Compliance Technology and Self-enforcing Agreements*

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Abstract

This paper analyzes a game in which countries repeatedly make emission and technology investment decisions. We derive the best equilibrium, i.e., the Pareto-optimal subgame-perfect equilibrium, when countries are insufficiently patient for folk theorems to be relevant. Relative to the first best, the best equilibrium requires countries to overinvest in technologies that are green, i.e., strategic substitutes for polluting, but to underinvest in adaptation and brown technologies, i.e., strategic complements to polluting. Technological transfers and spillovers might discourage investments but can be necessary to motivate compliance with emissions when countries are heterogeneous.

Keywords: environmental economics, green technology, repeated games, self-enforcing agreements

JEL: D86, F53, H87, Q54.

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1 Introduction

By lowering the relative cost of more environmentally sound technologies, technology policy can increase incentives for countries to comply with international climate obligations.

IPCC (2014:1035)

An international environmental treaty must address two major challenges to succeed. First, in the absence of international enforcement bodies, it must be self-enforcing. That is, countries will comply with the treaty in order to motivate other countries to do so in the future.\(^1\) This motivation, however, may not always be sufficiently strong. For example, for many years it was clear that Canada would not meet its commitments under the Kyoto Protocol and in 2011, it simply withdrew.

The second challenge is to develop new and environmentally friendly technology. The importance of new green technology is recognized in climate treaties, but traditionally they have not quantified the extent to which countries are required to invest in these technologies.\(^2\) Instead, negotiators focus on quantifying emissions or abatements and leave the investment decision to individual countries. Nevertheless, some countries do invest heavily in green technologies. The European Union has set itself the goal that 20 percent of its energy will come from renewable sources by 2020 and 27 percent by 2030. China is an even larger investor in renewable energy and has invested heavily in wind energy and solar technology.\(^3\) Other countries have instead invested in so-called “brown” technology: Canada, for example, has developed its capacity to extract oil from unconventional sources, such as tar sands, and it “risks being left behind as green energy takes off” (The Globe and Mail, September 21, 2009).

The interaction between the two challenges is poorly understood by both economists and policy makers. To understand how treaties can address these two challenges and how they are related, a model is needed that allows technology investment decisions and emission decisions to be made repeatedly. Since the treaty must be self-enforcing, strategies must constitute a subgame-perfect equilibrium (SPE).

\(^1\)The need for self-enforcement is recognized by the IPCC (2014:1015): “From a rationalist perspective, compliance will occur if the discounted net benefits from cooperation (including direct climate benefits, co-benefits, reputation, transfers, and other elements) exceed the discounted net benefits of defection.”

\(^2\)Chapter 16 of the Stern Review (2007) identified technology-based schemes as an indispensable strategy for tackling climate change. However, article 114 of the 2010 Cancun Agreement, confirmed in Durban in 2011, states that “technology needs must be nationally determined, based on national circumstance and priorities.” In contrast and as discussed in Section 7, some of the pledges following the 2015 Paris Agreement relate to technology.

\(^3\)For more details on the European Union’s climate and energy policy strategy, see ec.europa.eu/clima/policies/strategies/2030, and for that of China, see thediplomat.com/2014/11/in-new-plan-china-eyes-2020-energy-cap/.
There is no such theory in the literature and therefore many important questions are left unaddressed. First, what characterizes the “best” SPE, i.e., the best self-enforcing treaty? While folk theorems have emphasized that even the first best can be sustained if the players are sufficiently patient, what distortions occur if they are not? How can technologies be used strategically to ensure that the treaty is self-enforcing? Which types of countries ought to invest the most and in what kinds of technologies?

To address these questions, we present a repeated extensive-form game, in which countries can in each period invest in technology before deciding on emission levels. In the simplest version of the model, all decisions are observable and investments are self-investments, i.e., there are no technological spillovers. Consequently, equilibrium investments would have been first best if the countries had committed to the emission levels. The first best can also be achieved if the discount factor is sufficiently high, in line with standard folk theorems. For smaller discount factors, however, the best SPE requires countries to strategically distort their investment decisions in order to reduce the temptation to pollute more rather than less. We show that the distortions take the form of overinvestment in the case of “green” technologies, i.e., renewable energy or abatement technologies that can substitute for pollution. In the case of “brown” technologies, such as drilling technologies and other infrastructure investments that are strategic complements to fossil fuel consumption, investments must instead be less than the first-best amount in order to satisfy the compliance constraint. Our most controversial result states that countries should also be required to invest less than the first-best amount in the case of adaptation technologies, i.e., technology that reduces environmental harm in a country.

The comparative statics offer important policy implications. Of course, it is harder to motivate compliance if the discount factor is low or the environmental harm is on a small scale. This is also true when a small number of countries participate in the agreement, or when investment costs are high in the case of green technology or low in the case of brown or adaptation technologies. In these circumstances, the best SPE requires countries to invest more when the technology is green, and less when it is brown or when it is adaptation technology. If countries are heterogeneous, the countries that are most reluctant to cooperate because, for example, they face less environmental harm, are the most tempted to free ride. Thus, for compliance to be credible, such countries must invest the most in green technologies or the least in adaptation and brown technologies. This advice contrasts with the typical presumption that reluctant countries should be allowed to contribute less in order to satisfy their participation constraint. While incentives to participate require that a country’s net gain from cooperating be positive, incentives to comply with emissions also require that this net gain outweigh the positive benefit of free riding for one period, before the defection is observed. The compliance constraint at the
emission stage is therefore harder to satisfy than the participation constraint is.

Simplicity and tractability are two advantages of our baseline model. Our main results are derived in a pedagogical way with binary emission levels, while ignoring technological spillovers and investments in technology portfolios. However, when the model is extended to take into account these complicating factors, we obtain a deeper understanding of the interplay between agreements and technology. We show that technological spillovers make it harder to design self-enforcing treaties if countries are similar; however, spillovers are necessary to facilitate technology transfers if countries are heterogeneous. Our insight also extends to the situation in which a country can invest in a portfolio of different types of technologies. When green and brown technologies are strongly complements, we show that a complete shift to an economy based on green technology is unwarranted, even under the best climate agreement. Our working paper, Harstad et al. (2018), shows that when emissions are difficult to monitor, strategic investments in technologies can reduce the punishment or the risk that punishments are triggered by mistake, while still ensuring that countries are motivated to comply. That paper shows that our results also hold with continuous emission levels and if national governments regulate firms’ emissions and technology investments through taxes and subsidies. For this case, we show that optimal environmental regulation includes both emission taxes and investment subsidies if but only if the discount factor is small.

**Literature.** Our paper fills a gap between the literature on environmental economics and that on repeated games. As mentioned, it is widely accepted that international agreements must be self-enforcing.\(^4\) Thus, we draw heavily on the repeated games literature, although much of this literature has been concerned with folk theorems and conditions under which the first best can be sustained if only the players are sufficiently patient (see, e.g., Ivaldi et al., 2003; Mailath and Samuelson, 2006). In the context of international agreements, however, such a large discount factor is unrealistic and the gains from cooperation may depend on various national policies. We therefore extend the standard repeated prisoner’s dilemma game in two main respects: (i) we allow players to invest in technologies in each period, and (ii) we investigate the second-best equilibrium when the discount factor is so small that the folk theorem does not hold.\(^5\)

Our paper is not the first that studies self-enforcing environmental agreements. In previous papers, such as Barrett (1994; 2005) and Dutta and Radner (2004; 2006), tech-\(^4\)As Downs and Jones (2002:S95) observed, “a growing number of international relations theorists and international lawyers have begun to argue that states’ reputational concerns are actually the principal mechanism for maintaining a high level of treaty compliance.”

\(^5\)Note that neither of the two extensions would be interesting on its own, since with high discount factors, the folk theorem always holds, even in a model with technology. Without technology and with small discount factors, voluntary cooperation cannot be enforced in the repeated public good game.
nology investments are either not permitted or chosen as a corner solution at the beginning of the game. Harstad (2018a) shows how the bargaining game and the size of the emission cuts influence whether the agreement is self-enforcing. The contribution of this paper is to emphasize exactly how technological investments should (and will) be taken advantage of in the best self-enforcing agreement.

There is an emerging literature that examines the relationship between technology investments and international environmental cooperation. Most of it focuses either on the harmful effects of technology investments on a country’s bargaining position in the future, when new commitments are to be negotiated (see, e.g., Buchholz and Konrad, 1994; Beccherle and Tirole, 2011; Harstad, 2012, 2016; Helm and Schmidt, 2015), or on a country’s incentive to invest in the presence of positive international externalities (see, e.g., Barrett, 2006; de Coninck et al., 2008; Golombek and Hoel, 2005; Hoel and de Zeeuw, 2010). Our contribution to this literature is to stress how technology influences a country’s incentives to comply with emission abatements.

The structure of our model is related to the one of Harstad (2012; 2016) and Battaglini and Harstad (2016), where countries pollute and invest in green technologies in every period. These papers, however, assume contractible emission levels and study Markov-perfect equilibria, while we focus on self-enforcing agreements and subgame-perfect equilibria. This approach leads to a new strategic effect of technology—namely that technology should be chosen so as to make future cooperation credible.6

Theoretically, the paper is related to the industrial organization literature, in which strategic investments can deter entry (see, e.g., Spence, 1977; Dixit, 1980; Fudenberg and Tirole, 1984) or reduce production costs and therefore improve the competitive position vis-à-vis rivals (see, e.g., Brander and Spencer, 1983; Spence, 1984; d’Aspremont and Jacquemin, 1988; Leahy and Neary, 1997).7 These papers have, however, focused on static models and have ignored the influence of investments on the sustainability of cooperation.8

More closely related is the literature on the influence of capacity constraints on the sustainability of tacit collusion. In examining this question, Brock and Scheinkman (1985)

6Our investigation of bottom-up cooperation also complements the mechanism-design approach by, for example, Martimort and Sand-Zantman (2016).
7Papers on investment as entry deterrence show that incumbent firms may use strategic investment as a credible threat, since it modifies the incumbent’s ex post reaction function. Papers on cost-reducing R&D show that firms can invest strategically in R&D before the associated output is produced, if they anticipate that a lower marginal cost leads to a higher market share.
8Martin (1995) and Cabral (2000) contributed to the analysis of the role of strategic investment, by considering an infinite-period duopoly industry in which firms make R&D decisions as well as product market decisions. Both papers showed that R&D investments may encourage firms to tacitly collude on output, resulting in a welfare loss. However, the mechanism by which collusion is sustained occurs is very different from our mechanism, since Martin (1995) assumes that firms commit themselves to the joint profit-maximizing level of R&D, while Cabral (2000) assumes that R&D investments are hidden and therefore cannot be part of the agreement.
treated the capacity constraints as exogenous, while Benoit and Krishna (1987) allowed firms to collude on capacity investments as well as on price. When capacity investments are irreversible, firms overinvest in order to make retaliation harsh and credible; but this effect vanishes when investments are reversible, since firms can always adjust the retaliation capacity later. Our mechanism differs in that overinvestment in green technology or underinvestment in adaptation and brown technologies is necessary along the equilibrium path in order to undermine the short-run gain from deviation in the cooperative phase. This result holds even when investment decisions are fully reversible and is reinforced when they are not.

While similar mechanisms have fruitfully been applied in the relational contracting literature, we are the first to investigate the influence of investments on the sustainability of environmental agreements. The paper contributes to the more applied theory literature by, for example, predicting which types of players will invest in which types of technologies and by showing that technology spillovers can be beneficial or harmful for the sustainability of an agreement—depending on whether the players are similar or different.

The paper is organized as follows. The baseline model is presented in Section 2 and analyzed in Section 3. To shed further light on optimal climate change policy, we then allow for technological spillovers and transfers (Section 4), investments in more than one type of technologies (Section 5), and finally we discuss how the model can be reformulated to account for the accumulation of pollution and technology (Section 6). Section 7 concludes and Appendix contains all proofs.

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9 While Davidson and Deneckere (1990) do not allow firms to collude in capacity, they do allow them to collude on price. Like Benoit and Krishna (1987), they also show that excess capacity is present in all equilibria. The impact of asymmetry in capacity on self-enforcing collusion is instead analyzed by Lambson (1994) and Compte et al. (2002), who investigate how asymmetry in capacity influences whether collusion is self-enforcing. They conclude that, depending on parameters, asymmetric capacities may either encourage or discourage collusion.

10 The idea that technology investments can ex post relax the compliance constraint on individual contributions to a public good is also present in the relational contracting literature (see, e.g., Ramey and Watson, 1997; Halac, 2015). However, these papers study the impact of up-front investment by one party on the value of a relation between two parties and focus on the harmful effects of the holdup problem. Our model differs from theirs in that all countries invest and the investments are repeated. Repeated maintenance investments in the public good are allowed in Halonen and Pafilis (2018). However, in that paper, it is the ownership structure that is chosen to mitigate the temptation to free ride on individual contributions.

11 Building on our work, Kerr et al. (2018) study how the timing of transfers can facilitate compliance in a dynamic climate change game. Harstad (2018b) examines how green/brown technologies can be used as commitment devices for hyperbolic decision makers. Lancia and Russo (2018) study how agents exert effort strategically in order to signal their willingness to cooperate in a stochastic overlapping-generations model.
2 A Model of Compliance Technology

The model we construct is motivated by global environmental problems such as climate change. Since no world government can force countries to cooperate in solving such problems, the temptation to free ride must be mitigated. The possibility of free riding is a result of the fact that if a country increases its emissions, other countries will not retaliate immediately because, for example, emissions are observed with a lag. To capture this lag, we let time \( t \in \{1, \ldots, \infty\} \) be discrete and \( \delta \in (0, 1) \) be the common discount factor between periods.

Analogously, there is also a lag between the decision to invest in a technology and the point at which it begins to contribute to consumption. This lag leads us to use an extensive-form stage game, in which each country invests in technology before deciding on how much to consume or pollute. Furthermore, the infinite time horizon relevant for climate change implies that it is unrealistic to assume that a country can invest in the capacity to produce renewable energy once and for all, without later having to invest in maintenance. To capture this effect, we start out by assuming that technology fully depreciates, so that countries must invest in every single period. We also at first abstract from technological spillovers since, in contrast to environmental externalities, technological spillovers may be relatively small when the technology is a country’s capacity to produce renewable energy.

There are \( n \geq 2 \) players in the game, indexed by \( i \) or \( j \in N \equiv \{1, \ldots, n\} \). In each stage game, there is an emission stage in which countries simultaneously make a binary decision \( g_i \in \{g, \overline{g}\} \) between emitting less, i.e., \( g_i = g \), or more, i.e., \( g_i = \overline{g} > g \). Whenever it is not confusing, we omit the subscripts denoting time.

Let the benefit \( b_i(g_i, r_i) \) be an increasing function of country \( i \)'s emissions \( g_i \). The variable \( r_i \in \mathbb{R}_+ \) is meant to capture the fact that a country’s benefit and its environmental cost depend on the country’s technology, although \( r_i \) can in fact be any variable that influences the benefit and cost of emissions. The environmental cost from global emissions is \( h_i c(r_i) \sum_{j \in N} g_j \), where parameter \( h_i \) measures country-specific environmental harm.\(^{12}\) For simplicity, we assume that \( b_i(g_i, r_i) \) is increasing and concave in \( r_i \) and \( c(r_i) \) is decreasing and convex in \( r_i \). We also assume that the game at the emission stage is a prisoner’s dilemma, irrespective of the level of \( r_i \), as follows:

\(^{12}\) The assumption that the environmental cost is linear in emissions is common and relatively reasonable. As explained by Golosov et al. (2014:78): “Linearity is arguably not too extreme a simplification, since the composition of a concave \( S \)-to-temperature mapping with a convex temperature-to-damage function may be close to linear.” They also write (p. 67): “The composition implied by Nordhaus’s formulation is first concave, then convex; our function is approximately linear over this range. Overall, the two curves are quite close.”
**Assumption 1** For each $i \in N$ and $r_i \in \mathbb{R}_+$,

(i) $b_i(g, r_i) - h_i c(r_i) g < b_i(\bar{g}, r_i) - h_i c(r_i) \bar{g};$

(ii) $b_i(g, r_i) - h_i c(r_i) n g > b_i(\bar{g}, r_i) - h_i c(r_i) n \bar{g}.$

In words, country $i$ benefits from emitting more for any fixed emission from other countries, but every country would be better off if everyone emitted less. Hereafter, and unless otherwise specified, we use subscripts to denote derivatives. Moreover, we abuse the notation by defining $b''_{i,gr}(r_i) \equiv (b'_{i,r}(\bar{g}, r_i) - b'_{i,r}(g, r_i)) / (\bar{g} - g)$, which captures how the benefit of emitting more rather than less varies with the level of technology.

To illustrate the relevance of technology, we will occasionally refer to the following special types:

**Definition 1** For each $r_i \in \mathbb{R}_+$,

(A) Adaptation technology is characterized by $b''_{i,gr}(r_i) = 0$ and $c'_r(r_i) < 0$;

(B) Brown technology is characterized by $b''_{i,gr}(r_i) > 0$ and $c'_r(r_i) = 0$;

(C) Clean technology is characterized by $b''_{i,gr}(r_i) < 0$ and $c'_r(r_i) = 0$.

An adaptation technology is one that enables a country to adapt to a warmer or more volatile climate. Such technologies include agricultural reforms or more robust infrastructure and may in addition capture the effects of some geo-engineering practices that have strictly local effects. Adaptation technology is therefore complementary to polluting, since it reduces the environmental cost of emissions, i.e., $c'_r(r_i) < 0$. Brown technology can be interpreted as drilling technology, infrastructure that is beneficial in the extraction or consumption of fossil fuel, or some other technology that is complementary to fossil fuel consumption. The complementarity is captured by $b''_{i,gr}(r_i) > 0$. In fact, most investments made in polluting industries are brown, according to our definition. Clean technology, in contrast, is a strategic substitute for fossil fuel and reduces the marginal value of emitting another unit of pollution. This is the case for abatement technology or renewable energy sources, for example. Thus, $b''_{i,gr}(r_i) < 0$ for clean technologies. Of course, both brown and clean technologies may be beneficial in that $b'_{i,r}(g_i, r_i) > 0$.

We endogenize the technology level by permitting an investment stage, in each period, during which countries simultaneously and non-cooperatively decide on investment, before they decide on whether to emit less or more. As already noted, the sequential timing follows directly from the fact that there is a minimum length of time $l \in (0, 1)$ between the investment decision and the time at which the technology is operational. The lag implies
that if the actual marginal investment cost is, say, $\hat{k}_i > 0$, then its present discounted value, evaluated at the time of the emission, is $k_i \equiv \delta \hat{k}_i$. With this reformulation, we do not need to explicitly discount between the two stages within the same period. Note that assuming a linear investment cost is without loss of generality, since $r_i$ can enter a country’s benefit function in arbitrary ways.\(^{13}\) Country $i$’s per-period utility can then be written as:

$$u_i = b_i (g_i, r_i) - h_i c (r_i) \sum_{j \in N} g_j - k_i r_i.$$ 

**Benchmarks.** Before analyzing self-enforcing agreements, we examine two polar cases in which emissions and investments are chosen at every decision stage either non-cooperatively by each individual country or by a planner with full enforcement power.

Consider first non-cooperative investments. Suppose that each country is expected to pollute at the same level, that is, $g_i = g$ for each $i$. For every $g$, country $i$’s optimal investment level $r_i (g)$ is obtained by solving the following first-order condition:

$$b'_{i,r} (g, r_i) - h_i c'_{r} (r_i) n g - k_i = 0,$$ 

(1)

while the second-order condition holds trivially.

At the emission stage, Assumption 1 implies that $g_i = \overline{g}$ is a dominant strategy for every country. Thus, there is a unique subgame perfect equilibrium (SPE) of the stage game, that is, $(g_i, r_i) = (\overline{g}, r_i (\overline{g}))$. Using terminology from the literature on environmental agreements, we refer to this equilibrium as the business-as-usual (BAU) equilibrium and label it with the superscript $bau$. Note that BAU also coincides with the worst SPE, that is, the min-max payoff of the stage game, since every country is always guaranteed at least that utility level, i.e., $u_i^{bau} \equiv b_i (\overline{g}, r_i^{bau}) - h_i c (r_i^{bau}) n \overline{g} - k_i r_i^{bau}$ with $r_i^{bau} \equiv r_i (\overline{g})$.

The first-best outcome is characterized by $(g_i, r_i) = (g, r_i (g))$ for each $i$ and coincides with the case in which a benevolent planner makes all its decisions in order to maximize the sum of countries’ utilities. It follows that the first-best level of utility is $u_i^* \equiv b_i (g, r_i^*) - h_i c (r_i^*) n g - k_i r_i^* > u_i^{bau}$ with $r_i^* \equiv r_i (g)$. Since the first-best investment level also follows from condition (1), we can state the following preliminary result:

**Proposition 0** If all countries commit to the emission level $g_i = g$, equilibrium investments levels are first best (i.e., $r_i^*$).

Proposition 0 provides support for the presumption that it is not necessary to negotiate investments in addition to negotiating emissions. Under a commitment to $g_i = \overline{g}$, each

\(^{13}\)If the investment cost were a different function $\kappa_i (r_i)$, we could simply define $\tilde{b}_i (g_i, \kappa_i (r_i)) \equiv b_i (g_i, r_i)$ and $\tilde{c} (\kappa_i (r_i)) \equiv c (r_i)$, treat $\kappa_i (r_i)$ as the decision variable, and then proceed as we do in the paper.
country’s investment would be socially optimal and the first best would be sustainable
as an SPE. In what follows, we consider the more realistic scenario in which countries
cannot commit to low emission levels.

3 Self-enforcing Agreements

When actions are observable, an international environmental agreement can specify every
country’s levels of emission and investment at every point in time. For such an agreement
to be self-enforcing, the decisions must constitute an SPE. As in many dynamic games
with an infinite time horizon, there are multiple SPEs. When countries can communicate
and negotiate at the outset, it may be reasonable to assume that they will coordinate on
a Pareto-optimal SPE. Since the game is a prisoner’s dilemma at the emission stage, we
are especially interested in SPEs in which \( n \) countries emit less on the equilibrium path,
i.e., in which \( g_{i,t} = q \) for each \( i \in N \) and any \( t \geq 1 \).

Note that we do not require that all countries “in the world” emit less. Rather,
we can let \( N \) refer to the set of countries emitting less under the agreement. If there
exist other countries that always emit more, they will be irrelevant to the game and the
equilibrium subsequently analyzed, since the emissions of these other countries are not
payoff relevant when the environmental harm is linear in the sum of emissions. When
there is a unique Pareto-optimal SPE outcome among the \( n \) countries emitting less, we
refer to an equilibrium that supports it as a best equilibrium.

**Definition 2** An equilibrium is referred to as “best” if and only if it supports the unique
Pareto-optimal SPE outcome involving \( g_{i,t} = q \ \forall i \in N \) and \( t \geq 1 \) on the equilibrium path.

The best equilibrium must also specify the consequences if a country fails to emit less.
Since this never occurs on the equilibrium path, there is no loss in assuming that the
countries would respond by playing the worst SPE, i.e., BAU, forever. The observation
that punishments are never observed in equilibrium also implies that, in a setting with a
common discount factor, the best equilibrium outcome must be stationary, i.e., it supports
\( r_{i,t} = r_i \) for every \( t \geq 1 \) (Abreu, 1988). Therefore, we can omit the \( t \) subscripts for brevity.
The normalized (to one period) continuation value when complying with the best SPE is
\( u_i(r_i) \equiv b_i(g, r_i) - h_i c(r_i) n g - k_i r_i \).

Deviations can occur during either the investment stage or the emission stage. At
the investment stage, a country will compare the continuation value it receives from
complying with the SPE by investing in the \( r_i \) with the maximal continuation value it can
obtain by deviating. Since deviating at the investment stage implies that every country
will emit more starting from that period, the compliance constraint at the investment stage is as follows:

\[ \frac{u_i(r_i)}{1 - \delta} \geq \max_{r_i} b_i(\bar{g}, r_i) - h_i c (r_i) n\bar{g} - k_i r_i + \frac{\delta u_{i,bau}}{1 - \delta}. \] (CC\textsubscript{r\textsubscript{i}})

The right-hand side of constraint (CC\textsubscript{r\textsubscript{i}}) is maximized when \( r_i = r_{i,bau} \), implying that the compliance constraint at the investment stage simplifies to \( u_i(r_i) \geq u_{i,bau} \), which actually coincides with the participation constraint. If a country deviates at the investment stage, the penalty is imposed before the country can benefit from free riding on emissions. Thus, the temptation to free ride at the investment stage is weak since a country does not care about other countries’ investment levels per se, but only about its own emission levels.

At the emission stage, the investment cost in the current period is sunk and the compliance constraint becomes:

\[ \frac{u_i(r_i)}{1 - \delta} \geq b_i(\bar{g}, r_i) - h_i c (r_i) (\bar{g} + (n - 1) \bar{g}) - k_i r_i + \frac{\delta u_{i,bau}}{1 - \delta}. \] (CC\textsubscript{g\textsubscript{i}})

As \( \delta \) tends to one, (CC\textsubscript{g\textsubscript{i}}) approaches (CC\textsubscript{r\textsubscript{i}}). For any \( \delta < 1 \), however, (CC\textsubscript{g\textsubscript{i}}) is harder to satisfy than (CC\textsubscript{r\textsubscript{i}}) because of the free-riding incentive at the emission stage. It is not sufficient that the best equilibrium be better than BAU. In addition, the discount factor must be large or the temptation to free ride on emissions must be small. For notational convenience, we rewrite constraint (CC\textsubscript{g\textsubscript{i}}) as follows:

\[ \Delta_i (r_i, \delta) \equiv u_i(r_i) - u_{i,bau} - \frac{1 - \delta}{\delta} (\bar{g} - g) \psi_i(r_i) \geq 0, \] where

\[ \psi_i(r_i) \equiv \frac{b_i(\bar{g}, r_i) - b_i(g, r_i)}{\bar{g} - g} - h_i c (r_i) \]

relates to the one-period benefit from free riding on emissions, which is positive according to Assumption 1. For every \( i \), the equation \( \Delta_i (r_i, \delta) = 0 \) identifies a threshold discount factor \( \delta_i (r_i) \) that depends on the level \( r_i \). Let \( \overline{\delta}_i \) be defined as the level of \( \delta \) that solves \( \Delta_i (r_i^*, \delta) = 0 \). It follows that, if \( \delta \geq \max_i \delta_i \), every (CC\textsubscript{g\textsubscript{i}}) holds (even) for \( r_i = r_i^* \) and the best equilibrium is simply the first best. There is also a lower bound on the discount factor, denoted by \( \underline{\delta}_i \), such that if \( \delta < \underline{\delta}_i \), there is no \( r_i \) that satisfies both (CC\textsubscript{g\textsubscript{i}}) and (CC\textsubscript{r\textsubscript{i}}). In this case, there does not exist any \( r_i \) such that country \( i \) will emit less. When \( \delta \in [\underline{\delta}_i, \overline{\delta}_i] \), with \( \overline{\delta} \equiv \max_i \delta_i \), country \( i \) is willing to participate in the climate agreement, but compliance with less emissions is not satisfied if \( r_i = r_i^* \). To ensure that the compliance constraint at the emission stage is satisfied, the temptation to free ride must be reduced by ensuring that \( r_i \) is such that \( \delta_i (r_i) \leq \delta \). This requires that \( r_i > r_i^* \) if
\( \delta'_{i,r}(r_i^*) < 0 \), and \( r_i < r_i^* \) if \( \delta'_{i,r}(r_i^*) > 0 \). It is straightforward to verify that:

\[
\begin{align*}
\delta'_{i,r}(r_i^*) < 0 & \quad \text{if } b''_{i,gr}(r_i^*) < h_i c'_r(r_i^*) ; \\
\delta'_{i,r}(r_i^*) > 0 & \quad \text{if } b''_{i,gr}(r_i^*) > h_i c'_r(r_i^*) .
\end{align*}
\]

Condition \((G_i)\) stands for “green” technology and implies that making more investments relaxes the compliance constraint at the emission stage by reducing the threshold \( \delta_i(r_i) \). Clearly, this condition is satisfied in, for example, the case of clean technology as defined in Definition 1, since additional investment reduces the gain from emitting more rather than less. Condition \((NG_i)\) stands for “non-green” technologies and implies that making less investments relaxes the compliance constraint. Adaptation and brown technologies are special cases in which this condition holds. For these types of technologies, the benefit of emitting more is reduced if there is less investment in technology. When the benefit of emitting more is reduced, the compliance constraint \((CC_i^g)\) is relaxed and is satisfied for a larger set of discount factors. Since the results will depend on these two conditions, we henceforth will relate to green and non-green technologies, while occasionally discussing the relevant implications of the results for the specific types of technologies described in Definition 1.

Let \( r_i(\delta) \) be the level of \( r_i \) that maximizes \( u_i(r_i) \) subject to \( \Delta_i(r_i,\delta) \geq 0 \). The following proposition specifies the conditions under which the best equilibrium exists and characterizes the optimal distortion of the investment in technology from the first-best level.

**Proposition 1** There exists a best equilibrium if and only if \( \delta \geq \delta_i \). For each \( i \in N \), it supports \( r_i = r_i^* \) when \( \delta \geq \delta_i \). Otherwise,

\( (i) \) \( r_i = r_i(\delta) > r_i^* \) if technology is green;

\( (ii) \) \( r_i = r_i(\delta) < r_i^* \) if technology is non-green.

Furthermore, \( |r_i(\delta) - r_i^*| \) is decreasing in \( \delta \).

The result that the first best is achievable when the discount factor is sufficiently large is standard in the literature on repeated games.\(^{14}\) Thus, the contribution of Proposition 1 is to characterize the distortions that must occur if the discount factor is small. When the discount factor is so small that the first best cannot be achieved, countries are motivated to comply with an agreement and emit less only if they have previously invested more if

---

\(^{14}\)Rubinstein and Wolinsky (1995) show that Fudenberg and Maskin (1986)’s folk theorem can be generalized to repeated extensive-form games in order to account for subgame perfection within periods.
technology is green or less if technology is non-green. Investment levels are required to increasingly differ from the first-best level when $\delta$ declines from the level $\delta_i$ in order to reduce the temptation to deviate from the equilibrium.\textsuperscript{15}

For the special types of technologies described in Definition 1, the following result holds:

**Corollary 1** In the best equilibrium and relative to the first best, countries will:

(A) Underinvest in the case of adaptation technology;

(B) Underinvest in the case of brown technology;

(C) Overinvest in the case of clean technology.

### 3.1 Comparative Statics

The compliance constraints are not functions of only technology, but also depend on other parameters of the model. In this section, we consider the effect on investments in each type of technology of a change in these parameters. Compliance is particularly difficult to motivate if the cost of reverting to BAU is small, which holds true when there are few countries, i.e., when $n$ is small, or when the environmental harm is small, i.e., when $h_i$ is small. To satisfy the compliance constraint in these situations, it is necessary that country $i$ invest more in clean technology, and less in brown technology or adaptation technology. The comparative statics are summarized in the following proposition:

**Proposition 2** Suppose $\delta \in [\delta, \delta_i)$ and consider the best equilibrium:

(i) If $h_i$ or $n$ increases, $r_i$ decreases in the case of clean technology, increases in the case of brown technology, and, provided that $c(r_i) > (c'_r(r_i))^2 / c''_{rr}(r_i)$, increases in the case of adaptation technology;\textsuperscript{16}

(ii) If $k_i$ increases, $r_i$ increases regardless of the type of technology.

\textsuperscript{15}Note that it is not necessary to require that investment be sufficiently small or sufficiently large that emitting less becomes a dominant strategy; it is sufficient to ensure that the benefit of emitting more be smaller (though still positive) than the present discounted value of continuing cooperation. Requiring countries to invest at a level that is inefficient, conditional on the emission levels, must be part of the self-enforcing agreement, in the same way that low emission levels are, namely any deviation leads to BAU forever.

\textsuperscript{16}If this condition is violated, investing in adaptation technology is so productive that if $n$ or $h_i$ increases, country $i$’s environmental cost $h_i c(r_i) n g$ actually declines when the changes induce the country to invest more in adaptation technology, which seems unrealistic.
A surprising result is that investment in any type of technology will increase with the cost of investment \( k_i \). To see this, recall that \( r_i < r_i^{bau} \) for adaptation and brown technologies. For those technologies, a larger \( k_i \) reduces the value of BAU compared to the value of cooperating, i.e., \( u_i(r_i) - u_i^{bau} \) increases, and makes the compliance constraint easier to satisfy at the emission stage. Thus, when \( k_i \) increases, \( r_i \) can increase toward \( r_i^* \) without violating (CC\(_g^i\)). For clean technology, we have \( r_i > r_i^{bau} \), so that a larger \( k_i \) reduces the value of cooperating relative to the value of BAU. In that case, the compliance constraint becomes harder to satisfy when \( k_i \) increases and country \( i \) must invest even more to satisfy (CC\(_g^i\)).

Since countries are heterogeneous, the comparative statics are country specific. We can therefore differentiate between countries that are the most reluctant to cooperate from those that are the least. If country \( i \) has a lower level of environmental harm than country \( j \), or has a higher investment cost in the case of clean technology or has a smaller investment cost in the case of brown or adaptation technology, then \( \delta_i > \delta_j \), and we can say that \( i \) is more reluctant than \( j \). Since the most reluctant countries are tempted to emit more, it is more likely that their compliance constraints bind, i.e., \( \delta < \delta_i \), and that they must invest strategically to make compliance credible.

The result that countries which benefit less from cooperation ought to make greater sacrifices is in stark contrast to the idea that countries should contribute according to their ability and their responsibility for pollution and that they must be given a better deal to motivate cooperation. It is true, of course, that a reluctant country has a participation constraint, i.e., \( u_i(r) \geq u_i^{bau} \), which is more difficult to satisfy than are the constraints for other countries. However, as already shown, the compliance constraint (CC\(_g^i\)) is more difficult to satisfy than the participation constraint (CC\(_r^i\)). Although each country’s benefit from cooperating, relative to BAU, must certainly be positive, it must also be larger than the benefit from free riding for one period, before the deviation is detected.

### 3.2 Policy-relevant Extensions

The baseline model relies on a number of strong assumptions. While they have allowed us to present key results in a pedagogical way, the following sections make the model more realistic and policy relevant. The extensions make it possible to investigate the robustness of the results and also to obtain a deeper understanding of the relationship between technology and compliance. The reader is free to jump directly to the extension of interest, since they are independent and each is based on the baseline model.

While the baseline model considered only self-investments, Section 4 introduces technological spillovers and shows when they motivate compliance by facilitating technological
transfers from, for example, the North to the South. Section 5 allows countries to invest in a technology portfolio and shows how the elasticity of substitution between clean and brown technologies influences investment distortions. Section 6 discusses whether the results continue to hold when pollution or technology levels are permitted to accumulate over time.

4 Technological Spillovers

Cooperation on environmental policies may be plagued by free-riding problems arising from two types of externalities. The first is the environmental harm emphasized in the baseline model, while the second is technological spillovers, especially when the protection of intellectual property rights (IPRs) is relatively weak. Thus, one country’s investment in technology and R&D benefits other countries through technological trade, diffusion, and learning by doing. The weaker the protection of IPRs, the more other countries can benefit without having to pay, and the smaller will be the fraction of the total value enjoyed by the investing country. It turns out that these spillovers alter the strategic role of technology, and that this role is different if countries are homogenous than if they are not.

Let $e \in (0, 1)$ be the fraction of a country’s investment that benefits the others instead of the investor. A country’s per-period utility can then be written as:

$$ u_i = b_i(g_i, z_i) - h_i c(z_i) \sum_{j \in N} g_j - k_i r_i, \text{ where } z_i \equiv (1 - e) r_i + \frac{e}{n - 1} \sum_{j \neq i} r_j. $$

The term $(1 - e)$ is a normalization and can be removed without affecting the results. The term is natural, however, when a reduction in $e$ should be interpreted as stronger protection of IPRs, since in that case neighboring countries must pay the innovating country when using the new technology. In this context, the first-best investment level $r_i^*$ remains unchanged as $e$ varies, but the BAU investment level is lower when $e$ is small, since the innovating country is then capturing more of the total gain. Thus, it is no longer true that countries invest the efficient amount conditional on emissions. Moreover, if the spillovers are sufficiently large, it may be that $r_i^* > r_i^{bao}$ regardless of the type of technology.

Instead of letting each country decide on the expenditure $r_i$, we find it to be more realistic (and tractable) to assume that each country decides on its technology-level target,

\footnote{If we had $\hat{z}_i = r_i + \frac{e}{n - 1} \sum_{j \neq i} r_j$ instead of $z_i$, we could define $e$ from $e / (1 - e) \equiv \hat{e}$ and $z_i \equiv \hat{z}_i (1 - e)$ in order to write $b_i(g_i, z_i) - h_i c(z_i) \sum_{j \in N} g_j - k r_i$.}
Solving for the \( r_i \)'s in (2), we get
\[
r_i \equiv \frac{1}{n(1-e) - 1} \left[ (n - 1 - e) z_i - e \sum_{j \neq i} z_j \right],
\]
illustrating that \( j \)'s technology reduces \( i \)'s cost of achieving its target, \( z_i \), thanks to the technological spillovers.

Unlike in the baseline model, BAU is no longer the worst SPE, since a country could, in principle, invest less than \( r_i^{bau} \) as a punishment after defection.\(^{18}\) To facilitate comparison of the results to those in Section 3, we continue to focus on the Pareto-optimal SPEs that are enforced by trigger strategies in which defection leads to BAU forever.

### 4.1 Homogenous Countries and Intellectual Property

We start out with a situation in which countries are identical. Furthermore, we restrict our attention to symmetric SPEs in which every investment level is the same, so that a country’s equilibrium utility can be written as
\[
u(z) = b(g, z) - hc(z) n g - k z.
\]
The best equilibrium supports \((g, z_i = z)_{i \in N}\), where \( z \) maximizes \( u(z) \) subject to the compliance constraints. The compliance constraint at the emission stage is similar to the one in the baseline model, that is,
\[
\Delta^g (z, \delta) \equiv u(z) - u^{bau} - \frac{1 - \delta}{\delta} (\bar{g} - g) \psi(z) \geq 0,
\]
where \( u^{bau} \equiv b(\bar{g}, z^{bau}) - hc(z^{bau}) n \bar{g} - k z^{bau} \) and \( \psi(z) \equiv (b(\bar{g}, z) - b(g, z)) / (\bar{g} - g) - hc(z) \). The compliance constraint at the investment stage is:
\[
\Delta^z (z, \delta) \equiv u(z) - u^{bau} - (1 - \delta) \frac{e (n - 1)}{n(1-e) - 1} k (z - z^{bau}) \geq 0.
\]
Condition \((CC^z)\) is trivially satisfied if \( e = 0 \) or if \( z \leq z^{bau} \). When \( e > 0 \) and \( z > z^{bau} \), a country that deviates at the investment stage will not only enjoy its BAU continuation value, but will also benefit from the investments made by the other countries. In that case, countries may be tempted to deviate even at the investment stage. Thus, it is no longer true that it is always harder to motivate less emissions than to motivate investment.

To show this formally, let \( \delta^g (z) \) and \( \delta^z (z) \) identify the thresholds of discount factors associated with the binding constraints \((CC^g)\) and \((CC^z)\). The upper bounds \( \bar{\delta}^g \) and \( \bar{\delta}^z \) are defined as the levels of \( \delta \) that solve \( \Delta^g (z^*, \delta) = 0 \) and \( \Delta^z (z^*, \delta) = 0 \) at the first-best level \( z^* \). Thus, if \( \delta \geq \max \{ \bar{\delta}^g, \bar{\delta}^z \} \), both compliance constraints hold for \( z = z^* \) and the best equilibrium is simply the first best. When \( \delta < \max \{ \bar{\delta}^g, \bar{\delta}^z \} \), investment must be distorted away from its first-best level to ensure compliance with the agreement. Based on a comparison between \((CC^g)\) and \((CC^z)\), it is apparent that when \( e \) is sufficiently

\(^{18}\)Note that it is only when \( e > 0 \) that a reduced \( r_i \) can be used to punish other countries.
large, the compliance constraint at the investment stage is harder to satisfy than the compliance constraint at the emission stage. As we will show in the proof of the following proposition, there exists a threshold level \( \tilde{e} > 0 \) such that \( \tilde{\delta}^z \leq \tilde{\delta}^g \) for \( e \leq \tilde{e} \) and \( \tilde{\delta}^z > \tilde{\delta}^g \) otherwise.

If spillovers are small, i.e., \( e \leq \tilde{e} \), because of, for example, the presence of strong protection of IPRs, constraint \( (\text{CC}_e^g) \) binds first as \( \delta \) becomes smaller and investment distortions will be as described in Proposition 1: there will be overinvestment if technology is green and underinvestment if it is non-green. Formally, let \( z^g(\delta) \) be defined as the \( z \) that maximizes \( u(z) \) subject to \( \Delta^g(z, \delta) \geq 0 \). Analogously to the baseline model, the function \( z^g(\delta) \) is decreasing in \( \delta \) when the technology is green, but increasing when the technology is non-green.

If spillovers are large, i.e., \( e > \tilde{e} \), constraint \( (\text{CC}_e^z) \) binds first. To motivate compliance at the investment stage, the equilibrium investment levels must be lower in order to weaken the temptation to deviate. There must then be underinvestment, whatever the type of technology a country possesses. Formally, let \( z^z(\delta) \) be defined as the \( z \) maximizing \( u(z) \) subject to \( \Delta^z(z, \delta) \geq 0 \). When such a constraint binds, the function \( z^z(\delta) \) increases in \( \delta \) regardless of the technology type because a smaller \( \delta \) increases the gain from free riding on investments when \( z > z^\text{ban} \). Figure 1 provides an illustration of how different levels of technological spillovers affect strategic investments in the case of green technology.

As before, there exists a lower bound \( \underline{\delta}(e) \), equal to the largest \( \delta \), such that if \( \delta < \underline{\delta}(e) \),
then there is no \( z \) that can satisfy all compliance constraints.\(^{19}\)

**Proposition 3** There exists a best equilibrium if and only if \( \delta \geq \hat{\delta}(e) \). For each \( i \in N \), it supports \( z_i = z^* \) when \( \delta \geq \max \{ \delta^g, \delta^z \} \). Otherwise,

(i) if \( e < \tilde{e} \), then \( \delta^g > \delta^z \) and \( z_i = z^g(\delta) > z^* \) when the technology is green and \( z_i = z^z(\delta) < z^* \) when technology is non-green;

(ii) if \( e > \tilde{e} \), then \( \delta^g < \delta^z \) and \( z_i = z^z(\delta) < z^* \) regardless of the type of technology.

Compared to Proposition 1, the qualitative difference is that green investments decline with \( \delta \) if \( e > \tilde{e} \). When countries are homogenous, large spillovers discourage investments, since they impose a constraint on the investment levels that can be sustained as SPEs. Specifically, requiring a high level of investment in green technology to motivate compliance at the emission stage may not be possible if the spillovers are large. Thus, under a policy that reduces the spillover by, for example, strengthening the protection of IPRs, compliance can be motivated by requiring more investment in green technology without concern that the compliance constraint at the investment stage will be violated.

### 4.2 Heterogeneous Countries and Technology Transfers

The Paris Agreement encourages technology transfers to developing countries. Article 10 states that the countries “shall strengthen cooperative action on technology development and transfer.” In addition, “international trade and foreign direct investment are the primary means by which new knowledge and technology are transferred between countries” (IPCC, 2014:1035).

Thus, in terms of the model, technological transfers may require a larger \( e \). This type of technology transfer can be rationalized in our framework. To see this, note that when the critical assumption made about homogenous countries in the previous subsection is relaxed, spillovers may be beneficial to the agreement since the possibility of technology transfers emerges. Intuitively, if the countries with the weakest compliance constraints, i.e., the least reluctant countries, are willing to invest more, then, in the presence of technological spillovers, these investments relax the compliance constraints for other countries.

To show this formally, let \( \hat{\delta}_i(e) \) measure the smallest discount factor at which country \( i \)’s compliance constraints at the emission and investment stages hold if every country invests at the same level, say, \( \bar{z}_i(e) \). Without spillovers, we know that a best equilibrium

\(^{19}\)If \( e = \tilde{e} \), then \( \delta^g = \delta^z = \hat{\delta}(e) \), so that the first best is possible if \( \delta \geq \hat{\delta}(e) \); otherwise no equilibrium supports \( g_i = \bar{g} \) for each \( i \).
exists if and only if $\delta \geq \hat{\delta}(0) \equiv \max_j \hat{\delta}_j(0)$. Let $i = \arg \max_j \hat{\delta}_j(0)$ denote the most reluctant country in the absence of spillovers. We will say that country $j$ is less reluctant than country $i$ if, whenever $i$’s compliance constraints hold, $j$’s compliance constraints are non-binding. This implies that, at $\delta = \hat{\delta}_i(e)$, country $j$ can set any $z_j \in [\hat{z}_i(e), \hat{z}_i(e) + \theta_{j,i}]$ for some $\theta_{j,i} > 0$, without violating its own compliance constraints, even if the other countries specify only $\hat{z}_i(e)$. Since heterogeneity can originate from a variety of sources, $\theta_{j,i}$ is a measure of the degree of heterogeneity between $i$ and $j$, for any given $e$. The highest level of heterogeneity is defined as $\theta \equiv \max_j \theta_{j,i}$.

Let $\hat{\delta}(e)$ be the smallest discount factor at which we can sustain a best equilibrium, i.e., an SPE which involves less emissions by all countries, for some investment levels. With these definitions, we are able to show that spillovers can improve the possibility of sustaining a best equilibrium.

**Proposition 4** For every $e > 0$, we have:

(i) $\hat{\delta}(e) < \hat{\delta}(0)$ if the heterogeneity, $\theta$, is sufficiently large;

(ii) When $\delta \in (\hat{\delta}(e), \hat{\delta}(0))$, some countries will invest more in order to motivate the most reluctant countries to comply.

In other words, if countries are sufficiently heterogeneous, then the set of discount factors that support a best equilibrium can be expanded if the spillover is positive rather than zero. This is because spillovers allow countries taking advantage of the heterogeneity, so that the compliance constraints of the most reluctant country can be weakened by the investments of the less reluctant countries. In this way, technological transfers facilitate compliance.

### 5 Technology Portfolios

We have so far assumed that a country can invest in only one technology at a time, although we have been flexible regarding what type of technology that might be. In reality, however, technologies of different types are simultaneously available and each country invests in a technology portfolio. When the model allows for this, we can ask how different countries will invest in various types of technologies and when motivating compliance requires a transition from one type of technology to another.

This section extends the baseline model by permitting countries to invest in a technology portfolio $r_i \equiv (r^\sigma_i)_{\sigma \in \{A, B, C\}} \in \mathbb{R}_+^3$, which includes all three types of technology appearing in Definition 1. The superscript $\sigma = A$ will denote adaptation technology,
σ = B will denote brown technology, and σ = C will denote clean technology. We allow the cost of investment to vary across technologies. When all countries emit at a low level, country \( i \)'s per-period utility is given by:

\[
u_i (r_i) \equiv b_i (g, r_i^B, r_i^C) - h_i c (r_i^A) ng - \sum_{\sigma \in \{A, B, C\}} k_i^\sigma r_i^\sigma.
\]

Following the earlier notation, we define \( b''_{i,g\sigma} (r_i^B, r_i^C) \equiv (b'_{i,\sigma} (g, r_i^B, r_i^C) - b'_{i,\sigma} (g, r_i^B, r_i^C)) / (g - g) \) for every \( \sigma \in \{B, C\} \), where \( b'_{i,\sigma} (g, r_i^B, r_i^C) \) denotes the derivatives with respect to \( r_i^\sigma \).

Depending on which type of technology has the strongest impact on the benefit from emitting more rather than less, two alternative scenarios emerge, as embodied in the following relations:

\[
\begin{align*}
\frac{(b''_{i,gB}(\cdot))^2}{|b''_{i,BB}(\cdot)|} &\leq \frac{(b''_{i,gC}(\cdot))^2}{|b''_{i,CC}(\cdot)|}, & \text{(C-D)} \\
\frac{(b''_{i,gB}(\cdot))^2}{|b''_{i,BB}(\cdot)|} &> \frac{(b''_{i,gC}(\cdot))^2}{|b''_{i,CC}(\cdot)|}. & \text{(B-D)}
\end{align*}
\]

Condition (C-D) stands for “clean dominance” and holds when the benefit from emitting more is more sensitive to the level of \( r_i^C \) than to the level of \( r_i^B \). This condition is likely to hold in a country which has a specialized or targeted type of clean technology. In contrast, condition (B-D) stands for “brown dominance” and implies that it is the country’s brown technology that is dominant in determining the country’s benefit from polluting. This condition may hold for countries in which fossil-fuel-intensive industries are prevalent.

Since both brown and clean technologies enter the benefit function, they can be interdependent. Thus, they are technological substitutes if \( b''_{i,BC} (g, r_i^B, r_i^C) < 0 \) and complements otherwise. To facilitate exposition, we assume that \( (b''_{i,BC}(\cdot))^2 < b''_{i,BB}(\cdot)b''_{i,CC}(\cdot) \).

To illustrate the results, we will occasionally distinguish between weak and strong complementarity:

**Definition 3** Let \( \eta_i^{C-D}(\cdot) \equiv b''_{i,CC}(\cdot)b''_{i,gb}(\cdot)b''_{i,gC}(\cdot) / b''_{i,gb}(\cdot)b''_{i,gc}(\cdot) \) and \( \eta_i^{B-D}(\cdot) \equiv b''_{i,bb}(\cdot)b''_{i,gb}(\cdot)b''_{i,gC}(\cdot) / b''_{i,gb}(\cdot) \).

Under condition \( i \in \{C-D, B-D\} \), \( r_i^B \) and \( r_i^C \) are:

(i) strong complements if \( b''_{i,BC}(\cdot) > \eta_i(\cdot) \);

(ii) weak complements if \( b''_{i,BC}(\cdot) \in (0, \eta_i(\cdot)] \).

\(^{20}\)This restriction guarantees the existence of an interior solution but it is not essential to obtaining the result.
While all the cases are theoretically possible, each of them may be more or less realistic depending on the importance of specific industries.\textsuperscript{21}

The compliance constraints at the investment and emission stages are equivalent to the earlier constraints (CC\textsubscript{r}i) and (CC\textsubscript{g}i) with \( r_i \) replaced by \( r_1 \). If some of the technologies changed for exogenous reasons, the result would be intuitive: If the compliance constraint is binding, then (a) the level of abatement technology can increase with more clean or less brown technology, (b) the level of brown technology can increase with more clean or less abatement technology, while (c) the level of clean technology must increase if any of the other two technologies are enhanced. (We analyze this situation formally in Harstad et al., 2018).

Consider now the situation in which all investment levels are endogenously determined. Depending on the type of interdependence between the different technologies, the following proposition characterizes, for every country, the distortions of the technology portfolio in the best SPE, relative to the first-best level \( r_i^* \equiv (r_{i\sigma}^*)_{\sigma\in\{A,B,C\}} \). As in the baseline model, the upper bound \( \overline{\delta}_i \) is the level of \( \delta \) that satisfies (CC\textsubscript{g}i) with equality when all technology levels are first best, while the lower bound \( \underline{\delta} \) is the maximal \( \delta \), such that if \( \delta < \underline{\delta} \), there is no investment vector that can satisfy (CC\textsubscript{g}i) and (CC\textsubscript{r}i) for every country.

**Proposition 5** There exists a best equilibrium if and only if \( \delta \geq \overline{\delta}_i \). For each \( i \in N \), it supports \( r_i = r_i^* \) when \( \delta \geq \overline{\delta}_i \). Otherwise, \( r_i^A < r_i^{A*} \) and

1. \( r_i^B < r_i^{B*} \) and \( r_i^C > r_i^{C*} \) when \( r_i^B \) and \( r_i^C \) are substitutes or weak complements;

2. \( r_i^B > r_i^{B*} \) and \( r_i^C > r_i^{C*} \) under \((C-D)_i\) and \( r_i^B < r_i^{B*} \) and \( r_i^C < r_i^{C*} \) under \((B-D)_i\), when \( r_i^B \) and \( r_i^C \) are strong complements.

Proposition 5 generalizes the results of Proposition 1 to an environment in which countries invest in a technology portfolio. When countries are so impatient that distortions to investments are required in order to satisfy compliance with less emissions, there will be underinvestment in adaptation technology. As before, there will also be overinvestment in clean technologies and underinvestment in brown technologies, provided that they are substitutes or weak complements. The novel result obtained when countries invest in multiple technologies can be seen in the case of strong complementarity, i.e., when \( b_{i,BC}^C(\cdot) \) is sufficiently large. In this case, clean and brown technologies will

\textsuperscript{21}For example, one could argue that the different types of technology for the production of electricity (such as clean and brown) are complements. Since it is costly to store electricity produced from solar and wind power, and because electricity production from these sources varies considerably from hour to hour, they must be complemented by traditional sources in order to ensure a constant flow of electricity.
be distorted in the same direction, namely, there will be overinvestment in the case of clean dominance, but underinvestment in the case of brown dominance. Intuitively, in the case that the marginal benefit from polluting depends more on $r_i^B$ than on $r_i^C$, then $r_i^B$ must decrease in order to satisfy a binding compliance constraint, when the discount factor falls. Thus, $r_i^C$ will decrease together with $r_i^B$ in the best equilibrium. However, once clean technology is sufficiently specialized that $(C-D_i)$ holds, then the best way to satisfy the compliance constraint will be to distort $r_i^C$ upwards. In this situation, $r_i^B$ increases together with $r_i^C$ in the best equilibrium. Hence, an important implication of the analysis is that under strong complementarity, a complete shift to an economy based on clean technology is unwarranted, even under the best climate agreement.\footnote{Acemoglu et al. (2016) develop a growth model in which dirty and clean technologies compete in each of many product lines. As in the current paper, they also find that a shift toward clean technology is possible only when the two energy technologies are not complementary.}

The comparative statics described in Section 3.1 can be easily extended to the context of multiple technologies. Of particular relevance is the comparative statics of changes in investment costs. Countries that are more reluctant because they face a higher cost of investment in clean technology should invest more in clean technology and less in brown technology, unless the two technologies are strong complements.

### 6 Technology and Pollution as Stocks

In this section, we reformulate the model to treat technology, as well as pollution, as stocks. Suppose we let $r_{i,t}$ measure $i$’s technology stock at time $t$, where $q_i^r \in [0,1]$ is the fraction of past technology that survives, i.e., that has not depreciated, into the next period, and each unit of investment, $I_{i,t}$, costs $\tilde{k}_i$. Clearly, deciding on $I_{i,t}$ is equivalent to deciding on $r_{i,t}$ once $r_{i,t-1}$ is sunk. One benefit of investing today is that investments can be reduced in the next period. Naturally, we can account for the future cost saving already today:

$$\text{With } r_{i,t} = q_i^r r_{i,t-1} + I_{i,t}, \text{ let } k_i \equiv \tilde{k}_i (1 - \delta q_i^r).$$

be defined as the net cost of adding to the technology stock in period $t$, taking into account the future cost saving. If the $q_i^r$’s were small, the above analysis would remain unchanged since countries would need to invest in every period (even off the equilibrium path) in order to maintain the technology level that is necessary to satisfy the compliance constraint, and the net cost of investing would be equal to $k_i$. Small $q_i^r$’s are reasonable in the very long-run context of climate change, in which countries must expect to invest repeatedly, partly, for example, to maintain the infrastructure and the capacity to produce renewable energy. If the $q_i^r$’s are instead large, then a country cannot easily reduce a clean
technology stock to \( r_t \) after defecting and therefore defecting would be less attractive than assumed above. In this case, an agreement is more likely to be self-enforcing because of this irreversibility.

It is also straightforward to treat pollution as a stock. Suppose \( G_t \) is the pollution stock at time \( t \) and it depreciates at the rate \( q^g \in [0,1] \), and let \( \bar{h}_i \) be environmental harm to country \( i \)'s from each unit of \( G_t \) at each point in time. If \( \bar{h}_i \) is a constant, that is, independent of the technology level, then:

\[
\text{With } G_t = q^G G_{t-1} + \sum_j g_{j,t}, \text{ let } h_i \equiv \bar{h}_i / (1 - \delta q^g)
\]

be defined as the present discounted cost of emitting another unit, evaluated at the time of the emission, while taking into account that it will depreciate only gradually. The present discounted cost \( h_i \) of every unit \( g_{i,t} \) can be accounted for already at time \( t \), allowing us to represent \( i \)'s per-period payoff exactly as above.

Whether the technology or the pollution is a stock, the analysis continues to hold since the stocks are not payoff relevant, that is, they do not influence the marginal cost/benefit when deciding on \( r_{i,t} \) or \( g_{i,t} \). Thus, the stocks affect neither the equilibrium nor the first-best \( r_{i,t} \)'s or \( g_{i,t} \)'s. However, assuming away payoff-relevant stocks may not be satisfactory, since several interesting situations will arise when one country’s addition to a stock influences the incentives of other countries. Future research should take such considerations into account in order to deepen our understanding of how we can design the best self-enforcing agreement.

7 Conclusions and Climate Agreements

This paper presents a repeated prisoner’s dilemma game with endogenous technology. Investments over time in various types of technologies will influence the temptation to defect rather than cooperate. The best subgame-perfect equilibrium takes advantage of this influence by requiring investment levels that reduce the temptation to defect. We also permit technological spillovers and show that they make cooperation harder to sustain if the players are similar but cooperation becomes easier when the players are different.

The assumptions and extensions of the model are motivated by real-world international climate change policies. Countries invest in green and non-green technologies over time, and negotiations enable them to coordinate on the best self-enforcing agreement.

\[23\] Even with a convex investment-cost function for technology, the technology stock may be payoff irrelevant as long as it substitutes for emissions that have linear costs (which is the case in Battaglini and Harstad, 2016, for example).
The analysis provides positive predictions as well as policy recommendations. In order to motivate compliance with a climate treaty, it is necessary to ensure not only that the decisions be repetitive and observable: In addition, countries must invest sufficiently in green technology. The leading climate agreement to date, the relatively unsuccessful Kyoto Protocol of 1997, were not in line with this advice. The Protocol specified emission caps for two subsequent commitment periods and for relatively few (37) countries, without specifying investment targets. As discussed in the Introduction, China and the European Union have nevertheless invested substantially in environmentally friendly technology and, as predicted by our model, the European Union has been complying to a large extent with the Protocol. Other countries, such as Canada, were free to instead invest heavily in brown technology, and eventually withdrew rather than comply.

Relatively successful climate policies are more in line with our policy recommendations. For example, the 2020 Climate & Energy Package adopted by the European Union in December 2008 shares several features of the optimal self-enforcing treaty studied in this paper. First, in addition to setting emission targets, the European agreement also required countries to increase renewable energy sources to at least one fifth of the total energy mix by 2020. Second, it is possible that the effectiveness of the European agreement can be attributed to the sequential nature of investment and emission decisions. While member states were required to submit their national plans to meet the technology investment targets by 2010, they were required to limit their emissions to meet the annual limit starting only from 2013.24 Having installed technologies before the actual emission abatement, member states could then achieve enforcement by conditioning cooperation on prior technology installation, as our theory suggests.

The European Union has political institutions that facilitate enforcement and policy commitments. It thus differs from the 2015 Paris Agreement, which nevertheless vindicates the European approach by requiring countries to submit “their nationally determined contributions [...] at least 9 to 12 months in advance of the relevant meeting of the Conference of the Parties serving as the meeting of the Parties to the Paris Agreement.”25 Many of the countries have pledged technology investment targets for renewable energy, such as, India, China, Indonesia, Brazil, and the European Union, while Canada and the United States have made promises to regulate brown investments.26 The Paris Agreement is also more likely to succeed than the Kyoto Protocol was because it has a larger number of participants, it resembles a repeated game due to its periodic pledge-
and-review mechanism, and it emphasizes transparency by requiring similar reporting methods for all parties, components that are recommended by our theory in order to achieve an optimal and self-enforcing agreement.\textsuperscript{27}

\textsuperscript{27}For a comparison of the Paris Agreement with the Kyoto Protocol, see “The Paris Agreement: A new framework for global climate action,” retrieved from europarl.europa.eu/RegData/etudes/BRIE/2016/573910/EPRS_BRI(2016)573910_EN.pdf.
8 Appendix A

Proof of Proposition 1. Since \(u_{i,t}(r_{i,t})\) is concave and single-peaked in \(r_{i,t}\), the best equilibrium involving \(g_{i,t} = g\) for each \(i \in N\) and every \(t \geq 1\) requires \(r_{i,t}\) to be the closest to \(r_i^*\), subject to compliance constraints at both the investment and the emission stages being satisfied. Since deviations are never observed in equilibrium and the discount factor is common to all countries, the best equilibrium simply requires \(r_{i,t} = r_i\) at every date \(t\). Hence, we can remove \(t\) superscript and solve at any fixed \(\delta\) the following constrained optimization problem:

\[
\max_{r_i} u_i(r_i) \equiv b_i(g, r_i) - h_i c(r_i) n g - k_i r_i \quad \text{s.t.,}
\]
\[
u_i(r_i) \geq u_i^{bau},
\]
\[
\Delta_i(r_i, \delta) \equiv u_i(r_i) - u_i^{bau} - \frac{1 - \delta}{\delta} (\bar{g} - g) \psi_i(r_i) \geq 0,
\]

where \(\psi_i(r_i) \equiv (b_i(\bar{g}, r_i) - b_i(g, r_i))/(\bar{g} - g) - h_i c(r_i)\). Since \(u_i(r_i) \geq u_i^{bau}\) at \(r_i^*\), both constraints hold if \(\delta\) is close to 1. At \(r_i^*\), \(u_i(r_i)\) and condition \((CC_i^g)\) do not change when \(\delta\) falls, but \((CC_i^g)\) will eventually bind because \(\psi_i(r_i) > 0\) under Assumption 1. For each \(i\), a threshold \(\bar{\delta}_i\) is implicitly defined as the level of \(\delta\) that solves \(\Delta_i(r_i^*, \delta) = 0\). Thus, if \(\delta \geq \max_i \bar{\delta}_i\), \(r_i = r_i^*\) satisfies conditions \((CC_i^g)\) and \((CC_i^g)\) for all countries. If \(\delta < \bar{\delta}_i\), condition \((CC_i^g)\) is violated at \(r_i^*\). However, compliance with low emissions can be satisfied if \(r_i = r_i(\delta) > r_i^*\) for green technology, where \(r_i(\delta)\) maximizes \(u_i(r_i)\) subject to \(\Delta_i(r_i, \delta) = 0\), since \(\psi_i^{r_i} (r_i) < 0\). The opposite relation holds for non-green technology, i.e., \(r_i = r_i(\delta) < r_i^*\). As \(\delta\) declines further, condition \((CC_i^g)\) is satisfied only if the distortion \(|r_i - r_i^*|\) increases more. For each \(i\), there exists a lower bound \(\bar{\delta}_i \in (0, \delta_i)\), such that if \(\delta < \bar{\delta}_i\), conditions \((CC_i^g)\) and \((CC_i^g)\) cannot be satisfied for any \(r_i\) and no SPE supporting \(g_i = g\) for each \(i\) exists. 

Proof of Proposition 2. Recall that, conditional on \(g\), \(r_i^*\) and \(r_i^{bau}\) are given by the first-order condition (1). Differentiating such a condition w.r.t. \(g\) and \(r_i\), we get:

\[
\frac{dr_i}{dg} = \frac{-b''_i g r_i (g, r_i) + h_i c'_i (r_i) n}{b''_i (g, r_i) - h_i c''_i (r_i) n g},
\]

where the denominator is the second-order condition of \(u_i(r_i)\) w.r.t. \(r_i\), which is negative. Since \(g\) is discrete, we have:

\[
\int_{\bar{g}}^{\bar{g}} \frac{-b''_i g r_i (g, r_i) + h_i c'_i (r_i) n}{b''_i (g, r_i) - h_i c''_i (r_i) n g} dg.
\]
Hence, $r_i^* - r_i^b > 0$ if technology is clean, or negative otherwise. Furthermore, in the case of adaptation technology, Eq. (3) simplifies to $dr_i/dg = -c' (r_i) / (c'' (r_i) g)$ and in turn the term $c (r_i, g) g$ is increasing in $g$ if and only if $c (r_i) > (c' (r_i))^2 / c'' (r_i)$. If $\delta < \delta_i$, the best equilibrium satisfying $g_i = \bar{g}$ for each $i$ requires that $r_i = r_i (\delta)$, so that condition \((CC^g)\) binds. Differentiating $\Delta_i (r_i, \delta) \equiv u_i (r_i) - u_i^{bau} - \frac{1-\delta}{\delta} (\bar{g} - \bar{g}) \psi_i (r_i) = 0$ w.r.t. $r_i$ yields $\Delta_{i,r} = u_i^{bau} (r_i) - \frac{1-\delta}{\delta} (\bar{g} - \bar{g}) \psi_i (r_i)$. For $r_i \approx r_i^*$, we can then state the following results: (i) since $\Delta_{i,n} = -h_i (c (r_i) g - c (r_i^{bau}) \bar{g})$ and $\Delta_{i,h} = -n (c (r_i) g - c (r_i^{bau}) \bar{g}) + \frac{1-\delta}{\delta} (\bar{g} - \bar{g}) c (r_i)$, $dr_i/dn = -\Delta_{i,n}/\Delta_{i,r}$ and $dr_i/dh_i = -\Delta_{i,h}/\Delta_{i,r}$ are negative if technology is clean, and positive otherwise (for the case of adaptation provided that $c (r_i) > (c' (r_i))^2 / c'' (r_i)$); and (ii) since $\Delta_{i,k} = - (r_i - r_i^{bau})$, $dr_i/dk_i = -\Delta_{i,k}/\Delta_{i,r}$ is positive if technology is of any type. ■

**Proof of Proposition 3.** To determine the best equilibrium in the presence of technological spillovers and homogenous countries, we must solve at any fixed $\delta$ the following constrained optimization problem:

$$\max_z u (z) \equiv b (\bar{g}, z) - hc (z) n\bar{g} - kz \text{ s.t.,}$$

$$\Delta^z (z, \delta) \equiv u (z) - u^{bau} - (1 - \delta) \frac{e (n - 1)}{n (1 - e) - 1} k (z - z^{bau}) \geq 0,$$

$$\Delta^g (z, \delta) \equiv u (z) - u^{bau} - \frac{1-\delta}{\delta} (\bar{g} - \bar{g}) \psi (z) \geq 0,$$

where $\psi (z) \equiv (b (\bar{g}, z) - b (\bar{g}, z^{bau})) / (\bar{g} - \bar{g}) - hc (z) - z^{bau}$ is determined from $b' (\bar{g}, z^{bau}) - hc' (z^{bau}) n\bar{g} - (n - 1) e k/(n(1 - e) - 1) = 0$. The thresholds $\delta^z \equiv 1 - ((n (1-e) - 1)/(e (n - 1) k (z^* - z^{bau}))) (u (z^*) - u^{bau})$ and $\delta^g \equiv (\bar{g} - \bar{g}) \psi (z^*) / ((u (z^*) - u^{bau}) + (\bar{g} - \bar{g}) \psi (z^*))$ are equal to the levels of $\delta$ implicitly defined from $\Delta^z (z^*, \delta) = 0$ and $\Delta^g (z^*, \delta) = 0$, respectively. Hence, if $\delta \geq \max \{\delta^g, \delta^z\}$, conditions \((CC^z)\) and \((CC^g)\) are satisfied for $z_i = z^*$ and the best equilibrium is first best. Let $\delta < \min \{\delta^g, \delta^z\}$. Note that constraint \((CC^z)\) can bind first only if $z^* > z^{bau}$. Under this condition, since $d_z^{bau}/de = (k (n - 1)^2 / (n(1 - e) - 1)^2) (b'' (\bar{g}, z) - hc' (z) n\bar{g}) < 0$ and $d_u^{bau}/de = (((n - 1) k) / (n(1 - e) - 1)) (d_z^{bau}/de) < 0$, we have $d\delta^g/de < 0$ and $d\delta^z/de > d\delta^g/de$, which implies that there exists a threshold level $\bar{e} > 0$ implicitly defined from $\delta^z = \delta^g$, such that $\delta^z > (\delta^g < e \leq (\delta^g > \bar{e})$. Let $e \leq \bar{e}$ and $\delta \in [\delta^z, \delta^g]$. Then $z_i = z^g (\delta)$ where $z^g (\delta)$ is the level $z$ that maximizes $u (z)$ subject to $\Delta^g (z, \delta) = 0$. For $z \approx z^*$, $dz^g/d\delta \approx \psi (z) / ((1 - \delta) \psi' (z))$, which implies that $z^g (\delta) > z^*$ if technology is green and $z^g (\delta) < z^*$ otherwise. Let now $e > \bar{e}$ and $\delta \in [\delta^g, \delta^z]$. Then $z_i = z^z (\delta)$ where $z^z (\delta)$ is the level $z$ that maximizes $u (z)$ subject to $\Delta^z (z, \delta) = 0$. For $z \approx z^*$, $dz^z/d\delta \approx z - z^{bau}$, which implies that $z^z (\delta) < z^*$ if technology is of any type. Inspecting constraints \((CC^z)\)
and \((CC^z_i)\), it is easy to see that there exists a lower bound \(\hat{\delta}\) that is the largest level of \(\delta\) such that if \(\delta < \hat{\delta}\), there is no level of \(z\) that can simultaneously satisfy compliance with investments and emissions, i.e., \(g_i = \bar{g}\) for each \(i\) cannot be enforced for any \(z\).

**Proof of Proposition 4.** To determine the best equilibrium in the presence of technological spillovers and heterogeneous countries, we must solve, for any fixed \(\delta\), the following constrained optimization problem:

\[
\max_{z_i, z_{-i}} u_i(z_i, z_{-i}) \equiv b_i(g, z_i) - h_ic(z_i) n g - k_i \frac{(n - 1 - e) z_i - e z_{-i}}{n (1 - e) - 1} \quad \text{s.t.,}
\]

\[
\Delta_i^q(z_i, z_{-i}, \delta) \equiv u_i(z_i, z_{-i}) - u_i^{\text{ba}} - \frac{1 - \delta}{\delta}(\bar{g} - g) \psi_i(z_i) \geq 0,
\]

\[
\Delta_i^e(z_i, z_{-i}, \delta) \equiv u_i(z_i, z_{-i}) - u_i^{\text{ba}} - (1 - \delta) \frac{e}{n (1 - e) - 1} k_i (z_{-i} - z_{-i}^{\text{ba}}) \geq 0,
\]

where \(z_{-i} \equiv \sum_{j \neq i} z_j\) and \(\psi_i(z_i)\) is defined in the proof of Proposition 3. Let \(i\) be the country with the largest \(\hat{\delta}_i\). Suppose \(e = 0\) and \(\delta = \hat{\delta}_i\), and let \(\hat{z}_i(0)\) be the investment level that is satisfying with equality both compliance constraints for \(i\). Then, there is an SPE in which every country invests \(\hat{z}_i(0)\) and all compliance constraints are satisfied. Next, consider the situation in which \(e > 0\). When everyone continues to invest \(\hat{z}_i(0)\), \(u_i(z_i, z_{-i})\) is invariant in \(e\), \(u_i^{\text{ba}}\) decreases in \(e\), and thus both \(\Delta_i^q(z_i, z_{-i}, \delta)\) and \(\Delta_i^e(z_i, z_{-i}, \delta)\) decrease in \(e\). Obviously, the magnitude of these shifts is independent of \(\theta\), where \(\theta\) is defined in the main text. For some \(\theta > 0\), country \(j\) can choose \(z_j = \hat{z}_i(0) + \theta\) and still satisfy \(j\)'s compliance constraints. Thus, consider the SPE in which \(z_j = \hat{z}_i(0) + \theta\), while everyone else invests \(\hat{z}_i(0)\). The larger \(z_j\) benefits \(i\). This is because \(u_i(z_i, z_{-i})\) and \(\Delta_i^q(z_i, z_{-i}, \delta)\) increase by \(\theta k_i e / [n (1 - e) - 1]\), while \(\Delta_i^e(z_i, z_{-i}, \delta)\) increases by \(\theta \delta k_i e / [n (1 - e) - 1]\), according to the formulas above. Consequently, for a sufficiently large \(\theta\), the positive effects on \(\Delta_i^q(z_i, z_{-i}, \delta)\) and \(\Delta_i^e(z_i, z_{-i}, \delta)\) are larger than the direct negative effect following an increase in \(e\). For such a large \(\theta\), when \(z_j\) decreases by \(\theta\), both compliance constraints of the most reluctant country become nonbinding, i.e., \(\hat{\delta}_i\) declines.

**Proof of Proposition 5.** To determine the best equilibrium in the presence of multiple technologies, we must solve at any fixed \(\delta\) the following constrained optimization problem:

\[
\max_{r_i} u_i(r_i) \equiv b_i(g, r_i^B, r_i^C) - h_i c(r_i^A) n g - \sum_{\sigma \in \{A, B, C\}} k_i^\sigma r_i^\sigma \quad \text{s.t.,}
\]

\[
u_i(r_i) \geq u_i^{\text{ba}}, \quad \text{(CC}_m^r)\]

\[
\Delta_i(r_i, \delta) \equiv u_i(r_i) - u_i^{\text{ba}} - \frac{1 - \delta}{\delta}(\bar{g} - g) \psi_i(r_i) \geq 0, \quad \text{(CC}_m^c)\]

where \(\psi_i(r_i) \equiv (b_i(g, r_i^B, r_i^C) - b_i(g, r_i^{B}, r_i^C))/(\bar{g} - g) - h_i c(r_i^A)\). Constraint \(\text{(CC}_m^r)\) neces-
sarily binds at the optimum. Hence, the first-order conditions can be written as:

\[
\begin{align*}
\frac{h_t c_{i,A} (r_i^A)}{h_t c_{i,A} (r_i^A)} (ng + k_i^A) - \frac{b_{i,C} (g, r_i^B, r_i^C) - k_i^C}{b_{i,gC} (r_i^B, r_i^C)} = 0, \\
\frac{b_{i,B} (g, r_i^B, r_i^C) - k_i^B}{b_{i,gB} (r_i^B, r_i^C)} - \frac{b_{i,C} (g, r_i^B, r_i^C) - k_i^C}{b_{i,gC} (r_i^B, r_i^C)} = 0,
\end{align*}
\]

(4) (5)

with the second-order conditions being satisfied for \( u(r_i) \) sufficiently concave. Let \( \delta_i \) be the level of \( \delta \) solving \( \Delta_i (r_i^*, \delta) = 0 \), such that if \( \delta \geq \delta_i \), then \( r_i = r_i^* \). If \( \delta < \delta_i \), condition (CC\(_m\)) is violated at \( r_i^* \), and investments must be distorted from the first-best level to satisfy the compliance constraint on emissions. Using Eqs. (CC\(_m\)), (4), and (5) and differentiating w.r.t. \( r_i^\sigma \) for every \( \sigma \in \{A, B, C\} \) and \( \delta \) for \( r_i \simeq r_i^* \), we obtain:

\[
\begin{align*}
\frac{dr_i^A}{d\delta} &= \frac{c_{i,A} (\cdot)}{n g c_{i,A} (\cdot)} \left( \frac{b_{i,BC} (\cdot) dr_i^B}{b_{i,gC} (\cdot) d\delta} + \frac{b_{i,CC} (\cdot) dr_i^C}{b_{i,gC} (\cdot) d\delta} \right), \\
\frac{dr_i^B}{d\delta} &= \frac{b_{i,CC} (\cdot) b_{i,gB} (\cdot) - b_{i,BC} (\cdot) b_{i,gC} (\cdot)}{d\delta}, \\
\frac{dr_i^C}{d\delta} &= \frac{\psi_i (\cdot) b_{i,BB} (\cdot) dr_i^B}{b_{i,gC} (\cdot) d\delta} + \frac{\psi_i (\cdot) b_{i,BC} (\cdot) b_{i,gB} (\cdot) d\delta}{b_{i,gC} (\cdot) d\delta}.
\end{align*}
\]

Solving the above system of equations w.r.t. \( dr_i^\sigma /d\delta \) for every \( \sigma \in \{A, B, C\} \), yields:

\[
\begin{align*}
\frac{dr_i^A}{d\delta} &= \frac{c_{i,A} (\cdot)}{n g c_{i,A} (\cdot)} \frac{b_{i,CC} (\cdot) b_{i,CC} (\cdot)}{\Pi_i} \left( b_{i,BC} (\cdot) b_{i,BC} (\cdot) - (b_{i,BC} (\cdot))^2 \right), \\
\frac{dr_i^B}{d\delta} &= \frac{b_{i,CC} (\cdot) b_{i,gB} (\cdot) - b_{i,BC} (\cdot) b_{i,gC} (\cdot)}{d\delta}, \\
\frac{dr_i^C}{d\delta} &= \frac{\psi_i (\cdot) b_{i,BB} (\cdot) b_{i,BC} (\cdot) + b_{i,CC} (\cdot) (b_{i,gC} (\cdot))}{d\delta}.
\end{align*}
\]

where

\[
\Pi_i \equiv b_{i,CC} (\cdot) (b_{i,gC} (\cdot))^2 - b_{i,BC} (\cdot) b_{i,gB} (\cdot) b_{i,gC} (\cdot) + b_{i,CC} (\cdot) (b_{i,gB} (\cdot))^2 - \frac{h_i c_{i,A} (\cdot)^2}{c_{i,A} (\cdot)} (b_{i,CC} (\cdot) b_{i,CC} (\cdot) - (b_{i,BC} (\cdot))^2),
\]

which is negative under the assumption \((b_{i,BC} (\cdot))^2 < b_{i,BB} (\cdot) b_{i,CC} (\cdot))\). Hence, if \( \delta < \delta_i \), \( dr_i^A /d\delta > 0 \), which implies that \( r_i^A < r_i^{A*} \). Furthermore, the following cases hold: (i) if \( r_i^B \) and \( r_i^C \) are substitutes, i.e., \( b_{i,BC} (\cdot) \leq 0 \), or weakly complements, i.e., \( b_{i,BC} (\cdot) \leq \eta_i (\cdot) \) for any \( \iota \in \{C-D, B-D\} \), \( dr_i^B /d\delta > 0 \) and \( dr_i^C /d\delta < 0 \), which implies that \( r_i^B < r_i^{B*} \) and \( r_i^C > r_i^{C*} \); (ii) if \( r_i^B \) and \( r_i^C \) are strongly complements under (C-D), i.e., \( b_{i,BC} (\cdot) > \eta_i^{C-D} (\cdot) \),
$dr^B_i/d\delta < 0$ and $dr^C_i/d\delta < 0$, which implies that $r^B_i > r^B_i^*$ and $r^C_i > r^C_i^*$, while under (B-D$_i$), i.e., $u''_{i,BC} (\cdot) > \eta^B_{i-D} (\cdot)$, $dr^B_i/d\delta > 0$ and $dr^C_i/d\delta > 0$, which implies that $r^B_i < r^B_i^*$ and $r^C_i < r^C_i^*$.
References


