

Are Beginning Calculus and Engineering Students Adequately Prepared for Higher Education? An Assessment of Students' Basic Mathematical Knowledge

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Abstract

Are students transitioning from the secondary level to university studies in mathematics and engineering adequately prepared for education at the tertiary level? In this study, we discuss the prior mathematical knowledge and skills demonstrated by Norwegian engineering ($N = 1,537$) and calculus ($N = 626$) university students by using data from a mathematics assessment administered by the Norwegian Mathematical Council. The assessment examines students' conceptual understanding, computation skills and problem solving skills on the basis of the mathematics curriculum of lower secondary education. We found that calculus students significantly outperformed engineering students, but both student groups struggled to solve the test, with the calculus and engineering groups scoring an average of 60% and 46%, respectively. Beginning students who fail to master basic skills, such as solving arithmetic and algebra problems, will most likely face difficulties in their further courses. Although few female students enrol in calculus and engineering programmes compared with male ones and are thus underrepresented, male and female students at the same ability level achieved comparable test scores. Furthermore, students reported high levels of intrinsic and extrinsic motivation, and a positive relationship was observed between intrinsic motivation and achievement.

Keywords: basic mathematical knowledge, calculus students, engineering students, transition from secondary to tertiary education

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Introduction

Universities worldwide express growing concerns over students' preparedness to transition from secondary school, specifically in terms of their prior mathematical knowledge (Clark & Lovric, 2009; Gueudet, Bosch, DiSessa, Kwon, & Verschaffel, 2016; Kaiser & Buchholtz, 2014; Thomas, de Freitas Druck, Huillet, Ju, Nardi, Rasmussen, & Xie, 2015; Treacy & Faulkner, 2015). Scholars labelling the situation a 'mathematics problem' (e.g. Hourigan & O'Donoghue, 2007; Treacy & Faulkner, 2015) address issues caused by students' lack of adequate prior knowledge. Although they have completed advanced mathematics courses in upper secondary school, many students still experience difficulties at the tertiary level. Research aimed at investigating the source of student difficulties seek to relate the mastery of upper secondary mathematics content to difficulties in tertiary mathematics (see, for instance, Thomas et al., 2015). However, students at the tertiary level also demonstrate difficulties handling symbolic language and concepts introduced in compulsory education, so examining in depth students' competence in more basic mathematics might be interesting to see if they can handle simple arithmetic, algebra and problem solving. Vincent, Bardini, Pierce and Pearn (2015), for instance, found that tertiary mathematics students misuse the equals sign. Fluency in arithmetic and basic algebra is vital to flexibly handle more complex mathematics, and this competence is an important building block in the symbolic mathematical language (Kieran, 2007).

Issues on deficiencies in compulsory-level mathematical knowledge are not uniquely Norwegian concerns. In a similar study, Treacy and Faulkner (2015) analysed data from a diagnostic test taken by students transitioning from secondary education at an Irish university to identify deficiencies in compulsory-level mathematical knowledge demonstrated by beginning students enrolled in science and technology-based undergraduate courses. They

found that the proportion of students identified as at risk of failing their mathematics exams increased significantly from 2003 to 2013. The present study applies data from an assessment developed by the Norwegian Mathematical Council (NMR), which has been administered biannually since 2001, to assess beginning students' mastery of basic mathematical skills. The test is based on the Norwegian mathematics curriculum for compulsory lower secondary (NMR, 2016; Author, 2016). Each administration includes beginning students enrolled in university-level engineering, mathematics, economics, teacher education and other programs with a compulsory mathematics component of 60 European Credit Transfer and Accumulation System credits (ECTS) or more. The test typically requires students to perform paper-and-pencil computing tasks. Two groups of students are of special interest to this study because the maximum mathematics component in secondary school is either a prerequisite or the preferred background for these students' higher education studies. In Norway, students enlisted in calculus courses at traditional universities and who intend to study mathematics as a discipline, as well as engineering students in the three-year bachelor programme in engineering at university colleges and who intend to utilise mathematics as a professional tool, should have prior mathematical knowledge comparable to the maximum mathematics component in upper secondary school.

The research question addresses students' preparedness for tertiary education. More specifically, this study aims to discuss the competence in lower secondary school (basic) mathematics that engineering and calculus beginning students demonstrate in the assessment developed by the NMR. Here, competence should be broadly understood to encompass the procedural, conceptual and motivational aspects of knowing mathematics (see, for instance, Kilpatrick, 2014; Kilpatrick, Swafford, & Findell, 2001), and the assessment comprises items based on these three aspects of competence.

Previous Research on Students' Transition from Secondary to Tertiary Education

Transition from secondary to tertiary education has attracted the attention of many researchers in the past 20–25 years (Gueudet et al., 2016; Thomas et al., 2015). This research typically addresses issues regarding recruitment to science, technology, engineering and mathematics (STEM) education (Schreiner, Henriksen, Sjaastad, Jensen, & Løken, 2010), prior mathematical knowledge (Orpwood, Schollen, Leek, Marinelli-Henriques, & Assir, 2012; Thomas et al., 2015; Treacy & Faulkner, 2015), the gap between the mathematics cultures between secondary and tertiary education (e.g. didactical contract, see, for instance, Hourigan & O'Donoghue, 2007 or Luk, 2005) or other differences in teaching and learning at the secondary and university levels (Brandell, Hemmi, & Thunberg, 2008; Bingobali & Monaghan, 2008; Clark & Lovric, 2009; Hourigan & O'Donoghue, 2007; Kaiser & Buchholtz, 2014; Kajander & Lovric, 2005; Thomas & Klymchuk, 2012). The structure and quality of students' prior mathematical knowledge, their attitudes towards mathematics and the differences in teaching and conception of what it means to learn mathematics as a part of secondary education and education at universities and colleges can all contribute negatively to students' transition to and learning in higher-education mathematics courses. Calculus courses might be taught at the secondary or tertiary level, but the transition difficulties experienced by students are seemingly quite similar. In addition to research addressing transition, studies on student learning in calculus courses might also cast light on issues connected to transition. This research mainly focuses on the following four areas: 1) identifying student misconceptions, 2) investigating how students learn particular concepts, 3) classroom studies and 4) research on teacher knowledge, beliefs and practices (Rasmussen, Marrongelle, & Borba, 2014).

This study aims to enrich information on students' basic mathematical knowledge when transitioning to the tertiary level. Previous research has demonstrated that the

difficulties students experience in algebra are linked to their mistakes in arithmetic (Kieran, 2007). The lack of understanding and fluency in using the algebraic language will most likely hinder student learning at the tertiary level. For instance, Vincent et al. (2015) show that the misuse of the equals sign is an entrenched practice from the early primary years to tertiary mathematics. Whilst much research investigating transition issues mainly examine the differences between upper secondary and tertiary level mathematics teaching and learning, or the transition from calculus to analysis (see, for instance, Thomas et al., 2015), insights into student knowledge of more basic mathematics typically addressed in compulsory school might be important to understand the difficulties with developing symbolic literacy (Vincent et al., 2015) and fluency in the symbolic language. Similarly, the strong focus on argumentation and proof in higher education differs from that in secondary schools (Luk, 2005; Thomas et al., 2015). Students' difficulties in learning mathematics at the tertiary level may also be related to how mathematics is taught in secondary schools (Gueudet et al., 2016); for instance, a lack of focus on conceptual understanding can be found at the secondary level (Hourigan & O'Donoghue, 2007). Gueudet et al. (2016) propose that students transitioning from secondary education are insufficiently prepared for the autonomy that is expected of them at the tertiary level, and they are also unfamiliar with the use of multiple learning resources that can guide their learning.

A small amount of research on transition issues aims to investigate concerns related to the mastery of compulsory mathematics. One example of a study focusing on secondary mathematics is Treacy and Faulkner's (2015) longitudinal study of Irish students. The aim of their study was to identify those at risk of failing higher education because of the lack of prior mathematical knowledge. The diagnostic test administered to the students at Limerick University comprised topics from lower secondary mathematics (e.g. arithmetic and algebra) and upper secondary mathematics (e.g. differentiation and integration). By comparing data

for 2003–2013, the authors observed a marked decline in students' preparedness for university mathematics. This result is supported by a project conducted by Orpwood et al. (2012), who found that low numeracy skills lead to the mathematical difficulties faced by many entry-level college students in both technology- and business-oriented studies. The authors further stated that these numeracy skills, such as those relating to fractions, decimal numbers, percentages and algebra, are mostly taught as a part of compulsory education in grades 6–8, and entry-level university students' general command of these basic skills is rather weak. A plausible interpretation is that students do not develop a sound understanding of these basic concepts when such were taught in lower secondary education. The lack of numeracy skills and fluency in the symbolic language might also be an issue and lead to difficulties in learning at the tertiary level (Begg, Pierce, & McAndrew, 2017; Hourigan & O'Donoghue, 2007). For instance, students' difficulties with mastering the formal symbolic mathematical language affect their ability to communicate mathematically at the university level (Luk, 2005; Geuedet et al., 2016).

Various solutions have been proposed to help students be better prepared and to more easily transition from secondary to tertiary level. Gueudet et al. (2016) suggest that this transition can be viewed as a complex process in which difficulties are also associated with opportunities, and secondary-level approaches are combined with tertiary-level innovations that depend on the collaborations and common visions of teaching and learning mathematics. Hourigan and O'Donoghue (2007), for instance, propose that more emphasis should be given to developing conceptual understanding and establishing mathematical connections in secondary mathematics courses. Teachers should also strive to make mathematics meaningful to all students. Orpwood et al. (2012) suggest a different approach by stating that a numeracy strategy should be implemented to increase the mathematical levels of students enlisted in secondary and college courses. Some claim that to better predict the outcomes of higher

education mathematics courses, we should first fully understand the mathematical demands of college and university programs, and then align these demands with the courses offered in primary and secondary education (National Center on Education and the Economy, 2013). To this effect, Kajander and Lovric (2005) find that implementing a three-component transition ‘bridge’ between secondary and tertiary education, which involves students’ voluntary use of a mathematics review manual to prepare for university courses, administration of a mathematics background survey and the use of a redesigned first-year calculus course, is somewhat successful.

Evidently, numerous obstacles are associated with the transition from secondary to tertiary education, and indeed, a mathematical gap exists (Hourigan & O’Donoghue, 2007; Luk, 2005; Treacy & Faulkner, 2015). This brief review of the literature demonstrates that many students might graduate from secondary school knowing how to apply mathematical procedures, but lack sufficient conceptual understanding.

The Case of Norway

In 1972, at the time when the NMR was established as an independent board to advise the guild of university mathematics (NMR, 2016), many experienced university teachers reported concerns regarding the lack of mathematical competence in students transitioning from secondary education to universities in Norway. Similar claims were made internationally (see, for instance, Thomas & Klymchuk, 2012 or Luk, 2005). NMR decided to gather information on the declining mathematical competence of Norwegian students and, in 1984, conducted a first test to assess beginning students’ mastery of basic mathematical skills (NMR, 2016; Rasch-Halvorsen & Johnsbråten, 2002) that later was further developed into the current assessment. Whilst the Norwegian approach was to assess the students, other studies have surveyed mathematics professors’ views on transition to tertiary education (see, for instance, Thomas et al., 2015).

The Norwegian population is a small homogeneous population with respect to social structure. Norway has strong traditions regarding equity and equality in both society and education (Imsen, Blossing, & Moos, 2016), including free access to education at all levels from primary to tertiary. Norwegian students' scores on PISA, TIMSS and TIMSS Advanced are roughly at the international average (Mullis, Martin, Foy, & Arora, 2012; Mullis, Martin, Foy, & Hooper, 2016a; Mullis, Martin, Foy, & Hooper, 2016b; OECD, 2013b, 2016). Whilst the trend line for PISA has remained flat since 2003, indicating stable results, 15-year-old Norwegian students achieved a score above the OECD average for the first time in 2015 (OECD, 2016). International studies also provide some potential evidence of Norwegian students' preparedness for the tertiary level; for instance, the shortcomings of upper secondary students in physics were found to be related to their lack of formal mathematical knowledge, in general, and algebraic knowledge, in particular (Nilsen, Angell, & Grønmo, 2013). Overall, few students opt for advanced mathematics courses in the upper secondary level, and even fewer female students than male students do so for the most advanced courses (Mullis et al., 2016b). Bjørkeng (2011) finds that Norwegian girls aged 18 years probably needed to score higher grades in mathematics than do boys to opt for such courses. However and at the same time, Norwegian girls were found to significantly outperform boys in mathematics at the secondary level, both in exams and teacher grades.

In Norway, compulsory schooling comprises seven years of primary school, followed by three years of lower secondary school. Students are also expected to complete three (academic stream) or four (vocational stream) years of upper secondary education. Throughout their compulsory education, all students are taught the same mathematics curricula and receive the same amount of mathematics teaching (NDET, 2017a). Mathematics is compulsory even in the first two years of upper secondary school, although at this level, students can choose from various courses and content. Students interested in

mathematics and the natural sciences and who want to pursue higher education in these disciplines should choose a theoretical-oriented mathematics course in their first year of upper secondary school and opt for one or two years of further mathematics teaching, with the specialist courses consisting of geometry, algebra, functions, differential equations, probability and combinatorics (NDET, 2017b). Norwegian universities follow the Bologna model, in which a three-year bachelor's program might be followed by a two-year master's degree. To pursue mathematics and the natural sciences at the university level, students must complete at least one specialist mathematics course in secondary school. However, for mathematics-based theoretical studies, universities offer a calculus module that builds on the most advanced course students might take at the secondary level. Effective 2017, this maximum component will be made compulsory (MER, 2014b). The three-year engineering program follows the national framework for engineering education designed by the Council of Higher Education Institutions and adopted by the Ministry of Education (2014a). Despite the mandated maximum mathematics component, the framework offers alternative entry routes (bridges) that are accepted by local universities (Samordna opptak, 2017). These alternative routes are available to students who have completed vocational or academic training in upper secondary school. Engineering programs also build on the maximum mathematics component.

Methodology

This study examines data on the basic mathematical knowledge of undergraduate students ($N = 2163$) in their first year of tertiary engineering and calculus programs. We analysed responses to an assessment administered in the first two weeks of the first semester to identify students' prior knowledge and the differences between the two groups of interest. The students responded to a background questionnaire in addition to the mathematics test items.

Test

The construct for the entrance test is ‘basic mathematical knowledge’ (i.e. numbers, algebra, measurement and geometry) based on the lower secondary school/compulsory education curriculum. Several items are designed to measure conceptual understanding, whilst other items assess computational skills. The test form consists of 22 items distributed between 16 tasks, resulting in a maximum of 44 score points. A partial score was assigned to five items only. Non-responses are treated as wrong answers in the analysis and receive a score of zero. In addition to answering the test items, students are requested to fill out a background questionnaire (gender, age, and completion of upper secondary school mathematics courses) that includes items on attitudes towards mathematics; a scale of five interest-based questions and a question about calculator use. The interest items are taken from the 2012 PISA student questionnaire (ST29 statements b, d, e, f and g, OECD, 2013c, p. 230). The five statements were selected because they do not refer to a particular school context, but they ask students about their attitudes towards mathematics and their level of interest. The questionnaire and test are printed in the same booklet, with the test items found in the second half. The test reliability for the cognitive items, measured by Cronbach’s alpha coefficient, was .834, allowing for between-group comparisons of achievement based on sum scores (Cohen, Cohen, West, & Aiken, 2003). For the motivation scale, the Cronbach’s alpha coefficient was .734, indicating sufficient reliability.

Recruitment and Sample

Prior to the test administration, an invitation to participate in the study was sent to all universities offering study programs with a minimum of 60 ECTS mathematics component. Generally, each test administration includes 5,000–7,000 beginning students whose participation is decided by their respective universities. In total, 23 institutions and 6,044 students participated in 2011, including 626 calculus students and 1,537 engineering students,

who are considered the primary sample for this study. Table 1 presents the sample characteristics. Whilst many universities offer engineering programs, few provide traditional mathematics programs in which the entry course is entirely based on calculus. In 2011, 17 Norwegian universities offered a three-year engineering education, and eight universities, in total, offered general mathematics and natural sciences programs (calculus courses). In total, 11 and 5 universities submitted data on their engineering and calculus students, respectively. The sample is controlled for geographic diversity and represents different parts of Norway to a sufficient degree. However, in both samples, the number of male students is significantly higher than that of female ones and more prominently so among engineering students. According to Statistics Norway, 20% of the engineering students in 2016 were female, so the assumption is that the sample has a slight underrepresentation of female students (Statistics Norway, 2017).

Table 1

Student Group Characteristics: Engineering and Calculus Sample, 2011

	N	Male		Female		Universities	Average no. of students/institution	Min. group size	Max. group size
		N	%	N	%				
Engineering	1,532	1,296	85	236	15	11	139	28	270
Calculus	623	439	70	184	30	5	125	14	358
N	2,155	1,735		420					

Note: Some students did not report their gender.

The entrance test was also administered in 2013 and 2015. The test outcomes for engineering and calculus students are comparable across administrations (Author, 2014; Author, 2016). However, the average overall performance of Norwegian students when

students from other study programmes are included in the sample is slightly higher in 2015 than in the previous administrations. In the present study, the 2011 sample is used because it has better representativeness than the 2013 and 2015 samples. In the 2015 administration, the sample for engineering students was nearly half compared with that in the 2011 administration. As for calculus students, the 2013 and 2015 samples are comparable in size and even have a slightly higher number of participating students. However, fewer institutions are represented in the sample. We therefore included the 2013 and 2015 samples in the analysis of test outcomes over time (trend), but we used the 2011 sample to compare the knowledge structure and motivation of engineering and calculus students, as the 2011 sample represents a better geographical spread of Norwegian universities.

Procedures

The academic year begins in August, and the tests are administered within the first or second week of the semester, generally during the first or second lecture. The institutions print, administer and score the test papers in accordance with the written guidelines provided by NMR. The 2011 results were reported using an Excel template without student IDs, and sent to NMR. In 2013 and 2015, a web form was used to ensure the anonymity of data. The data cannot be traced back to the students or institutions. In accordance with the standards of good research practice, students were informed about the aim of the test, and they were not penalised if they opted not to participate.

Test scores are reported with one decimal and standard deviation as raw scores or percentages. The analysis is primarily performed by comparing the mean scores for different student groups with the use of SPSS; t-tests and one-way ANOVA with Scheffe's post-hoc tests are used to compare group means, with a 0.05 significance level applied, and the results are presented in tables and bar charts. Line diagrams are utilised to illustrate performance over time (trend). In addition, an IRT analysis applying Xcalibre is used to perform DIF

analysis and to plot test and item curves for investigating the differences between students at a similar achievement level. DIF-analysis (Mantel-Haenszel odds ratio) is used to investigate between group differences, as DIF-analysis allow for different construct effects, such as using different solution strategies (Wilson, 2005).

Finally, a correlation analysis was conducted to analyse the relationship between motivation and achievement.

Results

The entrance test was administered biannually from 2001 and 2015. Figure 1 shows the mean scores for engineering and calculus students for each administration. The data reveal that throughout this period, both groups have fairly stable results. This finding is in contrast to those of Treacy and Faulkner (2015) who observed a decline in the basic mathematical knowledge of Irish students during a similar period. However, the average achievement of both student groups is lower than what is aimed for, with engineering students scoring approximately 50% and calculus students approximately 60%. The test results shown in Figure 1 reveal that on average, neither student group sufficiently mastered the concepts and skills from the lower secondary curriculum included in the assessment. The concern regarding the preparedness of Norwegian calculus and engineering students is therefore warranted.

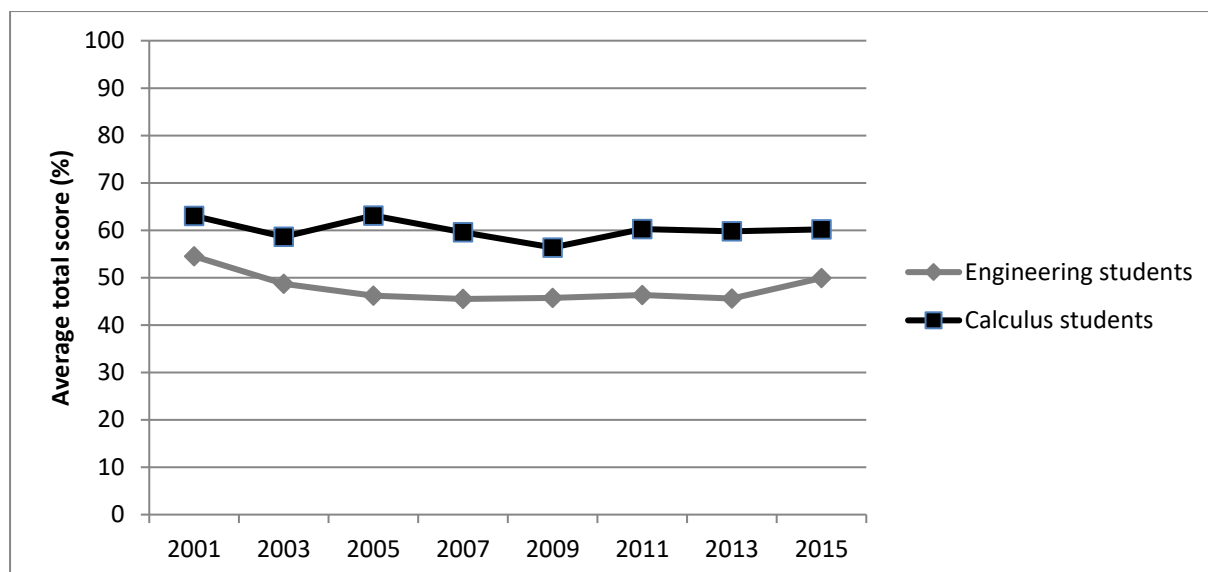


Figure 1. Results of 2001–2015 entrance tests as percentages of the maximum score. Reprinted using data reported in [37, 47].

Overall, the 2011 mean score was 26.5 ($SD = 8.61$) for calculus students and 20.4 for engineering students ($SD = 8.66$) out of a maximum of 44 points. This result indicated that engineering students scored less than 50%, on average, whereas calculus students scored 60%, on average. For the entire sample, including students in other programmes, the average score was 49%, with calculus students scoring significantly higher than the average and engineering students scoring close to the mean. Analysis of the trend data reveals that since 2001, calculus students have outperformed engineering students in every test administration. In 2011, the difference between calculus and engineering students was significant: $t(df = 2161) = -14.886, p < .01$. In addition, a decline was observed for the average results in 2003 for both groups, as indicated by the drop in the TIMSS Advanced mathematics results (Mullis et al., 2016). The results for engineering students are consistently low across the most recent test administrations. Some fluctuations can be seen in calculus students' results. The observed fluctuations in average results can most likely be attributed to the voluntary participation of both institutions and individual students.

Figure 2 shows the test response function for calculus and engineering students. A comparison of students who are on the same ability level (same Theta) reveals that a calculus

student, irrespective of his/her ability level, is most likely to have a higher score than an engineering student with the same ability level, although the difference is largest for students with average ability and smaller for students with low or high ability. Note that engineering students are expected to be better prepared for their university courses, as they are required to complete the maximum mathematics component during their upper secondary education. In addition, acceptance into engineering education programmes mostly needs a higher grade average than acceptance into general programmes in universities attended by calculus students (Samordna opptak, 2017). Treacy and Faulkner (2015) find that students who complete higher-level mathematics courses score higher on the diagnostic test than those students who complete ordinary-level mathematics courses. However, the entrance test results demonstrate that Norwegian engineering students are not better prepared than calculus students for tertiary education in terms of basic mathematical knowledge in the form of lower secondary mathematics content.

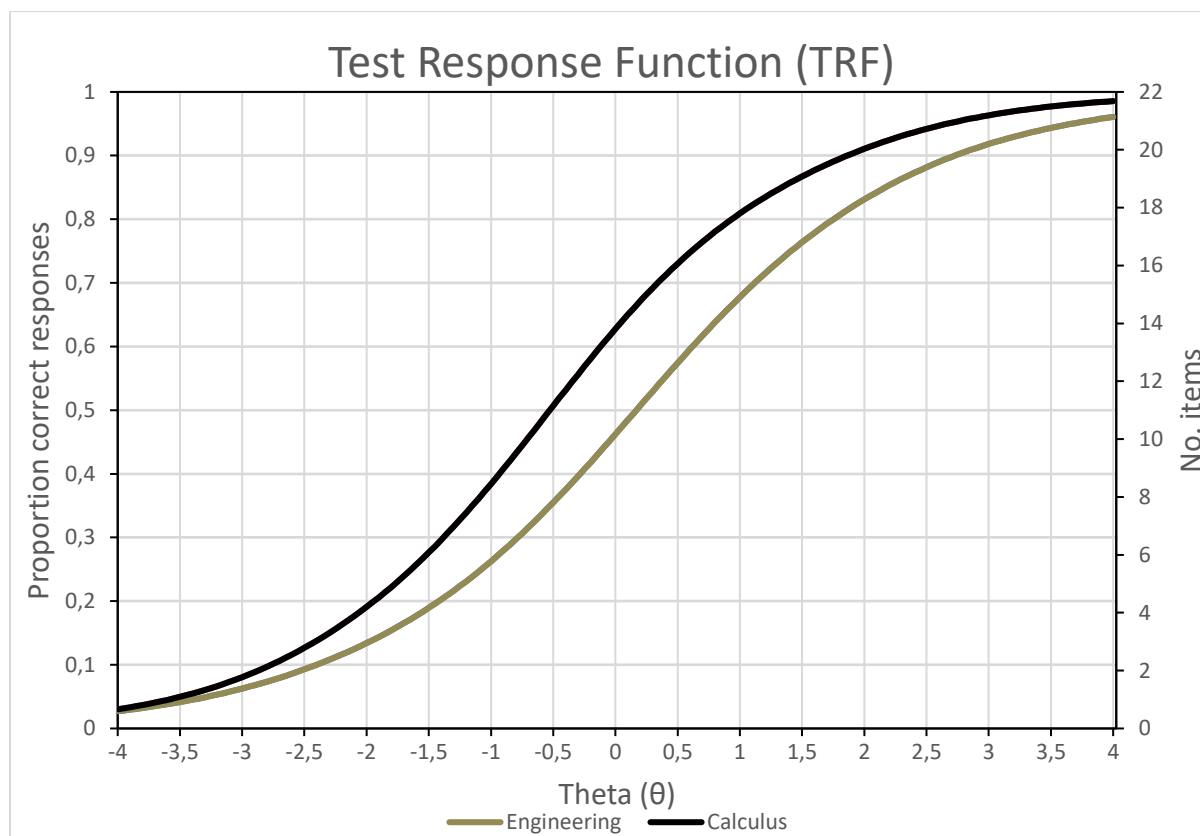


Figure 2. Test response function for calculus and engineering students.

The explanation for the consistent difference in the achievement of calculus and engineering students might be contributed to the courses they completed in secondary education. To be prepared for mathematics teaching and learning in tertiary education, students must have a solid knowledge base in mathematics, master the symbolic language and be well versed with arguments and proofs (Luk, 2005). This requirement also includes fully understanding basic mathematical competencies (Treacy & Faulkner, 2015). For Norwegian students, being formally prepared generally entails the completion of the maximum component of their mathematics courses during their secondary education. Although this component is not mandatory for calculus students, the first calculus course at the university level builds on it.

About 90% of calculus students and 62% of engineering students offered detailed information on their mathematical background from upper secondary education. Of these,

84% of calculus students and 70% of engineering students reported that they completed the most advanced mathematics course. This result might explain the differences in the achievement scores of the two groups of students, as students with a more formal mathematical training are expected to also have a better grasp of basic mathematical knowledge. Three out of four calculus students reported to have completed the maximum component, although this is not a formal requirement for admission to the calculus course. Theoretically, they should be well prepared for tertiary education. In comparison, also based on their self-reporting, at least 43% of engineering students completed the most advanced and, to them, obligatory course. About 30% of the engineering students who provided information on their background reported other combinations of secondary education courses and might be expected to struggle in their first courses in the engineering program.

For both calculus and engineering students, students who completed the maximum component had the highest scores, as expected, with an average of 28 ($SD = 7.98$) and 23.3 ($SD = 8.74$), respectively, out of the possible maximum score of 44 points. This finding means that calculus students with the maximum mathematics component from secondary education scored significantly higher than did engineering students who completed the same courses. This result is puzzling, perhaps indicating that other differences exist between calculus and engineering students. The students who apply for the two programmes might differ in other aspects of mathematical competence.

The question remains, however, whether engineering students with a vocational background are similarly well prepared for engineering courses at the university level compared with their academic counterparts. In total, 31% engineering students reported that they had vocational training, although they make up a diverse group: whilst some completed apprenticeship, others reported a combination of vocational and academic training during their secondary education. Dividing the engineering students into two groups, those with and

without vocational training, reveals differences in their overall score in the NMR assessment. Students with a vocational background scored 17.7 points, on average ($SD = 8.11$), whereas engineering students from the academic track in secondary education scored 21.2 ($SD = 8.39$). Seemingly, students with a vocational background have less basic mathematical knowledge compared with students with a combined vocational and academic or pure academic background, and, as a result, might be expected to struggle more in their first courses in the engineering programme.

Mastery of Basic Mathematical Knowledge

DIF-analysis revealed significant differences in favour of calculus students for a total of eight items, Mantel-Haenszel (M-H) coefficients ranging from 0.736 to 2.220 . These eight items comprise three arithmetic items, three algebra items, a measurement problem and one geometry item. Knowing how to manipulate numbers and symbols is a prerequisite for being algebraic literate (Vincent et al., 2015). Seemingly, calculus students, to some extent, master basic arithmetic and algebra better than engineering students do, as differences between the two groups of students are mainly caused by differences in items assessing basic knowledge in arithmetic and algebra. For two of the arithmetic items, students have to manipulate numbers without a calculator, whilst the third item asked the students to sort fractions by magnitude. This is basic mathematical knowledge taught in middle school, but students still lack fluency in it after finishing secondary education. The three algebra problems similarly asked the students to solve simple equations. The identified difference was largest for the problem that consisted of an unknown in the denominator. To successfully solve the measurement problem, students needed to understand ratio, something that also requires a good understanding of number operations.

Most studies on transition difficulties compare the curricula from the upper secondary level and the content in university courses to identify differences in approaches that may lead

to difficulties among students (see, for instance, Thomas et al., 2015). However, the more basic mathematics typically taught at the lower secondary levels, such as the general principles of arithmetic, simple algebra, and understanding and handling algebraic symbols and language, constitute building blocks necessary for students to develop a deep understanding of symbolic algebra and functions (Gueudet et al., 2016; Hourigan & O'Donoghue, 2007; Treacy & Faulkner, 2015). The identified differences and overall achievement of engineering and calculus students show that these two groups of Norwegian beginning students lack the fundamental mathematical knowledge needed for tertiary studies. For instance, the arithmetic item in which students were asked to order five fractions according to size was surprisingly challenging for both groups of students, with only 55% of engineering students and 69% of calculus students succeeding in solving it correctly.

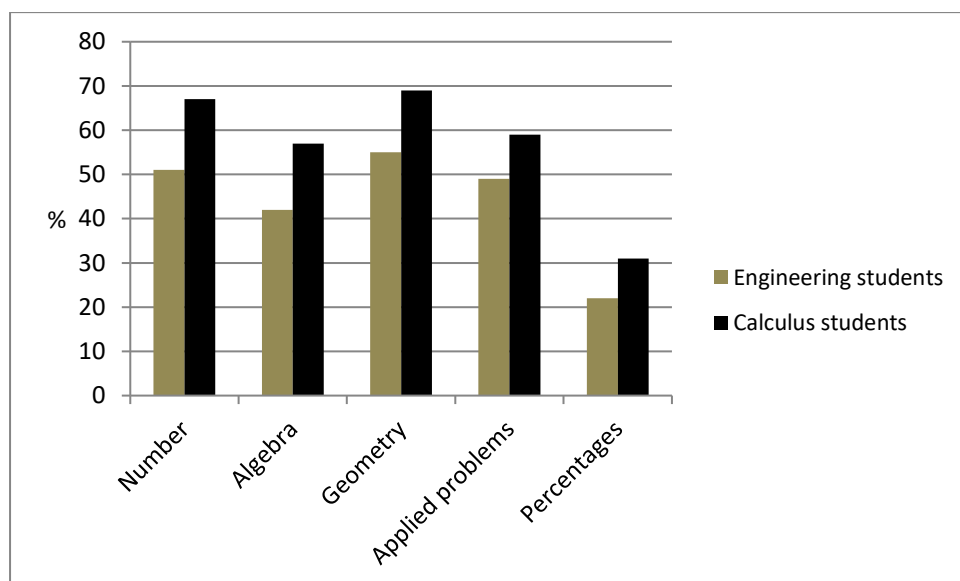


Figure 3. Percentages of correct responses by topic area.

Figure 3 displays the percentage of correct responses by topic area. The number of items in each topic area varies, and, thus, a comparison across areas should not be attempted.

As shown in Figure 3, calculus students outperformed engineering students in all content areas. All differences are significant.

Gender Differences

As shown in Table 2, a majority of the participating students are male both in the engineering and calculus samples. Although female students typically outperform male students in secondary education mathematics exams (Bjørkeng, 2011), male students, on average, outscore female students in the NMR assessment (see Table 2) [$t = (df = 1532) = 4.112, p < .01$ and $t = (df = 624) = 3.975, p < .01$]. The distribution in test scores is large in all four groups.

Table 2

Mean Scores for Female and Male Calculus and Engineering Students

	Male			Female		
	N	M	SD	N	M	SD
Calculus	439	27.3	8.54	184	24.4	8.42
Engineering	1,296	20,8	8,49	236	18.4	8.20

Gender differences were not expected, as no gender differences were found in the international PISA and TIMSS studies for lower secondary schools (Mullis et al., 2016a; OECD, 2013a, 2016), although significant, but small, differences were found in TIMSS Advanced at the upper secondary level, with males scoring higher average scores than females (Mullis et al., 2016b). A closer examination of the score patterns for male and female students in the NMR test shows that the differences can be mainly found among students with average and below average ability levels. Figure 4 shows the test response function for male and female students. It reveals that when comparing female and male students with the same ability level, the actual differences are small. In other words, the NMR test outcomes are

rather similar among female and male students. This result is a positive one, as no or small gender differences are the ideal situation in a society where equity and equality are the focus, as in the Nordic model (see, for instance, Imsen et al., 2016).

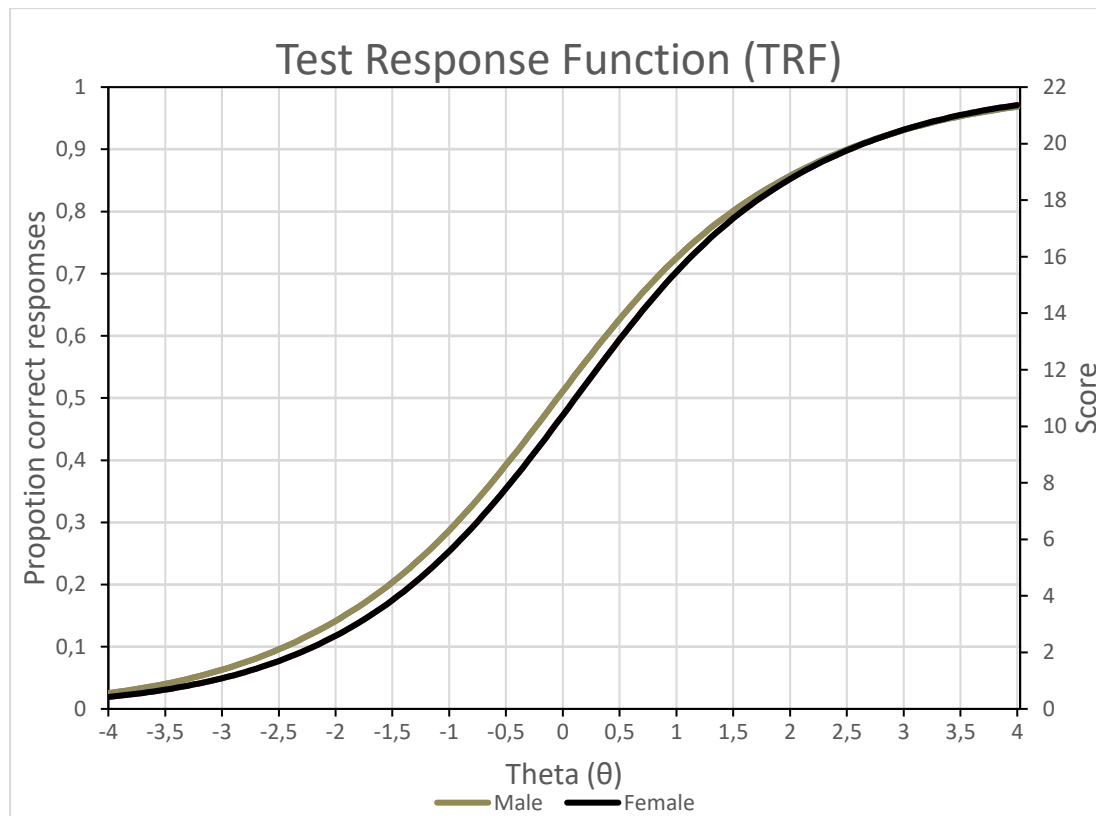


Figure 4. Test response function for male and female students.

Relationship between Use of Calculators and Test Scores

As a part of the background questionnaire, students were asked how often they use a calculator when calculating numbers. Surprisingly, 80% of engineering students and 62% of calculus students (see Table 3) stated that they use the calculator ‘often’ or ‘always’. As shown in Table 3, these students, on average, have a lower score in the NMR test than those who use the calculator less frequently, and, thus, assuming that the former group is less flexible in mental arithmetic or paper-and-pencil methods might be reasonable. However, understanding why engineering students who ‘almost never’ use a calculator achieved low

scores is difficult. This outcome might be due to size, few engineering students answer ‘almost’ never and as such represent a small proportion of the students and unaccounted third effects might be operational in this case. From this, using a calculator ‘sometimes’ can be concluded as the most constructive attitude to calculator use, as this construct is measured in the entrance test [$F(3, 1514) = 18.281, p < .01$; $F(3, 614) = 7.054, p < .01$]. It should be noted that the students were not allowed to use a calculator when solving items in the NMR test, in which many items required them to perform calculations and handle numbers. Thus, this result might, in part, be an artefact of the assessment.

Table 3

Students’ Mean Scores for the Use of Calculators by Category

	Always		Often		Sometimes		Almost never	
	N	M	N	M	N	M	N	M
	%	(SD)	%	(SD)	%	(SD)	%	(SD)
Calculus	9	23.8 (8.11)	53	25.7 (8.52)	34	27.8 (8.19)	3	30.9 (9.86)
Engineering	20	18.3 (7.83)	60	20.3 (8.34)	19	23.2 (9.03)	1	17.4 (7.78)

Motivation

Both calculus and engineering students chose to enrol in extra mathematics courses during their upper secondary education and have, in addition, chosen to enrol in an educational programme in higher education that comprises mathematics. These students can be assumed to enjoy mathematics or have an interest in it and are thus motivated to pursue mathematics. Motivation can be further divided into instrumental and intrinsic motivation (Ryan & Deci, 2000). Previous research has reported a positive relationship between

motivation and mathematical knowledge or competence (e.g. Gottfried, Marcoulides, Gottfried, & Olver, 2013; Kilpatrick et al., 2001; Marsh & Martin, 2011).

Table 4 presents the responses to the five attitude statements. As expected, a majority of the students 'agree' or 'strongly agree' with every statement. However, some differences can be observed for two statements measuring internal motivation; calculus students reported a higher degree of enjoyment of and interest in learning mathematics. The differences in response patterns of engineering and calculus students for these two statements are significant ($\chi^2 (df = 3) = 127.18, p < .001$ and $\chi^2 (df = 3) = 76.77, p < .001$).

The remaining three statements measure instrumental motivation—whether mathematics is viewed as a useful subject that can improve one's career or studies. The differences in response patterns between calculus and engineering students were smaller for these questions than with the statements on internal motivation. Significant differences in the response patterns ($\chi^2 (df = 3) = 10.15, p < .05$) were observed only for the last statement ('Mathematics is an important subject for me because I need it for what I want to study later on').

Table 4

*Calculus and Engineering Students' Responses to Internal and Instrumental Motivation**Statements*

<i>Statement</i>	<i>Group</i>	<i>Strongly agree (%)</i>	<i>Agree (%)</i>	<i>Disagree (%)</i>	<i>Strongly disagree (%)</i>
Making an effort in mathematics is worth it because it will help me in the work that I want to do later on.	E	66	32	1	0
	C	68	31	1	0
I do mathematics because I enjoy it.	E	15	62	22	2
	C	36	52	12	1
Learning mathematics is worthwhile for me because it will improve my career prospects.	E	52	44	3	0
	C	55	40	5	1
I am interested in the things I learn in mathematics.	E	19	67	14	1
	C	36	57	8	0
Mathematics is an important subject for me because I need it for what I want to study later on.	E	61	35	3	0
	C	64	30	5	1

Correlation analysis revealed an overall positive, although not strong, relationship for the two statements measuring internal motivation (enjoying mathematics and being interested) and achievement; $r_{xy} = .258$, $p = .01$ and $r_{xy} = .186$, $p = .01$). The relationship was somewhat stronger for calculus students, although it was significant at the .01 level for both groups. On the other hand, the results for the statements measuring instrumental motivation are somewhat different. We found a small positive correlation at the .01 level for the statement 'making an effort is important' for engineering students, and a correlation of .085 at the .05 level for 'mathematics is important for other subjects' for calculus students. No relationship was found for the third statement, indicating a weaker relationship between instrumental motivation and achievement.

Discussion

To become a ‘constructive, engaged and reflective citizen’, an individual must be mathematically competent (OECD, 2013a, p. 25). In the past 20 years, higher education has evolved from being elite education to mass education (Brandell et al., 2008), even for STEM subjects. It is usually believed that a large number of citizens with a high level of specialised mathematical knowledge is needed to develop a sustainable future. Thus, the outcome of mathematics teaching at the university level is crucial. Mathematics teachers, economists, engineers and natural scientists, in general, must possess a certain level of mathematical knowledge to fulfil the objectives of their professional life. The outcome is even more crucial for future specialists. However, many express concerns regarding transition problems and the level of mathematical competence among students entering universities (Brandell et al., 2008; Hourigan & O’Donoghue, 2007; Thomas et al., 2015).

The NMR assessment has shown that Norwegian engineering and calculus students have positive attitudes towards mathematics. This result is promising because as a nation, we need future experts in STEM professions. However, as shown in this study, beginning students did not demonstrate that they master the content assessed by the NMR entrance test, the entire content of which is covered in the mathematics curricula of lower secondary school; in fact, some of the items can even be solved by primary school students in their final years. Our findings are in line with those of previous research, although many other studies primarily investigated student understanding of upper secondary mathematics content (Hourigan & O’Donoghue, 2007; Kaiser & Buchholtz, 2014; Luk, 2005; Treacy & Faulkner, 2015; Geuedet et al., 2016; Thomas et al., 2015). Students’ difficulties in handling numbers might also be linked to their use of calculators.. Students who use a calculator to calculate with numbers are less likely to move flexibly between different mathematical representations, such as fractions, decimal numbers and percentages. This fluency is vital when working on

more complex mathematical tasks, such as those encountered in university courses. In addition to difficulties in correctly performing calculations, students struggled with simple algebra and geometry problems. Perhaps, they do not fully understand the underlying concepts, which can also lead to difficulties in learning mathematics content in their university courses (Thomas et al., 2015). The recent drop in the Norwegian TIMSS Advanced results between 1995(8) and 2011 led to increased demands for more formal mathematics learning in secondary education, such as a stronger focus on algebra in Norwegian compulsory education curricula (see, for instance, Grønmo, Onstad, Nilsen, Hole, Aslaksen, & Borge, 2012; Grønmo, Onstad, & Pedersen, 2010). This recommendation is in line with Thomas et al.'s (2015) and Luk's (2005) proposals, although we suggest that such an intervention be accompanied by a strong focus on conceptual understanding and learning of mathematical big ideas. Ideally, all students entering tertiary education should have mastered the concepts and skills needed to successfully solve such items in the entrance test. This lack of basic knowledge can have considerable consequences on individual students, who may struggle to complete their education. These struggles could be major enough to lead to failure in examinations, which can have both financial and academic consequences. However, the cost for a society that offers free tertiary education and needs skilled mathematicians and engineers might even be greater, as the investment made in education could possibly offer less returns than intended.

Some differences were found between the two groups of students in terms of basic mathematical knowledge and motivation, and we might be concerned with their differing level of preparedness for higher education. Even the preparedness of calculus students is questionable. Some of the differences observed between calculus and engineering students might plausibly be attributed to differences in the students themselves. Differences between engineering and calculus students were also found by Bingobali and Monaghan (2008) who

found that the two groups of students have different conceptual understandings of functions, which they linked to their exposure to different communities of learning. We argue that students are likely to pursue engineering and calculus courses for different reasons, and consequently, those deciding to enter an engineering program could have similar backgrounds and dispositions that are not shared by those entering general mathematical programmes.

Students enrolling in tertiary education should master the concepts and skills needed to successfully answer the NMR entrance test. The first-year mathematics courses in both engineering and calculus programmes build on the maximum mathematics component from secondary school. However, in both study programs, we found that students completed different combinations of mathematics courses. The maximum component was initially not obligatory to gain admission into a calculus course. As for engineering programs, students with a vocational training could, and still can, enrol in the program; bridging courses are offered as alternative routes. However, bridging activities are oriented towards upper secondary mathematics curricula, and all students are expected to be well versed with the mathematical concepts and skills of lower secondary school. Evidence that students' difficulties can be attributed to their experiences in secondary school can be supported by the outcomes of projects conducted in other countries to help students transition from secondary to tertiary level. To overcome shortcomings in basic knowledge, Orpwood et al. (2012) propose placing more emphasis on numeracy in secondary education. Other authors suggest bridging courses (Gueudet et al., 2016; Thomas et al., 2015), although Kajander and Lovric (2005) report that the bridging courses at McMaster University were not as successful as anticipated, and students did not achieve the expected increased performance. Also other initiatives at the tertiary level have lacked the desired success factor (Kajander & Lovric, 2005; Luk, 2005).

Concluding Remarks

This research gives a novel contribution to the field of mathematics education. Most of the earlier studies in this area have mainly investigated differences and similarities in mathematics education at the upper secondary level and in higher education. In this study, we address students' transition from secondary school to university and scrutinise their prior (basic) mathematical knowledge and interest. As such, this is a relative study of mathematical education in the sense that we examine university teaching in relation to secondary teaching.

One possible interpretation of our results is that engineering and calculus students need education designed particularly for their group, whilst there is little reason to do so for the two genders. In all test administrations, the test results differed substantially between the two groups, with calculus students outperforming engineering ones, although both groups have their own shortcomings and deficiencies. A risk of segregation in the student body might be found in many countries internationally, with engineering students mainly belonging to the working class and calculus students representing a high SES background, to a large extent. In this case, differences in the NMR assessment would reflect differences in student background and relationships between socio economic status (SES) and achievement. However, because of the homogeneity of the Norwegian population or the fact that Norwegian society is essentially not divided into social classes (Chan, Birkeland, Aas, & Wiborg, 2010), we argue that this is likely not the case in the present study. The relationship between mathematics achievement and SES is previously found to be smaller in Nordic countries than in other parts of the world (see, for instance, OECD, 2013). Consequently, this research can help identify the potential deficiencies in mathematical competence of Norwegian calculus and engineering students, which might affect their later progress in university mathematics, science and technology courses.

The engineering and calculus courses offered in Norwegian universities build on the maximum mathematics component in upper secondary school, although students with less mathematically oriented qualifications are also accepted. The outcomes of the NMR entrance test suggest that entry-level engineering and calculus students could encounter certain difficulties regarding the mathematics components in their university courses. Two serious social concerns are the gender imbalance in the recruitment for higher education and the low number of students choosing to study mathematics in secondary school.. Overall, few students opt for the maximum mathematics component in Norwegian secondary schools; in 2009, only 10% of the cohort chose to do so (Bjørkeng, 2011), in which case, even male students can be viewed as underrepresented. Consequently, recruitment must be conducted from the entire student population to ensure that a sufficient number of experts are available in the future, although this might contribute to issues relating to STEM education developing from elite to mass education. In addition, although more females than males enrol in university programs (Statistics Norway, 2012), fewer females choose to pursue STEM education (Schreiner et al., 2010). This gender difference might be an outcome of fewer female students choosing extra courses in mathematics and sciences in upper secondary school (Bjørkeng, 2011). This gender imbalance might present an equity issue if young female citizens have fewer opportunities to qualify for important positions and wield influence in society at large.

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