Solving the mysteries of $^{133}$Xe with inverse kinematics

*Nuclear level density and $\gamma$-ray strength function for $^{133}$Xe using the inverse-Oslo method*

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Thesis submitted for the degree of Master of Science in Nuclear Physics
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Printed: Reprosentralen, University of Oslo
Til mormor og morfar,
mine store forbilder og helter.
Abstract

When investigating statistical properties, the main route of investigation has been using a light ion beam on a stable target to produce the reaction of interest. This puts limits on the possible isotopes to explore. Using the Oslo method [1], it is then possible to extract the nuclear level density (NLD) and the γ-strength function (γ SF) for suitable targets. Alternatively, using the inverse-Oslo method, it is possible to study more exotic or unstable nuclei, using a heavy beam on a light target. A proof of principle experiment was performed in 2015 at iThemba LABS with a $^{86}$Kr(d,p)$^{87}$Kr to determine the NLD and the γSF of $^{87}$Kr [2].

In 2017, an experiment was carried out at iThemba LABS with $^{84}$Kr and $^{132}$Xe beams on a deuterated polyethylene target to undergo a (d,p) reaction, producing $^{85}$Kr and $^{133}$Xe. NLD and γSF were extracted from the measured particle-γ coincidences.

With the NLD and γSF, the nuclear structure of $^{133}$Xe has been investigated to determine if there is a low energy enhancement (LEE) in the γSF, along with any other resonances. Due to its location relative to doubly-magic $^{132}$Sn in the nuclear chart, $^{133}$Xe has been predicted to have an especially large LEE [3]. The extracted γSF showed a low energy enhancement for $^{133}$Xe. Shell model calculations are consistent with this and predicts that the LEE is caused by M1 transitions.

The statistical properties of $^{133}$Xe are of interest for (n,γ) calculations. Highly excited $^{133}$Xe* in high energy density plasmas has also been examined at the National Ignition Facility (NIF) at Lawrence Livermore National Laboratory to examine changes in angular momentum due to nuclear plasma interactions (NPI) [4], and LEE can cause an increase in the predicted NPI rate in $^{133}$Xe.
## Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Definition</th>
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<tr>
<td>AFRODITE</td>
<td>African Omnipurpose Detector for Innovative Techniques and Experiment</td>
</tr>
<tr>
<td>BGO</td>
<td>bismuth germanate</td>
</tr>
<tr>
<td>CT</td>
<td>constant temperature</td>
</tr>
<tr>
<td>ΔE-E</td>
<td>particle detectors Δ E and E</td>
</tr>
<tr>
<td>γSF</td>
<td>γ-strength function</td>
</tr>
<tr>
<td>gBA</td>
<td>generalized Brink-Axel hypothesis</td>
</tr>
<tr>
<td>GDR</td>
<td>giant dipole resonance</td>
</tr>
<tr>
<td>LEE</td>
<td>low energyhancement</td>
</tr>
<tr>
<td>NEEC</td>
<td>nuclear excitation by electron capture</td>
</tr>
<tr>
<td>NIF</td>
<td>the National Ignition Facility</td>
</tr>
<tr>
<td>NLD</td>
<td>nuclear level density</td>
</tr>
<tr>
<td>NPI</td>
<td>nuclear plasma interactions</td>
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<tr>
<td>OCL</td>
<td>Oslo Cyclotron Laboratory</td>
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<td>SM</td>
<td>shell model</td>
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Last of all, I would like to thank my parents and my family. You always cheer me on and I am grateful for all of you. Peter, you are the kindest.

\[^{1}\text{3}</3\]

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person I know, and that kindness can really help in difficult times. And thank you, for reading my thesis!
1

Introduction

"When did you become an expert in thermonuclear astrophysics?"
"Last night."

– Maria Hill and Tony Stark, Avengers I 54:45

Even though Lord Kelvin boldly claimed "There is nothing new to be discovered in physics now. All that remains is more and more precise measurement." in 1900 [5], there are still big unanswered questions in physics over a hundred years later. We still do not know what dark matter or dark energy is. Humans do not understand everything yet, and a hundred years from now, Lord Kelvin’s statement will probably still be false. The physical description of everything in the universe is still incomplete.

Scientists have made many advancements in determining where elements heavier than iron are produced. Data observed from the neutron star merger showed that this is indeed a site for the $r$-process [6]. However, there are still unanswered questions about how elements are made in the universe. For instance; we cannot correctly reproduce the solar system abundance yet. Abundance calculations rely on statistical properties of nuclei as input. When data is not available, phenomenological models for nuclear level density (NLD) and $\gamma$-strength function ($\gamma$SF) are used instead. The $\gamma$SF is dominated by the electric giant dipole resonance (GDR) and the model used is usually a smooth Lorentzian function. However, experimentally measured $\gamma$SF have revealed more structures like smaller resonances and sometimes a low energy enhancement (LEE).

When an experiment unveils the presence of resonances or a LEE for a nuclei, the calculated n-capture cross section will differ from the model based prediction. A change in the calculated n-capture rate, can affect the abundance calculations [7]. So, when nuclear properties are determined experimentally, our ability to simulate and calculate the abundance of matter in the universe gets better. One of the goals of this thesis is to contribute to solving this puzzle by providing experimentally measured NLD and $\gamma$SF for $^{133}$Xe.

When direct measurements are difficult, surrogate reactions can be used

---

1E.g. a scissors or a pygmy resonance.
The excited nuclei will emit $\gamma$-rays from the quasi-continuum region, which can be used to extract the nuclear level density and $\gamma$-strength function. The resulting $\gamma$-strength function might reveal resonance properties of the nuclei in the low energy tail of the giant dipole resonance. Illustration taken from Ref. [10].

to extract the desired statistical properties. There are some extra obstacles to measuring these statistical properties of $^{133}$Xe. Xenon is a noble gas and it is therefore an inconvenient target in an experiment, $^{133}$Xe also $\beta^-$-decays from the ground state with a half-life around five days [8]. Thus the usage of the inverse-Oslo method or the Oslo method in inverse kinematics is necessary.

In 2015, the first inverse-Oslo experiment was performed at iThemba LABS. As the inverse kinematics experiment on $^{87}$Kr was a successful proof of principle [2, 9], it showed that the method could be used to measure NLD and $\gamma$SF of the noble gas $^{87}$Kr. At iThemba LABS it is possible to create beams of heavy elements and use this to produce the isotope of interest. To produce the beam, a relatively small amount of the element is required.

Exciting $^{132}$Xe through a $d(^{132}{\text{Xe}},p)^{133}$Xe reaction, $^{133}$Xe* will decay towards the ground state via the emission of $\gamma$-rays. These $\gamma$-rays will be detected in coincidence with an outgoing proton. The proton energy is used to calculate the initial excitation energy of the nuclei, while the $\gamma$-rays in coincidence are used to extract NLD and the $\gamma$SF function, see fig. 1.1. Starting from a particle-$\gamma$ coincidence matrix it is possible to use the Oslo method [1, 11, 12] to extract the desired NLD and $\gamma$SF.

This thesis is focused on finding the statistical properties of $^{133}$Xe, both the NLD and the $\gamma$SF. The $\gamma$SF is particularly interesting since it can be used to predict nuclear plasma interactions (NPI) in these high energy density plasmas. In these stellar environments, high energy density plasma is expected to affect the formation of heavy elements from pre-existing nuclei [4]. Recreating stellar environments such as high density plasma here on earth is difficult and expensive, although possible. These experiments can be conducted at the National Ignition Facility (NIF) with a laser system.
delivering 1.8 MJ at 500 TW on a target \[13\]. In these high energy density environments, there can be NPI which can change the initial spin and parity distribution of the excited \(^{133}\)Xe. To observe nuclear plasma interaction, the isomer to ground state ratio of \(^{133}\)Xe can be used, as this ratio depends on the initial spin distribution. The predicted rate of NPI depends on the \(\gamma\)SF for \(^{133}\)Xe, which is measured in this thesis. The results from this thesis will be used to interpret the data from the NIF experiment to determine if NPI have taken place.

As mentioned previously, NLD and \(\gamma\)SF are important input parameters in n-capture cross section calculations using the Hauser-Feshbach model. The results are more reliable with measured statistical properties, instead of modeling all properties when based on the default models used for NLD and \(\gamma\)SF in reaction codes. The measured \(\gamma\)SF might vary from the predicted smooth tail of the GDR. Simulations on the impact of LEE have showed that is can increase the n-capture cross section up to two orders of magnitude for exotic neutron rich nuclei \[14\].

Since there are few known levels in \(^{133}\)Xe, a good way to benchmark the results from this work is comparison with theoretical calculations of \(^{133}\)Xe. Shell model calculations done on \(^{133}\)Xe to calculate transitions and statistical properties are presented and shown in comparison to the extracted level density and \(\gamma\)-strength function. Additionally, these calculations can be used to see if the assumptions of the Oslo method holds true for the spin and parity distribution of the \(^{133}\)Xe.

This thesis is structured as following: Nuclear physics and statistical properties are detailed in chapter 2 along with description of nuclear plasma interactions. Chapter 3 contains the details of the experiment and the detectors used. In chapter 4 calibration, time alignment, calculation of excitation energy, Doppler shift correction, event selection and background subtraction are described. In chapter 5 the Oslo method is presented with the extraction of the NLD and \(\gamma\)SF of \(^{133}\)Xe. Discussion of the statistical properties and comparison with shell model calculations is presented in chapter 6. Lastly, the results of this thesis are summarized in chapter 7 together with a future outlook.
Nuclear physics and statistical properties

"Do you guys just put the word "quantum" in front of everything?"
— Ant-man, Ant-Man II 33:46

Nuclear physics is the study of nuclei, nuclear properties, decay, and nuclear reactions. Through experiments and simulations, we can study nuclei that are abundant in nature, and nuclei that have a lifetime barely long enough that they can be measured to exist before they decay.

For low excitation energies, $E_x$, it is possible to study individual energy levels with spectroscopy. For high excitation energy, the levels spacing will be smaller than the individual width of each level, this is called the continuum region. Between the discrete region and the continuum region, we have the quasi-continuum. In the quasi-continuum, the transitions are more chaotic than for discrete levels [15].

Instead of measuring the transition strengths between individual levels, it becomes more fruitful to measure the statistical properties. Averaging over the levels, we can find functions that describe the nuclear level density (NLD) and the average transition strength called the $\gamma$-strength function ($\gamma$SF) in the regions of interest. These statistical properties are in focus for this thesis.

2.1 The chart of nuclides and nucleosynthesis

The chart of nuclides in fig. 2.1 shows all isotopes that have been measured, along with relevant properties such as the mass ($A$), proton ($Z$) and neutron ($N$) number of the isotopes. Other properties such as decay channels, lifetime and mass can also be shown. The different colors represent different decay modes. Additionally, the magic numbers are marked with blue ($Z$) and black ($N$) lines. In the shell model, magic numbers account for closed shells, and they are 2, 8, 20, 28, 50, 82 and 126, more on this in section 2.2.

Along the $x$-axis is the neutron number $N$, and along the $y$-axis is the proton number $Z$. The stable nuclei are colored black, the rest are
unstable nuclei that will decay towards the stable nuclei through different mechanisms.

From the chart of nuclides, it can be seen that for heavier elements, stable isotopes have $N > Z$, due to the Coulomb repulsion between protons. Isotopes are nuclei with same proton number $Z$, isobars are nuclei with same mass number $A$ and isotones are nuclei with same neutron number $N$.

An interesting question is how the elements in the nuclide chart are formed. All elements lighter than iron can be formed through fusion. When looking at heavier elements, it is no longer energetically favorable to create elements through fusion. One of the utmost important question in physics has been where elements heavier than iron are formed. There have been theories of supernova explosions [17, 18] and neutron star mergers [19] as the place of creation. In the fall of 2017, the first neutron star merger was observed and which confirmed that binary neutron star mergers are indeed the birthplace of many heavy elements [6] through the $r$-process, made possible with the neutrons available.

When simulating nucleosynthesis, there is a need for more precise input data. As the $s$-, $i$- and $r$-processes all include neutron capture, the $(n, \gamma)$-cross section is important for more accurate simulations of how elements are formed. Some nuclei are more important for the different processes, and an uncertainty in this cross section will then propagate through the entire abundance calculations.

For many nuclei, it is difficult or impossible to measure the $(n, \gamma)$-cross section directly. For example, if the nucleus of interest is gaseous or very short lived, it is difficult to make a target out of it. Instead, we can use
an indirect route to find the NLD and the $\gamma$SF to calculate the $(n, \gamma)$-cross section through codes like TALYS [20, 21]. In this thesis, the NLD and $\gamma$SF has been extracted for $^{133}$Xe.

### 2.2 The shell model

![Diagram of nuclear shells](image)

Figure 2.2: Illustration of the nuclear shells without spin-orbit interaction (left) and with spin-orbit (right). The numbers after the spin-orbit coupling is the number of particles in each orbital. The boxes show the total number of particles up to that shell closure. The levels are calculated by using a harmonic oscillator potential. Figure taken from Ref. [22].

Just like the electron subshells in atoms, the protons and neutrons in the nuclei can also be arranged in subshells. This is called the nuclear shell model and it illustrates why there are some magic numbers that will have more stable isotopes or isotones. In the chart of nuclides in fig. 2.1, the magic numbers are marked for both $N$ (black) and $Z$ (blue).

The existence of these subshells depends on the Pauli exclusion principle, as each single-particle state can only occur once. Using

---

1 or orbitals
a harmonic oscillator (HO) potential, the first three magic numbers can be calculated. To reproduce the rest of the magic numbers seen experimentally, calculations has to include a spin-orbit coupling, which Maria Goeppert Mayer got the Noble prize\footnote{Mayer’s Nobel lecture, Ref. \cite{23}, is a good introduction to the shell model.} for in 1963.

Table 2.1: Orbital angular momentum \( l \) in the shell model.

<table>
<thead>
<tr>
<th>( l )</th>
<th>Angular momentum</th>
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<tr>
<td>( s )</td>
<td>0</td>
</tr>
<tr>
<td>( p )</td>
<td>1</td>
</tr>
<tr>
<td>( d )</td>
<td>2</td>
</tr>
<tr>
<td>( f )</td>
<td>3</td>
</tr>
<tr>
<td>( g )</td>
<td>4</td>
</tr>
</tbody>
</table>

In fig. 2.2 the split levels are marked as \( Nl_j \), where \( N \) is the main oscillator quantum number from the HO potential. Here \( l \) is the orbital angular momentum with the number given in table 2.1 for the levels shown in fig. 2.2. The spin(s)-orbit(l) coupling \( j \) is given as \( j = l \pm s \).

For any orbit, if there an even number of nucleons with a total angular momentum quantum number \( j \), the nucleons will couple to give total spin \( J = 0 \) \cite{24}, with no contribution to the magnetic moment. For an even-even nuclei, the ground state spin and parity is \( J^\pi = 0^+ \). Generally, the levels will be filled with \( j = l + 1/2 \), then \( j = l - 1/2 \) \cite{25}. If there is an unpaired nucleon, that nucleon will determine the total spin and parity, where \( \pi = (-1)^l \), in the ground state. For an odd-odd nuclei, there is both an unpaired proton and an unpaired neutron, the total parity is the product of the unpaired nucleons and the total spin is in \( |j_p - j_n| \leq J \leq |j_p + j_n| \).

2.3 Radiation interaction with matter

Photons interact with matter primarily through three processes:

1. The photoelectric effect
2. Compton scattering
3. Pair production

The photoelectric effect is the emission of electrons caused by photons interacting with matter. In Compton scattering, a photon scatters off a particle and will end up with a lower wavelength. Pair production is the creation of an electron-positron pair from \( \gamma \)-rays with an energy over 1.022 MeV.
2.3.1 Photoelectric effect

Figure 2.3: Illustration of the photoelectric effect. Incoming light will cause the emission of electrons from matter. Figure taken from Ref. [26].

Even though most people have heard of $E = mc^2$, Einstein was only awarded the Nobel prize in Physics for fully explaining the photoelectric effect. It was his description of light as particles that finally could explain experimental results of the photoelectric effect. With Maxwellian theory, the energy of light can only be calculated as a continuous spatial function [27], not discrete.

For low-energy interactions with matter with a high $Z$, photoelectric effect is dominant below 50 keV [28]. With the photoelectric effect, we have that the maximum energy $E$ of the emitted electrons is

$$E = h\nu - \Phi,$$

(2.1)

Which means that a material will only emit electrons, see fig. 2.3, if the incoming light has a frequency $\nu$ which is above the photoelectric threshold of the material [29]. The incoming photons have an energy $h\nu$, which is greater than the work function $\Phi$ of the material, where $h$ is the Planck constant.

Scintillator detectors absorb the light and the production of electrons from the photoelectric effect produces signals.

2.3.2 Compton scattering

Figure 2.4: Illustration of Compton scattering. A photon scatters of a particle with a scattering angle $\theta$. Figure taken from Ref. [30].
Compton scattering was a discovery that at the time hinted at the particle nature of light. When a photon collide with a particle, e.g. an electron, the wavelength $\lambda$ and direction changes as energy is transferred to the particle, illustrated in fig. 2.4. The outgoing photon has a lower energy, and the wavelength increases [31].

As the energy from the photon is transferred to the electron, the scattered photon will have a wavelength $\lambda_\theta$ given as

$$\lambda_\theta - \lambda_0 = \frac{h}{m_e c} (1 - \cos \theta) \quad (2.2)$$

where $\lambda_0$ is the initial wavelength, $h$ the Planck constant, $m_e$ the electron rest mass, $c$ the speed of light in vacuum and $\theta$ as the scattering angle [32].

As we have $E = hc/\lambda$, the energy $E_\gamma$ of the scattered photon will be

$$E_\gamma = \frac{E_0}{1 + \frac{E_0}{m_e c^2} (1 - \cos \theta)} \quad (2.3)$$

Where $E_0$ is the energy of the incoming photon. If the photon scatters at a 180° angle, the particle will get the maximum amount of energy transferred, such that

$$E_{\text{max}} = E_0 - E_\gamma = E_0 - \frac{E_0}{1 + \frac{E_0}{m_e c^2} (1 + 1)} = E_0 \left(1 - \frac{1}{1 + \frac{2E_0}{m_e c^2}}\right) \quad (2.4)$$

This minimum energy $E_{\text{min}}$ transfer will cause the Compton edge, where there will be a background in the spectra that sharply drops at $E_{\gamma\text{max}}$.

The Compton effect is the principal absorption mechanism in the energy range 50 keV to 10 MeV [28].

### 2.3.3 Pair production

![Feynman diagram of the pair production of $e^-e^+$.](image)

Figure 2.5: Feynman diagram of the pair production of $e^-e^+$. A virtual $e$ is exchanged between the nucleus and the incoming $\gamma$, to produce the $e^-e^+$-pair.
If there is an incoming photon with more than 1022 keV, twice the rest mass of an electron, an electron-positron pair can be produced in the presence of a nucleus, illustrated in fig. 2.5. When a positron annihilates with an electron, the resulting two 511 keV γ-rays can escape the detector. If one escapes, it is called single escape, if both 511 keV γ-rays escape, it is called double escape. In the spectra this will be seen as two peaks with energy $E_s$ and $E_d$ at $E_f - 511$ keV and $E_f - 1022$ keV, when $E_f$ is the real energy of the peak.

This single escape and double escape is important when detecting γ-rays. This loss of efficiency in the detector can be recovered by adding the counts back to the full energy peak.

### 2.4 Radioactive decay

When a nucleus is unstable or excited, it wants to get rid of the excess energy. To do so, it can decay through various decay modes like α−, β± − and γ− radiation.

Assuming a potential for the nucleus on the form $H'$, solving the Schrödinger equation using $H$ gives stationary states. $H'$ is a very weak additional potential that can cause transitions between the states $i$ to $f$. Using this it is possible to calculate the transition probability $\lambda$ from the initial state $i$ to the final state $f$:

$$\lambda_{if} = \frac{2\pi}{\hbar} \left| \langle f | H' | i \rangle \right|^2 \rho(E_f),$$

(2.5)

where $\hbar$ is the reduced Planck constant, $\hbar = \frac{\hbar}{2\pi}$ and $H'$ is the perturbation. The transition probability, $\lambda$, will be large if there is a large number of final states $f$ accessible for the decay. This is given as $\rho(E_f)$, the density of final states. The probability $P(E)$ to observe the system in the energy interval between $E$ and $E + dE$ in the vicinity of energy of state $a, E_a$ is

$$P(E)dE = \frac{dE}{(E - E_a)^2 + \Gamma_a^2/4},$$

(2.6)

where the width of state $a$ is given as $\Gamma_a = \frac{\hbar}{\tau_a}$. Here, $\Gamma_a$ is our inability to precisely determine the energy of the state, and $\tau_a$ is the lifetime of state $a$.

### 2.5 γ-decay

Most α- and β-decays leave the nucleus in an excited state. If the nucleus is in an excited state, it can decay down to a lower state through emission of one or more γ-rays. If a state above the ground state has a long lifetime, it is called an isomeric state and is denoted as $\frac{1}{2}X^*$. 

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When studying $\gamma$-transitions, there are only certain angular momenta and parities which are possible. A $\gamma$-decay is illustrated in fig. 2.6. Since angular momentum has to be conserved, we have that

$$\vec{J}_i = \vec{L} + \vec{J}_f$$

(2.7)

As $\vec{J}_i$, $\vec{L}$ and $\vec{J}_f$ then must form a closed vector triangle, we get that

$$|J_i - J_f| \leq L \leq J_i + J_f \quad (L \neq 0)$$

(2.8)

The type of multipole radiation depends on if the parity $\Delta \pi$ changes with

$$\Delta \pi = \begin{cases} 
\text{no: even } E, \text{ odd } M, \\
\text{yes: even } M, \text{ odd } E.
\end{cases}$$

(2.9)

Additionally, the lowest possible multipole $X_L$ will generally dominate in transition strength. $EL$ transitions are stronger than an $ML$ transition. Generally, $L + 1$ is less probable than an $L$ multipole emission.

### 2.6 Nuclear level density (NLD)

When it comes to energy levels of the nucleus, lower energy levels are experimentally separable, where $D \gg \Gamma$, $D$ being the spacing between levels and $\Gamma$ the width of the levels. At higher excitation energy, for $D > \Gamma$, levels may be inseparable by detectors, as the resolution is on the same order or larger than the spacing $D$. This area in-between the discrete and the continuum is called the quasi-continuum. In the quasi-continuum, as the excitation energy increases, the nucleus will have transitions that goes from ordered to more chaotic. It is more fruitful to look at the nuclear level density (NLD). When $D < \Gamma$, each level has an inherent width $\Gamma$ that is greater than the spacing $D$, i.e. the continuum region.

The NLD ties into the de-excitation of a nucleus, as it is vital in the decay probability, $\lambda$, from one state to another. To know the decay probability...
from initial state to the final state, using Fermi’s Golden rule, we need to know the NLD $\rho(E_f)$, this is related by eq. (2.5).

2.6.1 Constant temperature model

When extracting the NLD, we need the level density at the neutron separation energy, $\rho(S_n)$ to fit the high energy part. There are many models that can be used to fit up to $\rho(S_n)$, and the constant temperature (CT) model is one of those models.

For excitation energies $E_x$ below 10 MeV, the constant temperature model \[34\] is good for fitting the experimental NLD up to the neutron separation energy $S_n$ with

$$\rho(E_x) = \frac{1}{T} e^{(E-E_0)/T},$$

(2.10)

where the free variables $T$ and $E_0$, the former fitted to a constant nuclear temperature and the latter fitted to energy shift \[15\]. Here the temperature $T$ is constant, as in the name of the model. In this thesis, we will use CT as a model for NLD.

2.6.2 Assumptions on spin and parity of nuclear level density

The total level density $\rho(E_x)$ is the sum of the partial level density dependent on spin and parity \[35\]. For the experimental part, we can only measure the NLD without spin and parity, due to experimental constraint. Instead, we assume that the NLD is independent of spin and parity because of the generalized Brink-Axel hypothesis (section 2.7.1).

The $\rho(E_x)$ is given as

$$\rho(E_x) = \sum_{J,\pi} \rho(E_x, J, \pi)$$

(2.11)

where $\rho$ is the number of nuclear levels per MeV around an excitation energy $E_x$, for a given spin, $J$, and parity, $\pi$. For the level density, some assumptions are made for the spin and parity distribution.

For spin, equiparity is often assumed, with

$$P(E_x, J, \Pi) = \frac{1}{2},$$

(2.12)

Equiparity is an equal distribution of positive and negative parity $P$ for the energy levels in the nucleus. Equiparity is assumed in the experimental analysis.

2.6.3 Spin-cutoff parameter

If the nucleus is assumed to be a rigid body, the spin dependence can be rewritten as \[34\]

$$\sigma^2 = I_{rig} \sqrt{\frac{U}{a}}$$

(2.13)
where the moment of inertia is approximated by

\[ I_{\text{rig}} = 9.65 \cdot 10^{-3} r_0^2 A^{5/3} \text{[h MeV}^{-1}], \]  

(2.14)

where \( r_0 = 1.2 \text{ fm} \) and \( A \) is the mass number. The moment of inertia can change to \( I_{\text{LQ}} \) if we instead look at the nucleus with the liquid drop model.

2.7 \( \gamma \)-strength function (\( \gamma \)SF)

From Ref. [36] the \( \gamma \)-strength function (\( \gamma \)SF) is defined as

\[ f_{J_i \lambda XL}^{I}(E_\gamma) = \frac{\Gamma_{J_i \lambda XL}^{I}}{E_\gamma^{2L+1}} \]  

(2.15)

With \( \Gamma_{J_i \lambda XL}^{I} \) being the averaged \( \gamma \)-ray partial width over \( \lambda \) states with spin and parities \( J \) close to \( E_\lambda \) with a level density \( \rho_J(E_\lambda) \). More easily it can be written as presented in Ref. [35]

\[ f_{XL}(E_\gamma, E_i, J_i, \pi_i) = \frac{\langle \Gamma_{\gamma}^{XL}(E_\gamma, E_i, J_i, \pi_i) \rangle_{XL}}{E_\gamma^{2L+1}} \rho(E_i, J_i, \pi_i) \]  

(2.16)

With \( \langle \Gamma_{\gamma}^{XL}(E_\gamma, E_i, J_i, \pi_i) \rangle_{XL} \) being the partial decay width for an excitation energy \( E_i \), with spin \( J_i \) and parity \( \pi_i \), with a \( \gamma \)-decay with energy \( E_\gamma \) for a multipolarity \( XL \). The \( \gamma \)SF \( f(E_\gamma) \) is related to the transmission coefficient \( T(E_\gamma) \) by [37]

\[ T_{XL}(E_\gamma) = 2\pi E_\gamma^{2L+1} f_{XL}(E_\gamma), \]  

(2.17)

The \( \gamma \)SF is dominated by the giant dipole resonance (GDR), seen in all nuclei across the chart of nuclides. This GDR is often described with a Lorentzian function. The focus of this thesis is the low energy tail of the GDR.

2.7.1 Generalized Brink-Axel hypothesis

One of the important assumptions for the Oslo-method is the generalized Brink-Axel (gBA) hypothesis [38, 39], which states that the properties of nuclei with collective excitation modes built on the excited states will be the same as those built on the ground state [15].

A consequence of the gBA hypothesis is that the \( \gamma \)-ray transmission coefficient, in eq. (2.17), is independent of the excitation energy of the nuclei. With that, \( T_{XL}(E_\gamma) \) is also independent of the nuclear temperature. Another consequence of the gBA hypothesis, is that the \( \gamma \)-strength function \( f \) should be independent of spin, parity and excitation energy, defined as:

\[ f(E_\gamma, E_i, J_i, \pi_i) \approx f(E_\gamma). \]  

(2.18)
2.8 Low energy enhancement (LEE)

Because of the GDR dominating the shape of the $\gamma$SF, it has been assumed that in the tail of the GDR, the $\gamma$SF decreases. Low energy enhancement (LEE), or upbend, is when the $\gamma$SF has large values for the low energy region, typically under 3 MeV. This has been found in several nuclei, depicted in the chart of nuclides in fig. 2.7. The energy trend for the upbend is similar for all Mo isotopes. Assuming the presence of a LEE similar to what was found in Ref. [40, 41] also applies for the neutron rich nuclei, the effect of the LEE on the n-capture rate has been calculated in Ref. [14], as presented in fig. 2.8.

From fig. 2.8 LEE can increase the $(n, \gamma)$ reaction ratios up to a factor 100 for Fe, Mo and Cd isotopes [14]. The figure shows the reaction rate using LEE/without LEE as a function of neutron number $N$. $S_n$ becomes smaller as $N$ increases and so the effect of the LEE increases. There is odd-even staggering, as even numbers of $N$ has a higher neutron separation energy, $S_n$.

This impact of a LEE in the $\gamma$SF can be seen in the $(n, \gamma)$-cross section in fig. 2.8. An increased n-capture rate could affect the calculated abundances of nearby nuclei. There is still no sufficient proof of what transition causes the LEE, but the majority of theories and shell model calculations done predicts that it is caused by $M1$ [3] transitions. However, there is one calculation using QRPA coupled to continuum that claim that a LEE is caused by the $E1$ transition [42].

An experiment by Jones et. al. [43], tried to measure the $E1/M1$ nature
of the LEE in $^{56}$Fe, which was the first discovered case of LEE \[40\]. The results were indecisive, but there was an indication that the LEE had a magnetic character \[43\]. There was another experiment at Argonne in February 2019 to investigate the polarization of $^{56}$Fe \[44\], also to determine the E1/M1 nature of the LEE. The results from this analysis are not yet finished.

![Figure 2.8](image_url)

**Figure 2.8**: Impact on (n,\(\gamma\)) reaction rates with and without LEE, using a generalized Lorentzian model. Figure adapted from Ref. \[14\].

### 2.9 Nuclear plasma interaction (NPI)

In stellar environments, nuclei in high energy density plasma are excited to thermal energies $^{iii}$ through photo-excitation, nuclear plasma interactions (NPI) and inelastic electron scattering $^4$ in the plasma. Formation of heavy nuclei from pre-existing nucleons are theorized to be greatly affected by these high energy density plasma environments $^4$ $^{15}$. A low energy enhancement in the $\gamma$SF can be used to extrapolate to what the $\gamma$ strength will be at below 10 keV. This could help predict the magnitude of the effect of nuclear plasma interactions, which have been measured at the National Ignition Facility (NIF) \[13\] at Lawrence Livermore National Laboratory.

There have been two experiments to measure the NPI with xenon at NIF, but none of them have been conclusive $^{iv}$ due to lack of statistics and fission product contamination from previous experiments using uranium

\[iii\]Where it is assumed that nucleosynthesis happen around 5-100 keV $^{45}$.

\[iv\]One in January 2017 measured the effect within $2\sigma$ $^{46}$. 

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hohlraums. As the nuclear transitions have a width of $\Gamma \leq 1\mu eV$, it is extremely difficult to measure experimentally [4]. With the results of this thesis work, the effect of the NPI can be estimated with more certainty using the measured $\gamma$SF.

### 2.9.1 Nuclear excitation by electron capture (NEEC)

![Diagram of NEEC process](image)

Figure 2.9: Illustration of how the NEEC process changes the decay intensity to the isomer and to the ground state, from the gray arrows to the yellow arrows, marked NPI. Figure adapted from Ref. [46].

One form of nuclear plasma interaction is nuclear excitation by electron capture. In a high energy density plasma, there are electrons that can easily interact with the nuclei. One form of NPI is by resonant capture of an electron from the plasma, into the atomic orbital. The free electron energy plus the binding energy [4] will then be transferred to the nucleus via a virtual photon, exciting the nucleus. The electron mediated NPI is assumed to change the angular momentum distribution of the $^{133}\text{Xe}^*$, leading to a different isomer population than when populated by the $(n, 2n)$-reaction outside of a plasma, ensuring that a significant amount of NPI has taken place. The NEEC process is illustrated in fig. 2.9.

### 2.9.2 Measuring nuclear plasma interaction

The double isomer to ground state (DIGS) ratio $R_{\text{DIGS}}$ can be used to measure the NPI. As the 11/2- isomeric state $^{133m}\text{Xe}$ decays with a $t_{1/2} = 2.198$ days, and 3/2+ $^{138}\text{Xe}$ has a $t_{1/2} = 5.248$ days [8], this ratio can be used to measure the NPI. By measuring the ground state to isomer ratio of a control sample versus the $^{135}\text{Xe}$ populated in the $(n, 2n)$-reaction in the plasma, the NPI can be measured.
\[ R_{\text{DIGS}} = \frac{\frac{N_{\text{plasma}}^{133m\text{Xe}}}{N_{\text{plasma}}^{133g\text{Xe}}}}{\frac{N_{\text{control}}^{133m\text{Xe}}}{N_{\text{control}}^{133g\text{Xe}}}} \quad (2.19) \]

\( N \) is the number of each state populated with the ratio of \(^{133m}\text{Xe} / ^{133g}\text{Xe}\) in the plasma over the control capsule outside of the plasma.

### 2.9.3 Rate of nuclear plasma interaction

The decay rate \( \lambda_d^{\text{NEEC}} \) of the NEEC process in between the initial state \( i \), assumed to be in the quasi-continuum, to all possible final states \( f \) is described [4, 47] as:

\[
\lambda_d^{\text{NEEC}} = \int dE_r \frac{d\Phi(E_r)}{dE_r} (1 - f_{FD}(E_r)) \quad \text{(plasma)} \\
\cdot \sum_{\text{all } b} \alpha(T_e) \ln(2) f_{FD}(E_b) \quad \text{(atomic)} \\
\cdot \frac{2J_f + 1}{2J_i + 1} \frac{E_3}{\bar{h}} S(E_\gamma) \quad \text{(nuclear)} \quad (2.20)
\]

The plasma part consists of integrating over all electron energies \( E_r \) with a differential electron flux \( \frac{d\Phi(E_r)}{dE_r} \) with the final and initial spin \( J_f \) and \( J_i \). Here \( f_{FD} \) is the Fermi-Dirac function, \( E_r \) and \( E_b \) are the free and bound electron energies. With the \( \gamma \)SF defined as

\[
S(E_\gamma) = \frac{\hbar \rho(E_i + E_\gamma, J_f)}{2 \langle T_{i \to f}^\gamma \rangle E_\gamma^3} \quad (2.21)
\]

Here \( E_\gamma = E_r + |E_b| \); \( \rho(E_i + E_\gamma, J_f) \) the level density for final nuclear states \( f \) possible; \( T_{i \to f}^\gamma \) the radiative lifetime of the transition \( i \to f \). Since NEEC is the inverse process of internal conversion, \( \alpha(T_e) \) is the internal conversion coefficient dependent on the electronic temperature of the plasma [47]. For low transition energies \( \alpha \) is important [4] and this is in the LEE-region.
Experimental setup

During the period 10th to the 19th of November, 2017, the data analyzed in this thesis was collected at iThemba LABS in South Africa. Detector calibration runs with sources and background radiation was measured before, during and after the experimental campaign. From the 1st to the 6th of November, data was collected for the inverse kinematics experiment to study the isotope $^{85}$Kr with similar experimental setup.

The goal of the experiment in this thesis is to determine the nuclear level density (NLD) and $\gamma$-strength function ($\gamma$SF) of $^{133}$Xe. We excite $^{133}$Xe through the reaction

$$d(^{132}\text{Xe}, p)^{133}\text{Xe}^*, \quad (3.1)$$

In addition to determining important information on NLD and $\gamma$SF, these quantities can be used to calculate the cross section of the reaction $^{132}\text{Xe}(n, \gamma)^{133}\text{Xe}$, (3.2)

which is not an easy reaction to study directly, as the beam of neutrons and the non-reactive nature of the target in this case would complicate the design of the experiment. Instead we use the n-capture reaction $^{132}\text{Xe}(n, \gamma)^{133}\text{Xe}$ to determine more of the properties of the reaction $^{132}\text{Xe}(n, \gamma)^{133}\text{Xe}$. This reaction has been studied before in Ref. [48, 49], but that was only for a narrow energy range.

The setup is sketched in fig. 3.1, where the beam and direction is illustrated with the red arrow, and some of the detectors are drawn for illustrative purposes. The Doppler shift has been sketched as well.
Figure 3.1: Sketch of setup of experiment. The beam is drawn with a double red line, with target, particle detectors and γ-ray detectors marked in the sketch. In front of the ΔE detector, the aluminum foil is drawn as a gray line. Depending on the emission angle of the γ-rays, they will have Doppler shifted frequencies, which is illustrated.

3.1 Beam facilities

iThemba LABS has a separated section cyclotron (SSC) facility. The cyclotron is used to produce radioactive isotopes for medical use, and in our case, nuclear physics experiments. The SSC can accelerate heavy nuclei up to the xenon mass region.

For the experiment the properties of the beam are given in table 3.1. Beam intensity was ≈ 0.5 pnA. The energy of the beam was chosen such that it would minimize fusion evaporation events [50].

<table>
<thead>
<tr>
<th>Rel. atomic mass [u]</th>
<th>131.9042</th>
</tr>
</thead>
<tbody>
<tr>
<td>Charge state Q</td>
<td>19</td>
</tr>
<tr>
<td>Energy (calculated from NMR field reading) [MeV]</td>
<td>529.91</td>
</tr>
<tr>
<td>Energy per nucleon [MeV]</td>
<td>4.0145</td>
</tr>
</tbody>
</table>

Table 3.1: Beam properties for the $^{132}_{54}$Xe$^{19+}$. The beam source was isotopically enriched $^{132}$Xe, which was 100 % pure after the acceleration in the cyclotron, as the mass and charge status makes it uniquely separable in the cyclotron. As the vacuum in the experiment

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was not perfect, the xenon was originally stripped to \( \text{Xe}^{19+} \), then it picked up two additional electrons, so it had a charge state of 17 once inside the cyclotron. This was a source of complication at the start of the experiment, as the beam was \( ^{132}\text{Xe}^{17+} \), therefore it was not able to go through the first turn of the cyclotron. This was sorted out by stripping the ion source of two additional electrons to \( \text{Xe}^{19+} \).

### 3.2 AFRODITE array

The AFRODITE array consists of eight collimated Clover(Ge) detectors with Compton suppressing BGO-shields. Six large volume LaBr\(_3\)(Ce) and six small volume LaBr\(_3\)(Ce) detectors were also in the array for this experiment. The AFRODITE array setup from the experiment can be seen in the fig. 3.2.

Figure 3.2: AFRODITE array, beamline through array with Clover and LaBr\(_3\)(Ce) detectors.

The detectors used all belonged to iThemba LABS, except three of the large volume LaBr\(_3\)(Ce) detectors, which we brought from the Oslo Cyclotron Laboratory (OCL) for the two inverse kinematics experiments that we conducted at iThemba LABS in November 2017.

---

\(^1\)African Omnipurpose Detector for Innovative Techniques and Experiment

\(^{ii}\)Bismuth germanate
3.3 Detectors

Detector positions and angle relative to beam direction is seen in figure fig. 3.3 where they have been drawn with angles relative to the beamline. Backward angle is $45^\circ$, normal to the beamline is $90^\circ$ and forward angle is $135^\circ$.

For the angles around the $z$-axis, that information is detailed in table 3.2, corresponding to the numbering in fig. 3.3. The positioning of some of the Clovers and LaBr$_3$(Ce) detectors relative to the chamber can be seen fig. 3.4, where the annular $\Delta E$-$E$ has been pulled apart.

Table 3.2: Detector position and spherical coordinates $\theta$ and $\phi$. Here, the $z$-axis is aligned with the beam axis. The $x$-axis is horizontal to the beam line, and the $y$-axis vertical. That leads the angles $\phi$ and $\theta$ defined as: $\phi = 0$ for detector 2, 7 and 14 as it is in the $yz$-plane. $\theta$ is normal to the beamline, in the $xz$-plane, where $\theta < 90^\circ$ is in backward angle from the beam direction.

<table>
<thead>
<tr>
<th>Position</th>
<th>Detector</th>
<th>$\theta$ [$^\circ$]</th>
<th>$\phi$ [$^\circ$]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Clover</td>
<td>135</td>
<td>90</td>
</tr>
<tr>
<td>1.5</td>
<td>Small LaBr$_3$(Ce)</td>
<td>135</td>
<td>22.5</td>
</tr>
<tr>
<td>2</td>
<td>Clover</td>
<td>135</td>
<td>0</td>
</tr>
<tr>
<td>2.5</td>
<td>Small LaBr$_3$(Ce)</td>
<td>135</td>
<td>337.5</td>
</tr>
<tr>
<td>3</td>
<td>Large LaBr$_3$(Ce)</td>
<td>135</td>
<td>270</td>
</tr>
<tr>
<td>3.5</td>
<td>Small LaBr$_3$(Ce)</td>
<td>135</td>
<td>200.5</td>
</tr>
<tr>
<td>4</td>
<td>Clover</td>
<td>135</td>
<td>180</td>
</tr>
<tr>
<td>4.5</td>
<td>Small LaBr$_3$(Ce)</td>
<td>135</td>
<td>112.5</td>
</tr>
<tr>
<td>5</td>
<td>Large LaBr$_3$(Ce)</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>6</td>
<td>Clover</td>
<td>90</td>
<td>45</td>
</tr>
<tr>
<td>7</td>
<td>Clover</td>
<td>90</td>
<td>0</td>
</tr>
<tr>
<td>8</td>
<td>Large LaBr$_3$(Ce)</td>
<td>90</td>
<td>315</td>
</tr>
<tr>
<td>9</td>
<td>Large LaBr$_3$(Ce)</td>
<td>90</td>
<td>270</td>
</tr>
<tr>
<td>10</td>
<td>Clover</td>
<td>90</td>
<td>225</td>
</tr>
<tr>
<td>11</td>
<td>Clover</td>
<td>90</td>
<td>180</td>
</tr>
<tr>
<td>12</td>
<td>Clover</td>
<td>90</td>
<td>135</td>
</tr>
<tr>
<td>13</td>
<td>Large LaBr$_3$(Ce)</td>
<td>45</td>
<td>90</td>
</tr>
<tr>
<td>13.5</td>
<td>Small LaBr$_3$(Ce)</td>
<td>45</td>
<td>22.5</td>
</tr>
<tr>
<td>14</td>
<td>Large LaBr$_3$(Ce)</td>
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<tr>
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<td>Large LaBr$_3$(Ce)</td>
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<td>270</td>
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<td>16</td>
<td>Large LaBr$_3$(Ce)</td>
<td>45</td>
<td>180</td>
</tr>
<tr>
<td>16.5</td>
<td>Small LaBr$_3$(Ce)</td>
<td>45</td>
<td>112.5</td>
</tr>
</tbody>
</table>
Figure 3.3: Positions and angles of detectors around beam. Here, the $z$-axis is aligned with the beam axis. The $x$-axis is horizontal to the beam line, and the $y$-axis vertical.

### 3.3.1 Si-detectors

Silicon detectors are used to detect charged particles as it is a semiconductor. The annular single sided silicon detectors with a central hole, (Micron Semiconductor model S2), have 16 sectors and 48 rings, such that the angular distribution of charged particles can be measured [51].

We used two silicon detectors, a $\Delta E$ and an $E$ detector. In fig. 3.4, the $\Delta E$-E is pulled apart. The silicon detectors both have a thickness of 1000 $\mu m$, though a $\Delta E$-E setup will usually have a thinner $\Delta E$ detector. To prevent noise from $\delta$-electrons in the energy spectra, the $\Delta E$-detector had a thin aluminum foil in front of it, with a thickness of $\approx 10 \mu m$. The targets were placed 30 mm upstream of the $\Delta E$-E, with an angular coverage from $\approx 19^\circ$ to $47^\circ$.

For the data analysis, we are only interested in events with particle-$\gamma$ coincidences. As all the sorting is done offline, the events from the $E$ detector will be used to gate on the $\gamma$-events so that if an event is detected in the $E$ detector, coincidence $\gamma$-rays will be sorted. The particle energy is used to determine the excitation energy of the $^{133}\text{Xe}$.
3.3.2 LaBr₃(Ce)

12 LaBr₃(Ce)-detectors were used in the experiment, consisting of six large volume detectors (3.5” × 8”) and six small volume detectors (2” × 2”). The large volume detectors have a higher efficiency compared to the small volume detectors, however, they will have slightly worse time resolution due to the larger volume. Thus, the small volume detectors can be

---

iii Along with a PhD student in their natural habitat.
used for fast-timing purposes and are used for time-calibration of the
other detectors. The LaBr$_3$(Ce)-detectors have lower resolution than the
germanium detectors, but for higher $\gamma$-ray energies, the detector efficiency
is much better than for the Clovers, where the efficiency decreases
significantly after 1-2 MeV [52]. We are interested in detecting $\gamma$-rays up
to the neutron separation energy $S_n = 6435.9$ keV [53], which makes the
LaBr$_3$(Ce) detectors crucially important.

3.3.3 Germanium detectors with BGO-shields

![Clover detector from backside, attached through clear plastic cables with black insulation to liquid nitrogen cooling.](image)

In the detector array, we had eight collimated Clover detectors with BGO-
shields. The Clover detectors each have four high purity germanium
crystals (HPGe) with dimensions $50 \times 50 \times 70$ mm. Since it is a HPGe-
detector, the energy resolution for Clover detectors are typically less than
1.05 keV at 122 keV and 2.1 keV at 1.33 MeV, with 2.3 keV at 1.33 MeV in
add-back mode. In fig. 3.5, a Clover used in the experiment is shown.

As there are four crystals, the energy of an event has to be the sum of
the energy deposited in all four crystals simultaneously, which are added
together in the offline sorting. Although unlikely, there can be an event
registered with two separate $\gamma$-events in a crystal, but we assume all energy
in one event is from a single $\gamma$-ray, the energy spread is assumed to be from
Compton scattering.

The BGO-shield is there to suppress the Compton scattering to increase
the peak to total energy ratio. If an event is detected in the BGO-shield
in coincidence with an event in the Ge-detector, the data is recorded, but discarded during the sorting routine as it would add to the Compton background.

For some semi-conductors like HPGe detectors, it is possible that the room temperature electrons in valence band have enough energy to jump to the conduction band in the Ge-detector, to prevent this, the HPGe detector is cooled using liquid nitrogen.

3.4 Data acquisition

For data acquisition, the events were collected with Pixie-16, a digital Gamma-ray processor from XIA. Each crate has a 16 channel data acquisition system, either with a 100 MHz or 500 MHz sampling rate [54]. The 100 MHz is used for the Clovers and the ∆E-E detectors, while the 500 MHz is used for all the LaBr₃(Ce) detectors.

The events collected are registered with a timestamp, therefore coincidences from the particle telescope and the γ-detectors must be sorted off-line. The crates were synchronized with a pulsar to ensure that they were in sync with each other, producing a singular line in the spectrum when in sync.

3.5 Targets

![Figure 3.6: Burned out C₂D₄-targets.](image)

The targets were made of deuterated polyethylene, C₂D₄, which had an enrichment of ≈ 99% and thicknesses of 0.5 – 1.1 mg/cm². The targets were made using a new technique developed at iThemba LABS [55]. The new production method made the targets with a low amount of pollution, as it did not rely on any release agents. Since the polyethylene was
highly enriched, the availability of deuterons was high, such that the wanted reaction $^{132}\text{Xe}(d,p)^{133}\text{Xe}$ could happen easily without unwanted contaminants.

Figure 3.7: Target ladder in chamber in front of $\Delta E$-detector. In top position is the ruby, which was used for beam tuning. Second, empty frame. Third and fourth, deuterated polyethylene targets. The $\Delta E$-detector is covered in aluminum foil to stop the $\delta$-electrons, which have low energy.

With a plastic target, it was hard to determine if the beam burned a hole immediately. As the beam hit the target, the reactions would take place in a halo around the beam. The beam was defocused on purpose, allowing for a larger area being irradiated, while reducing the risk of the target melting. The targets would then burn through gradually with counts dropping significantly when the target was burnt through. See fig. 3.6 with the burned out targets. This uncertainty was because we could not see into the chamber while the experiment was happening, only look at the targets when we opened the chamber.
When the counts dropped low enough, the person on shift would change the target position to a new fresh target. The target ladder with unused targets can be seen in fig. 3.7. To maximize our statistics, we had to optimize how often targets would get swapped out. Changing all targets in the ladder took at least half an hour, as the vacuum had to be aired out.
Data analysis

My machine requires the most delicate calibration. Forgive me if I seem overcautious.

– Dr. Arnim Zola, Captain America I 17:40

Before starting to extract the statistical properties of $^{133}$Xe, there are multiple steps that needs to be done. Detector calibration, time alignment, time gating, sort gated data into ROOT TTrees, Doppler correction and calculation of excitation energy. Only after all those steps are completed can the statistical properties be extracted successfully from the particle-$\gamma$ coincidences. With the ROOT trees it is easy to make the wanted gates on the data and access all relevant properties of the recorded events.

4.1 Detector Calibration

The first step of the data analysis is detector calibration. As the channel number from the data acquisition does not necessarily match the energy of the radiation, some calibration of the detectors is needed. To find the right energy for each channel, the gain and shift for each detector needs to be found using:

$$E = \text{ch\#} \cdot \text{gain} + \text{shift},$$

(4.1)

where $E$ is the energy of the characteristic radiation, ch\# is found from doing a Gaussian fit of the peak in the spectra. To find the gain and the shift, we need at least two peaks with known energies to solve a set of equations. For an optimal calibration, we choose several peaks spanning energies of interest.

4.1.1 $\gamma$-calibration

To calibrate the $\gamma$-spectra, the $^{60}$Co source was used first. $^{60}$Co emits to characteristic $\gamma$-rays with energies 1173 keV and 1332 keV. As this only gives two calibration points, it is necessary with multiple sources. We used $^{152}$Eu which have several visible peaks for calibration in the low energy part of the spectra. This calibration was done with a least squares fit of the

\footnote{I have chosen to write we instead of I throughout the thesis, as in "the reader and I".}
Table 4.1: Calibration peaks used for $\gamma$-detectors.

<table>
<thead>
<tr>
<th>Source</th>
<th>Energy [keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{60}$Co</td>
<td>1173</td>
</tr>
<tr>
<td></td>
<td>1331</td>
</tr>
<tr>
<td>$^{152}$Eu</td>
<td>121</td>
</tr>
<tr>
<td></td>
<td>1408</td>
</tr>
</tbody>
</table>

fitted peaks, see table 4.1, from both the $^{60}$Co and the $^{152}$Eu source.

The calibrated LaBr$_3$(Ce) spectra can be seen in fig. 4.1, where five of the detectors are well aligned, the last one have fewer counts, but is aligned to the degree it can be. Something must have blocked the source during the calibration run for LaBr$_3$(Ce) number 4, but in the data there was not a significant difference in the statistics for the LaBr$_3$(Ce) in the same angle.

The calibrated Clover spectra can be seen in fig. 4.2, for one Clover detector with four crystals. Each Clover detector has four different gain and shifts, for each crystal. The alignment is good for the energy range.

$^{60}$Co and $^{152}$Eu

With its two characteristic lines, $^{60}$Co was the natural place to start calibrating the $\gamma$-detectors. The Gaussian fit done for the 1173 keV line can be seen in fig. 4.3.

$^{152}$Eu primarily decays through electron capture [16] and was used to for a broader energy calibration in addition to the two characteristic lines from $^{60}$Co. This calibrations source has the advantage of many more distinct $\gamma$-energy lines.

4.1.2 Particle detector calibration

For the particle telescope, we used a $^{226}$Ra source, which undergoes $\alpha$-decay. This produces a cascade of decays, both $\alpha$ and $\beta^-$-decay. Multiple $\alpha$-decays makes it possible to do a sufficient calibration for a broad energy range, see table 4.2.

Table 4.2: Calibration peaks used for particle detectors from the $^{226}$Ra calibration source.

<table>
<thead>
<tr>
<th>Decay</th>
<th>Energy $\alpha$ [keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{226}$Ra$\rightarrow$ $^{222}$Rn $+ \alpha$</td>
<td>4784.34</td>
</tr>
<tr>
<td>$^{222}$Rn$\rightarrow$ $^{218}$Po $+ \alpha$</td>
<td>5489.48</td>
</tr>
<tr>
<td>$^{218}$Po$\rightarrow$ $^{214}$Pb $+ \alpha$</td>
<td>6002.35</td>
</tr>
<tr>
<td>$^{214}$Po$\rightarrow$ $^{210}$Pb $+ \alpha$</td>
<td>7686.82</td>
</tr>
</tbody>
</table>

The source measurements were done in vacuum for the $\Delta E$ and $E$ separately, otherwise the $\alpha$ particles from the source will be stopped in the
Figure 4.1: Calibrated LaBr$_3$(Ce) detectors with the $^{152}$Eu source.
Figure 4.2: Calibrated Clover detector with four segments aligned on top of each other with the $^{152}$Eu source.
ΔE detector.

ΔE detector

There was some drifting in the ΔE detector, so the gain and shift changed between the calibration runs, see fig. 4.4. Three weeks had passed between the first and last calibration round, which meant that the gain and shift had changed enough to make the first calibration round less accurate. This lead to using the calibration run after the $^{133}$Xe experiment, when it was possible. As there was a fault with one of the XIA cards, that lead to losing every even numbered ring from number 17-47 for more than half of the total beam time. This means that there is less data available from the experiment, since we lose so many rings. Less angular data means less coverage of high $E_x$ values. Since the energy registered in the rings are used for gating on the proton banana in the ΔE-E plot and only that, the energy calibration of the ΔE sectors are more important than the rings.

For the 16 ΔE sectors, see the calibrated peaks in fig. 4.5. The four peaks used for calibration are well aligned.
Figure 4.4: When the $\Delta E$ detector is calibrated with the run before the krypton experiment, the calibrated 7686.82 keV peak is not well aligned for the 16 different $\Delta E$ sectors. This is with the $^{226}\text{Ra}$ source.
Figure 4.5: Calibrated peaks for the 16 different $\Delta E$ sectors with the $^{226}\text{Ra}$ source.
**E detector**

The last calibration run for the E-sectors had a poor resolution, which meant that only half of the sectors had distinguishable spectra that could be properly calibrated. Which sectors are resolvable can be seen in table 4.3. To counteract this, the run before the krypton experiment had to be used to find a proper energy calibration.

Table 4.3: How resolvable the sectors of the E-counter from the $^{226}$Ra calibration source. Where the spectra were not resolvable, the calibration run before the krypton experiment had to be used instead.

<table>
<thead>
<tr>
<th>E-sector</th>
<th>Resolvable</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Yes</td>
</tr>
<tr>
<td>1</td>
<td>Yes</td>
</tr>
<tr>
<td>2</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>Yes</td>
</tr>
<tr>
<td>4</td>
<td>Yes</td>
</tr>
<tr>
<td>5</td>
<td>No</td>
</tr>
<tr>
<td>6</td>
<td>No</td>
</tr>
<tr>
<td>7</td>
<td>No</td>
</tr>
<tr>
<td>8</td>
<td>No</td>
</tr>
<tr>
<td>9</td>
<td>No</td>
</tr>
<tr>
<td>10</td>
<td>Yes</td>
</tr>
<tr>
<td>11</td>
<td>No</td>
</tr>
<tr>
<td>12</td>
<td>No</td>
</tr>
<tr>
<td>13</td>
<td>Yes</td>
</tr>
<tr>
<td>14</td>
<td>Yes</td>
</tr>
<tr>
<td>15</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Finding gain and shift

With multiple sources, gain and shift were found using the least-squares method, as there were many points to fit. This will ideally get adequate calibration for all energies relevant for the experiment. For $\Delta E$ sector 0, the fit can be seen in fig. 4.6. Here the calibration points align, which produces a good calibration of that sector. For each least squares fit on each detector, we plotted the equivalent plot to check if all the points were properly aligned (they were).

![Least squares fit](image)

**Figure 4.6**: Least squares for a $\Delta E$ sectors with the $^{226}$Ra source. $y = 2.05033x + 138.468$

The least square fit to find the gain and shift where done for each detector, for both $\gamma$ and particle detectors.

4.2 Time alignment

As the detectors are independently self-triggered, they have to be aligned in time. Proper time alignment allows for prompt event gating. For one run, the aligned $\Delta E$ sectors can be seen in fig. 4.7.

To get the timing calibration right, one of the fast timing LaBr$_3$(Ce) $2 \times 2$ detector is used as a reference start time, $t_{\text{start}}$, to which we align all other detectors, their time being $t_{\text{detector}}$ with $t = t_{\text{detector}} - t_{\text{start}}$. For each event, there can be more than one $\gamma$ registered in all of the detectors.
Figure 4.7: Time alignment for sectors in ΔE detector.
4.3 Mislabeled rings

Figure 4.8: When plotting the number of counts in the rings, one would expect the number to decrease with the angle, but in the sorted data, it did not. This error had to be fixed to get the right excitation energy. The staggering was due to a faulty XIA card about midway in the experiment, connected to every other ring.

For the $\Delta E$ detector, the ringID to channel number had been mislabeled for the outer rings. This was apparent because the count rate for the outermost rings were two orders of magnitude greater than the middle rings for run 84-99. This can be seen in fig. 4.8.

There was also more evidence that suggested mislabeled ringIDs. In the $\Delta E$-E matrix, the particles in the "outer" rings had punch-through, the "middle" did not. This would suggest that the particles with a greater $\theta$ had higher energy, which is contradicts what is expected based on kinematics. When the $^{133}$Xe is highly excited in the (d,p)-reaction, the proton will then have a lower energy and a greater $\theta$.

When mislabeled, the mismatch with the rings and the angles can be seen in fig. 4.9, where the angle was off by 5° in fig. 4.9a and 16° in fig. 4.9b. After correcting the ringIDs, the counts decreased when $\theta$ increased, as we would expect. From fig. 4.8, ringID = 1 has more counts, but there was a disproportionate amount of noise from that ring, thus it was excluded from the rest of the analysis.
(a) $\Delta E$-E for ringID = 26, before correcting the angle was sorted as $\theta_{\text{wrong}} = 37^\circ$. After correction, the angle was $\theta_{\text{right}} = 42^\circ$.

(b) $\Delta E$-E for ringID = 47, before correcting the angle was sorted as $\theta_{\text{wrong}} = 47^\circ$. After correction, the angle was $\theta_{\text{right}} = 31^\circ$.

Figure 4.9: $\Delta E$-E for two different rings for run 84–99. Here the events have been gated on multiplicity = 1 in $\Delta E$ and $E$ sectors, and multiplicity $\leq 2$ for $\Delta E$ rings. In the "outer" ring, the particles have more energy, more counts and there is also punch-through in the $\Delta E$-$E$, whereas for ringID = 26, there is significantly fewer counts, no apparent deuteron shape and no punch-through.
4.4 Get to the matrix

To get the $E_x$ vs. $E_\gamma$ matrix, there are some steps that needs to be done for all the runs, they are outlined below:

1. time align run
2. time gate $\Delta E$-E events for run
3. time gate particle-$\gamma$ coincidences for LaBr$_3$(Ce)-detectors
4. make proton cut for each ring
5. make $E_\gamma$ vs $E_x$ matrix for runs.
6. add everything to one chain in ROOT

Comparing the difference for the first and the last runs, there are not a significant differences in other than some slight adjustments in the time gating for each event.

4.5 Particle event selection

Before the final coincidence matrix is finished, we need to apply more gates on the data. To ensure we get the full proton energy, gates on the multiplicity is needed. Requiring that there is only one hit in the E-detector ensures we see events with one particle, which have enough energy to make its way through the $\Delta E$ detector. After that, the multiplicity of the $\Delta E$ can be restricted to one hit in the sectors, and a maximum of two hits in the rings. When two rings are hit simultaneously, the event is only kept if they hit neighboring rings, where the lowest ring number is used to calculate the angle of the proton.

4.5.1 Punch-through

The protons and the deuterons had enough energy to travel through the particle telescope, which gives a punch-through effect. It is visible in the $\Delta E$-E plot in fig. [4.11]. This lead to some unusable data where it doubles back and overlap, as a punch-through event cannot be separated from a stopped particle in that energy region. Only data without overlap was used.
In the $\Delta E$ versus $E$ plot, the punch-through of the particle telescope is visible as the reversal of the so-called bananas. This is for the innermost ring of the particle telescope (ringID = 0), not gated on time.

### 4.5.2 Time gate for $\Delta E$-$E$ events

Figure 4.11: $\Delta t$ for the $\Delta E$-$E$ sectors, plotted with the energy of the E-detector. Gating on this will help find the events in coincidence, as they have hit both detectors.

In fig. 4.11 we can gate on the events that happen around $\Delta t \approx 0$ in the $\Delta E$-$E$ counter to find the true particle events. Next, we can make a finer gate in time with the rings and the sector of the $\Delta E$ counter.
4.5.3 Calculating excitation energy

When calculating the excitation energy as a function of the proton energy, we have that

$$E_x(E + \Delta E) = a_0 + a_1 (E + \Delta E) + a_2 (E + \Delta E)^2$$  \hspace{1cm} (4.2)

To find $E_x(E + \Delta E)$ from eq. (4.2) we need to

1. Find the $E_{\text{loss}}$ of the beam, that is how much energy the beam deposits in the deuterated polyethylene target.

2. Find the energy of the reaction $E_{\text{reaction}} = E_{\text{beam}} - E_{\text{loss}}$, see appendix A and eq. (A.22).

3. Vary the proton energy $E_p$ from the minima, 9 MeV, required to pass through the $\Delta E$-E detector for each $\theta_i$ of the annular $\Delta E$ detector.

4. Find $E_x(E_p, \theta)$.

5. Fit a second degree polynomial of $E_p$ versus $E_x$ to find coefficients $a_0$, $a_1$, and $a_2$ on eq. (4.2).

When we have done that, we can finally calculate $E_x(E + \Delta E)$. Thus we can finally make an $E_x$ vs. $E_\gamma$ matrix for each run. More on the calculation of $E_x$ in appendix A.

4.6 Coincidence $\gamma$-rays

To produce the $E_x, E_\gamma$-matrix, the $\gamma$ events needs to happen within a small time window. After gating on the time window, we can apply a Doppler correction on the $E_\gamma$ before we do background subtraction.

4.6.1 Time gating particle-$\gamma$ events

With both a proton emitted and one or more $\gamma$-rays emitted for each recorded event, we need to find a suitable time limit for when the they happen in coincidence with each other. For each LaBr$_3$(Ce) detector we have to find the right time gate.

After finishing time alignment, we can plot the time difference from an event in the $\Delta E$ and E detector as a function of the energy deposited in the E detector. Doing so, we can make a graphical cut on the events in the coincidence. Then, plotting the time difference again with said cut, we can find a time interval for particle events. After that, we plot the time difference in the $\Delta E$ sectors and $\Delta E$ rings to narrow the time interval for a particle event. With the previous time condition, this will remove background caused by $\delta$ electrons.

With the particle interval in place, we can find the time interval for a coincidence event, that is an event with a particle detected, along with $\gamma$-rays.
4.6.2 Doppler correction

For a photon emitted from a moving source, there will be a change in frequency \( \nu \) from the inertial rest frame to another rest frame \( \nu' \). The Doppler-corrected energy will be given as

\[
E'_\gamma = \gamma (1 - \beta \cos \theta) E_\gamma
\]

where \( E_\gamma \) is the energy measured directly by the detectors, \( E'_\gamma \) is the energy of the \( \gamma \)-ray in the rest frame of \(^{133}\text{Xe}\). \( \theta \) is the angle to the beam in the lab frame, \( \gamma \) is the Lorentz factor, where

\[
\gamma = \frac{1}{\sqrt{1 - \beta^2}},
\]

and \( \beta = v/c \), where \( v \) is the relative velocity compared to the inertial rest frame. This works for non-relativistic velocities as well, as for \( v \ll c \), it is the classical Doppler shift.

As the \(^{133}\text{Xe}\) nucleus is moving at a high velocity, the emitted \( \gamma \)-rays from the nuclei will be Doppler-shifted. The Doppler-shift has to be corrected for after the experiment. The detectors that are 90° to the beamline will have no Doppler shift, but those in backwards and forward angles have to be adjusted. The mass of a \(^{133}\text{Xe}\) nuclei will be the rest mass plus the excitation energy of the nucleus.

To find Doppler corrected energy \( E'_\gamma \), the first step is finding \( \beta \). By varying the energy of the proton \( E_p \) registered in the \( \Delta E-E \), \( E_p = [9, 40] \)
MeV, and the angle of the proton $\theta_p \leq 47^\circ$, it is possible to calculate all possible excitation energies $E_x$ of $^{133}$Xe$^*$. With the proton angle and energy, the angle $\theta_{^{133}\text{Xe}^*,\text{lab}}$ can be calculated. From this we calculate $\beta$ and $\gamma$ with

$$\gamma = \frac{K_4 + m_{^{133}\text{Xe}} + E_x}{m_{^{133}\text{Xe}} + E_x},$$

(4.5)

where $K_4$ is the kinetic energy of $^{133}$Xe, $m_{^{133}\text{Xe}}$ the mass of $^{133}$Xe.

To find this $\beta$ value, looking at the possible $\beta$ values with $\theta_p \leq 47^\circ$ and $E_x < 12$ MeV$^\text{ii}$ the results can be seen in fig. 4.12 where $\beta \in [0.0860, 0.0875]$. Approximating to $\beta = 0.0865 \pm 0.001$ will simplify calculating the shifted Doppler energy, $E'_\gamma$.

### 4.7 Background

As the LaBr$_3$(Ce) has internal radiation, the $\gamma$-detectors have a clear background from this, visible in fig. 4.13. Gating on the $E_x(\Delta t) = E_x(t_{\Delta E} - t_{LaBr_3})$ can be used to find the $\gamma$-rays not from internal radiation. This can be done because of the great time resolution of the LaBr$_3$(Ce) detectors, as seen in fig. 4.14.

The gates on the prompt peaks can be seen in fig. 4.15a. Some issues with the XIA system has caused the small split of the prompt peak, but all those events are within the applied gates for particle-$\gamma$ coincidences and therefore used in this analysis.

Using the same gates, only shifted in $\Delta t$, this can be used for the background subtraction, as all the same conditions have been placed on the events. These gates are shown in fig. 4.15b. When gating on both of the prompt peaks, the resulting particle-$\gamma$ coincidence matrix is shown in fig. 4.14 but the $E_\gamma$ is not corrected for Doppler shift. Figure 4.14 has no prominent peak from the internal radiation after gating on prompt events and removing background.

---

$^\text{ii}$Though the excitation energy will most likely exclusively $E_x < S_n$ for all events.
Figure 4.13: Background from the internal radiation from the LaBr$_3$(Ce) detectors marked with red box.

Figure 4.14: Background subtracted and gated on prompt peak. No Doppler correction.
(a) $\Delta t$ to LaBr$_3$(Ce) with the $E_\gamma$. Making a cut on the prompt $E_\gamma$ shown as the black lines.

(b) $\Delta t$ to LaBr$_3$(Ce) with the $E_\gamma$. Making a gate in same shape for background subtractions shown as the red lines.

Figure 4.15: Gating on prompt particle-$\gamma$ events with background subtraction.
4.8 Coincidence matrix

With Doppler corrections, we end up at the starting point for the Oslo-method. Since it is possible to apply narrow time gates with the LaBr$_3$(Ce) detectors, the internal radiation is removed from the coincidence matrix. The particle-$\gamma$ matrix is in fig. 4.16 after background subtraction and gating.

![Coincidence matrix](image)

Figure 4.16: Starting point for Oslo-method, the particle-$\gamma$ coincidence matrix after gating on prompt $\gamma$-energies and Doppler correction.
The Oslo Method

"Just because something works doesn’t mean that it cannot be improved."

– Princess Shuri, Black Panther 14:45

The inverse-Oslo method is identical to the Oslo method after the $\gamma$-events have undergone Doppler corrections, see section 4.6.2, and the $\Delta E-E$ energy is used to calculate the excitation energy of $^{133}$Xe, in appendix A. The Oslo method was developed at the University of Oslo, Ref. [1], as a way to determine the statistical properties of nuclei as the $\gamma$SF and NLD.

5.1 The Oslo method in a nutshell

1. Unfolding of the $\gamma$-ray spectra [12] using the appropriate detector response function to account for how $\gamma$ interact with matter.
2. Find the first generation $\gamma$-matrix by extracting the primary $\gamma$-rays [11].
3. Extraction of the level density and strength function [1], assuming the Brink-Axel hypothesis [38, 39].
4. Normalization [56] of the NLD using neutron resonance spacing $D_0$.
5. Normalization of transmission coefficient for $\gamma$SF

5.2 Unfolding

Since detectors are not perfect, we need to find a way to unfold the detected $\gamma$-rays with the response of the detectors, so it will be closer to the true $\gamma$-ray energy.

The $\gamma$-spectra have a Compton contribution that can be corrected for, along with single escape and double escape caused by pair production. To correct for the Compton contribution, it is possible to smooth the Compton contribution and add its contribution back to the full energy peak from the $\gamma$-spectra [12]. This procedure even works with low statistics [12]. To do the unfolding, detector response functions are necessary.
5.2.1 Detector response function

For the unfolding to be done properly, there has to be some experimentally or simulated detector response function $R(E, E_{\gamma})$ available [12]. In this case, there were no detector response function available for the small volume LaBr$_3$(Ce) detectors, the data from them are not included from this point in the thesis work.

An event can be a part of five different processes in the spectrum. The ideal case is the full energy (f). If one 511 keV $\gamma$-ray escapes, it is a single escape (s), if two 511 keV escape, it is a double escape (d) and annihilation (a). The rest is of the spectrum its Compton events (c) [12]. Since this is assumed to be all type of events, the probabilities, $p$, of these events sum up to 1, we have

$$p_f + p_s + p_d + p_a + p_c = 1 \quad (5.1)$$

given that we can measure energy from zero energy, if not, extrapolation is needed. To get a new response function for an intermediate full energy $E_{\gamma}$, an interpolation between the known peak structures is performed. At the interpolated peak position, a Gaussian distribution is added with the proper intensity and energy resolution [12]. The case of the Compton background is more difficult, see fig. 5.1. For the full description of the interpolation method, see [12].

![Interpolation of Compton background. Figure from [12.]](image-url)
5.2.2 Iterative folding

Since unfolding directly is difficult, we can use iterative folding by guessing what the unfolded spectra \( u \) should look like. The first guess of \( u \) is the observed spectra \( r \).

\[
\begin{pmatrix}
  f_1 \\
  f_2 \\
  \vdots \\
  f_N
\end{pmatrix} =
\begin{pmatrix}
  R_{11} & R_{12} & \cdots & R_{1N} \\
  R_{21} & R_{22} & \cdots & R_{2N} \\
  \vdots & \vdots & \ddots & \vdots \\
  R_{N1} & R_{N2} & \cdots & R_{NN}
\end{pmatrix}
\begin{pmatrix}
  u_1 \\
  u_2 \\
  \vdots \\
  u_N
\end{pmatrix}
\] (5.2)

Equation (5.2) can be written as \( f = Ru \). Here \( f \) is the folded spectra, \( u \) the unfolded and \( R \) the response function matrix, where \( R_{ij} \) is defined as the response in channel \( i \) for a \( \gamma \)-ray in the detector with energy corresponding channel \( j \).

The iterative method works as follows [12]:

1) Start with a trial function
\[
u^0 = r
\] (5.3)

where \( r \) is the observed spectra.

2) Calculate the first folded spectra
\[
f^0 = Ru^0
\] (5.4)

3) Find a new trial function \( u^1 \) by
\[
u^1 = u^0 + (r - f^0)
\] (5.5)

4) Fold again to get \( f^1 \), then find the next trial function \( u^2 \):
\[
u^2 = u^1 + (r - f^1)
\] (5.6)

and repeat the iteration until \( f^i \sim r \), with \( i \) representing the iteration index.

After the iterative folding, we end up with the final unfolded spectra \( u \).

5.2.3 Compton subtraction method

The starting point of the Compton subtraction method is the unfolded spectrum \( u \). For the spectrum, the goal is to subtract the Compton distribution \( w(i) \) from the observed energy.

The Compton background spectrum \( c(i) \) is found by
\[
c(i) = r(i) - v(i)
\] (5.7)

with
\[
v(i) = p_f(i)u_0(i) + w(i) = u_f(i) + u_s(i) + u_d(i) + u_a(i)
\]
The Compton background is smoothed, often with a resolution of 1.0 FWHM, as it should be a slowly varying function of energy [12]. Afterwards, the structures $w$ and the smoothed Compton part $c$ is subtracted from the observed spectra $r$, with the unfolded part $u$ for full energy ($u_f$), single escape ($u_s$), double escape ($u_d$) and annihilation ($u_a$), then corrected for the probability of full energy $p_f$:

$$u(i) = \frac{r(i) - c(i) - w(i)}{p_f(i)}$$  \hspace{1cm} (5.8)

Lastly, the true $\gamma$-ray energy distribution $U(i)$ is calculated by correcting for efficiency $\epsilon_{\text{tot}}$

$$U(i) = \frac{u(i)}{\epsilon_{\text{tot}}}$$  \hspace{1cm} (5.9)

Further detail on the Compton subtraction method can be found in [12].

For the experiment, the unfolded spectra is shown in fig. 5.2. The detector response function was the same as used in [2], simulated using GEANT4. This response function only accounts for two LaBr$_3$(Ce), instead of six as were used in the experiment now, which might affect the environment enough to warrant a new simulation.

Figure 5.2: The unfolded coincidence matrix for the large volume LaBr$_3$(Ce) detectors.
5.3 First generation method

From highly excited states, γ-decay will usually happen in a cascade of transitions. As this happens faster than the available timing resolution of detectors, the first transition has to be found using the first generation method [11], as the spectra contains all generations of γ-decays. It is assumed that the populated states after the first γ transition will have the same properties as particle reactions directly populated states at the same excitation energy [11], since we assume that gBA hypothesis is valid for high level density regions.

From the particle-γ coincidence we produce bins \( f_i \) of excitation energy. For the highest excitation energy \( (f_1) \), the first generation γ-ray spectrum \( h \) is estimated as

\[
h = f_1 - g,
\]

where \( g \) is the sum of all spectra \( f_i \), weighted by the normalization factor \( n_i \):

\[
g = \sum_i n_i w_i f_i,
\]

where \( w_i \) is an unknown coefficient which describes the probability of decay from bin 1 to bin \( i \). After unfolding the primary coincidence matrix with the detector response function, \( w \) corresponds to the first generation γ-ray spectrum \( h \). Because of this relation, the \( w_i \) can be determined through a converging iterative procedure.

For bin \( i \), the normalization factor \( n_i \) is

\[
n_i = \frac{S_1}{S_i},
\]

with \( S_1 \) and \( S_i \) being the cross section of the singles particle for bin 1 and \( i \) respectively. Then the γ-ray multiplicity \( M_i \) is assumed to be known, the multiplicity normalization \( n_i \) is then

\[
n_i = \frac{M_i A(f_1)}{M_1 A(f_i)},
\]

with \( A(f_1) \) and \( A(f_i) \) representing the number of counts in the spectra \( f_1 \) and \( f_i \), \( M_1 \) is the multiplicity of bin 1.

Checking eq. (5.10) for area consistency, \( A(h) \) is the γ-ray multiplicity of one unit, where

\[
A(h) = A(f_1) - \alpha A(g)
\]

with an introduced correction factor \( \alpha \), close to unity. \( A(h) \) can also be expressed as

\[
A(h) = \frac{A(f_1)}{M_1}
\]

and with eqs. (5.14) and (5.15), \( \alpha \) can be expressed as

\[
\alpha = \left( 1 - \frac{1}{M_1} \right) \frac{A(f_1)}{A(g)}
\]
If a poor weighting function is chosen, the introduction of the correction factor $\alpha$ can be helpful to compensate.

The procedure as presented in Ref. [11]:

1. Apply trial function $w_i$.

2. Deduce $h$.

3. Transform $h$ to $w_i$ by unfolding.

4. Continue from 2. until $w_i(\text{new}) \approx w_i(\text{old})$.

The procedure is illustrated in fig. 5.3.

The first generation matrix for the experiment can be seen in fig. 5.4, where the matrix has 200 keV/bin for both excitation energy $E_x$ and $\gamma$-ray energy $E_\gamma$, such that the bin width is larger than the experimental uncertainty in calculation of the excitation energy, which is dependent on the angular resolution of the $\Delta E$ detector.
5.4 Extracting NLD and transmission coefficient

When the first generation method has been applied to the spectra, we have a normalized first generation matrix \( P(E_x, E_\gamma) \), and from that, we want to end get to the NLD, \( \rho(E_x) \), and the \( \gamma \)SF, \( f(E_\gamma) \).

From Fermi’s golden rule, given in eq. (2.5), we see that the probability of decay from state \( i \) to \( f \) depends on the available level density \( \rho(E_f) \).

From the first-generation matrix we have \( P(E_x, E_\gamma) \): the average \( \gamma \) decay probability with energy \( E_\gamma \) from an excitation energy \( E_x \). Assuming the Brink-Axel hypothesis is valid, section 2.7.1 the first-generation matrix is proportional to the level density and transmission coefficient \([1]\):

\[
P(E_x, E_\gamma) \propto \rho(E_x - E_\gamma) T(E_\gamma), \tag{5.17}
\]

where \( T(E_\gamma) \) is the \( \gamma \)-ray transmission coefficient, assumed to be independent of the excitation energy for low spin and low nuclear temperature. Here \( E_x, E_\gamma \) is assumed to be the “true” coincidence matrix.

In Ref. [1], the approximation to the experimentally normalized first generation matrix is done by:

\[
P_{th}(E_x, E_\gamma) = \frac{T(E_\gamma)\rho(E_x - E_\gamma)}{\sum_{E_\gamma - E_{\gamma_{\text{min}}}} T\rho(E_i - E_\gamma)}, \tag{5.18}
\]

Here, \( P_{th}(E_x, E_\gamma) \) is the theoretical first generation matrix. Starting from the first generation matrix \( P(E_i, E_\gamma) \), with the assumption of normalization
for each $E_x$ bin $E_i$, summing over from the minimum $\gamma$-ray energy $E_{\gamma}^{\text{min}}$ to maximum $\gamma$-ray energy at $E_i$. We assume that there is unity by:

$$\sum_{E_i = E_{\gamma}^{\text{min}}}^{E_i} P(E_i, E_{\gamma}) = 1. \quad (5.19)$$

When we compare the theoretical first generation matrix to the theoretical given by eq. (5.18), the general solution is found by a $\chi^2$ minimization \[56\]:

$$\chi^2 = \frac{1}{N_{\text{free}}} \sum_{E_i = E_{\gamma}^{\text{min}}}^{E_i} \sum_{E_{\gamma} = E_{\gamma}^{\text{min}}}^{E_{\gamma}^{\text{max}}} \left( \frac{P_{\text{th}}(E_i, E_{\gamma}) - P(E_i, E_{\gamma})}{\Delta P(E_i, E_{\gamma})} \right)^2 \quad (5.20)$$

where $N_{\text{free}}$ is the number of free parameters. The limits used in this thesis is given in table 5.3. With a global fit to the data, it is possible to construct an infinite functions of $\rho$ and $T$, which all fit the first generation matrix [1, 56] by using:

$$\tilde{\rho}(E_x - E_{\gamma}) = A \exp(\alpha(E_x - E_{\gamma}) \rho(E_x - E_{\gamma}) \quad (5.21)$$

$$\tilde{T}(E_{\gamma}) = B \exp(\alpha E_{\gamma}) T(E_{\gamma}) \quad (5.22)$$

For a solution of $\tilde{\rho}(E_x - E_{\gamma})$ and $\tilde{T}(E_{\gamma})$ depend on the transformation parameters $\alpha, A$ and $B$, which must be found from external data [56].

With the Oslo method [1], we can extract both the NLD and the transmission coefficient from the first generation matrix. With the transmission coefficient found, we can rewrite eq. (2.15) to find the $\gamma$SF $f_{XL}(E_{\gamma})$:

$$f_{XL}(E_{\gamma}) = \frac{T(E_{\gamma})}{2\pi E_{\gamma}^{2L+1}} = \frac{T(E_{\gamma})}{2\pi E_{\gamma}^3} \quad (5.23)$$

For the Oslo method, we look at dipole transition, where $L = 1$.

### 5.4.1 Normalization parameters

The relevant nuclear properties used in the Oslo method can be found in table 5.1 while the calculated parameters are in table 5.2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_n$</td>
<td>6.435 MeV</td>
<td>[57]</td>
</tr>
<tr>
<td>$D_0$</td>
<td>$750 \pm 230$ eV</td>
<td>[58]</td>
</tr>
<tr>
<td></td>
<td>$760 \pm 50$ eV</td>
<td>[59]</td>
</tr>
<tr>
<td>$\langle \Gamma_0 \rangle$</td>
<td>$70 \pm 10$ meV</td>
<td>[59]</td>
</tr>
<tr>
<td>$a$</td>
<td>12.804 MeV$^{-1}$</td>
<td>[60, 61]</td>
</tr>
</tbody>
</table>
Table 5.2: Calculated parameters used for extracting the NLD and $\gamma$SF.

<table>
<thead>
<tr>
<th>$\sigma(S_n)$</th>
<th>$T$</th>
<th>$\rho(S_n)$ [MeV$^{-1}$]</th>
<th>$E_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.610 ± 0.56</td>
<td>0.650</td>
<td>8.5·10$^4$ ± 3.1·10$^4$</td>
<td>-0.528</td>
</tr>
</tbody>
</table>

The first generation matrix was compressed to 200 keV/bin, shown in fig. 5.4. The limits drawn on the first generation matrix, is in table 5.3.

Table 5.3: Energy limits for first generation method.

<table>
<thead>
<tr>
<th>$E_{\gamma}^{\text{min}}$ [keV]</th>
<th>$E_{\gamma}^{\text{max}}$ [keV]</th>
<th>$E_x^{\text{min}}$ [keV]</th>
<th>$E_x^{\text{max}}$ [keV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1700</td>
<td>6435</td>
<td>3000</td>
<td>$S_n = 6435$</td>
</tr>
</tbody>
</table>

The average neutron resonance spacing is $D_0 = 750 \pm 230$ eV$^1$ for $^{132}$Xe$^{[58]}$. To find the level density at the neutron separation energy, we have that:

$$\frac{1}{D_0} = \frac{1}{2} [\rho(S_n, J = I_t + 1/2) + \rho(S_n, J = I_t - 1/2)], \quad \text{with } I_t = 0 \quad (5.24)$$

$$= \frac{1}{2} [\rho(S_n, 1/2) + \rho(S_n, -1/2)] \quad (5.25)$$

With the spin of the target nucleus $^{132}$Xe being $I_t = 0$. We assume that both spin $J = 1/2$ and $J = -1/2$ are accessible in the neutron resonance experiments, with equiparity. It is then possible to calculate the level density $\rho(S_n)$ with:

$$\rho(S_n) = \frac{2\sigma^2}{D_0} \frac{1}{(I_t + 1)} \exp \left(-\frac{(I_t + 1)^2}{2\sigma^2}\right) + I_t \exp \left(-\frac{I_t^2}{2\sigma^2}\right) \quad (5.26)$$

with $I_t = 0 \quad (5.27)$

$$= \frac{2\sigma^2}{D_0} \frac{1}{\exp \left(-\frac{1}{2\sigma^2}\right)} \quad (5.28)$$

From Ref. [56], we have an expression for the average total radiative width with s-wave capture resonances, expressed in terms of the experimental transmission coefficient $T(E_{\gamma})$:

$$\langle \Gamma_{\gamma 0} \rangle = \frac{B}{4\pi D_0} \int_{E_{\gamma}=0}^{S_n} dE_{\gamma} T(E_{\gamma}) \quad (5.29)$$

$$\times \rho(S_n - E_{\gamma}) \sum_{J=-1}^{1} g(S_n - E_{\gamma}, I_t \pm 1/2 + J)$$

where $B$ is a constant found for the normalization of the transmission coefficient, the process described in Ref. [62]. The spin and parity for the target nucleus are $I_t = 0$ and $\pi_t = +$. Here, $\rho(S_n - E_{\gamma})$ is the experimental

$^1$Though it is given in RIPL-3 as 750 000 eV by mistake, and as $D_0 = 760 \pm 50$ b in Ref. [59].
level density. The spin distribution \( g(E,J) \) is dependent on spin, \( J \), and energy, \( E \), given as:

\[
g(E,J) \simeq \frac{2J + 1}{2\sigma^2} \exp \left[ -\frac{(J + 1/2)^2}{2\sigma^2} \right], \tag{5.30}
\]

which is normalized such that \( \sum_J g \approx 1 \). When normalizing from eq. (5.29), we extrapolate the transmission coefficient with exponential functions.

The radiative width for s-wave resonance capture on \(^{132}\text{Xe}\) is \( \langle \Gamma_{\gamma 0} \rangle = 70 \pm 10 \text{ meV} \) \[^{59}\].

### 5.4.2 Spin-cutoff

The spin-cutoff parameter, \( \sigma(S_n) \), was calculated using the rigid moment of inertia formula \[^{60,61}\],

\[
\sigma^2 = 0.0146 \cdot A^{5/3} \cdot T, \tag{5.31}
\]

with an advanced Fermi gas model for temperature, where

\[
T = \left( 1 + \sqrt{\frac{1 + 4\alpha U}{2\alpha}} \right). \tag{5.32}
\]

With \( U \) is the excitation energy of the nucleus. The level density parameter is defined as \( \alpha = 12.804 \text{ MeV}^{-1} \) by Ref. \[^{60,61}\].

The rigid moment of inertia spin used in the analysis can be seen in fig. 5.5.
5.5 Extracted nuclear level density

The NLD for $^{133}$Xe is shown in fig. 5.6. For the high energy part, the constant temperature (CT) model is used to interpolate between the last data points until the $\rho(S_n)$ calculated from $D_0$ with eq. (5.26), with the arrows showing the regions used for the CT-extrapolation. With a $\chi^2$-minimization to the data in the region marked in fig. 5.6, the temperature that minimized the CT model up to $\rho(S_n)$ was $T = 0.650$, with a CT shift parameter $E_0 = -0.605$MeV.

![Figure 5.6: NLD of $^{133}$Xe with the constant temperature model for fit to the average resonance spacing $D_0$. The region used to fit are shown with arrows. The error bars are the statistical uncertainty.](image)

5.6 Extracted transmission coefficient

The extracted transmission coefficient can be seen in fig. 5.7, with arrows showing the regions used to fit the transmission coefficient.
5.7 Extracted $\gamma$-strength function

The $\gamma$SF for the experiment is shown in fig. 5.8. For the high energy part of the $\gamma$SF, there are few events that decay from the $S_n$ to the ground state, explaining large statistical uncertainty. This data above 5.6 MeV are most likely unreliable.

Figure 5.7: The transmission coefficient $T(E_\gamma)$ for $^{133}\text{Xe}$. The error bars are the statistical uncertainty.
Figure 5.8: $\gamma$SF of $^{133}$Xe. The error bars are the statistical uncertainty.
Discussion

“Going quantum. Three... two... one...”
– Bruce Banner, IV Avengers 164:18

6.1 Punch-through

As there was punch-through for the experiment, that was excluded from this analysis. To get better statistics, this can be included by extrapolating and calculating the excitation energy $E_x$ for the punch-through. With even more statistics of events at low excitation energy, as that part corresponds to events in the punch-through region, both for the LaBr$_3$(Ce) and the Clover detectors.

To avoid punch-through in future experiments, a thicker E detector can be used, or a third E detector could be added, i.e. $\Delta E-E-E$ could be considered. This would give better statistics, with no low excitation energy events lost due to punch-through.

6.2 Detector response function

In this thesis, the detector response function is based on GEANT4 measurements. A proper measured response function is currently being worked on at OCL to characterize the LaBr$_3$(Ce) detectors [63]. Additionally, with the LaBr$_3$(Ce) array OSCAR at OCL, it might be possible to measure the spin dependence of $\rho(E_x)$ by gating on the yrast-line [64].

6.3 Nuclear level density

Ideally, there would be even more known levels in $^{133}$Xe, such that the NLD can have a better fit due to more known levels. With only 29 known levels, and a complete level scheme only up to $\approx 1.4$ MeV, this will affect the NLD in the low energy region. However, with the new data from this thesis experiment, it might be possible to do spectroscopy with the $\gamma - \gamma$
coincidences to find more levels. Spectroscopy would have to be done with the \( \gamma - \gamma \) coincidences recorded in the Compton suppressed HPGe.

For benchmarking, the shell model calculations show the same trend for the NLD, but the calculations did not entirely reproduce the low lying level scheme with the correct spin and parity. To reproduce the low lying structure completely, the levels can be tweaked by changing the Hamiltonian of the shell model calculations [65].

For the fit, the CT-model provided a good fit for up to the experimental \( \rho(S_n) \) neutron resonance spacing bound from the data.

### 6.4 \( \gamma \)-strength function (\( \gamma \)SF)

The extracted \( \gamma \)SF for \(^{133}\)Xe shown in fig. 5.8 shows an upwards trend for the low energy region, a low energy enhancement (LEE). Observing a LEE was unexpected for \( A > 100 \), as this has not been seen for many isotopes, except for a few cases like \(^{138}\)La [66] and \(^{151,152,153}\)Sm [67, 68]. However, due to its location relative to the doubly-magic \(^{132}\)Sn, there have been predictions that this can cause a LEE [3]. With this increase from the estimated \( \gamma \)SF in [4], the NEEC rate given by eq. (2.20) will also increase.

Since it is possible to convert \((n, \gamma)\) cross sections to the downward \( \gamma \)SF, we can examine if this fits with the \( \gamma \)SF for \(^{132,134}\)Xe, to see if it fits as a tail to the electric giant dipole resonance.

#### 6.4.1 Comparing to \( \gamma \)SF from cross sections

To compare with cross section data, it is possible to convert cross sections for the \((\gamma, n)\) upward \( \sigma_{\gamma n} \) to the downwards \( \gamma \)SF with

\[
f(E_{\gamma}) = \frac{1}{3\pi^3\hbar^2c^2} \frac{\sigma_{\gamma n}(E_{\gamma})}{E_{\gamma}},
\]

where \( \frac{1}{3\pi^3\hbar^2c^2} = 8.674 \cdot 10^{-8} \text{ mb}^{-1} \text{ MeV}^{-2} \) [69].

Since there is no \( \sigma_{\gamma n} \) for \(^{133}\)Xe, the nearby \(^{132}\)Xe and \(^{134}\)Xe from Ref. [70] are plotted together with the \( \gamma \)SF extracted in this thesis, in fig. 6.1. For the high energy region, in the area where there is enough statistics to reliably extract the \( \gamma \)SF for \(^{133}\)Xe, it connects with the converted \( \sigma \). As \(^{132,134}\)Xe are even-even nuclei, the GDR part for \(^{133}\)Xe could be a bit different if \( \sigma_{\gamma n} \) changes for the even-odd \(^{133}\)Xe.

### 6.5 Shell model calculations

As we still are unsure of the \( M1/E1 \) nature of the LEE, it is interesting to see if shell model calculations on \( f_{M1} \) can reproduce the shape of the \( \gamma \)SF extracted in this thesis.
Figure 6.1: $\gamma$SF of $^{133}$Xe with converted $\gamma$SF for $^{132,134}$Xe$(\gamma, \gamma')$ from Ref. [70].
6.5.1 The shell model calculation code KSHELL

K SHELL [71] is a nuclear shell model calculation code which uses doubly-magic nuclei as a basis for particle-hole expansion to the desired nuclei. For these calculations, KSHELL was used with an interaction presented in Ref. [72]. This interaction is fine tuned with parameters to the closest doubly-magic core, $^{132}$Sn. For $^{133}$Xe, the calculations are done with four proton particles, and three neutron holes.

In KSHELL, the wave function is the sum of products of the proton and neutron configurations. To be effective, the Hamiltonian matrix elements are generated as needed, not stored, to solve for the eigenvalues "on-the-fly" [71]. As the model space needed for shell model calculations are quite large, the calculations are parallelized. The computational power needed is quite large due to the large model space, and shell model calculations are usually performed on a supercomputer. The shell model calculations presented in comparison with this work were done by J. E. Midtbø in May 2019.

\footnote{This work was performed on the Abel Cluster, owned by the University of Oslo and Uninett/Sigma2, and operated by the Department for Research Computing at USIT, the University of Oslo IT-department. \url{http://www.hpc.uio.no/}}
6.5.2 Comparing experiment to calculations

As there are few known levels in $^{133}\text{Xe}$, shell model calculations can be a good supplement to explore the results. All shell model calculations are from Midtbø, Ref. [65], on $^{133}\text{Xe}$. The basis for the calculations were the shell closure at the doubly-magic $^{132}\text{Sn}$-core, with added proton particles and neutron holes to reach $^{133}\text{Xe}$. For the calculations, with 200 levels per spin $J$ [65], yielding 4400 levels total, with over $1.2 \cdot 10^6$ transitions between those levels. The lower lying level of $^{133}\text{Xe}$ calculated by [65] can be seen in fig. 6.2, the measured levels are reproduced satisfactory.

Calculated nuclear level density

When calculating the level density, the levels are binned with $\Delta E = 200$ keV bins, which is the same bin size used in the experimental part of this work. The level density is found by counting the number of levels in each excitation energy bin, divided by the bin size, i.e.

$$\rho(E_x, J, \pi) = \frac{\text{(# levels within } E_x' \in E_x \pm \frac{1}{2} \Delta E, J' = J \text{ and } \pi' = \pi)}{\Delta E} \quad (6.2)$$

Where $E_x$ is the excitation energy, $J, J'$ the spin, $\pi, \pi'$ the parity and $\Delta E$ the energy bin size.

The calculated level density, $\rho(E_x)$, can be seen projected down from $\rho(E_x, J)$ with the spin distribution $J$. At around 4.5 MeV, it can be seen that the $\rho(E_x)$ drops off due to it running out of levels in the model space, but there are still some levels up to the neutron separation energy, $S_n$.

In the level density decomposed for spin $J$ and parity $\pi$ in fig. 6.3, it can be seen that equiparity is a good assumption for $^{133}\text{Xe}$. When plotting the shell model level density together with this work, see fig. 6.4, there is good agreement up until the thinning of the model space, and with the known levels of $^{133}\text{Xe}$. The CT-model extrapolation up to $\rho(S_n)$ fits well to the shell model calculations.

$^u$Complete scheme known only for the first 17 levels out of 28 above GS [16].
Figure 6.3: Shell model calculations of the level density $\rho(E_x)$ from [65].

*Top:* Spin $J$ distribution for each $E_x$ bin in 200 keV bins.

*Middle:* Decomposed level density $\rho(E_x, J)$ with parity and spin $\pi \cdot J$ distribution.

*Bottom:* Level density $\rho(E_x)$, summed over all spins $J$. 

$J$ [\hbar] $\pi \cdot J$ [\hbar] $\rho(E_x, J)$ [MeV$^{-1}$] $\rho(E_x)$ [MeV$^{-1}$] $E_x$ [MeV]
Calculated $\gamma$-strength function

Starting from Ref. \cite{35}, the $\gamma$SF for the shell model calculations is defined as:

$$f_{\gamma XL} = \frac{\langle \Gamma_{\gamma XL} \rangle (E_{\gamma}, E_{i}, J_{i}, \pi_{i})}{E_{\gamma}^{E_{L}+1}} \rho(E_{i}, J_{i}, \pi_{i})$$

(6.3)

$$= \frac{16\pi}{9\hbar^2 c^3} \langle B(XL) \rangle (E_{\gamma}, E_{i}, J_{i}, \pi_{i}) \rho(E_{i}, J_{i}, \pi_{i})$$

(6.4)

Here, $\Gamma_{\gamma XL}^X$ is the partial decay width of at a given level $i$ decaying with a $\gamma$-ray energy $E_{\gamma}$, with a multipolarity $XL$, where level $i$ has a given energy $E_{i}$, spin $J_{i}$ and parity $\pi_{i}$. The corresponding reduced transition strength, $\langle B(XL) \rangle$, is taken from Ref. \cite{35}.

Because of the limitation of working on a closed doubly-magic core, it is not possible to calculate the $\gamma$SF $f_{E1}$. For smaller, very light nuclei that does not require such a large model space, it is possible to incorporate $E1$ transitions as well \cite{65}. To include $E1$, there has to be more than one main shell that we can calculate transitions between, which increased the needed model space immensely.

Figure 6.4: Comparison of the shell model calculated level density $\rho(E_i)$ \cite{65} with this work.
In fig. 6.5 the experimental results from this work is compared with the shell model calculations, of the $f_{M1}$ contribution of the $\gamma$SF for the low energy region. The calculations reveal some expectations for a LEE in $^{133}$Xe. As the $\gamma$SF with LaBr$_3$(Ce) detectors only could be extracted down to 1.7 MeV for $E_\gamma$ due to the fit of the transmission coefficient, it will be interesting to see what the HPGe Closers will reveal. However, with the Compton suppression, if the $\gamma$-rays are considered statistical, it could be possible to extract down to 0.5 MeV.

With this data presented in fig. 6.5 the nature of the low energy enhancement in the $\gamma$SF of $^{133}$Xe can clearly be seen to be of M1 nature, in accordance with the shell model calculations of the M1 part of the $\gamma$SF. As $f_{E1}(E_\gamma)$ calculations require the inclusion of more shells, it is not possible to calculate $f_{E1}(E_\gamma)$ for heavy nuclei, the model space is far too great for calculations on supercomputers as well.

The generalized Brink-Axel (gBA) hypothesis assumes that the $\gamma$SF in spin-independent. In this work, we cannot measure if this is true experimentally. Shell model calculations does not have this limitation, and it is possible to calculate $f_{M1}(E_\gamma, J)$. In fig. 6.6 we can see that for low $E_\gamma$, $f_{M1}(E_\gamma, J)$ increases with a higher $J$. If this is accurate, that would be in violation of the gBA hypothesis in the low energy region of the $\gamma$SF.
Figure 6.6: Calculations of the $f_{M1}$ from [65] for different spin $J$ in the low energy region.
Summary and future outlook

"Can’t see into the future. I’m not a witch.”
– Loki, Thor III 14:31

7.1 Summary

In this thesis, we have extracted previously unknown experimental nuclear level density and γ-strength function of $^{133}$Xe, using the inverse-Oslo method. The inverse kinematics experiment was carried out at iThemba LABS in South Africa. Measured statistical properties have then been compared to shell model calculations done by J. E. Midtbø [65] using KSHELL [71]. Because a LEE has not observed for many nuclei with $A > 100$, the observation of the LEE for $^{133}$Xe was “unexpected”.

One of the main motivations of this work has been to better predict the decay rate from $^{133}$Xe, excited via nuclear excitation by electron capture. Extrapolation of the γSF towards very small $E_γ$ energies ($< 10$ keV [46]) can be used to calculate the expected decay rate. These results will give a better insight into nuclear plasma interactions. With the observed LEE, this will increase the upper bound of the decay rate. Exactly how much it changes depends on the future data analysis with the Clovers and the final error estimation of the γSF.

The observation of LEE in $^{133}$Xe will also affect the calculations of (n, γ)-capture rates as this may probably lead to an increase the (n, γ) cross section.

The NLD has the shape of the constant temperature model and was extrapolated to the $\rho(S_n)$ using the CT model within the uncertainty of the data. Compared to the NLD from shell model calculations, there is great agreement until the shell model calculation model space starts to run out at $\rho(E_x) \approx 4.4$ MeV. Decomposing the shell model level density in $\rho(E_x, I, \pi)$ is in agreement with the assumption of equiparity is valid for $^{133}$Xe made during the data analysis.

Compared to shell model calculations of $M1$ transitions, it would seem that the $f_{M1}$ is responsible the low energy part of the γSF the most. While
no experiment has concluded the $M1/E1$ nature of the LEE, this is very interesting.

When decomposing the shell model $\gamma$SF with spin dependence, $f_{M1}(E_\gamma,J)$ predicts that the generalized Brink-Axel hypothesis is violated when $E_\gamma$ goes towards zero. In chapter 6, $f_{M1}(E_\gamma,J)$ increases with the spin $J$. This is interesting, as one of the basic assumptions for the Oslo-method is that the generalized Brink-Axel hypothesis is valid.

7.2 Future outlook

To be able to extract the $\gamma$SF below 1 MeV, the analysis will be expanded to include the events from the Clover detectors. These have high resolution with Compton suppression, such that the noise is limited in the low energy region we are interested in. Comparing the $\gamma$SF from the Clovers with the shell model calculations can give more insight to the shape of the LEE observed in this work.

When the $\gamma$SF measured using the inverse-Oslo method has been extracted, we can compare it to the $\gamma$SF found with the ratio method \cite{73} by T. Seakamal\footnote{A PhD student at University of Johannesburg and iThemba LABS.} on the same $^{133}$Xe data from iThemba LABS.

In the near future, the analysis of this dataset might be expanded by looking at the (d,d')-reaction, as it is quite apparent in the $\Delta E$-plots. One of the remaining mysteries of $^{133}$Xe, is the incomplete level scheme above 1.4 MeV. With a level scheme only consisting of 29 known levels including the ground state, this information is incomplete and should be expanded on. Since the data was measured with HPGe, the $(\gamma,\gamma)$-matrix can be used for nuclear spectroscopy and information about more discrete levels below the quasi-continuum region.

New inverse kinematics experiments are planned to constrain the n-capture rates for the astrophysical i-process\footnote{The i stands for intermediate, neutron capture between the s- and r-process.} with the Oslo method at TRIUMF \cite{74}. Additionally, iThemba LABS are expanding their AFRODITE array with more large volume LaBr$_3$(Ce) detectors \cite{55}, which will lead to much better efficiency and statistics for inverse-Oslo experiments in the future.

To get better insights into the $^{132}$Xe(n, $\gamma$)$^{133}$Xe$^*$ cross section, it is possible to calculate the (n,$\gamma$) cross section in the Hauser-Feshbach framework in TALYS \cite{20,21}. With the NLD and $\gamma$SF for $^{133}$Xe, the output will be more accurate. With no statistical properties from experiments, TALYS relies smoothed theoretical models, which does not incorporate features such as small resonances, and most importantly, the observed LEE in the $\gamma$SF of $^{133}$Xe.

One of the main motivations of this work has been to better predict the nuclear excitation by electron capture rate on excited $^{133}$Xe. We have observed an unexpected LEE in $^{133}$Xe, the next step will be to include the observed LEE in the calculations on the effect of NPI on $R_{DIGS}$. As the previously predicted interaction rate used a low upper bound on the $\gamma$SF,
the new predicted interaction rate will increase due to the effect of the observed LEE.
Appendix A

Various derivations

A Kinematics of nuclear reactions

In relativistic physics, the center-of-mass system is characterized as

\[ \mathbf{P} = \sum_k \mathbf{p}_k = 0 \] (A.1)

In this thesis, we are only dealing with two-body reactions. As the beam is accelerated, the reactions can be relativistic. The following derivation is modified from [76].

The reaction is

\[ m_1(m_2, m_3)m_4 \] (A.2)

This means that for a case with a mass \( m_1 \) as the stationary target, \( m_2 \) the relativistic four-momentum will be

\[ P_1 = (m_1, 0, 0, 0) \] (A.3)

For the system, the Lorentz invariant is given as

\[ s = \left( \sum_k E_k \right)^2 - (\mathbf{p}_k)^2 \] (A.4)

\[ = (m_1 + m_2)^2 + 2m_1 T_k \] (A.5)

\[ = (m_{\text{target}} + m_{\text{beam}})^2 + 2m_{\text{target}} T_k \] (A.6)

For the center-of-mass system, we have that \( \mathbf{p}_1 = -\mathbf{p}_2 \) and \( p_1^2 = p_2^2 = p_{\text{cm}}^2 \), such that the invariant \( s \) is unchanged:

\[ s = \left( \sqrt{m_1^2 + p_{\text{cm}}^2} + \sqrt{m_2^2 + p_{\text{cm}}^2} \right)^2 \] (A.7)

Combining eqs. (A.4) and (A.7)

\[ p_{\text{cm}} = \sqrt{\frac{(s - m_1^2 - m_2^2)^2 - 4m_1^2m_2^2}{4s}} \] (A.8)
The target is boosted to a momentum $p_{cm}$ with the center of mass rapidity, with that, it is possible to determine the rapidity of the center-of-mass frame

$$P_1 = (m_1, 0) \quad \text{(A.9)}$$

$$→ P'_1 = (E_1, p_{cm}) = \left( \sqrt{m_1^2 + p_{cm}^2}, p_{cm} \right) \quad \text{(A.10)}$$

$$= (m_1 \cosh \chi, m_1 \sinh \chi) \quad \text{(A.11)}$$

Which leads to an expression for the rapidity

$$\chi = \ln \left( \frac{p_{cm} + \sqrt{m_1^2 + p_{cm}^2}}{m_1} \right) \quad \text{(A.12)}$$

In the same manner as eq. (A.8), the center-of-mass momentum $p'_{cm}$ for the reaction products will be

$$p'_{cm} = \sqrt{\frac{(s - m_3^2 - m_4^2)^2 - 4m_3^2m_4^2}{4s}} \quad \text{(A.13)}$$

For the ejectile $m_3$, we have that in the lab frame

$$E_{3_{lab}} = \sqrt{p'_{cm}^2 + m_3^2 \cosh \chi + p'_{cm} \cos \theta \sinh \chi} \quad \text{(A.14)}$$

and

$$p_3 \cos \theta_3 = p'_{cm} \cos \theta_{cm} \cosh \chi + \sqrt{p'_{cm}^2 + m_3^2 \sinh \chi} \quad \text{(A.15)}$$

and

$$p_3 \sin \theta_3 = p'_{cm} \sin \theta_{cm} \quad \text{(A.16)}$$

For the recoil $m_4$, the boost will have an opposite sign from eq. (A.14)

$$E_{4_{lab}} = \sqrt{p'_{cm}^2 + m_4^2 \cosh \chi - p'_{cm} \cos \theta \sinh \chi} \quad \text{(A.17)}$$

and

$$p_4 \cos \theta_4 = -p'_{cm} \cos \theta_{cm} \cosh \chi + \sqrt{p'_{cm}^2 + m_4^2 \sinh \chi} \quad \text{(A.18)}$$

$$p_4 \sin \theta_4 = p'_{cm} \sin \theta_{cm} \quad \text{(A.19)}$$

From that, the momenta can be expressed with the lab angle $\theta$ of the particle as

$$p_3 = \frac{\sqrt{m_3^2 + p'_{cm}^2 \cos \theta_3 \sinh \chi} \pm \cosh \chi \sqrt{p'_{cm}^2 - m_3^2 \sin^2 \theta_3 \sinh^2 \chi}}{1 + \sin^2 \theta_3 \sinh^2 \chi} \quad \text{(A.20)}$$

$$p_4 = \frac{\sqrt{m_4^2 + p'_{cm}^2 \cos \theta_4 \sinh \chi} \pm \cosh \chi \sqrt{p'_{cm}^2 - m_4^2 \sin^2 \theta_4 \sinh^2 \chi}}{1 + \sin^2 \theta_4 \sinh^2 \chi} \quad \text{(A.21)}$$
With all of this, it is possible to calculate what energy the ejectile has then in turn calculate what the energy of the recoil has. Then we can get to the most important part, the excitation energy $E_x$ of the $^{133}$Xe. We assume that the reaction will lead to an increase in mass with $m_4 \rightarrow m_4 + E_x$ as the excitation energy. The excitation energy can then be expressed as:

$$E_x = \sqrt{s - 2\sqrt{s}E_{3_{im}} + m_3^2} - m_4$$ (A.22)
Bibliography


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