Study on Seismic Data Sensitivity on Pressure-Saturation Discrimination

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Study on Seismic Data Sensitivity on Pressure-Saturation Discrimination

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Preface

This thesis was written as part of a two year master study in Geoscience at the University of Oslo (UiO), to complete an MSc degree in Geophysics. The thesis represents 60 ECTS credit points, which is equivalent to a one-year study.

The study was conducted in collaboration with the Norwegian Geotechnical Institute (NGI), under supervision of Joonsang Park (NGI), Guillaume Sauvin (NGI), and Professor Valerie Maupin (UiO). The data for the study was provided by The Northern Lights Project and NGI. The study is sponsored by The Norwegian CCS Research Centre (NCCS), under grant 257579/E20.
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Robin David Kifle, June 1st, 2019
Abstract

The effects of pore pressure and fluid saturation on the seismic velocity changes are coupled and important to discriminate one from the other in the context of time-lapse monitoring of fluid injection into the subsurface such as the CO₂ storage case. The purpose of this thesis is to compare the use of conventional streamer data to multi-component seismic data. Explicit linear expressions for pressure and saturation discrimination that are used for petroleum application are tested for a CO₂ injection scenario. The explicit linear expressions are found to be oversimplified for CO₂ injection; hence the problem is solved by the use of non-linear systems of equations. The multi-component method is demonstrated to give a more sturdy pressure-saturation discrimination compared to conventional streamer data.

Several limitations need to be taken into account when using the methods, as several factors affect the result. These include the reliability on a priori information of spatially variant parameters, a reasonable certainty in measured seismic parameters, and good initial information about the in situ pressure and saturation. Typical uncertainties in the measured seismic parameters are tested. The results show the use of multi-component seismic data to be superior to the conventional streamer data.

The methods in this thesis utilize an existing reservoir simulation for the Smeaheia area in order to investigate the issues mentioned above (i.e. better discrimination). In this course, there are some uncertainties related to the existing reservoir simulation. The uncertainties are explored through a simple reservoir simulation to deduce the certainty in discrimination for other scenarios than the provided Smeaheia fluid flow simulation.

It is learned from this study that the conventional amplitude versus offset (AVO) method is applicable for CO₂ saturations less than 50%. For increasing saturation and pressure, which case is more important for the integrity of reservoir and the cap rock, the multi-component seismic data seem necessary for pressure-saturation discrimination.
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1 | Introduction

1.1 Motivation and background

Carbon capture and storage (CCS) within offshore saline aquifers is one of the developing technologies for reducing the carbon footprint and, according to the intergovernmental panel on climate change (Rogelj et al., 2018), is a necessity to achieve the ambitious goals of the Paris Agreement. Gassnova (2016) published a feasibility report on a full-scale CCS project in Norway. The report proposed three locations for CO\textsubscript{2} injection, with the Smeaheia site as the preferred site. Smeaheia has not been exploited as a petroleum production field, thus available information and experience in this area is limited. Recently, the Smeaheia project has been postponed and a nearby storage site, Aurora, has become the new target site for the first full-scale project (Rørvik et al., 2018).

The part of the full-scale CCS project covering ship transport, CO\textsubscript{2} receiving terminal, seafloor pipeline, and injection well was named The Northern Lights CCS project. The industry partners of the Northern Lights project are Equinor, Shell, and Total. As part of the Northern Lights CCS project, Equinor conducted the storage part of the study. The feasibility study focused on the area covered by the 3D seismic cube GN1101, presented in Figure 1.1. Within the Sognefjord- and Fensfjord formations, two suitable structures were mapped. Both structures (Alpha and Beta) were advertised as suitable to accommodate up to 1.3 Mt of CO\textsubscript{2} injected per year over 25 years (Statoil, 2016). Reservoir simulations from Statoil (2016) suggest that both structures have the capacity of storing roughly 100 Mt CO\textsubscript{2} each.
Figure 1.1: Position of the GN1101 seismic cube. The Alpha (A) and Beta (B) structures are marked with black contour lines on the seismic cube. Notice that the seismic cube does not fully capture the Alpha nor the Beta structure. There are two well logs, 32/4-1 and 32/2-1, marked as a red and blue star, respectively. Arrow indicates North.

The use of multicomponent acquisition is becoming more accessible for research. Data from NPD (2019) show that Permanent Reservoir Monitoring systems (PRM) at the Snorre, Grane, Ekofisk, and Valhall fields account for 11 of the 17 contractual time-lapse surveys (4D), during 2015 and 2016 (Johnston, 2017). Availability of such data will induce research, leading to optimized survey layouts, and maximization of the quality of data. For the second quarter of 2019, there are plans for acquisition of 23 3D seismic surveys, five of which are Ocean Bottom Seismic (OBS) (NPD, 2019).

Successful monitoring of carbon storage is vital for reducing risks induced on wildlife, groundwater quality, as well as reducing the human footprint. The cost of monitoring is typically less than 2% of the full-scale CCS project costs (Ringrose et al.,
A balance between the benefits of ensuring successful and continuous CO₂ storage and cost is still needed. Successful monitoring will also lead to reduced operational costs and prevent additional costs such as additional wells and periods of suspended injection (Ringrose et al., 2018). Equinor proposed in their feasibility study from 2016 in addition to direct injection monitoring using pressure and temperature gauges at the wellhead and downhole, 4D seismic as the main method to ensure conformity in injection and to detect migration or leakage of CO₂ out of the storage complex.

The challenges and opportunities presented in this thesis work are relevant for CCS projects on the Norwegian Continental Shelf. Pressure maintenance is an important part of enhanced oil recovery (EOR), which also makes this study interesting for oil and gas monitoring as well.

1.2 Objective of this study

The purpose of this thesis work is to establish means of discriminating the effects of pressure and saturation changes on seismic data during and after CO₂ injection and storage in a reservoir, compare different methods, and apply these methods to the specific case of Smeaheia.

The use of converted wave data is supplemented to pure compressional wave data. The methods used will be based on amplitude variations with respect to offset (AVO), and multicomponent data (PP & PS reflections). Reliable pressure and saturation discrimination is vital for the integrity of the seal, which is NGI’s main contribution to CCS.

Pressure and saturation effects often interact in reservoirs, making pressure-saturation discrimination a key issue in analyzing 4D seismic data. To solve this, multiple schemes have been proposed based on travel time variations and amplitude variations (Brevik, 1999; Tura & Lumey, 1999; Landrø, 2001). This thesis is based on methods augmented from Landrø (2001), such as Meadows (2001), Landrø et al. (2003), and Stovas & Landrø (2004), utilizing that pressure and saturation have different effect on time-lapse AVO data (Grude et al., 2013) and on PP and PS reflections. The method presented in Landrø (2001) have been modified for optimization by multiple authors (e.g. Meadows (2001); Angelov et al. (2004); Trani et
al. (2011); Bhakta & Landrø (2014)). This thesis will aim to analyze the perks, or need, of converted P-S wave reflectivity in regards to carbon storage.

1.3 Carbon capture and storage (CCS)

Carbon capture and storage involves capturing carbon dioxide (CO$_2$) from large industrial sources before it is emitted to the atmosphere where it may contribute to climate change. Once captured, CO$_2$ can be stored in offshore saline aquifers or in mature oil and gas reservoirs.

CCS is technically complex and expensive, and involves a three-step process, as follows:

- CO$_2$ is captured at an industrial plant. There are several methods for capturing. The most common is to extract CO$_2$ from gas outlet by contact with amine-based solvents, which are then heated to release the CO$_2$ at low pressure.

- The second stage involves the transportation of separated CO$_2$ to the intended use or storage site. The Northern Lights CCS project, as mentioned in the introduction, covers ship transport, CO$_2$ receiving terminals, and seafloor pipelines to the injection sites for storage.

- The third stage involves storing CO$_2$ in secure geological formation. On the Norwegian Continental Shelf (NCS), there a number of potential storage sites including offshore saline aquifers and depleted or mature oil and gas reservoirs which will also have additional benefit of pressure maintenance for EOR purposes.

Independently of the technical challenges, marketing of CO$_2$ storage represents an important aspect, to offset the costs of capture and storage. The development of a CO$_2$ storage market represents an opportunity to offset some of the costs of capture and storage.
1.4 Thesis structure

This first chapter gave an introduction to the motivation and objective of the study presented in this thesis work. Chapter 2 gives an overview of the available data, such as previous surveys, well logs, the synthetic geological model, and laboratory data. Chapter 3 links the injection of CO$_2$ with changes in seismic parameters. Chapter 4 explains the methodologies used in the thesis and goes through the uncertainties in the measured seismic data. Chapter 5 tests a simplified pressure and saturation discrimination approach from Landrø (2001) for the given CO$_2$ scenario. The simplified approximation is deduced to be insufficient for CO$_2$ injection, and the method is extended to a non-linear system for pressure and saturation discrimination in Chapter 6. Two scenarios for initial pressure and saturation information are tested. After this, the seismic data is given uncertainty and the discrimination is tested by Monte-Carlo simulations. In Chapter 7, a simple fluid flow simulation is performed to deduce the reliability of the inversion for larger pressure build-up scenarios. The concluding remarks drawn from the study are presented in Chapter 8.
Figure 1.2: Workflow for the thesis. In "Preface", the available data and connecting injection of CO₂ with change in seismic parameters is analyzed. In the "AVO and PP-PS method", the synthetic geological cube is subject to forward modeling and inversion by the use of different schemes. In "Scenarios with other pressure build-ups", other scenarios than the Eclipse simulation are tested to get insight in the outcome from other possible pressure build-up scenarios.
2 | Available data

2.1 Previous surveys

The available data at Smeaheia include a substantial amount of 2D seismic surveys, two 3D seismic cubes, and some well logs. The survey area is shown in Figure 1.1. Two reservoirs in the GN1101 cube were proposed for injection, the Alpha structure (A) in the west and the Beta structure (B) in the east (Figure 1.1). The GN1101 cube does not cover the full extent of the Alpha structure, nor the Beta structure. A larger 3D cube, the CGG17M01, extends over the full structures. However, the CGG17M01 cube is not released to the research partners in NCCS. Full coverage of the structures is needed for the baseline seismic survey, new volume calculations, and to define an optimal drilling location. Two wells have been drilled inside the cube and will be further discussed in Section 2.2.

2.2 Well log data

As illustrated in Figure 1.1, there are two wells in the area. Data from the wells are available from the Norwegian Petroleum Directorate (NPD), provided by The Northern Light Project. The data have been used in this study for acoustic impedance inversion of the baseline seismic survey, and 1D AVO inversion. The 32/4-1 T2 well is located in the Alpha structure and was drilled in 1996. The 32/2-1 well is close to the Beta structure and was drilled in 2008. Both wells are P&A (plugged and abandoned). The well used for the 1D studies is the 32/4-1 T2. Figure 2.1 shows logs from the 32/4-1 T2 well between 1000 m and 1450 m measured depth (MD). The formations in this section are the Draupne formation, Heather formation, and the two upper layers of the reservoir zone, consisting of Sognefjord and Fensfjord. The properties of the Heather formation have been up for debate. Previous studies such as the feasibility study by Statoil (2016) and Dupuy et al. (2018) have discussed two different scenarios:

- Heather formation is acting with low to moderate reservoir properties, allowing CO$_2$ to migrate until it reaches the Draupne formation, the cap rock seal.
- Heater formation is acting as seal, making Sognefjord the top of the reservoir.
Reservoir properties such as initial porosity, permeability, viscosity, temperature, and pressure would be different for the two scenarios. For simplicity, this study will assume the second scenario, making Heather the cap rock of the reservoir.

From the 32/4-1 T2 log caliper log, a washout effect (enlarged region of wellbore), is noticed in the Draupne formation at 1130 m to 1200 m MD, Figure 2.1. This effect lowers the reliability of the density and neutron porosity in this layer. Gamma-ray (GR) values above 100 API are apparent in the Draupne and Heather formation at 1110 m to 1240 m MD. Draupne formation displays low P-wave and S-wave velocities, 2350 $\frac{m}{s}$ and 850 $\frac{m}{s}$, respectively. S-wave velocity for the baseline is estimated using the empirical relation of Castagna et al. (1985). Castagna’s equation, also known as the Mudrock line, was determined on the basis of log data, as well as seismic, and laboratory measurements of brine saturated siliciclastic rocks, such as sandstones and shales, making it a suitable estimation for the brine saturated baseline model. Separation between the density and neutron porosity logs, or shale effect, is marked as green in the second log. A noticeable shale effect is observed around the Heather formation (1210-1240 m MD), indicating a shale zone.

At the Draupne-Heather formation boundary, an increase in density and P-wave velocity is observed. This increase in both properties might reduce the acoustic impedance contrast between these layers. There is also a slight drop in the gamma-ray values between the two layers, while the borehole environment seems good for the relevant layers below the base Draupne formation.

The Sognefjord formation is defined by lower GR values. Some small drops in the GR values, with peaks in the density neutron porosity log, and increase in P-wave velocity, correspond to highly calcite cemented rocks (1290 m, 1310 m, and 1330 m MD) (Dupuy et al., 2018). The lateral extent of these carbonate layers are uncertain, as they are not noticed by seismic resolution.
Figure 2.1: Well logs from 32-4 T2 from 1000 m to 1450 m MD. From left to right: caliper and gamma ray, density-neutron porosity, sonic P-wave, S-wave derived from P-wave, total porosity derived from neutron-density combination, Shale volume derived from gamma ray. Top formations are marked on the right-hand side.

2.3 Synthetic geological model

Figure 2.2 shows the extent of the synthetic geological model provided by NGI. The model is developed from inversion of the GN1101 cube and the well logs marked in Figure 1.1. The synthetic geological model extend further than the GN1101 cube from Figure 1.1 and over the full Alpha and Beta structure.
Figure 2.2: Extent of the synthetic geological model. Structural closures of the Alpha and Beta structure are contoured in black on the synthetic geological model. The data in the colormap is the P-wave velocity before injection. Arrow indicates North.

2.4 Laboratory data

Multiple parameters extracted from laboratory measurements are needed to approximate the behavior of CO₂ injection on seismic parameters. These relations will be further discussed in Chapter 3. However, there are no laboratory data available from the Smeahea field, as well as a limited amount of data from drainage and imbibition flooding test with CO₂ and brine on reservoir sandstones. Therefore the use of ultrasonic velocity measurements done during compaction and shearing of an unconsolidated brine saturated sandstone, and imbibition and drainage tests from a study by Alemu et al. (2013) done on sandstone from Rothbach. Using core data from another field is far from optimal, and will be discussed in Section 3.4.
The geophysical methods presented in this thesis are designed to depend on laboratory data. In this section, the effect on seismic parameters by injecting CO$_2$ into a brine aquifer will be quantified and the validity of the data will be discussed.

### 3.1 CO$_2$ saturation effect on seismic parameters

Rock properties are affected when CO$_2$ is injected and stored into an aquifer, and start replacing the brine. Hence, the seismic parameters are affected. Rock physics is used to link these fluid effects to the measured seismic parameters. Gassmann’s equation for fluid substitution (Gassmann, 1951) is widely used. It relates the change in saturating fluid with change in seismic parameters:

\[
K_{sat} = K_d + \frac{(1 - \frac{K_s}{K_f})^2}{\frac{\phi}{K_f} + \frac{(1-\phi)}{K_s} - \frac{K_s}{K_f}^2},
\]

were $K_{sat}$ denotes the effective bulk modulus, $K_d$ denotes the bulk modulus of the solid frame (non saturated porous rock), $K_s$ denotes the bulk modulus of the effective matrix (0 porosity solid rock), $K_f$ denotes the effective fluid bulk modulus, and $\phi$ denotes the porosity.

There are six main assumptions for using the Gassmann model for fluid substitution (Gelius, 2019a). These are as follows:

- The rock is homogeneous and isotropic at a macroscopic scale.
- All the pores are connected.
- The fluid within the rock is frictionless.
- The system is closed, i.e. no flux.
- The fluid does not alter the rock properties of the frame.
• The velocity of the wave is much larger than the relative motion between the fluid and the rock.

The estimations in this thesis also assume that the saturation changes only affect the bulk modulus and the density, while the shear modulus is independent in regards of saturation. This can be deduced from the second Gassmann assumption. All pores are connected, hence the fluid will flow freely without affecting shear deformation.

An issue not taken into account by the Gassmann model is the mixture of multiple fluids, which is the case with CO$_2$ injection (Sen & Dvorkin, 2011). This makes lab data important for modeling the effect of CO$_2$ saturation on seismic parameters. However, as previously mentioned in Section 2.4, laboratory data from the Smeaheia field is not available. Hence, a study by Alemu et al. (2013) on data from a sandstone outcrop from Rothbach is used. This makes it important to work on normalized relationships between the change in velocity and change in saturation.
**Figure 3.1: Relative change in P-wave velocity caused by change in CO₂ saturation.** The uppermost curve is calculated with Brie’s fluid substitution with exponent=1, and is the Voigt upper bound. With increasing exponent, the curves are approaching Gassmann’s equation (dark blue curve). When \( e \to \infty \) we get the Reuss lower bound (dark red curve), below Gassmann’s equation.

The Alemu et al. (2013) laboratory measurements (red dots in Figure 3.1) plot between Gassmann’s equation and the Voigt upper bound (Figure 3.1, orange line). These data indicate a patchy fluid saturation. Brie et al. (1995) suggested an empirical equation for patchy fluid distribution:

\[
K_f = (K_w - K_{co2})(1 - S_{co2})^e + K_{co2},
\]

where \( K_w \) and \( K_{co2} \) denote the bulk modulus for the in situ brine and CO₂, respectively. \( S_{co2} \) denotes the CO₂ saturation in the pore space. \( e \) denotes an empirical value which will be referred to as the Brie exponent.

The rock parameters used for calculating the different mixing laws in Figure 3.1 are from different origins. Grain properties are from Sognefjord lithology (Dupuy et al., 2018), and the dry bulk modulus is approximated using the empirical relation
from Nur et al. (1998),

$$K_d = K_s \left(1 - \frac{\phi}{\phi_{crit}}\right),$$  \hspace{1cm} (3.3)

where the critical porosity of the sandstone $\phi_{crit}$, is assumed to be 40%. Shear bulk modulus for the Sognefjord lithology is not available, therefore the measured shear bulk from the Rothbach sandstone was used. The use of shear bulk modulus from the Rothbach sandstone is a flaw and will be discussed in Section 3.4.

The laboratory data from Alemu et al. (2013) does not align perfectly with any of the mixing laws, but a modest fit can be found using Brie’s equation with an exponent between 3 and 5. Brie’s equation with exponent set to 3 is chosen for further calculations based on studies such as Grude et al. (2013), and a decent fit with the available laboratory data.

The relationship between CO$_2$ saturation and change in P-wave is non-linear (Figures 3.1 and 3.2), but the methodologies from Landrø (2001) and Landrø et al. (2003) assumes a linear relationship between saturation changes and seismic parameters (Section 4.1.1). A second order approximation would describe the observed variation in P-wave velocity better, but this will introduce non-linearity to the inversion (Grude et al., 2013), as will be shown in Chapter 6.

The linearized approximations are found using the least squares method. The gradient of the linear approximation is conserved, and the constant $b$ in the linear approximation $f(x) = ax + b$, is set to 0 to resemble the curve presented in Landrø (2001). The approximated linear relative change in P-wave velocity related to saturation change is illustrated as a blue dotted line in Figure 3.2. The relative change in S-wave (red line in Figure 3.2) is already linear. The reason for this is the assumption mentioned in Section 3.1, stating that saturation changes only affect the bulk modulus and the density, while the shear modulus is independent. This makes the change in S-wave in regards of saturation purely dependent on the change in density.
From Figure 3.2, the linear approximation of the relative change in seismic parameters, $\alpha$, $\beta$, $\rho$ in regard of fluid effect, can be represented as:

$$\frac{\Delta \alpha^F}{\alpha} \approx -0.062 \Delta S,$$

$$\frac{\Delta \beta^F}{\beta} \approx 0.027 \Delta S,$$

$$\frac{\Delta \rho^F}{\rho} \approx -0.0517 \Delta S,$$

(3.4)

where the exponent ”$F$” denotes the change to be from fluid effect.
3.2 Injected CO₂ pressure effect on seismic parameters

Injection of CO₂ into an aquifer will increase the pore pressure, \( P_p \), in the reservoir. This results in a decrease in effective (or net) pressure, \( P_n \), assuming a constant overburden pressure, \( P_o \) (Gelius, 2019b).

\[
P_n = P_o - \eta P_p, \tag{3.5}
\]

where \( \eta \) is the Biot coefficient (Biot, 1941), for internal deformation, which is usually set to 1.

A study by Grande et al. (2011) describes the pressure effect on the reservoir sandstone, were they performed ultrasonic velocity measurements on two unconsolidated sands during compaction and shearing at high stresses. The measurements are made on brine saturated core samples. The different values for the quartz dominated sand is plotted in Figure 3.3. The error bar represents maximum and minimum interpreted velocities, and not the error in the measurements. It is noticeable in Figure 3.3 that S-wave velocity is more affected by changes in net pressure than P-wave velocity. How well this data can represent the fairly consolidated Sognefjord formation is debatable, however the curves in Figure 3.4 will be assumed sufficient for the study at hand. The relative change in P-wave and S-wave velocity caused by a change in net pressure (Figure 3.4) is normalized to an initial pressure of 13 MPa, which is the assumed net pressure at the Smeaheia reservoir. The methodologies expects a second degree approximation to the relative change in velocity caused by an increase in pore pressure, and the average curve in Figure 3.4 will be used for further modeling and inversion.
Figure 3.3: P-wave (blue) and S-wave (red) velocities plotted for different effective stresses. The errorbars represent maximum and minimum interpreted velocities.
Figure 3.4: Relative change in P-wave (blue) and S-wave (red) velocity caused by change in net pressure, quadratic approximations. Normalized for a net pressure of 13MPa, the assumed net pressure in the Smeaheia reservoir. Dotted line represent lower values, black line represent average values, and solid blue line represent upper values.

The P-wave and S-wave curves in Figure 3.4 are found by a second degree least square approximation of the average change in relative velocity regarding pressure change. These approximated curves can be expressed as:

\[
\frac{\Delta \alpha^P}{\alpha} \approx -0.0081\Delta P + 0.0003\Delta P^2, \\
\frac{\Delta \beta^P}{\beta} \approx -0.0331\Delta P + 0.0011\Delta P^2, 
\]

where the exponent \( P \) denotes the change to be from pressure effect.
3.3 Total effect of saturation and pressure

The total relative change in seismic parameters in regards of pressure and saturation changes can then be expressed by 7 coefficients:

\[
\begin{align*}
\frac{\Delta \alpha}{\alpha} & \approx k_\alpha \Delta S + l_\alpha \Delta P + m_\alpha \Delta P^2, \\
\frac{\Delta \beta}{\beta} & \approx k_\beta \Delta S + l_\beta \Delta P + m_\beta \Delta P^2, \\
\frac{\Delta \rho}{\rho} & \approx k_\rho \Delta S.
\end{align*}
\]

(3.7)

Using the values found in equations 3.4 and 3.6 we get:

\[
\begin{align*}
\frac{\Delta \alpha}{\alpha} & \approx -0.062\Delta S - 0.0081\Delta P + 0.0003\Delta P^2, \\
\frac{\Delta \beta}{\beta} & \approx 0.027\Delta S - 0.0331\Delta P + 0.0011\Delta P^2, \\
\frac{\Delta \rho}{\rho} & \approx -0.0517\Delta S.
\end{align*}
\]

(3.8)

3.4 Discussion of CO\textsubscript{2} effect

There are numerous uncertainties related to the coefficients for change in seismic properties derived in this chapter.

The use of pure rock physics models will give a variety of experimental parameters ranging from the Gassmann-Reuss lower bound to the Voigt upper bound for saturation related seismic parameter changes, illustrated in Figure 3.1. The uncertainty in the laboratory data is not given. Plotting laboratory data together with rock physics models with values from other fields increases model uncertainty.

The shear modulus used for Gassmann’s equation in Section 3.1 should be calculated from the synthetic inverted P-wave, density, and given bulk modulus \( \left( \mu = \frac{3(V_p^2 \rho - K_{sat})}{4} \right) \), which would slightly change the form of the approximated Gassmann curves.

In regards to pressure changes, empirical relations such as Hertz-Mindlin (Mindlin,
1949) and Hashin Shriktman (Hashin & Shtrikman, 1963) are widely used (Stovas & Landrø, 2004). The laboratory data correlates poorly with these rock physics models. By solemnly using empirical models such as Hertz-Mindlin and Hashin Shriktman a physical basis is absent. Empirical models can often fail to predict the pressure dependence of the elastic properties outside the limited range of measured pressure values (Saul & Lumley, 2013). A noticeable difference between the core data from Grande et al. (2011) and the rock physics models from Mindlin (1949) and Hashin & Shtrikman (1963), is the similarity in relative P-wave and S-wave change for the rock physics models. The laboratory data meanwhile, present distinct difference in P- and S-wave characteristics from pressure changes. An acceptable fit between the pressure curves from laboratory tests and empirical methods is desired.

Using the early Triassic Rothbach sandstone to model the upper Jurassic sandstone Sognefjord is bound to cause issues. However, a sandstone which has undergone imbibition and drainage testing on CO₂ is needed. The parameters should be changed when more relevant data is available. The velocity information from Grande et al. (2011) is normalized and the relative changes in velocities are considered to reduce this poor approximation.

The method of correlating lab data and comparing it with rock physics models are done to show the concept. Using data from a single core is inadequate, as the parameters are spatially variant and cores from close by wells can deviate from one another (Landrø, 2001). Landrø (2001) suggest averaging data from multiple cores to be a more reasonable procedure. Depending on spatially variant measurements is a substantial flaw in the scheme.

It is also worth mentioning that ultrasonic core measurements are done with frequencies above the seismic frequencies, and a problem with upscaling of the data arise. The calculation of acoustic impedance from sonic and density logs illustrates the issue of upscaling. Upscaling (or blocking) based solely on the sonic or density log might result in poor scaling of the other log. However, calculating the acoustic impedance, and then block the acoustic impedance, would distort the amplitudes. One solution is to use sequential Backus averaging (Backus, 1962), which takes a stack of thin layers into account and averages the properties in the thin layers, making them similar to the average properties of the stacked layer.
Core samples are damaged during the coring process (Nes et al., 2000). The samples are reloaded to simulate the *in situ* stress conditions, however the original state of the sample is not likely to be re-established (Landrø et al., 2001).

The value of coefficient $\eta$ in equation 3.5 causes another uncertainty. The coefficient is usually unknown, but close to 1 (Landrø, 2002). Using another value than 1 will result in stretching of the curve in Figure 3.1, resulting in an increased uncertainty.

Approximating the non-linear saturation curve in Figure 3.1 with a straight line is not desirable. The issues with this will be prominent in Section 6.1. Landrø (2002) suggest reducing this uncertainty by fitting a straight line only to represent the saturations expected in the specific case, e.g., let the constant $k_\alpha$ vary for different injection timespan.

Injection of CO$_2$, compared to an oil production scenario, increase the complexity in velocity. The approximated constant $k_\alpha$ will be a worse representation of a complex velocity relation (CO$_2$), than for a more elementary scenario (oil). This is investigated in Chapter 6 by using a quadratic approximation of the saturation curve.
4 | Discrimination using AVO and PP-PS timelapse

In this section, the methods from Landrø (2001) and Landrø et al. (2003) will be presented. Subsequently, uncertainties not taken into account in these methods will be reviewed.

4.1 PP AVO data

For the methods at hand, injection-related changes such as temperature, porosity, and permeability, are assumed constant. The first approach to estimate pressure and saturation from seismic parameters is by the use of pure PP-reflection data. The method exploit the PP-reflection data by regarding the near and far offset stacks to calculate PP AVO parameters. The PP AVO parameters are employed as independent parameters (Landrø, 2001).

4.1.1 Landrø’s method

![Diagram of two-layered model and layer parameters](image)

The method for discriminating between pressure and fluid saturation effect on seismic data is explained in Landrø (2001). The technique is explained here, as it is fundamental in this study.

A simple two-layered model consisting of a cap rock overlaying a reservoir layer,
as illustrated in Figure 4.1, is considered. We define interval P-wave velocity, S-wave velocity, and density, in layer 1 $\alpha_1$, $\beta_1$, and $\rho_1$, respectively, as constant during injection. The porous reservoir layer parameters, before injection, is defined as $\alpha_2$, $\beta_2$, and $\rho_2$. The same parameters, after injection, will be defined as $\alpha'_2$, $\beta'_2$, and $\rho'_2$.

Zoeppritz (1919) derived plane wave amplitudes of reflected and transmitted seismic waves as a function of angle. These formulas are exact, but the complex nature makes it difficult to relate these amplitude variations to the seismic parameters (Gelius, 2019a). This has lead to the derivation of multiple sophisticated approximations such as Bortfeld (1961), where the change in elastic parameters are linearized. The approximated equations were further polished by Aki & Richards (1980). With simple assumptions, such as a known $\frac{V_p}{V_s}$ ratio, the Aki and Richards approximation can be written in three terms, independently involving the seismic parameters $\alpha$, $\beta$, and $\rho$. This makes the Aki and Richards approximation appealing for this study.

From Aki & Richards (1980):

$R_{pp}(\theta) = \frac{1}{2}(1 - 4\beta^2 p^2) \frac{\Delta \rho}{\rho} + \frac{1}{2\cos^2 \theta} \frac{\Delta \alpha}{\alpha} - 4\beta^2 p^2 \frac{\Delta \beta}{\beta}$, \hspace{1cm} (4.1)

where $\theta$ denotes the angle of incidence, $p$ denotes the slope of the time-distance curve, or the ray parameter (Yilmaz, 2001). $\alpha = (\alpha_1 + \alpha_2)/2$, $\beta = (\beta_1 + \beta_2)/2$, $\rho = (\rho_1 + \rho_2)/2$, $\Delta \alpha = \alpha_2 - \alpha_1$, $\Delta \beta = \beta_2 - \beta_1$, and $\Delta \rho = \rho_2 - \rho_1$. Substituting $p$ with $\frac{\sin \theta}{\alpha}$, and the identity $\frac{1}{\cos^2 \theta} = 1 + \tan^2 \theta$, we get the given equation:

$R_{pp,0}(\theta) = \frac{1}{2}(\frac{\Delta \rho}{\rho} + \frac{\Delta \alpha}{\alpha}) - 2\gamma^2(\frac{\Delta \rho}{\rho} + \frac{2\Delta \beta}{\beta})\sin^2 \theta + \frac{\Delta \alpha}{2\alpha} \tan^2 \theta$, \hspace{1cm} (4.2)

where $R_{pp,0}$ represents the PP-reflection coefficient before injection. $\gamma$ is the velocity ratio of seismic compressional- and shear-wave ($\frac{\beta}{\alpha}$). In regards to fluid effect from the injection of CO$_2$ in the reservoir layer, the PP-reflection coefficient, $R_{pp,1}$, can be defined as:

$R_{pp,1}(\theta) = \frac{1}{2}(\frac{\Delta \rho'}{\rho'} + \frac{\Delta \alpha'}{\alpha'}) - 2\gamma^2(\frac{\Delta \rho'}{\rho'} + \frac{2\Delta \beta'}{\beta'})\sin^2 \theta + \frac{\Delta \alpha'}{2\alpha'} \tan^2 \theta$, \hspace{1cm} (4.3)

where $\alpha'$, $\beta'$, $\rho'$ denotes the same parameters after fluid substitution in the reservoir layer, and $\Delta \alpha' = \alpha'_2 - \alpha_1$, $\Delta \beta' = \beta'_2 - \beta_1$, $\Delta \rho' = \rho'_2 - \rho_1$.

We define the change in seismic properties in the reservoir layer, induced by CO$_2$ injection, as $\Delta \alpha^F = \alpha'_2 - \alpha_2$, $\Delta \beta^F = \beta'_2 - \beta_2$, $\Delta \rho^F = \rho'_2 - \rho_2$. By assuming $|\frac{\Delta \alpha}{\alpha}| \ll 1$
and $|\Delta \alpha^F| \ll 1$, higher order combinations of these can be neglected, as for a variable $x \to 0$, $x^2 \ll x$, and we may assume w.l.o.g., $x^2 = 0$.

For separating the term $2\gamma^2(\Delta \rho^F + 2\Delta \beta^F)\sin^2 \theta$ from equation 4.3 to the representative PP-reflection coefficient term prior to injection $2\gamma^2(\frac{\Delta \rho}{\rho} + \frac{2\Delta \beta}{\beta})\sin^2 \theta$, and the change induced by CO$_2$ injection $2\gamma^2(\frac{\Delta \rho^F}{\rho} + \frac{2\Delta \beta^F}{\beta})\sin^2 \theta$, an approximation of the velocity ratio is utilized. The velocity ratio can be rewritten as:

$$\gamma' = \frac{\beta'}{\alpha'} = \frac{\beta_2 + \beta_1}{\alpha_2 + \alpha_1} = \frac{\Delta \beta^F + \Delta \alpha^F}{\Delta \alpha^F + \alpha_2} = \frac{\beta + \frac{\Delta \beta^F}{\beta_2}}{\alpha + \frac{\Delta \alpha^F}{2\alpha}} = \frac{\beta(1 + \frac{\Delta \beta^F}{2\beta})}{\alpha(1 + \frac{\Delta \alpha^F}{2\alpha})},$$

(4.4)

where the delta terms $\frac{\Delta \beta^F}{2\beta}$ and $\frac{\Delta \alpha^F}{2\alpha}$ are to be multiplied with the delta terms in $(\frac{\Delta \rho^F}{\rho} + \frac{2\Delta \beta^F}{\beta})$. This multiplication results in second degree terms, which are neglected. Hence, the resulting approximated velocity ratio can be used for the $\sin^2 \theta$ term:

$$\gamma' = \gamma.$$  

(4.5)

Introducing these technicalities to 4.3, we obtain the following equation:

$$R_{pp,1}(\theta) \approx R_{pp,0}(\theta) + \frac{1}{2} \left( \frac{\Delta \rho^F}{\rho} + \frac{\Delta \alpha^F}{\alpha} \right) - 2\gamma^2 \left( \frac{\Delta \rho^F}{\rho} + \frac{2\Delta \beta^F}{\beta} \right) \sin^2 \theta + \frac{\Delta \alpha^F}{2\alpha} \tan^2 \theta. \quad (4.6)$$

As mentioned in Section 3.1, the shear modulus $\mu$, is assumed unaffected by saturation changes. Because of this assumption:

$$\frac{\Delta \rho^F}{\rho} + \frac{2\Delta \beta^F}{\beta} \approx \Delta \ln(\rho) + 2\Delta \ln(\beta) = \Delta \ln(\beta^2\rho) = \Delta \ln(\mu) = 0. \quad (4.7)$$

The resulting change in reflectivity, to the lowest order, due to saturation change is:

$$\Delta R_{pp}^F(\theta) \approx \frac{1}{2} \left( \frac{\rho^F}{\rho} + \frac{\Delta \alpha^F}{\alpha} \right) + \frac{\Delta \alpha^F}{2\alpha} \tan^2 \theta. \quad (4.8)$$

The density can be written as the arithmetic average:

$$\rho = \phi \rho_f + (1 - \phi) \rho_s,$$

(4.9)

where $\rho$ denotes the bulk density, $\phi$ denotes the porosity, $\rho_f$ denotes the average fluid density, and $\rho_s$ denotes the grain density. The average fluid density is dependent on the given mixture of fluids. The changes in porosity from pressure change is assumed to be small; hence the change in density from pressure change will be
neglected (Landrø, 2001; Meadows, 2001).

Following the same derivation as for equation 4.8, the change in reflection coefficient, to the first order, in regards of pressure changes can be written as:

$$\Delta R_{pp}(\theta) \approx \frac{1}{2} \frac{\Delta \alpha}{\alpha} - 4\gamma^2 \frac{\Delta \beta}{\beta} \sin^2 \theta + \frac{\Delta \alpha}{2\alpha} \tan^2 \theta.$$

(4.10)

The total change in PP-reflection coefficient dependent on the coefficients found in Section 3 can be expressed as the sum of equations 4.8 and 4.10:

$$\Delta R_{pp}(\theta) \approx \frac{1}{2} \left( \rho F \rho + \Delta \alpha \frac{F}{\alpha} \right) + \frac{1}{2} \left( \frac{\Delta \alpha}{\alpha} + \frac{\Delta \alpha}{\alpha} \right) \tan^2 \theta - 4\gamma^2 \frac{\Delta \beta}{\beta} \sin^2 \theta.$$

(4.11)

Equation 4.11 is written in an expanded manner for later use in Section 4.1.2.

### 4.1.2 Analytic solution for linear AVO method

The total change in PP-reflection coefficient, $\Delta R_{pp}(\theta)$, can be expressed by inserting the parameters from equation 3.7 into equation 4.11:

$$\Delta R_{pp}(\theta) \approx \frac{1}{2} \left( \rho F \rho + \Delta \alpha \frac{F}{\alpha} \right) + \frac{1}{2} \left( \frac{\Delta \alpha}{\alpha} + \frac{\Delta \alpha}{\alpha} \right) \tan^2 \theta - 4\gamma^2 \frac{\Delta \beta}{\beta} \sin^2 \theta.$$

(4.12)

Using the small angle approximation $\sin^2 \theta \approx \tan^2 \theta$, and the AVO intercept and gradient formula:

$$R_{pp}(\theta) = A + B \sin^2 \theta,$$

(4.13)

where $A$ denotes the conventional AVO intercept parameter and $B$ denotes the AVO gradient, equation 4.12 can be split into one intercept and one gradient term:

$$\Delta A \approx \frac{1}{2} \left( k_{p} \Delta S + k_{a} \Delta S + l_{a} \Delta P + m_{a} \Delta P^2 \right),$$

$$\Delta B \approx \frac{1}{2} \left( k_{a} \Delta S + l_{a} \Delta P + m_{a} \Delta P^2 \right) \tan^2 \theta,$$

$$-4\gamma^2 \left( l_{\beta} \Delta P + m_{\beta} \Delta P^2 \right) \sin^2 \theta.$$

(4.14)

Equations 4.14 and 4.15 can be solved to find explicit expressions for changes in pressure and saturation.

Solving equations 4.14 and 4.15 for $\Delta S$ results in, respectively:

$$\Delta S = \frac{2\Delta A - \frac{1}{2} l_{a} \Delta P - \frac{1}{2} m_{a} \Delta P^2}{k_{p} + k_{a}},$$

(4.16)
\[ \Delta S = 2B - \frac{1}{2}(l_\alpha \Delta P + m_\alpha \Delta P^2) + 4\gamma^2(l_\beta \Delta P + m_\beta \Delta P^2) \frac{k_\alpha}{k_\alpha} . \]  
(4.17)

Setting equation 4.16 equal to equation 4.17 result in a second degree equation with variable \( \Delta P \). The change in pressure can be expressed as:

\[ \Delta P \approx \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} , \]  
(4.18)

where the constants \( a, b, \) and \( c \) are defined as:

\[ a = m_\alpha - 8\gamma^2m_\beta - \frac{m_\alpha k_\alpha}{k_\alpha + k_\rho} , \]  
(4.19)

\[ b = l_\alpha - 8\gamma^2l_\beta - \frac{k_\alpha l_\alpha}{k_\alpha + k_\rho} , \]  
(4.20)

\[ c = \frac{2k_\alpha \Delta A}{k_\alpha + k_\rho} - 2\Delta B . \]  
(4.21)

For zero change in \( \Delta A \) and \( \Delta B \), the resulting change in pressure should be zero. Hence, the negative root is neglected.

By assuming a velocity ratio of 0.5, and small changes in \( \Delta A \) and \( \Delta B \), the following approximation for pressure changes is derived:

\[ \Delta P \approx p_1 \Delta A + p_2 \Delta B , \]  
(4.22)

\[ p_1 = \frac{2k_\alpha}{p_3} , \]  
\[ p_2 = \frac{-2(k_\alpha + k_\rho)}{p_3} , \]  
\[ p_3 = 2k_\alpha l_\beta - k_\rho l_\alpha + 2k_\rho l_\beta . \]

The change in saturation can be expressed in the same regards:

\[ \Delta S \approx s_1 \Delta A + s_2 \Delta B , \]  
(4.23)

\[ s_1 = \frac{2l_\beta - l_\alpha}{s_3} , \]  
\[ s_2 = \frac{l_\alpha}{s_3} , \]  
\[ s_3 = \frac{k_\alpha + k_\rho}{2} . \]

The provided formulation is sufficient for an analytical data test of the method. However, previous studies have demonstrated that AVO curves obtained after 3-D prestack time migration are heavily contaminated by noise (Landrø, 2001; Meadows, 2001). Near and far stacks are less noise contaminated than prestack gathers (Landrø, 2001). Utilizing near and far stacks might provide a better estimation of the change in pressure and saturation. By exploiting the near- and far-offset stacks to
estimate the AVO parameters, the change in pressure and saturation can be derived from equations 4.22 and 4.23:

\[
\Delta P \approx p_1(\Delta N - c(\Delta F - \Delta N)\sin^2\theta_N) + p_2c(\Delta F - \Delta N),
\]

(4.24)

\[
\Delta S \approx s_1(\Delta N - c(\Delta F - \Delta N)\sin^2\theta_N) + s_2c(\Delta F - \Delta N),
\]

(4.25)

where \(\Delta N\) denotes the change in the PP-reflected near stack around an average near angle \(\theta_N\), and \(\Delta F\) denotes the change in the PP-reflected far stack around an average far angle \(\theta_F\). The constant \(c\) is defined as:

\[
c = \frac{1}{\sin^2\theta_F - \sin^2\theta_N}.
\]

(4.26)

It is worth mentioning that the formulation expects a change in saturation and pressure in the underlying layer. Hence, the values will have an opposite sign, and show a negative saturation and pressure change, at the bottom of the reservoir.

### 4.2 PS data

The PS-converted wave has a conventional P-wave source (e.g. explosive or marine air gun) which converts to a S-wave at the reflector, and is recorded at the seabed. This conversion is illustrated in Figure 4.2. The recording of a converted wave require at least 3 recording channels per receiver. The PS-converted wave derived in Aki & Richards (1980) manifest that the amplitudes recorded for a PS-converted wave is related only to density and shear reflectivities. Having input parameters with less amount of dependent unknowns is advantageous. PS-reflectivities can also be used in the manner of PS AVO. Gray (2003) emphasise that density effects is measurable for much shorter offsets by the use of PS AVO, compared to the corresponding PP AVO. A reason for this is that the PS-wave is reflected at a smaller incidence angle than the corresponding PP wave, as can be seen in Figure 4.2. Previous studies have found PS AVO to give reliable estimates of the density, and shear reflectivities (Gray, 2003).
Figure 4.2: Figure to highlight the issue of CDP points for PP-reflection and PS-reflection not being at the same position, even for a horizontally layered earth. Weak gray line show the regular CMP and shot-reciever distance for a streamer survey.

4.2.1 Incorporating PS-data in Landø’s method

The method is extended to use PP and PS stacks as input in Landrø et al. (2003). The underlying assumptions previously mentioned in Section 4.1.1 are present in the incorporation of PS-data as well, making the PP AVO method and the PP-PS method very similar in regards of assumptions. This generates a way to compare AVO from streamer data with seabe monitoring of PP- and PS-reflections. The method for estimating saturation and pressure changes from PP-PS data is explained in Landrø et al. (2003); however, it is a vital part of this study and will be repeated in this section.

Equation 4.12 is used for the total change in PP-reflectivity from saturation and pressure changes, and a consistent equation for the PS-reflectivity from saturation and pressure changes is derived in Landrø et al. (2003).

From Aki & Richards (1980) we have:

\[ R_{ps}(\theta) = \frac{-p_{12}}{2\cos\psi} \left[ \left( 1 - 2\beta^2 p^2 + 2\beta^2 \frac{\cos\theta \cos\psi}{\alpha \beta} \right) \frac{\Delta \rho}{\rho} - 4\beta^2 \left( p^2 - \frac{\cos\theta \cos\psi}{\alpha \beta} \right) \frac{\Delta \beta}{\beta} \right], \quad (4.27) \]
where $\psi$ denotes the angle of the reflected converted wave. Landrø et al. (2003) gives a weak contrast and small angle approximation for the PS-reflection coefficient:

$$R_{ps,0}(\theta) = -\frac{1}{2}\left((1+2\gamma)\frac{\Delta \rho}{\rho} + 4\gamma \frac{\Delta \beta}{\beta}\right)\sin \theta + \gamma\left(\left(\gamma + \frac{1}{2}\right)\left(\frac{\Delta \rho}{\rho} + 2\frac{\Delta \beta}{\beta} - \frac{\gamma}{4}\frac{\Delta \rho}{\rho}\right)\sin^3 \theta.\right)$$

(4.28)

In equation 4.28, the small angle approximation $\tan^2 \theta \approx \sin^2 \theta$ is made. This is not consistent with the change in PP-reflection coefficient found in equation 4.12, where this approximation is not applied. However, equations 4.12 and 4.28 will be used collectively.

The change in PS-reflection coefficient due to pressure and saturation changes can be calculated in the same manner as equations 4.8 and 4.10:

$$\Delta R_{ps}^F(\theta) = -\frac{1}{2}\frac{\Delta \rho^F}{\rho}\sin \theta - \frac{\gamma^2}{4}\frac{\Delta \rho^F}{\rho}\sin^3 \theta, \quad (4.29)$$

$$\Delta R_{ps}^P(\theta) = -2\gamma\frac{\Delta \beta^P}{\beta}\sin \theta + 2\gamma\left(\gamma + \frac{1}{2}\right)\frac{\Delta \beta^P}{\beta}\sin^3 \theta. \quad (4.30)$$

### 4.2.2 Analytic solution for linear PP-PS method

The change in reflectivity due to both saturation and pressure changes (equations 4.29 and 4.30) can be written for $R_{ps}$ in the same manner as for $R_{pp}$ in equation 4.12:

$$\Delta R_{ps}(\theta) \approx -\frac{1}{2}k_p\Delta S\sin \theta - \frac{\gamma^2}{4}k_p\Delta S\sin^3 \theta - 2\gamma(l_\beta \Delta P$$

$$+m_\beta \Delta P^2)\sin \theta + 2\gamma\left(\gamma + \frac{1}{2}\right)(l_\beta \Delta P$$

$$+m_\beta \Delta P^2)\sin^3 \theta. \quad (4.31)$$

Equations 4.12 and 4.31 are still angle dependent and could be used in the manner of PP and PS AVO. Using the equations in this form will not mitigate the main concern with the exploitation of AVO parameters, e.g. non-identical wavelet and multiple residual energy, which is discussed in Section 4.3. A more potent approach would be to work on full stacks. Equation 4.12 and 4.31 are integrated within the minimum ($\theta_1$) and maximum ($\theta_2$) incident angles:
\[ \Delta R_{pp}^\Sigma \approx \frac{1}{2} (k_\rho \Delta S + k_\alpha \Delta S + l_\alpha \Delta P + m_\alpha \Delta P^2) i_0 \\
+ \frac{1}{2} (k_\alpha \Delta S + l_\alpha \Delta P + m_\alpha \Delta P^2) i_3 \\
- 4\gamma^2 (l_\beta \Delta P + m_\beta \Delta P^2) i_2, \]  
\[ (4.32) \]

\[ \Delta R_{ps}^\Sigma \approx -\frac{1}{2} k_\rho \Delta S i_1 - \frac{3}{4} k_\rho \Delta S i_4 - 2\gamma (l_\beta \Delta P \\
+ m_\beta \Delta P^2) i_1 + 2\gamma \left( \gamma + \frac{1}{2} \right) (l_\beta \Delta P \\
+ m_\beta \Delta P^2) i_4, \]  
\[ (4.33) \]

were \( i_n \) is defined as:

\[ i_0 = \frac{1}{\theta_2 - \theta_1} \int_{\theta_1}^{\theta_2} d\theta = 1. \]

\[ i_1 = \frac{1}{\theta_2 - \theta_1} \int_{\theta_1}^{\theta_2} \sin \theta d\theta = \frac{1}{\theta_2 - \theta_1} (\cos \theta_1 - \cos \theta_2), \]

\[ i_2 = \frac{1}{\theta_2 - \theta_1} \int_{\theta_1}^{\theta_2} \sin^2 \theta d\theta = \frac{1}{\theta_2 - \theta_1} \left( \frac{1}{2} (\theta_2 - \theta_1) - \frac{1}{4} (\sin 2\theta_2 - \sin 2\theta_1) \right), \]

\[ i_3 = \frac{1}{\theta_2 - \theta_1} \int_{\theta_1}^{\theta_2} \tan^2 \theta d\theta = \frac{1}{\theta_2 - \theta_1} (\tan \theta_2 - \tan \theta_1 - (\theta_2 - \theta_1)), \]

\[ i_4 = \frac{1}{\theta_2 - \theta_1} \int_{\theta_1}^{\theta_2} \sin^3 \theta d\theta = \frac{1}{\theta_2 - \theta_1} \frac{1}{12} (\cos 3\theta_2 - \cos 3\theta_1) - \frac{3}{4} (\cos \theta_2 - \cos \theta_1). \]  
\[ (4.34) \]

Rearranging these terms gives:

\[ \Delta R_{pp}^\Sigma \approx a_1 \Delta S + a_2 \Delta P + a_3 \Delta P^2, \]  
\[ (4.35) \]

\[ \Delta R_{ps}^\Sigma \approx b_1 \Delta S + b_2 \Delta P + b_3 \Delta P^2, \]  
\[ (4.36) \]

were the coefficients are defined as:

\[ a_1 = \frac{1}{2} k_\rho i_0 + \frac{1}{2} k_\alpha i_0 + \frac{1}{2} k_\alpha i_3, \]

\[ a_2 = l_\alpha i_0 + \frac{1}{2} l_\alpha i_3 - 4\gamma^2 l_\beta i_2, \]

\[ a_3 = \frac{1}{2} m_\alpha i_0 + \frac{1}{2} m_\alpha i_3 - 4\gamma^2 m_\beta i_2, \]

\[ b_1 = -\frac{1}{2} k_\rho i_1 - \frac{\gamma^2}{4} k_\rho i_4, \]

\[ b_2 = -2\gamma l_\beta i_1 + 2\gamma (\gamma + \frac{1}{2}) l_\beta i_4, \]

\[ b_3 = -2\gamma m_\beta i_1 + 2\gamma (\gamma + \frac{1}{2}) m_\beta i_4. \]  
\[ (4.37) \]
Equation 4.35 and 4.36 can be solved for saturation and pressure changes as:

\[
\Delta P = \frac{b_2 a_1 - a_2 \pm \sqrt{(a_2 - \frac{b_2}{b_1}a_1)^2 - 4(a_3 - \frac{b_2}{b_3}a_1)(\frac{a_3}{b_3} \Delta R_{ps} - \Delta R_{pp})}}{2(a_3 - \frac{b_2}{b_3}a_1)}, \quad (4.38)
\]

\[
\Delta S = \frac{\Delta R_{pp} - \frac{a_3}{a_2} \Delta R_{ps} - (a_2 - \frac{b_2}{b_3}a_3) \Delta P}{a_1 - \frac{b_1}{b_3}a_3}. \quad (4.39)
\]

For zero change in \( \Delta R_{pp} \) and \( \Delta R_{ps} \), there should be no change in \( \Delta P \). This results in neglection of the negative root in equation 4.38, in the same manner mentioned for equation 4.18.

### 4.3 Noise not taken into account in this synthetic study

In this thesis, 1D models for synthetic geological models are considered with perfect repeatability. Noise will be defined in the same manner of Cambois (2000), "(...) as anything that does not fit the assumed model (...)". The assumed model in this study will be the Aki & Richards (1980) equations 4.1 and 4.27.

#### 4.3.1 PP AVO noise

The PP AVO method is prone to error. As shown in equations 4.24 and 4.25, differencing of near and far stacks is required. For this to be viable, the near and far stacks need identical wavelet. Identical wavelet for the near- and far-angle stacks might be achieved through complex processing (Cambois, 2001). The final resolution will be controlled by the far offset, decreasing the vertical resolution, in comparison to using full PP and PS stacks. One improvement to the method in Landrø (2001) might be to implement more optimal PP AVO approximations. Efforts for improving the PP AVO parameters are still being made (e.g. Causse et al. (2007) and Dupuy et al. (2017)).

By utilizing Meadows (2001) enhancement of Landrø’s method, the wavelet effects are eliminated as \( \Delta B \) can be calculated exactly from \( \Delta I_p \), \( \Delta I_s \), and \( \Delta \rho \). The differencing of the seismic parameters will then be done on inverted data. Meadows (2001) reasons that \( I_p \) and \( I_s \) are derived by inversion of PP AVO data over the
entire range of offsets, while in this thesis, the PP AVO parameters are calculated from near and far stacks, disregarding the mid stack. Disregarding the mid-stack decreases the signal-to-noise ratio (S/N) as the full stack is not taken advantage of. The two-step method by Meadows (2001) could be an improvement to the method.

Another disadvantage of PP AVO is the need for identical arrival time of the near- and far-stack. Regular normal move-out (NMO) and dip move-out (DMO) corrections approximate the travel time by a hyperbolic move-out (Yilmaz, 2001), which is not applicable for larger offsets. Neglecting larger offsets can result in residual multiple energy (Cambois, 2001), decreasing the S/N ratio.

Another issue with 4D seismic is the need to align identical acquisition geometries (Gelius, 2019a). Multiple factors affect the survey, such as sea level, sea state, swell noise, water temperature, and salinity (Lumley, 2001). Variable near-surface conditions can result in statics and receiver coupling variations. Variation in ambient and shot-generated noise will also decrease the repeatability.

Landrø et al. (2003) argues that many of the acquisition and processing uncertainties are reduced in time-lapse seismic by committing the same error for the baseline survey and the monitor survey, and then subtracting one from the other. Landrø (2001) mentions that by increasing the amount of independent parameters, such as either taking the curvature parameter of PP AVO into account or by the use of other seismic survey measurements, e.g. converted wave reflections or time-shift information, other reservoir changes than pressure and saturation could also be estimated. Given the uncertainties in the data, estimating more than two parameters is on the verge of too ambitious.

4.3.2 PP-PS noise

By utilizing the PP-PS method, full stacks are exploited. The two inputs do not need identical wavelets, as the PP and PS stacks are viewed as two independent inputs. However, processing of PS-data is complex. One issue with processing PP- and PS-reflections is highlighted in Figure 4.2; PP- and PS-reflections with the same common midpoint (CMP) do not share the same common depth point (CDP), even for a horizontally layered earth model. The PP-PS method in Section 4.2.1 assumes the same CDP for both stacks. Even though the geometry can be corrected in pro-
cessing, it will still be another source of uncertainty added to a real data example. Figure 4.2 also show that CMP and PP-reflection CDP’s are not the same point for a horizontally layered earth when using ocean bottom logging (OBS). Even though this can be corrected, it is not a necessity, as the same calculation will be done for both the baseline- and monitor-survey, correcting the assumed simplification. Ocean bottom logging with good receiver coupling increase the repeatability as the receiver end of the survey will be stable.

Yilmaz (2001) approximate the NMO correction of PS data by averaging the P- and S-wave velocity and assuming a hyperbolic form. PS data does not follow a hyperbolic form (Yilmaz, 2001), causing uncertainty to the method.

PS data have a low amplitude for small offsets. This effect decreases the S/N of PS data compared to PP data.
5  |  Linear saturation case

To point out the issues with the simplified approximation in Landrø (2001), estimated monitor surveys are forward modeled in Petrel by the linear approximation in Section 5.1 and by a higher order approximation in Section 6.1. The models are analytically inverted using the simplified Landrø (2001) approximation.

Figure 5.1: Position of line L1, used in Chapter 5 is marked in red, and extent of cube used in Chapter 6 is marked in black. The CO₂ saturated plume is marked as yellow. Arrow indicates North.
5.1 Forward modeling from the provided simulation

The synthetic geological model with seismic parameters is provided by NGI under NCCS. As a quality control, an inversion of the GN1101 cube together with the two well logs, 32/4-1 and 32/2-1 was performed in HampsonRussell. The inversion done in HampsonRussell is marked as "1" in Figure 5.2. The provided P-wave cube is marked as "2". There are a lot of similarities between the two inversions. The HampsonRussell inversion is close to constant in 2D. The reason for this is the use of only two well logs for an inversion of a full 3D cube. The basement in the HampsonRussell inversion should be neglected as the wells do not extend to this depth. Multiple anomalies in the NGI inversion are not apparent in the HampsonRussell inversion. The anomalies in the NGI inversion suggest the use of other wells, the use of stacking velocities, or an inversion where the seismic data have been weighted in comparison to the well logs. The provided S-wave model follow the Castagna mudrock line.

![Figure 5.2: Figure illustrating the Vp inversion done in HampsonRussell by using the GN1101 seismic cube and the wells 32/4-1 and 32/2-1, marked as "1". Inverted Vp cube from NGI is marked as "2". There is a lot of similarities, however the extent of cube "2", and layers in the overburden of the NGI inversion suggest the use of lines or wells not taken into account in the HampsonRussell inversion, stacking velocities, or an inversion where the seismic data have been weighted in comparison to the well logs. Arrow indicates North.](image-url)
The saturation and pressure model provided by Equinor is made in Eclipse (Statoil, 2016). It has a maximum injection rate of 1.3 MTPY with an injection period of 25 years. A workflow was constructed in Petrel, calculating expected seismic parameters from the background seismic parameters, as well as saturation and pressure cubes, using the relationships provided in equation 3.8.

The background cube and selected time frame are exported from Petrel with the seismic parameters, the modeled saturation, pressure, as well as indices. Figure 5.3 show line L1, which slices through the Alpha structure. The position of the line is illustrated in Figure 5.1. The line L1 has indice i=54 in the synthetic matrix, and x coordinate 558633m in the given Petrel coordinate reference system (CRS). An issue with the imported data is that some indexes are missing. These missing indexes are linearly interpolated, making the method applicable for different lines. Some reshaping and sorting of the data is done before inverting it using the two methods at hand. The sorted data is used to calculate analytical PP AVO coefficients, as well as PP and PS stacks.

Figure 5.3: P-wave velocity at the chosen seismic line L1 for year 2020 (top) and year 2045 (bottom). Some effect of the plume is noticed in the Figure for year 2045 around and y=6740 km.
PP AVO

PP AVO parameters are calculated from the seismic parameters by utilizing the small angle approximation $sin^2 \theta \approx tan^2 \theta$, and separating equation 4.2 into an intercept $A$ term, and a gradient $B$ term:

\[
A = \frac{1}{2}(\Delta \rho / \rho + \Delta \alpha / \alpha), \\
B = -2 \gamma (\Delta \rho / \rho + 2 \Delta \beta / \beta) + \Delta \alpha / 2 \alpha,
\]

where $\gamma$ is set to 0.5.

PP-PS

Full PP- and PS-stacks need to be calculated from the baseline and monitor seismic parameters. These can be calculated by integrating the reflection coefficients from Aki & Richards (1980) over a given angle span:

\[
R_{pp}^{\Sigma} = \frac{1}{2}(\Delta \rho / \rho + \Delta \alpha / \alpha) - 2 \gamma (\Delta \rho / \rho + 2 \Delta \beta / \beta)i_2 + \frac{\Delta \alpha}{2 \alpha}i_3, \\
R_{ps}^{\Sigma} = \frac{1}{2} \left((1 + 2 \gamma) \Delta \rho / \rho + 4 \gamma \Delta \beta / \beta\right)i_1 + \gamma \left((\gamma + 1/2) \left(\frac{\Delta \rho}{\rho} + 2 \frac{\Delta \beta}{\beta} - \frac{\gamma \Delta \rho}{4 \rho}\right)\right)i_4,
\]

where $i_n$ is defined in equations 4.34.

Results from inversion of the provided simulation

The PP AVO parameters, PP- and PS-reflection coefficients are used together with the constants in equation 3.8, to solve for pressure and saturation changes using equations 4.22, 4.23, 4.38, and 4.39.
Figure 5.4: Subplots showing the change in saturation from 2020 to 2045 for the provided simulation. a) Change in saturation from simulation, b) Change in saturation from PP AVO, c) Change in saturation from PP-PS timelapse, d) Saturation at top of reservoir from the 3 other plots.
From Figures 5.4 and 5.5 it can be deduced that the pressure- and saturation effect can be discriminated for a model following the assumptions provided in Chapter 3. The maximum saturation in the simulation in 2045 (Figure 5.4, a) is 80.79%. The PP AVO inversion (Figure 5.4, b) overestimates this saturation to 82.74%, while the PP-PS inversion (Figure 5.4, c) slightly underestimates this saturation to 78.85%. The extent of the saturated plume is well defined in both of the schemes. There is a moderate pressure effect at the top and bottom of the reservoir, which is barely noticeable on the imaged data, away from the plume. However, this effect results in an assumed CO₂ saturation of less than 0.5%.

The maximum pressure build-up at the top of the reservoir in Figure 5.5 is 1.07 MPa. Figure 5.5 d shows that the inversion schemes notice the changes in pressure, while underestimating the values. This is assumed for small pressure build-ups, as the P-wave curve in Figure 3.4 barely change for small changes in pressure. The PP AVO method results in a maximum pressure build-up of 0.686 MPa, while the PP-PS method provides a maximum pressure build-up of 0.734 MPa. A drop in pressure
is noticeable in Figure 5.5 at the injection well (horizontal distance 6740km). The reason for this is an initial saturation in the the provided simulation at the injection cell in year 2020.

Bhakta & Landrø (2014) found the first order Landrø approximation to under-predict both PP AVO parameters. Trani et al. (2011) found the change in the intercept AVO parameter to be estimated well, while the change in gradient is under-predicted. The under-prediction of the gradient term for a field data example is expected as Landrø's method originate from Aki & Richards (1980) equations. The Aki & Richards equations deviate from Zoeppritz (1919) equations for angles larger than 30°. The small angle approximation utilized in equation 5.2, is found to deviate for angles over 45° (Grude et al., 2013). This lead to a slightly under-estimated pressure and a slightly over-estimated saturation for the PP AVO method. However, this is not the main issue at hand, and the given Aki & Richards approximation is regarded as an insignificant reason for uncertainty.
6 | Non-linear saturation estimation

6.1 Non-linear saturation modeling solved with linear approximated inversion

Even though the results from Section 5 appear applicable, the scenario is oversimplified. The relationship between saturation and P-wave velocity is far from linear. As previously mentioned, using a non-linear approximation of the relative change in P-wave velocity versus saturation was introduced in the manner of Landrø’s method by Meadows (2001). This results in a change in equation 3.7:

$$\frac{\Delta \alpha}{\alpha} \approx j_\alpha \Delta S^2 + k_\alpha \Delta S + l_\alpha \Delta P + m_\alpha \Delta P^2,$$  \hspace{1cm} (6.1)

Approximating the curve from Brie’s equation (equation 3.2), with a Brie exponent equal to 3, and finding the best fit non-linear equation using least squares, results in $j_\alpha = 0.12$ and $k_\alpha = -0.18$. The curve is illustrated in Figure 6.1.
Relative change in P- and S-wave velocity from CO$_2$ saturation change estimated using Brie’s fluid substitution. The red curve is the relative change in S-wave and is already linear. The solid blue line is the relative change in P-wave with a Brie exponent equal to 3. The dotted blue line is the linear approximation for the relative change in P-wave. The black dotted curve overlaying the solid blue line is the approximated non-linear relation.

When forward modeling the synthetic cube from 5.1 using the updated change in P-wave velocity (equation 6.1) and solving the system for $\Delta P$ and $\Delta S$ using the linear approximation from Landrø (2001) (equation 4.22 and 4.23), the results become misleading which are illustrated in Figures 6.2 and 6.3.
Figure 6.2: Subplots showing the change in saturation from 2020 to 2045 for the provided simulation with a non-linear forward and linear solver. a) Change in pressure from simulation, b) Change in pressure from PP AVO, c) Change in pressure from PP-PS timelapse, d) Pressure at the top of reservoir from the 3 other plots.
Figure 6.3: Subplots showing change in pressure from 2020 to 2045 for the provided simulation with a non-linear forward and linear solver. a) Change in pressure from simulation, b) Change in pressure from PP AVO, c) Change in pressure from PP-PS timelapse, d) Pressure at top of reservoir from the 3 other plots.

Figure 6.2 shows that the saturation is overestimated in a greater extent than in the more straightforward case from Section 5.1. The maximum saturation in the simulation in 2045 was 80.74%, while response from the PP AVO and PP-PS methods are now 99.53% and 95.27%, respectively.

There is an considerable error in the pressure estimation. The true maximum pressure build-up of 1.07 MPa is estimated to a pressure depletion of 0.22 MPa and 0.28 MPa for the PP AVO and PP-PS method, respectively. By utilizing the linear approximation method by Landrø (2001) and Landrø et al. (2003) the plume is visible, but the pressure effect outside of the plume is not visible. One can conclude that inverting using the linear approximation by Landrø (2001) and Landrø et al. (2003) is oversimplified, and a more refined scheme is needed to discriminate between pressure and saturation effects for the CO₂ injection scenario.
6.2 A two layered model

Another way to interpret the data is to plot all pressure- and saturation pairs with the corresponding amplitude change in AVO parameters and reflection coefficients for a two layer model consisting of a cap rock and a reservoir layer. By placing a proposed well along the line in Section 5.1, the seismic properties of the cap rock and the reservoir layer are chosen.

In Stovas & Landrø (2004), Figure 6.4 is used to illustrate that orthogonality between the curves is desired for pressure and saturation discrimination. The optimal parameters for discriminating between two fluid effects is two independent measurements, each purely dependent on one fluid effect. The two dense lines in Figure 6.4 represent a given, constant value for the change in stacked PP and PS data. The dotted lines represent the error in the parameters. If the angle $\alpha$ in Figure 6.4 is 90 degrees, the stacks are orthogonal and sensitive to completely different fluid effects. Figure 6.4 express a production scenario, with increasing water saturation in the x-axis. This is in contrast to the CO$_2$ injection case, where the plot goes from 0% to 100% CO$_2$ in the x-axis. In extension this means that the x-axis will be flipped for an injection scenario.

![Figure 6.4](image)

**Figure 6.4:** Figure retrieved from Stovas & Landrø (2004) illustrating the importance of an angle between the input seismic parameters relative to saturation and pressure. $\delta C_n$ illustrate error in stacks, $\delta S$ and $\delta \frac{P}{P_0}$ illustrate error in output saturation and pressure, respectively.
Figure 6.5: Contour plots of $\Delta R_{pp}^\Sigma$ and $\Delta R_{ps}^\Sigma$ (left), and $\Delta A$ and $\Delta B$ (right), for a linear relationship between saturation and $\frac{\partial^2 \alpha}{\partial x^2}$. The horizontal lines in the plots are $\Delta R_{pp}^\Sigma$ (left plot) and $\Delta A$ (right plot). The vertical lines are $\Delta R_{ps}^\Sigma$ (left plot) and $\Delta B$ (right plot).

Figure 6.6: Contour plots of $\Delta R_{pp}^\Sigma$ and $\Delta R_{ps}^\Sigma$ (left), and $\Delta A$ and $\Delta B$ (right), for a non-linear relationship between saturation and $\frac{\partial^2 \alpha}{\partial x^2}$. The horizontal lines in the plots are $\Delta R_{pp}^\Sigma$ (left plot) and $\Delta A$ (right plot). The vertical lines are $\Delta R_{ps}^\Sigma$ (left plot) and $\Delta B$ (right plot).
Figure 6.5 a and b shows a similar discrimination by the use of stacked reflection coefficients (Figure 6.5, right) and AVO parameters (Figure 6.5, left). The angle between the contour lines are both close to orthogonal. It can be deduced that a good discrimination between saturation and pressure would be prominent, if there was a linear relationship between saturation and $\Delta\alpha$. However, this was established as a fallacious approximation in Sections 3.1 and 6.1.

Another noticeable characteristic in the figures, is the distance between the contour lines. In Figure 6.6, the lines are closely spaced for low values of CO$_2$ saturation and pressure, and increases for larger change in saturation and pressure values. For closely spaced contour lines, the coefficients have a more abrupt change, which is good for discrimination. Figure 6.6 shows that for both the PP-PS and AVO method, the graph becomes more complex when you reach a higher saturation than 50%.

For the PP stacked reflection coefficient, an abrupt change is noticed between 90-100% change in saturation. This is because of the approximated non-linear curve in Figure 6.1, where there non-linear curves diverge from the trend of a decreasing $\Delta\alpha$, and have a small increase from 80-100 % saturation. The plots seem to conclude that there will be an issue with pressure- and saturation discrimination for saturations over 50%. Using the relation of the plots in Figure 6.6 to solve for the relations in Figure 6.5 was shown to be a bad approximation in Section 6.1. The conclusion was that a more refined scheme is needed. From the plots in Figure 6.6 it is not definite if there will be any difference in pressure- and saturation discrimination by the use of PP-PS compared to the PP AVO parameters. This will be tested for the rest of this chapter.

### 6.3 Non-linear solution for saturation and pressure

The non-linear system of equations can be solved by root finding algorithms. Trani et al. (2011) utilized PP AVO attributes together with PP and SS time shifts. Trani et al. (2011) solved the overdetermined system using Gauss-Newton algorithm. Bhakta & Landrø (2014) utilized PP AVO attributes and solved the non-linear system by using the Levenberg–Marquardt (LM) algorithm. In this study, the LM algorithm is utilized. The algorithm is reviewed in Section 6.3.2. The process of making multiple synthetic seismic cubes from the synthetic geological model and
the provided simulation is described in Section 6.3.1.

6.3.1 Forward modeling

Method

A multi-trace convolutional model from Buland & Omre (2003) is used to create pre-stack seismic traces. The method utilize the fact that a 1D convolution can be done by converting the angle-dependent coefficients from equations 4.1 and 4.27 to a Toeplitz matrix, and simply multiply with a wavelet to get the reflectivity series efficiently. By assuming slowly varying seismic parameters, the relative contrasts can be approximated as \( \frac{\Delta \alpha}{\alpha} \approx \Delta x \ln(\alpha) \), \( \frac{\Delta \beta}{\beta} \approx \Delta x \ln(\beta) \), and \( \frac{\Delta \rho}{\rho} \approx \Delta x \ln(\rho) \), where \( \Delta x \) is the depth increment. The relative contrast can be defined as \( c = Dm \), where 

\[
D = \begin{bmatrix}
D_0 & 0 & 0 \\
0 & D_0 & 0 \\
0 & 0 & D_0 \\
\end{bmatrix},
\]

(6.2)

where

\[
D_0 = \begin{bmatrix}
-1 & 1 \\
& \ddots \\
& & -1 & 1 \\
\end{bmatrix},
\]

(6.3)

and \( m \) is defined as

\[
m = \begin{bmatrix}
\ln \hat{\alpha} \\
\ln \hat{\beta} \\
\ln \hat{\rho} \\
\end{bmatrix},
\]

(6.4)

where \( \hat{\alpha} \), \( \hat{\beta} \), and \( \hat{\rho} \) are column vectors with seismic parameters for full 1D profiles. A depth-angle gather, \( r \), for the given trace is found by taking the product of the relative contrasts, provided by \( c \), and the matrix \( M_{pp} \) or \( M_{ps} \). The matrices \( M_{pp} \) and \( M_{ps} \) contains the angle-dependent coefficients \( M_{\theta, \phi}^{(1)}, M_{\theta, \phi}^{(2)}, M_{\theta, \phi}^{(3)} \) for \( \frac{\Delta \alpha}{\alpha}, \frac{\Delta \beta}{\beta}, \frac{\Delta \rho}{\rho} \), respectively, from equation 4.1 or 4.27, for angle \( \theta_1 \) to \( \theta_n \).

\[
M_{pp} = \begin{bmatrix}
M_{pp,\theta_1}^{(1)} & M_{pp,\theta_1}^{(2)} & M_{pp,\theta_1}^{(3)} \\
& \ddots & \ddots \\
M_{pp,\theta_n}^{(1)} & M_{pp,\theta_n}^{(2)} & M_{pp,\theta_n}^{(3)} \\
\end{bmatrix}, \quad M_{ps} = \begin{bmatrix}
M_{ps,\theta_1}^{(1)} & M_{ps,\theta_1}^{(2)} & M_{ps,\theta_1}^{(3)} \\
& \ddots & \ddots \\
M_{ps,\theta_n}^{(1)} & M_{ps,\theta_n}^{(2)} & M_{ps,\theta_n}^{(3)} \\
\end{bmatrix},
\]

(6.5)
\( M_{ps}^{(1)} \) is always zero as \( R_{ps} \) do not depend on \( \frac{\Delta \alpha}{\alpha} \).

The convolved trace \( d \) is then the product of a strictly diagonal matrix \( W \), containing the given wavelet on the diagonal, and the depth-angle gather \( r \):

\[
\begin{bmatrix}
d_1 \\
\vdots \\
d_i \\
\vdots \\
d_p \\
\end{bmatrix} = \begin{bmatrix}
w & \ldots & 0 & \ldots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \ldots & w & \ldots & 0 \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \ldots & 0 & \ldots & w \\
\end{bmatrix} \begin{bmatrix}
r_1 \\
\vdots \\
r_i \\
\vdots \\
r_p \\
\end{bmatrix},
\]

(6.6)

The matrices \( W \) and \( D \) are made in scripts by Buland (2003).

\[\text{Figure 6.7: Angle gather PP-reflection series. Every second trace from 0°-30° is shown from 1350m-1750m from sea surface. The reservoir is brine filled. Top reservoir is the trough at 1530m.}\]
Figure 6.8: Angle gather for a PS-reflection series. Every second trace from 0°-30° is shown from 1350m-1750m from sea surface. The reservoir is brine filled. Top reservoir is the peak at 1530m.

Figure 6.7 show the angle gather of the pre-stack PP-reflection at the first j position for line L1 (i,j,k=54,1,:). The wavelet, w, is a Ricker wavelet with center frequency of 45 Hz. Every second angle from 0 incidence to 30 degrees is displayed from a depth of 1350m to 1750m. Figure 6.8 shows the corresponding pre-stack PS-reflection. The wiggle-plotting is made in a script by A.K. Booer (1993), were the input is seismic and depth. There is no noise or attenuation added as we consider noise an academic challenge outside the scope of this thesis work. Normal moveout is not taken into account as the kinematics part of the modeling is not a part of this thesis. The synthetic pre-stack data is stacked into full PP and PS stacks, as well as near and far offset PP-stacks. The far offset stack does not need scaling as the wavelet matrix in equation 6.6 was defined as angle independent.

The background survey and given monitor surveys from the synthetic geological
Petrel model is imported into MatLab. The data is decimated to the black cube illustrated in Figure 5.1. The missing area is extrapolated from the rest of the cube, as the area of interest is mainly the plume. This resulted in an artefact that can be seen in all plots in Section 6.4, where the top 3 lines have no pressure effect, making the calculation of the pressure and saturation easy for the inversion. Using the line portrayed in Figure 5.3 as a centerpoint, 20 lines were used, and is contoured as the black rectangle in Figure 5.1.

Different sets of the synthetic seismic cubes representing the background and monitor seismic are imported into a Levenberg–Marquardt inversion scheme, which is explained in Section 6.3.2.

![Figure 6.9: Inversion workflow](image)

### 6.3.2 Levenberg–Marquardt method

The Levenberg-Marquardt method is a root finding algorithm for solving non-linear least squares problems. The method can be explained as an extension of the Newton-Raphson method, adding damping to the system. The Newton-Raphson method uses the tangent line of a given initial point $\tilde{x}_0$, of a function $f$, to calculate the zero point of the tangent line. This is done to, hopefully, give a better approximation
to the root of the given function. The algorithm iterates the approach until some
tolerance of change in the previous points, or a maximum number of iterations is
met. The root of a function can be found by iterating:
\[ \tilde{x}_{n+1} = \tilde{x}_n - \frac{f(\tilde{x}_n)}{f'(\tilde{x}_n)}, \]  
where \( \tilde{x}_n \) is the initial guess.

For \( k \) non-linear equations \( f \), with \( m \) variables \( x \), and \( m \leq k \), the system can
be solved by Gauss-Newton method:
\[ \tilde{X}_{n+1} = \tilde{X}_n + Q G, \]  
\[ Q = -H^{-1}(\tilde{X}), \]  
\[ G = 2J^T F(\tilde{X}_n). \]

Q is the scaling factor, where \( H \) denotes the Hessian matrix for the set of non-linear
equations. \( G \) is the gradient, \( J \) is the Jacobian matrix, and:
\[ \tilde{X}_n = \begin{bmatrix}
\tilde{x}^{(1)}_n \\
\tilde{x}^{(2)}_n \\
.. \\
\tilde{x}^{(m)}_n
\end{bmatrix}, \quad F(\tilde{X}_n) = \begin{bmatrix}
f^{(1)}(\tilde{X}_n) \\
f^{(2)}(\tilde{X}_n) \\
.. \\
f^{(k)}(\tilde{X}_n)
\end{bmatrix}. \]

The Hessian matrix can be approximated as \( H = 2J^T J \), where \( J^T \) is the trans-
pose Jacobian matrix (Trani et al., 2011). The Levenberg–Marquardt (LM) method
(Levenberg, 1944; Marquardt, 1963) adds a damping parameter (\( \lambda \)) to the Hessian
matrix:
\[ H = (2J^T J) + \lambda^2 I, \]  
where \( I \) is the identity matrix. For iterated values of \( \tilde{X}_{n+1} \) closer to the minimum,
\( \lambda_{n+1} \) decreases. While \( \lambda_{n+1} \) increase when \( \tilde{X}_{n+1} \) overshoot. The resulting LM al-
gorithm is as follows:
\[ \tilde{X}_{n+1} = \tilde{X}_n - ((J^T J) + \lambda^2 I)^{-1} J^T F(\tilde{X}_n). \]
6.3.3 Solving the specific system using Levenberg-Marquardt method

The PP AVO parameters $\Delta A$ and $\Delta B$ from equations 4.14 and 4.15 is introduced to the non-linear expression for change in saturation (equation 6.1):

$$
\Delta A = \frac{1}{2}(k_p \Delta S_n + k_a \Delta S_n + j_a \Delta S_n^2 + l_a \Delta P_n + m_a \Delta P_n^2),
$$

$$
\Delta B = \frac{1}{2}(k_a \Delta S_n + j_a \Delta S_n^2 + l_a \Delta P_n + m_a \Delta P_n^2) - 4\gamma^2(l_\beta \Delta P_n + m_\beta \Delta P_n^2).
$$

The PP-reflectivity from equation 4.32 can be expressed in the same manner:

$$
\Delta R_{pp}^\Sigma \approx \frac{1}{2}(k_p \Delta S + j_a \Delta S^2 + k_a \Delta S + l_a \Delta P + m_a \Delta P^2)i_0
$$

$$
+ \frac{1}{2}(j_a \Delta S^2 + k_a \Delta S + l_a \Delta P + m_a \Delta P^2)i_3
eq \frac{4\Delta^2}{\alpha^2}(l_\beta \Delta P + m_\beta \Delta P^2)i_2.
$$

The PS-reflectivity is not dependent on $\frac{\Delta s}{\alpha}$, and will remain as described in equation 4.33. Equation 6.14 can be rewritten in the same manner as seen in equation 4.35:

$$
\Delta R_{pp}^\Sigma \approx q_1 \Delta S^2 + a_1 \Delta S + a_2 \Delta P + a_3 \Delta P^2,
$$

where:

$$
q_1 = \frac{1}{2}j_a i_0 + \frac{1}{2}j_a i_3.
$$

$i_0$ and $i_3$ in equation 6.16 are defined in equations 4.34.

In regards of pressure- and saturation discrimination, $\tilde{X}$ is iterated approximations for $\Delta P$ and $\Delta S$. For PP AVO, the functions of $F(\tilde{X})$ are equations 6.12 and 6.13 solved for 0:

$$
F(\tilde{X})_{avo} = \left[ \begin{array}{c}
\frac{1}{2}(k_p \Delta \tilde{S}_n + k_a \Delta \tilde{S}_n + j_a \Delta \tilde{S}_n^2 + l_a \Delta \tilde{P}_n + m_a \Delta \tilde{P}_n^2) - \Delta A \\
\frac{1}{2}(k_a \Delta \tilde{S}_n + j_a \Delta \tilde{S}_n^2 + l_a \Delta \tilde{P}_n + m_a \Delta \tilde{P}_n^2) - 4\gamma^2(l_\beta \Delta \tilde{P}_n + m_\beta \Delta \tilde{P}_n^2) - \Delta B
\end{array} \right].
$$

Utilizing PP-PS stacks, equations 4.36 and 6.15 results in $F(\tilde{X})$:

$$
F(\tilde{X})_{stacks} = \left[ \begin{array}{c}
q_1 \Delta \tilde{S}^2 + a_1 \Delta \tilde{S} + a_2 \Delta \tilde{P} + a_3 \Delta \tilde{P}^2 - \Delta R_{pp}^\Sigma \\
b_1 \Delta \tilde{S} + b_2 \Delta \tilde{P} + b_3 \Delta \tilde{P}^2 - \Delta R_{ps}^\Sigma
\end{array} \right].
$$

The Jacobian matrices can be found as:

$$
J_{avo} = \left[ \begin{array}{cc}
\frac{1}{2}(k_p \Delta \tilde{S}_n + k_a \Delta \tilde{S}_n + j_a \Delta \tilde{S}_n^2 + l_a \Delta \tilde{P}_n + m_a \Delta \tilde{P}_n^2) & \frac{l_\gamma}{2} \\
\frac{1}{2}(k_a \Delta \tilde{S}_n + j_a \Delta \tilde{S}_n^2 + l_a \Delta \tilde{P}_n + m_a \Delta \tilde{P}_n^2) & \frac{l_\beta}{2} - 4\gamma^2(2m_\beta + l_\beta)
\end{array} \right].
$$

53
\[ J_{\text{stacks}} = \begin{bmatrix} 2q_1 \Delta \tilde{S}_n + a_1 & 2a_3 \Delta \tilde{P}_n + a_2 \\ b_1 & 2b_3 \Delta \tilde{P}_n + b_2 \end{bmatrix}. \] (6.20)

By the use of initial guesses for saturation and pressure, together with equations 6.17, 6.18, 6.19, and 6.20, the LM algorithm in equation 6.11 can be iterated to find approximate solutions for saturation and pressure.

### 6.3.4 Limitations of the Levenberg-Marquardt Method

Second order derivatives are neglected in the approximation of the Hessian matrix. This approximation makes the method only valid for weakly non-linear functions (Trani et al., 2011). In this case, the functions of \( F(X) \) (equations 6.17 and 6.18), are assumed to be adequate.

An issue that was encountered when using the LM method was \( \lambda \) becoming very small without meeting the tolerance of change in \( X \). The main reason for the use of LM method instead of the Gauss-Newton algorithm, was the issue of the Hessian matrix becoming a singular matrix. By using the LM method, \( Q \) should never become singular, as long as \( \lambda \neq 0 \).

\( \lambda \) could also become a dominant value after few iterations. A large \( \lambda \) results in the scaling factor \( Q \) becoming insignificant, and \( \tilde{X}_{n+1} = \tilde{X}_n \), without iterating further. Fletcher (1971) modified the LM scheme by scaling the components of the gradient in correspondence to the curvature. This is done by replacing the identity matrix \( I \) with a diagonal matrix consisting of the diagonal elements of the Hessian \( H \). The resulting scaling factor \( Q \) can be written as:

\[ Q = -((J^T J) + \lambda^2 \text{diag}(J^T J)), \] (6.21)

which is the Levenberg–Marquardt-Fletcher method (LMF). However, using the LMF method resulted in new problems. Scaling by the use of the diagonal of the Hessian matrix resulted in the algorithm failing as it was frequently trying to invert a singular matrix. \( \lambda^2 \text{diag}(J^T J) < \lambda^2 I \), making the LMF method less successful compared to the LM method in this case.

Optimizing the algorithm is not the scope of this thesis, even though efforts were made. Inverting a singular matrix is a flaw. The programming is done in MatLab, which approximate the matrix inverse of a singular matrix as a pseudoinverse.
6.4 Top reservoir results using true saturation, baseline, and previous survey

The figures in this section is marked with indices from the chosen cube in Figure 5.1. Line number 10 is the line L1 from Figure 5.1. The x-axis extend from x=561 km (line number 1), to x=555 km (line number 20) in the Petrel CRS. The y coordinates extend from y=6680 km (j=1) to y=6770 km (j=300).

6.4.1 Good initial guess saturation

The results from employing the LM algorithm with good initial guesses are shown in Figures 6.10, 6.11, 6.12, and 6.13.

![Figure 6.10: Saturation at top reservoir for 3-(top left), 13-(top right), and 25-(bottom left) years of injection, and 25 years after injection (bottom right). Difference between the provided simulation and the monitored using PP AVO method. Initial guesses are set to 0.95 of the provided simulation.](image)
Figure 6.11: Saturation at top reservoir for 3-(top left), 13-(top right), and 25-(bottom left) years of injection, and 25 years after injection (bottom right). Difference between the provided simulation and the monitored using PP-PS method. Initial guesses are set to 0.95 of the provided simulation.
6.4.2 Good initial guess pressure

Figure 6.12: Pressure at top reservoir for 3-(top left), 13-(top right), and 25-(bottom left) years of injection, and 25 years after injection (bottom right). Difference between the provided simulation and the monitored using PP AVO method. Initial guesses are set to 0.95 of the provided simulation.
6.4.3 Discussion and observations for the good initial guess scenario

The initial values $X_0$ was found by setting the initial guess of saturation and pressure to 95% of the simulated saturation and pressure. This is an unreasonable small error for an initial guess. However, a substantial amount of calculation steps have been done between the Eclipse simulation and the synthetic data. The data have been forward calculated in Petrel from the models in Chapter 3, forward modeled to pre-stack data through the convolutional model in Section 6.3.1. The data was subject to a slowly varying seismic parameters approximation. The pre-stack data was sorted and stacked into full PP and PS stacks, and the AVO parameters were calculated from near and far stacks. Hence, this is done to check and accentuate for the differences.

Good saturation estimates are found by using the PP AVO method (Figure 6.10), and the PP-PS method (Figure 6.11). The figures give a quantifiable measure of
the pressure and saturation estimation.

Both methods slightly under-estimate the extent of the plume, resulting in under-estimates on the periphery of the plumes. The PP AVO method in Figure 6.10 over-estimate the saturation in some areas. The PP-PS method also over-estimates the saturation, but to a smaller degree.

The pressure front in Figures 6.12 and 6.13 is well described outside the plume. A slight under-estimation can be seen for 13- and 25-years of injection by the use of the PP-PS method. This is outside of the saturated plume, and the system of equations becomes simple when one of the parameters (the CO\textsubscript{2} saturation), is zero. There is a noticeable issue when approximating the pressure inside of the saturated plume. As seen in Figure 6.12 the PP AVO method under-estimates the pressure more than the PP-PS method in Figure 6.13.

The poor pressure discrimination inside the plume in Figure 6.12 result in the same effect as in Section 6.1, where the saturation became over-estimated as a result of under-predicted AVO parameter. An over-estimation of saturation goes conjointly with an under-estimation of pressure.

**6.4.4 Monitor Saturation**

The results from employing the LM algorithm by utilizing the previous survey as initial guess are shown in Figures 6.14, 6.15, 6.16, and 6.17.
Figure 6.14: Saturation at top reservoir for 3-(top left), 13-(top right), and 25-(bottom left) years of injection, and 25 years after injection (bottom right). Difference between the provided simulation and the monitored using PP AVO method. Previous monitor survey is set as the initial guess.
Figure 6.15: Saturation at top reservoir for 3-(top left), 15-(top right), and 25-(bottom left) years of injection, and 25 years after injection (bottom right). Difference between the provided simulation and the monitored using PP-PS method. Previous monitor survey is set as the initial guess.
6.4.5 Monitor Pressure

Figure 6.16: Pressure at top reservoir for 3- (top left), 13- (top right), and 25- (bottom left) years of injection, and 25 years after injection (bottom right). Difference between the provided simulation and the monitored using PP AVO method. Previous monitor survey is set as the initial guess.
Figure 6.17: Pressure at top reservoir for 3- (top left), 13- (top right), and 25- (bottom left) years of injection, and 25 years after injection (bottom right). Difference between the provided simulation and the monitored using PP-PS method. Previous monitor survey is set as the initial guess.

6.4.6 Monitoring plan discussion

The first monitoring survey does not have a previous saturated survey. Because of this, the initial guess for this survey will in this experiment be the true saturation, with a small error, by analogy to Section 6.4.3. The subsequent surveys use the previous one as the initial guess. By employing this approach, the extent of the plume becomes slightly under-estimated for both the PP AVO and the PP-PS method. This can be seen in Figures 6.14 and 6.15. The plume has a blue edge for all time spans, caused by the monitored survey showing 0 saturation, resulting in a negative difference between monitored and true saturation. The PP AVO method results in noticeably worse saturation discrimination when comparing the use of a good initial guess and the use of the previous monitor survey (Figures 6.10 and 6.14). This is noticed by lower values in the middle of the plume, and overall higher variance. For the PP-PS method (Figures 6.11 and 6.15), the result after using the monitoring plan, compared to the good initial guess, does not change much. In regards to the saturation, the PP-PS method seem more tolerable to worse initial guesses, in direct comparison to the PP AVO method.
In regards to the pressure front, both methods under-estimate the pressure front outside of the plume. Figure 6.16 shows an under-estimated pressure inside the saturated plume for the PP AVO method. The PP-PS method in Figure 6.17 show a better pressure discrimination in the plume, compared to the PP AVO method (Figure 6.16). The worst pressure and saturation discrimination is seen in the plot for 25 years after injection. There is a substantial change in pressure from the end of the injection period to 25 years after. This change can be seen from the bottom two plots in Figure A.2. There is a pressure build-up at the end of the injection period (Figure A.2, bottom left). 25 years later, the pressure has depleted (Figure A.2, bottom right). The system is minimized into two different scenarios in the pressure estimates from the monitoring plan by the use of PP AVO and PP-PS methods (fig 6.16 and 6.17). The PP AVO method (fig 6.16), estimates an increased depletion. The PP-PS method (fig 6.17), over-estimate the pressure on the edge of the plume. The PP-PS method under-estimate the pressure in the center of the plume. The poor estimation of the pressure front outside the plume indicate that the method is not sensitive enough to notice the pressure front. Hence the starting values when using a good initial guess in Section 6.4.1 and 6.4.2 are accepted as a root of the system, while in Section 6.4.4 and 6.4.5 the initial pressure of 0 is accepted as the root of the system.

Good initial guesses are vital for correct solutions of the method. Previous results indicates that leakage between $\Delta S$ and $\Delta P$ is induced by poor approximation of changes in gradient reflectivity (Meadows, 2001). According to multiple studies, such as Meadows (2001); Trani et al. (2011); Grude et al. (2013); Bhakta & Landrø (2014), the gradient term is by far the least accurate input. One reason for this is the neglection of higher order terms in the already linearized Aki and Richard approximation (equation 4.6). However, as mentioned in Section 5.1, this is not an issue in this thesis study. As previously mentioned in Section 6.3.4, the Hessian approximation is only valid for weakly non-linear functions. When comparing the PP-PS method to the PP AVO method in Figure 6.6, the PP AVO method is non-linear for a larger area of the plots. The issue in the PP AVO method lies in the relationship between the parameters illustrated in Figure 6.6. This is a reason for the PP-PS method being more robust in regards to pressure-saturation discrimination by the use of non-linear root finding.
6.5 Probability calculations for top reservoir 2045

After examining the sensitivity of initial saturation and pressure information in Section 6.4, the uncertainties in the measured seismic parameters are tested. The study is first sought out for two given points at the top reservoir. The points are chosen as one point in the plume were the PP AVO method in Section 6.4 resulted in a good estimate for pressure and saturation, and one point where the PP-PS method in Section 6.4 resulted in a good estimate for the pressure and saturation.

The PP AVO parameters and the PP-PS stacks are Monte Carlo simulated over $10^4$ cycles. The parameters are varying between different uncertainties. The uncertainties are set in correspondence with the paper by Stovas & Landrø (2004). $\delta \Delta A$ is set to 0.002, $\delta \Delta B$ to 0.008, and $\delta \Delta R_{pp}$ and $\delta \Delta R_{ps}$ to 0.005. The parameters are inverted into saturation and pressure using the LM method reviewed in Section 6.3.2. The initial saturation and pressure is given as 95% of the saturation and pressure from the Eclipse simulation.

In Section 6.5.2 the same study is done on the full top reservoir. Monte Carlo simulating over the full top reservoir generates a lot of data. Because of this, "good discrimination" is set to ±20% of the true saturation and ±0.5 MPa from the true pressure. Three different outputs for good discrimination is made; values giving good saturation discrimination, good pressure discrimination, or parameters giving good results for both pressure and saturation. The groups are divided by the number of Monte Carlo simulations, giving the probability of getting a sufficiently good measured pressure, saturation, or both, in regards of uncertainty in the input parameters. The workflow is portrayed in Figure 6.18.
Trani et al. (2011) tested the pressure-saturation discrimination for 4D PP AVO with ±15% uncertainty in $\Delta A$ and $\Delta B$ for two production sites. They also tested an uncertainty of ±15% in $\Delta A$ and ±30% in $\Delta B$ for the same scenarios. The concluding result gave reasonable pressure-saturation discrimination for both reservoirs. Landrø (2002) set the uncertainty of $\Delta A$ and $\Delta B$ to 50%. However, using PP-PS was not investigated in the papers. The main object of this thesis is to compare PP AVO and PP-PS methodology. Hence uncertainties for both methods are needed, and most importantly relative to one another. Stovas & Landrø (2004) set the uncertainty of $\delta \Delta A$ to 0.002, $\delta \Delta B$ to 0.008, and $\delta \Delta R_{pp}$ and $\delta \Delta R_{ps}$ to 0.005. In this study, the uncertainties from Stovas & Landrø (2004) are set as the standard variation to make normal distributions of the input parameters $\Delta A$, $\Delta B$, $\Delta R_{pp}$, and $\Delta R_{ps}$.
6.5.1 Probability for given points

Probability plots for an PP AVO biased point

Figure 6.19: Histogram of $\Delta A$ and $\Delta B$ input Gaussian distributions for a given point where PP AVO is assumed to give a good saturation estimate.
Figure 6.20: Histogram of saturation and pressure outputs using PP AVO input, for a given point where AVO is assumed to give a good saturation estimate. Zero pairs of input-parameters generated diverging results. Probability for saturation estimate between $\pm 0.2 \Delta S_{true} = 0.57$. Probability for pressure estimate between $\pm 0.5 \Delta P_{true} = 0.61$. Probability of pairs of input-parameters giving good estimate in both saturation and pressure $= 0.56$. 
Figure 6.21: Histogram of $\Delta R_{pp}$ and $\Delta R_{ps}$ input Gaussian distributions for a given point where PP AVO is assumed to give a good saturation estimate.
Figure 6.22: Histogram of saturation and pressure outputs using PP-PS input, for a given point where AVO is assumed to give a good saturation estimate. 3% of input-parameter pairs generated diverging results. Probability for saturation estimate between $\pm 0.2 \Delta S_{\text{true}} = 0.52$. Probability for pressure estimate between $\pm 0.5 \text{ MPa} = 0.59$. Probability of pairs of input-parameters giving good estimate in both saturation and pressure $= 0.50$. 
Probability plots for a PP-PS biased point

Figure 6.23: Histogram of $\Delta A$ and $\Delta B$ input Gaussian distributions for a given point where PP-PS is assumed to give a good saturation estimate.
Figure 6.24: Histogram of saturation and pressure outputs using PP AVO input, for a given point where PP-PS is assumed to give a good saturation estimate. Zero pairs of input-parameters generated diverging results. Probability for saturation estimate between $\pm 0.2 \Delta S_{true} = 0.51$. Probability for pressure estimate between $\pm 0.5 \text{ MPa} = 0.51$. Probability of pairs of input-parameters giving good estimate in both saturation and pressure $= 0.50$. 
Figure 6.25: Histogram of $\Delta R_{pp}$ and $\Delta R_{ps}$ input Gaussian distributions for a given point where PP-PS is assumed to give a good saturation estimate.
Figure 6.26: Histogram of saturation and pressure outputs using PP-PS input, for a given point where PP-PS is assumed to give a good saturation estimate. 6% of input-parameter pairs generated diverging results. Probability for saturation estimate between $\pm 0.2 \Delta S_{true} = 0.54$. Probability for pressure estimate between $\pm 0.5$ MPa = 0.59. Probability of pairs of input-parameters giving good estimate in both saturation and pressure = 0.52.

Discussion and observations regarding the probability plots

The inverted saturation and pressure estimates in Figures 6.20, 6.22, 6.24, and 6.26 give probabilities below 60%. However, the variance in output measurements are small.

For the PP AVO biased point, both methods have a peak at the true saturation and pressure (Figures 6.20 and 6.22). A more prominent saturation measurement at around 0.8 $\Delta S$ is noticed from the PP AVO method in Figure 6.20, compared to the PP-PS method in Figure 6.22. Another noticeable effect for the PP AVO method is the probability of measuring a pressure depletion of 0 to -5 MPa (Figure 6.20).
In direct comparison to the PP-PS method, the PP AVO method has a slightly higher probability of giving a good estimate pressure- and saturation for this given point. However, the probability of good results are still below 60% for both methods.

Utilizing PP AVO parameters at the PP-PS biased point present a multivariate saturation distribution (Figure 6.24). This is an effect that can cause saturation in the plume to be over-estimated. The results from Sections 6.1 and 6.4, as well as Figure 6.20 and 6.24, imply the connection between over-estimated saturation and under-estimated pressure. The probability of good saturation and pressure results are between 50-60%. The PP-PS inverted data (Figure 6.26), have a distinct peak close to the true saturation and true pressure.

After testing given points at the top of the reservoir in Section 6.5.1, both methods struggle with discriminating saturation and pressure effects with the uncertainties given in Stovas & Landrø (2004). In the following Section, the method of quantifying the likeliness for good discrimination (Figure 6.18) is extended to the full top of the reservoir.
6.5.2 Probability map for top reservoir 2045

Figure 6.27: Probability map for good saturation and pressure at top reservoir using the AVO method with $\delta A = 0.002$ and $\delta B = 0.008$. 
Discussion and observations regarding probability maps of top reservoir

The PP AVO and PP-PS methods show varying pressure-saturation discrimination. The discrimination is quantified in Figures 6.27 and 6.28. The figures show the probability of a set of input parameters (ΔA and ΔB, or ΔR_{pp} and ΔR_{ps}), resulting in a "good" saturation and pressure discrimination. "Good" discrimination was defined in Section 6.5 as ±0.2 ΔS_{true} and ±0.5 MPa ΔP_{true}. Red indicates a higher probability of good discrimination, decreasing to white, while blue indicates a low probability of good discrimination.

Both the AVO method in Figure 6.27 and the PP-PS method in Figure 6.28 show decent discrimination of the pressure front outside the plume. However, there are significant differences inside the plumes. The AVO method (fig 6.27) show a decent discrimination in the middle of the plume, but decreasing discrimination further
from the plume. Independent saturation and pressure plots can be found in Appendix B. The independent discrimination by the PP AVO method is plotted in Figure B.1 and B.3 in Appendix B. Figures B.1 and B.3 show a higher probability for correct saturation estimation than pressure estimation in Figures B.1 and B.3. The leakage between saturation and pressure is visible in Figures B.1 and B.3. The effect of the pressure smearing onto the saturation was also observed in Sections 6.1 and 6.4. The effect is not as prominent in the saturation and pressure probability maps for the PP-PS method in Figures B.2 and B.4. The probability for "good" discrimination is better by the use of the PP-PS method, with the given uncertainties. Certainty in the seismic parameters is vital for good pressure- and saturation discrimination. The PP-PS method is demonstrated as more robust than the PP AVO method, in this thesis.

The provided fluid flow simulation used is one of multiple possible scenarios. Other scenarios are taken into account in an article by Lothe et al. (2018). In an attempt to mimic the pressure build-up scenarios from Lothe et al. (2018), a simplified fluid flow simulation is done in the next Chapter. This is done to test how the methods will react to a larger pressure build-up than the one from the the provided simulation.
7 | Simplified scenarios

There are multiple uncertainties not taken into account in the provided simulation. The model from Equinor is based on the GN1101 cube, representing only 50% of the Alpha structure (Statoil, 2016). The volumetric analysis in the Statoil (2016) rapport is based on this data, and doubled to get the full potential of the closure. The vertical resolution of seismic data is limited by the tuning thickness, \( \frac{\lambda}{4} \), where \( \lambda \) is the dominant wavelength of the pulse (Gelius, 2019c). The calcite cemented rocks, mentioned in Section 2.2, is not visible in seismic data. This makes the extent of the calcite cemented rocks uncertain. Another issue in regards to faults is discussed in Lothe et al. (2018). A problem with the potential of relay ramps between two normal faults, is that a seismic line seem continuous in one orientation, while the fault is evident in another direction.

Lothe et al. (2018) simulates a new model, taking multiple scenarios into account. One of the models considers the extension of two faults in two relay zones along the Vette Fault Complex, sealing faults in the area, and some depletion caused by the Troll field into account. This model has an increase in bottom hole pressure (BHP) of 5 MPa. Scenarios with an increased bottom hole pressure, and the possibility of local pressure build up, make it interesting to analyze other scenarios than the simulation presented by Equinor.

7.1 Fluid flow simulation

Simulations from Statoil (2016) and Lothe et al. (2018) are simulated using ECLIPSE, a 3-dimensional, 3-phase flow, reservoir simulator by Schlumberger. Mathias et al. (2011) describes an axisymmetric, 2-dimensional, 2-phase fluid flow solution. This method provides an explicit approximate solution for estimating pressure build-up due to injection into a closed brine aquifer. It solves a system of equations consisting of the conservation of mass and Darcy’s law. The model is axisymmetric, and simulates a fully developed flow profile (i.e. the saturated plume development in time is not simulated). A sharp interface between the wetting fluid (brine), and the non-wetting fluid (CO\(_2\)) is assumed. Figure 7.1 b) illustrates the plume front associated with a 5 MPa pressure build up. The area above the black line is fully saturated.
by the non-wetting fluid, while the area below is fully saturated by the wetting
fluid. Saturation, relative permeability, and viscosity are assumed to be constant
and uniform within both the wetting- and non-wetting fluid. The fluids, as well as
the reservoir rock, are assumed to have small compressibility and are independent
in regards to pressure change. Saturation, relative permeability, and viscosity are
assumed to be constant and uniform within both the wetting- and non-wetting fluid.
The fluids, as well as the reservoir rock, are assumed to have small compressibility
and are independent in regards to pressure change.

The time of interest, outer aquifer formation radius, injection flow rate, forma-
tion thickness, and porosity of the reservoir rock is explored to get the desired BHP
build-up of 5 MPa.

Figure 7.1: Plot of pressure distribution in the simulated reservoir (a) and the CO₂ plume front
(b) for the 5 MPa pressure build-up scenario. The porosity in the reservoir is set to 0.3. The
injection time is 25 years, outer aquifer radius is 20 km, and reservoir formation thickness is 150
m. Injection flow rate is set to 57 $\frac{kg}{s}$.

7.2 Simplified background model

A simplified background model is made to apply the simulated fluid flow in Section
7.1. A proposed well is placed along the line presented in Section 5.1. Seismic
properties, along with the corresponding depth in the proposed well, are exported. A
horizontally homogenous 2D line is made by copying these properties in the column
dimension, resulting in a simplified background model as shown in Figure 7.2.
Figure 7.2: Simplified P-wave velocity background model using a column from the Petrel model.

The fluid flow simulation from Section 7.1 is then applied to the background model, using the expected relative changes in seismic parameters from equation 3.8, resulting in a projected model with a 5 MPa maximum pressure build-up, which can be seen in Figure 7.3.
Figure 7.3: Simplified model showing expected P-wave velocities from a simulated plume with a maximum pressure build-up of 5 MPa. The plume is noticeable at about 1200m depth and a horizontal distance of 16000m. There is also an abrupt change from the pressure front at horizontal distance 8000m. This sudden change is a result of padding of the background model on the left side of the model.

Figure 7.4: Plot showing a) saturation and b) pressure at the top of reservoir from the fluid flow simulation, PP-PS method, and PP AVO method.

From the results illustrated in Figure 7.4, the same saturation trend as in the pro-
vided model (Figure 5.4) is apparent. The PP-AVO method slightly overestimates the true saturation (100% becomes 104%), while the PP-PS method slightly underestimates the saturation to be 98%. The true maximum pressure increase of 4.87 MPa is underestimated by the PP AVO method to 4.27 MPa, while the PP-PS method overestimates the pressure build up to 5.26 MPa. In the pressure plot (Figure 7.4 b) it is visible that the PP-PS slightly overestimate the pressure, while the PP-AVO method slightly underestimates the pressure. The area corresponding to the saturated plume is noticed in the pressure plot, as a slight dip in the estimated pressure trend, from 14-18 km on the x-axis. As this is an analytical result for a simplified scenario, disregarding a lot of both structural and fluid dynamic parameters, this is a good indication that full discrimination of pressure and saturation effects might be far-fetched. However, the collected information from data and expected output might make it possible to estimate a better fitting pressure and saturation.

### 7.3 Different pressure scenarios

The injection flow rate in the fluid flow simulation was changed to get five desired pressure build-up scenarios, listed in Table 7.1. The porosity of the reservoir layer was set to 0.3, the injection time to 25 years, outer aquifer radius to 20 km, and reservoir formation thickness to 150 m. Figure 7.5 illustrates the measured maximum pressure plotted against the true maximum pressure for the top and bottom of the reservoir. As previously mentioned in Section 4.1.1, the PP AVO and PP-PS methods assume that the change due to injection appears in the layer below the interface. This corresponds to the method showing a negative pressure change at the bottom of the reservoir. Consequently, the absolute value of the measured pressure build-up is used in Figure 7.5 and 7.6.

<table>
<thead>
<tr>
<th>Pressure</th>
<th>Injection flow rate ( \frac{\text{kg}}{\text{s}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 MPa</td>
<td>9.5</td>
</tr>
<tr>
<td>2 MPa</td>
<td>20.6</td>
</tr>
<tr>
<td>3 MPa</td>
<td>32.3</td>
</tr>
<tr>
<td>4 MPa</td>
<td>44.4</td>
</tr>
<tr>
<td>5 MPa</td>
<td>57</td>
</tr>
</tbody>
</table>

**Table 7.1**
Figure 7.5: Measured absolute maximum pressure vs true absolute maximum pressure. Black dotted line is the true pressure. Red circle for top of reservoir using PP AVO method. Red cross for bottom of reservoir using PP AVO. Blue circle for top of reservoir using PP-PS method. Blue cross for bottom of reservoir using PP-PS method.

Figure 7.5 shows that the measured maximum pressure appears to be more accurate for low pressures, and seem to deviate for higher pressures. This is investigated in Figure 7.7, where the median relative error at the top of the reservoir for the pressure is calculated for the different scenarios. The observation of a more accurate measured maximum pressure is also visible in Figure 7.7. Figure 7.6 illustrates a constant measured saturation for the PP AVO method for the various pressure build-ups. The PP-PS method appears to give a more precise measured saturation compared to the PP AVO method, emphasized in Figure 7.7, but seem to deviate more for more significant pressure build-ups.
Figure 7.6: Measured absolute maximum saturation for different maximum pressure build-ups. Black dotted line is the true saturation. Red circle for top of reservoir using PP AVO method. Red x for bottom of reservoir using PP AVO. Blue circle for top of reservoir using PP-PS method. Blue x for bottom of reservoir using PP-PS method.
The results from the simplified model might seem reasonable. The method seems stable for these simplified scenarios, with a slight increase in error for higher pressures. However, when the data in Figures 7.5, 7.6, and 7.7 is analyzed and compared with the results from the more advanced simulation from Chapter 6, we deduced that the results from a more advanced simulation with a relaxed interface between the wetting- and non-wetting fluid would deviate even more from the true pressure build-up. This is also supported by Figures 6.5 and 6.6. The distance between contour lines in Figures 6.5 and 6.6 increase with the increase in pressure. As mentioned in Section 6.2, closely spaced contour lines would be better for discrimination, as the coefficients have a more abrupt change. The PP-PS method barely overestimates the pressure in the simplified model.
Concluding remarks

The main objective of the thesis was to compare, and test pressure and saturation discrimination methods by geophysical means, and the implications for carbon storage. Specifically, synthetic seismic data was forward modeled and inverted with different schemes originating from Landrø (2001).

This study finds that the Landrø (2001) method relies extensively on *a priori* information on spatially variant parameters to express changes in seismic parameters. The validity of this assumption and limitations have been discussed and is regarded a vulnerability of the method.

The linear saturation model from Landrø (2001) gave decent estimates for the production scenario at Gullfaks. For the CO\textsubscript{2} injection simulation by The Northern Lights CCS project, forward modeled by the use of laboratory data and rock physics models, the method resulted in a small pressure build up to resemble a depletion scenario. The use of a multicomponent approximation from Landrø et al. (2003) resulted in the same observation. The estimated pressure depletion results in saturation being over-estimated.

A non-linear inversion scheme was subsequently tested. We found high quality initial guesses (close to the true saturation and pressure) a necessity. Both the PP AVO method and the PP-PS method under-estimate the extent of the plume. The non-linear PP AVO method has a tendency to over-estimate the saturation and under-estimate the pressure in the plume. The PP-PS method results in better pressure-saturation discrimination. The reason for this can be summarized as Figure 6.6, where the PP-PS method is weakly non-linear over a larger area than the more heavily non-linear PP AVO method. Also, as discussed in Section 4.3, ocean bottom logging increase the repeatability and full stacks are a more robust measurement compared to PP AVO data.

This inversion scheme was tested by including uncertainties (from Stovas & Landrø (2004)) in the full stacks, as well as in the AVO parameters. The PP-PS method gave better inversion results compared to the PP AVO method.
Finally, the use of a simple fluid flow simulator suggests that monitored pressure and saturation will deviate from the true value for larger pressure build-ups. This deviation correspond to the contour plots in Figure 6.6, where a more complex relationship between PP-PS stacks and PP AVO parameters was found for saturations over 50%, and increasing complexity with increasing pressure.
Further work

The methods presented in this thesis take into account changes in reflection coefficient between a baseline to a monitored survey. The methods could be extended to regard a region of multiple reflections. This would give a more stable estimation of the saturation and pressure. Further, as the methods in this thesis are greatly dependent on core measurements, regions could be gridded in regards to properties provided by seismic interpretation and inversions (i.e. extended elastic impedance inversion for shale zones).

To compare the methods, the 1D forward modeling presented in this study is sufficient. However, to move away from the 1D assumption, it would be welcome to test the methods with 2D forward modeling by the use of ray tracing, finite difference, finite volume, or/and finite element. Additionally, the forward modeling could also be more realistic by taking multiple injection related changes, such as temperature, porosity, and permeability into account. A study by Sylte et al. (1999) showed that porosity change can be expressed as a function of change in pressure and saturation. Hence, as mentioned by Bhakta & Landrø (2014), porosity could be taken into account in the forward modeling, without increasing the amount of required number of output parameters from the inversion.

The non linear systems can be solved for \( k \) non-linear equations \( f \), with \( m \) variables \( x \), and \( m \leq k \). Trani et al. (2011) implemented time-shift data in the 4D AVO inversion. Landrø (2002) found time-shift data more important than changes in AVO parameters for increasing reservoir thickness. Adding more known data to the method will always cause a more certain output.

A real data example is needed. This will make it possible to test different rock physics models, as well as analyzing the need of higher certainty in seismic data.
References


Appendices

Appendix A: Plots for section 6.4

Top reservoir from the provided simulation

Figure A.1: Saturation at top reservoir for 3-, 13-, and 25-years of injection, and 25 years after injection. The pictured data is from the provided simulation.
Figure A.2: Pressure at top reservoir for 3-, 13-, and 25-years of injection, and 25 years after injection. The pictured data is from the provided simulation.
Monitored saturation with good initial guess

![Image of saturation at top reservoir for 3-, 13-, and 25-years of injection, and 25 years after injection. Monitored using PP AVO method and initial guesses 95% of the simulation.](image1)

**Figure A.3**: Saturation at top reservoir for 3-, 13-, and 25-years of injection, and 25 years after injection. Monitored using PP AVO method and initial guesses 95% of the simulation.

![Image of saturation at top reservoir for 3-, 13-, and 25-years of injection, and 25 years after injection. Monitored using PP-PS method and initial guesses 95% of the simulation.](image2)

**Figure A.4**: Saturation at top reservoir for 3-, 13-, and 25-years of injection, and 25 years after injection. Monitored using PP-PS method and initial guesses 95% of the simulation.
Monitored pressure with good initial guess

**Figure A.5:** Pressure at top reservoir for 3-, 13-, and 25-years of injection, and 25 years after injection. Monitored using PP AVO method and initial guesses 95% of the simulation.

**Figure A.6:** Pressure at top reservoir for 3-, 13-, and 25-years of injection, and 25 years after injection. Difference between the provided simulation and the monitored using PP-PS method and initial guesses 95% of the simulation.
Monitored saturation from monitoring plan

**Figure A.7:** Saturation at top reservoir for 3-, 13-, and 25-years of injection, and 25 years after injection. Monitored using PP AVO method and monitor initial guesses.
Figure A.8: Saturation at top reservoir for 3-, 13-, and 25-years of injection, and 25 years after injection. Monitored using PP-PS method and monitor initial guesses.
Monitored pressure from monitoring plan

Figure A.9: Pressure at top reservoir for 3-, 13-, and 25-years of injection, and 25 years after injection. Monitored using PP AVO method and monitor initial guesses.
Figure A.10: Pressure at top reservoir for 3-, 13-, and 25-years of injection, and 25 years after injection. Monitored using PP-PS method and monitor initial guesses.
Appendix B: Plots for section 6.5.2

Probability maps for saturation after 25 years of injection

Figure B.1: Probability map for $\pm 20\% \Delta S_{true}$ at top reservoir of reservoir using the AVO method with $\delta \Delta A = 0.002$ and $\delta \Delta B = 0.008$. 25 years of injection.
Figure B.2: Probability map for $\pm 20\% \Delta S_{\text{true}}$ at top reservoir of reservoir using the PP-PS method with $\delta \Delta R_{pp} = 0.005$ & $\delta \Delta R_{ps} = 0.005$. 25 years of injection.
Probability maps for pressure after 25 years of injection

Figure B.3: Probability map for $\pm 0.5$ MPa from the true pressure at top reservoir of reservoir using the AVO method with $\delta \Delta A = 0.002$ & $\delta \Delta B = 0.008$. 25 years of injection.
Figure B.4: Probability map for ±0.5 MPa from the true pressure at top reservoir of reservoir using the PP-PS method with δΔR_{pp} = 0.005 & δΔR_{ps} = 0.005. 25 years of injection.