Wake potential of a dust particle in magnetised plasmas

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Abstract. The electric potential distribution around a dust particle immersed in a magnetised supersonic plasma flow is studied by numerical simulations. It is shown that with increasing magnetisation of plasma, the peak in the wake potential gets smaller and moves upstream. For strong magnetisation, the trailing peak in the potential distribution vanishes and the potential becomes more isotropic. The results agree qualitatively with the linear response approach. The numerical simulations are carried out with a particle-in-cell code.

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1. Introduction

A dust particle immersed in plasma will be electrically charged and the resulting potential on the dust will be screened by electrons and ions [1, 2]. Under usual plasma conditions, due to high mobility of electrons, the dust particle will be charged negatively. Dust charge is a fundamental problem in studies of complex (dusty) plasmas, where a large number of dust particles immersed in plasma interact with each other through screened Debye-Hückel (Yukawa) potentials. These interactions can be reflected in collective phenomena, such as crystallisation, phase transitions, or low frequency plasma waves that include the dynamics of dust [2, 3].

Dust charging becomes anisotropic when the dust particle is exposed to a plasma flow [3, 4, 5]. The relative motion of plasma and dust leads to formation of wakes in potential and density. For high electron to ion temperature ratios, which is often the case in laboratory dusty plasma experiments, a typical feature observed downstream of a dust particle is ion focusing, which leads to a locally enhanced charge density due to ions being electrostatically lensed into the wake [6, 7]. A corresponding feature is oscillations in the wake potential that can extend several Debye lengths downstream [4, 8, 9]. Ion
focusing and potential oscillations have important implications on structuring of dust clusters, which are often levitated above an electrode, i.e., they are exposed to the ion flow. The wakefield can align dust particles in the direction of the flow and give rise to non-reciprocal interactions between dust particles [10, 11, 12, 13]. Note that these effects can be significantly modified by plasma-neutral collisions and also by the electric fields present in the sheath of the electrode [14].

While electrical charging of dust particles in flowing, unmagnetised plasmas has been subject to extensive theoretical, experimental and numerical studies, the effects of the magnetic field on dust charging under such conditions, and the resulting potential distributions, are still not well understood [15, 16]. Through the Lorentz force, the magnetic field will restrict the dynamics of electrons and ions and thus impose yet another restriction on the charging process. Experiments that have been carried out with dust in magnetised plasmas have revealed formation of striations, filaments, or concentric circles in plasma, which influence the dust dynamics [15, 17]. A magnetic field will also modify the wake pattern and hence also the wake interactions between dust particles [18, 16]. Recent theoretical works and numerical studies, which employ the formalism of linear response approach for the plasma flow aligned with the magnetic field, reveal reduction of amplitudes in the trailing peak in the wake potential and increase in the periodicity of the wake extrema [19, 20, 21, 22]. For strong magnetisations, the topology of the wake changes significantly and the oscillatory wake potential is either expected to vanish or to be amplified, depending on the approach [18, 21, 23].

Since the magnetic field increases the complexity of the studied system, it is advantageous to address the problem of dust charging in magnetised plasma with first-principle particle simulations, where the plasma particle trajectories are followed in self-consistent force fields with minimal assumptions. Such an approach can be realised with the particle-in-cell (PIC) method, in which the dynamics of a large number of simulation plasma particles can be studied in complex geometries at a reasonable computational cost, and dust charging can be treated self-consistently [24, 5].

In this work we study the potential wake downstream of a dust particle in flowing magnetised plasmas with full PIC simulations, where the dynamics of electrons and ions are followed in self-consistent force fields. By considering different magnetisation regimes for plasma flowing in the direction of the magnetic field, we can compare results from our PIC simulations with solutions given by the linear response approach.

2. Numerical approach

Numerical simulations are carried out with the DiP3D code, which is a three-dimensional electrostatic PIC code designed for studying in particular dust charging in plasma [5, 7, 13]. The code has been described in detail in a previous work [25], and here we provide only a summary of features that are essential for this study.

DiP3D is an electrostatic code that operates in a 3D Cartesian coordinate system
and simulates dynamics of electrons and ions in self-consistent force fields. In this study, the particles are subject to electric and magnetic fields, and their trajectories are advanced according to the Boris algorithm [26] combined with the leap-frog method. The leap-frog method is characterised by a staggered temporal mesh and spatial mesh:

\[
\vec{r}_i(t + \Delta t) = \vec{r}_i(t) + \vec{v}_i(t + \Delta t/2)\Delta t \\
\vec{v}_i(t + \Delta t/2) = \vec{v}_i(t - \Delta t/2) + \vec{f}_i(t)\Delta t/m_i
\]

where \(\vec{r}_i\) and \(\vec{v}_i\) are the position and velocity of the \(i\)-th plasma particle of mass \(m_i\) and charge \(q_i\), \(\vec{f}_i = q_i\vec{E}\) is an electric force projected from the nearest grid points on the particle, and \(\Delta t\) is the computational time step. The Boris algorithm accounts for the velocity rotation due to the Lorentz force \(\vec{f}_L = q(\vec{E} + \vec{v} \times \vec{B})\). It rotates velocity \(\vec{v}\) in the plane perpendicular to the magnetic field \(\vec{B} = m\vec{\Omega}/q\), where \(\vec{\Omega}\) is the gyrofrequency vector, to a new velocity \(\vec{v}\):

\[
\vec{w} = \vec{v}_- + \vec{v}_- \times \vec{\xi} \\
\vec{v}_+ = \vec{v}_- + \vec{w} \times \vec{\zeta}
\]

where: \(\vec{\xi} = \frac{\vec{\Omega}}{\|\vec{\Omega}\|} \tan \frac{\|\vec{\Omega}\|\Delta t}{2}\), and \(\vec{\zeta} = 2\vec{\xi}/(1 + \|\vec{\xi}\|^2)\).

To combine it with the standard leap-frog method, we first half-accelerate the particle due to the electric force (i.e., advance its velocity for a half-time step \(\Delta t/2\)), then rotate the velocity due to the magnetic field according to Eqs. (2), and again half-accelerate the particle due the electric force using the leap-frog method. Thus, the mean particle velocity is used for the Lorentz force calculation.

The magnetic field \(\vec{B}\) is constant in each simulation and oriented in the \(x\)-direction, which is also taken to be the direction of the plasma flow in this study. The electric field is calculated from the electrostatic potential \(\Phi\), which is updated in each time-step to account for the actual charge density distribution. The Poisson equation \(\nabla^2\Phi = -\rho/\epsilon_0\), where \(\epsilon_0\) is the electric permittivity of vacuum, is solved with a multigrid method, with Dirichlet boundary conditions on the edges of the simulation box. To build the charge density \(\rho\) on the grid, the electric charges of plasma particles are weighted to the grid with first order linear weighting, and a similar approach is used for projecting the electric force back to the particles. The plasma particles are introduced according to the shifted Maxwellian distribution, and the box boundaries are open for plasma. In this study the plasma is collisionless, i.e., we do not consider collisions with neutral background.

The dust particle of radius \(r_p \sim 0.1\lambda_{De}\) is placed inside the simulation domain far away from the external boundaries of the domain. Here, \(\lambda_{De} = \sqrt{\epsilon_0 k T_e / n_e e^2}\) is the electron Debye length, where \(k, T_e, e, n_e\) are respectively the Boltzmann constant, electron temperature, electron charge, and the electron number density. Initially, the dust particle is electrically neutral, and it acquires charge due to plasma currents during the simulation. It takes typically a few ion plasma periods \(\omega_{pi} = \sqrt{n_i q^2/m_i \epsilon_0}\) for the charge on the dust particle and the wake to reach stationary conditions. Plasma particles that hit the dust surface are lost and contribute to the net charge of the dust. In this study, we consider conducting dust, and thus the accumulated charge is redistributed on the dust surface such as to cancel the internal electric fields.
We consider supersonic plasma flows with the flow velocities of $v_d = 1.2c_s$ and $v_d = 2.4c_s$, where the sound speed is given by $c_s = \sqrt{(kT_e + 5T_i/3)/m_i}$. The ion to electron mass ratio is set to $m_i/m_e = 120$, in order to reduce the computational cost, but still maintain significant mass separation between the species, and the electron to ion temperature ratio is $T_e/T_i = 100$. We use a background plasma density of $n = 10^{13} \text{m}^{-3}$ and an electron temperature of $T_e = 3 \text{eV}$. The electron Debye length is $\lambda_{De} = 4.03 \cdot 10^{-3} \text{m}$, and the simulation box extends ca. 0.025 m in each direction. By varying $|\vec{B}|$, we consider different magnetisations of the plasma, $\beta = \omega_{ci}/\omega_{pi}$ within the range $\beta \in (0.09, 10)$, which corresponds to magnetic fields of approximately $|\vec{B}| \in (1, 100) \text{mT}$ for our choice of parameters. Thus, in the low $\beta$ regime, the gyroradius of both plasma species $r_L$ is larger than the dust radius $r_p$, while for high $\beta$, $r_L \sim 0.1r_p$. This magnetic field is strong enough to magnetise the plasma since we use reduced ion mass in the simulations. Note that stronger magnetic fields need to be used in the laboratory experiments. We run the simulations until the steady state conditions are reached, and perform the analysis under such conditions.

3. Results and Discussions

For low magnetisation of plasma, we expect similar results as for the unmagnetised control case, which is shown in Figure 1. For clarity, in the figure only the shallow potential variations are shown, while all potential values below $\Phi = -0.2kT_e/e$ are coloured white. The dust particle is at the origin of the coordinate system, and the flow is in the positive direction. The potential is characterised by a significant enhancement in the potential distribution in the wake, with the maximum of the trailing peak at $d \approx 1.6\lambda_{De}$ downstream of the dust particle. Such a potential distribution is typical for supersonic flows with large electron to ion temperature ratio, $T_e/T_i \gg 1$, and it reflects the ion focusing effect due to electrostatic lensing in the vicinity of the dust [13]. Such potential distributions have been shown to be important for the interactions between dust particles in flowing plasmas, where the downstream dust is being aligned with the flow and controlled by the upstream dust particle via nonreciprocal forces [7].

With an increase in the magnetisation of plasma, the first maximum in the potential distribution (the trailing peak) is gradually modified and finally disappears at high $\beta$. In Figure 2, the potential variations around the dust are shown for selected $\beta$ values: $\beta \approx \{0.1, 0.5, 1.0, 5.0\}$. For $\beta \approx 0.1$, the result is essentially the same as in the unmagnetised case. Here the gyroradii of plasma particles are much larger than the size of the dust and the plasma can be considered unmagnetised or only weakly magnetised, which does not significantly affect the dust charging. With increasing $\beta$, a stronger magnetisation of the plasma leads to a less pronounced but more localised trailing peak. The peak is shifted in the upstream direction and is followed by an enhanced minimum in the wake. A significant difference in the topology of the potential distribution is seen for $\beta \approx 5.0$, when the maximum in the potential distribution disappears, the potential distribution becomes more isotropic with an extended potential depletion in the wake. In
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Figure 1. (a) The normalised electric potential distribution around a dust particle in unmagnetised plasma ($\beta = 0$) flowing in the $x$-direction at $v_d = 1.2c_s$. Only shallow variations in the potential distribution are coloured. (b) The cut through the potential in the $x$-direction at $y = z = 0$ emphasising the potential variation in the wake.

In this regime, both plasma species are magnetised, and the charging currents are strongly restricted by the Lorentz force.

The potential distribution in the wake of the dust particle that we obtain with the full PIC simulations can be directly compared to recent results obtained with a linear response approach and other analytical methods [21, 23]. For this purpose it may be instructive to study the potential slices in the direction of the flow. We thus present in Figure 3 slices through the potential along the $x$-direction for different $\beta$ for the plasma flow of $v_d = 1.2c_s$ (a) and $v_d = 2.4c_s$ (b). It is evident that for weakly magnetised plasma and highly supersonic regime ($v_d = 2.4c_s$), the maximum is located farther downstream as compared to $v_d = 1.2c_s$, which is in accordance with previous studies [6]. The effect of magnetisation is similar for both cases: the trailing peak becomes smaller and shifts upstream. The extent of the peak is smaller, and a pronounced potential minimum in the wake appears. Already at $\beta = 1$ the wake is characterised by an overall negative wake potential with superposed oscillations, which is clear for $v_d = 1.2c_s$. For $\beta > 1$ the trailing peak eventually disappears, and weak and short-wavelength oscillations are present on the enhanced potential depletion in the wake. In general, due to a relatively small simulation domain, these effects are more clear for a slower plasma flow.

The results from our PIC simulations agree qualitatively very well with the linear response calculations by Joost et al. [21]. The wake potential structure has also a
Figure 2. The normalised electric potential distribution around a dust particle in a plasma flowing in the $x$-direction at $v_d = 1.2c_s$ at different magnetisation regimes: $\beta \approx 0.1$ (a), $\beta \approx 0.5$ (b), $\beta \approx 1.0$ (c), $\beta \approx 5.0$ (d). Only shallow variations in the potential distribution are coloured.

similar behavior as in Salimullah et al. [27]. We observe the same trend for the position and amplitude of the maxima in the wake, and finally the change in the character of wake for the strong magnetisation (see Figures 6 and 7 in [21]). Since our simulation
domain is relatively small, we only observe the first peak in our simulations for weakly magnetised plasmas and we can not directly assess the enhancement in the subsequent maxima in the wake. While the enhanced minimum for $\beta = 1$ suggests that this should be the case, extensive simulations for larger domains should be carried out to confirm it, and this is suggested as future work.

In the linear response calculations, the charge of the point-like dust particle is fixed and the wake potential can be scaled, where a larger dust charge implies stronger maxima in the wake \[4, 21\]. In our PIC simulations the dust charging is self-consistent and thus one-to-one comparison between the two methods can be intricate. We observe variations in the dust potential with increasing flow velocity. For weakly magnetised plasma $\Phi \approx -1.15 kT_e/e$ for $v_d = 1.2c_s$, and $\Phi \approx -1.0 kT_e/e$ for $v_d = 2.4c_s$. However, the dust potential remains essentially unchanged for $\beta < 5$ for a given $v_d$. Thus, for $\beta < 5$ the observed trends in the wake potential are mainly attributed to the magnetisation of plasma. For stronger magnetisations, the dust potential varies by about $15 - 20\%$. However, in this strongly magnetised plasma regime the wake pattern is significantly modified, and these variations of the dust potential do not alter significantly the wake topology or potential maxima.

The agreement between the two studies is encouraging even though there are certain differences in the approach. In the linear response approach the dust particle is considered as a point-like disturbance, and thus the characteristic length that needs to be considered is the Debye length, which can be directly obtained from $\beta = \omega_a/\omega_{pi} \approx \lambda_D/eL_i$. In our PIC simulations, the dust radius is a fraction of the electron Debye length, but it is comparable to the ion Debye length. The high $\beta$ regime in the linear response model can be related to a strongly magnetised plasma regime in our PIC simulations. In our first principle calculations the kinetic and nonlinear effects are included, but for the chosen set of parameters the potential response in the studied system can be well approximated by the linear approach.

On the other hand, there is some discrepancy between our results and other analytical studies employing the linear response approach with other approximations \[23\]. In that work it has been noted that the amplitudes of the wakefield maxima increase with $\beta$, while the periodicity decreases. While we also observe a decrease in the distance between the extrema, the amplitude of the trailing peak decreases with $\beta$ in our simulations. Notably, the kinetic effects need to be included in the analysis of the problem to capture all important effects, such as Landau damping of ion-acoustic waves in the case of small temperature ratios.

The effects of the magnetic field on the wake formation, which have been demonstrated here, will have implications on the dust interactions in systems comprising several dust particles. One can expect smaller equilibrium distances between the particles aligned with the flow, as well as modified coupling between them. The magnetic field can strongly influence charging of a downstream dust and can lead to a non-monotonic dust charge distribution in larger dust clusters. Thus, to fully understand such a system, one needs to study the self-consistent charging of several dust particles.
in magnetised plasma flows. Furthermore, a systematic parameter study for various flow velocities and temperature ratios will shed light on the wakefield formation and is needed for detailed comparison with analytical models. These are subjects of our ongoing studies of dust charging in magnetised, collisionless plasma flows.

4. Conclusions

With the full PIC simulations we have studied the potential distribution around a single dust particle in streaming plasmas for different magnetisation regimes. With increasing magnetic field, the amplitude of the first potential maximum in the wake becomes weaker and shifts upstream, while the subsequent extrema might be enhanced. At high magnetisation, the trailing peak in the wake disappears. Our results agree with recent linear response calculations and suggest that kinetic effects are important for the wakefield formation in magnetised plasma flows.

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6. References

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