics of the RSA, and those that comment on the context of the cartoon. Based on this broad classification, we reviewed and refined our codes and coded the data set again. The final set of themes and codes is presented in Fig. 3.

The first author conducted the first two steps of the thematic analysis and identified the relevant responses for the metaphor analysis. To ensure the validity of the analysis, both authors then discussed the mappings and the codes over several rounds while reviewing all responses together until they reached agreement. Particular focus was put on the interpretation of the findings that were critically re-examined in light of the literature findings.

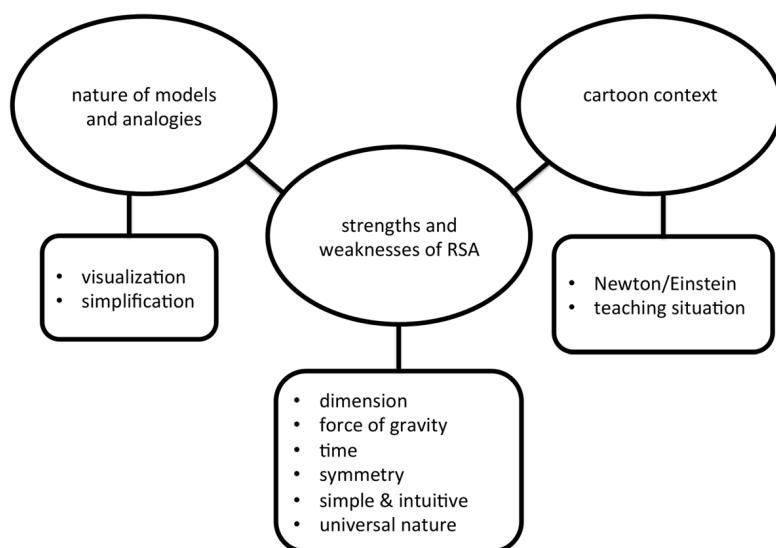


Fig. 3 Map of themes and codes of the thematic analysis of student responses

## 4 Results

In this section, we attempt to spell out the nature of the RSA and characterize upper secondary students’ understanding of it based on a metaphor analysis of relevant literature and a combined metaphor and thematic analysis of students’ written accounts. Following this dual approach, we present results from the literature analysis first and use these results to contextualize the empirical findings from students’ responses.

### 4.1 Metaphor Analysis of the RSA According to the Literature

The goal of our systematic metaphor analysis was to structure the RSA on the level “target-is-source” and to identify and reconstruct the conceptual metaphors that guide this classification. Based on our analysis of relevant literature, we were able to unpack the metaphorical network of the RSA by identifying four different conceptual metaphors. Each of these conceptual metaphors affords understanding of a different aspect of the concept of gravity as curved spacetime by highlighting and hiding various features of the scientific concept. In Table 1, we give an overview of the systematic metaphor analysis of the literature.

**Table 1** The RSA encompasses four conceptual metaphors each of which can be exemplified by specific analogies. The conceptual metaphors are synthesized from a systematic metaphor analysis of relevant literature. The examples come from analogies found in the literature. The conceptual metaphors that comprise the dynamical mapping can be formulated either from the spacetime or the mass perspective

Conceptual metaphor		Analogies that exemplify the conceptual metaphor
Static mapping	Spacetime is a <b>fabric</b> that is <b>malleable</b> .	Spacetime is a <b>piece of rubber</b> that is distorted. Spacetime is a <b>trampoline</b> that is stretched. Spacetime is a <b>pillow</b> that is deformed. Spacetime is a <b>membrane</b> that is warped.
	Spacetime is a <b>2D-surface</b> that has <b>geometrical features</b> .	Spacetime is <b>flat</b> . Spacetime is <b>curved</b> . Spacetime is <b>bumpy</b> . Spacetime has <b>slopes</b> .
Dynamical mapping	Spacetime is a <b>background</b> that <b>responds</b> to the presence of massive objects.	Massive objects <b>distort</b> spacetime. Spacetime <b>stretches down</b> under the weight of an object. Spacetime <b>bends in</b> towards an object. Objects <b>create cavities, slopes and depression</b> in spacetime.
	Spacetime is an <b>actor</b> that <b>influences</b> the movement of objects.	Objects <b>move under the influence</b> of spacetime. Objects <b>roll across</b> spacetime. Objects <b>fall in</b> towards heavy objects. Spacetime curvature <b>alters the path</b> of objects. Objects <b>deviate from their trajectory</b> in response to deformation.

Bold emphasis corresponds to the target and source areas of the specific metaphorical mapping

Before looking closer at the four conceptual metaphors that the RSA encapsulates, we want to make two preliminary remarks. First, it is important to note that there are mappings in the RSA that seem to be less interesting with respect to the characterization of gravity as the geometry of spacetime. While we have identified many examples of objects that are commonly placed on the rubber sheet such as bowling balls, golf balls, marbles, and rocks, these objects do not reflect a relevant imaginative principle that characterizes one thing in terms of another, but are just examples of massive objects that exert a gravitational effect. Even though we might say that the *bowling ball* curving the rubber sheet is like the *sun* curving spacetime, this comparison is mostly an upscaling from everyday size objects to cosmic scale objects. However, the intrinsic feature of being a massive object does not change when going from a ball to the sun. Thus, the mapping is qualitatively different from the mapping that takes place on the level spacetime-is-rubber sheet. When identifying conceptual metaphors for gravity, we therefore focused on the target-is-source mappings that deal with spacetime itself.

Second, as noted already in the methods section, the RSA entails two different kinds of mappings. First, there is a static mapping that maps an experience-based source area like the rubber sheet and marbles to the target area of spacetime and planets. Second, there is a dynamical mapping that encodes the dynamic interplay between the different actors of the mapping, i.e., how masses curve spacetime just like marbles curve a rubber sheet. It is this dynamical interplay that gives rise to the phenomenon of gravity. We argue that both types of mappings are important and constitute a metaphorical network of gravity as curved spacetime. The static mapping settles the underlying structure of the RSA, whereas the dynamical mapping employs the “basic logic” (Niebert and Gropengießer 2014, p. 299) of the source domain to make sense of the physical mechanism of gravity.

To exemplify the four identified conceptual metaphors below, we will use the following example from a physics education article:

(...) let us now think of spacetime as though it were a rubber sheet stretched on a frame hanging over the ground. If there is no matter in it, spacetime is flat. If a particle, a marble or a light ray, were rolled across flat space time, it would go straight. If, on the other hand, matter, a star for example, is present, it acts like a weight on the sheet and creates a distortion. The sheet would stretch down under the weight; the greater the weight the greater the indentation. Now when a marble or light ray is rolled across the sheet it curves into the depression. In this picture the particle is moving rapidly enough to bend in toward the lump and continue to move on out. Another particle might circle and eventually fall into the depression. (Chandler 1994, p. 171)

While spacetime as the target domain remains the same, we have found a variety of source domains that get mapped onto this abstract domain: a rubber sheet, a pillow, a membrane, a trampoline. However, all source domains have one feature in common which leads to the reconstruction of the first conceptual metaphor: *Spacetime is a fabric that can be stretched and deformed*. This conceptual metaphor captures the idea that all source domains are fabric-like objects that are malleable. Evidence for this conceptual metaphor includes the use of a source domain that either implicitly displays this property (as for example a rubber sheet does) or explicitly mentions the stretching and deforming of the source domain: “(...) let us now think of spacetime as though it were a **rubber sheet stretched** on a frame hanging over the ground.”; “The sheet would **stretch down** under the weight; the greater the weight the greater the indentation.”

Moreover, most mappings did not stop at the level of comparing spacetime to a fabric. The internal logic of this mapping invites us to deduce further characteristics of the target domain, which leads to the formulation of the second conceptual metaphor: *Spacetime is a two-dimensional surface that has geometrical features*. In the literature, we found analogies that characterized spacetime via a source object that is flat, curved, bumpy, twisted, has a slope, and



which, accordingly, has geometrical features. These characterizations imply in particular that spacetime is a two-dimensional surface embedded in three-dimensional space: “If there is no matter in it, spacetime is **flat**. If a particle, a marble or a light ray, were rolled across flat space time, it would go straight.”; “In this picture the particle is moving rapidly enough to bend in toward the **lump** and continue to move on out.”

These two conceptual metaphors make up what we call the static mappings of the RSA. They characterize spacetime in terms of more familiar notions, but do not yet explain how gravity arises. The explanation of this phenomenon is captured by two additional conceptual metaphors that make up the dynamical part of the mapping. Note that each of these two conceptual metaphors can be formulated either from the spacetime or the mass perspective: **Spacetime is a background that responds to the presence of massive objects/Massive objects distort spacetime**. In the literature, the RSA is used to explain how gravity arises by saying that spacetime stretches down under the weight of objects, spacetime bends in towards objects, or that it is distorted by objects. On the other hand, it is said that objects create cavities, slopes, or depressions. Mappings were considered to have evidence of this conceptual metaphor if they discussed either the way that spacetime reacts to the presence of massive objects or the distortion effect of massive objects on spacetime: “If, on the other hand, matter, a star for example, is present, it **acts like a weight** on the sheet and **creates a distortion**. The sheet would **stretch down under the weight**; the greater the weight the greater the indentation.”

Finally, we have the metaphor: *Spacetime is an actor that influences the movement of objects/Objects move under the influence of spacetime*. Evidence for this conceptual metaphor entails the way objects react to the geometry of spacetime or the way curvature alters their paths: objects deviate from their trajectories, they curve or fall in towards massive objects, and their motion changes in response to deformation: “Now when a marble or light ray is rolled across the sheet **it curves into the depression**. In this picture the particle is moving rapidly enough **to bend in toward the lump** and continue to move on out. Another particle **might circle and eventually fall into the depression**.”

By definition, conceptual metaphors capture the underlying relationships that guide analogical mappings between the target and source domain. Thus, breaking down the ways that gravity is conceptualized in the RSA helps to identify the strengths and weaknesses of the analogy. In order to do so, we compared the strengths and weaknesses of the four conceptual metaphors that we had identified that were mentioned explicitly in the literature. This comparison allowed us to supplement the literature collection of strengths and weaknesses with our own findings. To answer our first research question, we synthesized the features that the RSA brings into focus and obscures in Table 2.

## 4.2 Students' Understanding of the Rubber Sheet Analogy

We found a big variety in students' written responses in terms of length, depth of reflection, and the range of issues addressed. This variety shows that students engaged with the task in many different ways. The task was an open one: by asking students to use their knowledge of GR to discuss the cartoon and to summarize their discussion in written form afterwards, we challenged them to figure out what they felt was important. In 39 of 65 responses, we identified instances of metaphorical language that were accessible to metaphor analysis, whereas 42 of 65 responses addressed limitations and strengths of the analogies. Those responses encompassed elaborations on the need to employ analogical reasoning in science,

**Table 2** Strengths and weaknesses of the RSA. Findings in the table were synthesized from literature examples supplemented with our own findings based on a metaphor analysis of the literature

Strengths of the RSA	Weaknesses of the RSA
<ul style="list-style-type: none"> <li>• Spacetime is dynamic and not static</li> <li>• Spacetime is influenced by objects</li> <li>• Spacetime alters the movement of objects</li> <li>• Gravity exhibits a universal nature: Spacetime responds universally according to the weight of the objects—the more massive an object, the more distortion of spacetime it will create</li> <li>• Gravitational phenomena involve no “mysterious” action at a distance</li> <li>• Gravity is geometry; the RSA provides a mechanism of how gravity arises</li> <li>• The RSA is a simple, intuitive model</li> <li>• The RSA has great explanatory power; it is suitable to show orbital motions, curved space, and photon trajectories</li> </ul>	<ul style="list-style-type: none"> <li>• The RSA obscures that spacetime is four-dimensional</li> <li>• The RSA obscures that spacetime has a temporal dimension</li> <li>• The RSA obscures that curvature is an intrinsic feature; it depicts curved spacetime as if there was an unseen dimension into which spacetime curves</li> <li>• The RSA obscures that curvature around massive objects is symmetric in all dimensions</li> <li>• The RSA makes use of the force of gravity to explain the distortion of the rubber sheet</li> </ul>

as well as pointed out specific shortcomings of the RSA in the context of GR. In addition, 12 responses dealt with the instructional context of the cartoon and how the interaction between teacher and student contributed to understanding GR.

It was interesting to see how students incorporated different parts of the learning environment in order to solve the task. Many connected the cartoon to explanations previously presented, such as our inability to visualize four dimensions except mathematically. Thus, the format of the question seems to have been successful in engaging students to piece together the different bits of explanations that convey the complex scientific concept of gravity as curved spacetime.

#### 4.2.1 Systematic Metaphor Analysis of Student Responses

To characterize students' understanding of the RSA, we first looked at the ways in which students talked about the RSA. Analyzing the language they employed through the lens of metaphor analysis allowed us to approach our second research question.

In the metaphor analysis of the literature, we found four conceptual metaphors that describe the relationships between spacetime and massive objects. Conducting a similar analysis of students' responses, we found that students displayed a wider range of target-source-mappings (Table 3, Fig. 4). While the literature only identified productive target source mappings, students had not acquired a complete understanding of the analogy yet, and were therefore likely to produce mismatches between target and domain areas of the analogy. Naturally, students produced more mappings because there are many possibilities to create mappings between target and source objects. However, on a deeper level, these mismatches allowed us insight into the challenges that students face when conceptualizing gravity and spacetime.

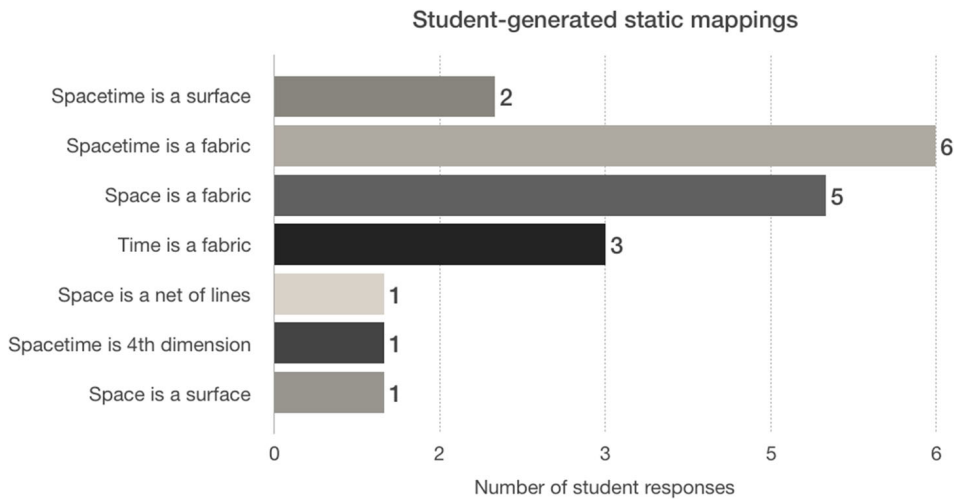
Similarly to the characterization of the literature findings, we divided the student-generated mappings into static and dynamic ones (Table 3). In general, occurrences of static mappings were less frequent than dynamical ones and there was a greater variety of mappings in the dynamical domain. This difference in frequency provides a first hint that students displayed more misconceptions in the dynamical mappings of the RSA. They might struggle most with

**Table 3** Student-generated mappings between target and source domains of the RSA. The shaded conceptual metaphors are the ones found in the literature. Examples are translations from student responses retrieved from the learning environment

Conceptual metaphor	Examples of student response.	
Static mappings	Spacetime is a fabric that is malleable.	In this cartoon we see that the teacher tries to explain spacetime by comparing it to a <b>rubber pad</b> where heavier masses fall further down than smaller masses. (Student group 19)
	Spacetime is a 2D-surface that has geometrical features.	Spacetime is influenced by gravity, therefore <b>the rubber sheet gets twisted</b> . (Student group 20)
	Space is a fabric that is malleable.	In the cartoon there is a question what pulls the object downwards such that the <b>space gets curved</b> . But you should not see this as a force, but that the <b>space “curves itself around”</b> . (Student group 21)
	Time is a fabric that is malleable.	If you put a mass on a sheet it will <b>bend</b> and create a <b>curved</b> spacetime around the mass like that we have seen before where <b>the sheet was time</b> . (Student group 22)
	Space is a surface that has geometrical features.	(...) space (...) can almost be viewed as a <b>sheet around the object</b> . (Student group 23)
	Space is a net of lines.	Here it is introduced that the “force” of gravity pulls the <b>lines</b> down. This is wrong according to Einstein. (Student group 24)
	Spacetime is the fourth dimension.	Einstein thinks that objects with mass curve spacetime, the <b>fourth dimension</b> . He thinks that people live in a four-dimensional reality where <b>the fourth dimension is spacetime</b> . (Student group 1)
	Dynamical mappings	Spacetime is a <b>background</b> that <b>responds</b> to the presence of massive objects.
Spacetime is an <b>actor</b> that <b>influences</b> the movement of objects.		Mass curves spacetime and <b>spacetime determines therefore the movement</b> of the masses in spacetime. (Student group 26)
Space is a background that responds to the presence of massive objects.		The point is not that the mass is “pulled down” in space. <b>Space curves itself around the mass</b> . (Student group 27)
Time is a background that responds to the presence of massive objects.		<b>Big masses curve all of time</b> and space and do this in several dimensions. (Student group 9)
Mass is pulled down.		This can be difficult to visualize, so we usually look at this in two dimensions. Then it looks as if there is <b>something that pulls the mass down</b> . But it is the mass itself that curves the space. (Student group 9)
A force curves spacetime.		We discussed what this <b>force that curves spacetime</b> could be since it is not a force. (Student group 2)
Gravity influences spacetime.		<b>Spacetime is influenced by gravity</b> , therefore the “rubber sheet” gets twisted. (Student group 20)
Mass influences force.		The ball sinks down into the sheet because of gravity and heaviness, but in the outer space heaviness will not make it fall down. This is because there is no force of gravity in outer space. Instead, <b>the mass of an object will tell how much force of gravity</b> it has. How much it attracts other objects. (Student group 28)

Bold emphasis corresponds to the target and source areas of the specific metaphorical mapping

the actual mechanism of gravity (i.e., the dynamical interplay between target and source components that give rise to the physical phenomenon of gravity) than with the static mapping between spacetime and rubber sheet as such.



**Fig. 4** Conceptual metaphors that students generated in the static mapping between source and target domain. The height of the bar represents the number of student responses. In total, 39 responses featured various metaphorical instances of which 19 corresponded to static mappings

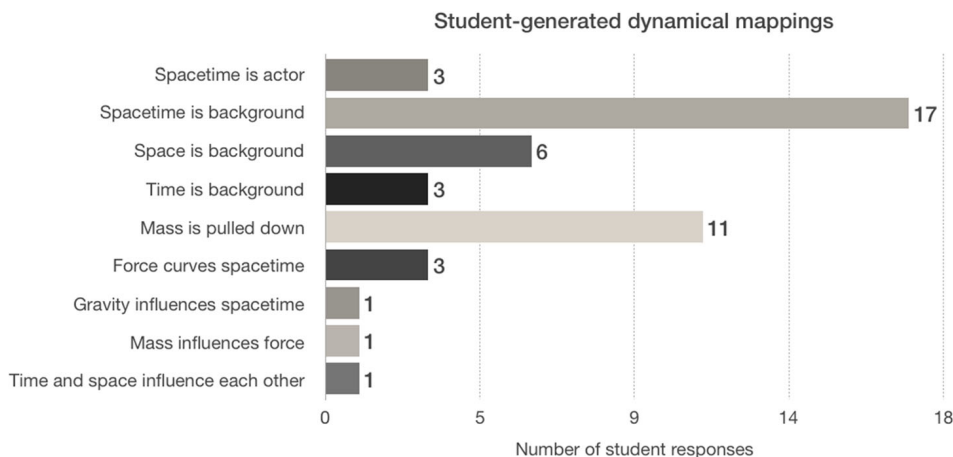
Most of the static mappings only broke down spacetime into space and time components. Students mapped both the space and the time component onto a fabric-like object that resembled a surface with geometric features. This object could be a rubber sheet, a sheet, a trampoline, a tablecloth, a paper, or a rubber pad; but no matter what actual source domain the students chose, their mappings resembled the two static conceptual metaphors we found in the literature. In one instance, students chose “lines” as the source domain to describe spacetime with. We interpret this choice as borrowing from a common way of depicting spacetime with the help of a deformed mesh (Fig. 1).

The only novel mapping we found that differed significantly from the common comparison of a rubber sheet to space, time, or spacetime involved the fourth dimension:

Einstein thinks that objects with mass curve spacetime, the **fourth dimension**. He thinks that people live in a four-dimensional reality where **the fourth dimension is spacetime**. (Student group 1)

This response shows how students confused the new terminology, which seems to be particularly challenging. Even though the formulation “mass curves spacetime” is a correct one, it becomes clear that this group of students still struggled with the abstract notions of the fourth dimension and spacetime both of which are equated in this response. Thus, using the right terminology could in some cases mask students’ lack of conceptual understanding and a metaphor analysis allowed exploring whether this was indeed the case.

One would expect an added level of complexity when students have to describe the physics of gravitation that is captured by the dynamic relationships between target and source domains. Our findings align with this speculation, as students generated a greater variety of dynamical mappings (Fig. 5). While the most common mappings corresponded to the ones identified in the literature—namely that spacetime (or space or time separately) is a background that responds to the presence of massive objects and an actor that influences the movement of these—we identified various other novel conceptual metaphors. These conceptual metaphors concerned mainly the interplay of force, mass, and spacetime.



**Fig. 5** Conceptual metaphors that students generated in the dynamical mapping between source and target domain. The height of the bar represents the number of student responses. In total, 39 responses featured various metaphorical instances of which 46 corresponded to dynamic mappings

Not surprisingly, the most common novel conceptual metaphor addressed the problem that is featured in the cartoon: in order for the mapping to work, the RSA relies on the force of gravity that pulls a massive object down, thus explaining the relativistic notion of gravity with its classical counterpart. Of course, the task invited students to observe this. Accordingly, almost all responses that employed the passive (or Newtonian) perspective that the mass is pulled down into the sheet instead of the active (or Einsteinian) view that mass curves spacetime expressed criticism towards this idea. We come back to this observation in the next section when looking closer at students’ awareness of the analogical nature of the RSA.

Less frequent but crucially related to the Newtonian conception of gravity is the idea that spacetime is curved by a force or by gravity acting on it:

We discussed what this **force that curves spacetime** could be since it is not a force. (Student group 2)

This response and the general mappings that conflated forces with the analogical mappings show that students still used the force concept in their reasoning even though they “knew” and were told that gravity is not a force. These conceptual metaphors thus point towards a conceptual struggle that students faced when attempting the transition from classical to relativistic theories of gravitation. They confused cause and effect in the analogy: the force of gravity does not curve spacetime, but it arises from the curvature of spacetime. Finally, we would like to comment on the implicit mappings that we already mentioned in the methods section. Many students used the RSA implicitly—22 out of 39 metaphorical phrases remained either in the target or the source area. This observation could first of all be simply a sign of the fact that students inferred from the given context that the mapping was there without seeing the need to actually spell it out. But it could also indicate an insufficient understanding of what the target and the source domains were and might display lacking of mastery of the domain specific language. Possibly, the usefulness of analogical mapping was not clear to them—the productive use requires explanations of the relationship between target and source.

In Table 3, we list all student-generated conceptual metaphors. Each metaphor is exemplified by a student response. It is important to note that student responses often comprise several

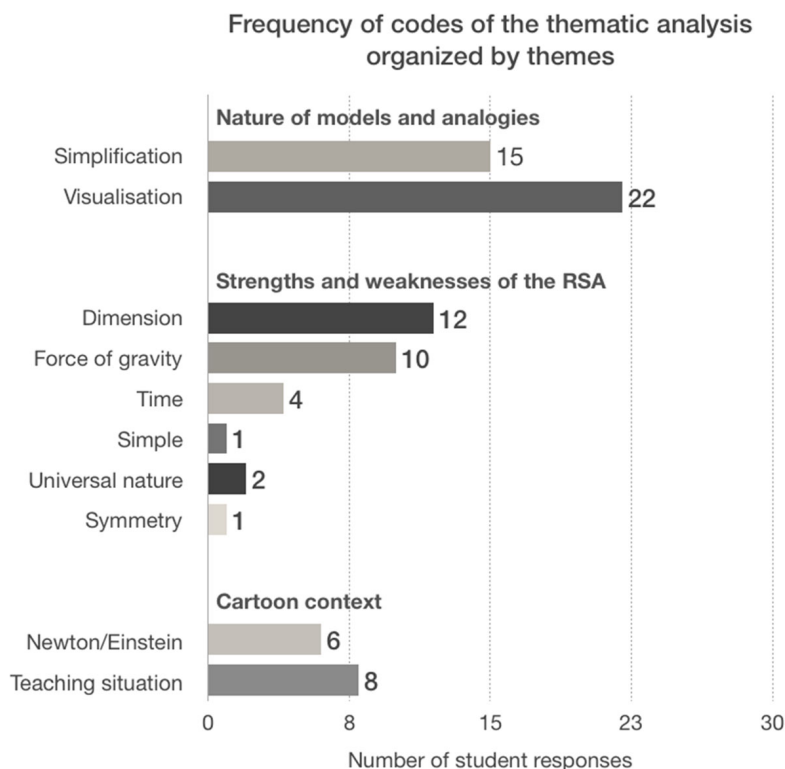
conceptual metaphors, so our choice of examples does not necessarily reflect just one particular conceptual metaphor.

#### 4.2.2 Thematic Analysis of Student Responses

The metaphor analysis of students' language served as a starting point from which we further explored students' understanding of the RSA. The thematic analysis of student responses allowed us to move beyond the structural linguistic level by taking into account how students showed awareness for the analogical nature of the RSA. An overview of the frequency of codes is displayed in Fig. 6. In what follows, we explain the findings in detail.

In the two most frequent types of responses, students displayed a general understanding of the role of analogies and analogical models in science, which we turned into the theme "nature of models and analogies" and that consists of the codes "visualization" and "simplification." The code "simplification" encompasses written accounts that express the insight that analogies are always limited in their explanatory power and that they inevitably simplify or approximate a phenomenon to a certain extent:

A useful tool to understand physical phenomena are models. The problem with the models is that they are simplifications. In this case the models become actually wrong. You could think that it is the force of gravity that pulls the object down, but there is no force of gravity. The alternative is to explain the phenomenon purely mathematically, but then you don't have any illustration. (Student group 3)



**Fig. 6** The thematic coding of student responses comprises three themes and ten codes. In total, 42 responses featured student talk about analogies

Here, students displayed awareness of the limited nature of the RSA and expressed the understanding that models of gravity can only be an approximation, as well as that it is only through mathematics that one can fully describe GR. Other students were more explicit in relating the need for visualizations to their understanding of the simplifying function of models:

We make models to describe physical phenomena, but these models are simplifications and not quite precise. They help us to visualize, even though they don't tell the whole truth. Mathematically, we get the correct results just by using calculations, but to understand curvature of spacetime we need to visualize it with help of simplifications. (Student group 4)

In those two examples, we can also identify another important issue that got mentioned repeatedly: the inability to visualize curved spacetime. Students expressed their awareness of their inability to visualize more than three dimensions:

*It is impossible to make a precise three-dimensional representation of a four-dimensional phenomenon.* (Student group 5)

*It is impossible to visualize<sup>1</sup> four-dimensional spacetime, and you need to use two- and three-dimensional analogies that approximately can give an understanding of how four dimensions work.* (Student group 6)

*We live in a four-dimensional world where three of them can be understood by human beings. To understand the concept of curvature of spacetime we can use analogies, but analogies will never make you visualize time, this can only describe the effect of spacetime curvature.* (Student group 7)

While students showed a quite sophisticated understanding of the need for analogies and visualizations in the domain of GR, many of the responses remained on a rather general level and only about half explained specific shortcomings of the RSA. These strengths and weaknesses that relate directly to the RSA are summarized in the second theme that encompasses six codes which we contrast with the significantly longer list of strengths and weaknesses as synthesized based on the literature analysis in Table 2.

The most common limitation of the RSA that students identified was the reduction of a four-dimensional phenomenon to a lower-dimensional representation tagged by the code “dimension.” While this weakness of the RSA is closely related to the general inability to visualize four dimensions, some students touched upon the problem of “intrinsic” curvature versus “extrinsic” curvature:

We discussed how curvature does not happen within the dimensions the object is in, but in a new such that we cannot observe that space itself gets curved. (Student group 8)

This response reflects a common criticism brought forward by physicists and physics educators (e.g., Gould 2016), namely that the RSA suggests that spacetime curves into an unseen additional dimension. Indeed, it seems that students struggled with this depiction of spacetime and were not necessarily aware that the unseen dimension is an artifact of the analogy that does not correspond to a real physical phenomenon.

The second most common analogical weakness identified by students was the problem related to the force of gravity:

Large masses curve everything of time and space, and do this in several dimensions. This can be difficult to visualize, so we usually look at this in two dimensions. Then it looks as if there is something pulling the mass. But it is the mass itself that curves space. (Student group 9)

<sup>1</sup> The original Norwegian “å se for seg” can be translated as “to visualize,” “to envision,” “to see in your mind’s eye” or more literally “to see in front of you.” In our translations, we chose the expression “to visualize.”

Here, students summarized the key problem of the RSA addressed in the cartoon: that “something” is needed to exert a pull on the massive object. In the source domain of the rubber sheet, this pull is provided by the force of gravity—which does not have an analogue in the target domain of abstract spacetime.

Almost all of the ten responses tagged by the code “force of gravity” addressed the incorrect assumption that the mass is being pulled down:

We discussed that this model is a bit wrong to use, because it refers to a force of gravity that holds the ball down. It does not work like this according to Einstein. (Student group 10)

However, student discussions about the analogy suggest that many still thought along the lines of classical physics or struggled with reconciling how masses can exert an influence without the mediating force of gravity:

*It is difficult to describe spacetime. It is also difficult to visualize that mass influences spacetime just by being mass. That mass is not influenced by a force and that that's the reason it exerts an influence.* (Student group 11)

*We discussed how mass can influence spacetime, if spacetime does not have mass itself.* (Student group 12)

This finding shows us that, for students, the phenomenon of gravity seems to be deeply associated with the concepts of force and mass. In particular, the second quote is interesting, as it expresses the idea that spacetime itself might have mass in order for it to be influenced by other masses. This finding gives insight into students’ ability to use their existing knowledge to deduce characteristics of novel scientific concepts. In this case, the justified conclusion that spacetime must have mass because it reacts to the presence of other masses was discarded by the students themselves.

Another important feature that the RSA hides is that spacetime has a temporal dimension—masses curve time as well. Even though the learning environment introduced the RSA by pointing out that this is a weakness of the analogy, only few students addressed this weakness:

*The first analogy does not take the time coordinate into account and there is a simplified model. Therefore, there arise questions concerning imprecisions of the analogy. We have to use simplified models because we cannot visualize four dimensions.* (Student group 13)

*We discussed Einstein's model where curvature in spacetime and geometry around it lead to what we call gravity. The most difficult to understand is the time parameter in the model and how also this is curved.* (Student group 14)

The relatively few responses that mentioned the time dimension suggest that, generally, students were not aware that the time dimension plays an important role in the origin of gravitation; those that showed such awareness admitted that this part of the theory was difficult to understand. This observation suggests that the role of time might have posed a conceptual challenge for students when dealing with gravity in the setting of GR. Alternatively, the cartoon might have set students on a different track by emphasizing the force aspect of the analogy, making them neglect the time aspect.

Interestingly, students usually only addressed one flaw of the RSA. Of the 26 responses that addressed a specific strength or weakness, only four mentioned two specific flaws/strengths and three of those responses mentioned the dimension problem. Even though the task was open-ended, thus allowing students to explore different problems that come with the use of the RSA, most seemed to have settled on one problematic issue. This observation suggests that there might be instructional potential to facilitate conceptual understanding of GR by presenting various strengths and weaknesses of the RSA explicitly. The last theme, “cartoon context,” does not



directly relate to the RSA, but offers interesting insights into students' conceptualization of gravity in GR nonetheless. The theme contains the codes "teaching situation" and "Newton/Einstein."

Many students picked up on the teaching situation illustrated in the cartoon that emphasized the role of the teacher when learning GR. They stated that it is difficult to teach GR and that falling back to mathematics might be a convenient way for teachers to avoid facing difficult questions by students:

*We discussed that the teacher didn't have a good response to the question of the student and responded with a really theoretical calculation to stop the questions. The reason for this can be that it is impossible for us to visualize four dimensions, and therefore it is also difficult to teach this. (Student group 15)*

*First the teacher explains via drawings, the student does not understand this, so it gets explained via formulas and logic and the student thinks this is boring. The topic is possibly also too difficult for the student to understand if you just jump right into it. (Student group 16)*

Surprisingly, interpretations of the teaching situation produced an interesting response in five cases: students compared the teacher to Einstein and the student to Newton. This is in line with the presentation of GR in the program where GR is presented in opposition to Newtonian physics. Students projected the Newtonian and the Einsteinian view on the two protagonists in the cartoon. Here, we recognize an observation made already during the metaphor analysis: students drew on previous presentations of GR in the program and several students seemed to remember the contrast between Einstein and Newton well:

*The teacher is Einstein, while the student is Newton. (Student group 17)*

*First the teacher tries to explain how time and space can be curved by objects. The student doesn't understand this and he tries to explain it with equations instead. He thinks this is boring. These are two persons that maybe have two different ways to look at spacetime. The teacher looks at it in the same way as Einstein and the student in the same way as Newton. Therefore, they don't quite understand each other. (Student group 18)*

We have used the thematic analysis of student responses to explore student awareness of the analogical nature of the RSA and to answer our third research question. In summary, we can see that students displayed a sound understanding of the scope and limitations of analogies as one particular model in the domain of GR. Nonetheless, students addressed specific strengths and weaknesses less often. The reduction of the number of dimensions and the incorrect mechanism of the curving of the rubber sheet by means of the classical force of gravity were the weaknesses that students mentioned most. Less common was the observation that the RSA only depicts curved space and thus neglects the curvature of the time component in spacetime.

## 5 Discussion

We began with the goal of understanding the RSA and the affordances it provides for students to conceptualize gravity as curved spacetime in the domain of GR. In this section, we want to summarize our findings in light of our research questions and discuss instructional implications related to the approach of embodied cognition.

Two rounds of independent analyses of student responses (coding for conceptual metaphors and coding for strengths and weaknesses of the RSA) showed that students generated more conceptual metaphors than the ones found in the literature. The greater part of the conceptual metaphors had much overlap with the ones employed by experts in the field and merely deconstructed spacetime into its space and time components. However, we observed novel mappings between the target and

source domains as well, and those mappings led to essentially different conceptions: whereas GR posits that force is a consequence of the curvature of spacetime (we interpret geometrical properties as forces acting on objects), students turned this reasoning upside down. They described a force that curves spacetime or talked about gravity curving spacetime. Thus, metaphor theory suggests that students might confuse cause and effect when working with the RSA.

Niebert and Gropengießer (2014) made a related observation concerning students' conceptions of the greenhouse effect. They found that students and scientists used the same source and target schemata but mapped them differently, leading to different conceptions of the greenhouse effect. Selecting those mappings that will be fruitful when conceptualizing scientific concepts is thus an intricate task in abstract domains such as climate change or general relativity.

We casted our investigations into the framework of embodied cognition, which assumes that conceptual understanding requires grounding in experience (Niebert et al. 2012). According to this framework, it is not enough to relate instructional analogies to everyday life. Students use their embodied experience to understand analogies, something that instructors need to be aware of. For analogies to be successful in communicating scientific concepts, the chosen source domains need to be embodied in such a way as to not conflict with the target domain. A metaphor of gravity should not depend on student's embodied experiences with gravity.

In light of our findings, we would like to put this observation further into perspective. Even though the source domain of the RSA draws on students' embodied experience, it seems that exactly this conceptualized experience of gravity often got in the way of inferring the right analogical mappings. In order to conceptualize the physical mechanism of gravity in the domain of GR, students need to develop awareness of the tension between the physical force of gravity in the everyday experiential sense and the curved spacetime explanation.

Nonetheless, the RSA gives students a concrete object to visualize and interact with. If we make students become aware of the scope and limitations of their imaginative capacities, this analogical visualization could fill in a link in the chain of reasoning leading from experiential understanding of gravity towards a more sophisticated understanding in the context of GR. After all, many students linked their understanding of curved spacetime to their ability to visualize it. This finding resonates with a shared interest in visualizations among science educator who have called attention to the significance of developing students' skills of visualization more systematically (Gilbert 2005).

More generally, we argue that GR is a domain in which students can benefit from a teaching approach with a greater emphasis on the nature of science and scientific models, in particular on the scope and limitations of scientific models. While many students displayed a good understanding of the role that analogies and models play in GR, significantly fewer identified specific limitations of the RSA. There seems to be untapped potential in creating awareness for exactly those misleading features of the RSA in order to foster conceptual understanding of relativistic phenomena. We have thus identified several specific instructional strategies for improving the introduction of GR in classroom settings. First, we suggest that teachers might provide an explicit classroom discussion of the flaws of the RSA as listed in Table 2. Identifying the shortcomings of a two-dimensional, spatial representation of four-dimensional curved spacetime can help prevent the formation of mismatches and incorrect mappings between target and source domains.

The RSA is one way of visualizing the physics of curved spacetime. To prevent the one-sided presentation of the concept of curved spacetime as a deformed rubber sheet, teachers can supplement this analogy with other models of spacetime such as the world map model that compares the geometry of spacetime to the geometry of two-dimensional maps (Gould 2016;

Stannard et al. 2017). Seeing that the time dimension tends to be a neglected feature in the RSA, it is moreover important to emphasize that curvature and movement in spacetime entail both curvature and movement in space and time. The role of time as a crucial part of teaching and instructional in GR is taken up in a related study of project ReleQuant (Steier and Kersting n.d.). In this case study, that reports on the first trial of the ReleQuant project learning environment, students struggled to use Einstein's model, and in particular the RSA, to explain gravitational phenomena from everyday life. While they could explain planetary movement according to GR, students failed to draw on Einstein's model to explain why they were pulled towards the ground. It seemed that students related curvature to movement and lacked an understanding of their continuous movement along the time dimension. Teachers should thus pay particular attention to the role of time when using the RSA to teach GR.

In their discussions, students frequently juxtaposed Newton's explanations of gravity to Einstein's and identified the stickmen in the cartoon with Newton and Einstein respectively. Thus, another fruitful way for teachers to introduce the physics of GR might be to address the historic development of GR and Einstein's struggle to overcome Newtonian physics. Helping students contrast their own classical conceptions of gravity with the novel relativistic ones can serve as a fruitful addition to the use of the RSA.

More generally, linking the concept of spacetime and other key concepts of GR to students' life worlds is one design principle for learning resources that project ReleQuant has identified as important in the domain of GR (Kersting et al. 2018). To counteract the lack of experience with relativistic phenomena, visualizations in form of digital simulations and animations can supplement static representations of spacetime.

In this study, students encountered the analogy as part of a learning sequence that guided them through different explorations of curved spacetime in form of interactive simulations. Each separate task provided a slightly different perspective on spacetime, which constitutes one conceptually important part of GR. The way Einstein modeled gravitational phenomena through geometric reasoning extended his original ideas about the principle of relativity and the relation of space and time in special relativity. A broader account of this development and students' understanding of other concepts in GR is given in (Kersting et al. 2018).

Finally, we would also like to discuss what we view as two important limitations of this study. First, our data consist of written responses from five physics classes in three Norwegian upper secondary schools. The analysis of students' metaphorical language allowed us to gain insight into the ways that students conceptualized curved spacetime through the RSA. These insights are, however, often only supported by a small number of responses. While our results are thus not generalizable per se, we think that knowledge of the student-generated mappings between target and source domain can help teachers to identify possible sources of conflict with the RSA. This knowledge has thus the potential to be quite broadly applicable in teaching and instruction of GR.

Second, the discussion task featuring the cartoon of the RSA was open-ended. Students were thus not necessarily interpreting the cartoon in a way that aligned with our research questions. Asking students to use their knowledge of GR to discuss and comment on the cartoon can of course only give a partial insight into their conceptual understanding of gravity and curved spacetime. Keeping this in mind and viewing our study as a first step towards a more holistic understanding of learning processes in GR, however, the format of the task had advantages as well: the cartoon addressed explicitly a particular flaw of the RSA, thus prompting students to comment on its analogical nature. Also, by leaving the task open, students could not merely repeat back answers from other parts of the learning environment—a common behavior that we had observed in the first trialing of the learning environment.

## 6 Conclusion

Addressing the controversy around the use of the RSA in the teaching and learning of GR, this study presents empirical evidence of upper secondary school students' reasoning when conceptualizing gravity as curved spacetime. First, we performed a metaphor analysis of the literature to identify four conceptual metaphors that comprise the fundamental relationships between target and source domains of the RSA. Based on this analysis, we identified strengths and weaknesses of the RSA. A second metaphor analysis of students' written responses revealed a greater variety of student-generated conceptual metaphors than held by experts in the field and a thematic analysis gave insight into students' awareness of the analogical nature of the RSA.

We hope that knowledge of students' different conceptions of gravity as curved spacetime and our compilation of strengths and weaknesses of the RSA can give guidance for teachers and science educators alike. Making students become aware of the strengths and weaknesses of the RSA might be as important as introducing them to the physics of gravitation according to the relativistic framework. Moreover, teaching should be explicit about identifying the source and target domains in order to make it clearer to students what a metaphor is and how it is used. One area to be explored in future work would be to teach students simultaneously about gravity with the RSA along with an introduction to the structural features of metaphors more generally so that they can better interpret and apply the RSA.

Moreover, our study contributes to a growing body of recent research on metaphor analysis and embodied cognition in the field of science education. While previous research has found that it takes more than connecting analogies and metaphors to students' everyday life, namely an analogy that employs embodied sources (Niebert et al. 2012), we present findings that give important nuances to this observation. We observed a conflict between students' embodied understanding of gravity and the abstract description of GR. Even though the source domain of the RSA draws on students' embodied experience, it seems that exactly this conceptualized experience of gravity can get in the way of inferring the right analogical mappings. Thus, even if an analogy builds on an embodied source domain, it can fail in communicating scientific concepts fruitfully. It is therefore crucial that students probe their imaginative skills when conceptualizing abstract scientific concepts such as curved spacetime to build awareness of the processes of their own metaphorical reasoning.

Despite some inherent conceptual flaws, the RSA has the potential to serve as a good metaphor. Teaching GR can be successful if approaches build on students' understanding of the limited nature of scientific models, communicate explicitly target and source domain and strengths and weaknesses of the RSA, and point out the disagreement between students' experiential understanding of gravity and the reliance of the RSA on exactly this experiential understanding to explain gravity in more abstract terms.

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### Compliance with Ethical Standards

**Conflict of Interest** The authors declare that they have no conflict of interest.

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## Free fall in curved spacetime—how to visualise gravity in general relativity

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# Free fall in curved spacetime— how to visualise gravity in general relativity

Magdalena Kersting<sup>1,2</sup> 

<sup>1</sup> Department of Physics, University of Oslo, Oslo, Norway

<sup>2</sup> Department of Physics, OzGrav-UWA, The University of Western Australia, Perth, Australia

E-mail: [magdalena.kersting@fys.uio.no](mailto:magdalena.kersting@fys.uio.no)



## Abstract

The first direct observation of gravitational waves in 2015 has led to an increased public interest in topics of general relativity (GR) and astronomy. Physics teachers and educators respond to this interest by introducing modern ideas of gravity and spacetime to high school students. Doing so, they face the challenge of finding suitable models that visualise gravity as the geometry of curved spacetime. Most models of GR, such as the popular rubber sheet model, only address spatial curvature. Yet, according to Albert Einstein, gravitational phenomena stem from deformations both in space and time. This paper presents a new model that builds on a relativistic generalisation of Newton's first law. We use Einstein's free fall thought experiment and a classical height-time diagram to explain how warped time gives rise to gravity. Our warped-time model acts as a convenient supplement to the rubber sheet model. To support teachers in integrating the model into their classroom practice, we have implemented the model as an interactive simulation that is freely accessible. The model is the result of a three-year period of developing and trialling digital learning resources in Norwegian high schools. Based on these trials, we suggest specific instructional strategies on how to use the warped-time model successfully in science classrooms.

## 1. Introduction

Einstein's general theory of relativity is our current best description of gravity. According to general relativity (GR), gravity is the result of the dynamic interplay between space, time, and the mass and energy content in the universe. Spacetime curves and ripples under the influence of massive objects. The first direct detection of gravitational waves in 2015 [1] has led to a new interest in topics of gravity and gravitational astronomy. This interest leads to new opportunities for teachers and educators to engage students

and the general public [2–4]. Indeed, topics of GR and astronomy seem to motivate high school students to a great extent [5–7].

However, with great opportunities come great educational responsibilities. Physics teachers face the challenge of having to translate an abstract scientific theory into a qualitative description without oversimplifying the concepts too much. This paper responds to the challenge of educating and engaging high school students in topics of GR by presenting an interactive warped-time model. While the popular rubber sheet model uses curved



space to explain planetary movement in an intuitive way, the model ignores deformations in time. Our warped-time model presents an alternative strategy to explain gravity. The model thus acts as a useful supplement to the rubber sheet to visualise how warped time makes objects fall.

The presentation of this paper follows a three-fold structure: First, Einstein's key ideas on gravity and spacetime are summarised by presenting two models of GR: the traditional rubber sheet model and our warped-time model. The presentation lists advantages and limitations of each model as well. Second, the development of the warped-time model are contextualised as part of the greater design-based research project ReleQuant that develops digital learning resources in modern physics [8]. Finally, the last section reports on students' experiences with the warped-time model and discusses instructional implications to improve teaching and learning of GR.

## 2. Gravity and spacetime

This section summarises key ideas of GR and relates these ideas to two instructional models. The warped-space model has become synonymous with GR, whereas the warped-time model is our novel approach to visualising curved spacetime.

### 2.1. Warped-space model

At the heart of GR lies Einstein's field equation that describes the interplay between space, time, and massive objects [9]. The popular phrase 'spacetime tells matter how to move, matter tells spacetime how to curve' aptly encapsulates this equation [10]. The widely used rubber sheet model visualises this dynamic interplay through an intuitive hands-on activity [11].

The analogy compares the fabric of the universe to a stretched rubber sheet. Gravity is illustrated by placing a bowling ball and marbles on the rubber sheet. The bowling ball produces a warp of the rubber, which results in an inward tug that influences the movement of the marbles. It is the warp of the rubber sheet that creates the gravitational tug.

The rubber sheet model, sometimes also denoted spacetime simulator or pillow model [11, 12], offers an intuitive explanation of gravity.

The deformed sheet provides a mechanism of how gravity arises and the model has great explanatory power: it is suitable to show orbital motions, curved space, and photon trajectories [13]. Yet, no instructional model comes without limitations. Research suggests that the rubber sheet might be misleading despite its visual power and simplicity: The rubber sheet obscures that spacetime is 4D; in particular, the model obscures that spacetime has a temporal dimension [13].

### 2.2. Warped-time model

The warped-time model addresses limitations of the rubber sheet model by offering a strategy to visualise gravity as an effect of warped time. The warped-time model builds on another important equation of GR, the geodesic equation. The geodesic equation is an equation of motion that can be thought of as a generalisation of Newton's first law. In an attempt to introduce the geodesic equation to science classrooms, physics educators recently coined the term 'Einstein's first law' [14]: Objects that are not influenced by forces move along geodesic curves in spacetime.

A geodesic curve is the spacetime generalization of a straight line. The usefulness of geodesic curves in GR is that they are the paths followed by particles in free fall [15]. There is one important thing to note when formulating Einstein's first law: In contrast to classical mechanics, Einstein did not consider gravity to be a force. Thus, objects in free fall are indeed free—no force in the classical sense acts on them. Einstein's happiest thought, namely that a person in free fall will experience a state of weightlessness, is an everyday example of Einstein's first law: Objects in free fall follow geodesic curves in spacetime.

Building on Einstein's first law, a new teaching strategy makes the warping of time visible. The interactive warped-time model is part of a digital learning environment in GR that is freely accessible at [www.viten.no/relativity](http://www.viten.no/relativity). The warped-time model invites students to explore the physics of free fall both from a classical and from a relativistic perspective. As starting point, the model takes a digital height-time diagram and presents students with two different scenarios (figures 1 and 2): Einstein stands on top of a 45 m high tower and ponders the nature of gravity. In the first case, he remains standing on top of the

tower. In the second case, he steps off the tower in line with his famous thought experiment. To familiarise students with the digital height-time diagram, they are asked to draw trajectories into the height-time diagram. This task serves as a warm-up: Remaining on top of the tower corresponds to a straight line in the height-time diagram and stepping off the tower corresponds to a parabola.

The second part of the warped-time model shifts the two scenarios to a relativistic setting. This time, students have to take warped spacetime into account. Before they can draw trajectories students have to move a slider to warp the time-axis (figures 3 and 4). In this warped diagram, remaining on top of the tower corresponds to a curved line and stepping off the tower corresponds to a straight line.

The difference between the classical height-time diagram and its warped counterpart is that free-fall trajectories either look curved or straight. Students learn that a straight path through spacetime does not necessarily look like a ‘straight line’ in a given representation. Students learn to shift their perspective to understand that objects in free fall follow the straightest possible path through spacetime. Is it a force that pulls objects towards the ground? According to Einstein, there is no force pulling objects to the ground—it is the geometry of curved spacetime.

In a last step, the warped-time model invites students to move between the Newton and Einstein models of gravity (figure 5). By moving a slider up and down, students can compare how both physicists explain the physics of free fall in two different ways: Newton treats gravity as a force that accelerates objects in free fall towards the centre of the Earth. The corresponding trajectory in the space-time diagram is a parabola. Einstein treats gravity as a geometric phenomenon. Objects in free fall follow geodesic curves in spacetime. In a warped height-time diagram trajectories are straight indicating that there is no force acting on the object.

To help teachers use the warped-time model successfully, it is important to list its strengths and limitations. One important limitation of the warped-time model relates to the depiction of curvature. First, the warping of the time-axis is greatly exaggerated. Relativistic effects of warped time are very small on the surface of the

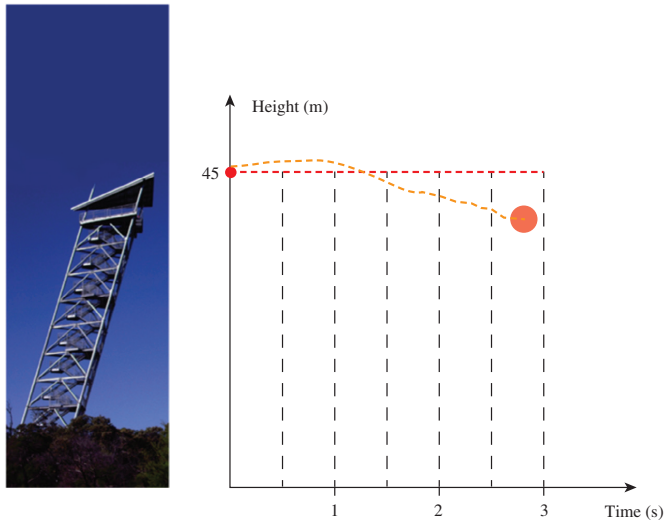
Earth [16]. Second, the curvature of the time-axis is chosen in such a way as to make a free-fall trajectory in the height-time diagram straight. Thus, the time-axis curves somewhat arbitrarily and the curvature does not accurately correspond to the way spacetime is warped around the Earth<sup>3</sup>.

Another limitation of the warped-time model relates to the double nature of gravity. The model does not distinguish between the two aspects of gravity that affect an object—one aspect due to acceleration and one part representing tidal forces. The free fall thought experiment demonstrates the principle of equivalence: gravity and acceleration are locally indistinguishable: To describe a single idealised object in free fall one does not have to evoke curved spacetime explanations. In this case, one can describe gravity by shifting to an accelerated frame of reference. Yet, in reality, objects have an extension and will experience tidal forces. Tidal forces arise from non-uniformities in the gravitational field and cannot be removed in free fall. These forces relate to spacetime curvature. A more thorough discussion of tidal forces can be found in [17].

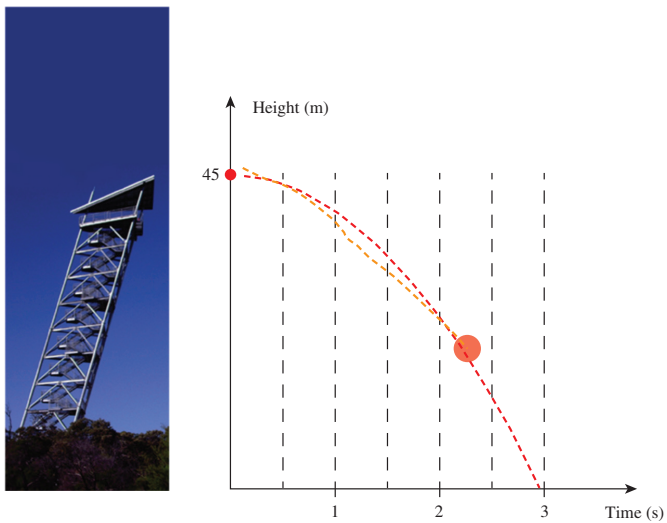
Despite its limitations, the warped-time model has several strengths that make it an ideal supplement to spatial visualisations of GR:

- (1) The warped-time model makes use of one of Einstein’s most famous thought experiments and thought experiments are powerful tools to communicate relativistic concepts to high school students [18].
- (2) The model explicitly addresses and illustrates the time dimension in its depiction of curved spacetime. Moreover, the model links time dilation to the phenomenon of gravity: The warped height-time diagram illustrates that clocks run faster (or tick more often) higher up in a gravitational field (figure 6).
- (3) The model compares Einstein and Newton’s theories of gravity by using a representational tool that high school students are familiar with. Height-time diagrams allow students to link their previous knowledge of movement in a gravitational field to a relativistic model of gravity.

<sup>3</sup> ‘How Gravity Makes Things Fall’ is an award-winning video by Edward Current that presents a different way of warping time to illustrate gravity: <https://youtube.com/watch?v=jITVIM0ix3I>.

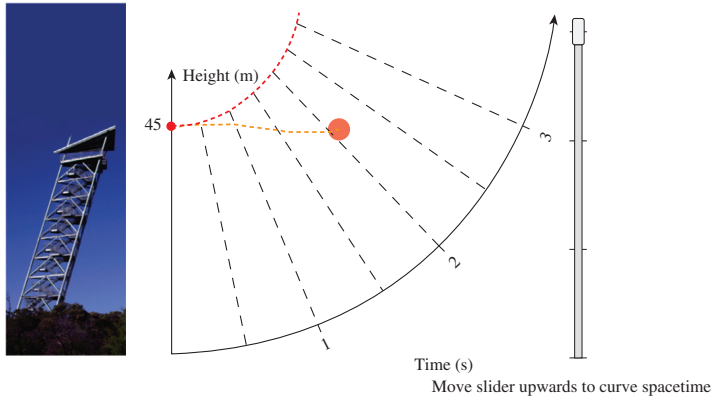


**Figure 1.** The warped-time model invites students to explore Einstein's free fall experiment from a classical perspective: remaining on top of the tower corresponds to a straight line.

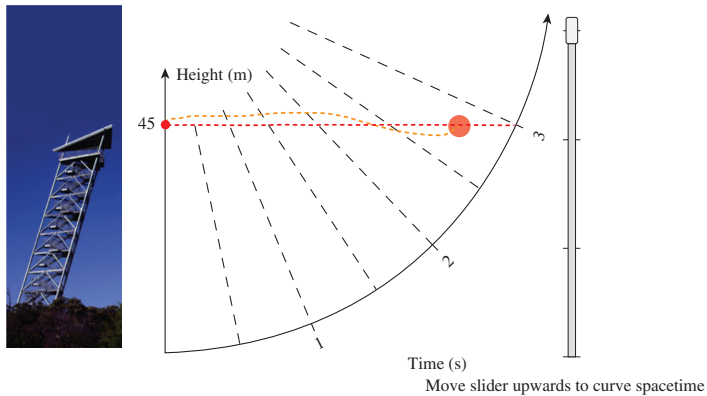


**Figure 2.** The warped-time model invites students to explore Einstein's free fall experiment from a classical perspective: stepping off the tower corresponds to a parabola in a height-time diagram. The interactive model can be found at [www.viten.no/relativity](http://www.viten.no/relativity).

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**Figure 3.** The warped-time model invites students to explore Einstein’s free fall experiment from a relativistic perspective: remaining on top of the tower corresponds to a curved line. The interactive model can be found at [www.viten.no/relativity](http://www.viten.no/relativity).



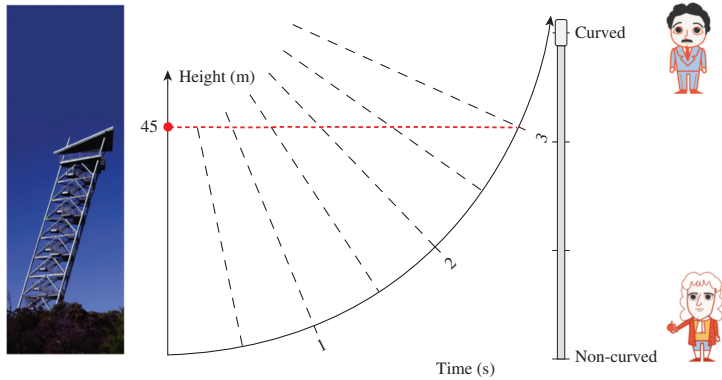
**Figure 4.** The warped-time model invites students to explore Einstein’s free fall experiment from a relativistic perspective: stepping off the tower corresponds to a straight line.

- (4) The warped-time model is flexible in its mode of presentation: Even though this paper presents a digital model of a warped height-time diagram, similar ideas can be implemented using hands-on activities only. It is for example possible to draw height-time diagrams on curved surfaces such as balloons or balls to illustrate the effect of a curved geometry on straight lines<sup>4</sup>.

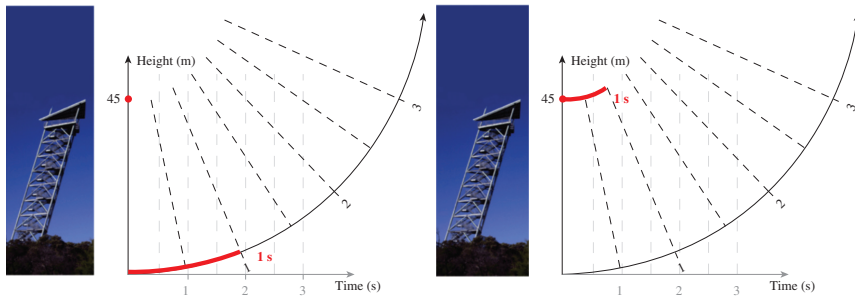
<sup>4</sup>The Perimeter Institute Outreach Program presents a hands-on activity that illustrates the warping of spacetime using beach balls: <https://resources.perimeterinstitute.ca>.

### 3. Educational context

The warped-time model is the result of a design-based research approach to developing learning resources in modern physics [6]. Topics of modern physics place high demands on students’ understanding of abstract and often counter-intuitive concepts. In response to these challenges project ReleQuant was established to study novel and innovative ways of teaching modern physics in Norwegian high schools [8]. In close collaboration with teachers and teacher students, the ReleQuant team developed a digital learning



**Figure 5.** The warped-time model allows students to compare Newton and Einstein’s theories of gravity by moving a slider between a classical and a warped height-time diagram.



**Figure 6.** The warped-time model illustrates that clocks run faster higher up in a gravitational field: there are more ticks of a clock in the same stretch of time.

environment in GR. The learning resources were trialled in 12 upper secondary physics classes over a 3 year period. The Norwegian Centre for Science Education hosts the learning environment that is freely available in English and Norwegian on the open-source learning platform Viten: [www.viten.no/relativity](http://www.viten.no/relativity)

In addition to having been developed within ReleQuant, the warped-time model pools experience from Einstein-First and the Gravity Discovery Centre. Einstein-First is an Australian educational project that aims to introduce young learners to topics of relativity and quantum physics by developing simple models and hands-on activities [19]. The Einstein-First team coined the notion of ‘Einstein’s first law’ in reference to the geodesic

equation [14]. The Gravity Discovery Centre is an outreach facility and science museum co-located at the Australian International Gravitational Research Centre in Gin Gin, Western Australia. The centre features the so-called ‘Leaning Tower of Gin Gin’ which allows visitors to recreate free fall experiments [20]. The warped-time model presented in this paper takes a digital version of the Leaning Tower as a setting to explore Einstein’s law and free-fall motion in curved spacetime.

### 3.1. Student experiences

The final design of the warped-time model is a result of three iterative rounds of developing and testing learning resources in 12 Norwegian

physics classrooms. In this section, key insights from the classroom trials are summarised to guide instruction based on the warped-time model.

Generally, the classroom trials showed that students felt motivated and engaged by curved spacetime even though many admitted that the concept was challenging [6]. The first trial of the learning resources suggested that students struggled to conceptualise movement along the time-dimension [21]. The warped-time model makes movement along the time dimension more visible for students by asking them to draw the trajectory of an object that remains spatially at rest. Understanding that objects always move in spacetime is an important insight that helps students integrate ideas of time and gravity into a relativistic framework.

The second trial of the learning resources targeted a prototype of the warped-time activity specifically. Analysis of small group discussions showed that even though many students seemed to be comfortable with the idea of movement in space and time, only few groups were able to connect geodesic curves to the physical state of being in free fall [22]. Thus, successful instruction should aim to link the geometric description of GR to the physics of free fall. Focus group interviews supported the findings from the classroom discussions during the second trial. Students perceived a gap between relativistic and classical descriptions of free fall. Moreover, the interviews revealed that students continued to find it difficult to visualise time even though the warped-time model helped them to get a better picture of this abstract concept.

Not all students approved of the warped-time model though. Some criticised the model for not being representative of relativistic phenomena. This criticism reveals an understanding of the limitations of this model as well as of the scope of Einstein's theory. Successful instruction of warped time should therefore complement the warped-time model with other examples from cosmology and astrophysics where relativistic phenomena have a more significant effect.

#### 4. Discussion and conclusion

Every instructional model has limitations. In learning domains such as GR where concepts are very abstract or impossible to visualise, it is crucial to develop different models that can

complement each other [23]. This paper presents a new instructional model to visualise gravity as a manifestation of warped time. The model acts as a supplement to spatial models of GR such as the rubber sheet model. In addition to addressing the time dimension, the model introduces students to Einstein's first law.

Based on our classroom trials, we suggest four specific instructional strategies to use the warped-time model successfully:

- (1) It is important to emphasise that every object moves both in space and in time. The insight that objects always move in time (in other words, they age) helps students link geometric descriptions of gravity to their everyday experience of gravity.
- (2) Einstein's thought experiment of freely falling objects is a popular introduction to GR. We suggest capitalising on this thought experiment and using the warped-time model as a second step to explain gravity as a manifestation of curved spacetime.
- (3) In everyday life, relativistic phenomena cannot be observed directly. To help students make sense of warped time, we suggest using the warped-time model to discuss gravitational time dilation as well.
- (4) The warped-time model allows for a direct comparison between the Newton and Einstein models of gravity by showing how two different models can describe the same physical phenomenon. Teachers should use this opportunity to help students build awareness of the nature of scientific models.

The detection of gravitational waves and applications of GR create a fantastic vision of physics for the future. It is up to teachers and physics educators to bring this vision into science classrooms. By offering novel instructional models that visualise gravity and curved spacetime, we hope to support teachers in engaging and inspiring the next generation of scientists.

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### ORCID iDs

Magdalena Kersting  <https://orcid.org/0000-0003-3568-8397>

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**Magdalena Kersting** is a science educator and mathematical physicist who has developed a digital learning environment about Albert Einstein's general theory of relativity. She works as a PhD researcher in the physics education group at the University of Oslo and with the ARC Centre of Excellence for Gravitational Wave

Discovery at the University of Western Australia. She studies learning processes in Einsteinian physics to understand how learners make meaning of abstract scientific concepts.

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## Navigating four dimensions – upper secondary students' understanding of movement in spacetime

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# Navigating four dimensions – upper secondary students' understanding of movement in spacetime

**Magdalena KERSTING**

*University of Oslo, Department of Physics, Postboks 1048, 0316 Oslo, Norway*

**Abstract.** In contrast to classical physics, general relativity (GR) interprets gravity as geometry. Objects follow geodesic curves in four-dimensional spacetime and curvature creates the illusion of a gravitational force. Despite the scientific relevance of this theory, educational research in the domain of GR is scarce. This study contributes to a growing body of knowledge concerning secondary students' understanding of GR. Based on a thematic analysis of audio records of student discussions of 97 Norwegian physics students (18-19 years), we present results on students' understanding of geodesic movement in spacetime. Our findings can give guidance to improve learning and instruction of a key concept of GR at the upper secondary school level.

## 1. Introduction

In the last years, physics educators have made first attempts of introducing Albert Einstein's general theory of relativity to secondary school curricula [1]. Initial efforts have focused on developing appropriate teaching approaches that rely on qualitative understanding [2], geometrical approaches [3], or simplified mathematical treatments [4]. However, to make teaching and learning successful, there remains the need to study students' knowledge and conceptual understanding of key features of GR.

### 1.1. Movement in spacetime

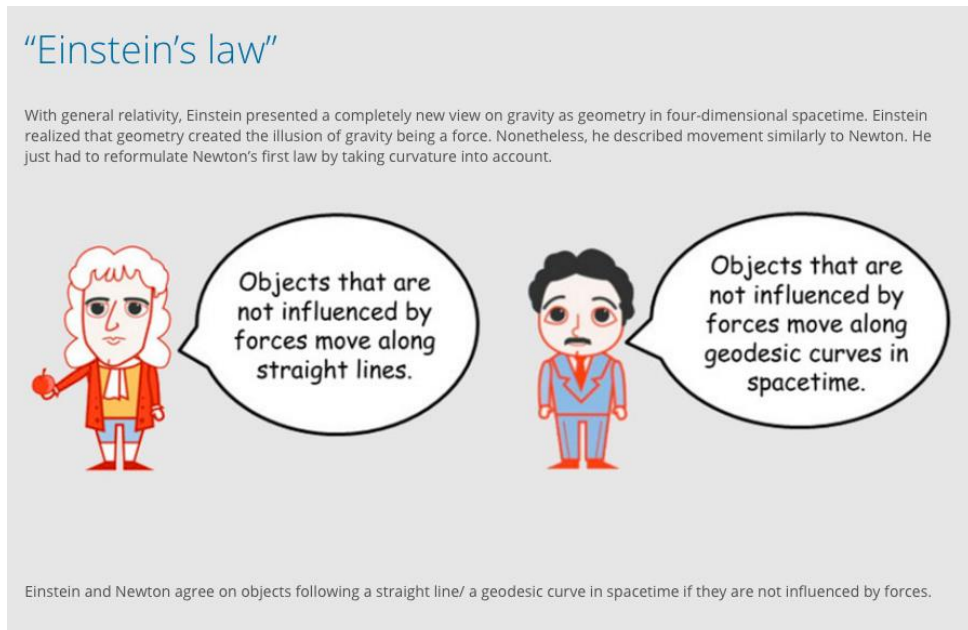
One central feature of GR relates to the motion of objects in four-dimensional spacetime. Spacetime provides the dynamic setting in which GR takes place and in which the phenomenon of gravity arises. Famously, John Archibald Wheeler summarized the relativistic model of gravity by saying that "spacetime tells matter how to move; matter tells spacetime how to curve" [5]. Einstein explained this dynamic interplay between the geometry of spacetime and the movement of objects by drawing on the concept of geodesic curves. Geodesic curves generalize the notion of straight lines to the realms of curved spaces. According to Einstein, free objects that are not subject to any external force will follow geodesic curves through spacetime. It is the curvature of spacetime that manifests itself as gravitational phenomena: There is no force of gravity acting on a falling object – the object follows a geodesic path in a curved universe.

To illustrate Einstein's model of movement in spacetime, physics educators recently introduced "Einstein's first law" in analogy to Newton's first law [4]. While Newton's first law states that objects move along straight lines if no external forces act on them, Einstein's first law translates this statement to a four-dimensional setting. The law takes into account the reality of curved space and warped time: Objects move along geodesic curves in spacetime if no external forces act on them (Figure 1).

There are two crucial differences between Newton and Einstein's model of gravity. First, Newton models space and time as static entities against which the laws of physics unfold. Einstein merges



space and time into a dynamic fabric that takes an active role in the unfolding of these laws. Second, in classical physics, objects fall down to Earth because the force of gravity acts on them; objects resting on the ground experience no net force. In GR, this model is turned upside down. There exists no force of gravity: Objects at rest are subject to an upward-pointing normal force and it is only in free fall that objects are free of any force acting on them. Thus, according to Einstein, a ball falling to the ground is not subject to a downward-facing force, but it follows a geodesic curve which is the straightest possible path through curved space and warped time [4,6]. The force of gravity is an illusion created by the geometry of our universe.



**Figure 1.** Einstein's law serves as a generalisation of Newton's first law in the setting of curved space and warped time. Screenshot is taken from [www.viten.no/relativity](http://www.viten.no/relativity).

### 1.2. Literature review: student understanding of movement in spacetime

Since topics of GR are relatively new in teaching and instruction at the upper secondary school level, there is not much empirical evidence on how secondary school students conceptualize four-dimensional spacetime or geodesic motion. Yet, research with college and undergraduate students can give helpful guidance because upper secondary school students often are of comparable age to freshmen students at the college or university level. A recent investigation into undergraduate students ideas about the curvature of the universe concluded that the majority of students drew on their previous knowledge of geometry: Students understood curvature similar to how scientists used the term by referring to properties such as the dimensions, shape, or amount of bend [7].

Yet, whereas the curved geometry of the universe does not seem to pose significant problems, other findings suggest that undergraduate learners struggle to explain motion in spacetime when shifting between classical and modern perspectives of gravity. To describe the motion of freely-falling objects in gravitational fields according to Newton and Einstein, students have to "crosswalk" between two

conceptual frameworks [8]: Undergraduate students struggle with this transition because of the altered view of gravity and the altered view of freely falling systems [9,10]. The few available studies on secondary school students understanding of GR, mainly conducted in Norway [2] and Australia [11], conclude that learners find the concept of spacetime and topics of “Einsteinian physics” engaging and motivating. However, these findings allow not much insight into the detailed learning processes of students, their conceptual struggles, and successful ways of teaching relativistic concepts.

In an attempt to give a more comprehensive account of secondary school students’ understanding of spacetime, a recent study looked at the metaphorical language of final year physics students that worked with concepts of gravity and spacetime [12]. Even though many students reasoned about spacetime similar to experts, Newtonian views proved to remain strong: Several students confused cause and effect in the domain of GR by drawing on the force of gravity to explain curvature of the universe. Moreover, an exploratory case study with a pair of upper secondary physics students in Norway found that movement was a key issue when students conceptualized gravity and curvature. While the students could draw on the Einsteinian view of gravity to explain planetary movement, they struggled to grasp the gravitational influence on objects at rest [8]. Thus, while researchers have started to investigate upper secondary students’ conceptual understanding of movement in spacetime, there is clearly the need to gain a more comprehensive understanding to make teaching and instruction of GR successful.

### *1.3. Research questions*

The starting point of this study was the scarcity of comprehensive accounts of secondary school students’ conceptual understanding in GR. The aim was to gain deeper insight into upper secondary students’ understanding of movement in spacetime related to gravity and curvature. Two research questions guided the investigations:

- 1) What characterizes upper secondary school students’ understanding of movement in four-dimensional spacetime?
- 2) What are difficulties and challenges that upper secondary school students face when conceptualizing movement along geodesic curves?

## **2. Project background and methods**

This study is part of the larger design-based research project ReleQuant that develops digital learning environments and studies students’ learning processes in modern physics [2,13]. The project takes a sociocultural stance towards learning [14] and emphasizes the use of language in physics education. In particular, the learning environments invite students to discuss key topics repeatedly through structured interactions with peers and teachers.

This study reports from the second of three consecutive classroom trials of the learning environment. The trial was implemented in five upper secondary physics classes with in total 97 students (18-19 years) in three Norwegian schools in spring 2017. To collect data, ReleQuant researchers employed a novel approach of fostering collaborative learning: In several built-in activities, students were asked to discuss in pairs or small groups, record their conversations with mobile phones, and send the records to the teacher afterwards. In line with design-based research principles [15] and based on results from the first trial that showed that students struggled to relate gravity to movement in spacetime [8], the second version of the learning environment presented students with the discussion task shown in figure 2.

## Discuss in pairs

Are you moving in spacetime right now? Do you follow a geodesic curve through spacetime?

According to Newton, why do you experience a pull towards the ground? And according to Einstein?



**Figure 2.** Students were asked to discuss movement in spacetime through structured interactions within a collaborative learning environment. Screenshot is taken from [www.viten.no/relativity](http://www.viten.no/relativity).

The data comes from 21 audio-recorded discussions of small groups of 2-5 students. After transcribing the audio files, methods of thematic analysis [16] were used to unpack students' understanding of movement in spacetime. The codes emerged inductively and were related to spatial and temporal movement as well as Newtonian and Einsteinian conceptions of spacetime. In a second step and in line with the methodology of thematic analysis, the codes were then reviewed and grouped into three themes. Table 1 shows a summary of the final themes and codes. In addition to the thematic analysis, each group discussion was tagged by whether or not the group came to the correct conclusion: Students in the classroom did not follow a geodesic curve because they were not in free fall – the external normal force acted on them.

**Table 1.** Overview of the themes and codes of the thematic analysis of student discussions.

Theme	Codes	Explanation
Physical Conception	<ul style="list-style-type: none"> <li>Newtonian conception</li> <li>Einsteinian conception</li> </ul>	This theme comprises the two different physical frameworks students drew on to explain gravity. The codes tag instances of Newtonian and Einsteinian explanations.
Nature of Movement	<ul style="list-style-type: none"> <li>spacetime movement</li> <li>temporal movement</li> <li>spatial movement</li> </ul>	This theme categorizes the ways students talked about their movement in spacetime. The theme encompasses three codes - movement in space, movement in time, and movement in spacetime.
Movement along geodesic curves	<ul style="list-style-type: none"> <li>shortest path</li> <li>fastest path</li> <li>straight path</li> <li>geodesic force</li> <li>free fall</li> <li>geodesic Einstein</li> </ul>	This theme categorizes the ways students talked about geodesic movement. Students conceptualized geodesic curves in various ways and the codes correspond to the different classifications of geodesic curves.

### 3. Results

The presentation of the results follows the three main themes of the thematic analysis: The presentation moves from students' general understanding of gravity to their understanding of movement in spacetime to their understanding of movement along geodesic curves.

Generally, most groups were able to state the difference between Newton and Einstein's theories of gravity. Students displayed a sound understanding of Newton's force model and related the strength of the gravitational force to the mass of objects. Einstein's model invited the groups to address a broader range of topics: First, students were able to state that gravity, in the Einsteinian framework, is not considered to be a force. Second, students discussed the geometry and curvature of spacetime to explain gravity. Moreover, the groups talked about concepts of gravity and spacetime in visual ways, often acknowledging their wish or need to visualize these concepts further, and often referred to the popular rubber sheet analogy:

**Student 1:** *According to Newton, why do you experience a pull towards the ground? And according to Einstein? So, according to Newton, there is*

**Student 2:** *force of gravity*

**Student 1:** *the force of gravity that is from the mass of the Earth that pulls on us.*

**Student 2:** *Yes.*

**Student 1:** *And we, but why do you experience a pull towards the ground according to Einstein? And then there is something with...no, that's actually a good question. Because, even though if the rubber sheet analogy, right?*

**Student 2:** *Mhmm.*

**Student 1:** *It stood actually further up [in the learning environment] that we are pulled down by warped time actually.*

**Student 2:** *Mhmm.*

**Student 1:** *We don't stand, in a way, in the curved spacetime, but if we think that spacetime is like continuous with a lot of layers on top of each other or many of those layers. That each layer curves, that if you have a layer right on top of the North Pole, which curves as well, then it makes maybe sense. [...]*

Students broadly conceptualized their own movement in spacetime according to movement in space and movement in time. The codes for both classifications appeared equally often even though students seemed to be more certain to explain their movement in space than their movement in time. Space was mostly understood in terms of the universe and spatial movement was often exemplified via planetary movement such as the elliptical movement of the Earth around the Sun or via the movement of galaxies in an expanding universe.

**Student:** *[...] we go around the Sun, and we go around our own solar system, and other solar systems, and we go around our own axis.*

The groups acknowledged temporal movement by the observation that they did not stand still in time. Yet, students admitted that it was difficult to visualize movement in time.

**Student 1:** *Yes, we move along the time dimension all the time.*

**Student 2:** *Yes.*

**Student 1:** *Also when we stand still.*

**Student 2:** *Yes, and...*

**Student 1:** *This is nothing we see, so it is difficult to...maybe difficult to visualize, but it is a form of movement.*

While many groups seemed to be comfortable with the idea of generic movement in spacetime, very few understood the subtleties of movement along geodesic curves. Indeed, only four out of 21

groups came to the right conclusion that they did not follow a geodesic curve. And only one of those four groups gave a correct explanation drawing on the concept of free fall:

**Student 1:** *We move in spacetime right now.*

**Student 2:** *We do this all the time.*

**Student 1:** *We accelerate upwards in spacetime.*

**Student 2:** *We do that. We resist the curvature of the Earth.*

**Student 1:** *Yes.*

**Student 2:** *And we move forward in time all the time.*

**Student 1:** *Yes.*

**Student 2:** *Yes.*

**Student 1:** *And then we follow also a geodesic. No, we don't do that. We don't do that because if we had been in free fall then we would have followed a geodesic curve. Now we are influenced by the normal force as well.*

**Student 1:** *Yes, yes, we accelerate upwards.*

Other groups tried to reason with forces and tried to connect geodesic movement to an absence of forces but did not succeed in finding the right answer.

**Student 1:** *Do you follow a geodesic curve in spacetime right now?*

**Student 2:** *Hmmm.*

**Student 1:** *[...] objects that are not influenced by forces follow a geodesic curve in spacetime.*

**Student 2:** *Are we influenced by forces? We are influenced by forces, aren't we? If gravity is not a force.*

**Student 1:** *Is it not a force?*

**Student 2:** *That's what we started with.*

**Student 1:** *The force of gravity is not a force, but then we don't move along a curve. No, we move along a curve because we are not influenced by forces.*

**Student 2:** *Gravity is just a geometric phenomenon after all.*

**Student 1:** *So we move along a curve now.*

**Student 2:** *We move along a geodesic curve now.*

**Student 1:** *Curve in spacetime, that's what we do you know.*

**Student 2:** *That's what we do you know.*

**Student 1:** *Yes, but then we say this.*

Interestingly, students often stated that they were following a geodesic curve because Einstein said so. There seemed to be a common (mis)conception that movement in spacetime is the same as movement along geodesic curves:

**Student 1:** *Are we moving in spacetime right now? And do we follow a geodesic curve?*

**Student 2:** *Yes, one does that.*

**Student 1:** *Yes, because all objects with mass do this.*

**Student 2:** *Anyway, according to Einstein.*

**Student 1:** *But not according to Newton. So according to Newton, why do we experience a pull towards the Earth?*

**Student 2:** *That's because of the force of gravity after all.*

Even though students struggled to answer the question whether or not they moved along geodesic curves, they were able to draw on various different ways of describing a geodesic. The two most common ways of characterizing geodesic curves were through the concept of the straightest and shortest path through spacetime. Many students discussed whether they were literally following the straightest or shortest path through spacetime:



**Student 1:** *Do we move along a geodesic curve? I don't know.*

**Student 2:** *What do you think?*

**Student 3:** *No, we surely don't do that.*

**Student 2:** *We don't necessarily move along the straightest path. We get bent a bit here and bent a bit there.*

**Student 1:** *As far as I know, we don't get that. We have to have a goal to follow the shortest path.*

Indeed, this geometric understanding of a geodesic curve might have prevented the groups from drawing a connecting to the physical state of being in free fall.

#### 4. Discussion and Conclusion

With the aim of gaining deeper insight into upper secondary school students' understanding of movement in four-dimensional spacetime, this study set out to characterize difficulties and challenges that students faced when conceptualizing movement along geodesic curves in relation to gravity and curvature.

The findings show that students were able to explain their movement in spacetime by drawing on movement in space and movement in time. Common examples of students' spatial movement included the Earth circling around the Sun or the movement of our galaxy in an expanding universe. Even though students realized that they move in time because they grow older, they admitted that temporal movement was hard to visualize. While generic movement in spacetime did not pose significant challenges to students, the concept of movement along geodesic curves did. Students displayed a broad variety of different ways of characterizing geodesic curves such as the shortest, straightest, or fastest path between two points or as a curve on which no forces act. This variety in responses shows that students were able to use different characterizations of geodesics, which, in turn, suggests that they had a good understanding of the concept of a geodesic curve as such. Yet, *movement* along geodesic curves in spacetime challenged almost all groups. Only few groups were able to connect the geometric description of a geodesic curve as the straightest path in a curved space to the physical state of being in free fall or alternatively, to the state of not being affected by external forces. Thus, there seems to be a gap in students' understanding between the geometric framework of GR and the physics of gravity. This finding challenges an observation by Bandyopadhyay and Kumar [9] who stated that the separation between the conceptual and technical aspects of GR is possible in a meaningful way. While it seems that one can separate the technical from the conceptual features in a learning sequence on GR, students eventually struggle to stitch the pieces together again.

It is important to address some obvious limitations of this study. First, the data of this study comes from five upper secondary physics classes in three Norwegian schools with in total 97 students. While the analysis of the audio records of the group discussions allows insight into students' spontaneous ideas and reasoning with concepts of gravity and spacetime, the findings are of course not generalizable to other contexts. Moreover, letting students record their own discussions could have led to a bias in the responses that were collected. It is likely that not all groups of students submitted their records and the data set might consist of those discussions that were done by high-achieving or motivated groups of students. However, the great variation in the responses suggests that data came from a variety of achievement levels.

Identifying student challenges in relation to concepts of gravity and geodesic movement can not only give insight into learning processes of upper secondary physics students. The findings allow improving teaching and instruction of GR as well. The results of this study influenced the design of the ReleQuant learning environment that addresses free fall and geodesic movement in spacetime explicitly in its final unit on spacetime.<sup>1</sup>

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<sup>1</sup> The Norwegian Center for Science Education launched the final version of the learning environment *General Relativity* in January 2018: [www.viten.no/relativity](http://www.viten.no/relativity)

When presenting Einstein's model of gravity it is important to be explicit about Einstein's and Newton's description of free fall. This study supports a conclusion of Bandyopadhyay and Kumar [10] that the "most important cognitive transition that needs to be affected [...] is the altered view of an inertial frame wherein a freely falling frame in uniform gravity (which is an accelerating non-inertial frame in the Newtonian view) is inertial". Indeed, the findings of this study corroborate the importance of a sound instructional emphasis on the concept of free fall. However, contrasting Newton's and Einstein's views of gravity should not lead to oversimplified presentations of the topic. In this study, students displayed a common misconception that every object follows a geodesic curve through spacetime. Instructional units should thus help students connect the idea of free fall and geodesic movement while taking into account these common misconceptions.

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## How history and philosophy of science can inform teaching and learning of general relativity in upper secondary school

Magdalena Kersting

*Department of Physics, University of Oslo,  
Postboks 1048, Oslo, 0316, Norway*

*\*E-mail: magdalena.kersting@fys.uio.no  
www.mn.uio.no/fysikk/english/people/aca/magdak*

Science educators have to move beyond traditional content-focused instruction to teach concepts of Einsteinian physics. This work presents a design-based research project that introduced general relativity (GR) to upper secondary school students in Norway. The educational approach invited students to explore the historical development and philosophical aspects of GR within a digital learning environment. Results based on focus group interviews show that students were particularly motivated by such an approach because it emphasised the cultural and social relevance of physics and it linked GR to their previous knowledge of physics. Employing history and philosophy of science in the service of physics education can serve as a successful approach to making GR more accessible to young learners.

*Keywords:* Physics Education; History & Philosophy of Science; General Relativity

### 1. Context and Background

Few scientific discoveries have had a bigger impact on our understanding of the universe than GR. Einsteins revolutionary theory of gravity did not only herald a new scientific age but fostered a new interest in the philosophy of space and time as well. Yet, despite its scientific, philosophical, and technological importance, GR and other topics of Einsteinian Physics<sup>5</sup> lack from most school curricula around the world. This work presents a design-based research project that introduced GR to upper secondary physics students in Norway<sup>6</sup>. The educational approach invited students to explore the historic development and philosophic aspects of GR within a digital learning environment. Research suggests that history and philosophy of science (HPS) can be fruitful in teaching and learning of physics: HPS perspectives can foster awareness of the nature of science and they help to promote deeper understanding of scientific ideas<sup>4</sup>. This work aims to study how such approaches might foster understanding of and motivation for GR.

### 2. History and Philosophy in General Relativity Education

Science educators promote the use of HPS as a fruitful teaching strategy in learning domains where students have to fundamentally revise their understanding of space and time<sup>9</sup>. This strategy can be traced back to Einstein who was guided by the wish to understand the intrinsic value of physics when developing his theories of relativity<sup>8</sup>: The physicist wished for the human mind to understand the reality of nature. Indeed, by presenting a new view towards space, time, and gravity, GR invites students to view reality from a new philosophical viewpoint. This philosophic

perspective can be complemented with historic approaches to teaching physics. According to Einstein, knowledge of the history of physics is important because it emphasises connections between a new theory and its rich environment:

To use a comparison, we could say that creating a new theory is not like destroying an old barn and erecting a skyscraper in its place. It is rather like climbing a mountain, gaining new and wider views, discovering unexpected connections between our starting point and its rich environment. But the point from which we started out still exists and can be seen, although it appears smaller and forms a tiny part of our broad view gained by the mastery of the obstacles on our adventurous way up.<sup>3</sup>

In the next sections, this work explores how HPS can inform teaching and learning of GR in upper secondary schools. This exploration is guided by the following research question: **How can a historic and philosophic approach to teaching and learning of Einsteinian Physics foster understanding of and motivation for GR?**

### 3. Theoretical Framework and Methodology

To bridge the gap between educational research and the reality of physics classrooms, methods of design-based research informed the design and development of a digital learning environment about GR.<sup>1</sup> The goal was to find workable ways of making GR accessible at the upper secondary school level. In line with the iterative nature of the design-based research framework, two rounds of field studies were conducted - one in spring 2016 and one in spring 2017. In total, eleven physics classes in five upper secondary schools in Norway participated in the trials. To gain insight into students experiences with the digital learning resources, the author of this paper conducted seven semi-structured focus group interviews with 5 to 8 participants per group and 46 students (18-19 years) in total. The teachers chose the students to allow for a balance of gender and to include both stronger and weaker students. The interviews were based on an interview guide that included the design of the learning activities, the use of history and philosophy, and challenges and motivation in the learning domain of GR. The interviews were transcribed and analysed using thematic analysis.<sup>2</sup> The themes of the thematic analysis focused on the use of history and philosophy; the individual codes unpacked students' motivation, interests, and challenges.

### 4. Results and Design Examples

In the focus group interviews, students expressed their curiosity towards learning about GR at the high school level and without advanced calculations. Students had a general expectation that the topic would be difficult and they showed respect towards Einstein's achievement. At the same time, they were looking forward to

learning more about relativistic ideas such as gravitational waves that they had encountered in popular culture and in the news before. Generally, the findings show that students approved of an HPS approach in GR education. Four factors turned out to be particularly motivating or insightful for the participating students:

- (1) Einstein is a popular figure in contemporary culture and students mentioned repeatedly that they enjoyed learning more about Einstein. The learning environment presented Einstein as a real person who struggled to find a new theory of gravity. Being able to follow some of these struggles helped students to identify with Einstein. One student in the interview expressed that relating to Einstein's reasoning helped him to understand GR more qualitatively:

**Student:** *You got into the same reasoning that made Einstein and his friend, it's the same that got into the thoughts of them that resonate within me a bit as well. That you could use this knowledge in a somehow different way than to calculate.*

- (2) Physics was presented as an important part of our cultural heritage with social relevance for our daily lives. Students liked to get a broader and more comprehensive perspective on physics and to make connections to the society as a whole. In particular, they appreciated to learn that physics is an ongoing endeavour and that physicists do not have answers to everything.
- (3) Students learned that physics was a modern field that has more to offer than just textbook knowledge. Many students appreciated these insights into perspectives of modern researchers as the following excerpt shows:

**Interviewer:** *So do you feel you have learned something?*

**Student:** *Yes, I think it was really nice that you got, like, the perspective of recent scientists who made their new discoveries so that you get a bit more recent perspectives. Because it is a bit easy to think that physics is just somehow in the book at school.*

- (4) Finally, the historic contextualisation helped students to link new ideas in GR to their knowledge of classical physics and to their previous experiences. Students appreciated to be able to follow the reasoning that led to GR; this presentation helped them to understand why a theory actually works as the following excerpt from an interview illustrates:

**Interviewer:** *So you mentioned this historic approach. [...] What do the others think about it?*

**Student 1:** *I think it was good.*

**Student 2:** *But that it became clear why it is like that. And it is good that they*

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explained how they ended up there, why it is actually like that.

**Student 3:** Yes, it gets maybe easier to make connections. Or the theories up to now, if you get, somehow, you know a bit about the history and what happened before. And you get something you think - because you think the history is fascinating - you make better connections this way than just getting the theory or the formula straight into your face without any evidence for understanding why it came out like that.

**Student 4:** And if you also learn about how they reached this, then it can be easier to understand why it actually works.

The findings from the focus group interviews informed revisions of the learning environment that took place over the course of two consecutive years in 2016 and 2017. These revisions led to a series of activities that draw on historic events and philosophic questions (Figures 1-3). The learning environment *General Relativity* is accessible at the Norwegian open-access learning platform Viten: [www.viten.no/relativity](http://www.viten.no/relativity)

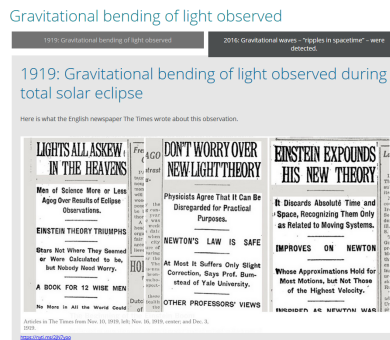


Fig. 1. The learning environment contrasts the first experimental confirmation of GR to the recent breakthrough in gravitational wave astronomy. Students get to explore the gravitational bending of light observed during the solar eclipse in 1919 and they can move between newspaper headlines from 1919 and 2016.

## 5. Discussion and Conclusion

Science educators have to move beyond traditional content-focused instruction to teach concepts of Einsteinian physics. This work explored how perspectives of history and philosophy of science can inform teaching and learning of GR at the upper secondary school level. Specifically, this work presented a design-based research project that invited students to explore the historic development and philosophic

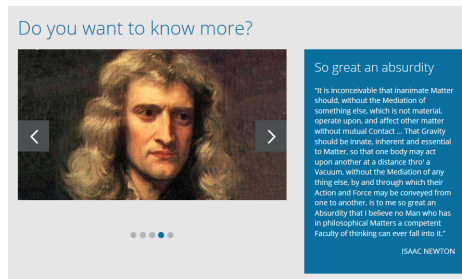


Fig. 2. Newton felt uncomfortable with the mysterious force of gravity that exhibits the principle of action at a distance. In a letter to a colleague, Newton called the gravitational force a ‘great absurdity’.

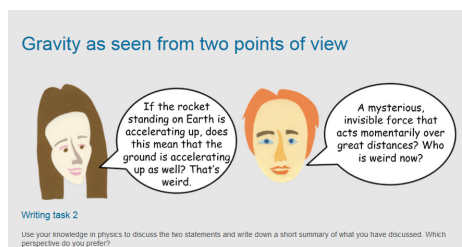


Fig. 3. In a discussion task, students are asked to discuss the epistemological implications of Einstein's and Newton's interpretation of gravity.

aspects of GR within a digital learning environment<sup>6</sup>. Previous research suggests that HPS approaches can be an effective way to help students revise their understanding of space and time in line with relativistic ideas<sup>7,9</sup>. Findings of this study shift the focus away from novel conceptualisations of space and time towards more personal and societal aspects of GR: Guided by the question of how an HPS approach to teaching and learning of GR might foster understanding of and motivation for GR, the findings show that students were motivated by the presentation of Einstein as a real and struggling person instead of an all-knowing genius. Moreover, students appreciated perspectives that emphasised the cultural and social relevance of physics. Being able to gain insights into modern researchers' perspectives and seeing that there are still many unresolved questions in physics was another motivating factor. HPS perspectives did not only foster motivation among students; the historic contextualisation of GR helped students to link relativistic concepts to their previous knowledge of physics. The findings thus support an observation by Einstein that awareness of the history of physics can facilitate deeper understanding

of physics<sup>3</sup>. In conclusion, the findings of this work suggests that employing history and philosophy of science in the service of physics education can serve as a successful approach to making GR more accessible to learners at the upper secondary school level.

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# Appendices



# Appendix A

## Focus Group Interview Guide

**Before recording** (presentation of researcher and project, goals of the focus group interview, practical information)

Welcome to this conversation about teaching and learning activities in general relativity. My name is... and I am...

We will now have a conversation to learn more about your experiences with the GR learning environment that you have used in your physics lessons during the past week. The interview will take around 45 minutes. I would like to hear what was useful and less useful for your learning and what activities you think were more (or less) motivating. Participation in this discussion is voluntary and you can withdraw at any time, without further explanation.

We want to take audio recordings of the discussion. The recording is for research purposes only and will not affect your assessment or your grades in physics. The recording will be treated confidentially: you will not be identified by name or otherwise recognized in research reports. For the audio recording, it would be nice if you talked just one at a time (although we realize that you can get excited!)

Does everyone still want to participate? In that case, we start the discussion and audio recording. We want you to discuss with each other as much as possible on the basis of relatively open questions and topics that we raise.

**Introduction** (goals of the interview, opening questions to get you started)

What has it been like to work with general relativity in class during the past week?

How do you think the last few days have been?

Have you learned something?

What was / was not exciting?

### **Educational Reconstruction**

- **design / evaluation of the learning environment**

The learning environment was a digital one. In what ways does this might have affected your learning of GR?

Was it easy / difficult to follow along?

Do you remember parts of the material you liked particularly well or not at all?

## A. Focus Group Interview Guide

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You have worked individually, in pairs / small groups, and have had discussions within the whole class. How did it work for you to learn GR in these different formats? Why did it work well / less well?

The learning environment uses text, illustrations, animations and videos. How did this combination work for you? Why did it work well / less well? How did you like the different tasks?

- **student perspectives and motivation**

What do you think about GR? (exciting, interesting, motivating, difficult, weird, boring, frustrating, . . . ?)

Did you have expectations about learning GR before the start of the unit?

How do you experience GR now?

Would you like to learn more about GR?

How has this unit on GR affected your motivation for physics?

If you compare GR to other topics in physics: Do you think GR was particularly easy / difficult to understand? Why?

What is GR all about? What is the main idea of GR? Can you sum it up in a few words?

- **language, speaking / writing physics**

What do you think of talking / writing so much in physics lessons?

In what ways does this affect the understanding of the concepts?

- **historical-philosophical approach**

Do you remember an example where we used thought experiments to explain something? (equivalence principle, rocket / elevator in free fall, light deflection, red offset, time extension, ...)

How do thought experiments work for you when you try to learn physics?

What do you think of using historical events as examples? (solar eclipse, exchange of letters, ...)

- **conceptual understanding**

What thoughts do you have around the concept of time?

How is gravity related to time and its structure?

What do we mean by curvature of time? How can we imagine that?

What makes time curve?

What is gravity? Is it a force?

How did you like the module on time and curvature?

What do we mean by Einstein turning classical physics upside down?

Can you come up with one or more phenomena that confirm GR?

Can you describe what the principle of equivalence tells us?

What are inertial frames in GR?

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**Conclusion** (summary, suggestions for improvement)

How would you summarize the difference between GR and Newton's gravitational theory? Are they similar in any way?

Then, just to come to an end, I want to hear if there was something that left a particular impression on you?

We will continue to develop this learning environment in the future. What do you want us to change?

I have no more questions. Is there anything else you want to say before we end the interview?

Thank you for participating.



Appendix B

# Consent Form



## ReleKvant: Læringsressurser og elevers begrepsutvikling i relativitetsteori og kvantefysikk

### TIL ELEVER:

## Forespørsel om deltakelse i forskningsprosjekt

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### Bakgrunn og formål

*ReleKvant - Læringsressurser og elevers begrepsutvikling i relativitetsteori og kvantefysikk* (<http://www.mn.uio.no/fysikk/forskning/prosjekter/relekvant/>) er et forskningsprosjekt i samarbeid mellom Universitetet i Oslo, NTNU, Naturfagsenteret, og fire videregående skoler i Oslo og Akershus. I prosjektet utvikles og utprøves læringsressurser innen kvantefysikk og generell relativitetsteori for Fysikk 2 på plattformen *Viten.no*, og vi ønsker å finne ut mer om elevers læring og motivasjon knyttet til disse temaene. Målet er på sikt å bidra til bedre læring og motivasjon i fysikk. Du som får dette brevet, går på en av ReleKvants samarbeidsskoler, og din lærer deltar i ReleKvants prosjektgruppe.

### Hva innebærer deltakelse i undersøkelsen?

I ReleKvant ønsker vi å observere klassen mens dere arbeider med kvantefysikk og generell relativitetsteori i Fysikk 2. Dersom elevene i klassen samtykker, vil vi gjøre observasjoner (i noen tilfeller også med et videokamera) i klasserommet, få tilgang til elevenes skriftlige svar i Viten, samle inn lydopptak av korte elevgruppe-diskusjoner om fysikk, og i noen tilfeller intervju noen elever enkeltvis eller i smågrupper om hvordan dere opplever å arbeide med ReleKvant-ressursene. Ditt arbeid i fysikktimene (både skriftlig og muntlig) vil selvsagt bli vurdert av læreren din på vanlig måte. Det at en forskergruppe får tilgang til lyd- og skriftlig materiale, vil ikke påvirke lærerens vurdering eller karaktersetting av din innsats i fysikk. Forskergruppen vil bruke opplysningene kun til videreutvikling av undervisningsressursene samt til forskning på elevers forståelse og læring innen disse fysikktemaene.

### Hva skjer med informasjonen om deg?

Forskningsgruppen vil behandle alle personopplysninger konfidensielt og vil bare benytte de innsamlede dataene til forskningsformål. Ingen andre enn forskergruppen vil ha tilgang til dataene, og du vil ikke kunne gjenkjennes i noen rapporter fra prosjektet. Prosjektet skal etter planen avsluttes i 2019, og da vil alle opptak og alt skriftlig materiale der du kan identifiseres med navn eller på annen måte, slettes eller anonymiseres. Innsamlede opplysninger som er anonymisert, kan lagres også etter 2019 med tanke på mulig etterprøving eller oppfølgingsstudier. Som forskere forholder vi oss til etiske regler om hvordan personopplysninger kan brukes. Prosjektet er meldt til Personvernombudet for forskning, Norsk samfunnsvitenskapelig datatjeneste.

### Frivillig deltakelse

Det er frivillig å delta i studien. Klassen din vil uansett følge ReleKvants undervisningsopplegg og få besøk av forskere fra prosjektet, men data om deg vil bare samles inn til forskningsprosjektet dersom du gir samtykke til det. Hvis du ikke samtykker til å delta i undersøkelsen, vil bare læreren ha tilgang til arbeidene dine, som vanlig, og du vil ikke bli filmet eller bedt om lydopptak. Deltar du i undersøkelsen, kan du når som helst trekke ditt samtykke uten å oppgi noen grunn. Dersom du trekker deg, vil alle opplysninger om deg bli slettet eller anonymisert.

Dersom du har spørsmål om undersøkelsen, ta kontakt med prosjektlederne (se nedenfor) eller snakk med læreren din, som også kjenner og deltar i prosjektet.

### Kontaktinformasjon prosjektledere:

Ellen K. Henriksen og Carl Angell, Fysisk institutt, UiO, Postboks 1048 Blindern, 0316 OSLO  
[e.k.henriksen@fys.uio.no](mailto:e.k.henriksen@fys.uio.no), [carl.angell@fys.uio.no](mailto:carl.angell@fys.uio.no)



## Appendix C

# Compilation of Epigraphs

Alice: “Would you tell me, please, which way I ought to go from here?”

The Cheshire Cat: “That depends a good deal on where you want to get to.”

Alice: “I don’t much care where – ”

The Cheshire Cat: “Then it doesn’t much matter which way you go.”

— *Lewis Carroll, Alice in Wonderland*

(Carroll, 2019, p. 60)

In every field of creative activity, it is one’s taste, together with ability, temperament, and opportunity, that determines one’s style and through it one’s contribution. That taste and style have so much to do with one’s contribution in physics may sound strange at first, since physics is supposed to deal objectively with the physical universe. But the physical universe has structure, and one’s perceptions of this structure, one’s partiality to some of its characteristics and aversion to others, are precisely the elements that make up one’s taste. Thus it is not surprising that taste and style are so important in scientific research, as they are in literature, art, and music.

(Ning, 2005, p. 4)

After several unsuccessful attempts to weld my results together into such a whole, I realized that I should never succeed. (...) my thoughts were soon crippled if I tried to force them on in any single direction against their natural inclination. And this was, of course, connected with the very nature of the investigation. For this compels us to travel over a wide field of thought criss-cross in every direction.

(Wittgenstein, 1997, p. vii)

It should be possible to make Einstein’s view of the Universe as much a part of the intellectual equipment of ordinary people as is that of Newton.

(Durell, 1926, p. vii)

Everybody knows that Einstein did something astonishing, but very few people know exactly what it was. It is generally recognised that he revolutionised our conception of the physical world, but the new conceptions are wrapped up in mathematical technicalities. (...) What is demanded is a change in our imaginative picture of the world - a picture which has been handed down from remote, perhaps pre-human, ancestors, and has been learned by each one of us in early childhood. A change in our imagination is always difficult, especially

## C. Compilation of Epigraphs

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when we are no longer young. The same sort of change was demanded by Copernicus, who taught that the earth is not stationary and the heavens do not revolve about it once a day. To us now there is no difficulty in this idea, because we learned it before our mental habits had become fixed. Einstein's ideas, similarly, will seem easier to generations which grow up with them; but for us a certain effort of imaginative reconstruction is unavoidable.

(Russell, 1925, p. 9)

All science is either physics or stamp collecting.

(Bernal, 1939, p. 9)

Anyone who has ever tried to present a rather abstract scientific subject in a popular manner knows the great difficulties of such an attempt.

(Barnett, 2005, foreword)

Frameworks have a tendency to disappear when they are intuitive and carefully planned, because our attention is on the wonderful fruits of the process.

(Chimero, 2012, p. 91)

To use a magnifying glass is to pay attention, but isn't paying attention already having a magnifying glass? Attention by itself is an enlarging glass.

(Bachelard, 1969, p. 158)

Historically, some of the best minds in the world have addressed themselves to education; for example, Plato, Rousseau, Dewey, Bruner and Illich. But they have addressed education essentially as theorists, even where they have tried to design schools or curricula to implement their ideas. What is different today is that some of the best minds in the world are addressing themselves to education as experimentalists: their goal is to compare different designs to see what affects what. Technology provides us with powerful tools to try out different designs, so that instead of theories of education, we may begin to develop a science of education. But it cannot be an analytic science like physics or psychology; rather it must be a design science more like aeronautics or artificial intelligence. For example, in aeronautics the goal is to elucidate how different designs contribute to lift, drag, manoeuvrability, etc.. Similarly, a design science of education must determine how different designs of learning environments contribute to learning, cooperation, motivation, etc.

(Collins, 1992, p. 158)

Knowledge (...) is an ocean of alternatives channelled and subdivided by an ocean of standards. It forces our mind to make imaginative choices and thus makes it grow. It makes our mind capable of choosing, imagining, criticising.

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(Feyerabend, 1975, p. 3)

Words and chronological time create all these total misunderstandings of what's really going on at the most basic level.

(Foster Wallace, 2014, p. 427)

When the ideas involved in Einstein's work have become familiar, as they will do when they are taught in schools, certain changes in our habits of thought are likely to result, and to have great importance in the long run.

(Russell, 1925, p. 149)

When faced with a totally new situation we tend always to attach ourselves to the objects, to the flavor of the most recent past. We look at the present through a rear-view mirror. We march backwards into the future.

(McLuhan and Fiore, 1967, p. 75)

(...) marching is no way to go into the future. It is too methodical and restricted. The world often subverts our best laid plans, so our road calls for a way to move that is messier, bolder, more responsive. The lightness and joy afforded by creating suggests that we instead dance.

(Chimero, 2012, p.XIII)



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