Mathematically gifted adolescents in Norway:

Exploring mathematically gifted adolescents’ experience with the school system in Norway

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PhD Thesis
Department of Education
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University of Oslo
2019
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Acknowledgements

First, I would like to thank Professor Ella Idsøe for providing guidance and co-authoring my first publication in this thesis and for sharing her expertise on the topic. She has been a constant support and inspiration during an occasionally trying process, for which I am truly grateful. I would also like to direct a special thanks to Professor Thormod Idsøe for his tutelage and analysis of my work. His contribution in this regard was far beyond what might be considered the call of duty.

I am very grateful for the support, guidance, and critical feedback from my supervisor Ove Edvard Hatlevik, who took on the main supervisor responsibilities at a critical time and guided me through the writing process. Without his help, I do not think that I would have managed to deliver on time. My sincere gratitude also goes to Kristinn Hegna, who helped throughout the months of my PhD work and provided critical comments during the final drafts of my thesis. I also want to express my gratitude to my co-supervisor Stein Erik Ulvund for critical comments on drafts of the first publication. Furthermore, I am thankful to my mentor, Professor Kjell Skogen, who always made time for a coffee, pep-talk, or discussion. Despite being an inspirational authority in the field of pedagogy, you always treat your students like peers and fellow researchers.

Also, I would like to thank my university roommate Kristin Rogde, as well as Professor Anders Nordahl Hansen, for the many discussions, borderline supervisions, and sharing of laughter. Professor Hansen taught me the Queens Gambit during one of our many sessions of Chess, although I suspect that this was because he was bored of the lack of competition on my part. My sincere thanks also goes to Annette Hjelmevold and Yngvild Dahl for our encouraging morning coffees at the Institute for Pedagogy as well as to Professors Tone Kvernbekk and Helge Ivar Strømsø for always having their office doors open to frustrated Ph.D. students.

Finally, a great thanks to my closest family, Dagny, Anders, Marthe, and Per, for supporting me through the writing process and for tolerating my moodiness and absentmindedness during my period at university, especially during my final months. I am also grateful for being part of the workspace at IPED and all the experiences gained from teaching at the University of Oslo.
To my brother Anders, in particular, thank you for your critical thoughts and for providing scrutiny even though the feedback was sometimes personal. Like in our youth, you are always willing to support me in all my, not always productive, endeavors.

Special mentions go out to my mother, Dagny, for believing in me despite all the evidence pointing you to the contrary. Between the many work hours and day-to-day responsibilities of a mother, you always made time to encourage me to better myself and to believe in the advantages of education.
Sammendrag

I de senere årene har det blitt et økt fokus på evnerike elever i Norge. Likevel har vi lite kunnskap om identifiseringspraksisen knyttet til de elevene som forserer og hvor godt vi klarer å møte deres individuelle behov i skolen. Det er viktig at vi utvikler en definisjon som kan legge grunnlaget for en god identifiseringspraksis som også sikrer gode utdanningsmuligheter for evnerike elever, siden det er en risiko for at de kan bli underyttere.

Denne artikkelbaserte doktorgraden undersøker hvordan matematisk evnerike elever opplever skolesystemet i Norge og hvordan vi kan forstå definisjonene innenfor dette fagfeltet gjennom vitenskapsfilosofi. Det overordnede forskningsspørsmålet for denne avhandlingen er «Hvordan reflekterer over og opplever matematisk evnerike elever deres skolegang i Norge?». For å besvare denne problemstillingen er det utarbeidet en filosofisk, en kvalitativ og en kvantitativ studie, hvor disse tre artiklene sammenfaller for å besvare den overordnede problemstillingen.

Hvordan vi definerer evnerikdom har vært en sentral debatt innenfor dette fagfeltet i flere år. Det filosofiske studiet er inspirert av de to empiriske studiene ved at det diskuterer den konseptuelle rekkevidden ved blant annet de to definisjonene som er brukt i dem. Gjennom den filosofiske artikkelen argumenterer jeg for at konseptet er vagt og at vi må definere evnerikdom innenfor den kulturelle konteksten det forekommer og at de forskjellige definisjonene (intelligensbasert, prestasjonsbasert eller multidimensjonale) kan ha forskjellig konseptuell rekkevidde. Samtidig som det er viktig å definere evnerikdom i forskning og praksis slik at vi har like identifiseringsverktøy slik at vi identifiserer de studentene som vi faktisk ønsker å identifisere.

I det kvalitative studiet rekrutterte jeg 11 evnerike elever i matematikk som deltok i en gruppe ved universitetet i Oslo. Disse elevene hadde også forsert (akselerert) i løpet av skolegangen. Formålet med studien var å undersøke om skoler i Norge gir et optimalt læringstilbud til denne elevgruppen i matematikk. I studien brukte jeg semistrukturerte intervjuer for å utforske hvordan elevene hadde opplevd skolegangen sin. Kategoriene i intervjuene omhandlet blant annet motivasjon, faglige utfordringer i skolen, forhold til læreren og medelever og akademisk selvbilde. Studien indikerer at det fortsatt er mye arbeid som bør gjøres før vi er i stand til å møte disse elevenes behov i skolen. Elevene opplevde få faglige utfordringer i matematikk og mottok lite støtte når de arbeidet med faget. Sistnevnte
var tydeligere i tidlig skolegang. Elevene sin motivasjon for matematikk var lavere enn hva jeg forventet. Det var også færre jenter som deltok på tilbudet enn hva jeg forventet gjennom det kvantitative studiet.


Summary

In recent year’s gifted education have revived more attention in Norway. Still, we have little knowledge about the identification practice for acceleration programs in Norway for gifted students, and if we are able meet their individual needs in school. It is important to develop a definition that can guide the identification practice and ensure proper educational opportunities for this group as they might be at risk for underachievement.

This article-based thesis investigates how mathematically gifted students experience the Norwegian school system and how we can understand the definitions of gifted education through the philosophy of language. The overarching research question in this thesis is “How do mathematically gifted adolescents perceive and reflect on mathematics tutoring during their school years in Norway?”. A philosophical, a qualitative and a quantitative article collectively seek to answer the overarching aim of the thesis.

How we define giftedness have been a central debate in the field of giftedness over time. The philosophical article draws on ideas from the two empirical studies by discussing the conceptual range of the definitions used in the respective studies. Through the philosophical article I argue that giftedness is a vague concept that needs to be defined within its cultural context and that different definitions (e.g., intelligence-based, performance-based and multifactorial) can have different conceptual ranges. At the same time, it is important to define giftedness in research and practice so that we employ similar identification methods and to identify those students we seek to identify.

In the qualitative article I recruited 11 mathematically gifted students participating in an ability group at the university in Oslo. The students had also received acceleration opportunities in school. The aim of the study was to see if schools in Norway can provide an optimal learning environment for gifted students in mathematics. In this study I used semi-structured interviews to explore how these students had experiences school. The categories were connected to motivation, challenges in school, peer and teacher relationship and academic self-concept. The study indicated that there is still much work to be done in meeting gifted students needs in Norwegian schools. The students felt little challenges in math and received little support in mathematics. The latter was clearer at the earlier stages in school. Further, the student’s motivation for mathematics were lower than what I expected and there was fewer girl in the program than what we could expect from the quantitative survey.
The quantitative study draws its conclusions from a large dataset received from the Norwegian armed forces. It consists of intelligence scores, school grades and a survey assessing pedagogical and psychological questions. This study indicates that there is a correlation between intelligence and achievement in mathematics as well as a gender difference in mathematical achievement across levels of intelligence. It seems that the gifted (students defined through IQ scores) rated their relationship with their peers as more negative than what is considered average. An important finding in the study is that teachers seem to influence mathematical achievement across levels of intelligence, which indicates that the teacher is important for both high-achieving and low-achieving students.

Through mixed-methods the three studies collectively represent a methodological approach that ensure a deeper understanding of mathematical giftedness in Norway. As the philosophical article clarify conceptual issues in defining giftedness, both a performance-based (article 2) and intelligence-based (article 3) definition seem to access some of the same students. The qualitative and quantitative article together provide a more enlightened understanding of mathematically gifted students school experiences. The integration articles indicate some differences between how gifted adolescents experience the school system at a qualitative level and how they describe their teacher through the questionnaire. The students seem to rate the teacher’s pedagogical knowledge as better than their academic knowledge in the qualitative study. The latter can explain why the students in general rated their relationship with their teacher as higher in the quantitative study. Finally, this thesis concludes that even though there is more focus on this group in Norway, there is still much work to be done before we have developed a system that provides the best learning opportunities for this group. We should place emphasis on designing programs and identification methods that reflect a coherent definition of giftedness. The latter is important so that we can identify students who might underperform in school. Further, it seems that teachers lack the relevant competency to challenge students in mathematics, which means that we need more knowledge about the groups in both teacher education and special needs education.
Background

My interest in the field of gifted education stems from my background as a BA and MA student in special needs education. In general, we learned nothing about how we, as special needs teachers, should teach or help those with exceptional talent. In later years, I learned that we do not focus on this group of students in Norway and that our egalitarian ideology about inclusion and equal opportunities sometimes works against them. Gifted children can develop psychosocial difficulties or become underachievers if their needs are not sufficiently met in school (Montgomery, 2009). Research suggests that underachievement is a complex aspect of gifted education (Reis & McCoach, 2000). It is possible that some gifted students drop out of high school, and/or can become unsuccessful as adults (Rubenstein, McCoach, Reis, & Siegle, 2012), and that underachieving students share familiar personal and academic traits (Kim, 2008). Thus, our limited knowledge about this group of children in Norway prevents us from providing for them within our current school system. For this reason, understanding how gifted students experience school and whether they receive enough tutoring became my field of interest.

Arnold Hofset (1968) was the pioneer on gifted students in Norway. He concluded that we needed to increase our focus on this group and expressed concern about how they experience school. After Hofset, there was a long silence in the field until the early 2000s. Today, with several master’s theses having been written and with the field advancing through some empirical research publications, individuals and organizations recognize the need to conduct further research and focus on this group. More recently, an Official Norwegian report (NOU, 2016, p. 14) published extensive work on mapping Norwegian intensifiers from private organizations, individuals (bloggers, speakers, and others), and researchers engaged in the field. The report emphasized that all students should receive equal and individual opportunities in line with the Norwegian ideology of public education. Thus, we should be
able to identify gifted children early on and focus on classroom and organizational differentiation that meets the needs of gifted individuals (NOU, 2016:14). The report states that:

If students with higher learning potential are to be given differentiated instruction that will motivate their learning and give them challenges in the subjects, it is important to identify their needs and abilities. This requires research-based knowledge on the needs the students have and what characterizes their challenges and strengths. (NOU, 2016, p. 74)

The report also states that our knowledge about acceleration in school is insufficient to draw definite conclusions. Moreover, the report underlines the importance of not making permanent divisions between groups of students. This latter point is one of the areas that this doctoral thesis explores.

For example, a more recent Scandinavian-based doctoral thesis by Mattsson (2013) also sought to expand on and conduct similar work in Norway. Mattsson presented an in-depth description of mathematically gifted students in Sweden, advancing the field by contributing original work and research about the way mathematically gifted students’ needs are meet and understood in Sweden. Even though there are many similarities between Norway and Sweden, there might be differences in terms of how we are able to meet the needs of gifted adolescents. Mattsson’s doctoral thesis explores both the concept of mathematical giftedness and how teachers perceive mathematically gifted students in Sweden. Another researcher worth mentioning is Laine, Kuusisto, & Tirri (2016), who has conducted extensive work on Finnish gifted education through her doctoral thesis. The research from Finland shows that teachers associate cognitive features and creative and motivational characteristics with giftedness and that they would like to receive more information about giftedness. Furthermore, teachers’ conception of giftedness might also affect the recruitment and education of gifted students (Laine et al., 2016).
There are several reasons why we should focus on gifted students in Norway (e.g., economic, innovation, and recruitment to the welfare state). However, individual underachievement might be the strongest argument for a greater focus on this group, in general, and on Norway in particular. Underachievement can be described as not performing in line with your potential, whether you are intellectually gifted, highly creative, or you have more domain-specific abilities in mathematics, arts, human science, or language. All student needs ought to be met in ways that stimulate their abilities by teachers who have the competence to guide them and who can design academic challenges that meet their individual needs. As my own studies show, teachers might need special competences and high mathematical knowledge to foster these needs. Therefore, while many gifted students in Norway might be underachievers, there is no Norwegian study focusing specifically on underachievement.

As the NOU (2016:14) states: “If the educational system had succeeded nationally and locally in providing differentiated instruction for all students, it would have been unnecessary to produce an NOU concentrating on students with higher learning potential” (NOU, 2016, p. 8). Therefore, there is a need for a discussion on the various types of interventions for gifted students to guard against underachievement and to successfully motivate students who already underachieve in school.
# Part I

1. Introduction .................................................................................................................. 1  
   1.1 Gifted education .................................................................................................... 1  
   1.2 Giftedness in the Norwegian context .................................................................. 3  
   1.3 Acceleration opportunities in Norway .................................................................. 5  
      1.3.1 Summary ........................................................................................................ 7  
   1.4 Delineation of the research field: Giftedness ...................................................... 7  
   1.5 Delineation of the research field: Gifted in mathematics .................................. 9  
      1.5.1 Academic needs of mathematically gifted students ...................................... 10  
      1.5.2 Summary ...................................................................................................... 12  
   1.6 Overarching aim and research objectives ............................................................ 12  
   1.7 How the articles collectively address the main research question .................... 15  
2. Different Approaches to Giftedness .......................................................................... 18  
   2.1 Intelligence and giftedness ................................................................................... 18  
   2.2 Multifaceted models of giftedness and talent ..................................................... 22  
   2.3 Summary ............................................................................................................. 28  
3. Mathematically Gifted Adolescents .......................................................................... 29  
   3.1 Understanding mathematical giftedness ............................................................... 29  
      3.1.1 The SMPY: Studies of mathematically precocious youth ...................... 30  
      3.1.2 Mathematical ability ..................................................................................... 32  
      3.1.3 Creativity among mathematically gifted students .................................. 34  
      3.1.4 Gender differences in mathematically gifted adolescents ..................... 37  
      3.1.5 Motivation in gifted students ...................................................................... 39  
   3.2 Summary ............................................................................................................. 40  
4. Acceleration and Ability Groups for Gifted Students ............................................. 41  
   4.1 Academic benefits of acceleration ...................................................................... 41  
   4.2 Academic benefits of ability groups for gifted students .................................. 43  
   4.3 Social issues related to acceleration and ability groups for gifted adolescents ... 44  
   4.4 Discussion ........................................................................................................... 48  
5. Methodological Considerations .............................................................................. 50  
   5.1 Mixed methods .................................................................................................... 50  
   5.2 Validity and reliability in Article 1 ...................................................................... 53
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.3</td>
<td>Validity, reliability, ethics, and generalization in Article 2</td>
<td>55</td>
</tr>
<tr>
<td>5.4</td>
<td>Validity and reliability in Article 3</td>
<td>58</td>
</tr>
<tr>
<td>5.5</td>
<td>Summary</td>
<td>62</td>
</tr>
<tr>
<td>6</td>
<td>Results</td>
<td>64</td>
</tr>
<tr>
<td>6.1</td>
<td>Article 1</td>
<td>64</td>
</tr>
<tr>
<td>6.2</td>
<td>Article 2</td>
<td>66</td>
</tr>
<tr>
<td>6.3</td>
<td>Article 3</td>
<td>70</td>
</tr>
<tr>
<td>6.4</td>
<td>Summary</td>
<td>73</td>
</tr>
<tr>
<td>7</td>
<td>Overall Discussion</td>
<td>74</td>
</tr>
<tr>
<td>7.1</td>
<td>Originality of the research articles in this thesis</td>
<td>75</td>
</tr>
<tr>
<td>7.2</td>
<td>Defining giftedness in the Norwegian context</td>
<td>75</td>
</tr>
<tr>
<td>7.3</td>
<td>Identifying gifted adolescents</td>
<td>77</td>
</tr>
<tr>
<td>7.4</td>
<td>Acknowledging gifted adolescents’ perspectives</td>
<td>78</td>
</tr>
<tr>
<td>8</td>
<td>Contribution, Limitations, Further Research, and Concluding Remarks</td>
<td>83</td>
</tr>
<tr>
<td>8.1</td>
<td>Contribution</td>
<td>83</td>
</tr>
<tr>
<td>8.2</td>
<td>Limitations and further research</td>
<td>86</td>
</tr>
<tr>
<td>8.3</td>
<td>Concluding remarks</td>
<td>89</td>
</tr>
<tr>
<td>9</td>
<td>References</td>
<td>91</td>
</tr>
</tbody>
</table>
Part I I The Articles

Presentation of the articles:

Article 1.

Smedsrud, J., (accepted). Explaining the variations of definitions in gifted education through the philosophy of language. Nordic Studies in Education.

Article 2.


Article 3.

1 Introduction

In this chapter, I will briefly introduce gifted education at the international level, how we can understand or undermine gifted education in Norway, and a broader perspective on what it means to be gifted and mathematically gifted. In the last section of the chapter, the research questions and overarching aim of this thesis will be presented. To understand giftedness in a broad context, it is important to introduce gifted education both internationally and in Norway. Although there is no specialized gifted education in Norway at the teacher or student level, I choose to focus on gifted education in Norway because, in many ways, the chapter summarizes how we conceptualize giftedness in the Norwegian context.

1.1 Gifted education

China, England, Russia, and the United States are among the countries associated with education for gifted children and adolescents, and they are generally more focused on individuals than in Scandinavia. There is extensive research on the subject from the United States (Assouline & Lupkowski-Shoplik, 2012; Robinson & Campbell, 2010; Smutny, 2003; Tannenbaum, 2000). The study of mathematically precocious youth (SMPY) is one of a few studies on mathematically gifted students receiving what we would describe as special opportunities in Norway (I will describe this study in detail in Section 3.1.1). Even though the focus on gifted education in the US changed in the twentieth century, the US continues to be one of the countries with the highest focus on gifted education. In the 1960s, the US focused on whether gifted children and adolescents should be included and given a special place in the Education Act or not (Tannenbaum, 2000). The congressional mandate of 1979 added provisions related to gifted and talented children and, thus, expressed a legislative decision to include gifted and talented students among those receiving help through the Elementary and
Secondary Educational Amendments (Tannenbaum, 2000). In the same way that general education practice and educational practice for students with special needs have changed paradigmatically, gifted education has witnessed change (Gallagher, 2002a). Gallagher (2002b) emphasizes four issues regarding the new paradigm in gifted education: (1) the changing views of intelligence; (2) the paradigm of creativity; (3) identification of gifted students; and (4) the challenges of cultural differences and various models of instructional strategies for gifted students. Even though the educational options for gifted students are numerous and teacher practices for gifted students have changed, we have little knowledge about what works and what the effects of current practices are (Persson, Joswig, & Balogh, 2000). This has resulted in a variety of identification methods, definitions, and focus in gifted education. As discussed in Article 1, the way we define giftedness have profound impacts on how gifted education develops and on the extent of the focus on gifted education in a country. Although the general focus on gifted education has increased, the lack of proper legislation to secure provisions for gifted students has resulted in resource scarcity and, consequently, dissatisfied parents and teachers (Shaughnessy & Persson, 2009). An ongoing debate in gifted education has been whether gifted children should be included as part of the group of children who might be in a position of extra risk in school, thus qualifying for special attention and being included in special legislation aimed at securing educational outcomes for these groups (Tannenbaum, 2000). Besides the suggestion to change the Federal Bureau of the Handicapped to the Bureau of Exceptional Persons to include gifted children (Tannenbaum, 2000), no country that I am aware of has included giftedness as part of this understanding, which contrasts with the UNESCO 1994 Salamanca Statement. The statement places gifted children in a position of equality with special needs children in school. Since this group is at risk, they should also receive special attention (UNESCO, 1994). In the next section, I will focus on gifted education in Norway. As discussed later, there is a need to address the fact
that there is no legislation in Norway guiding educational provisions for gifted students. There are merely general education guidelines.

1.2 Giftedness in the Norwegian context

The Norwegian pre-university education system is organized through three stages: primary school (6–12 years old), lower secondary school (13–16 years old), and upper secondary school (16–19 years old). Participating in school is considered a right and is stated in the Norwegian Education Act (opplæringslova). The Norwegian school system is based on inclusion and equality (NOU, 2009).\(^1\) In general, this means that in school, all students have equal learning rights and the right to a positive learning experience. Furthermore, there is no segregation with regard to students’ socioeconomic status, ethnicity, or ability (Ministry of Education, 2009\(^2\)). However, Persson (2010) describes the Scandinavian school as a danger for gifted adolescents, because of its egalitarian ideology. The egalitarian education philosophy aims to compensate for individual differences that might occur in pre-school years due to, for example, social and economic differences (Moon & Rosselli, 2000). By ensuring equal opportunities in school, and equal opportunities for higher education, the given school model serves to ensure equality across social and economic backgrounds. In the case of gifted education, this might lead to a greater focus on weaker students in school and little to no focus on gifted students (Moon & Rosselli, 2000). To ensure inclusion in school, Norway have a strong educational act that aims to assure individual, cultural, and economic equality and that makes learning a privilege and an obligation for all students (Lovdata.no\(^3\)). The idea of

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\(^1\) See https://www.regjeringen.no/no/dokumenter/nou-2009-18/id570566/sec2

\(^2\) https://www.regjeringen.no/no/dokumenter/stmeld-nr-030-2003-2004-/id404433/sec8

\(^3\) https://lovdata.no/dokument/NL/lov/1998-07-17-61
inclusion is aimed at impacting students socially, academically, and physically (Norwegian Directorate for Education and Training)\textsuperscript{4}. The concept of inclusion is often misunderstood and can lead to schools treating every student in the same way, instead of cultivating differences and offering individual talent support (Skogen, 2010). As a result, in Norway, few systems are in place to tutor children and adolescents with a high potential for learning. According to Skogen and Idsøe (2011), the fear of elitism has also influenced the lack of optimal learning opportunities for gifted adolescents in Norway. This fear is derived from the belief that focusing on gifted adolescents is the same as cultivating a cognitive over-class and an over focus on individualism.

The Norwegian Education Act (opplæringslova\textsuperscript{5}) contains two paragraphs that are meant to secure an optimal learning environment for all children and adolescents in school: § 1-3 and § 5-1. § 1-3 can be described as fostering adaptive training and are meant to secure individually tutored education for all students in Norway, regardless of their cognitive level or subject understanding. Adaptive training should emphasize three student-related aspects: level of subject understanding, learning capacity, and learning style (The Norwegian Education Act, 2008).\textsuperscript{6} In practice, it is often the case that students at the lower end of the achievement scale receive far more attention than those at the upper end. Furthermore, since Norway has no tradition of using intelligence testing in school, we often have little knowledge about students’ IQ level. Information about students’ learning capacity is important to ensure an individually tutored learning pace or progress in subjects, especially in the case of gifted underachievers (Montgomery, 2009).

The background of § 5-1 is to secure the output of § 1-3, which means that if students

\textsuperscript{4} See https://www.udir.no/laring-og-trivsel/tilpasset-opplaring/inkludering-og-fellesskap/

\textsuperscript{5} See https://lovdata.no/dokument/NL/lov/1998-07-17-61

\textsuperscript{6} See https://lovdata.no/dokument/NL/lov/1998-07-17-61/KAPITTEL_1#%C2%A71-1
do not receive enough provisions through adaptive training or regular learning, they might have the right to receive special tutoring, which is then documented through § 5-1. Earlier interpretations of § 5-1 opened this law to justifications of provisions of special education for gifted learners and other groups that might be receiving insufficient learning or tutoring in school. The Norwegian Directorate for Education and Training (2009) later added a section clarifying that this part of the Education Act cannot be used to provide faster, better, or more learning for students who might learn more rapidly than the average student. This effectively exclude gifted learners from receiving special tutoring through § 5-1. If a gifted student has a learning disability, psychosocial difficulties, or other forms of behavioral problems, § 5-1 cannot be used to provide a more optimal learning environment based on his or her high learning potential. The act focuses exclusively on disadvantages or disabilities. This notion within § 5-1 is of concern because gifted students can develop learning and behavioral problems if they do not receive enough tutoring (Montgomery, 2009). Skogen (2010) goes further in describing the political resistance against supporting gifted students. He concludes that even though the Norwegian school system generally has relatively good standards, it often fails gifted students. Some possible reasons for this failure could be the lack of knowledge among teachers, the absence of a quality legislation to meet these students’ individual learning needs, and we do not provide a learning environment aimed at fostering gifted students’ abilities. Despite the increased focus on professional knowledge in schools, gifted learners are precisely those who are most likely to suffer (Skogen & Idsøe, 2011).

1.3 Acceleration opportunities in Norway

Acceleration is generally defined as moving students through an educational program at faster rates or younger ages than the norm (Colangelo, Assouline, & Gross, 2004). The belief that

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students can become social misfits or emotionally unstable from participating in classrooms
that are typically aimed at higher age groups has served to hinder the acceleration of gifted
students (Colangelo, Assouline, & Lupkowski-Shoplik, 2004; Karnes & VanTassel-Baska, 2005). Empirical evidence suggests the opposite, at least for segments of the gifted
population. Many gifted children feel isolated and have social needs that are more comparable
to those of older children. Therefore, acceleration serves to at least meet their academic needs.
This can be crucial to preventing them from becoming underachievers (Coleman & Cross, 2000; Gross, van Vliet, Teach, & Australia, 2003). However, it is also vital that gifted
students are sufficiently challenged in their acceleration classes. In Norway, we have, for
some years, offered acceleration (in Norwegian: forsering) opportunities for “students with an
interest in and talent for science” (Norwegian Directorate for Education and Training, 2016).8
The typical way to tailor or compress the syllabus for gifted and high-achieving learners in
Norway is by moving the student one year ahead in the given topics in which he or she is
talented. However, we lack research on whether these acceleration programs are successfully
meeting the needs of gifted learners. Geographical differences across Norway also make it
difficult to secure these opportunities for all students. Students who live near cities receive
such opportunities, and those who live in smaller communities do not. For example, Oslo and
Bergen, two of the largest cities in Norway, have special opportunities for mathematically
gifted students that are not found anywhere else in the country. For example, students in
junior high school can participate in mathematics classes for high school students in Oslo.
These opportunities do not present themselves before students are in junior high school
(Norwegian Directorate for Education and Training, 2016),9 which can too late for many

8 See https://www.udir.no/kvalitet-og-kompetanse/nasjonale-satsinger/realfagsstrategien/tilbud-til-elever-som-
trenger-ekstra-utfordringer-i-realfag/

9 See https://www.udir.no/kvalitet-og-kompetanse/nasjonale-satsinger/realfagsstrategien/tilbud-til-elever-som-
trenger-ekstra-utfordringer-i-realfag/
gifted learners. A recent white paper (NOU, 2016, p. 14) describes similar concerns about the situation of gifted learners in Norway, with the authors suggesting several strategies for meeting the needs of gifted learners, including more acceleration opportunities in school. To ensure the materialization of these strategies, we should also discuss proper legislation to secure their implementation.

1.3.1 Summary

In the last section of this chapter, I presented a broad perspective of the engine driving gifted education at the international level, especially in the United States. As this thesis is centered on the Norwegian context, I also presented gifted education from a Norwegian perspective. As discussed in the chapter, we are in the early stages of developing proper learning activities and educational opportunities for this student group. Furthermore, some of the laws developed to ensure equality of opportunity in the Norwegian educational system can, in some cases, have an opposite effect, in that, gifted students might not be sufficiently academically challenged in school. Furthermore, we tend to focus more on weaker students, than we do on gifted students, when we interpret the content of paragraph § 1-3 of the Education Act.

Importantly, there is little research in Norway on how gifted students perceive and experience the school system. Before presenting the overarching research objectives that guide the thesis, the following section will delineate the research field of giftedness and the mathematically gifted.

1.4 Delineation of the research field: Giftedness

In the early stages of my literature review for this thesis, I discovered that the field of gifted education is replete with contradictions and different definitions. There is no consensus regarding how we should define “giftedness,” “high-achieving students,” “exceptional
“students,” or “students with high learning potential.” Instead, these terms are often used synonymously existing research. The Marland Report of 1972 conducted extensive work on mapping giftedness, concluding that there were six broad categories of gifted behavior: 
general intellectual ability, specific academic aptitude, creative or productive thinking, leadership ability, ability in the visual and performing arts, and psychomotor ability (Marland, 1972; Stephens & Karnes, 2000). Some definitions are geared toward practice, whereas others are geared toward culture, research, or individual differences (Ambrose, Van Tassel-Baska, Coleman, & Cross, 2010; Freeman, 2003 2005). Giftedness is a complex concept and can vary at the qualitative and quantitative levels (Passow, 2004). The understanding of giftedness also varies among teachers (Laine et al., 2016; Moon & Brighton, 2008), parents, and researchers (Ambrose et al., 2010; Mann, 2006b). The shift in internationally gifted education suggests that the current conception should emphasize more malleable and contextualized definitions than was previously the case (Ambrose, et al., 2010; Cross & Coleman, 2014), although some professional educators still view giftedness as a fixed and innate characteristic of a person (Cramond, 2004; Laine et al., 2016; Kaufman & Sternberg, 2008; Mattsson, 2010). New concepts of giftedness have emerged through several paradigms and systematics reviews and individual or multidimensional definitions have been suggested or expanded in the literature (Kaufman & Sternberg, 2008). Research emphasize that gifted students might experience social and emotional issues during school or in specific environments (Mönks & Van Boxtel, 1985; Rimm, 2002; Vialle, Heaven, & Ciarrochi, 2007; Winner, 1996). To become successful adults, the social, emotional, and academic needs of gifted students must be met, both in school and in their surroundings (Cross & Coleman, 2014). However, the search for an “essence” and a unified definition of giftedness might be misleading. To argue that teachers, have a misconception about what giftedness is, one must
be able to ascertain that the definition at hand reflects the concept more than any other understanding.

To investigate these conceptions, a broad spectrum of research methods should be applied (Kaufman & Sternberg, 2008; Ziegler, 2009), particularly to understand the essence of giftedness and how gifted individuals experience life. Qualitative and quantitative research methods utilize different perspectives when engaging and integrating the social, emotional, and motivational characteristics of gifted adolescents:

An understanding of what constitutes giftedness shows the importance of drive and hard work in achievement of any kind and reveals that high abilities in some domains do not require a high IQ. A fundamental question that is not yet resolved is whether gifted children differ from average ones only in a quantitative way or whether they differ qualitatively, in which case, new principles are required to account for their performance. (Winner, 2000a, p. 155)

This means that research that integrates qualitative and quantitative perspectives of giftedness can enhance our understanding of the concept, which is what I do in this doctoral thesis.

Furthermore, these approaches can outline the importance of understanding different aspects or definitions of giftedness. In the next section, I will outline some central aspects of mathematically gifted students and their academic needs.

1.5 **Delineation of the research field: Gifted in mathematics**

Some researchers suggest that the tendency of becoming unmotivated underachievers is higher among adolescents who are gifted or profoundly gifted in mathematics than among adolescents who are gifted in other subject areas (Pettersson, 2008). As the discipline of mathematics shares common traits across nations, one would expect that mathematically gifted individuals—and gifted adolescents in general—would share some common traits (Dauber & Benbow, 1990). In the same way that no specific criterion has been established to determine giftedness, no coherent definition of mathematical giftedness exists. Therefore,
Mathematical giftedness is often described through a series of qualitative terms such as problem-solving abilities, metacognitive abilities, creative mathematical thinking, and high abilities/performance in mathematical problem-solving (Leikin, Koichu, & Berman, 2009; Leikin, Leikin, Baruch, Waisman, & Lev, 2017).

Mathematical abilities are regarded as dynamic characteristics that are developed if provided enough opportunities and support by the environment. Mathematical giftedness is seen as an inherent potential for mathematical knowledge and a deeper understanding of mathematical concepts (Leikin et al., 2017; Subotnik, Pillmeier, & Jarvin, 2009). The mathematically gifted possess intellectual characteristics such as curiosity, the ability to visualize abstract models, quick thinking, and metaphorical thinking (Deary, 2000; Silverman, 1997). Furthermore, people who are mathematically gifted possess mathematical creativity. There is no commonly accepted definition of what mathematical creativity is (Plucker, Beghetto, & Dow 2004; Singer, Sheffield, & Leikin, 2017). A focus on mathematically gifted and creative students has been neglected over time, which has generally been due to the overriding focus on students with challenges in understanding mathematical concepts in school (Singer et al., 2017). In the next sub-section, I will discuss the academic needs of mathematically gifted students.

1.5.1 Academic needs of mathematically gifted students

Mathematical giftedness and creativity are typically used synonymously in the literature (Hoth et al., 2017; Presmeg, 1986). In general, some students are identified by their ability to move through school material at a rapid pace, while others are categorized according to the complexity of their situated thinking and mathematical abstraction. It is more difficult to identify students in the latter group because students’ abstraction and complex thinking do not always reflect their level of performance in school. Hong and Aqui (2004) suggest a range of
features that distinguish mathematically gifted students from their non-gifted peers.
Mathematically gifted students have stronger cognitive abilities and are instinctively motivated. They think more strategically and are more likely to have control over their own solution process. Mathematically gifted students use more strategies for organizing and transforming information and use them more effectively. They can also transfer these strategies to novel tasks and use more rereading, inferring, analyzing structure, predicting, and evaluating strategies (Hong & Aqui, 2004; Hoth et al., 2017). Students who score highly on all or some of these characteristics can probably become highly creative workers and high-achieving students if they receive individual tutoring in school.

In contrast to their non-gifted counterparts in the classroom, mathematically gifted learners often perform well above what is expected for their age group (Mann, 2009). Empirical evidence suggests that learning opportunities are the main element in fostering mathematically talented students (Hoth et al., 2017; Nadjafikhah, Yaftian, & Bakhshalizadeh, 2012). To experience school as meaningful, all students need appropriate challenges and differentiation based on their skill level, and these differentiations should occur on an individual level.

Research suggests that teachers lack the knowledge to provide gifted students with appropriate cognitive challenges in the general classroom (Hoth et al., 2017). Instead, teachers’ instructions typically focus on tasks aimed at the overall student population (Rotigel & Fello, 2004). A learning environment that can meet the needs of gifted students in mathematics is recognized by an appreciation for alternative ideas and discussion regarding alternative or multiple solutions (Nadjafikhah et al., 2012). Teachers should provide guidance for students to explore their own ideas, define and make hypotheses, refute and adapt heuristic strategies, reason and justify conclusions, and reflect on them on a metacognitive level (Hoth et al., 2017; Nadjafikhah et al., 2012).
1.5.2 Summary

As outlined in the previous section, giftedness is defined in different ways across cultures and research. The lack of consensual definitions potentially represents a research gap, as it might be difficult to compare the current research in the field. The international shift in gifted education emphasizes a focus on more contextualized definitions than the earlier fixed and often intelligence-based definitions. If Norway is to follow the international trend, we should allow research and practice to guide us toward a definition that is representative and reflective of the Norwegian context. We encounter the same issue in defining mathematically gifted students, as research has tended to focus more on the common traits shared by mathematically gifted individuals than on possible individual differences. To meet the academic needs of gifted students, we should establish an educational practice that are grounded in empirical evidence. The gap in research between Scandinavian countries and, for example, the United Stated makes it difficult to make definitive conclusions because the educational practices between the countries vary considerably. In the following section, I will outline how the overarching aim and research objectives of the thesis can reduce some of the current research gaps in Norway.

1.6 Overarching aim and research objectives

This thesis addresses mathematically gifted adolescents in secondary school in Norway. As presented in the introduction, there is limited research on this topic, and therefore, more research is required. It is important to address how gifted students experience school in Norway so that future interventions for this group can be empirically driven and, thus, can be based on these students’ perception of the school system. The above-mentioned knowledge gap between practice, policy, and research cannot be filled through a doctoral thesis.
However, this thesis can serve to fill some of the knowledge gaps in Norway and to guide future research. The overarching aim of this thesis is expressed through the following question:

**How do mathematically gifted adolescents perceive and reflect on mathematics tutoring during their school years in Norway?**

To answer the overarching aim, three research questions were developed, each focusing on different aspects of the overarching aim. The research questions are essential in terms of exploring giftedness in Norway. Collectively, they represent the overarching aim by describing different perspectives of gifted students. An article was developed to answer each of the following three research questions:

**Research question 1:**

The first research question challenges the notion that we should understand and explore the “essence” of giftedness. For example, intelligence-based definitions of giftedness propose that intelligence is the “essence” of giftedness. Thus, instead of asking the “what is” question, I pose the research question:

**How can variations in the definition of giftedness in the literature be discussed through the philosophy of language?**

**Research question 2:**

The second research question deals with the dissemination of adolescents’ perceptions of tutoring in upper secondary school. Due to the notion that giftedness varies at the qualitative and quantitative levels (Winner, 2000b), qualitative studies can serve as a tool to quantitative research at a deeper level. This is particularly relevant in exploring motivation, peer relationships, and teacher relationships among gifted adolescents, as these factors may affect
gifted adolescents’ performance and feelings of meaning. Therefore, the second research question is as follows:

**How do mathematically gifted adolescents perceive accelerated learning opportunities in mathematics at the university level and lower in Norway?**

**Research question 3**

To ascertain whether gifted individuals are motivated, whether their social and emotional needs are met at an individual level, whether their academic needs are met in school, and how it “feels” to be gifted, a qualitative approach appears more appropriate. When we want to explore some of these aspects among mathematically gifted students in more general terms, a quantitative approach is more fit. The aspects explored in the qualitative and quantitative study is based on each other. The quantitative material was gathered before the qualitative study was conducted. Therefore, the qualitative and quantitative study explore somewhat similar aspects of giftedness, thus, at different levels. The qualitative study strengthens the interpretations of the quantitative, and the quantitative can help to build stronger conclusions from the qualitative study. The third research question addresses how gifted adolescents experience peers, motivation, teacher relationship and academic achievement a quantitative level:

**How do mathematically gifted adolescents experience motivation, peer relationships, and academic achievement at secondary school?**

In this thesis, I explore giftedness in philosophical, qualitative, and quantitative terms. As mentioned earlier, a combination of qualitative and quantitative approaches can provide a better and more nuanced understanding of the current situation involving this student group in Norway. The integration of these methods in this thesis allows for the possibility to enlighten
individual differences and to describe general tendencies between some of the characteristics presented earlier. Thus, we might come closer to understand giftedness in the Norwegian context. As argued in the introduction of this thesis, it is important to explore giftedness through both qualitative and quantitative studies. In the following section, I will outline how the articles in this thesis collectively address the main research question.

1.7 How the articles collectively address the main research question

In Article 1, I explore the most central variables and definitions of gifted education, with the purpose of understanding their usefulness in reflecting practical, cultural, individual, and research purposes. The goal of the article is not to describe all existing definitions of giftedness but, rather, to discuss some general aspects of traditional and newer ways of conceptualizing giftedness in students. The article serves as a guide for the definitions in the two other articles and, thus, as an argument regarding how the definitions presented can represent a valid understanding of mathematically gifted adolescents.

Smedsrud, J., (accepted). Explaining the variations of definitions in gifted education through the philosophy of language. Nordic Studies in Education.

Article 2 raises several questions that are further outlined in the thesis. It explores whether students received any form of special tutoring (e.g., acceleration or enrichment activities) during their previous schooling. It employs qualitative semi-structured interviews to examine their general conceptions about their teacher, peers, and academic self-concept. To access their own experiences and explore possible descriptions that cannot be achieved through quantitative research methods, qualitative research methods were employed.

Article 3 takes advantage of access to a large dataset among highly gifted and average adolescents and explores the connection between intelligence and academic achievement in mathematics, perceptions of peer relationships, teacher relationships, and gender differences. This makes it possible to explore some group differences within the gifted population and between gifted and average students.


Together, these three articles present a multifaceted understanding of mathematically gifted students in Norway through qualitative and quantitative research methods (see figure at the end of this section). The integration of methods (mixed) to address a specific question can describe and possibly explain misinterpretations of the quantitative data or ascertain whether the tendencies in the quantitative survey are also included in the qualitative study. As an exploratory study, I took advantage of the available data within the timespan of a doctoral study. Since Norway does not provide accelerated education programs, and neither does it implement gifted ability groups or allow for intelligence testing of children and adolescents, quality data that follow a definition based on empirical evidence are difficult to come by. In the next part of the thesis (chapters 2–4), I will present the overarching theoretical framing used for the articles. The focus will be on the most central concepts: individual giftedness, mathematical giftedness and acceleration, and ability groups and individual differences in mathematically gifted adolescents. Furthermore, several attempts have been made to separate
giftedness from inherent potential, which have failed to predict gifted behavior through variables that are not correlated with intelligence.

Figure 1. Display of the articles in this thesis on exploring different aspects of giftedness in Norway.
2 Different Approaches to Giftedness

In this part of the thesis, I will present the overarching theoretical framework used for the articles and the doctoral thesis. The chapter outlines some general and specific topics that are essential for understanding giftedness in the twenty-first century and mainly focuses on understanding giftedness and mathematical giftedness from an individual perspective. While gifted education was described in the introduction in general, the remainder of the thesis will mainly discuss education for the gifted from the perspective of students. To understand the different approaches and definitions of giftedness, it is important to describe several aspects and trends in understanding giftedness and mathematical giftedness. As I understand giftedness to include mathematical giftedness, though not always the other way around, I begin by outlining what giftedness is. It is necessary to describe the fundamental discussions regarding understandings of giftedness to appreciate the discussion in Article 1. In what follows, the various theories, models, and conceptions used as theoretical framing in this thesis will be presented and discussed in the next two sections: (2.1) Intelligence and giftedness and (2.2) Multifaceted models of giftedness and talent.

2.1 Intelligence and giftedness

There are several definitions of intelligence and giftedness in the literature, with a corresponding disunity among researchers. A rather common understanding of intelligence is that it involves an individual’s ability to adapt to the environment and learn from experience (Sternberg, 2010). In the traditional view of giftedness, intelligence was the fundamental variable with which to predict gifted behavior and high abilities. In the earlier stages of gifted research, researchers examined whether intelligence was associated with the divergence hypothesis, which assumed that high scores on intelligence tests were correlated with negative
personal traits such as stress, emotional instability, and negative physical traits (Stoeger, 2009; Terman & Oden, 1948). Intelligence—and especially its possible relationship with academic achievement and specific personal traits—has been at the forefront of debates over the last hundred years. It is one of the most valid measures of human potential (Warne, 2016) and has a major impact on academic achievement (Colom, Escorial, Shih, & Privado, 2007; Deary, Strand, Smith, & Fernandes, 2007; Kuncel, Hezlett, & Ones, 2004; Watkins, Lei, & Canivez, 2007; Wellisch & Brown, 2012). Past debates have focused on the best construct and measure of intelligence rather than the existence of a valid measure of intelligence (Stoeger, 2009). This means that while several intelligence theories exist in the literature, they are based on somewhat different assumptions about the measurement of intelligence (Undheim, 1981).

In the more traditional intelligence theories—Spearman’s (1923, 1946) model of general intelligence and Horn and Cattell’s (1966) theory of crystalized and fluid intelligence—intelligence is understood as a psychological construct that affects abilities such as learning and problem-solving (Colom et al., 2007). The three-stratum theory (Carrol, 1991, 1997) combined the existing intelligence theories and added a general measure of intelligence (g-factor, g, or general intelligence measure) to summarize the cross-correlations between several cognitive tasks. The predictive validity of the general intelligence factor (g) has been well documented in several studies (Duckworth & Seligman, 2005; Kuncel et al., 2004). Originally developed by Spearman (1946), Spearman’s g is a highly useful psychological construct, which is measurable through psychometric tests such as Raven’s Progressive Matrices or the Wechsler Intelligence Scale (WISC) (Warne, 2016). Thus, the underlying assumption is that one construct can predict several of the seemingly overlapping performance measures in school. In later years, competitive theories of intelligence emerged through, for example, Gardner’s (1983, 1993, 2003) multiple intelligence theory (MI) and
Sternberg’s (1996) *triarchic theory of human intelligence and successful intelligence*. Gardner’s (1983, 1993, 2003) MI theory is not based on a unitary view of intelligence (Davidson, 2009). According to MI theory, at least eight interactive “types” of intelligence have been valued across a variety of cultures. Linguistic, logical-mathematical, and spatial intelligence are like the categories measured through traditional intelligence tests (Davidson, 2009), while musical, bodily kinesthetic, interpersonal, and intrapersonal address latent physical talents as well as intrapersonal abilities and emotionality (Hatch & Gardner, 1993). MI theory is grounded in empirical evidence that, in contrast to earlier ideas, interprets the human mind as modular in design, which means that separate psychological processes appear to be involved in dealing with linguistic, numerical, pictorial, gestural, and other symbolic systems (Hatch & Gardner, 1993; Gardner & Wolf, 1983). The theory suggests that there does not need to be any correlation between the categories; however, many occupations and capabilities emphasize the use of several performance abilities. MI theory has been criticized for its lack of empirical evidence (Gardner & Connell, 2000; Sternberg & Grigorenko, 2004; Waterhouse, 2006). In response, Gardner and Moran (2006) have suggested that the traditional way of measuring intelligence is flawed and that the theory cannot be validated or evaluated through traditional pen-and-paper tests. In the context of gifted education, MI theory can help us explain the diversity of talent and personality traits we see in the gifted population (Davidson, 2009). In the same way, the g-factor can explain why we observe some of the same behaviors when we identify giftedness though IQ and achievement.

Sternberg (1996, 1999; Sternberg & Lumbart, 2000) suggests another approach to intelligence, described as the *triarchic theory of successful intelligence*. He argues that the traditional way in which researchers have operationalized and narrowed intelligence—through fluid, crystalized, and general intelligence—is incomplete (see also Sternberg & Grigorenko, 2006). Instead of understanding intelligence as adaptation to the environment and as
something that can be operationalized and measured through narrow intelligence tests, he
defines intelligence as the “ability to achieve success in life, given one’s personal standards,
within one’s sociocultural context” (Sternberg, 2003b, p. 42). The theory does not neglect
adaptation to the environment; rather, he envisions the following three interacting aspects that
contribute to successful intelligence: (1) analytical skills that help individuals evaluate, judge,
critique, or analyze information; (2) practical abilities that can create an optimal match
between individual skills and the environment and creative intelligence, which serve to
maximize experiences in order to generate new products and solve problems; and (3) the
ability to achieve success. The third point depends on an individual’s ability to capitalize on
inherent strengths and compensate for weaknesses through a balance of analytical, creative,
and practical abilities (Sternberg, 1999, 2003a, 2005). The idea is that some of the intelligence
scores that differ across cultures can be accounted for by this model and that intelligence is
not a context-free phenomenon. To define the most successful or gifted individuals, both
culture and individuals’ ability to adapt and perform within that specific context should be
addressed. In contrast to MI theory, several studies have validated Sternberg’s theory (Dai,
2009; Sternberg, 2003a). However, his theory has also been the subject of criticism his
argument that practical intelligence and performance in practical occupations are not
predicted by g and that, collectively, there is much evidence in favor of the g-factor
(Gottfredson, 2003a).

The WISC model of giftedness is a possible common basis for identifying gifted
individuals (Dai, 2003; Sternberg, 2006, 2010). According to this model, wisdom,
intelligence, and creativity are the fundamental traits of gifted individuals; they are necessary
if individuals are to contribute to society in the future. Furthermore, these traits can be
identified through abilities within academic domains or other fields of expertise and/or
performance. The understanding of this model is linked to the WISC test, which is commonly
used in several societies to identify or test children and adolescents in school. Sternberg (2010) conducted an in-depth analysis of the traits presented in the model. Although creativity, as an aspect of giftedness, is promoted in many models, it is not an easily understood aspect of human behavior and will be described in greater depth later in this thesis. In the next section, I will explain the multifaceted models of giftedness and talent.

2.2 Multifaceted models of giftedness and talent

In one sense, there is an intuitive and logical connection between environment, inherent potential, and the potential for performance. Although there are many theories of how giftedness develops and what factors lead to gifted behavior, few, if any, explain the complexity of the behavior we observe in gifted adolescents. In response to understanding giftedness using a uniform construct measured by high scores on intelligence (Davidson, 2009), other more multifaceted models have emerged. Several of these models can be understood as multifaceted; however, the theories outlined here do not necessarily define intelligence. According to these models, giftedness is often a result of several variables—connected to the environment and one’s inherent potential in terms of performance—that work together and can result in exceptional performance in one or more domains. The theories emphasize different factors in relation to giftedness. However, they are unified in their assumption that outstanding performance is far too complex to be explained exclusively through the lens of intelligence. One of the most influential theoreticians in this field is Joseph S. Renzulli. According to Renzulli’s (1978, 1986, 2002, 2005, 2012) three-ring conception of giftedness, gifted behavior results from three interacting variables: above-average ability, creativity, and task commitment. The three rings cluster together to illustrate that gifted behavior is a result of several interacting personality and cognitive traits that must work together. Only with enough stimuli from the environment can gifted behavior develop.
Above-average ability refers to a person’s cognitive ability, which, in Renzulli’s view, is found in the top 15–20% of a given age group compared with the average. High levels of creativity are associated with originality of thought, while task commitment refers to a person’s special interest in or commitment to a subject(s) (Renzulli, 1978, 2002b, 2012). Renzulli’s understanding of giftedness was developed through an extended literature review (for details, see, e.g., Davidson, 2009).

Renzulli’s (1978) three-ring conception of giftedness:

![Three-ring conception of giftedness](image)

**Figure 2.** The interaction between the three components leads to gifted behavior, according to Renzulli’s (1978, 2002, 2012) three-ring conception of giftedness.

Building on the three-ring conception of giftedness, Mönks (1992) developed the *multifactor model* of giftedness. This model adds three components to Renzulli’s theory, which Mönks (1992; Mönks & Katzo, 2005) argues are necessary for gifted behavior to occur. In contrast to Renzulli’s model of giftedness, Mönks adds three environmental factors, namely, *family*, *peers*, and *school* (Mönks, 1992; Mönks & Katzo, 2005). The justification is that a definition that neglects the interactive nature of human development does little justice to the dynamic nature of development, and thus, the only appropriate framework is a multidimensional approach that includes those interactive components (Mönks & Katzo, 2005). In this view, giftedness can only be transformed into outstanding performance when all
these variables are working together and when children or adolescents experience a meaningful environment. The opposite of this notion can also be inferred: If one or more of these variables do not occur, underachievement can ensue.

The three-ring conception of giftedness (Renzulli, 1978, 2002b, 2012) and the multifactor model of giftedness (Mönks, 1992; Mönks & Katzo, 2005) have several factors in common (Davidson, 2009). Although both models aspire to take different aspects of giftedness into account, they also include abilities that are assumed to be possessed by all high-performing adolescents or adults in terms of intellectual ability, creativity, and environmental influences. For example, Davidson (2009) argues that it is unlikely that a child who grows up with a high IQ in an under-stimulating environment will become highly successful in a domain that has not been part of his or her life. However, it would be difficult to distinguish between nature and nurture if a highly successful adolescent grew up in a stimulating environment. Nevertheless, while it is probably safe to assume that the reason for success lies somewhere in between, these assumptions do not amount to empirical evidence.

The similarities between the terms giftedness and talent illustrate a conceptual issue in gifted education, as these terms are associated with different concepts across cultures, especially in Norway, where we are known to appreciate outstanding performance in sports. The terminology used in the field can, in some cases, create more confusion than clarity, especially when we define giftedness through the given terms. Thus, every term can create more confusion than clarity. The terms do not answer the question “what giftedness is” or create a general definition of giftedness. For many, talent is a sub-category of giftedness, or a pre-category, whereby talent in a field can refer to the potential for performance within this field or domain (Feldhusen, 1986).

Gagné (1985, 2004) suggests a different approach to giftedness and talent. The developmental model of giftedness and talent (DMGT) can, in my view, be described as
performance-oriented, in contrast to earlier models, which focused on inherent ability and intelligence (Gagné, 2005). In this model, a distinction is made between realized achievement and the potential for achievement. Furthermore, giftedness is associated with natural or innate abilities, which should reflect the top 10% of a given age group (Gagné, 2005). Nevertheless, the focus of the model is on the development of and transformation between gifts and talent, whereby individuals can transform genetic allocations into outstanding performance in one or more fields of expertise. In contrast to other theories, expertise in a field is described as talent, and the potential for expertise can be described as giftedness. Furthermore, a talented individual should demonstrate expertise in at least one field of human activity, and Gagné (2005) proposes at least 30 fields in which the given activity can unfold. The model demonstrates a five-level system that differentiates between the levels or hierarchies of giftedness and talent in individuals. These levels are like Gross’ (2009) categories of prevalence in the gifted population:

**Table 1.** Gross’s (2009, p. 337) categories of prevalence in the gifted population

<table>
<thead>
<tr>
<th>Level</th>
<th>IQ-range</th>
<th>Prevalence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mildly (or basically gifted)</td>
<td>115–129</td>
<td>1:6-1:40</td>
</tr>
<tr>
<td>Moderately gifted</td>
<td>130–144</td>
<td>1:40-1:1000</td>
</tr>
<tr>
<td>Highly gifted</td>
<td>145–159</td>
<td>1:1000-1:10 0000</td>
</tr>
<tr>
<td>Exceptionally gifted</td>
<td>160–179</td>
<td>1:10 000-1:1 million</td>
</tr>
<tr>
<td>Profoundly gifted</td>
<td>180+</td>
<td>Fewer than 1:1 million</td>
</tr>
</tbody>
</table>

Gagné’s (2005) prevalence is similar to that of Gross (2009); however, he is somewhat more optimistic about the percentages in a given category (1:10 mildly gifted, 1:100 highly gifted, 1:1000, exceptionally gifted 1:10 000, and extremely gifted 1:100 000).
The *Munich Model of Giftedness* (MMG) draws its theoretical perspective from one of the largest surveys of gifted adolescents in Europe (Heller, 2001; Heller & Hany, 1986; Heller, Perleth, & Lim, 2005). The Munich longitudinal study was based on a psychometric classification of several different types of giftedness (Heller et al., 2005). This multidimensional model consists of seven relatively independent ability factor groups (predictors), various performance domains (criterion variables), and personality (motivational, etc.) and environmental factors that serve as moderators for the transition of individual potentials into excellent performance (Heller et al., 2005). This model has been internationally validated in several studies (Heller, 2001). The combination of cognitive (intelligence) and non-cognitive (motivation, control, expectations, self-concept) traits and social moderators was developed for diagnostic purposes and, thus, serves as an identification tool. The development of expertise is not understood as static; instead, exceptional performance is developed through interaction between traits, and non-cognitive personality traits are given increased significance in the development of achievement. Personality traits can also explain the difference between achievement and underachievement. The MMG model is process-oriented, and it views inherent ability as one of several categories that should be nurtured for the development of expertise (Heller et al., 2005).

The *actiotope model of giftedness* considers giftedness as a characteristic that changes over time. In this model, giftedness (or gifts) is not seen as a personal attribute but, rather, as an attribute constructed by scientists (Ziegler, 2005; Ziegler, Vialle, & Wimmer, 2013). Thus, similar to the MMG model, the emphasis lies on a repertoire of actions needed to fulfill the potential for achievement through systematic training, years of progress in school, and awareness of what can be done with one’s potential (Ziegler, 2005). Individual students act and change the environment, and their actions need to be considered as reflecting changes in talents or gifts over time. According to this model, six distinctions between subjective and
objective influences are needed for giftedness: (1) actions: These consist of a sequence of partial actions, each of which is a composition of parallel and multiple actions; (2) the action repertoire: understood as sustainable possibilities for actions that an individual is capable of executing; (3) the subjective action space: what people believe they are able to do; (4) the goals: what people want to do; (5) the environment: characterized by a rapid alteration of domains; and (6) the interactions among the components: resulting in a constant quest for equilibrium and the progressive adaptation of the individual to the environment as well as the ability to realize when an action is successful (Ziegler, 2005).

Few models of giftedness neglect the connection between cognitive abilities and success in life. However, whether above-average cognitive abilities predict success in life more so than average cognitive abilities is still up for debate, as researchers recognize the complexity of interactions that lead to expert achievement. For example, Plomin (1999) found a strong statistical correlation between the measured intelligence quotient (IQ) and high achievement or success later in life. Thus, empirical evidence affirms Renzulli’s (1978, 2002b, 2012) emphasis on task commitment by illustrating that Asian students with an IQ of 110 can achieve at the same level as American students with an IQ of 120 when studying in the US (Baumeister, Tierney, & O’Hare, 2011). They suggest that hard work, motivation, and task commitment can compensate for intelligence when comparing normal populations across nations.

The introduction of domain-specificity in gifted education emphasizes specific areas of aptitude and focuses on the needs of those who show potential in those areas and receive acceleration or enrichment at an appropriate skill level (Kaufman & Sternberg, 2008). In domain-specific models, it is often not deemed necessary to include additional psychological traits or processes (Brody & Stanley, 2005; Karnes & VanTassel-Baska, 2005). Accordingly, creativity is an output of knowledge and can only come to light when enough content is
mastered (Karnes & VanTassel-Baska, 2005; Kaufman & Sternberg, 2008). In sum, there seems to be some evidence in support of all the proposed models of giftedness, and it could be that each explains some aspect of giftedness, though perhaps not the whole picture. Thus, Feldhusen (1998) sought to accommodate all the existing models of giftedness by formulating a developmental model of giftedness based on talent development and by synthesizing the various aspects illustrated in the different approaches to giftedness (see also Kaufman & Sternberg, 2008).

2.3 Summary

As I have shown throughout this chapter, there are a multitude of definitions of giftedness, and several models describe how giftedness develops. Some of these models explicitly define giftedness through one or more variables (for example, intelligence-based definitions), while others implicitly define giftedness through a combination of several variables (for example, motivation, task commitment, and above-average ability). The problem is not the mere existence of so many definitions but, rather, how they are communicated and used in gifted education, such as when some of these communicate an “essence,” even though they do not define this essence. In Article 1, the problem of essentialist definitions is addressed in gifted education, along with an exploration of why there are so many definitions of giftedness. In the following chapter, I present the concept of mathematically gifted adolescents, which shows that the same issues experienced in defining giftedness, in general, also apply to definitions of mathematical giftedness.
3 Mathematically Gifted Adolescents

This chapter presents the most common ways of understanding mathematical giftedness (3.1). Furthermore, the central study of mathematically precocious youth (SMPY) will be described in the context of defining mathematical giftedness (3.1). Aspects of mathematical ability (3.2), creativity (3.3), gender (3.4), and motivation (3.5) will also be described, as these are central to articles 2 and 3 in this thesis.

3.1 Understanding mathematical giftedness

Mathematical giftedness is generally affected by the same issues as giftedness. Although mathematical giftedness is domain-specific, no precise criteria have been suggested that adequately define mathematical potential and/or mathematical giftedness (Mann, 2006a). While there are several definitions and models for understanding mathematical giftedness (Mann, 2006), to the best of my knowledge, few, if any, studies explore mathematical giftedness in Norway. For giftedness to materialize, chance, cultural values, biological disposition, pedagogical variables, and cognitive and psychosocial variables are of tremendous importance (Mayer, 2005; Subotnik, Olszewski-Kubulus, & Worrel, 2012; Tannenbaum, 1986). Thus, while inherent abilities do play a role, they are interchangeable with environmental influences when giftedness is realized and becomes expert performance. Nevertheless, the opportunities provided and the ability to act on these opportunities might be the leading factor contributing to the realization of domain-specific and expert performance (Subotnik et al., 2012; Barnett & Durden, 1993).

In mathematics, one of the most important studies of this observation is Bloom (1985). In his study, he demonstrated that an educational practice that identifies and stimulates the needs of gifted learners is of tremendous importance. Furthermore, the study was conducted
retrospectively, with Bloom interviewing 120 talented individuals, their parents, and teachers. His goal was to understand which processes led to high mastery and expert performance within, for example, mathematics. Bloom concluded that support from the environment, the ability to work over time, training, endurance, and receiving support and encouragement from the environment were all important factors in developing talent in mathematics. Several variables are seen to influence the performance of mathematically gifted students, especially creativity, self-efficacy, motivation, and more specific problem-solving strategies (Geary & Brown, 1991; Mann, 2006a; Pajares, 1996; Presmeg, 1986, 2008). The differentiation between mathematically talented and mathematically creative students is not distinguishable in the literature. However, empirical evidence suggests an individual differentiation between students who are creative and those who do not possess the same levels of abstract and creative thinking (Sriraman, 2005). Furthermore, surveys suggest a differentiation between the types of educational needs these different types of students might have (Banfield, 2005; Pierce et al., 2011). I describe these individual and educational differences later in this chapter (Section 3.5).

3.1.1 The SMPY: Studies of mathematically precocious youth

The SMPY study offers important insights into the minds of mathematically gifted adolescents. More than 5,000 mathematically gifted participants were studied over the course of five SMPY cohorts (Lubinski & Benbow, 2006). The study started in September 1971 by observing one intellectually superior 13-year old (Stanley, 1996). The initial goal was to identify, through objective tests, youths who reasoned exceptionally well in mathematical and/or verbal areas. Through the SMPY database, researchers have been able to better understand the complex minds of highly gifted individuals as well as engage in follow-up studies about the long-term effects of interventions on the gifted. Moreover, the results of
these studies have led to policy changes (see Benbow, 2012; Lubinski & Benbow, 1994, 2006; Stanley & Benbow, 1982; Swiatek, 2002). Drawing on the large empirical evidence from SMPY, some conclusions can be made about the development of gifted individuals. First, cognitive ability seems to play an important role in developing mathematical expertise (Benbow, 2012; Lubinski & Benbow, 2006). Second, the field of giftedness has developed beyond the one-dimensional approach to gifted education. In SMPY, the students within cohort 3 (exceptionally high intelligence scores) scored above the other groups (cohorts 2 and 1) in performance later in life. Third, although there were individual differences in the top one percent, they generally outperformed the bottom quartile within the gifted population (Lubinski et al., 2001). Fourth, the findings from the SMPY also seemed to serve as evidence against the “ability threshold hypothesis” (that beyond a certain point, ability does not change how high you score on a test or how high an IQ you might have) (Robertson, Smeets, Lubinski, & Benbow, 2010). In the words of Lubinski and Benbow (2006): “Nevertheless, other things being equal, more ability is always better, and there was evidence to suggest this long before the letter appeared in science” (p. 324). The study also revealed rather interesting covariance among verbal, mathematical, and spatial ability.

The relevance of SMPY to this thesis is that high levels of mathematical and spatial abilities, relative to verbal abilities, characterized college students who favored math, engineering, or computer science (Lubinski & Benbow, 2006). It is essential to also take ability patterns into account when seeking to understand individual differences among gifted adolescents, as these patterns can have profound effects on how a person learns or on what his or her interests are. Ability patterns also seem to be critical to the choices that gifted individuals make regarding their future education (Gottfredson, 2003b; Lubinski & Benbow, 2006). The extensive research conducted through the SMPY has also concluded that if
mathematically gifted children and adolescents are to fulfill their potential, it is essential that they receive appropriate academic challenges and nurturing (Lubinski & Benbow, 2006).

3.1.2 Mathematical ability

In theories describing mathematical constructs, we encounter several perspectives and distinctions regarding mathematical ability (Bishop, 1980; Presmeg, 1992). Many of the newer constructs of mathematical knowledge draw their ideas from Krutetskii (1976), who distinguishes between highly capable mathematics students and those who are less capable. In addition to the broader conceptions above, researchers have also investigated more specific characteristics of mathematical ability and giftedness.

One of the most influential researchers in the area is Krutetskii (1976), whose work in seeking to understand the components of mathematical ability have resulted in the distinction between levels of mathematical ability, determined by a verbal-logical component of thinking, and types of mathematical cognition, determined by a visual-pictorial component. Thus, it is not only the ability to use visual-spatial components; it is also the case that the preference for their use in solving mathematical problems determines the type of mathematical cognition (Krutetskii, 1976; Presmeg, 2008). According to Krutetskii (1976), spatial ability is not enough to ensure an individual’s preference for solving mathematical problems through visualization. Thus, the logical component is the defining factor in mathematical success.
Figure 3: Quadrant separating logical and visual components of mathematic ability (Presmeg, 2008, p. 88).

With stimulating environmental conditions, all individuals can develop their mathematical ability, although individuals with a mathematical mind use up less time and energy in developing mathematical skills. In addition to the visual representation of mathematical ability presented above, Krutetskii (1976) found six characteristics that are essential for high mathematical ability (gifted): (a) the ability to comprehend the formal structure of a mathematical problem; (b) the ability to generalize numerical and spatial relations; (c) the ability to operate with numbers and other symbols; (d) the ability to switch from one mental operation to another; (e) the ability to grasp spatial concepts; and (f) the mathematical memory for mathematical generalizations and structures (Cited in: Sak, 2008, p. 54). These characteristics were later supported by other empirical evidence (Presmeg, 2008, Sak, 2008, Sriraman, 2003).

Krutetskii (1976) was also one of the first to conduct research about the special abilities of the mathematically gifted, emphasizing their peculiarity in terms of structure of mathematical thinking and the fact that some components can be compensated by others (Singer & Ohmer, 2018). Other than the characteristics mentioned above, mathematical
giftedness in childhood is recognized by speed of thinking process; counting skills; a distinct memory of symbols, numbers, and formulas; visual thinking; and the ability to vividly imagine abstract mathematical relations and dependencies (Krutetskii, 1976; Singer & Ohmer, 2018). For adolescents, one can identify three components: geometric, analytic, and harmonic. The harmonic component is based on the relation between the visual and abstract-logical components (Singer & Ohmer, 2018).

In recent years, international surveys such as PISA and TMMS have defined mathematical ability and the ability to teach mathematics through several levels. These surveys have also standardized measures for comparison across countries (OECD, 2013; Onstad & Grønmo, 2012). Further, mathematical giftedness is a complex structure that some researchers associate with general giftedness (Leikin, 2013; Leikin & Lev, 2013; Lubinski & Benbow, 2006). As high achievement in school often reflects high levels of problem-solving proficiency in certain topics, it can be an indicator of high mathematical ability (Leikin, 2013).

3.1.3 Creativity among mathematically gifted students

There is no single and authoritative perspective on creativity or mathematical creativity (Mann, 2006; Sriraman, 2005). Instead, creativity is often defined through the battery of tests available for measuring it (Torrance, 2004). Some researchers distinguish between convergent and divergent thinking, referred to as production (Leikin, Berman, & Koichu, 2009). Convergent thinking is connected to the generalization of multiple answers to a specific problem and is referred to as flexible thinking (Guilford, 1967). Sternberg and Lubart (1996) suggest defining creativity as the ability to produce unexpected original work that is useful and adaptive. Furthermore, most definitions involve the ability to create and develop or
produce something new or different, to change a field or view of a problem, and the interaction between intrapersonal variables and the environment (Sriraman, 2005). However, there may be a distinction between general creativity and domain-specific creativity in a specific field of expertise, even though the separation between the two is obscure. Sriraman (2005) has gathered some of the most common understandings of creativity, defining it as follows: “Creative individuals are prone to reformulating a problem or finding analogous problems. These individuals are also different from their peers in that they are fiercely independent thinkers who tend to persever and reflect a great deal” (p. 24). It remains unclear whether there are specific differences—except for the specific subject of interest—between general creativity and domain-specific creativity in terms of how these abilities are applied. Thus, mathematical creativity involves the application of these individual characteristics to a given task.

Mathematical giftedness involves the individual’s abilities in mathematical processes, which comprise (1) the ability to abstract, generalize, and discern mathematical structures; (2) the ability to manage data and the ability to master principles of logical thinking and inference; (3) the ability to think and infer and the ability to think analogically and heuristically; (4) the ability to pose related problems as well as flexibility and reversibility of mathematical operations and thought; (5) awareness of mathematical proof; (6) independent discovery of mathematical principles; (7) the ability to make decisions in problem-solving situations; (8) the ability to visualize problems and relations; (9) the ability to infer behaviors that test for the truth or falsity of a construct; and (10) the ability to distinguish between empirical and theoretical principles and the ability to think recursively. As Sriraman (2005) notes, most of these abilities are cognitive, and thus, they do not involve the learning environment or other variables that might affect the development of mathematical giftedness.
Some truly mathematically gifted adolescents possess abilities that can be difficult for the common person to understand. Usiskin (2000) presents an eight-tier hierarchy and maintains that not all mathematically gifted individuals are highly creative (see also Sousa, 2009). The hierarchy presented in Figure 4 suggests that there are several levels of mathematical giftedness (levels 5–7) and that creativity occurs at the top two levels (i.e., exceptional mathematician and genius). In contrast, one can be gifted and “only” be able to produce mathematics. Usiskin’s (2000) model advances our understanding of the levels of mathematical comprehension, not only among highly gifted individuals, but also among the general population. Moreover, it explains why we cannot expect all mathematically gifted adolescents to be creative. Thus, mathematical creativity implies mathematical giftedness (Sousa, 2009; Sriraman, 2005; Usiskin, 2000). We are much better at identifying mathematical giftedness than mathematical creativity, suggesting that students operating at the highest levels of mathematical knowledge might not be identified in school (Sriraman, 2005).

(Figure 4: A representation of Usiskin’s (2000) eight-tier hierarchy, presented in Sousa (2009, p. 170).
In the next section, I will discuss gender differences in mathematically gifted adolescents.

### 3.1.4 Gender differences in mathematically gifted adolescents

The topic of possible gender differences in mathematical ability or spatial intelligence has been hotly debated, and it has been claimed that this topic is affected by political correctness and burdened by ideology (Lippa, 2006). Although it may seem harsh to claim that there is a difference in mathematical ability or mathematical reasoning skills between girls and boys, which are not connected to environmental factors, such as interest or gender roles, the **people-versus-thing** dimension has been documented in several studies (Lippa, 2006; Lubinski, 2000; Lubinski & Benbow, 2006). Females tend to display greater interest in learning about and working with people (organic content), whereas males, on average, tend to prefer learning about and working with things (inorganic content). For example, Lippa (2006) notes a difference between gender preferences for **people vs. things**, with an effect size of 1.20 standard deviation points. Undoubtedly, these differences contribute to helping us understand why—among highly gifted adolescents—fewer females have an interest in becoming engineers or mathematicians (Lubinski, 2006). The findings of Halpern and LaMay (2000) suggest that, on average, there are no differences in general cognitive ability between males and females. Thus, the difference seems to be in the subtests of intelligence tests. For example, males seem to do better at manipulating images in working memory, and females seem to have an advantage in the use of verbal information and long-term memory tasks (Halpern & LaMay, 2000).

The gap in mathematical achievement between genders seems to persist in some countries but not in others (Else-Quest, Hyde, & Linn, 2010). Furthermore, opportunities, gender stereotypes, and differences in stress levels in test-related situations might all contribute to differences in mathematical performance (Else-Quest et al., 2010; Halpern &
LaMay, 2000; Halpern et al., 2007). A meta-study by Hyde, Fennema, and Lamon (1990) supports these findings. In their study, females slightly outperformed males the early years of schooling, while differences in math performance favoring men emerged in high school and college (Hyde et al., 1990). Hyde, Lindberg, Linn, Ellis, and Williams (2008) found a small tendency favoring boys at the upper end of the intelligence scale (99th percentile), thereby supporting the notion that there may be cognitive characteristics that favor boys in outperforming girls in mathematics and science, further supporting the gender similarity hypothesis (Hyde et al., 2008).

Differences in mathematical ability seem to increase with age and are greater between male and female adolescents than between male and female children (Skaalvik & Skaalvik, 2004). The same pattern has also been found for mathematics self-concept and mathematics self-efficacy (Pajares & Miller, 1994; Skaalvik & Skaalvik, 2004). According to a study by Junge and Dretzke (1995), there are significant differences in self-efficacy between gifted males and females in mathematical subjects. Furthermore, the mathematical confidence of females, relative to that of males, was weakest with respect to mathematics-related college subjects. This is consistent with the findings of Heller and Ziegler (1996), that gifted female adolescents reported less confidence, greater anxiety, and fewer interests in subjects related to “hard science” and, therefore, might not unleash their full potential in mathematics.

Academic self-concept in mathematics shows a stable tendency that favors males among both gifted and non-gifted individuals (Eccles, Wigfield, Harold, & Blumenfeld, 1993; Marsh & Yeung, 1997; Skaalvik & Skaalvik, 2004; Wilgenbusch & Merrill, 1999). Preckel, Goetz, Pekrun, and Kleine’s (2008) study indicates that, in general, gifted students showed higher achievement in mathematics and that gifted males and females achieved the same grades. However, gifted males showed a significantly higher interest and self-concept in mathematics.
This was true for both gifted and non-gifted students; thus, the gender difference was higher in the gifted population (Preckel et al., 2008).

3.1.5 Motivation in gifted students

As described in the last section, gifted students might experience school as un-motivating if we do not meet their individual academic needs (Phillips & Lindsay, 2006). Therefore, loss of motivation is often connected to underachievement. Several studies indicate that gifted students score higher in intrinsic motivation than others (Gottfried, Cook, Gottfried, & Morris, 2005; Gottfried & Gottfried, 1996; Olsewski-Kubilius, Kulieke, & Krasney, 1988). It has also been suggested that motivation determines the difference between high-achieving and underachieving gifted students (McCoach & Siegle, 2003a; Reis & McCoach, 2000; Whitmore, 1981), along with negative attitudes toward teachers, learning disabilities, and low self-regulation (McCoach & Siegle, 2003a). Intrinsic motivation can be understood as a person’s drive and capacity toward assimilation, mastery, and spontaneous interest (Ryan & Deci, 2000). How a person values an activity can have direct consequences for the effort and time spent on that activity (Ryan & Deci, 2000). Clinkenbeard (2012) found that only 20% of several thousand articles published on motivation and gifted students were empirical, thus making it difficult to determine group differences regarding motivation in the gifted population.

Much of the argument for developing special programs or ability groups for gifted students is connected to boredom (Little, 2012). Boredom can be understood as an emotion that reflects an individual’s inability to value a topic or activity, a corresponding desire to avoid this topic or activity (Pekrun, Goetz, Daniels, Stupnisky, & Perry, 2010), and a lack of interest in and enjoyment of an activity (Little, 2012). Furthermore, boredom and motivation are also connected to how meaningful the task is experienced by the student (Little, 2012).
Yet, in general, the curriculum and learning phase in school are aimed at the general student population.

3.2 Summary

In the same way that giftedness generally embodies several conceptual issues, I have shown in this and the previous chapter that this also applies to mathematical giftedness. In the following chapter, I will present the most central research on acceleration and ability groups for gifted adolescents. Even though these are not specifically linked to mathematics, it is important to address these issues in general. I will also discuss whether or not gifted students might suffer any social, academic, or emotional harm due to their participation in such programs. This is especially relevant, since Norway is at the starting point of providing such opportunities. I shall also present research on acceleration and ability groups for gifted adolescents, as the two are important in terms of how we secure learning outcomes for this group internationally as well as in Norway.
4 Acceleration and Ability Groups for Gifted Students

In this chapter, I will discuss the current research on acceleration as a way of meeting the needs of gifted students. I will also provide an overview of acceleration opportunities (4.1) and ability groups (4.2), in addition to social issues related to these opportunities (4.3).

Although, internationally, several reports have indicated that acceleration is a simple and effective way of meeting the needs of gifted adolescents in school, especially those who already achieve at a high level (Colangelo & Assouline, 2009), there is a lack of research on acceleration and ability groups in Norway

4.1 Academic benefits of acceleration

In 2004, The Templeton National Report on Acceleration was published (Colangelo, Assouline, & Gross, 2004). It focuses on published seminal research on acceleration titled “A Nation Deceived: How Schools Hold Back America’s Brightest Students.” The publication presents extensive work on the unravelling of the most central myths connected to acceleration for gifted students, especially regarding social harm. Acceleration is defined simply as moving students through the traditional curriculum at a faster rate than is typical for their age group (Colangelo & Assouline, 2009). A common and long-held misunderstanding of academic acceleration is that students are, at some level, socially and/or emotionally disadvantaged by participating in a group with older students at school. The fear of social loss has had a great impact on whether educators provide acceleration opportunities, especially in

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10 The types of acceleration presented by Colangelo and Assouline (2009) are grade-based forms and subject-based forms. Grade-based forms are: (1) early admission to kindergarten; (2) early admission to first grade; (3) grade-skipping; (4) early entrance into middle school, high school, or college; and (5) early graduation. Subject-based forms are: (1) continuous progress; (2) self-paced instruction; (3) subject-matter acceleration/partial acceleration; (4) combined classes; (6) telescoping curriculum; (7) mentoring; (8) extracurricular programs; (9) correspondence courses; (10) concurrent/dual enrollment; (11) advanced placement; (12) credit by examination; and (13) acceleration in college (p. 1088).
early grades. There is a major difference between social loss and no social gain from acceleration. In particular, whole-grade acceleration has been the focus of these assumptions (Colangelo & Assouline, 2009).

Both arguments are based on a misconception about the positive effects of acceleration for gifted students. The academic effects of acceleration for gifted students have been documented in several studies (Colangelo et al., 2004; Colangelo & Assouline, 2009). According to Colangelo and Assouline (2009), there are more than 18 standard types of acceleration for gifted students, the most visible being whole-grade skipping. Southern and Jones (2004) identify five dimensions of acceleration that are useful in distinguishing among the 18 types. These dimensions involve pacing, salience, peers, access, and timing. Salience refers to the degree to which the accelerative options are noticeable by others (for example, grade skipping). Colangelo and Assouline (2009) note that the remaining types of acceleration are self-explanatory. In comparisons between academic outcomes for accelerated gifted learners and their non-accelerated counterparts, the empirical evidence suggests that acceleration is one of the best ways of meeting the needs of gifted learners (Kulik, 2004; Kulik & Kulik, 1982). The results of Kulik’s (2004) meta-study on academic acceleration indicate that acceleration as an intervention for gifted learners is highly effective and contributes substantially to the academic achievement of bright students. Among gifted students, boredom represents a great risk that may lead to the development of adjustment difficulties or underachievement (Gallagher, Harradline, & Coleman, 1997; Kanevsky & Keighley, 2003; Phillips & Lindsay, 2006; Reis & McCoach, 2000; Swiatek & Benbow, 1991). Boredom can often be the result of having to wait to learn something or of being far ahead of one’s peers in the learning process. It can occur in one special subject or across several subjects. Therefore, curriculum compacting, or other forms of acceleration serve as
important ways of meeting some of the needs of gifted students in school (Renzulli, Smith, & Reis, 1982).

4.2 Academic benefits of ability groups for gifted students

In the same way that opposition to acceleration has been a central theme in gifted education, the promotion of homogeneous ability groups has been met with strong opposition (Feldhusen & Moon, 1992; Slavin, 1990). Radical acceleration appears to be an inappropriate way of meeting gifted learners’ needs in schools based on inclusion. Thus, full-time ability groups seemingly represent the same issue, especially when educators gather a rather homogeneous group with the goal of providing tutoring catered to students’ individual academic abilities. Ability groups can be understood as special classrooms for gifted students, used in combination with ability tracking, and with changes to the curriculum aimed at increasing the quality of education (Preckel, Götz, & Frenzel, 2010). However, ability groups are designed to meet the needs of high-achieving students as well as the needs of low-achieving students or students achieving at the middle range in school (Steenbergen-Hu, Makel, & Olszewski-Kubilius, 2016). Several studies suggest that for gifted students, there is great academic benefit in participating in such groups (Goldring, 1990; Hattie, 2002; Kulik & Kulik, 1982; Rogers, 2007; Shields, 2002). Still, there is substantial misconception regarding what ability groups are in comparison with tracking, streaming, setting, sorting, or classroom organization (Steenbergen-Hu et al., 2016).

Ability groups can be defined on the basis of three components: (1) Students are placed into different classrooms or small groups based on their initial skill levels, readiness, or abilities; (2) the main purpose of such placements is to create a more homogeneous learning environment so that teachers can provide instruction that is better matched to students’ needs, and students can benefit from interactions with their academically
comparable peers; and (3) the placements are not permanent school administrative arrangements that lead to restrictions on students’ graduations, destinations, or career paths (Steenbergen-Hu et al., 2016).

Furthermore, there is a narrower categorization into four ability groups: (1) The between-class ability grouping involves assigning students of the same grade into high, average, or low classes based on their achievement; (2) the within-class ability grouping involves teachers assigning students within a class to several small homogeneous groups for special instruction based on their achievements; (3) the cross-grade ability grouping involves grouping different grade levels together to learn a particular subject based on their achievement or potential for achievement; and (4) the special grouping for the gifted refers to a special grouping designed specifically to meet the needs of gifted and talented learners (Steenbergen-Hu et al., 2016). A recent meta-study suggested that the overall effect of special ability groups on academic achievement was positive, moderate, and statistically significant and that the groupings were homogeneous (Steenbergen-Hu et al., 2016). Thus, it can be concluded that ability groups for gifted students might be beneficial. These findings are in line with Kulik and Kulik’s (1982) meta-analysis, which reported an overall small effect for ability groups and a beneficial effect on the performance of gifted and talented students.

4.3 Social issues related to acceleration and ability groups for gifted adolescents

Although the academic benefits of acceleration and ability groups can be measured through randomized controlled trials and meta-analysis, few studies on acceleration and ability groups have engaged with the social and emotional aspects of acceleration and the effect that acceleration might have on academic self-concept in school (Colangelo & Assouline, 2009). Thus, the investigation of social dimensions is complicated in many ways. Kulik (2004) argues that social satisfaction can be investigated through other variables. Kulik’s meta-study
concluded that although the academic effects of acceleration are positive, the social benefits are less clear. In comparing boredom between gifted students who participated in ability groups with students who participated in a regular classroom, Preckel et al. (2010) found that ability groups had no effect on whether the students felt bored. One argument against ability groups is the big-fish-little-pond effect (BFLPE) (Marsh, 1987), which explains why students of equal ability may have lower academic self-concept in classes where the average ability or achievement is high—and vice versa if they have high ability in a low-achieving environment (Marsh, 2005; Marsh et al., 2008; Marsh & Parker, 1984). This effect seems to also hold true when comparing gifted students participating in special classes with those participating in regular classes (Craven, Marsh, & Print, 2000; Preckel & Brüll, 2010; Preckel et al., 2010; Shields, 2002).

To understand the BFLPE, Marsh and Parker (1984) and Marsh et al. (2015) suggest that the concept of one’s self cannot be adequately understood without a frame of reference and that the same objective characteristics and accomplishments can lead to different self-concepts depending on the frame of reference or standards of comparison that individuals use to evaluate themselves (Marsh et al., 2007). Although the BFLPE has been observed as a stable phenomenon, the effect seems to be higher when students transition from participating in an average group to participating in a high-achieving and more homogeneous group (Preckel & Brüll, 2008, 2010). The same tendency was found in a longitudinal study in which the largest effects were found in the first year of participating in a more demanding class with higher average ability (Huguet, Dumas, Monteil, & Genestoux, 2001). Although the negative peer-competition and lower academic self-concept associated with ability groups can serve as an argument against providing such opportunities in school, research suggests that other components of such programs have a positive effect on both the academic and social needs of gifted individuals. These components include a different curriculum, enrichment experiences,
and meeting with highly trained and competent teachers (Dai & Rinn, 2008; Hattie, 2002; Preckel et al., 2010). Moreover, gifted pupils are known to thrive when grouped with other gifted students with whom they are more inclined to form deeper friendships, which might enhance feelings of pride and togetherness. To establish a coherent picture of the social outcome of ability groups, factors that might be specifically related to gifted learners need to be fully understood and controlled for in research.

Students may experience a boost in their academic self-concept when accepted into special programs and a decrease in their academic self-concept over time as they compare themselves with other participants. This effect is also referred to as the “basking-in-reflected-glory effect” (Marsh et al., 2008; Trautwein, Lüdtke, Marsh, Köller, & Baumert, 2006). There might also be a distinction between full-time and part-time ability groups or students participating in an enrichment program that can also be defined as an ability group. For example, Preckel et al. (2010) found a significant decrease in academic self-concept in their examination of full-time ability groups in mathematics. Although the short-term social effects of acceleration or ability groups might affect the academic self-concept of gifted students, the long-term effects of the same option might paint a clearer picture (Colangelo & Assouline, 2009).

In understanding the long-term effects of acceleration or ability groups on gifted students’ social lives, few works are as extensive as the SMPY, which investigated the long-term effects of acceleration and/or ability matching over ten years. Through the SMPY, there have been extensive longitudinal publications addressing both the academic and social dimensions of acceleration (Lubinski, 2004; Lubinski & Benbow, 1994; Richardson & Benbow, 1990; Swiatek & Benbow, 1991). At age 18, few (6.4%) students had reported detrimental effects from acceleration on their social and emotional development, and the effects seemed to decrease with age (Richardson & Benbow, 1990). During adulthood, the
former students were asked to recall their acceleration experiences, which they described as positive (Lubinski, 2004). Furthermore, these studies offer important insights into the long-term effects of acceleration interventions on the social and emotional lives of gifted students and, thus, present a compelling argument for accelerating gifted students. In comparison, those students who were not accelerated perceived their school experiences as less positive than those of their accelerated counterparts (Colangelo & Assouline, 2009; Lubinski, 2004).

Some researchers critique policymakers for being guided by single studies or by emotional and heated debates that might be interpreted as indicating that acceleration or ability groups might have negative social consequences (Rogers, 2002). However, there is a major difference between something having a profoundly negative effect, no effect, or a positive effect on the social lives of gifted students. The study by Lubinski (2004) was a follow-up to Rogers’ (1992) extensive review and investigated over 80 studies on the social or emotional impacts of acceleration. According to the measure of Slavin’s best-evidence synthesis, Rogers found that acceleration had a positive effect on both social (effect size = 0.46) and emotional (effect size = 0.12) aspects of gifted students’ lives (Neihart, 2007; Rogers, 1992). Furthermore, he found a significant positive emotional effect for subject-based acceleration. Nevertheless, high-achieving gifted students are known to report high levels of academic self-concept (McCoach & Siegle, 2003a, 2003b; Plucker & Stocking, 2001; Pyryt & Mendaglio, 1994; Reis & McCleach, 2000). The possible decrease in academic self-concept by the start of an acceleration program might be due to immediate social and academic comparisons at the start of the program.

Research addressing the social and emotional advantages for ability groups is somewhat more obscure. Thus, while acceleration is typically aimed at gifted or high-achieving students, ability groups is usually aimed at students who exhibit several levels of performance in school. For example, many countries have a long tradition of special classes
for students with learning disabilities or other problems that might hinder their learning in school. According to Neihart (2007), no conclusions can be made about the social effectiveness of ability grouping. “The overall conclusion of ability grouping has differential effects for gifted students. Peer ability grouping seems to have positive socioaffective effects for some gifted students, a neutral effect for others, and a detrimental effect on a few” (p. 334). The difference in academic self-concept between acceleration and ability groups might also be explained through the fact that a student can be accelerated in a specific subject without a change of environment or peers. For example, a competent teacher may be able to provide sufficient challenges in a regular classroom. Kulik and Kulik (1992) and Rogers (1992) found a generally strong, positive effect in support of all forms of ability groupings for gifted students, and they attribute the lack of ability grouping possibilities for gifted students to myths about this type of grouping for such students (Banfield, 2005).

4.4 Discussion

There are major differences in the way giftedness is defined across cultures, including in terms of the qualitative and quantitative levels, as seen through the models presented for both mathematically gifted adolescents and giftedness in general. Moreover, no specific criteria have been established to determine what giftedness is. Some might argue that one of the main issues in gifted education is that we have so many distinctive definitions and, thus, a range of identification methods that lead to a range of approaches in practice. There are several seemingly contradictory statements about giftedness. For example, we emphasize the importance as well as the danger of possible underachievement within the group. Thus, some models view performance in school as one of the main variables predicting giftedness. Furthermore, the best way of predicting underachievement is through intelligence testing because a significant difference in ability and performance might be an indication of
underachievement in school. Whether the different approaches suggest that the concept of giftedness is vague, or that we are yet to identify the essence of giftedness, remains unanswered.

The studies addressing gender differences in mathematical ability should be interpreted with caution because there could be differences between equality and opportunities between countries. Even Western countries have differences in culture and practices of equality, which can affect outcomes. Further, motivation, creativity, and above-average ability all seemed to play an important role in understanding giftedness and mathematical giftedness. In Norway, we tend to identify giftedness through performance. This has arguably led us to develop a limited understanding of giftedness as synonymous with performance, which prevents us from engaging adolescents who might have a high IQ but do not perform at a high level in school and, thus, need extra attention. Educational efforts in the form of acceleration and ability groups are generally designed to meet the academic needs of mathematically gifted adolescents. While ability groups for gifted adolescents might, in some cases, be beneficial, they can also be of no academic or social benefit to these students. Nevertheless, the range of definitions in the gifted education literature also presents a problem in comparisons involving this type of research. Researchers have followed different definitions or criteria regarding how they view various groups and have generally yielded divergent results. Some studies have focused exclusively on underachieving students or have tried to control for them. Environmental and cognitive factors also affect how adolescents master school. Among highly gifted adolescents, it seems that individual factors and stress play an important role in how these students use their “gifts.” In the next chapter, some methodological considerations regarding the articles as well as validity issues in qualitative and quantitative research will be discussed.
5 Methodological Considerations

This thesis seeks to understand giftedness through at least two definitions (Article 2 through academic achievement and Article 3 through intelligence). The thesis is of an exploratory nature, and I seek to understand and explore the field through the available contextual data. Because the three articles herein employed different methodological approaches, the methodological considerations will be discussed separately after the next section, which engages in an overall methodological discussion. Before I present the methodological approach, it is necessary to repeat the overarching aim and the three research questions guiding this thesis: *How do mathematically gifted adolescents perceive and reflect on mathematics tutoring during their school years in Norway?*

By employing three different approaches to giftedness, I provide insights on some of the hypothetical misconceptions at the qualitative and quantitative levels of giftedness and, at the same time, discuss some overarching and fundamental views of giftedness through philosophy. Because I have collected data through different sources and methods, a collective methodological approach (mixed) is suited to answering the overarching aim of the thesis.

5.1 Mixed methods

The combination of the different methods and philosophical or conceptual approaches can be described as eclectic. As I used different methods to explore different aspects of giftedness, this thesis is eclectic in its methodological standpoint. Mixed methods can be defined using both a wide and narrow range (Yin, 2006). For example, Johnson, Onwuegbuzie, and Turner (2007) identified 19 definitions used by practitioners of mixed methods. These ranged from broad to more specific and from methods to designs. The narrow approach to mixed methods emphasizes the *mixing or integration* of qualitative and quantitative data and analyzes
techniques *within a specific study* (Johnson & Onwuegbuize, 2004; Yin, 2006). Conversely, broader definitions involve the *integration of results* from several studies (Creswell & Plano Clark, 2011), combining qualitative and quantitative research approaches for *breadth* and *depth* in understanding (Johnson et al., 2007). The three most central arguments in this thesis are that, first, *one data source might be insufficient*. Second, there is *a need to explain the initial results*. Finally, *there is a need to understand a research objective through multiple research phases*.

The overall design of the thesis can be described as convergent (Creswell & Plano Clark, 2011). Convergence is a central topic in mixed methods, the purpose of which is to ascertain how the data used in studies relate to each other. In a convergent design, there is no hierarchy between studies in which one method is the main source of information (*qualitative/quantitative or quantitative/qualitative*). As illustrated in the introduction (see the model on p. 19), the relationship between the studies is linear. In other words, they influence each other, and no single study embodies the *main* method of the thesis; they are complementary and draws ideas from each other. To exemplify, the quantitative data used in the thesis were the first to be gathered. Part of the reason for conducting the qualitative study was to further understand some of the quantitative data and, thus, to draw conclusions from both sets of studies and the philosophical article discuss ideas from both studies. This is one way of meeting an insufficient data source from one study and exploring the overall research objective of the thesis, which cannot be described through one method. Further, the need to explain the definitions (first article) relates to both methods in terms of understanding how the different definitions might have validity in identifying gifted adolescents. This is an important discussion that relates to construct validity in all three articles.

Even though the design of the thesis fulfills the broad definition of mixed methods, it is not without methodological issues. For example, Yin (2006) sees this type of combination
(or integration) of results as merely complementary and not as an instance of mixed methods. As I discuss later in the limitations, there is no guarantee that I have researched the same population of students. In relation to this problem, a philosophical discussion is of tremendous importance in explaining whether both identification criteria identify students from the same population or if, they are in fact two different populations.

Further, Creswell and Plano Clarck (2011) argue that the overall research question guides whether researchers should mix methods in their overall discussion and understanding of the phenomenon at hand. The overall research question in this thesis is best answered through several methods. The integration of methods can help in building stronger conclusions because one method can outline the weaknesses of the other (Bazeley & Kemp, 2012; Johnson & Onwuegbuzie, 2004), or the different designs can, in some cases, explain differences within the same research topic. None of the articles presented in this thesis combines different approaches within a single article; the integration of methods is applied in the overall thesis, and the combination and discussion of the different results and methods are applied to understand the same phenomenon from different angles. I discuss the field of gifted education in Norway through different methodological lenses. The conceptual issue of defining giftedness cannot be adequately addressed through qualitative or quantitative means because the issue is illustrated through cultural or social differences. As mentioned earlier, Article 3 represents what Creswell and Plano Clarck (2011) describe as an insufficient data source. For example, intelligence and results in mathematics are known to correlate (Rohde, & Thompson, 2007; Spinath, Spinath, Harlaar, & Plomin, 2006). Therefore, the results connected to research question five (What are the associations between IQ and grades in mathematics?) might be artificial. The different theoretical models of giftedness emphasize that intelligence is limited in its descriptions of gifted adolescents. To understand whether these results are artificial or whether there are other quantitative descriptions that might be
explained in an in-depth manner by the qualitative study, the integration of the results is important to answer the overall research objective. The next section will present a discussion on validity and reliability in the individual articles.

5.2 Validity and reliability in Article 1

The research enterprise consists of many tests of validity (Shadish, Cook, & Campbell, 2002), all of which vary in their design, method, and strategy for communicating research. This article acts as an overall discussion of the theoretical framing of giftedness and how different definitions can serve different purposes in research and practice. The overall validity of the constructs in the field of giftedness and the validity of the constructs represented in the definitions of giftedness are of great importance for a general discussion in the field, not only for the development of the field, but also for the credibility of the different results in the other articles. The traditional and reductionist approach to giftedness deems giftedness an inherent, interchangeable, and sometimes a genetic property (Dai, 2005). As an alternative to reductionism, emergentism posits that the properties of a system are emergent if they represent a new outcome of some other property of the system and/or the interaction between properties (Baker, 2013). Emergentism strongly argues for the functional autonomy of subjective contents that have been engendered through the interaction of the inner environment (Dai, 2005). This tradition explains the multidimensional models of giftedness. An emergentistic approach to giftedness allows much more uncertainty and ambiguity in explaining the concept, while the reductionist’s goal is to explain giftedness in one or a few terms. The following subsection presents some conceptual issues of giftedness.
The criterion problem in gifted education

Renzulli and Delcourt (1986) argue that the common problem in the concept of giftedness is the criterion problem. The criterion problem stems from the lack of social agreement regarding an external criterion that can be used as a benchmark against which comparisons can then be made. No empirical evidence can establish the validity of such criteria, and there is no objectivity that a researcher can apply to explain the phenomenon. Because there is no objective verifiable means to establish the validity of values, scientists cannot answer definite questions that will lead to the establishment of a universally accepted criterion. In vague and ambiguous concepts such as giftedness, these problems might be quite clear. However, these problems can also be found in rater-established concepts, such as intelligence (Renzulli & Delcourt, 1986). Whether a single criterion for giftedness exists remains uncertain, raising the question about whether giftedness exists as a concept or it has been created as a concept. As discussed, I cannot adequately argue for the existence of giftedness outside cultural, social, and/or educational dimensions. Therefore, the empirical search for a single criterion of giftedness might not be meaningful in establishing a definition. Rather, the goal of the researcher, teacher, or culture should be reflected in the definition.

Construct validity of intelligence for giftedness

According to Renzulli and Delcourt (1986), the test-score-as-criterion addresses several conceptual issues in studying giftedness. The problem refers to using a single criterion for portraying or predicting a concept with multiple meanings and/or criteria. General intelligence is a highly useful psychological construct (Warne, 2016), often measured through psychometric tests such as the WISC. The construct has been shown to be durable and compatible with many different theories of intelligence (Deary et al., 1996). Further, general
intelligence seems to correlate with academic achievement (Deary et al., 2007; Kuncel et al., 2004; Watkins et al., 2007). However, most researchers would agree that giftedness is constructed through something more than intelligence. This also relates to test validity because many studies use intelligence as one or the single measure for identifying gifted students. Here, two problems should be addressed. First, there are several theories of intelligence (see Chapter 1), which means that the measurement of giftedness can vary according to the theory employed. Second, the different tests have different loadings of what we often refer to as general intelligence. Silverman (2009) argues that the tests best suited for identifying giftedness are those that have a high loading of \( g \). For example, the WISC test has a lower loading of \( g \) than Raven’s Progressive Matrices (Silverman, 2009). These differences in tests apply to the identification of gifted individuals, and as I argue in Article 1, we indirectly define giftedness through the identification process.

5.3 Validity, reliability, ethics, and generalization in Article 2

Qualitative research is suited to exploring underlying meanings and individual differences and to communicating rich descriptions of the participants’ experiences, allowing the researcher to generate hypotheses for testing (Golafshani, 2003; Kvale & Brinkmann, 2009).

As the researcher’s inference could cause validation biases in quantitative research, the researcher is a natural part of qualitative research. One might argue that reliability is a concept used for testing or evaluating quantitative research and, thus, is irrelevant for all qualitative studies (Stenbacka, 2001; Golafshani, 2003). The term reliability is important in qualitative research, although it must be redefined to fit the purpose of such studies (Golafshani, 2003). Therefore, reliability in qualitative research is related to the quality, credibility, neutrality, consistency, and transferability of the study (Golafshani, 2003; Lincoln & Guba, 1985). The quality of a study relates to whether the data collection process is credible and whether the
subjects you talk to reflect the subjects that a researcher wishes to better understand or explore. Credibility can be understood on several levels in qualitative research, the foremost of which is whether the research is conducted in a trustworthy manner and is in line with ethical guidelines. Furthermore, credibility can be understood by the conclusions that the researcher reaches from the given materials. For example, does the article communicate the qualitative research in a quantitative way? Does the method allow the researcher to make cross-case generalizations?

In the current study, only cross-case generalizations could be made because of the limited number of informants. Moreover, the transferability of the study at hand should be questioned. Semi-structured open-ended interviews serve an important purpose in allowing informants to talk freely and to describe their own life situation without interruptions from the researcher. However, they are also reliant on the researcher’s ability to ask follow-up questions. Furthermore, the reproducibility of a study becomes impossible because other researchers cannot replicate the interview or the questions asked. The theoretical background of the researcher also affects his or her ability to take advantage of and ask questions that elaborate important and meaningful statements that might be revealed during an interview. In this study, the informants were undoubtedly mathematically gifted and perceived mathematics at a high level; however, the researcher (me) has a background in pedagogical psychology and, thus, automatically tended to focus on those aspects of the groups. Another researcher (such as a mathematician) might have been able to provide good follow-up questions to explore the participants’ mathematical abilities and understanding. Thus, the researcher’s own theoretical perspective always affects the interview and analysis process (Kvale & Brinkmann, 2009).
Ethical considerations

As the categories in the interview guide were broad and relied on a conceptual framework that understands giftedness using only a few criteria, open-ended interviews seemed to be the best approach. Open-ended interviews can present special ethical considerations because the researcher has little control over what informants focus on. Therefore, even though there are few sensitive categories in the interview guide, sensitive information can be described by the informants. I needed to contact the Norwegian Centre for Research Data (NSD) to ensure that I would follow the official guidelines to collecting data when conducting interviews. The NSD provided guidelines that concluded that the project did not concern any special provisions. The project was approved by the NSD, and all the data were anonymized by February, as requested by the NSD.

In terms of research ethics, there is also a concern with how one communicates his or her research. As the only person besides the informants who knew what was said in the interviews, it can be tempting to provide results that misrepresent the statements made in the interviews or to overrate a single statement. Several measures were taken to meet formal ethical guidelines in this study. First, all the participants were told of their right to withdraw their participation from the entire process, even after the analysis process had begun. Second, all the informants participated voluntarily, and their participation was anonymous to both the head teacher in their ability group at the university and their head teacher at their regular school. Third, whenever difficult subjects were addressed in the interviews, the participants were given the opportunity to ask questions or have discussions with the interviewer following the interview.
Generalizability of qualitative research

In the qualitative research context, neither the participants nor the questions asked can be reproduced. The questions asked by the interviewer, the background of the interviewer, as well as the context all influence the analytical process and can infer the conclusions drawn. The assumption about the need to generalize research is that it should provide knowledge across cultures and time (Kvale & Brinkmann, 2009). In qualitative research, the goal of the descriptive provision is often the opposite. The goal is to describe the context and not to generalize from “it.” In social science, the goal of generalization is not to make rules and laws or to generalize across cultures; rather, it is to discuss whether the findings of the specific interview or case study can provide knowledge that can enhance the general understanding of a topic or provide information that can be adopted into practice for a given student group or a group with special needs. In qualitative research, generalization concerns whether the study is theoretically generalizable. In other words, can the descriptions in the interviews be explained using existing theory, or does a new theory have to be developed to explain the phenomenon? In this specific study, it seemed that some of the individual experiences could be generalized among the informants. Furthermore, because the school system in Norway generally provides the same opportunities for every student, it is possible that the experiences connected to the school system might also have some validity in other contexts.

5.4 Validity and reliability in Article 3

Regarding Article 3, some validity questions should be discussed when reading the research. There are several threats against validity in research (Shadish et al., 2002). In the quantitative survey, three categories deserve special attention: construct validity, internal validity, and the generalizability of the results.
Validity issues and bias in quantitative research

Drawing on a large dataset from the Norwegian armed forces, we controlled for both mathematical ability and intelligence in the study; however, we had no control over the constructs measured. In terms of construct validity, we referred to the way in which the concepts of operationalization and measurements represented the variables that we sought to measure (Cook & Campbell, 1979; Lund, 2002). Operationalized variables raise serious questions of weakness when they are measured and defined by how they are measured. In this particular study, intelligence was estimated through the measurements available in the military tests, which means that the definitions of the construct were created and estimated by the scores on those tests. The scores relied on a specific understanding of intelligence (general mental ability [GMA]), and thus, alternative results might have been encountered using other tests or by implementing several measures of intelligence. The results regarding the individuals in the study were a function of what we (the test) wished to measure and might be a systematic measurement error (Kleven, 2002a). While random measurement errors might level out in the long run, systematic measurement errors can create a skewed picture and measure irrelevant factors connected to the construct we seek to measure (Kleven, 2002a). Thus, the tests used by the military represent valid measures for IQ and are based on a standardized test with long traditions in both the military and child psychology services.

Cook and Beckman (2006) argue that the results of psychometric assessments have meaning or validity only in the context they purport to assess. Therefore, the validity of the instrument scores hinges on the construct, and thus, a clear definition of the intended construct is the first step in any validation evaluation (Cook & Beckman, 2006). One issue in this study is construct under-representation, especially the constructs extracted to measure teacher relationships through the questionnaire from the Norwegian military service. The issue with these constructs can be due to the low number of questions in the survey, poor
inter-relatedness between the items, or heterogeneous constructs (Tavakol & Dennick, 2011). In this study, a two-item scale was used to measure the specific construct. Moreover, a two-item scale should be applied instead of a single-item scale whenever possible to decrease standard measurement errors (Gliem & Gliem, 2003). The low Cronbach alpha value(s) related to the above-mentioned issue suggest that the results of the study should be interpreted with caution.

Furthermore, in this article, the construct of “highly intelligent adolescents” or “gifted adolescents” represents another issue of construct validity. As the reader might have noticed throughout this text, there are several models in which we try to explain “what” giftedness is and that no unified definition follows. By setting a limited definition of giftedness in this article (e.g., intelligence and mathematical performance), we rule out some adolescents who might have been included as gifted in other studies. We try to account for some of the issues by treating intelligence as a continuous variable, not as a dichotomous variable, so as to rule out the unnatural “effects” of intelligence. Thus, giftedness is often defined as an attribute of those scoring at the top 2–5 percent, measured by an intelligence test (Winner, 2000). Moreover, the study might control for correlations that occur naturally between mathematical knowledge and intelligence because mathematical knowledge is known to predict high scores on at least parts of the sub-tests in a given test battery (Diezmann & Watters, 2000; Kvist & Gustafsson, 2007). Thus, by creating a correlation that can be predicted by the nature of intelligence tests and that has high loadings of g, artificial correlations in research are often related to internal validity.

There are several tests of internal validity in the literature (Lund, 2002). For example, structural equation modeling (SEM) estimates and controls for measurement errors. The most common issue in research is that the researcher selects the variables that he or she wishes to control for (Kleven, 2002a). This type of bias might occur when researchers choose not to
control for variables that might affect the results of the research. In the discussion of internal validity in quantitative research, the *direction of possible causal relationships* is a test of the conclusions that can be reached from the results. Regarding the statistical conclusions, there might be other underlying variables interfering with the results; however, if these variables are not accounted for, or if we have no information about them, we cannot determine whether that is the case.

The goal of quantitative research is often to generalize and, in some cases, to describe the effects of an intervention (Lund, 2002). In relation to generalization, external validity is important in determining the strength and validity of the conclusions made by the researcher as well as which conclusions are supported by the material at hand. External validity concerns whether the research results can be generalized to other settings and populations (Cook & Campbell, 1979). A single study can seldom provide enough external validity on its own; thus, reproducing a study result or re-testing a hypothesis can strengthen the external validity of a study. Additionally, a known theory might support the generalizability of a given result. Thus, it is important to strive for a representative selection of participants in a research project.

This study (Article 3) had high statistical power in its statistical assumption. However, *pre-selection bias* might have significantly influenced the results. The military service in Norway pre-selects recruits based on physical and motivational characteristics. Moreover, physical disadvantages have little effect on intelligence and/or school grades. Therefore, there might have been a selection bias that we were unable to control for in the study. Furthermore, we did not engage directly with the subjects and had no information on their *motivation, social and economic background, or geographical location*, which might have also skewed

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11 If you have mental or physical issues that will render you unfit for service, you might not be invited to the induction procedure, and thus, you would not be included in the given data materials.
the statistical conclusions drawn in the article. A positive aspect of the study is that we were able to sample both males and females because induction was made mandatory for both genders in 2009, making it possible to test models that engage in gender comparisons. Because there are no comparable studies in Norway, the conclusions drawn from the study should be interpreted cautiously. However, the correlation between intelligence and academic performance has significant empirical support in several nations. We could have chosen to treat intelligence as a latent dichotomous variable, instead of a continuous variable, using outsourcing, i.e., those who scored in the top 2–5 percent, which would have produced artificial highly intelligent vs. normal groups (see discussion in Article 1: continuous vs. latent approach\textsuperscript{12} to intelligence and the arbitrary “cutoffs” in some definitions). The latter would probably have created artificial group differences, which would have represented a larger selection bias than what existed before.

5.5 Summary

As we saw in the discussion at the beginning of this chapter, giftedness can be explored on several levels. Giftedness and mathematical giftedness in Norway are little explored, and no quantitative surveys exist, with the sole exception of Article 3 in this doctoral thesis. Even though the three articles explore different sides of giftedness in Norway, collectively, they represent a methodological approach that enables us to better understand the overarching aim of this thesis. As the quantitative and qualitative articles define giftedness using two fundamentally different views (potential and performance), there is no guarantee that they are

\textsuperscript{12} The dichotomous variable is referred to as a variable with only two “categories” or “levels”: you are either in the A or B group. A continuous variable has an infinite number of possible values, and thus, we could determine whether intelligence moderates’ differences in the groups (e.g., gender and mathematics).
exploring the same group. However, the correlation between math achievement and IQ in Article 3 validates the definition of giftedness through the lens of performance, as argued in Article 2. By applying mixed methods, we were able to understand the research in a broader context than would have been possible through a single study. Furthermore, this enabled us to come closer in our understanding of giftedness, especially mathematical giftedness in the Norwegian context. For example, mixed methods helped us understand the motivational questions raised by the quantitative survey. The strong motivation between school grades and IQ was limited in explaining whether students felt motivated. However, the results from the qualitative survey indicated that this varies throughout school and depends on the teacher’s ability to engage and teach the subject. Moreover, the discussion of definitions in Article 1 serves as an argument for applying two different identification methods in the two surveys, as giftedness should be understood contextually, and the definition should be based on the goal of the survey. The different results, especially those connected to motivation (see Chapter 6), underlined that qualitative and quantitative surveys occasionally yield different conclusions. When we integrated the results from the qualitative study, we also speculated that gender roles would still exist in Norway: While the boys were overrepresented in the acceleration program, they were underrepresented in performance and IQ in the quantitative survey. The results suggest that girls chose different educational options or experienced such programs as competitive. Thus, the results are in line with the theory presented in Chapter 3. Nevertheless, the differences between the two studies suggest that there is a conceptual difference in giftedness between genders, which is not currently addressed in the different models.
6 Results

In this chapter, I start with a summary of each of the three articles incorporated into the thesis and discuss each article’s findings. These three articles are presented as follows: Article 1 (6.1); Article 2 (6.2); and Article 3 (6.3).

6.1 Article 1


Aim and objectives. In gifted education, educators tend to rely heavily on IQ scores, even though research has emphasized the limited nature of this view of giftedness (Gottfredson, 1997; Jensen, 1998; Kaufman & Sternberg, 2008). To appreciate how giftedness is understood across cultures, one cannot rely on a single definition or model because educational practices, educational ideologies, and cultures vary significantly across nations, and definitions in the field are geared towards these social or political practices. In my review, I encountered several definitions of giftedness, more specifically, mathematical giftedness, both directly and indirectly. The definitions and models of giftedness are vague because they draw on different data and explain giftedness through different terms and models. Furthermore, similar issues are detected in domain-specific giftedness (e.g., mathematics, language, and arts). Some researchers have tried to come up with a specific definition of giftedness, while others have sought to explain giftedness through a set of given terms or one specific term (IQ)—the so-called reductionist approach. The aim of this article was to explore why we encounter several definitions and understandings of giftedness across research and cultures so as not to reduce giftedness to a specific definition but, rather, to provide an
explanatory discussion about when and how different definitions can serve different purposes in research and practice. Moreover, the question of “what is” might not be valid in educational science because the answer might vary substantially. Thus, from a language philosophical point of view, this article serves as an overview of the entire thesis, including the two additional articles, and explores the different ways in which we can define giftedness, depending on the goal of the research and the data to which we have access. For example, if we define giftedness in Norway through high IQ scores (as in Article 3), several problems for the educational system may occur because IQ testing is not traditionally performed for children and adolescents in Scandinavian schools. The goal of this article was not to describe all the different definitions in the field but, rather, to explore the origins and benefits of the general competing conceptions within the field. At the end of the article, I provide a specific approach to concept formation that might be of use to researchers or practitioners within the field of gifted education.

**Methods.** Although no specific qualitative or quantitative analyses were applied in this article, it follows a specific theoretical definition, which is connected to Scheffler’s (1974) ideas about definitions in social science. Scheffler argues that when we try to define abstract entities in social science, they cannot be understood in a vacuum or without emphasizing the context. He suggests that the definitions we encounter can be organized through three different layers according to the degree to which they reflect prior usage, explain a phenomenon, or have direct impact on practice (e.g., recruitment to gifted programs, diagnoses, receiving special needs education, scholarships). This organization does not represent a hierarchy, and one cannot argue that one definition or view of giftedness is superior to another. Through Scheffler’s thoughts, I argue that we understand why there are several useful definitions and why teachers, researchers, and policymakers might have somewhat different understandings of “what giftedness is.” Gerring’s (2002) min-max
strategy of concept formation serves the purpose of defining concepts in the social sciences. In gifted education, definitions relying on IQ have received much scrutiny for being limited or unable to identify all types of giftedness. Furthermore, through Gerring (2002), we can explore the minimal properties needed for general definitions in social science, thus enabling us to discuss the conceptual range of different definitions of giftedness.

**Results.** In this article, I found that in contrast to expectations, IQ-based definitions seem broader in their conceptual range than definitions grounded in several criteria that are defined as gifted. In general, the concept of giftedness is vague and cannot be adequately defined through a single definition. Furthermore, researchers should choose definitions that are grounded in their data. Thus, researchers should be aware of their own definitions because how we communicate our definitions in research can impact how, for example, the reader understands the concept. In the same way, how we define giftedness in educational practice can have a direct impact on which children are recruited into the program we wish to employ. An ideal-type definition includes several attributes that together define the concept in its purest and most ideal form. While minimal definitions are minimal in their concept formation, i.e., minimal in the attributes that form the concept, they become maximal in their conceptual range. Maximal definitions (ideal-type) are maximal in their concept formation but minimal in their phenomena range. The strategy of concept formation serves to bind this particular concept in a semantic and referential space, providing the most satisfactory general definition of that concept.

**6.2 Article 2**

**Aim and objectives.** While Article 1 discussed why we encounter different types of definitions in gifted education and why different definitions can be useful in education and research, Article 3 explored the association between intelligence and different social aspects and academic achievement. The aim of Article 2 was to explore in further depth information about a smaller group of high-achieving mathematically gifted students to better understand both the results from the quantitative article and how these students experience the Norwegian school system. However, this aim hardly paid attention to identifying or providing optimal learning opportunities for the group. The definition in this article followed Scheffler’s (1974) description as “keyed into practice,” in that, the participants were recruited based on their previous performance. In a way, the students were defined though performance, which can be limited. However, the correlation between mathematical performance and intelligence is well documented (see Deary et al., 2007; Geary, Nicholas, Li, & Sun, 2017; Kuncel et al., 2004; McCoach et al., 2017; Neisser et al., 1996; Watkins et al., 2007). Moreover, this particular study is unique in the Norwegian context, as all the students had been accelerated through school, with some skipping classes, some skipping years, and a few skipping several years.

**Methods.** Exploratory qualitative semi-structured interviews were conducted to gather information about the students’ experiences. The interview guide was influenced by Renzulli’s (2002) *three-ring concept of giftedness* and Mönk’s (1992) *multifactor model of giftedness* (see Article 2 or the introduction of this thesis). These models are interrelated, as Mönks developed Renzulli’s model to account for environmental factors. This study explored both individual and environmental factors among the informants. Open interviews can be demanding to implement in practice because informants are encouraged to talk freely, and the interviewer has to ask follow-up questions that are not necessarily represented in the interview guide. Thus, their strength is that the focus of the interviews is driven by the informants, not the interviewer.
Participants. To gather informants for this study, purposive sampling was applied (Robinson, 2014). The study participants were recruited through their participation in a special program at the university (defined as an ability group). In this program, they receive mathematical tutoring and can take study points in mathematics at the university level. The participants voluntarily took part in the program during their free time and simultaneously participated in their regular school activities during the day. All the students had passed math $r_2$ in high school. Their ages ranged from 16–19 years, and the sample size was $N = 11$ (3 girls and 8 boys), which is considered small (Robinson, 2014). All the participants demonstrated high levels of mathematics mastery and showed well-above normal achievement compared with their age-group counterparts. Significantly fewer girls than boys participated in the program, which is interesting because in Article 3, girls were equally represented in the top group of mathematical performers.

Data collection and analysis. The interviews were collected over a short period of time following approval from the NSD. Because there was no qualitative information about this group in Norway, it was important to use exploratory open-ended interviews. In this way, the informants could focus on what they deemed important in their lives instead of being led to answers or guided by the researcher’s notions. Although the interview guide was influenced by the three-ring concept and the multifactorial model of giftedness, the participants spoke freely within these categories. Moreover, the questions in the interview guide served as options and were not followed strictly by the interviewer. NVivo was used to analyze the material, and the analysis process was data-driven, inductive, and thematic. Thus, the categories in the interview guide served as a natural starting point for the analysis process. Triangulation was applied between the theory, interviews, and context to improve the validity and reliability of the study.
Results. The main findings of the study were discussed within the following categories: motivation and interest in the subject, whether high-ability students were sufficiently challenged, and teacher and peer relationships.

All the students in the study had experience with acceleration through school and were positive in their descriptions of acceleration in mathematics. Thus, they hoped that acceleration could be applied to all subjects in school. They did not experience negative peer competition within their ability group at the university. Their motivation for mathematics was generally high, and they enjoyed working with mathematical problems at high levels. For some of the students, acceleration was crucial to maintaining their motivation and interest in mathematics in school. However, none of the students was accelerated in their earlier years of schooling, and they had felt bored in school during this time. In particular, they felt that elementary school and secondary school were boring or were not sufficiently challenging. An important finding in this study is that the Norwegian school system does not have sufficient guidelines for how acceleration should be implemented for highly able or gifted adolescents. These students often underwent acceleration in only part of their schooling, with no acceleration in later years. Furthermore, there were no follow-up strategies if the students finished school or a specific subject earlier than planned. Although they generally liked their teachers, many did not receive tutoring in mathematics through school, which is alarming because a large number of mathematically gifted students become underachievers or unmotivated much earlier than junior high school or high school. It was clear through the study that the importance of social relationships varied among the students. Some of them lived very social lives, while others did not feel the need to have many friends, preferring to have few close friends. However, few girls participated in the program in this study, and there seemed to be a difference between boys and girls in how they perceived stress and academic
self-concept. In particular, stress in test situations seemed to be experienced differently between the few girls and boys in this study.

6.3 Article 3


Aim and objectives. In this article, mathematically gifted adolescents are understood through the intelligence lens. The article aimed to understand the connection regarding intelligence, mathematical achievement, gender, social relationships, and teacher relationships between gifted adolescents and the normal population. As discussed earlier (Article 1), intelligence was treated as a continuous variable, not a dichotomous variable, to ascertain whether possible differences were moderated across levels of intelligence. Thus, drawing on a large sample, this article was exploratory in the sense that we had little information about this group in Norway, and larger quantitative surveys could help us determine whether intelligence had a major impact on academic achievement in Norway.

Research question/aim: The research questions for this article were as follows:

1. Are student-teacher relationships associated with grades in mathematics?
2. Are peer relationships associated with grades in mathematics?
3. Is gender associated with grades in mathematics?
4. Are these possible associations moderated by intelligence?
5. What are the associations between IQ and grades in mathematics?

Methods. In Norway, there is no tradition of intelligence testing for children and adolescents that does not consider school-related difficulties. Thus, over time, the military has
used intelligence testing and a questionnaire as part of its recruitment to service. Drawing on a rather large sample from the 2009 induction to military service questionnaire, we were able to extract intelligence scores and answers to questions about peer relationships, teacher relationships, gender, and grades.

**General mental ability (GMA)** was measured through a computer-based test battery, which consists of three different sub-tests measuring numerical ability, word similarity, and general reasoning ability. These scores were again recorded and transformed into a stanine score. The scores represented a coherent picture of the recruits’ mental capabilities (Skoglund, Martinussen, & Lang-Ree, 2014). These tests correlated better with the known WAIS-FSIQ measure ($r = .72$), with the latter often being used by the school psychology service in Norway.

**Mathematical achievement** was measured using the students’ grades at the end of the school semester in 10th grade. The school grades were self-reported; however, it is illegal to misrepresent grades, as this can have consequences for serving in the Norwegian armed forces.

**Social and peer relationships** were measured by combining items from the “level of social and life skills” questionnaire from the military induction center. The items extracted to measure peer relationships were *initiates contact with peers easily, enjoys the company of peers, and the frequency of social peer contact* (Cronbach’s alpha: .69). Teacher relationships were measured through two items: *cooperates well with the teacher and withstands criticism* (Cronbach’s alpha, .5) (see Article 3 and methodological considerations for a discussion on the low and medium-low alpha coefficients.)

**Conventional analyses** were conducted using SPSS. Moreover, we used SEM to test the conceptual model. The measurement models (confirmatory factor analysis [CFA]) were estimated and evaluated separately using Mplus (Muthén & Muthén, 1998–2014). The latent
variable approach allowed us to test the relationships among variables that were free of measurement error, reducing the bias of the coefficients (Jöreskog & Sörbom, 1988). Each independent variable was investigated separately to determine whether it interacted with IQ in its effect on mathematical achievement. Finally, we conducted a step-wise procedure to avoid unnecessary complexity, as numerical interrogation becomes increasingly more computationally demanding as the number of factors and the sample size increase.

**Results.** The results from this quantitative study not only validated the intelligence-based definition (explored in Article 1) but also confirmed the correlation between intelligence and academic achievement in mathematics, thus endorsing the performance-based definitions of mathematically gifted adolescents. All the independent variables had significant effects, except peer relationships. The effect of mathematics was low but significant, and the same was true for the effect of teacher relationships on mathematical achievement. The effect of peer relationships was the lowest, though not significant, and as expected, IQ had a very high effect. Altogether, the variables in the models explained 41% of the variance in mathematical achievement. While gender and social relationships both demonstrated significant interactions with IQ in their effect on mathematical achievement, this was not the case for teacher relationships. The effect of gender was still significant after the interaction term IQ was introduced. This means that gender interacted differently with IQ across levels. Social achievement had no significant effect on mathematical achievement. Thus, the significant effect of the interaction between social relationships and IQ indicates that higher scores in math are associated with lower ratings of peer relationships for adolescents with the highest IQ levels. Finally, teacher relationships were associated with mathematical achievement, and this effect was the same for all IQ levels, as we found no significant interaction effect. In general, this study followed a limited view of giftedness (defined by IQ). Nevertheless, the participants who scored at the higher end of the scale could also be defined...
as mathematically gifted (high IQ score and high mathematical knowledge). Thus, drawing on Scheffler’s (1974) research, this definition is broader in its conceptual range and can serve to explore categories that should be explored in greater depth through a more qualitative approach.

6.4 Summary

Article 1 discusses how we can use the different approaches to giftedness and still study the same group of gifted adolescents. It seems that the two definitions applied in articles 2 and 3 have some validation due to the strong statistical correlation between IQ and academic achievement. Collectively, the results from articles 2 and 3 fulfill the purpose of explaining one important factor, namely, how the results from qualitative and quantitative studies can show different results. Even more, they seem to validate both an intelligence-based view of giftedness as well as a performance-based understanding. Even though it seems that gifted adolescents are motivated by achieving high grades, the qualitative study (Article 2) seems to suggest that grades are not a good measure of motivation. The collective results and different methods across the three articles broaden our understanding of how mathematically gifted adolescents experience school in Norway. Moreover, the articles also explain why we achieve different results from qualitative and quantitative research methods in research on giftedness. At the macro level, however, mathematically gifted adolescents seem to be motivated (Article 3). Nevertheless, addressing micro-level behaviors explain that several factors lead to motivation and achievement and that IQ is a limited definition of giftedness. In the following chapter, I will address some overall discussion points for the thesis based on the results across the three articles.
Overall Discussion

In this chapter, I shall discuss the results from the articles in line with the overarching aim of this thesis:

**How do mathematically gifted adolescents perceive and reflect on mathematics tutoring during their school years in Norway?**

In my doctoral thesis, differences emerged between the qualitative and quantitative results. The results from the quantitative study indicated that, overall, the gifted adolescents performed well in school and reported to have had good relationships with their mathematics teacher (Article 3). The integration of the two empirical articles (articles 2 and 3) revealed some differences between how the gifted adolescents experienced the school system at a qualitative level and how they described their teacher through the survey. However, there was uncertainty regarding the representativeness of the informants in the two studies and the fact that I selected the participants through two different definitions of giftedness (IQ and school performance). In the articles, I recruited gifted students through two different approaches to giftedness, and there seemed to be validity in both ways of defining these students. The students participating in the ability group at the university level were gifted in mathematics and showed high performance in that specific field (Article 2). It might be that these students could be profoundly gifted. However, I could not validate this assumption (e.g., through an intelligence test). In mixed methods, one should draw “meta-inferences” (Creswell, & Clark, 2011), which involve conclusion across the qualitative and quantitative strands. Therefore, we could assume that I have studied the same group because there are similarities in their academic performance.
7.1 Originality of the research articles in this thesis

The research presented in the thesis is original in the sense that no such research has been conducted in Norway. I would argue that Article 1 represents an original view of the definitions used in gifted education. While I earlier discussed whether one definition represents the “essence” of giftedness more so than others, this article explains the variations in definitions using philosophy of language and definition theory. This thought process can, in principle, be applied to any topic in social science. The results presented in both empirical studies are original, given the context in which these studies were conducted. The qualitative study represents an interesting and new insight into how mathematically gifted adolescents experience the Norwegian school system. It is original in the sense that few of these topics have been explored in this specific context, and therefore, they can be used to identify these students’ pedagogical and psychological needs. The last instance of originality represented in this thesis is the use of several methods to describe a single phenomenon. Philosophy combined with Quantitative and qualitative methods serve to represent a more coherent picture of mathematically gifted adolescents than is possible under a single approach. Methodological approaches inherently apply different analyses and, thus, force the researcher and reader to understand the topic in a broader context. In that way, the scientific quality of the research was arguably enhanced by using several methodological approaches.

7.2 Defining giftedness in the Norwegian context

As discussed in this thesis and expressed specifically in Article 1, the constructs of giftedness can be understood from several perspectives (for example; individual, systematic and cultural). In line with Article 1, articles 2 and 3 used two different methodological approaches to define giftedness in research. Both approaches seem to have validation as I and engage and
gather information about the same groups in both studies. Thus, the information about both intelligence and school grades in Article 3 might be a stronger identification method because it fulfills more criteria (potential and achievement). Article 3 showed a high correlation between mathematical ability and intelligence and article 2 that mathematical performance at one school level (secondary school) pointing to performance at a higher level (university), suggesting a level of predictability in mathematical giftedness. Thus, the limitation of the performance-based definition lies in its conceptual range. By defining gifted individuals through performance, we overlook the possibility of underachievement, which some researchers emphasize as important for this group (Montgomery, 2009). In Norway, we recruit individuals into acceleration programs or acceleration groups at university through performance. Therefore, we probably miss out on a large proportion of gifted learners who might have become underachievers at an earlier age in school. The main goal of a discussion on the definitions of giftedness is to understand why there are so many ways of understanding these students (Stephens & Karnes, 2000). The definitions being used in a certain context (here, the Norwegian school system) are also limited by information, and thus, performance-based information seems to be preferred. We have little information about other attributes in school when it comes to highly gifted students. We should strive to broaden our view of gifted students in Norway so that we can be in a better position to reach a definition that might have broader conceptual range. As demonstrated in Article 1, the definition implemented in practice in specific contexts guides the pedagogical practice for the students in question. As shown in articles 2 and 3, different results are obtained when gifted adolescents are defined in different ways (see methods).
7.3 Identifying gifted adolescents

In exploring giftedness in the Norwegian context, some challenges in identifying gifted adolescents should be discussed. First, I identified some gifted adolescents through an accelerated mathematics group that was established to facilitate the academic needs of gifted adolescents, and I recruited informants through one of these acceleration programs (Article 2). However, these programs do not aim to provide an optimal learning environment (social/emotional) for gifted adolescents or to recruit potential underachievers. The programs start late (secondary school), and thus, many gifted children might have become underachievers by this time. A second and more general issue connected to these programs is that they recruit students exclusively on the basis of their performance. Therefore, the adolescents participating in these programs might inherently be characterized by individual traits such as motivation and task commitment (see Article 2) because these traits are often necessary to be recruited into accelerated programs in mathematics. This might lead to selection bias in these programs and, by extension, in research, as discussed in Article 2, because recruits voluntarily participate in these programs during their leisure time. To participate in such a program under such conditions, they would probably need to have a special interest in mathematics from the beginning, a point revealed by some participants.

Thirdly, it is generally difficult to obtain data on the gifted population in Norway because we have no tradition of identifying them. To obtain data about a large proportion of these gifted adolescents, I chose to identify them through available intelligence tests (article 3), which also can have validity issues.

A possible solution to some of the issues presented above is to implement research-based identification methods in gifted education in Norway, which go beyond the potential vs. performance dichotomy. One such method is the Scales for Rating the Behavioral Characteristics of Superior Students (SRBCSS; Renzulli, Siegle, Reis, Gavin, & Reed, 2009).
The SRBCSS are used for identifying superior students and their strengths. The method consists of both teacher ratings and identification of strengths in a specific content area (Renzulli et al., 2009). The scales can identify both specific talent in, for example, mathematics, and more general learning characteristics connected to motivation, communication, leadership, and/or technology. However, we have no such scales in use in Norway per now. Nevertheless, the SRBCSS\textsuperscript{13} is research-based, seems to have validity across cultures (Ali Mahdi et al. 2014), and is a good tool for identifying gifted students in several domains (Renzulli et al., 2009). Further, the scales would be beneficial in Norway, since we do not use IQ instruments for identifying potential and, therefore, urgently require other tools through which practitioners can identify gifted students. At the same time, similar identification method could help us identify student that is gifted in other areas than traditional science subjects.

7.4 Acknowledging gifted adolescents’ perspectives

Perhaps the most important implication for further practice is that it seems that the students experienced no social or emotional harm from acceleration or from participating in ability groups. In fact, they expressed that they enjoyed it and sometimes felt it necessary for them to participate in such groups outside school to feel motivated in school (Article 2). However, with a relatively small population, conclusions could not be drawn about gifted adolescents in general. While the results from the qualitative study indicated that highly gifted students do

\textsuperscript{13} A possible scale for implementation is the SRBCSS-R. The protocol consists of learning characteristics, creativity characteristics, motivation characteristics, leadership characteristics, artistic characteristics, musical characteristics, dramatics characteristics, communication characteristics (precision), communication characteristics (expressiveness), planning characteristics, mathematical characteristics, reading characteristics, technology characteristics, and science characteristics. For an example of the protocol (not for reproduction), see Renzulli et al. (2002): https://www.researchgate.net/publication/234718050_Scales_for_Rating_the_Behavioral_Characteristics_of_Superior_Students_Technical_and_Administration_Manual_Revised_Edition
not experience negative peer competition from participating in the accelerated mathematics programs at university, it is important to note that these students could choose whether they wanted to be tested or to sit for a final examination. As some of the participants explained, individual school-related stress is often related to test situations and not to meeting others who perform at an equally high level. Therefore, acceleration opportunities and the opportunity to participate in ability groups should be voluntary, which is the case in Norway. Although, in Article 3, I sought to understand why adolescents with the highest intelligence scores rated their peer relationship lower than those with lower scores, the gifted students in Article 2 did not describe such tendencies. Therefore, the nature of a quantitative questionnaire might not capture nuances in explanations because respondents are, to some extent, “forced” to choose between four alternative answers. Thus, my empirical contribution was also influenced by my methodological choice of integrating quantitative and qualitative methods in collecting self-reported perspectives from gifted adolescents.

As discussed in the introduction to this thesis, one paragraph in the Norwegian Educational Act (§ 1-3) assures that all students in Norway receive adapted education in school based on their individual ability and learning capability. The high statistical correlation between mathematical achievement and intelligence (see Article 3) indicate that gifted students at least accomplish high grades and that high intelligence scores are an indication of high performance in school.

Conversely, the interviews in Article 2 indicate that students who receive high grades in mathematics are not always motivated in school and, in some cases, are unmotivated. The gifted students described pre-school and junior high school as hardly challenging. Although some of the answers can be interpreted as the teacher not having time to meet individual students’ needs, the overall assumption made when integrating the results from articles 2 and 3 is that teachers do not have the necessary competence to meet gifted students’ needs in
mathematics. The results from both articles 2 and 3 indicate that gifted students generally rated their teachers highly. In Article 3, there was no significant difference across levels of intelligence regarding their relationship with their former teachers, which means that the students generally expressed a good relationship with their teachers.

Another empirical contribution from the thesis concerns adapted education. In Article 2, the gifted students described that in the primary grades, they received tasks that they had to solve independently. In this way, even though § 1-3 in the Norwegian Education Act\textsuperscript{14} should secure individual tutoring, it seems that, at least in mathematics, the teacher’s subject knowledge plays an important role in whether students receive individual guidance on their level of knowledge. This is in line with empirical evidence from the TEDS-M reports, which indicated that Norway has few teachers at the highest levels of mathematical knowledge (Hoth et al., 2017; Onstad & Grønmo, 2012). Empirical evidence further suggests that there are associations between the dimensions measured through TEDS-M and the ability to communicate mathematics in the classroom as well as the teacher’s ability to deconstruct knowledge and offer support to both high achievers and low achievers (Ball, Lubienski, & Mewborn, 2001; Baumert et al., 2010). Therefore, it is tempting to suggest that a potential lack of academic support and individual tutoring for gifted students might be connected to teachers’ knowledge in mathematics in Norwegian schools.

As mentioned in the introductory chapter, a recent white paper (NOU, 2016) described concerns about the situation regarding gifted learners in Norway and suggested several strategies for meeting the needs of gifted learners, including acceleration. In Norway, we have been offering acceleration opportunities since 2006 for “students with an interest in and talent

\textsuperscript{14} See https://lovdata.no/dokument/NL/lov/1998-07-17-61/KAPITTEL_1#%C2%A71-3
Tailoring or compressing the syllabus for gifted learners in Norway typically involves moving the students one year ahead in the given topics in which they show talent. As mentioned earlier, such opportunities do not present themselves before students enter junior high school (Norway’s Directorate of Education and Training, 2006), which might be too late for many highly gifted learners. In the quantitative study (Article 3), I had no information regarding whether the participants had previously participated in acceleration programs. Therefore, I considered it of utmost importance to gather information about early acceleration in my qualitative study, which revealed that the Norwegian school system does offer some opportunities for gifted adolescents in Norway. However, our school system does not seem to provide sufficient frameworks to ensure that gifted students are followed up or offered further opportunities for acceleration throughout their schooling. My research shows that acceleration opportunities often end when the gifted student changes teacher in mathematics or when he or she starts at a higher school level, which works against a natural flow toward progress in the subject.

A final empirical contribution concerns gender distribution among gifted adolescents. The integrated findings from articles 2 and 3 reveal a rather interesting gender distribution. The analyses in Article 3 indicate that girls score better than boys in mathematics across levels of intelligence and that the gender effect is strongest at the highest levels of intelligence. This result was a surprise because mathematics has traditionally been a subject in which boys have dominated (Lubinski, 2006). Looking at the recruitment to the ability group in question at the university, significantly more boys than girls participated. One expectation might be that if girls do better in mathematics throughout school, more girls than boys would

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15 See https://www.udir.no/kvalitet-og-kompetanse/nasjonale-satsinger/realfagsstrategien/tilbud-til-elever-som-trenger-ekstra-utfordringer-i-realbug/

16 See https://www.udir.no/kvalitet-og-kompetanse/nasjonale-satsinger/realfagsstrategien/tilbud-til-elever-som-trenger-ekstra-utfordringer-i-realbug/
also be participating in these ability groups. However, it might be that in Norway, boys have a greater interest in mathematics as a school subject than girls, although Norway generally has a high level of gender equality compared to other countries. It could also be that boys (and their teachers) perceive boys as better at mathematics than girls. This would be similar to the results from the SMPY studies (described in Chapter 4), which concluded that girls tend to pursue other careers, even though they do well in mathematics and science in school (Lubinski, 2006). However, an important factor for these programs is that they are voluntary, and it might be that girls’ choices outside school are different from those of boys.
8 Contribution, Limitations, Further Research, and Concluding Remarks

In this closing section of my doctoral thesis, I will present the contributions, limitations, and conclusions drawn from the research. The doctoral thesis also includes the three articles presented earlier. Before discussing the contributions, it is important to note that even though Norway is in the embryonic stage of identifying and providing tutoring for gifted students, we have an educational system that should make it possible to secure these possibilities. We also have an educational system based on inclusion, which means that it is not only possible but also important to secure these opportunities in the public school system in order to maintain the spirit of the Norwegian ideology.

8.1 Contribution

As mentioned in the methods section of the thesis, qualitative and quantitative studies have strengths and weaknesses based on their designs. For this reason, I chose to combine the methods, as doing so enabled me to engage in smaller nuances, illuminate possible differences, and explain tendencies in the quantitative study through more detailed and individual descriptions. As discussed in the introductory chapter, we had little knowledge about how gifted adolescents experience school in Norway. There are a growing number of master’s theses, especially in pedagogic and special needs education, that focus on gifted students or children. Further, the limited existing research mainly focuses on gifted children in pre-school and primary school, which makes it important to gather information about how adolescents experience school and how those in secondary and junior high school reflect on their previous experiences during their pre-school and primary school years. The combination of individual experiences about their current situation and earlier experiences makes it
possible to gather more nuanced information about how they see their own experiences with teachers, peers, motivation, and school in general.

On one hand, the research presented in this thesis shows that school grades are not always a good measure of motivation. For example, the informants in Article 2 often received high grades in school (mainly the highest grade), yet some lacked motivation, especially in primary and secondary school, and the teacher seemed to lack mathematical knowledge to guide them in several aspects of mathematics. On the other hand, there is a need for more information to enable us to draw conclusions about how mathematically gifted students experience school in general, and we know little about which teachers are able to motivate and tutor mathematically gifted students. It seems that to thrive in school, students need teachers with high mathematical knowledge and the ability to create tasks that challenge them.

The qualitative study showed that the mathematically gifted students appreciated acceleration as an opportunity in school. They experienced the increased learning pace as important; thus, normal content knowledge did not seem to meet these students’ individual needs in mathematical thinking, and their teachers lacked the competence to guide them in mathematical thinking. Further, they experienced their teachers as generally well intentioned, albeit lacking the competence in mathematics to challenge them in class. This also meant that the teachers were unable to guide the students through a more advanced curriculum and the creative side of mathematics. Many did not feel that this was the case before they came to the university, where mathematics was more “theoretical” than before.

Therefore, this study emphasizes the importance of teacher competence in mathematics. Mathematics competence among Norwegian teachers has been in focus for some time (Kjærnsli & Lie, 2004), and this thesis has emphasized the importance of this focus. It might even be more important in Norway because opportunities to accelerate
(forsere) are found in central areas or cities in Norway, not in the countryside. Thus, acceleration opportunities are merely coincidental, not a general possibility for all students in school. Nevertheless, one its own, acceleration does not appear to be sufficient in meeting the needs of the gifted.

Mixed methods allow us to nuance some of the results from Article 3, one of which is that the mathematically gifted perceived peers and teachers more negatively than average students did. First, the mathematically gifted students negatively perceived the content knowledge and mathematical ability of their teacher, not their teachers’ intrapersonal abilities (see Article 2). Peer relationships might be explained by the fact that the mathematically gifted wished to spend their time on other subjects or felt that they did not need many friends. If we want to meet the needs of mathematically gifted adolescents in school, teachers should be highly competent and should be able to challenge these students through creative strategies.

Both empirical studies (articles 2 and 3) contributed original research in the area of giftedness in Norway. Article 1 contributed with an alternative understanding of giftedness from earlier discussions regarding how we should define giftedness and what giftedness is. As a conclusion from Article 1, giftedness cannot be distanced from the cultural context in which it occurs. Furthermore, the concept is vague, and therefore, we should spend more energy on developing definitions that reflect the purpose for which they are needed instead of searching for a possible essence of the concept. Collectively, it seems that both performance-based and potential-based definitions are grounded in valid understandings of the group (see Article 1); thus, despite the critique, defining giftedness through intelligence has a broader conceptual range than is the case with other definitions. Therefore, it might be less elitist than other definitions, especially those focusing on performance. The large dataset in Article 3 is, to my knowledge, the only of its kind that focuses on this group in Norway. The study revealed that
the same tendency between genders (girls outperform boys) also applied within the gifted population. This is important for two reasons. First, the gender distribution in the special program at the university was dominated by boys, which arguably indicates our inability to recruit the smartest and highest-achieving girls to these programs. Second, there is also the likelihood that girls experience emotional stress or social stigma as a result of participation in these programs and choose not to participate. In this way, it is even more important that these students, especially mathematically gifted girls, receive sufficient challenges and emotional support in their regular classroom.

To my knowledge, no similar study (articles 1–3) has been done in Norway about giftedness, which makes this contribution important in understanding how these students experience school. At a systematic level, the study indicates that we have some issues. As mentioned earlier, acceleration seems to be a good way of meeting some of the needs of the mathematically gifted, but we learned through Article 2 that the school system sometimes works against these students. Because acceleration is not sanctioned under the Education Act, it becomes a mere coincidence that determines whether the gifted receive this possibility, with few students being followed up in a way that sufficiently meets their needs. Second, they often reach a point where learning opportunities dry up, and as such, they must wait until they start university. This is important because stratifying these opportunities in the Norwegian Education Act might serve as a solution to the problem. Furthermore, it might be that students who have accelerated share some common personal traits that are not shared by their non-accelerated counterparts.

8.2 Limitations and further research

As qualitative research allows for a more limited population, gender distribution could be addressed in all three studies. Although gender distribution in Article 3 was closer to 50%
than it was in Article 2, this might represent a skewed picture about the groups under study. First, I knew little about the characteristics of the females represented in Article 3, and it might be that the gender differences we found in the study can be explained by factors other than those discussed in the article (for example, characteristics of females wanting to severe in the military). In Study 2, there were more boys. As discussed earlier, the fact that more boys than girls participated in the group at the university seems puzzling, since girls generally do better than boys in mathematical subjects. For this reason, it could be that we are not successful at recruiting girls to these types of programs or that other underlying factors influence female’s choice to participate.

Moreover, the gender discussions in Article 2 are somewhat tentative. However, it is normal in qualitative research to operate with smaller populations, as the goal is not to generalize. Rather, it is to create hypotheses (Stenbacka, 2001). Few informants could also affect whether the students’ experiences with ability groups were generally positive, and it might be that those students who did not experience ability groups and/or acceleration as something positive simply chose not to participate in these types of programs. Both studies seemed to have only accessed information about gifted students who had already achieved at a high level in mathematics, although it could be that the students had underachieved or were currently underachieving. This is because grades are not always a good measure of motivation in students, especially if their individual needs are meet.

Thus, we know little about how gifted students who underachieve experience the Norwegian school system, even though the subject is much described internationally. In order to understand the group as a whole, studies that control for or interview underachieving gifted students could contribute with important empirical findings regarding the existing understanding of the group in Norway. We need to know which factors contribute to underachievement in gifted adolescents or whether one factor might be more important than
others in Norway. As mentioned earlier, underachievement can be understood in line with the three-ring concept and Mönks’ multifactorial model, whereby a lack of stimuli regarding one or more of those factors can lead to underachievement. Nevertheless, it is generally difficult to obtain data about underachievement, and this might be especially difficult in Norway and other Scandinavian countries because we do not engage in identifying underachievement through, for example, IQ measures. For instance, the students participating in Article 2 were recruited through achievement measures (school grades), and we knew little about their cognitive level. They described that their needs were sometimes being met through school but that they were bored other times.

As mentioned in the overall discussion of the results from the three articles, the thesis addresses several aspects of giftedness and mathematical giftedness in Norway. Because the instruments used to define and measure teacher and student collaboration in Article 3 were developed by the military service, there was no way of manipulating the items in the questionnaire to see whether we could obtain a higher reliability for the items used. Furthermore, there was restricted access to the sub-test scores for IQ. Unrestricted access might have yielded a better measurement model, since part of the IQ score (numerical ability) probably correlates with math achievement, which might have led to overestimation and possible artificial results. Grounded in the two empirical studies (articles 2 and 3), we should develop research that can engage in the quality of teaching in the classroom and ascertain how the long-term effect of ability groups and/or acceleration is experienced from the student’s perspective. We should also engage in studies on the characteristics of teachers who can meet mathematically gifted students’ needs’ in the regular classroom in Norway. It is especially important to explore the teacher’s perspective, from which we can determine whether there is coherence between what researchers describe as giftedness and what teachers describe as gifted students.
It is worth mentioning the newly published master’s thesis by Kassandra Petsas and Katarina Steigen, which found that teachers generally seem to be positive about gifted students in Norway. However, they receive less attention than other student groups, and teachers have a greater desire to meet their needs in the regular classroom rather than providing acceleration (Petsas & Steigen, 2018). Further, the quality of teaching instructions could be an important factor in terms of how gifted students experience school. It is important to provide optimal teaching strategies and high levels of competency so that gifted students do not become underachievers. We know little about what characterizes teachers who are able to provide a sufficient learning environment for gifted adolescents in the regular classroom. While in later years we may have more studies addressing giftedness in Norway, we may still lack studies that address the teacher’s perspective.

8.3 Concluding remarks

We cannot draw general conclusions from this doctoral thesis. However, following from the empirical studies presented, it seems that acceleration does meet the needs of some mathematically gifted student. Further, the Norwegian school system ought to address some issues in its organization in terms of meeting the needs of these students. The organization of the various school levels can work against the provision of acceleration opportunities. It seems that acceleration often works within the same school (across grades in, for example, junior high school) and, thus, can become difficult across school levels (junior high school to high school). Therefore, there seems to be an urgency to discuss the extent to which acceleration could be ratified in the Norwegian Education Act to secure it as a right—not only a possibility or opportunity. This could be a dramatic change and could also secure the quality and equality of these types of programs. Based on Article 1, we should also discuss how we define giftedness in the Norwegian context or whether we should adapt an existing definition.
or create an understanding of giftedness that is grounded in our cultural context. To answer this question, we should first address the purpose of gifted education in Norway and which students to identify and recruit.
9 References


Baumeister, J., Kunter, M., Blum, W., Brunner, M., Voss, T., Jordan, A., ... & Tsai, Y. M. (2010). Teachers’ mathematical knowledge, cognitive activation in the classroom, and student progress. American educational research journal, 47(1), 133–180.


Carrol, J. B. (1991). No demonstration that g is not unitary, but there’s more to the story: Comment on Kranzler and Jensen. *Intelligence, 15*, 423–436.


Blank International Center for Gifted Education and Talent Development (NJ1). New South Wales, Australia.


students. *ZDM, 49*, 107–120.


Renzulli, J. S. (2002). Expanding the conception of giftedness to include co-cognitive traits and to promote social capital. *Phi Delta Kappan, 84*(1), 33–58.


doi:10.1080/14780887.2013.801543


Stanley, J. C. (1996). In the beginning: The study of mathematically precocious youth (SMPY). In C. P. E. Benbow & D. J. E. Lubinski (Eds.). *Intellectual talent:
Psychometric and social issues (pp. 225–235). Maryland: Johns Hopkins University Press.


Ziegler, A., & Heller, K. A. (2000). Approach and avoidance motivation as predictors of achievement behavior in physics instructions among mildly and highly gifted eighth-
doi:10.1177/016235320002300402

Part I I The articles
Article II
**Mathematically Gifted Accelerated Students Participating in an Ability Group: A Qualitative Interview Study**

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Schools in Norway often emphasize heterogeneous groups in education. The postulated negative effects of homogeneous ability groups on motivation and academic self-concept have long been debated. This study uses semi-structured interviews to investigate how a group of 11 accelerated and ability-grouped high-ability students (gifted) in math have experienced school. All students were interviewed individually. This study explores categories connected to the students’ motivation for the subject, challenges in school, peer and teacher relationships, and academic self-concept. The aim of the study is to investigate whether the school system is able to provide an adequate learning environment for high-ability students, both in ordinary class and in ability groups. The findings show that although some of the needs of high-ability students in Norway are being met, there is much work to be done before an optimal learning environment is established for these students. For example, students do not receive sufficient challenges in math. Furthermore, teachers in the early years of school lack sufficient mathematical knowledge to challenge and support mathematically gifted students, students’ motivation for the subject is lower than expected, and boys’ self-concept seems to be higher than that of girls.

**Keywords:** mathematically gifted, high-ability students, acceleration, ability groups, giftedness, education for gifted students

**INTRODUCTION**

Research indicates that teachers lack the relevant pedagogical skills and specific content skills to challenge highly gifted learners in school (VanTassel-Baska and Stambaugh, 2005). Empirical evidence suggests that the main element in fostering mathematically gifted students is learning opportunities (Nadjafikhah et al., 2012; Hoth et al., 2017). Although grouping students according to their ability in a specific subject has not been a traditional way of challenging high-achieving students in Scandinavia. In the last few years, more attention has been aimed at gifted education and gifted research. The Official Norwegian report (2016) concludes that more research is needed on gifted children and high-achieving students within the Norwegian context as well as educational provisions and teacher competence with regard to this group (Børte et al., 2016). One of the most important factors in developing the potential of gifted students is meeting their academic needs (Winner, 2000; Ziegler and Heller, 2000; Clark and Callow, 2002; Montgomery, 2009). There are currently few local schools in Norway with curricula adapted to this group, few programs with opportunities for acceleration, and few ability groups for gifted adolescents who achieve highly. However, some universities and high schools give high-ability students the opportunity to participate in accelerated programs or ability groups.
This study aims to investigate how high-ability (gifted) students in Norway who participated in a special accelerated math program at a university experienced motivation, relationships with their teachers, peer relationships, and perceived self-concept in math. The students participating in the program followed 10 study points from a mathematical course at the university and were tutored one evening a week over a year. At the end of the semester, the students could choose whether to be examined and receive the study points or to follow the curriculum for their own interest. The qualitative interviews were based on a broad understanding of gifted adolescents grounded in the three-ring concept of giftedness and Mönks (1992) multifactor model of giftedness, which builds on Renzulli's thoughts. This approach facilitates the exploration of both the underlying micro and macro factors that lead to gifted behavior and the realization of high potential.

Micro levels are connected to individual differences, such as personality, motivation, and emotional stability, while macro levels are connected to school, teacher, parents, or peers (or other environmental aspects that might have an influence on individual performance). The combination of the concepts allows us to understand giftedness at both broad individual and environmental levels (Davidson, 2009). Renzulli (2002a) understands gifted behavior as a combination of above-average ability, creativity, and task commitment, whereas Mönks (1992) and Mönks and Mason (2000) include school, family, and peers. Mönks’ model has been extended to include time perspective, planning, and emotional factors (Mönks and Katzko, 2005). Above-average ability, which Renzulli considers the top 15–20% of a given age group, refers to a person’s cognitive ability. In Renzulli’s model, high levels of creativity are associated with originality of thought, and task commitment refers to a person’s special interest or commitment to a subject (Renzulli, 2002b, 2003). In Renzulli’s view, giftedness occurs when all three components described are present. Mönks describes how individual (stress, social, and motivational factors) and environmental (school, peer, and parental factors) also influence gifted behavior. In Mönks’ view, task commitment also includes motivation. The top levels of performance involve the top 5–10% of performance in any given domain (Mönks and Mason, 2000). Because gifted behavior only occurs with sufficient stimuli from the environment, it is interesting to explore both individual and environmental traits for a better understanding of how giftedness develops.

The current study seeks to collect information about the micro and macro levels connected to gifted behavior in one specific domain (mathematics). Macro levels include school, peers, or other environmental factors, whereas micro levels are connected to individual traits, such as motivation, emotions, and task commitment. The levels examined are motivation, peer relationships, relationships with teachers, and experiences connected to accelerated and ability groups. To obtain information about individual traits that lead to performance in one academic area, qualitative interviews can provide interesting information about the underlying individual differences or similarities in gifted students. Furthermore, interviews can provide deeper explanations of how these adolescents experience school in Norway.

**Mathematically Gifted Students**

Continuum research on the topic of giftedness shows that no single specific criterion can be used to determine giftedness (Sternberg, 1993; Gagné, 1995, 2000; Morelock, 1996; Wellisch and Brown, 2012). Newer interactional models of giftedness see giftedness as a pattern of cognitive, motivational, and social variables needed for high achievement in one or more domains (Vlahovic-Stetic et al., 1999). Thus, giftedness can be understood as a result of several interacting variables that lead to gifted behavior. Factors connected to the micro and macro levels of individuals are important to understand how the needs of gifted adolescents can be met (Vlahovic-Stetic et al., 1999; Reed, 2004). Sowell et al. (1990) suggest two types of mathematically gifted students. The first type of student is typically able to work with mathematical problems at a level of difficulty well above what is normal for their age. The second type is able to solve mathematically complex problems by employing different thinking processes (Reed, 2004). Mathematically gifted students are often capable of high levels of problem solving and inductive thinking. They display high levels of logical reasoning, high self-efficacy, and intrinsic motivation for the subject (Pajares and Graham, 1999; Sriram, 2003; Koshy et al., 2009; Leikin, 2014; Leikin et al., 2017). Furthermore, mathematically gifted students are often recognized by their ability to solve complex tasks and engage in mathematical thinking that far exceeds that of their relative age group (Sowell et al., 1990; Reed, 2004). The Study of Mathematically Gifted Youth (SMPY) model of mathematically giftedness was developed to identify mathematically gifted students and to help them develop their full potential (Brody and Stanley, 2005; Lubinski and Benbow, 2006; Brody, 2009). In the SMPY model, mathematically gifted students are those who reason exceptionally well in mathematics. The students are identified using the Scholastic Aptitude Test (SAT-M), which consists of 60 multiple-choice tasks that make it possible to discriminate among students who would score high on in-grade-level tests and those who understand mathematics well above their given grade (Lubinski and Benbow, 2000, 2006). Several longitudinal studies of mathematical giftedness suggest that personality traits such as motivation, individual stress factors, interests, and emotional stability seem to play important roles in developing exceptional talent in mathematics (Lubinski and Benbow, 2006). Thus, the individual traits are not guided by a specific definition of giftedness; rather, they serve as the baseline for identifying mathematically gifted students.

**Motivation in Gifted Students**

Motivation, self-efficacy, individual stress levels, and academic self-belief are all important factors leading to performance across different levels of intellectual ability (Bandura and Schunk, 1981; Zimmerman, 2000; Pajares, 2003). For gifted students, appropriate challenges in school have a large influence on motivation (Winner, 2000; Phillips and Lindsay, 2006). According to self-determination theory, motivation is connected to intentions, energy, direction, persistence, and endurance.
Academic Self-Concept and Ability Groups

One way of fulfilling some of the needs of gifted students is the use of ability groups or acceleration. Some studies suggest a negative effect of ability groups or acceleration on motivation and academic self-concept. This phenomenon is known as the “big-fish-little-pond effect” (Marsh et al., 2008; Nagengast and Marsh, 2012). The theory suggests that it is better to be a high-achieving student with average peers than a high-achieving student among other high-achieving students in a high-ability group (Zeidner and Schleyer, 1999; Marsh and Hau, 2003; Preckel et al., 2008a). The research indicated that academic self-concept, anxiety, and stress related to achievement are lower for students participating in a normal classroom than in a high-achieving classroom. In general, self-concept and academic self-concept are associated with adolescents’ ability to self-regulate and with their individual happiness, self-esteem, and emotional and cognitive outcomes. Cognitive outcomes are typically measured as academic achievements (Shields, 2002; Preckel and Brüll, 2010; Marsh and Martin, 2011). Nevertheless, academically gifted students are typically found to have a higher academic self-concept than average students, which occurs even after participating in ability groups (McCoach and Siegle, 2003). Furthermore, empirical evidence suggests that academically gifted students can experience a decrease in academic self-concept at the beginning of their participation in special programs but that they eventually experience an increase (Dai and Rinn, 2008; Preckel et al., 2010).

Acceleration and Ability Groups for Gifted Students

Academic acceleration in school refers to opportunities that allow students to move through the school system or curriculum more quickly than the standard pace. Acceleration can take the form of grade skipping, accelerated groups, and compressed curricula (Steenbergen-Hu et al., 2016). In general, previous research has assumed that academically gifted or high-achieving students can experience social and emotional harm from acceleration (Boaler, 2008, 2014). It is suggested that countries that do not promote ability groups but rather allow all their students to follow the same curriculum within a heterogeneous context achieve the best results on PISA (Boaler, 2008, 2014). For example, Finland has received considerable attention for its PISA results and has chosen to group students heterogeneously (Sahlberg, 2014). Research from Britain suggests that ability grouping has no positive effects on children’s mathematical performance and, for some children, even has a detrimental effect (Nunes et al., 2009; Hattie, 2012). However, a meta-study suggests that negative social and emotional problems are an exception (Steenbergen-Hu et al., 2016). Further, the ways in which researchers and schools define ability groups varies greatly (Boaler et al., 2000; Steenbergen-Hu et al., 2016). Empirical evidence suggests that heterogeneous groups favor low-achieving students’ more than gifted students (Fuligni et al., 1995; Linchevski and Kutscher, 1998; Nunes et al., 2009; Boaler, 2014). Zevenbergen (2003) found that high-ability students grouped by their ability were far more positive about their learning environment and had higher attitudes toward mathematics compared with students with lower mathematical ability participating in lower ability groups. Gifted youth who participate in accelerated programs can experience positive impact on their long-term achievement, social and emotional well-being, and enhanced learning compared with their non-accelerated counterparts (Steenbergen-Hu and Moon, 2011; Steenbergen-Hu et al., 2016). Even more important, the assumption that acceleration carries the risk that individuals develop social and emotional issues does not seem to be driven by empirical evidence (Colangelo et al., 2004). However, academic acceleration is not always the right answer for every academically gifted student. Some might not want to participate in an accelerated class or do not wish to change their environment. Nevertheless, the option should be available and should be evaluated as equivalent to regular tutoring (Steenbergen-Hu and Moon, 2011).

Aims of the Study

The aim of this study was to explore how mathematically gifted students participating in a special ability group at a university in Norway experience their school situation. The understanding of giftedness in this study was guided by the three-ring concept of giftedness and the multifactor model of giftedness, which have similar categories. Other discussions of interest occurred in the interviews. Because little knowledge exists regarding how this student group experiences Norwegian school, open-ended semi-structured interviews seemed to be the best way to capture their individual experiences. This approach allows information to be gathered about how students experience programs intended to meet their learning needs and whether these programs are sufficient in their aims. The categories in the results section include both macro and micro factors described by the participants in the interviews. They reflect the informants’ general focus during their interviews.

MATERIALS AND METHODS

Participants

In this study, the students participated in the accelerated group in their own free time and by their own free will while simultaneously participating in regular school. The participants ranged in age from 16 to 19 years old. All of the students had accelerated by one or more years in mathematics within the Norwegian school system, and they passed math r2 in high school (r2 is the most difficult level of math in the second year)
of Norwegian high school). Their home schools varied greatly; however, all students were located in communities near Oslo. Thirteen high-ability adolescents – nine boys and four girls – chose to sign an agreement to be interviewed. However, two withdrew their participation for a final sample of \( N = 11 \), three girls and eight boys. The sample reflects participation in the program, which includes significantly fewer girls than boys in general. The sample is sufficient for making cross-case generalizations but is fairly small compared with samples used in some other qualitative studies (Robinson, 2014). The participants all showed high levels of mathematical mastery though their school years, and they achieved well above the norm compared to their age group. However, no intelligence testing was included to further validate their intellectual level. Empirical evidence suggests that high intelligence scores and academic performance correlate (Neisser et al., 1996; Geary et al., 2017; McCoach et al., 2017). This type of selection process is referred to as purposive sampling, in which the participants are recruited within a specific context (Robinson, 2014).

**Procedure**

To secure volunteer participation, during a lecture at the university, information was provided to all students participating in the program. Some weeks later, letters were given to the respondents with information about the project; anonymity and confidentiality were ensured. The letters were based on the ethical guidelines of the NSD (Norwegian center for research data). The students who were willing to participate signed the letter and were later contacted by the researcher for interviews. The participants provided both oral and written consent; the written consent was saved according to the NSD’s guidelines. All of the interview data were gathered in a fairly short time span (within 1 month). The interviews were intended to last about 40 min, but in reality, most of them lasted more than an hour, and some lasted up to 1.5 h. A semi-structured schedule was used with open-ended questions based on theoretical descriptions of mathematically gifted students and information about the context of the program. Because the interviews were open-ended, the categories in the interview guide were broad and addressed school history, experience in accelerated classes, how the students perceived mathematics, if they felt motivated toward mathematics, how they experienced the school system in relation to being talented in Norway and their experience with the ability group. The follow-up questions differed between the interviews. The aim was to not guide the interview subjects in their discussion and explanation. Whenever something interesting was brought to light, they were asked to further elaborate. The categories were drawn from the theoretical frameworks of Mönks and Renzulli, which emphasize both the individual and environmental factors leading to gifted behavior. The categories were so broad to allow the interview subjects to focus on their experiences rather than the researcher narrowing and guiding their answers by following a rigid set of questions. After every interview, field notes were made about unclear questions or observations that could influence the meaning of a specific statement. Two tape recorders were used to ensure that all statements were clear and to ensure that a backup was available if any technical issues arose. The transcription process was completed in two stages. In the first stage, one person transcribed all the interviews in Microsoft Word. In the second stage, the same person read through the interviews and corrected any misunderstandings that could have occurred during the interview by also listening to the tape. All of the interviews were conducted and transcribed in Norwegian. The end results were presented in English, which represent a danger of “loss of meaning” (Hammersley, 2010). The problem of meaning affects all types of interviews, and translating from one language to another represents a special issue. To compensate for this problem, metaphors that were not meaningful in English and local idioms were not represented in the citations. To ensure accuracy, the data were checked with the interviews (audio file) several times to improve understanding of the subjects’ meaning (Dalen, 2011).

**Analysis**

Castleberry (2012) NVivo version 11™ was used to further analyze the material. Thematic analysis is one of the broadest and most frequently used methods of analyzing qualitative data in educational science (Braun and Clarke, 2006; Kvale and Brinkmann, 2009). By first exploring the material for themes other than those already presented in the interview guide, the study was open to the development of new information and new ways of understanding the concept at hand. Interesting new discussion points, such as lack of challenges in school and the system fails the students, emerged as a result of the focuses of the informants in the interviews. The analysis was theory-driven (as structural interviews often are). Triangulation is a strategy that is typically used to improve the validity and reliability of qualitative research (Maxwell, 1992; Golafshani, 2003). The triangulation process in this study mainly proceeded from context to informant to theory. The theories represented in the interview guide (e.g., the categories’ micro/macro levels) were understood in line with the informants’ statements and then considered in a broader theoretical framework of gifted education and mathematically gifted students. Triangulation also involves questioning findings that might contradict previously established models or theoretical frameworks. This process was used throughout the entire analytic process. A triangulation process can enhance the validation of the eventual findings and eliminate misinterpretations of the transcribed material. However, a weakness of this research is that one person translated and analyzed the interviews. Thus, the research is guided by that individual’s personal theory and understanding of the participants in the study.

**RESULTS**

**Motivation and Interest in the Subject**

Intrinsic motivation is often emphasized as an important factor in students’ engagement and performance in academic subjects (Preckel et al., 2008b; Klapp, 2017; Liu and Hou, 2017). Intrinsic motivation is also seen as one of the hallmarks of high-achieving gifted students (Al-Dhamit and Kreishan, 2016; Leroy and Bressoux, 2016). In this study, the students did not perceive math as a subject in which they were more motivated than other
subjects, even though they had skipped grades – and, in some cases, several years – in mathematics. First, they often described a lack of opportunities for acceleration or enrichment activities in other subjects as a result of choosing math as their subject. Second, many described math as a subject they understood easily; therefore, they used little effort when studying or preparing for a test.

“It is not really math I like the best of the subjects. I like chemistry and physics, however, we did not receive any tutoring in those subjects during elementary school or junior high school. We didn’t get a chance to accelerate in those subjects either, so only math was left…” (Boy).

Norway has no tradition of enrichment activities for high-ability students outside of school in science or other subjects. Few of the participants were participating in any activities connected to their talent outside of school. The informants described a mathematical understanding as something different from understanding within other subjects. They often described mathematical knowledge as connected not necessarily to motivation but rather to understanding, and math was the only subject in which teachers could easily identify their ability. Interestingly, many of the students identified the reason for their understanding of math as outside their own control, describing themselves as lucky “to be born with a brain to understand math.” Often, they found that they thought differently about math when they compared themselves to other students.

“It’s not really allowed to say; however, I feel that when I talk to others, they have a totally different understanding of mathematics than me. . . For me, a certain answer just makes sense. . . I feel like I have a different approach and understanding, but if someone asks, I can’t really describe why; my brain just works in that way…” (Boy).

Because they interpreted mathematics in a way that differed from other students, they often did not feel they could discuss mathematics with other students and therefore had to stimulate their interest in arenas other than school. In this way, most of their motivation was driven from within.

Mathematical Ability and Academic Self-Concept
A majority of the students described math as something they had a talent for in other ways than their particular interest in the subject. In general, the students needed little time to understand mathematical problems in high school. Furthermore, most felt that there was a general problem with the way math is taught in Norwegian schools. The general approach did not meet their understanding of mathematics. The informants highlighted two different approaches to math: they felt that the understanding of math was under-communicated in school and that teachers focused on memorizing models before a specific test.

“One thing is to use the theory to calculate something. One must focus on understanding the theory; then you can understand why the results are the way they are. . . The way I understand math is like a language just like in other topics. . . Especially when I talk to other students, I explain math as a language. . . A language you can use to explain all other sciences… Therefore, you have to explain and emphasize a more underlying understanding of math in school because understanding math as a language can give you a deeper understanding of physics, chemistry and biology.” (Girl).

In the students’ view, a greater focus on the underlying theoretical perspective of math would be a much better way of teaching the subject in school. When asked why they thought this was a better approach to teaching math, they explained that if you memorize something, you do not really understand the concept. However, if you truly understand a concept, you can apply the understanding to a broad range of problem-solving strategies. “If you spend more time on understanding the concept, even though some might not understand it, I think it would make math easier in the long run for everyone, even the weaker students” (Boy).

When comparing themselves to average students, some students felt it was somewhat unfair that they needed to use so little effort before a test or to understand a certain topic. One student felt she had to lie to the other students about how long she actually studied before a test: “… I can feel guilty if my friends tell me they have worked really hard before a test and I have not. Then I often just say I also have used a lot of time to study before the test. I have friends that work a lot more than me and I get much better grades” (Girl). The few girls in this study seemed to be more focused on what their peers might think about their performance. This statement can be connected emotional support. It seems that several of the participants described a lack of emotional support from the environment, from their teachers, and, in this case, from friends. Thus, they received emotional and social support from the other participants in the ability group – mostly because they did not perceive their participation as competition.

“It was such a good experience. I don’t really care whether or not I receive a good grade on the exam now. I met others that thought like me; it was really fulfilling. Of course, I like my other friends also; however, it was so cozy to have them around me in the group. It is nice to feel that math is cool; we are not exactly perceived as cool in ordinary school” (Girl).

For most of them, their most positive experience at the university was related to the perception of their talent as something positive and that they did not need to “hide” their engagement: With my other friends who are not so good at mathematics, I cannot just talk out loud about how good I am. They would probably think I was bragging about my abilities (Girl). Their academic self-concept seemed high in general. However, in contrast to what one might expect, the statement above can be understood as an increase in academic self-concept after participating in an ability group. In their view, their mathematical talent was normalized by mirroring her interests with other like-minded students.

Are High-Ability Students Sufficiency Challenged?
In the interviews, all of the participants were asked how they felt their academic needs were met through school. Most of the students reported receiving far too few challenges in primary school and middle school. All of the informants had skipped one
or two grades in math during middle school or later, but none of the students had been invited to skip grades or to participate in another classroom while in primary school. Some of the students had been allowed to work alone with a book used by higher grades, but often with little tutoring from the teacher in class. The most significant issue was the lack of individualized challenges and how this affected the students’ learning. As the main problem, they described feeling a lack of motivation and boredom because of repetition and too few challenges: “I think it is a good idea not to let those who really have a talent or a genuine interest in a subject just sit there and get frustrated by repetitive tasks, which they don’t need to do” (Boy). In some cases, the students even read books on other topics or focused on other tasks during math lectures:

“…As I said, I was sitting and reading a book; it was probably because I did not feel it was necessary to pay any attention to the teacher. It’s like this: if the teachers ask any control questions, you just answer right on the question, and the teacher gets kind of astounded and quiet, and then you just keep sitting there.” (Girl).

When the students were asked what the school or the teacher should have done to better challenge them in math, two discussion points emerged from the material. The informants felt that subjects in school were taught at a slow pace and had little depth. The fact that schools often emphasize group work also stands out as negative for the high-ability students. “Slow is probably a good word for describing it. You use a lot of time to learn very little and to understand very little” (Girl). They did not feel the teacher was able to explain the depth of mathematical knowledge or that they individually received a broad explanation of the subject through participation in the regular classroom. Some explained the lack of challenges as a result of the lack of teacher competence. In the students’ view, it should be easy to individualize the speed at which the teachers move through a subject, especially compared with individualized one-on-one tutoring.

“When you are really good in a subject, you feel like you only learn about the surface of the subject. It is way too slow… Slow on the surface. You are in a way just rowing in a canoe on the surface without any oars, and you need to use only your hands to get forward. It is too slow.” (Girl).

It is evident that in the students’ experience, there is too little acceleration and a dearth of customized learning opportunities. The Norwegian system is not properly structured to attend to the needs of accelerated students because they fall between school levels or teachers. All of the students were very satisfied with their speed of learning more deeply. However, in many cases, the system did not function properly. Therefore, the students had to make choices between subjects they liked, or they received challenges for a period of time that then stopped.

“In my case, I have skipped two years of math. However, there is nothing to do next year because I’ve skipped two grades. Maybe I could go on another course at the university, but during daytime I need to be at my regular school. I can’t just drop out of regular school to go on a course, and it probably would not be customized to fit my needs” (Boy).

In general, the students were very satisfied with the opportunity to accelerate in math. For some, it was life changing to be recognized for their abilities and to have the opportunity to learn at their own pace. Several commented that if their needs had not been properly recognized at some point or if they had not at least received some opportunity to enrich their understanding of certain subjects, they probably would have suffered throughout their entire school career. It is interesting to note that most of the students describe it as important to be “allowed” to learn at their individual pace; in Norway, it is the law that students have the right to receive tutoring customized to meet their individual needs.

It is extremely important that you actually are allowed to work at your level. That you get to experience the joy of challenges… Of course, it is a joy to be ahead of everyone else and work with others who enjoy the subject as much as yourself. If I had to continue working on the same level as everyone else, I would probably continue to experience math as boring, so, in a way, it has changed my entire life that I have been allowed to be ahead of everyone else (Boy).

Some students felt a certain level of anxiety related to time pressure because they participated in the accelerated group in addition to everything else. Other students enjoyed working at their own pace and learning more within the subject. At the same time, they felt the advantage of being accelerated was overrated because they still had to wait to start at the university.

**Teacher and Peer Relationships**

In their ability to develop positive personal relationships with high-ability students, most teachers were rated highly by the informants, with the exception of some individual teachers. The students generally described their social relationship with their teacher as positive. “I feel I’ve had a good relationship with my teachers in general; however, I felt that math was pretty easy early on in school” (Boy). Some students had experienced negative feedback or negative attitudes toward their personality or negative reactions when they asked difficult questions in class. They did not necessarily need the feel for a teacher to instruct them, and it was more important to them that the teacher not hinder their progress than support them academically. However, with regard to academic knowledge and whether they felt supported by – or needed academic support from – the teacher, most students felt the teacher-student relationship to be somewhat overrated. The students’ little need for academic support in the classroom could be explained by the fact that they were given few individual challenges and that most of the challenges they did receive were things they already understood on their own.

I remember that a friend and I were taken out of class and sat in a room and were given some harder tasks, but without any explanation or introduction to them. And then it was suddenly too difficult, so that didn’t really help much. Then we had to participate in the regular classroom again, where I already knew everything. The teacher probably thought, “Oh, so they were not that smart after all.” (Boy).
When describing whether academic support or their intrapersonal relationship with the teacher was more important to them, all the students agreed that academic progress was important to their thriving at school. They all explained that the teacher was important for personal support and understanding. However, in their earlier stages of schooling, they had not received any specific academic challenges. In some cases, they had served as an assistant teacher or were asked by the teacher to explain a difficult concept. Furthermore, their subjective experience with teachers in school reflected the teachers' poor understanding of the academic needs of high-ability students. However, individual teachers made a significant difference in helping the students explore their talent and in introducing them to new and exciting subjects. From the participants’ perspective, the most important teacher characteristics for learning were dedication and deep knowledge of mathematics. Nevertheless, there was a general consensus among the students that the largest difference between the university and regular school was that at the university, they actually needed the teacher if they wanted to understand the concept at hand.

A positive relationship with peers was important to all informants. All the students experienced the opportunity to participate in acceleration programs as positive in general, and they were excited to meet others who were even better than they were at mathematics. All of them described their interpersonal skills as sufficient and saw no negative associations between their talent and developing friendships; rather, they perceived their talent as something positive. Furthermore, their attitude toward the other participating students was positive, and many explained that they were socially compatible in another way compared with their average peers. For example, they shared a common understanding of and interest in mathematics, which they had not previously experienced in the same way with other peers, even though they had previously participated in acceleration groups. However, most of them had one or two friends who were also talented in mathematics. The fact that they shared this interest meant a lot to them: Without my two friends, I think it would have been very difficult without them, even though my family supports me (Boy). Others felt the fulfillment of academic challenges was more important than friends or friendships: From my perspective, I would much rather have sufficient challenges academically than participate in a classroom or... with people at my age (Boy). The last quote underlines the importance of academic challenges if high-ability students are to thrive and feel that their talent is accepted. Furthermore, in this category, it is also clear that the need for social relations differs among the students. Although contact with other academically talented students is seen as important for gifted and high-ability students, some expressed the importance of heterogeneity in their friendships.

DISCUSSION

The general gender distribution of the program was interesting. Significantly, more boys than girls participated in the group, which does not reflect newer research showing that in Norway, girls tend to do better in math than boys. There is little information about differences in interest in math between boys and girls in Norway. Although several trends can be identified in the way boys and girls describe their experiences, there is a lack of sufficient data to draw conclusions about these trends in this particular study. Halpern et al.'s (2007) study found gender differences in the interest in science and mathematics that favored boys at the highest levels of cognitive ability. This study might explain why there are so few girls in this particular group. A study by Hyde et al. (2008) described how gender differences in mathematics emerge in high school and college, which is the same age group as the informants in this study. It might be that the gender stereotypes are stronger in this age group. Furthermore, the greater man variability hypothesis is one explanation for why boys tend to have a greater variance in performance and test scores than girls do (Hyde and Mertz, 2009; Else-Quest et al., 2010). In general, boys seem to be overrepresented at the very top or bottom of any topic, even if there is no average gender difference (Hyde, 2005). In some cases, it might be that girls are more drawn to subjects other than science and mathematics. Nevertheless, the few girls in the study described more stress connected to high performance situations, especially in test situations. This tendency can be interpreted as the girls displaying a lower academic self-concept. Van Boxtel and Mönks (1992) related inner stress in performance situations to lower academic self-concept, whereas the stress related to tests can affect their general academic performance and academic self-concept. Moreover, it can also be understood in the context of math anxiety; girls tend to show higher anxiety when performing mathematical tasks than boys do (Devine et al., 2012). Wu et al. (2012) found that math anxiety was present in both high-achieving and low-achieving students, although the reported anxiety was strongest in the low-achieving students. Dowker et al. (2016) related math anxiety to expected performance, and these students already perform at a high level, thus also displaying a high level of expectations regarding their own performance. It might be that the interest and enjoyment in the subject is overridden in specific test situations. The students were interviewed only about mathematical knowledge compared with self-concept; it is possible that their descriptions could be different for other subjects.

All the students participating in this study showed above-average capability in mathematics. Whether the students were creative was more difficult to detect through the interviews. However, most of the participants described mathematics as a creative subject. They considered math as a tool for understanding other subjects in science, and a deep assimilation of math and understanding its applications can be viewed as creative. Furthermore, most of the participants showed high levels of task commitment when working with mathematical problems; they had a genuine interest in searching for an answer and did not describe the process as exhausting, but enjoyable. In contrast, the fact that most of them did not describe themselves as more motivated toward math than other school subjects was interesting. In gifted research, motivation is often described using two categories: first, as a stable personality trait or characteristic
and second, as an environmentally induced transitory state (Dai et al., 1998). Beliefs, values, and attitude are all important factors that determine whether gifted individuals achieve in a certain domain (McCoach et al., 2013; Siegle et al., 2017), along with goal orientation and mindset (Dweck, 2012). In this study, the boys in particular showed high levels of goal orientation in math; they found their motivation in earning good grades, whereas the girls were more driven by interest for the subject. The high levels of self-assurance in their own capabilities in mathematics displayed by the boys were in line with these descriptions. They were unafraid of failure and enjoyed competition. However, most of the students also described an increase in motivation when they participated in ability groups in school. If motivation is a personality trait, it might explain why the students did not lose their motivation for the subject during the earlier stages of school.

Motivation seemed to be stable among the students and appeared to be based on their innate interest in learning. It is possible that the groups of high-ability students who display high levels of performance in their early years but do not display this stable motivation are at a higher risk of becoming underachievers at an early stage in school, which could occur before any acceleration opportunities or ability groups are presented to them. A typical understanding of underachievement is a rather large discrepancy between potential and performance (Dowdall and Colangelo, 1982; Whitmore, 1986; McCoach and Siegle, 2003). If gifted students are not sufficiently challenged, they are at great risk of becoming underachievers, and this can happen in the early stages of school (Clinkenbeard, 2012).

One of the most important factors for motivating gifted students is to provide them with opportunities to learn at their level of pace and based on their interests (Phillips and Lindsay, 2006). Challenges based upon individual levels of understanding are important for maintaining motivation in general (Wallace, 2000). Many underachieving gifted students experience problems with peers, teachers, and self-regulation (Reis and McCoach, 2000; McCoach and Siegle, 2003). In this study, no such problems were described. To determine whether a student is underachieving, specific information about the student’s potential is necessary. The students in this study had never been tested on their abilities. They had only been identified through high performance in math. It might be that if the students had received individual tutoring in their early schooling, they could have perceived their current level of performance. The study consequently ruled out any information about underachievers because the selection for the study only favored those already performing at a high level.

In general, it seems, at least for this group, that the early years of schooling in Norway are the crucial years in which their needs were not met. The classroom setting is homogeneous, and the teacher’s focus is on the general student population. The students felt that much of their learning in the early years was a waste. In high school, the students had fewer negative experiences, and their needs were met more sufficiently. However, few of the informants felt their needs were met through participation in the regular classroom; rather, acceleration opportunities made a difference for these students. The tendency described above can be explained through international surveys such as TIMMS, which indicates that Norwegian teachers score low on mathematical knowledge (Hoth et al., 2017). A teacher’s mathematical knowledge might affect their ability to provide sufficient challenges and communicate mathematics aimed at highly gifted learners. Nadjafikhah et al. (2012) emphasized that a teacher needs the ability to discuss complex ideas and understand mathematics at a high level to provide support and guide mathematically gifted adolescents in their learning process. Assouline et al. (2011) noted that even though acceleration is a good way of meeting gifted adolescents’ needs, the teacher must be able to guide them through the more challenging curriculum and must have high mathematical understanding to identify gifted students. The teacher must have a broad repertoire of mathematical problem-solving activities and strong pedagogical content knowledge to foster mathematically gifted student’s needs (Goldin, 2017). It is one thing to align the curriculum for gifted students; it is another to foster and recognize mathematical creative thinking. Creative and divergent thinking can be understood as the ability to generate new and numerous ideas in a given field (Preckel et al., 2006). Mathematically gifted students often perceive and process mathematics in a complex way, and the teacher must be able to do the same (Leikin et al., 2017).

Despite the empirical evidence suggesting that gifted adolescents participating in ability groups might experience negative peer competition and lower academic self-concept, in this study, the informants described their experiences as positive, and they experienced competition as something positive and driven from within. This finding indicates that they mostly competed with themselves and did not perceive any negative peer competition. This tendency is in line with research on highly gifted achievers and ability and/or acceleration (Colangelo et al., 2004, 2010). With regard to acceleration in Norway and whether it is a positive experience for high-ability students to be accelerated, there was consensus among the informants. None of the students in the study described acceleration as something negative. Their descriptions of the perceived positive experiences with acceleration are in line with the findings of Hornstra et al. (2017). In their study, high-ability students participating in acceleration programs had generally positive experiences, both academic and social. Furthermore, part-time programs were even more sufficient for high-ability students than full-time programs (Hornstra et al., 2017). They all described their teachers as successful in developing positive relationships with students. However, they questioned their teachers’ lack of mathematical knowledge. Therefore, teachers with high levels of knowledge may be important for high-ability students, and having teachers who are better at providing emotional support may be more important for students with lower performance in math. If other opportunities, such as more enrichment activities in school or in the classroom, were available, they may have felt differently. These students have few other opportunities; therefore, they are likely to take whatever option is available. The negative aspects associated with acceleration and ability groups were connected to the school system and not to the fact that the students had an opportunity to learn at a faster pace. Furthermore, because the system often does not recognize or offer acceleration at a younger age, the participating students might represent an opportunistic
and highly motivated and devoted group, whereas many gifted children might have lost their interest and motivation in earlier years.

It might be that we in Norway are too afraid of implementing acceleration and ability groups as a general option for students who perform at high levels. The fact that at least one girl did not feel she could be honest even with her friends about how she perceives and understands mathematics may also be connected to the egalitarian ideology and fear of elitism in Norway. Norway is a country that tends to respect talents and abilities developed through hard work more than those that can be explained by inherent potential. Fear of elitism and egalitarian ideology is strong in Norway and can, in some cases, explain both why we do not support gifted adolescents and why politicians do not discuss ways of meeting these students’ needs in school (Skogen, 2010). The students who were accelerated in mathematics took all of the possible opportunities that they received (i.e., they chose to participate whenever they got the chance), which is more than we can expect from all students in school. Therefore, it is likely that there are large numbers of high-ability students in Norway who do not receive the same opportunities. Coincidences lead to student's participation in accelerated classes or ability groups in Norway, meaning that individual teachers or geographic affiliations make a major difference with regard to introducing students to acceleration opportunities. Opportunities to learn should not be limited to the students who are located in a certain geographical area or who have the luck of having a teacher that knows about a specific program. Since there is so little history of facilitating special learning opportunities for high-ability students in Norway, it could be that personality traits connected to motivation, self-beliefs, and goal orientation are even more important for these students over time. Therefore, highly gifted students who are at risk of becoming underachievers might have lost their motivation and, in a worst case scenario, might have already dropped out of school due to insufficient challenges. Studies of high-ability students suggest that boredom predicts underachievement in school (Obergriesser and Stoeger, 2015). Boredom can occur at the very beginning of school and as early as kindergarten in some cases (Mooij, 1999; Little, 2012). The students in this study also described boredom. However, their goal orientation and ability to work with math over time seemed to play an important role in protecting them against becoming underachievers.

LIMITATIONS

Several limitations of this study should be addressed. First, the conceptual framework of Renzulli and Mönks is broad in its conceptual understanding of giftedness. Their model does not clearly define gifted behavior or explain important differences between gifted individuals (Kaufman and Sternberg, 2008). In this study, the categories presented in the results section are broad and might not be explicitly connected to the models presented in the theory section. Thus, the models guide the categories by describing environmental and individual aspects in the population at hand and describing key points in the students’ interviews. No single study can be generalized, especially not a qualitative study. The study presents results from a few interviews with students who were high performing in math. The results may have been different with more participants or with interviews with students who did not perform at a high level. It is not possible to replicate this type of interview study. In particular, it is not possible to duplicate studies that have open designs, such as open-ended interviews, because the context and the interviewer affect the results. Open-ended interviews make it possible for the participants to focus on how they experience their own life situation. These interviews are limited by the researcher’s ability to engage and ask follow-up questions that take advantage of moments of interest in the interview. In this study, higher mathematical knowledge and more conceptual understanding of giftedness by the researcher could have enhanced the quality of the follow-up questions and may have provided more interesting discussion points. The selection criteria in this study were rather limited; although the students were gifted, we should have obtained more information. Specifically, it may have been interesting to obtain intelligence scores to determine whether students were exceptionally gifted. If students were exceptionally gifted, other theoretical aspects could have been added to enlighten the findings and guide the interviews. In the study, underachievers were automatically ruled out because the study only recruited participants through performance mathematics. The results might have been different if underachieving gifted students or gifted students who had not received similar opportunities were interviewed. The results of the study only permit speculations about the general population of mathematically gifted adolescents in Norway. Although the current study raises many interesting discussion points, more thorough research is necessary to better understand the situation of gifted adolescents.

CONCLUSION

Despite the limitations presented above, some conclusions can be drawn from this study. The most important finding of this study was that all of the students perceived acceleration and ability groups as something positive. These groups should be part of a changing paradigm in which all gifted students are at least given such opportunities. The students did not experience any negative pressure or competition in the accelerated groups. During the first 10 years of schooling in Norway, too little effort is put into meeting these students’ needs; some of them receive more support when they enter high school. The students seemed to display characteristics associated with the three-ring concept of giftedness, and task commitment was seemingly a very important variable for their success throughout school. Although all of the students displayed above-average ability, creativity was more difficult to identify. More studies are needed that examine ways to create learning opportunities for high-ability students in Norway before it can be concluded that acceleration and ability groups are the best way to meet their needs. We should address how we recruit adolescents to these groups to ensure that we include...
all students who may need extra activities to feel stimulated in school.

Students’ understanding and enjoyment of working with complex and deep problems was not reflected in the way they were treated by their teachers in the early stages of school. Even though the students in this project were high achievers, many wondered whether they could have learned more and done more throughout their school years. It is evident that too little is done for high-ability students in Norway, and acceleration may be one way of meeting some of the needs of gifted students in Norway.

REFERENCES


Appendix
Interview Guide
Intervjuguide:

Intro:


Skole og forsering

1. Hvordan vil du beskrive matematikk som fag i skolen?
2. Fortell om hvordan du har opplevd skolen oppgjennom
3. Hvordan vil du beskrive den faglige støtten fra lærerne dine gjennom skolegangen?
4. Når fikk du muligheter for akselerasjonsmuligheter den første gangen? Og hvordan opplever du slike tilbud?
5. Er du fornøyd med din avgjørelse om å forsere opp igjennom skolegangen?
7. Kunne du tenkt deg å ta lignede tilbud igjen? Hvorfor/hvorfor ikke?
8. Påvirket deltagelsen din på programmet hvorvidt du vil studere matematikk videre eller ikke?

Kreativitet

1. Opplever du deg selv om kreativ innenfor de fagområdene du har vært best i?
2. I så fall, hvordan opplever du at dette har blitt møtt?
3. Hva tenker du om at matematikk kan beskrives som et kreativt fag?

Sosiale relasjoner og støtte fra Læreren

1. Hvilke tanker har du rundt det sosiale på skolen? Opplever du det som viktig/uviktig?
2. Vil du beskrive deg selv om en sosial person?
3. Er det viktigere for deg å være i et sosialt fellesskap enn å få faglige utfordringer?
4. Opplever du fellesskapet på slike organiserte tiltak på en annen måte enn på den vanlige skolen?
5. Fikk du noen nye venner på akselerasjonstiltaket(ene). (Hvordan er disse eventuelt annerledes)
6. Har du opplevd støtte og forståelse fra familien din for at du ønsker mer utføring i matematikk?
9. Did any teacher meet your needs better than another, if so, how and when?
10. What could the teacher have done differently, in both cases (the ordinary and the acceleration).

Motivasjon/oppgaveforpliktelse
akademisk selvoppfatning

1. Vil du beskrive deg selv som en spesielt motivert person i matematikk?
2. Hva skal skole/lærere gjøre for at du skal opprettholde motivasjonen din?
3. Hvordan har eleven opplevd motivasjonen sin for faget(fagene) gjennom skolegangen? På en skala fra 1-5. Beskriv hvorfor

4.
1. Hvilke tanker har du rundt å bli kalt evnerik/akademisk talentfull?
2. vil du beskrive deg selv som spesielt flink i matematikk?
3. Har du opplevd noen negative sider ved at man har deltatt på spesielle programmer og/eller fått ekstra tilbud fordi man er faglig sterk?
4. Hvordan vil du beskrive deg selv om du sammenligner deg med de andre på forseringstilbudene og på tilbudet på universitetet?

Avsluttende:

Har du noen tanker om hvordan skolen burde vært organisert for din gruppe elever?
NSD
Tilbakemelding på melding om behandling av personopplysninger

Vi viser til melding om behandling av personopplysninger, mottatt 23.08.2017. Meldingen gjelder prosjektet:

55436 M athematically gifted students in N orway
Behandlingsansvarlig Universitetet i Oslo, ved institusjonens øverste leder
Daglig ansvarlig Jørgen Smedsrud

Personvernombudet har vurdert prosjektet, og finner at behandlingen av personopplysninger vil være regulert av § 7-27 i personopplysningsforskriften. Personvernombudet tilrår at prosjektet gjennomføres.

Personvernombudets tilråding forutsetter at prosjektet gjennomføres i tråd med opplysningene gitt i meldeskjemaet, korrespondanse med ombudet, ombudets kommentarer samt personopplysningsloven og helseregisterloven med forskrifter. Behandlingen av personopplysninger kan settes i gang.


Personvernombudet har lagt ut opplysninger om prosjektet i en offentlig database.


Dersom noe er uklart ta gjerne kontakt over telefon.

Vennlig hilsen

Marianne Høgetveit Myhren

Dokumentet er elektronisk produsert og godkjent ved NSDs rutiner for elektronisk godkjenning.
Kontaktperson: Pernille Ekornrud Grøndal tlf: 55 58 36 41 / pernille.grondal@nsd.no
Vedlegg: Prosjektvurdering
FORMÅL
Formålet med studien er å undersøke hvordan matematisk begavede elever opplever sin skolesituasjon.

UTVALG
Du opplyser at utvalget rekrutteres gjennom at deltakerne er med i et program om matematisk begavede personer på UiO. Personvernombudet legger til grunn at taushetsplikten ikke er til hinder for rekrutteringen, og at forespørsel rettes på en slik måte at frivilligheten ved deltagelse ivaretas.

INFORMASJON OG SAMTYKKE
Utvalget informeres skriftlig om prosjektet og samtykker til deltakelse. Informasjonsskrivet er noe mangelfullt utformet, og vi ber deg gjøre følgende endringer før intervjuene gjennomføres:

- at UiO er den behandlingsansvarlige institusjonen,
- oppgi kontaktinformasjonen din, herunder e-post og telefonnummer,
- oppgi dato for prosjektslutt (18.10.2017) og hva som skal skje med opplysningene etter prosjektslutt (anonymisering)

Revidert informasjonsskriv sendes på e-post til personvernombudet@nsd.no.

SENSITIVE PERSONOPPLYSNINGER
Det er ikke krysset av for at du skal samle inn sensitive personopplysninger om elevene. Personvernombudet tar imidlertid høyde for at det, på grunn av spørsmålenes utforming, kan registreres sensitive personopplysninger om helse, se spesielt intervjuguidens kategori 5.

INFORMASJONSSIKKERHET
Personvernombudet legger til grunn at forsker etterfølger Universitetet i Oslo sine interne rutiner for datasikkerhet. Dersom du bruker din private pc/mobil lagringsenhet ved transkribering av intervju legger vi til grunn at bruken er i tråd med disse retningslinjene.

PROSJEKTSLUTT OG ANONY MISERING
Forventet prosjektslutt er 18.10.2017. Ifølge prosjektmeldingen skal innsamlede opplysninger da anonymiseres. Anonymisering innebærer å bearbeide datamaterialet slik at ingen enkeltpersoner kan gjenkjennes. Det gjøres ved å:
- slette direkte personopplysninger (som navn/koblingsnøkkel, e-post, telefonnummer)
- slette/omskrive indirekte personopplysninger (identifiserende sammenstilling av bakgrunnsopplysninger som f.eks. skole, klassetrinn, karakterer, alder og kjønn)
- slette digitale lydopptak