An Investigation of Ski-Snow Friction

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The front page depicts a section of the root system of the exceptional Lie group $E_8$, projected into the plane. Lie groups were invented by the Norwegian mathematician Sophus Lie (1842–1899) to express symmetries in differential equations and today they play a central role in various parts of mathematics.
Abstract

Traditional cross country skiing is an interplay between the active motion of the body of the skier and the physical interaction between the surface of the skis and the snow. Friction and glide between the solid substrate and the snow is a function of the snow temperature, surface chemistry and roughness, dry friction and liquid film flow. It is a truly multi-scale phenomenon with physics from the thickness of the water film (< 100 nm) to a snow crystal (∼ mm) to the length of the skis (1-2 m). Although a great amount of resources have been spent on understanding glide and friction — the Norwegian National team in skiing plan to use 15 million NOK in 2014/2018 alone [22], a basic physical understanding of the phenomenon is still at large and limits novel development of new and robust methods to control glide. This thesis work focuses on the basic mechanical effects that can help to better understand the phenomena of glide and friction that occur between snow and skis.

Two experimental systems were developed, one in the laboratory and one in the field. Both experiments features versatile instruments assembled from off-the-shelf sensors and open-source electronics. The laboratory setup is a novel linear tribometer (3 m long and 0.1 m wide) that allows studies of various friction regimes. The setup has been validated against known coefficients of kinetic friction. Furthermore, the investigation of other frictional regimes, e.g. hydrodynamic lubrication, yields results in compliance with theory. Seeing that a lot of the same instruments were to be used in both experiments, the laboratory setup also functioned as a preparation for the experiment to be conducted in the field.

The field setup features a mobile, novel linear tribometer allowing investigation with whole skis in realistic cross-country conditions. The coefficient of kinetic friction between ski and snow is measured by a load cell, i.e. the force necessary to pull a ski sled along a flat track. Thermistors measures the temperature in the ski-snow interface. In addition, a GPS measures displacement and velocity (accuracy ± 0.06 m/s). Coefficient of frictions ranging from $\mu_k = 0.033$ during cold weather ($T = -11^\circ C$) to $\mu_k = 0.0145$ during warm weather ($T = 0^\circ C$) were measured. The highest temperature increase ($\Delta T \approx 5^\circ C$) was measured 40 cm from the ski tip at the highest velocity of 27 km/h. The obtained results are in compliance with several reported properties from similar experiments, e.g. coefficient of friction, temperature increase in ski-snow interface and the frictions dependence on velocity. Furthermore, cold and wet grinded skis were instrumented with thermistors enabling us to differentiate the difference in
Abstract

...temperature in the interface due to different ski surface structure. These results provide continued support for improving cost-efficient, versatile instrumental design for investigation of ski friction.
First of all I would like to thank my supervisors Professor Atle Jensen and Professor Andreas Carlson for providing me with an interesting topic and giving me the opportunity to carry out an experimental thesis. I would also like to offer my special thanks to Lab-Engineer Olav Gundersen who offered great help in the experimental setup in addition to helping me carry out the field experiment. I am particularly grateful for the assistance given by PhD Candidate Jean Rabault whom has been of great help throughout the work of this thesis. Without his expertise in various fields this thesis would not have been possible. Thanks to all the people in the lab for contributing to an inspiring working environment.

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CHAPTER 1

Introduction

In this thesis there were designed and developed two experimental setups; one in the laboratory and the other one in the field. Both setups were validated against relevant, reported values. Experiments with various frictional regimes were carried out and the obtained data agreed with previously published results and was quite repeatable. The experiments will be presented in the following chapters, starting with the linear tribometer setup in the laboratory.

1.1 Kinetic friction on snow and ice

The aim of this thesis is to investigate the phenomena of low friction when sliding on snow, especially w.r.t. cross-country skiing. Frictional heating is widely considered the main mechanism causing low friction while skiing. The heat melts a tiny layer of snow which results in a thin water film that acts as lubricant between the ski and the snow. The goal is to see if one can detect a phase transition - from dry to wet friction - while sliding on snow. Since the existence of a thin water film generated by frictional heating is debated [25, 28], both force, velocity, acceleration and temperature will be measured in order to have several ways to investigate the problem. The force will be measured by a load cell, which measures the force at which the winch pulls the slider. A GPS attached to the sled enables displacement and velocity measurements. Thermistors are attached in the ski base, enabling temperature measurements in the ski-surface interface.

Given that a phase transition occurs, this would ideally be detected by our measurements techniques in the following manner:

1. A transition from dry to wet friction would result in a drop in the load cell measurements since the pull from the engine and the winch is constant.

2. Lower friction would yield higher velocity, hence we should be able to detect an acceleration from our GPS.

3. If the frictional heating causes the phase transition the thermistors should measure an increase in temperature. Ideally they should measure a temperature close to zero if melting occurs.

Validation of the experimental setup is done by comparing the obtained measurements to other relevant, published results. This involves investigating the
1. Introduction

influence of different parameters on the coefficient of friction and the temperature evolution in the ski-snow interface. Thus I will briefly summarize the most important findings in the investigation of sliding friction on snow and ice.

Bowden and Hughes (1939) [9] found that the low kinetic friction of ice is due to a partial water layer formed by frictional heating. It had been previously considered that pressure melting was alone responsible for the formation of this water film. They found that the coefficient of kinetic friction was independent of the load, apparent area of contact, and speed of sliding over a certain range, but decreases with decreasing temperature. Furthermore, they performed experiments on snow that showed similar trends at higher coefficient of friction. The increase in friction was attributed to the extra work done in displacing and compressing snow grains.

Ambach and Mayr (1981) [3] developed a probe for measuring the water film between a gliding ski and snow. They measured the water film to be several micrometers thick (5-30 µm), but this is generally considered to be too high, due to the fact that their measurement device had a hydrophilic coating.

Colbeck (1992) [13] is the one who has done the most extensive work regarding the processes that control snow and ice friction. He developed mathematical models to describe the sliding process and proposed that the total friction could be expressed as the sum of a series of terms representing each mechanism. He assumed contributions from plowing, solid deformation, water lubrication, capillary attraction and surface contamination. Under most circumstances of subfreezing conditions, he found that the heat available was insufficient to generate enough meltwater to separate the surfaces completely. Thus he suggested that solid-to-solid interaction and meltwater lubrication are the dominant processes when meltwater is generated by phase change at the interface.

Strausky et al. (1998) [38] attempted to detect possible water films using fluorescence spectroscopy combined with a pin-on-disc tribometer. Seeing that the detection limit of the experiment was 0.1 µm and no film was detected, it was concluded that water films, if present, must be below this limit. This value is much smaller than predicted earlier, however, their experiment was limited to velocities below 0.1 m/s.

Moldestad (1999) [30] did research on the ski base structure and the factors that affects ski base sliding friction. A Ski base Structure Analyser (SSA) utilizing laser technology was developed to analyze the detailed structure of a stone-ground ski base.

Buhl et. al (2001) [11] examined the gliding of polyethylene on snow in laboratory experiments and in field tests under racing conditions. Their main investigation was the influence of snow temperature and load during gliding. Both laboratory and field experiments showed that kinetic friction was lowest around -3°C and increases for low temperatures as well as for snow temperatures close to 0°C. As the snow temperature increased, the influence of pressure decreased, and for temperature above -6°C there was no detectable difference between different loads.
1.2 Experiments

Bäurle (2006) [4] used a hydrodynamic approach to explain the measured friction forces and temperature evolution. He found that the main factors determining friction are the thickness of the water films and the relative real contact area. The low friction observed is due to unevenly distributed thin water films. Bäurle’s calculations suggest that the films have thickness of 30-250 nm along the slider at -5°C. He further states that the behavior of water films and size of the real contact area can explain the friction process, no capillary attachments are needed. Bäurle reports that the real contact area is the most critical parameter determining friction between skis and snow or ice.

Bäurle (2007) [6] developed a numerical model for sliding on ice including dry friction and generation of lubrication by water films. To verify the model they compare it with experimentally determined friction coefficients and slider temperatures. Real contact area estimates based on actual topography can explain the experimentally observed temperature dependence, while squeeze out of water at the junctions can explain the observed load dependence of friction.

Thiele et al. (2009) [40] investigated the deformation mode of the snow during contact and the real contact area between snow and ski base. Displacement-controlled loading experiments of hard packed snow samples were carried out. Before and after the loading experiment a high resolution 3D image of the snow microstructure was captured by X-ray CT.

Schindelwig et al. (2014) [36] studied the temperature of snow under skating skis with infrared sensors and measured the coefficient of kinetic friction using a linear tribometer. The highest increase in temperature (4°C) was measured 10 cm behind the ski binding.

The most recent research is done by Lever et al. (2018) [28]. They used high-resolution (15 μm) infrared thermography to observe the warming of stationary snow under a rotating polyethylene slider. They did not observe any melting at contacting snow grains despite low friction values. Such a result challenge whether meltwater produced by frictional heating is the dominant mechanism underlying the low kinetic friction at snow. However, their sliding speed (0.3−1 m/s) and slider pressure (0.8 − 3.6 kPa) were relatively low compared to that of cross-country skiing.

1.2 Experiments

Much has been learned about the physics and mechanisms of ski-snow friction from laboratory experiments. Such experiments enables study of each parameter in a controlled environment, however, the results are limited to the conditions in which the experiments were conducted. Devices or machines that are used to measure the coefficient of friction - or other tribological quantities - are called tribometers.

The most used laboratory setups for investigating ski-ice/snow friction are the pin-on-disc tribometers. These are rotational tribometers where a stamp...
1. Introduction

carrying a sample of some material is pressed against a snow/ice-filled rotating disk and the frictional force acting on the sample is measured. [9, 38, 11, 5]. The rotational tribometers have several advantages. It easy to obtain high velocities over any given duration of time and it is effortless to change the normal force of the sample on the surface. On the other hand, both these features yields obvious drawbacks. The sample slides over the same surface numerous times which will result in an increasingly polished surface as temperature and water films accumulates. In addition there will be a problem at very high velocities, namely the centrifugal force, resulting in melt water and snow moving radially outwards altering the measured friction force. These effects may be negligible at low velocities and load, but not if one are to conduct experiments with realistic values w.r.t. cross-country or alpine skiing. In order to provide measurements involving whole skis at sports-specific speeds, several different field experiments have been carried out over the years [14, 29, 19, 10]. [10] developed a test apparatus for determining the coefficient of friction for skis in the field. This setup involved a sled which was given an initial velocity with the aid of potential energy stored in a set of springs. Several sensors were placed along the path of the sled (≈ 10 m) which gave time and position information of the sled. They state that their setup is able to the detect the difference in friction between two pairs of skis with a resolution of 0.001. The most realistic laboratory setup is reported in [21]. This full-scale linear tribometer allows studies with skis at reasonable cross-country speeds in a controllable environment.

1.3 Channel experiment

In order to gain further knowledge and insight on sliding friction, a smaller experimental setup was built in the laboratory. This setup is a smaller version of what would be done in the field experiment. The setup features a lot of the same equipment and techniques that will be used outdoor, hence it is a great opportunity to test the instruments thoroughly before doing full scale experiments. The channel setup enables experiments with dry and wet friction in a more controlled environment.

1.4 Field experiment

Briefly put, the field experiment setup consists of a sled being pulled by a winch. In order to be in full control of the setup and keeping costs low, the measurement system is composed of off-the-help equipment and instruments.

To perform experiments in as natural conditions as possible, we had to build a setup which could be used outside and tested in an actual ski-course. The device is portable and straightforward to set up, making it easy to conduct experiments at different locations at a limited amount of time. The field experiment will be further presented in its own section.

1.5 Arduino

A central part in both experiments is the use of Arduino microcontrollers. Arduino is an open source electronics prototyping platform based on flexible
software and hardware which are easy to use. It features single-board microcontrollers and microcontrollers kits for building digital devices and interactive objects that can be used to control equipment, instruments and digital loggers in a very efficient way and at a low cost. All devices and software are open source and the user community offers detailed documentation which makes it easy to tailor the setup to ones needs. The Arduino setup which has been used in my experiments will be further explained in their respective chapters and sections.

1.6 Outline

This thesis comprise mainly the two experimental setups. The design of the laboratory setup - the channel experiment - is presented in chapter 2. This includes equipment, devices, logging procedures and how to carry out an experiment, among other things.

The physical properties to be investigated in the laboratory experiment along with the parameters that can be varied are given in chapter 3.

The results obtained in the laboratory experiment will be discussed and compared with theory in chapter 4.

The design of the field experiment is described in chapter 5.

Measurements procedures, handling of obtained data and adjustable parameters regarding the experiments conducted in the field are given in 6.

A brief summary of the conducted field experiments along with the obtained results are provided in 7. The findings will be compared and rationalized on the basis of hydrodynamic lubrication.

Summary and possible future work for both experimental setups are given in chapter 8.
CHAPTER 2

Channel Experiment Design

2.1 General overview

To achieve full control over the experiment the setup was built in the lab. To limit the cost and maintain control and high flexibility we used off-the-shelf sensors and open-source electronics to carry out the measurement. With this setup we can measure the kinetic coefficient of friction for both dry and greased conditions. In addition, changing the following parameters are no problem with our setup:

- velocity
- load
- material
- dry and wet friction regimes

The setup consists of a long, slender channel in which a slider is pulled by an electrical engine. A load cell measures the force at which the engine pulls the slider and a camera records the motion such that velocity and trajectory can be determined. Thermistors were meant to measure the temperature in the slider-surface interface, but due to negligible frictional heating this feature was removed. Further details about the experiment will be presented in the following subsections.

2.2 The channel

The slider is pulled in a long, slender channel made entirely out of Plexiglas (PMMA - polymethyl methacrylate), with dimensions

- length = 3.00 m
- height = 0.15 m
- width = 0.10 m

In the one end, the channel has a small hole in the wall where the rope/string pulling the slider goes through.
2. Channel Experiment Design

This channel was a couple of years old and had been used for wave-experiments earlier, hence it had to be polished and cleaned to ensure no dust, plastic, debris, etc. were left on its surface which could negatively effect the friction experiment.

![Sketch of channel setup. The winch to the right pulls the slider through the channel with force $F_P$. The friction force, $F_F$ works in the opposite direction. A camera mounted above the channel records the motion. The slider is easy to track due to a red light attached to its top.](image)

2.3 Microcontrollers

Microcontrollers were used to control and log all necessary measurements, and to some extent enable communication between our modules. All of the devices used were either developed by or for Arduino. With the Arduino products being open-source hardware and software in addition to being well documented online, their devices are easy to tailor to ones needs. The Arduino products used in the setup will be listed and explained further in this section.

**Arduino devices**

1. Arduino Uno
2. ARD-LTC2499
3. LC-81 MegaMoto GT
4. Adafruit Feather 32u4 LoRa Radio (RFM9x)
5. Adafruit Assembled Data Logging Shield for Arduino

**Features**

6. Logging Details
2.3. Microcontrollers

1. Arduino Uno

The Arduino Uno is a microcontroller board based on the high-performance microchip ATmega328P. The Arduino Uno has several digital input/output pins and analog pins which can be used to conduct readings from sensors or other devices. The Arduino Uno is built in such a way that other boards can easily be stacked on top of it, allowing communication between them. This device offers high sampling frequency (∼10 kHz on analog pin) and a 10-bit ADC (analog-to-digital converter).

2. ARD-LTC2499

The ARD-LTC2499 is a 16-Channel 24-bit ADC Data Acquisition Shield for Arduino. With a 24-bit ADC this shield enables much higher resolution than the Arduino Uno. On the downside, this shield offers only 15 Hz sampling frequency, and even slower if we are reading from several pins, hence we need to stack multiple shields on top of each other to ensure a high enough sampling rate.

3. LC-81 MegaMoto GT

The Robot Power MegaMoto GT is a low-cost robust H-bridge shield which can handle high current loads without overheating. The H-bridge is an electronic circuit which enables a voltage to be applied across a load in opposite direction. Such a circuit allow DC motors to run forwards and backwards. The MegaMoto GT can control up to 50A (5 sec) of current at 24V. This device was used to drive the motor with full variable-speed control.

4. Adafruit Feather 32u4 LoRa Radio (RFM9x)

In order to enable communication with the slider (to signal start/stop logging) we used the Feather 32u4. This is a microcontroller with a "Long Range (LoRa)" packet radio transceiver with built in USB and battery charging. Since the Feather 32u4 does not have a built-in antenna, a wire antenna is attached to the device. The wire is cut according to the quarter-wave whip principle, i.e. the wire is approximately one-quarter of a wavelength long. A wire of length 16.5 cm was chosen, which corresponds to a radio wave frequency of 433 MHz.

5. Adafruit Assembled Data Logging Shield

Since the slider is not connected to a computer we used this dedicated data logging shield to log the data obtained from the thermistors and the load cell. This shield is easily stacked on top of other shields and the data can be saved on a SD card.

6. Logging Details

When writing to a SD-card there is a possibility for running into write speed problems, i.e. problems that occur if you try to write data to SD card too fast. If one are to store data from a sensor onto a SD-card, the naïve and straightforward approach would be to write the data received from a given sensor at its sampling rate. This will work fine if the time between readings from the sensor
2. Channel Experiment Design

is long relative relative to the latency between the microcontroller receiving a string, reading and writing it to the SD card. Such a latency may typical be a few milliseconds. If however the sampling rate is high, such an approach will lead to problems, e.g. the latency heavily delays the sampling frequency. The solution is to write larger chunks of data at a lower speed, this is done by storing several readings from the sensors before writing it to the SD Card. In addition, to further reduce latency, one should avoid opening and closing the file which is written to while the logging is occurring, i.e. the file can be left open while waiting on a new value and should only be closed when there are no more values to write and/or the logging is finished.

To further improve the logging sequence the code was written such that one file is generated per run (start/stop signal from Adafruit Feather unit). It is then important that the filenames are of ascending order such that the user easily can keep track of the files. This order in the filenames must hold even if there is a loss of power. To enable such a feature one must use the microcontrollers EEPROM, i.e. electrically erasable programmable read-only memory, which is a non-volatile memory. This memory is like a tiny hard drive in which bytes can be stored even if the microcontroller is turned off.

2.4 Instruments, devices and equipment

The rest of the setup consists of the following instruments and devices:

1. Winch
2. GPS-3030D - Bench Power Supply
4. Slider rack
5. Bow string
6. Ambient Light Sensor
7. Thermistors
8. Full-Brudge Thin-Beam Load cell

1. Winch
To keep costs down and make sure we have a device that fits our need, the lab-engineer built a winch system which pulls the slider.

GPS-3030D - Bench Power Supply
The GPS-3030D was our motors power supply. This device can deliver output volts in the range 0 ~ 30 V which is sufficient to utilize the full range of our MegaMoto GT.
2.4. Instruments, devices and equipment

2. Webcamera Logitech Brio 4k Stream Edition
A webcam was attached 2.2 meters above the channel and was used to record the motion, which in turn was used to determine trajectory, velocity and acceleration.

During the experiment the nearby lights in the room were turned off. The only visible light source during the camera recording was a light-emitting diode. This made it easy to track the slider movement - frame by frame.

The Logitech Webcam offered $720 \times 1080$ resolution with 60 frames per second, which is more than enough to track the sliders movement. In addition, using the ffmpeg library makes it easy to control the webcam from the command line or in a script. This made it possible to set all necessary camera settings (such as focus, saturation, exposure time, etc.) with a python script, ensuring that the settings are equal for every run.

3. Slider rack
A small rack made out of aluminum is used to hold the logging equipment (microcontrollers) in place. On the bottom of the rack there is a detachable layer (attached with screws). With this feature we only need to change a small part of the rack when changing to a different slider base material. The bottom also has four small holes, in which the thermistors can be placed.

4. Bow string
To avoid external forces - from other than the pull from the motor - the rope pulling the slider needed to be as light as possible. Furthermore, the rope had to be static in order to avoid stick-slip effects while pulling. A rope satisfying this condition was a bow string made out of Kevlar. The rope was attached to the slider in the one end and to the winch in the other end.

5. Thermistors
Thermistors were used to measure temperature. This is a type of resistor whose resistance changes significantly with a change of temperature. We used NTC thermistors, i.e. the resistance decreases as temperature rises. This device features a glass-encapsulated sensor. It performs well in an operating range -50 to 250°C and offers high accuracy. The thermistor have high resistance change per degree of temperature which provides excellent resolution and its small size means fast response to temperature changes. The thermistors were placed through the holes in the bottom of the slider rack and their values read by the ARD-LTC2499. A thermistor used in the experiment is depicted in Figure 2.2.

6. Full-Bridge Thin-Beam Load Cell
The load cell we used had a capacity of 454 g, and was used to measure the force at which the slider was pulled. The thin-beam load cell is mounted with clamps to create a 'double bend' during loading. An electrical output is generated as the double bend causes tension and compression on the sensor strain gage.
2. Channel Experiment Design

Figure 2.2: Image of thermistor used in channel experiment and in field experiment. The thermistor legs are placed in ceramic tubes to prevent contact.

7. Ambient Light Sensor

A light sensor is used to determine when to stop the motor (and recording and logging). The light sensors detects the slider when it passes and signals the motor (via the computer) to stop. This device incorporates a photo-diode and a current amplifier IC in DIP package.

2.5 Python Code

As will be presented in the next section, a run in the channel setup involves several units. Three of these units - two Arduino microcontrollers and one web-camera - had to be initiated simultaneously, hence the need for a program combining them arose. This was solved by writing a Python code which handles all the devices within the same script. This program also works as a user interface, making it easy for others to conduct an experiment. By implementing the python subprocess module the script can spawn new processes (Arduino devices and web-camera), connect to their input/output pipes, and obtain their return codes.

2.6 Performing an experiment

To illustrate how all the components work together I will describe how an experiment was carried out.

A run is started from the computer and is initialized by a python script. The computer has three USB outputs which goes to

1. Webcam.
2.6. Performing an experiment


3. Radio Signal Transmitter (Feather 32u4 LoRa).

1. Starting an experiment.
A Python script is used to run the experiment.

```python
python Run_Experiment.py
```

From the python script the user will first be asked to specify the name of the file which will store the camera recording.

- **Name of video file:**

Further, the user must send an integer which gives the duty cycle of the motor, $v \in [0, 255]$, where $v = 0$ is zero force and $v = 255$ is maximum force:

- **Give duty cycle value (0 to 255):**

This value is sent to the MegaMoto GT which is stacked on top of an Arduino Uno. The Arduino Uno uses the command `analogWrite()` to send an analog value (PWM wave) to one of the MegaMoto GTs current sense pins. The pin will generate a steady square wave of the specified duty cycle until a new call to `analogWrite()` on the same pin is made. The duty cycle is between 0 (always off, zero speed) and 255 (always on, maximum speed), hence the requested input range in the script.

In order to make the experiment easy to run and the post-processing easy to handle the Feather 32u4 radio shield was used for communication between the devices. After having sent a velocity as input we are ready to start the experiment. The following line will appear in the terminal (Linux machine):

- **Send char S to start, other to reboot:**

When $S$ is typed and sent, the Feather 32u4 LoRa unit transmits a signal to the slider which tells it to start logging data. At the same time the webcam is signaled (connected to the computer via USB) to start recording and the motor to start running.

The motor amps up to the requested speed and the slider is pulled across the channel.

2. Ending an experiment.
At 2.7 m down the channel the light-sensor is attached to the channel wall. The light sensor is connected to the motor and measures the light at a frequency of 15 Hz. When the slider passes the light sensor, it will block most light which results in a lower measured value which in turn will signal the engine to stop. This is signaled to the computer which in turn signals the webcam to stop recording and the slider instruments via the Feather 32u4 radio transmitter to stop logging. This setup makes it easy to keep track of the data in the post-processing, since each run generates one file.
2.7 The Slider

The slider consists of a rack which stores the logging equipment, i.e.

- Arduino Uno
- Adafruit Logging Shield
- Feather 32u4 LoRa (Receiver unit)

and a slider base which is the material we want to test. The slider base can be changed according to the material we want to investigate. The load cell is attached in front of the slider, and a rope (bow string) is connected to the load cell and to the winch which pulls the slider. The slider has a steering fin at the back to ensure a steady motion. The friction contribution from the fin is negligible due to its geometry and weight. The slider is depicted in Figure 2.3.

Figure 2.3: Slider used in the channel experiment. A 15 cm ruler is placed on the side to show size.
2.8 Error Analysis

2.8.1 Power supply

The slider had two 9 V batteries on board, one supplying the load cells amplifier and the other one supplying the stack of microcontrollers; Arduino Uno, Adafruit Logging Shield and Feather LoRa (Radio unit). The load cells amplifier was especially sensitive to the batteries voltage in the sense that ADC value read from the load cell for equal load increased as the battery voltage dropped. This was noticeable after a sequence of 15-20 runs. The readings from the load cell were also affected by the voltage drop of the battery supplying the stack of microcontrollers, though not as much. Still, these two together had an unwanted impact on the measurements.

2.8.2 Wear of the channel

The slider sliding across the channel will eventually cause some wear on the channel surface. The number of runs are not that drastic, neither are the load, but some deformation of the bottom must be expected.

2.8.3 Wear of sliders

The sliders experienced different amount of wear during the numerous experiments. High density polyethylene and acrylic were the slider bases that was used the most. The slider base made out of high density polyethylene had no visible scratches or dent. The acrylic slider on the other hand, which has a much lower abrasion and wear resistance than high density polyethylene, [12], had several scratches and clear signs of wear. Such changes to slider base may impact the frictional properties of the system. Ideally, the slider bases should be replaced after a certain number of passings to ensure equal experimental parameters.

Figure 2.4: Clearly visual wear of the acrylic slider base.

2.8.4 Unstable motions

The pull from the rotary device is not perfectly smooth due to the gear teeth of the micro-motor. This will cause some vibrations in the rope which results
2. Channel Experiment Design

in oscillations in the load cell readings. Also, the slider itself is not perfectly stable, it will be wobbling during a run. Furthermore, the slider will be in contact with the channel wall to some extent.

The string pulling the slider did on some occasions come in contact with the channel wall (the hole at the end of the channel where the string goes through). This may have caused additional friction and vibrations in the load cell measurements. The hole should be made bigger to avoid such contact.

To further limit vibrations during a run the string should be attached to the slider via two knuckle eyes to prevent the application of torsional moments, bending moments, lateral and oblique loadings on the transducer.

The electronics taking care of the devices and logging equipment on the slider demanded more space than anticipated. The slider rack that was supposed to store these units was made in advance and should have been made bigger to better store the microcontrollers in a more stable manner.

2.8.5 Slider bases

The slider should have been made in a way that allows additional sections of some material to be added in between the slider base and the slider rack in order to obtain the same height of the load cell w.r.t to the surface for every material. This was not done in the experiments conducted, hence the vertical position of the load cell was not identical for all materials.
CHAPTER 3

Channel Experiment Measurements

This chapter will present the physical phenomena/properties that will be investigated in the channel experiment, the parameters that these depends on and which parameters that can be adjusted by the user.

3.1 Physical properties measured

Seeing that this channel experimental setup is a linear tribometer, the most relevant property is the kinetic coefficient of friction. This value can be determined based on the measurements from the load cell. To put the results in a more detailed framework the velocity will be calculated based on particle tracking made possible from video recording. The third and final property is the temperature which is to be measured in the slider-surface interface by thermistors. The details regarding these measurement techniques will elaborated further (in this section).

3.1.1 Temperature

First off, all handling of the data is done in the post-processing, hence we only store the raw data, i.e. ADC values, on the SD-card when reading from the sensors. The thermistors are connected to the ARD-LTC2499 shield which has a 24-Bit ADC, thus the data obtained will be integers in the range $[0, 2^{24}-1]$.

To obtain accurate temperature values we must use the Steinhart-Hart equation to describe the temperature-resistance curve. The equation is

$$
\frac{1}{T} = A + B \ln(R) + C (\ln(R))^3
$$  \hspace{1cm} (3.1)

where

- $T$ is the temperature (in Kelvins).
- $R$ is the resistance at $T$ (in ohms).
- $A, B$ and $C$ are the Steinhart-Hart coefficients. These vary on the type and model of thermistor and the temperature range of interest.
3. Channel Experiment Measurements

This is a much used third-order approximation which accounts for the highly nonlinear relationship between resistance and temperature. But first, we must convert the raw ADC values from voltage to resistance. The equation which tackles this is the resistive divider equation

$$V_{out} = \frac{R_2}{R_1 + R_2} V_{cc}$$

(3.2)

where

- $V_{out}$ is the voltage output
- $R_2$ is the variable resistor.
- $R_1$ is the fixed resistor. In our case $R_1 = 47000$.
- $V_{cc}$ is the power supply voltage. In our case $V_{cc} = 4.096$ when connected to the ARD-LTC2499.

When we measure a voltage $V_m$ into an Arduino ADC, we will get a number

$$ADC_{val} = \frac{V_m \times 2^{24}}{V_{aref}}$$

(3.3)

With the Ard2499, we have $V_{aref} = 2.048$ For the Arduino Uno, we have $V_{aref} = V_{cc} = 5V$.

Furthermore, since $V_{out} = V_m$ we can combine the equations (3.3.) and (3.2.) to get

$$ADC_{val} = \frac{R_2}{R_2 + 20000} \frac{V_{cc} V_{aref}}{2^{24}}$$

(3.4)

From now on, we write $R = R_2$ for simplicity. Finally, we want to solve for $R$ - the unknown resistance. We rearrange (3.4) and we get

$$R = \frac{20000 \alpha}{1 - \alpha}$$

(3.5)

where

$$\alpha = \frac{ADC_{val} \times V_{aref}}{V_{cc} \times 2^{24}}$$

(3.6)

Furthermore, we must obtain the Steinhart-Hart coefficients where we need to know three values of resistance data for three known temperatures. These can be found in the thermistors data sheet and must be in the temperature range of interest. This gives us three equations with three unknown and can be solved by the following linear system

$$\begin{bmatrix} 1 & \ln(R_1) & \ln^3(R_1) \\ 1 & \ln(R_2) & \ln^3(R_2) \\ 1 & \ln(R_3) & \ln^3(R_3) \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 1/T_1 \\ 1/T_2 \\ 1/T_3 \end{bmatrix}$$

Having obtained the coefficients we can plug in the measured resistance $R$, and solve equation (3.1) for $T$. 

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Temperature measurements in the slider-channel interface

The plan was to conduct experiments with thermistors installed in the slider. But after carrying out several runs in the channel I found it best to remove the thermistors from the channel setup and rather focus on the load cell and camera measurements (loads, acceleration and velocities). The thermistor did not show any significant change in temperature, most likely due the small amount of forces involved - both friction, loads and velocities were relatively low. A quick and coarse calculation can give further insight of the forces at play.

Let's consider the channel experiment setup where the slider slides over the channel surface. The slider has load $m$ and velocity $v$ relative to the surface, the coefficient of friction of the channel-slider system is $\mu_k$. It is widely held view among tribologists that nearly all of the energy dissipated in frictional junctions is transformed into heat [42]. For simplicity, let's assume that all dissipated energy in the sliding contact is dissipated as heat on the sliding surfaces within the real area of contact. According to [24], the heat generation per unit area of contact, $q$, is given by

$$q = \frac{\mu_k F_N v}{A}$$  \hspace{1cm} (3.7)

where $A$ is the area of the slider and $F_N = mg$ is the normal load. Parameter values taken from a standard run is $v = 0.16 \text{ m/s}$, $m = 0.5 \text{ kg}$, $\mu_k = 0.20$, $A = 0.006 \text{ m}^2$. This yields

$$q_{channel} \approx 33 \text{ W/m}^2$$  \hspace{1cm} (3.8)

Such a heat generation is too small to be detected by the thermistors. The thermistors were expected to detect a temperature gradient during the field experiment due to much larger frictional heating. A typical run in the field experiment had the following parameter values; $m = 75.0 \text{ kg}$, $v = 8 \text{ m/s}$, $\mu_k = 0.025$ and $A = 1.8 \text{ m} \times 0.05 \text{ m} = 0.09 \text{ m}^2$. This yields the following heat generation

$$q_{ski} = 1635 \text{ W/m}^2$$  \hspace{1cm} (3.9)

Note that the real area of contact is much smaller than apparent area of the sliders, hence the heat generation within the contacting asperities is much higher in both cases. However, the goal of this example was to show the relative difference in frictional heating between the two setups.

### 3.2 Coefficient of Friction

The ARD-LTC2499 shield used to read the thermistor values has great resolution, but for the load cell measurements we need higher sampling frequency, hence we must use the Arduino Uno. With this shield we could increase the sampling rate to $1 \text{ kHz}$. The load cell has a rated output of $2\mu\text{V/V}$. From the battery supplying the load cell we have $V_{cc} = 6\text{V}$, hence the maximum voltage that can be measured is

$$V_{max} = V_{cc} \times \text{rated output} = 6\text{V} \times 2\mu\text{V/V} = 12m\text{V}$$  \hspace{1cm} (3.10)
3. Channel Experiment Measurements

Further, our reference voltage is $V_{ref} = 5\text{V}$, which gives the smallest possible change in voltage we can measure

$$dV = \frac{5\text{V}}{2^{10}} = 0.00488\text{V}$$ (3.11)

This means that our range consists of

$$n_{tmp} = \frac{12 \times 10^{-3}\text{V}}{5\text{V}} \times 2^{10} \approx 2.45$$ (3.12)

number of steps. This is of course useless, so we had to install an amplifier in our setup. The amplifier has a gain of 350, hence, the number of steps we can achieve is

$$n_{steps} = n_{tmp} \times 350 = 860$$ (3.13)

The load cells capacity is 454 g, this yields the following theoretical resolution

$$\text{res} = \frac{454\text{g}}{860} \approx 0.53\text{g/step}$$ (3.14)

Handling the load cell data

Due to the delicate structure and design of the load cell it was important that a calibration was performed before and after each test sequence (a sequence of runs consists of 5-10 runs with the same load, material and surface) to ensure that the load cell had not taken any damage during the test sequence. A force applied on the load cell in an unfortunate manner may cause permanent deformation which alters the mechanisms in the strain gauge. 5 loads were hung from a pulley and the corresponding ADC values were measured (as seen in Figure 3.1.). These values were then used to make a polynomial fit such that we have a function mapping ADC values to load. The loads used in the calibration were weighed by a high precision scale (Precision Balances ML303T/M00) which has an accuracy of $\sim 1\text{ mg}$. The load cell data sampled at 1000 Hz was stored to a SD-Card which was extracted by a python script. A typical calibration is shown in Table 3.1. According to the theoretical calculations in section 3.2 we would expect a resolution of 0.53 g/step over the entire range of the load cell.

<table>
<thead>
<tr>
<th>Mass (gram)</th>
<th>ADC Value Before</th>
<th>ADC Value After</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>272.4</td>
<td>273.0</td>
</tr>
<tr>
<td>107.3</td>
<td>620.2</td>
<td>617.1</td>
</tr>
<tr>
<td>157.3</td>
<td>780.7</td>
<td>774.5</td>
</tr>
<tr>
<td>197.3</td>
<td>896.0</td>
<td>894.4</td>
</tr>
<tr>
<td>247.3</td>
<td>903.1</td>
<td>903.7</td>
</tr>
</tbody>
</table>
3.2. Coefficient of Friction

(from 0 g to 454 g). As seen from the Table 3.1. and Figure 3.2., the actual calibration is different. In the range from 0 g to approximately 200 g we have a quite linear fit, but for higher loads the load cell becomes non-linear. In the range \( l \in [0, 197.3] \) we have a resolution of

\[
\text{res} = \frac{197.3g}{896 - 272.4} \approx 0.32g/\text{step} \quad (3.15)
\]

The load range could be increased by changing the configuration of the amplifier, but this would result in a lower resolution. Hence the shown range was kept, but then it was important that we held the measurements within this range. These data were then used to make a third degree polynomial to be used in the post-processing of the conducted experiment. The raw ADC data gathered in the experiments was then given as input in the polynomial function \( p_{\text{cal}}(ADC_{\text{val}}) \) to obtain the corresponding load values. A typical dataset obtained from a run in the channel experiment is shown in Figure 3.3. In this figure we have converted the load values (gram) from the calibration procedure into newtons, i.e. \( F_P = mg \), where \( m \) is the measured mass in kilogram and \( g = 9.81 \text{ m/s} \). In the section with (approximately) constant pull, \( F_P \), and constant velocity the net force is zero. Furthermore, the cross-sectional area and velocities used during the experiment are very small, thus the contribution from air drag becomes negligible. Hence the measured pull, \( F_P \), must be equal to the friction force, i.e. \( F_F = |F_P| \). The coefficient of friction is given by

\[
\mu_k = \frac{\bar{F}_F}{F_N} \quad (3.16)
\]

where \( F_F \) is calculated as the arithmetic mean of the values in the measurement section and \( F_N \) is the normal load of the slider.
3. Channel Experiment Measurements

![Figure 3.2: Calibration points for ADC values and loads.](image)

3.3 Velocity

The recorded path of the slider enabled velocity calculations. The camera was mounted 2 meters above the channel in order to cover the entire path of the slider in its field of view. The frames were extracted from the video using FFmpeg which is a free software project which has libraries and programs for handling multimedia data. It features (among others things) a ffmpeg command line program for transcoding multimedia files. The extracted frames were then cropped to only include the channel to ease the image processing. A particle tracking procedure inspired by Jean Raubault’s code for tracking seeds in a water tank [34] was used to track the position of the slider. The procedure consists of the following steps:

1. **Mean Image**
   Compute the mean image based on all the frames extracted from the video’s valid range. This will result in an image which is pitch black where no motion has occurred and slightly lighter in the path of the sliders (the light-emitting diode).

2. **Change in images**
   For each image a 'change image' is computed by subtracting the change image from itself.

3. **Threshold**
   The 'change image' - which is of RGB-format - is used to calculate a threshold, which is an image consisting of only 1 (change) and 0 (no change).

4. **Convolution**
   A convolution disc of relevant size is used to estimate the point of the
3.3. Velocity

(a) Raw dataset converted to values in newton.

(b) Smoothed data set with measurement section.

Figure 3.3: Calibration used on load cell data (a) and the same data (smoothed) (b) where the shaded area indicates the measurement section.
3. Channel Experiment Measurements

slider. The center of the disc is then chosen as the pixel position of the slider.

5. Grid
A grid which relates the image to real-life coordinates is used to obtain the spatial position \((x_i, y_i)\) of the slider in image \(I_i\).

Calculating velocity

Based on the position \((x_i, y_i)\) at each frame/image \(I_i\) the spatial path, velocity and acceleration can be obtained. The spatial resolution in the direction of motion is given by the number of pixels (1280) in the same direction and the length of the channel (3 m). Thus we have \(dx \approx 0.0023\) m. The resolution in time is given by the cameras frames per second, i.e. \(dt = 1/60\) s. The horizontal velocity at each point is obtained by the simple equation

\[
v_i = \frac{x_{i+1} - x_i}{\Delta t}
\]

(3.17)

The resolution in time is quite high compared to the sliders speeds, thus noise can be reduced by using a bigger span, e.g.

\[
v_i = \frac{x_{i+5} - x_i}{5\Delta t}
\]

(3.18)

Since the motor is pulling at (ideally) constant force and the channel surface is assumed uniform, the velocity should also be constant, hence this crude way of estimating the velocity is sufficient for our goal. To further ease the visualization of the velocity profile, the data was smoothed by a Savitzky-Golay filter (Appendix D). The measurement section of the velocity was the same time interval as the one chosen in the load cell measurement section procedure (see Fig. 3.3), hence an interval with constant values for both velocity and pull force was obtained. The corresponding velocity plot is depicted in Figure 3.4.

3.4 Parameters

The channel setup features the ability to experiment with several parameters. The different possibilities will be presented in the following subsections.

3.4.1 Load

The loads that could be experimented with had an upper and lower limit. The lower limit was the minimum weight of the slider which was approximately 450 g, depending on the slider base material. The load cells capacity was 454 g, but in practice the measurements became inaccurate due to bad resolution at loads above ~ 250 g. Given that materials to be used should have a coefficient of friction \(\mu < 0.3\) (obtained from preliminary tests), the maximum loads tested were 750 g to make sure we did not end up in an area with inaccurate measurements.
3.4. Parameters

Figure 3.4: Velocity profile for a slider in the channel experiment. The blue line shows the raw data and the orange line is smoothed.

### 3.4.2 Velocity

The velocity of the slider (given equal load) was varied by changing the motor power through the DC motors duty cycle. The motor power could be set in the range $p \in [0, 255]$ where 0 is minimum and 255 is maximum. Given the specifications of the power supply and the motor control shield, quite high velocities can be obtained if the slider’s weight is kept to a minimum. However, too high velocities will cause unstable motions, in addition there will be no room left for an interval with constant velocity since the entire channel length will be spent on acceleration and deceleration. In order to have a longer interval with constant speed the motor power in the experiments were held in the range $p \in [30, 80]$ which corresponds to velocities in the range $v \in [0.1, 0.4]$ m/s.

$p<20 \sim 30$ will not give any motion. The power is too low to overcome the static coefficient of friction ($\mu_s$) and pull the slider. $p > 80$ is problematic due to the limitations of the load cell and the channel. It will take too much time to accelerate up to this motor power and a faster acceleration may damage the load cell.

### 3.4.3 Slider Base Material

The lab-engineer cut appropriate pieces of different kinds of plastics, in addition we had two types of ski base material. The following materials were used in the channel experiments.

**PMMA** (Polymethyl Methacrylate), also known as acrylic or acrylic glass. This is the same material as the channel.
3. Channel Experiment Measurements

**PETG** (Polyethylene Terephthalate Glycol). This is plastic material with similar mechanical qualities as PMMA, they are both thermoplastics.

**HDPE1000** (High Density Polyethylene) is a subset of the thermoplastic polyethylene. It is considered a very though material with the highest impact strength of any thermoplastic presently made. Its key benefits is high abrasion and wear resistance in addition to low friction due to its self-lubricating character [12]. Self-lubrication is characterized by the materials ability to transfer microscopic amounts of material to the mating surface. In the case of HDPE1000, given enough friction during sliding, small bits of polymer debris will break off. The generated bed of particulate will provide lubrication and reduce the friction.

**UHMW** (Ultra High Molecular Weight Polyethylene). Two different grinds of ski base material were tested - green and blue grind, the green grind has a finer structure than the blue. The exact composition of the ski base material is not known, but its main component is UHWM, hence its frictional qualities should be similar to that of HDPE1000.

### 3.4.4 Dry Friction

Dry friction experiments were conducted to validate the experimental setup. These experiments comprise a block sliding over a surface in the channel. The coefficient of kinetic friction $\mu_k$ is equal to the friction force $F_F$ divided by the force normal to the surface $F_N$, i.e. $\mu_k = F_F/F_N$. There are three empirical laws in dry friction:

- The friction force is independent of the apparent area of contact.
- The friction force is directly proportional to the load.
- The friction force is independent of the sliding velocity.

These laws are in general well obeyed for reasonable loads, contact area, surface roughness and velocities.

### 3.4.5 Wet Friction

Having a channel made entirely out of Plexiglas makes it easy to perform experiments in the regime of wet friction. A given fluid may be applied to wet the surface in order to investigate hydrodynamic lubrication. If an appropriate lubricant is applied the friction should be greatly reduced. It is then essential that ratio

\[ \epsilon = \frac{h}{L} \]

is small, i.e. $\epsilon \ll 1$, where $h$ is the characteristic film thickness and $L$ is the length of the slider.

For some $H$ we should have a minimum of friction. Too much fluid will increase friction, since the slider may sink in it and end up plowing the fluid instead of sliding on top of it. Too little friction will eventually result in dry
friction, which we know from theory is not optimal either (w.r.t to minimizing friction).

**Wetting of the channel**

A spray bottle was used to wet the channel surface. In order to estimate the amount of fluid applied, the average weight of a spray was calculated. A high precision scale was used in this procedure. The fluids used to wet the surface were

- Tap water.
- Tap water with soap added. The mixture consisted of 450 ml tap water with 8 gram of diluted Zalo added (soap from Zalo spray bottle).

**3.4.6 Surface Material and Structure - Contact Area and Rolling Friction**

The surface of the tribometer setup could be altered by adding another material in the bottom of the channel. Whole pieces of a material could be cut to cover the entire surface or one could add particles, spheres, granular material, etc. This feature enables the possibility to conduct testing with contact area as the parameter to investigate. If the material covering the surface is spheres which are free to wander one could carry out simple experiment involving rolling friction.

**Area Of Contact**

The area of contact is a fundamental quantity in the science and engineering of interacting surfaces in relative motion. Understanding the contact of solid bodies is essential when trying to understand friction, but the real area of contact is difficult to determine, especially for dynamic processes ([7] ch. 3). There are two terms that are central in this matter, i.e. the apparent and real area of contact. From Amontons’ Second Law regarding kinetic friction we have that the force of friction is independent of the apparent area of contact. The area of real atomic contact between two solids is usually proportional to the load ([33], ch. 5). This means that if the geometric area is doubled while the normal force is held constant, the fraction of area in atomic contact is halved while the real area of contact and friction force remains the same. As depicted in Figure 3.5, the area of real contact is (in most cases) much smaller than the apparent area of contact.

**Rolling Friction**

Experiments with rolling friction can be conducted by applying a layer of spheres to the channel surface. This friction regime is defined as the resistance to motion that occurs when a surface is rolled over another surface [7]. For relatively hard materials, the coefficient of rolling friction ($\mu_r$) between a spherical body against a flat body generally is in the range of $5 \times 10^{-3}$ to $10^{-5}$. For regular sliding friction of dry bodies the coefficient of sliding friction ($\mu_k$) ranges typically from 0.1 to 1 [7]. In our setup one cannot expect purely rolling, it will more likely be a
3. Channel Experiment Measurements

Figure 3.5: Illustration showing the apparent and real contact area for two surfaces.

combination of rolling and sliding. The situation is also more complicated than what is stated in the definitions above since our setup involves three different materials, i.e. the channel surface, slider and the spheres separating them.

3.5 Conclusions

The experimental setup enables determination of the coefficient of friction for various material combinations by measuring the force necessary to pull a block over a surface. The load cell has an accuracy of $\pm 0.003$ N in the range $0 \leq F \leq 2$ N. This should be sufficient to measure differences between different materials. A camera records the motion of the slider and provides displacement and velocity. The spatial resolution in the direction of motion is $dx = 0.002$ m and the time resolution is $dt = 1/60$ s. The setup also provides the ability to measure the temperature in the interface of the contacting bodies. This feature was omitted from the conducted experiments due to the relatively low loads and velocities used, which in turn yields low frictional heating. By applying fluids or particles to the channel surface one can also investigate frictional regimes such as fluid friction and rolling resistance.
CHAPTER 4

Channel Experiment - Results

Having build this setup from scratch it was important to perform several tests to make sure that everything worked as planned. The aim of this channel experiment was to test the equipment (a lot of it were to be used in the field experiment) and to see if we could obtain results that matches that of friction and lubrication theory.

In this chapter I will present the different experiments conducted in the channel, from basic dry friction experiments to experiments with glass particles and glycerol. This setup was meant to function as a linear tribometer, hence it was important to conduct dry friction experiments to check if the setup could produce results comparable to what has been done earlier. The results from dry friction will be presented first and will serve as a baseline for each material.

Table 4.1: Combinations of materials and fluids investigated.

<table>
<thead>
<tr>
<th>Surface</th>
<th>Slider base material</th>
<th>PMMA</th>
<th>HDPE1000</th>
<th>Blue ski base</th>
<th>Green ski base</th>
<th>PETG</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMMA</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>Glass plate</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4 mm glass spheres*</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1 mm glass spheres*</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-2 mm plastic spheres</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PMMAA with water</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PMMA with soap water</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glass plate with water</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glass plate with soap water</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Glass plate with glycerol</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*the glass particles were used in two ways: 1) Laying freely in the channel, resulting in rolling friction. 2) Glued to the surface below, making a solid, rough surface.

4.1 Dry Friction Experiments

Dry friction experiments were conducted for all 5 sliders bases. They were all tested against the same surface - PMMA, the material of the channel. Furthermore, some of the materials were tested against various glass surfaces to investigate rolling friction and the dependence of contact area. The coefficient of friction should be independent of load and velocity, hence tests where these parameters were varied were conducted.
4. Channel Experiment - Results

4.1.1 Dry sliding

Table 4.2: Coefficient of friction for different slider materials against PMMA (dry friction).

<table>
<thead>
<tr>
<th>Slider material</th>
<th>( \mu_k )</th>
<th>( \sigma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMMA</td>
<td>0.233</td>
<td>0.006</td>
</tr>
<tr>
<td>PETG</td>
<td>0.220</td>
<td>0.003</td>
</tr>
<tr>
<td>HDPE1000</td>
<td>0.197</td>
<td>0.005</td>
</tr>
<tr>
<td>Blue Ski Base</td>
<td>0.201</td>
<td>0.006</td>
</tr>
<tr>
<td>Green Ski Base</td>
<td>0.195</td>
<td>0.006</td>
</tr>
</tbody>
</table>

The sliding friction of polymer-steel combinations is well reported. However, corresponding basic literature on kinetic friction behavior of polymer material combinations is largely lacking. Of the materials tested it was only for PMMA sliding on PMMA that reported values for \( \mu_k \) was found, hence this was the combination that was first tested. According to [16], PMMA vs. PMMA should yield a coefficient of kinetic friction of \( \mu_k = 0.23 \). Nine runs were performed with PMMA vs. PMMA in the channel. As can be seen in Table 4.2., the obtained result fits quite well with the reported value.

The next material on the list was PETG. We could not find any reported values for PETG sliding on PMMA. But seeing that PETG have similar mechanical properties as PMMA, it is reasonable to believe that their frictional behavior should be similar as well. Hence a value similar to \( \mu_k \) for PMMA was expected. Nine runs were conducted and the result was in compliance with the expectations.

Of the three "normal" plastic materials tested, it was HDPE1000 that should have the lowest frictional properties. HDPE1000 has the best wear resistance of the polyethylene materials due to its low coefficient of sliding friction and self-lubricating qualities. The last samples tested during dry sliding against PMMA was two different grinds of ski base. The exact details of the material is not known, but all ski bases are made primarily out of Ultra High Molecular Weight polyethylene (UHMWP), hence a coefficient of friction similar to that of HDPE1000 is expected. The different grinds are meant for different temperatures. The blue grind is used for temperatures in range \( t \in [-2, -8]^{\circ}C \) and the green grind is for even colder conditions, \( t < -8^{\circ}C \). The green grind has a finer structure than the blue, hence we would expect the blue grind to yield slightly lower friction due to fewer contact points between the slider and the surface. As can be seen in Table 4.2., \( \mu_k \) for both grinds are quite similar to that of HDPE1000. Even though the green grind has a slightly higher value than that of the blue grind, they are statistically indistinguishable. Figure 4.1. presents a graphical view of the results.
4.1. Dry Friction Experiments

Figure 4.1: Mean and standard deviations (2σ) of the kinetic coefficient of friction for different materials vs. PMMA measured in the channel experiment.

Load and Velocity

According to the empirical laws concerning dry friction the coefficient of friction should be independent of the sliders load and velocity (for reasonable values). Two test sequences were conducted to see if this holds for PMMA. In the first sequence the load was varied; three different loads where three runs were performed for each load. The same procedure was executed with velocity as the varying parameter. In both cases the results were statistically indistinguishable, i.e. \( \mu_k \) is independent of load and velocity. The results are presented in Figure 4.2.

Investigation of area of contact

The area of contact is of great importance when it comes to friction. The aim of this section is to investigate this property. The idea was to see if we could obtain results in compliance with theory, i.e. lesser area of contact should yield lesser friction. Glass particles of different sizes were used to perform this experiment. Three different surfaces - all with the same material but with different structure were to be tested during sliding friction.

1. Glass particles with 0.4 mm diameter.
2. Glass particles with 0.1 mm diameter.

The glass particles were to be distributed evenly across the channel surface. Ideally, the largest glass particles would yield lesser friction since they would give a surface with fewer points of contact. A continuous glass surface should yield the highest friction. The glass plate was cut to perfectly fit the channel. All surfaces were to be tested against two different slider bases; PMMA and HDPE1000. The rest of the slider bases were left out of this experiment in order to limit the number of parameters due to time constraints.
Figure 4.2: Mean and standard deviations ($2\sigma$) for the coefficient of friction for a) different loads and b) different velocities. The slider base material is PMMA.
In order to test sliding friction with such a setup the glass spheres had to be glued to the surface. If they were simply pored in the channel the spheres would be free to wander which would result in rolling friction. Hence double-sided tape was glued to the channel surface and glass particles were applied. The surface was vacuumed and brushed to ensure a solid surface without any debris. The results from these tests can be seen in Table 4.3 and in Figure 4.3 where standard deviation is listed. All of these experiments were conducted twice with up till 7 days apart for redundancy, the results were the same. Both materials experienced increased friction with increased size of glass spheres which can be attributed to a larger area of contact. In compliance with theory, PMMA experienced an ever higher friction while sliding on the glass plate. A most interesting result is seen for HDPE1000 when sliding on the glass plate. The coefficient of friction is the lowest for all the six tested combinations. This result is most likely due to the self-lubricating qualities of HDPE1000. In general, this kind of feature require a certain amount of friction in order to obtain sufficient self-lubrication. It is the wear from friction that breaks of the debris. From the results for HDPE1000, it seems plausible that the wear from sliding over the glass spheres is insufficient to generate enough debris. However, the glass plate is a very smooth surface which results in high frictional wear which - from the looks of it - breaks of debris to that extent that it lubricates the motion.

4.1.2 Rolling Friction

The next friction phenomena to investigate was rolling friction. This physical property is difficult to handle, since it is dependent on the structure of the surface, the rolling objects/particles and the slider on top. According to (Bhushan, 2013) [7] (ch. 5.2.7.2), the coefficient of friction for a sphere of radius $R$ rolling freely on a plane (elliptical contact) is given as

$$\mu_r = \frac{3\alpha a}{16R}$$

(4.1)

where $\alpha$ is a factor related to hysteresis loss and $a$ is the half width of contact. A similar dependency of $R$ during rolling friction was found by [20, 17]. Three combinations were tested. A PMMA slider base were tested against two different monolayers of glass spheres (diameter equal to 0.1 mm and 0.4 mm). The third combination was HDPE1000 sliding on plastic spheres with diameter 1 mm $\le d \le 2$ mm. As can be seen in Table 4.4, $\mu_r$ decreases for increasing sphere diameter. A more accurate estimation of the parameters involved in rolling

### Table 4.3: Coefficient of friction for different slider material and glass surfaces.

<table>
<thead>
<tr>
<th>Slider material</th>
<th>Surface material and structure</th>
<th>0.4 mm spheres</th>
<th>0.1 mm spheres</th>
<th>Glass plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMMA</td>
<td></td>
<td>0.174</td>
<td>0.207</td>
<td>0.337</td>
</tr>
<tr>
<td>HDPE1000</td>
<td></td>
<td>0.178</td>
<td>0.209</td>
<td>0.156</td>
</tr>
</tbody>
</table>
4. Channel Experiment - Results

Figure 4.3: Mean and standard deviations (2 σ) of the kinetic coefficient of friction for PMMA and HDPE1000 against three different glass surfaces.

Table 4.4: Coefficient of rolling friction for different slider material and sphere diameter.

<table>
<thead>
<tr>
<th>Slider material</th>
<th>Surface material and size</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.1 mm spheres</td>
</tr>
<tr>
<td>PMMA</td>
<td>0.075</td>
</tr>
<tr>
<td>HDPE1000</td>
<td>-</td>
</tr>
</tbody>
</table>

friction is beyond the scope of this thesis, but the trend - as depicted in Figure 4.4 - is in compliance with equation 4.1.

4.2 Wet Friction Experiments

To get a further notion of friction, this section aims to explore the regime of wet friction. At first there were conducted experiments with water added to the channel surface, but because of the hydrophobic behavior of PMMA (channel material) it was hard to get a uniform layer of water across the channel, the water gathered in small puddles instead. The slider ended up plowing the water instead of sliding on top of it. To break up the surface tension a small amount of soap (Zalo) was added to the spray bottle which was used to cover the channel surface with fluid. A small amount of soap is assumed to have a negligible effect on the viscosity of water. Hence this parameter is not altered w.r.t. calculations regarding hydrodynamic lubrication. Two surfaces were used in these test sequences, the PMMA channel surface and a glass plate.
4.2. Wet Friction Experiments

Figure 4.4: Mean and standard deviations $2\sigma$ of the coefficient of rolling friction for PMMA and HDPE1000 against different surfaces.

**Hydrodynamic Lubrication**

Lubrication theory can be applied when there is a thin fluid film separating two parallel surfaces/two rigid boundaries $z = 0$ and $z = h(x, y)$. $U$ is the velocity of the slider and $L$ is the horizontal length scale of the flow, i.e. the length of the slider. In order to use lubrication theory we must have

$$h \ll L$$  \hspace{1cm} (4.2)

If the channel surface is completely wetted by a thin water film, the wet friction force can be written as

$$F_w = \mu AU$$  \hspace{1cm} (4.3)

where $\mu$ is the dynamic viscosity of the fluid, $A$ is the contact area, $U$ is the velocity of the slider and $h$ is the fluid film thickness. The force, $F_w$, is tangential with the slider and the size of the slider is assumed to be large enough so that one need not consider what happens near the edges. Equation (4.3) is deduced in Appendix B. It is difficult to measure the exact thickness of the water film in the channel, but given the amount of water we add, an estimated value can be provided. In addition we measure the force during this wet friction regime. Hence there are two parameters that can be compared to theory. Please note that both the coefficient of friction and the dynamic viscosity are denoted with the Greek letter $\mu$. In order to prevent future confusion, I want to emphasize the difference in notation between the two in this thesis.

- $\mu_k$ denotes the coefficient of kinetic friction, sometimes just called the coefficient of friction.
- $\mu_s$ denotes the coefficient of static friction.
4. Channel Experiment - Results

Figure 4.5: Load cell data for PMMA sliding on PMMA surface wetted with water.

- \( \mu \) denotes the dynamic viscosity of a fluid in general. In the case of performing calculations for a specific fluid, e.g. water, this will be denoted by subscript, i.e. \( \mu_{\text{water}} \).

4.2.1 Results and discussion

The main results from the different material and fluid combinations are presented in Table 4.5. The first experiment with fluid had PMMA as slider base and Table 4.5: Coefficient of friction for different slider material against wetted glass surface.

<table>
<thead>
<tr>
<th>Slider material</th>
<th>Fluid wetting the PMMA surface:</th>
<th>Fluid wetting the glass surface:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Water with soap</td>
<td>Water</td>
</tr>
<tr>
<td>PMMA</td>
<td>0.20</td>
<td>0.209</td>
</tr>
<tr>
<td>HDPE1000</td>
<td>0.05</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Surface and water was used to wet the surface. Due to the hydrophobic character of PMMA, the water gathered in puddles and did not spread out evenly. This resulted in the slider plowing the water instead of sliding on top of it. As can be seen in Figure 4.5, the load cell reached its maximum value (≈200 g) almost during the entire run. To further investigate the hydrophobicity of the material, an attempt was made to measure the contact angle. The drop and its shape are depicted in Figure 4.6. A pipette was used to set a drop on the surface and a camera (Niko D7500) was used to obtain a high resolution image. The angle of contact was measured to approximate 75°, which fits well with Accu Dyne Test’s [39] reported value of 70.9°.
4.2. Wet Friction Experiments

(a) Image of drop.

(b) Detect shape of drop.

Figure 4.6: Process image of drop to calculate angle of contact ($\theta \approx 75^\circ$).

Wetted PMMA surface

Adding soap to the water greatly reduced the surface tension and made it much easier to wet the surface with a relatively thin film. For the combination of PMMA as slider base and surface, the wetting of the surface did not have much effect and the result was quite similar to that of dry sliding. This setup was repeated three days later. 10 runs were conducted with the same settings and the result was the same. For the same case with HDPE1000 as slider base the friction was much lower than that of dry sliding. This result is more in compliance with theory. Equation 4.3 can be used to calculate the theoretical friction force given that the surface is wetted. 10 g of fluid was added, given that it is uniformly distributed across the channel surface this would correspond to a film thickness $h \approx 33\mu m$. We assume that the small amount of soap does not change the viscosity of water, hence $\mu_{\text{water}} = 0.001 \text{ Ns/m}^2$. Furthermore, the area of the slider is $A = 0.0068 m^2$, the load is $m = 0.535 \text{ kg}$ and the sliding velocity is $U = 0.4 \text{ m/s}$. This yield

$$F_F = 0.082 \text{ N}$$

which gives $\mu_k = F_F/F_N = 0.015$. The measured coefficient of friction for HDPE1000 ($\mu_{k,\text{pmma}} = 0.05$) is much closer than the result for PMMA, but it is still off by a factor 3.
4. Channel Experiment - Results

Wetted Glass Surface

Adding water to the glass surface gave much lower friction for the PMMA slider base than what was obtained at the PMMA surface with water. The water still formed puddles, but the plowing effect was less present, hence a valid measurement of this combination was obtained. The result was similar to that of water with soap wetting the PMMA surface. A significant drop in friction occurred when the same water-soap mix was added to the glass surface. The coefficient of friction decreased from $\mu_k = 0.209$ to $\mu_k = 0.082$. The difference in $\mu_k$ between the two fluids was not as striking for HDPE1000. The coefficient of friction was quite low in both cases. For the combination of HDPE1000 with soap-water mixture added to the surface a coefficient of friction of $\mu_k = 0.019$ was obtained, which is quite close to the theoretical solution ($\mu_k = 0.015$).

Experiments with glycerol

There were also conducted experiments with glycerol wetting the surface. Due to the high viscosity of this fluid, the friction force was outside the measurement range of the load cell. Experiments with fluids with high viscosity require a load cell with much higher capacity.

4.3 Limitations

The main limitations for this setup are the lack of data regarding the real area of contact and the thickness of the water film. Potential improvements will be discussed in chapter 8.2.1.

4.4 Conclusion

The channel experiment has been carried out and the overall results are promising. In most cases, the results from the experiments involving dry friction were in compliance with theory. The results where load and velocity were varied fits well with the basic empirical laws of dry friction. The results concerning rolling friction are broadly in line with theory, given that the rolling resistance should decrease with sphere diameter. However, to fully understand the physics at work in this regime requires further and a more accurate investigation than what is presented here. Furthermore, the area of real contact was investigated. Experiments with surfaces with varying roughness were conducted. The assumption was that the surface with the finest roughness would yield the highest friction due to an increase in contact spots. The findings for the PMMA slider base was in line with said assumption. In the case of the HDPE1000 slider base, a distinct decrease in friction was found for the surface with the finest roughness compared to the two rougher surfaces. This result is attributed to the self-lubricating quality of HDPE1000. Regarding the experiments with a wetted surface, it is difficult to obtain accurate input values for the equation concerning hydrodynamic lubrication. Still, the results were promising. The general trend was lower friction when compared to dry sliding. A small amount of soap added to the water gave a more uniformly distributed water film since it greatly lowers the surface tension. For the HDPE1000 slider base sliding on a glass surface wetted with soap-water the friction force was quite close to
the theoretical solution. In the following chapters the design, measurement methods and results concerning the field experiment will be presented.
CHAPTER 5

Field Experiment Design

5.1 General Overview

A setup to be used in the field was built parallel to the channel experiment. Engineers from the Physics Department built a winch to pull the slider and the Lab-Engineer at the Fluid-Mechanics Department built a sled and made sure that the right equipment were ordered. The field experiment consisted of

- A winch which pulls the sled.
- A motor with full speed control, handled from a python script.
- A sled mounted on skis.
- Measuring equipment and data-logging system.

The measuring equipment consists of microcontrollers and instruments suitable for the physical properties we want to measure. The microcontrollers handles the logging sequence which is very similar to that of the channel experiment. Thermistors were used to measure the temperature evolution beneath the ski, a load cell measured the force at which the winch pulled the sled and a GPS measured velocity. The setup will be documented in detail in the following sections.

5.2 The Winch

The winch consists of a winch drum (diameter 31.5 cm) mounted on support legs. The setup is mobile, but it was important that the construction was solid and heavy enough to withstand the motion of the drum when pulling the sled. It is crucial that the winch is stable such to ensure that the measurements we obtain are due to the motion of skiing and not the wobbling from the winch. Three L-shaped aluminum profiles were attached to the bottom of the rack on which the winch was mounted. These profiles reached 10-15 cm into the snow and held the winch in place. In addition, the three car batteries were placed on the same rack, further securing that the rack holding the winch did not move during the experiments. The winch had a steering mechanism for the rope enabling the possibility to guide the rope while reeling in the sled, thus enabling a uniform distribution of the rope over the winch drum. The rope used in the experiment was a 150 m long static nylon rope with tensile strength equal to 100 kg.
5. Field Experiment Design

Figure 5.1: Image from field testing. The three car batteries are placed behind the winch, keeping the construction in place. The computer is controlling the motor via a microcontroller.

Figure 5.2: The winch used in the experiment. Guiding the rope while a run is conducted is made possibly by the pin seen in the lower right part of the image.
5.3 Motor/engine

The motor used in the setup was a Rimfire 65CC (90-85-160) brush-less electric motor with 7500W as max continuous power, more than enough to accelerate the sled to realistic cross-country velocities within a reasonable amount of time given conservative estimation of the coefficient of friction. A quick calculation may be useful to give the reader a better perspective.

Motor power and acceleration - an example

- Assume that a flat, straight test track is 100 meters long. We want to have a long section with constant velocity to obtain good data for calculating the coefficient of friction, but we also need a smooth acceleration phase to ensure stability of the sled and avoid excessive tension in the rope. A acceleration of 1 m/s from \( v_0 = 0 \) m/s to \( v_1 = 8 \) m/s will result in a section of approximately 30-35 m of the track being spent on acceleration, this leaves us with plenty of track for measuring and deceleration. There are many reported values of the coefficient of friction, but to be safe we use a conservative value of \( \mu_k = 0.25 \) (approximately 10 times higher than expected). Furthermore, the mass of the sled is \( m = 70 \) kg. The pull force needed for acceleration is given by

\[
\sum F = ma = F_{\text{pull}} - F_{\text{friction}} \Rightarrow F_{\text{pull}} = ma + F_{\text{friction}}
\]

If we insert the values we get \( F_{\text{pull}} = 245 \) N. The maximum power needed in the end of the acceleration phase is \( P = Fv_1 = 1960 \) W, which is far within the specifications of the motor.

The setup was designed in such a way that we had full speed control, enabling the user to give motor power as digital input. The motor originally came with a controller used to handle the speed of the motor. This was a controller similar to those used to control model-cares, boats, etc., and was ready for use straight from the box. The downside with this controller is - like with all hand controllers - that the velocity is given by the force at which you squeeze your finger inwards - not very accurate when trying to keep a constant velocity, and also hard to reproduce. Hence, Arduino microcontrollers and scripts were used to achieve full control over the motor.

Arduino offers a handy library - Servo - to handles such cases. The Servo Library allows an Arduino board to control RC servo motors. These have integrated gears and a shaft that can be precisely controlled. This library works for both standard and continuous rotation servos. Standard servos allow the shaft to be positioned at various angles, normally between 0 and 180 degrees, while continuous rotation servos allow the rotation of the to be set to various speeds.

A velocity is set by writing a value in microseconds to the servo which controls the shaft accordingly. In the case of a standard servo, the written value sets the angle of the shaft. This value is usually in the range of \( v \in [1000, 2000] \), where 1500 is equal to zero movement, 1000 is fully counter-clockwise and 2000 is fully clockwise. Since our motor and motor controller is meant for aircraft, the
5. Field Experiment Design

configuration is different; 1000 microseconds and 1500 microseconds corresponds to zero movement and mid-throttle, respectively.

5.4 Thermistors

Thermistors were used to measure the temperature evolution in the ski-snow interface. The thermistors are the same as those used in the channel experiment. 7 thermistors were mounted along the left ski on the sled. To obtain a detailed picture of the temperature situation under the ski it was decided to place all thermistors under one ski. A more detailed description of the procedure follows below.

To ensure that the full accuracy of the thermistors were utilized the ARD-LTC2499 shield was used to read from the sensors. As mentioned earlier, this shield offers a 24-bit ADC, but only 15 Hz sampling frequency (at the most). Seven thermistors connected to the same shield would greatly reduce the sampling frequency. To remedy this, seven shields were stacked on top on each other, such that each shield was connected to only one thermistor. With this setup a sampling frequency of 10 Hz was achieved. The reduction from 15 Hz to 10 Hz is caused by the time it takes to read from the different shields. To control all ARD-LTC2499 boards with one object (and one script) is made possible by using different addresses for each board. An address is set by manually configuring the order of a set of jumpers on the board. To ensure that thermistors measurements were synchronized with the other measurements a radio-signal setup similar to the one used in the channel experiment was implemented.

5.4.1 Ski and thermistors

We had two pairs of skis at our disposal (given to us/the department by Olympiatoppen). The main questions of this part of the setup were how many thermistors that should be used and where to place them. This became a conflict of interest between resolution in time and in space. It is expected that the increase in temperature will occur as soon as the skis start gliding at reasonable cross-country velocities, hence a high sampling rate is desirable. On the other hand, it is also of great interest to be able to measure the temperature evolution across the entire length of the skis at the same time as we want to be sure to have sufficient thermistors in the section of the ski where the highest increase in temperature is expected. Hence, it is desirable to have as many thermistors as possible. The number of thermistors that could be used were limited by the number of ADC shields. As mentioned earlier, one thermistor per shield would result in 10 Hz sampling frequency. Since the change in temperature is expected to happen quite fast I did not want to reduce the sampling rate further. Hence the number of thermistors was seven. The next question was whether to place all thermistors on one ski or place them on both right and left ski. We assume that there are no difference between the right and the left ski, nor the left or right side of the track, hence all thermistors were placed on the left ski (for both cold and wet grind). A pressure distribution of the ski was performed to determine where to place the thermistors. A thickness ruler
was used to detect where the two area areas of contact started and ended for different loads. For normal load equal to 75 kg the following intervals - starting from the tip of the ski - were in contact with the floor:

- Area 1 (in front of foot): $x \in [22, 49]$ cm.
- Area 2 (behind foot): $x \in [130, 150]$ cm.

For a lighter normal load, 25 kg, the areas were

- Area 1 (in front of foot): $x \in [22, 36]$ cm.
- Area 2 (behind foot): $x \in [144, 178]$ cm.

The areas obtained for $m = 75$ kg is the one of most interest since this load corresponds to the weight of an average, grown man. The pressure distribution for the lighter load was also taken into consideration when determining the thermistor positions in case there would be carried out runs with less load. Based on the pressure distributions performed the following positions were determined: An image displaying the thermistors along a left skating ski is depicted in Figure 5.3.

Table 5.1: Position of thermistors in cm from ski tip.

<table>
<thead>
<tr>
<th>Thermistor number</th>
<th>Position in cm from ski tip</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>25 cm</td>
</tr>
<tr>
<td>T2</td>
<td>40 cm</td>
</tr>
<tr>
<td>T3</td>
<td>130 cm</td>
</tr>
<tr>
<td>T4</td>
<td>140 cm</td>
</tr>
<tr>
<td>T5</td>
<td>150 cm</td>
</tr>
<tr>
<td>T6</td>
<td>160 cm</td>
</tr>
<tr>
<td>T7</td>
<td>170 cm</td>
</tr>
</tbody>
</table>

5.4.2 Attaching thermistors

The lab engineer drilled seven holes in the left ski for both pairs. The thermistor legs were put in insulation caps (ceramic tubes, Figure 2.2) to prevent contact and soldered to the wires connecting them to the amplifier. These wires were put in heat shrink tubes for further protection. The thermistors were then carefully placed in the hole and positioned such the top of the thermistor was aligned with the ski base surface ($\pm 1$ mm). The top of the holes were glued to keep the wires and thermistors in place. The glue could easily be heated up to a liquid state, making it possible to adjust the thermistors vertical position if necessary. A ski with wires attached is depicted in Figure 5.4. The wires connected to the thermistors came from the same internal parallel cable which was attached to a headboard adapter. This headboard adapter could then be attached to the amplifier, connecting all thermistors at once. The same procedure was done for the left ski on the other pair. This setup made it easy to change skis without having to rearrange many wires.
5. Field Experiment Design

Figure 5.3: Thermistor positions along the ski. The positions are given in cm from ski tip.

5.4.3 Calibration

As described in 3.1.1, the thermistors must first be calibrated for their non-linear behavior using the Steinhart-Hart equation. It is essential that the known resistance values to be used in equation (3.1) are in the range of the temperature that will be measured. Hence the following temperatures resistance values were chosen: $R_1 = -10^\circ$C, $R_2 = 0^\circ$C and $R_3 = 5^\circ$C. To further ensure that the sensors worked properly, tests were conducted in both laboratory and field. The calibration test in the laboratory was performed by placing the instrumented skis in a long and slender container filled with snow. The skis were loaded with 75 kg to make sure that all thermistors were in contact with the snow. The snow temperature is assumed to converge to 0$^\circ$C as its melting. The result can be seen in Figure 5.5. The sensor at 130 cm measures a much higher temperature due to the fact that it is not in contact with the snow. This was discovered by
Figure 5.4: Wires connecting thermistors to ADC. The holes are glued on the top to keep the wires in place.
inspection. The same goes for the sensor at 140 cm. The five remaining sensors
measures a constant temperatures in the range $t \in [-0.09, 0.14] ^\circ C$. It should
be noted that if the snow is contaminated, e.g. it contains salt, its melting
point may be slightly higher than $0 ^\circ C$. To further investigate the behavior of
the thermistors a measurement was carried out in the field. The sled (with
the skis attached) was tilted such that the thermistors measured the ambient
temperature in the shadow. As can be seen in Figure 5.6, they all converge
to approximately the same temperature. The differences may be explained
by their positions along and inside the ski. Overall, these results shows that
the thermistors works as expected. All sensors in contact with melting snow
measures approximately $0 ^\circ C$ and they all tend to measure the same temperature
given equal circumstances in the field.

5.5 Load Cell

Larger forces are at play in the field experiment, hence a larger load cell was
used. The load cell - S2M - is a S-shaped force transducer which measures tensile
and compressive forces. The measuring body is an aluminum bending beam on
which strain gauges are installed. The strain gauges are arranged so that the two
are stretched and the other two compressed when a force acts on the transducer.
The rope which pulls the sled is attached to the load cell through two knuckle
eyes. The use of one knuckle eye prevents the application of torsional moments
on the transducer. Having a second knuckle eye prevents bending moments,
lateral and oblique loadings. The S2M load cell has a capacity of 200 N and a
overload stop with a force limit of 1000 $\%$ times the max capacity. Furthermore,
it has a nominal sensitivity of $2 \text{ mV/V}$.

5.5.1 Calibration

Since the load cell has a nominal sensitivity of $2 \text{ mV/V}$ we need an amplifier
to achieve reasonable readings for the Arduino Uno shield. The measuring
amplifier - Clip AE301 - was from the same vendor (HBM) as the load cell,
hence they are suitable for measurements of such devices.

The Clip AE301 features several clips and jumpers which can easily be config-
ured to fit with the chosen load cells capacity. The Arduino Uno can handle
input voltage between 0 V and 5 V, hence the load cell cells capacity has to be
fit accordingly. By adjusting the pins of the amplifier the appropriate measuring
range was achieved. 0 kg was set to give 0 V as output and 20 kg was set to
give 5 V as output. This was achieved by using the same pulley mechanism
as shown in Figure 3.1. The load cell was loaded with said weights while the
output voltage was read by a multimeter. The pins on the amplifier were then
adjusted in order to obtain the desired voltage. The voltage for additional loads
in the current range was checked and these points were used to make a linear fit
that converts voltage values to load. With these features it is easy to change to
the voltage output accordingly if one is to use a load cell with higher or lower
capacity.
Figure 5.5: Temperature profile of test in laboratory. The skis are placed on top of melting snow. (b) shows the marked section from (a).
5. Field Experiment Design

(a) Temperature profile.

(b) Closeup.

Figure 5.6: Temperature profile of measurement in field. (b) shows the marked section from (a).
5.6 GPS

The GPS model used in our field experiment was Copernicus II GPS Module. This is a 12-channel receiver from Trimble, and it has a small form factor which makes it a great device for applications requiring precise GPS control. This device offers output in the form of standard NMEA 0183 sentences, which includes latitude, longitude, GPS quality indicator, number of satellites in use and speed over ground. The sampling rate for the NMEA protocol is 1 Hz and the velocity measurements has an accuracy of 0.06 m/sec.

In the conducted experiments we used the Copernicus II GPS Receivers default configuration which outputs two messages: GGA and VTG. These messages are output at a 1 second interval with "GP" ID and checksums. These messages are output at all times during operation, with or without a fix. To make sure the GPS had a fix when conducting the experiments it had to be turned in advance to get sufficient signal from the satellites.

5.6.1 Output

The GGA message includes time, position and fix related data for the GPS receiver. A typical output is shown below.

- GPGGA,092355.00,5956.40026,N,01043.34929,E,1,04,7.59,00100,M,040,M,,*62

<table>
<thead>
<tr>
<th>Outout</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>092355.00</td>
<td>UTC of Position</td>
</tr>
<tr>
<td>5956.40026,N</td>
<td>Latitude (North)</td>
</tr>
<tr>
<td>01043.34929,E</td>
<td>Longitude (East)</td>
</tr>
<tr>
<td>1</td>
<td>GPS Quality Indicator: 0=invalid fix, 1=GPS fix</td>
</tr>
<tr>
<td>04</td>
<td>Number of Satellites in Use</td>
</tr>
<tr>
<td>7.59</td>
<td>Horizontal Dilution of Precision (HDOP)</td>
</tr>
<tr>
<td>00100, M</td>
<td>Antenna Altitude in Meters, M = Meters</td>
</tr>
<tr>
<td>040, M</td>
<td>Geoidal Separation in Meters, M=Meters</td>
</tr>
<tr>
<td>-</td>
<td>Age of Differential GPS Data.</td>
</tr>
<tr>
<td>*62</td>
<td>Checksum</td>
</tr>
</tbody>
</table>

The output of interest is the position data and it is convenient to have a timestamp. The second output is the VTG. This message conveys the actual track made good (COG) and the speed relative to the ground. A typical output is shown below.

- GPVTG,148.1,T,145.8,M,002.6,N,004.9,K,A+*2E

The output of interest from the VTG message is the speed. To have an additional measure of speed one can calculate the velocity from the position data from the GPGGA message. This is achieved by using the LatLon package that Python offers. The LatLon package have methods for representing geographic
coordinates including the ability to calculate distance between lat/lon points using the WGS84 approximation. The World Geodetic System (WGS84) is the reference coordinate system for The Global Position System. It comprises of a reference ellipsoid, a standard coordinate system, altitude data and a geoid.

5.7 Microcontrollers

The microcontrollers that handled the logging were put in a box on top of the sled. This box contained

- GPS connected to Adafruit Assembled Data Logging Shield mounted on top of an Arduino Uno.
- CLIP AE301 connected to Adafruit Assembled Data Logging Shield mounted on top of an Ardunio Uno.
- Thermistor wires from the left ski connected to an amplifier which in turn was connected to the 7 24-bit ADC’s. These 7 seven shields were stacked on top of an Ardunio Uno. On top of the entire stack was a Adafruit Assembled Data Logging Shield.
- Feather 32u4 (receiver).

Since thermistors, GPS and load cell were all sampled at different rate each device had its own Adafruit Assembled Data Logging Shield. This made the obtained data easy to post-process, since each SD-card only contains data from one device at a fixed sampling rate. All three data logging shields were connected to the Feather 32u4 unit. This Feather unit worked as a radio signal receiver. When this unit received a signal from base it transmits a signal to the logging units (to either start or stop logging). Making use of the Feather 32u4 in such a manner greatly simplifies the post processing since this setup produces one file per run per SD-Card.

5.8 A note about logistics and logging

A field trip had to be planned a long time in advance. There were a limited number of locations where this experiment could be carried out and these tracks were frequently occupied. Furthermore, the setup consists of many parts and the total load was almost 200 kg, hence a car with a large trunk was needed. Ideally,
there should be at least two persons present when conducting the experiment for both safety reasons and efficiency. The gear had to be carried 200-300 meters from the parking lot to the designated location which was quite heavy and time-consuming for only one person. Due to all these logistic challenges there were a limited number of trips that could be made, hence mistakes and delays could not be afforded once an experiment was to be conducted. The most important feature, besides that the mechanical setup is working, was the logging sequence. It would be a disaster if a whole day was spent in the field only to realize that there was no valid data stored on the SD-card. This is where the use of the Feather LoRa unit becomes extra handy. As mentioned earlier, to have a unique file for each run makes the post-processing easier, but its features is even more important for securing the logging itself. A simple, quick and naive approach would be to log the entire field experiment as one, i.e.
5. Field Experiment Design

let the sensors log the entire day where all data is saved in one file (for each instrument). This could cause problems if the microcontrollers should lose power while a file on the SD card is open and being written to. Such an event would result in the entire file being corrupted and lost. By using the Feather LoRa unit and writing to a new file for every run, only the current file would be lost if anything were to happen (accidental power loss, disconnection of the power input, bug).

5.9 Power Supply

The winch was powered by three car batteries (\(3 \times 12 \text{ V} = 36 \text{ V}\)). These batteries also functioned as power supply for the computer used to control the motor and for the thermometer used to measure temperature at site. Given the maximum expected power consumption calculated in Example 5.3., we can calculate how long the power supply is expected to last.

\[
I = \frac{P}{V} = \frac{1960 \text{W}}{36 \text{V}} \approx 54 \text{ A}
\]

The car batteries have an ampere hour of approximately 80 Ah each, hence we had at least 90 minutes of running time at our disposal. This estimation is highly conservative, and given that a run seldom takes more than one minute, we were never in danger of running out of power while conducting the experiments. The logging equipment and the load cell were powered by two MC batteries (2 \(\times 12 \text{ V}\)). These batteries were put on top of the sled, next to the box containing the logging equipment. Both the load cell and microcontrollers had very low power consumption. The two MC batteries were more than enough to power the devices throughout the day. All batteries were charged overnight after every field trip.

5.10 Arming the motor - initialization sequence

The motor controller (Phoenix EDGE HV) is actually meant for controlling air crafts such as helicopters or planes. Hence this speed controller needs to go through an arming procedure before it can operate the motor (unlike controllers meant for cars). The initialization sequence consists of the following steps:

1. Connect the EDGE controller to the throttle channel on the receiver (in our case the Arduino Uno, orange and dark purple wire).

2. Turn the transmitter ON and set the throttle stick to mid-throttle. This corresponds to sending a value of 1500 microseconds to the servo (\(\text{servo.writeMicroseconds(1500)}\)).

3. Connect the motor battery to the speed controller (turn switch located on the side of the winch clockwise). The speed controller will remain disarmed and will not operate the motor until it receives the 0 % throttle signal, i.e. \(\text{servo.writeMicroseconds(1000)}\).
5.11 Track (Field Experiment Location)

The field experiments were conducted at Sognsvann Snøpark and Holmenkollen. In both places the most straight part of the course was chosen.

Sognsvann Snøpark

The course at this location was approximately 120 meters long. Even though the course seemed straight in both horizontal and vertical direction at first sight, it became clear that it would have been a great improvement to have a more flat and straight track when conducting the experiment. The most critical downside with the slope was the uneven elevation profile which made it difficult to find an appropriate interval to use as measurement section for the coefficient of friction. 6 out of 7 field experiments were conducted at this location.

Holmenkollen

The track used at Holmenkollen was far more straight in both horizontal and vertical direction than the one used at Sognsvann Snøpark. A section of approximately 80 meters behind the Biathlon Stadium was used as test track. Ideally, most of the testing should have been done here, but only one field experiment was carried out at this location. Due to the many ski events (including FIS and IBU World Cup races) held at Holmenkollen it was hard to find available time for the experiments.

5.12 Performing an experiment

All equipment, devices, mechanics and electronics which makes the experiment have been presented. To give the reader a better understanding of how it all works together a walk-through of an experiment will be given.

5.12.1 Preparations

The platform containing the winch and the batteries is placed right beside the track. It is important to make sure that the aluminum profiles under the platform are packed as deep as possible into the snow to ensure stability. If a track can be prepped for the purpose of the experiment, it is desirable to have the track turn to either side of the winch. In that way the platform can be in perfect alignment with the track and there is no risk of the sled colliding with the winch (the sled slides past the winch instead).

The sled is mounted with the desired load and placed at an appropriate distance from the winch. The distance is limited by the length of the rope and the length of straight part of the track. The computer is connected to an Arduino Uno which in turn is connected to the motors controller system. The computer is also connected to a Feather 32u4 (radio transmitter). Before the experiment is started the current time (hh.mm), temperature and throttle is written down. This makes it effortless to keep track of the different files in the post processing since the GPVTG output from the GPS features a time stamp.
5. Field Experiment Design

5.12.2 Running the experiment

In order to handle both Arduino units (motor controller and radio transmitter) at the same time a Python script is used. This script communicates with both Arduino Uno and the Feather 32u4 and incorporates the arming sequence described in section 5.10. The program consists of the following steps:

1. **Arm motor.**
   The power can be turned on once this step is executed.

2. **Set desired throttle.**
   \[ t \in [1000, 1500] \text{ microseconds} \]. 1000 microseconds corresponds to 0% throttle and 1500 is full throttle.

3. **Start motor.** When this step is executed the Feather 32u4 signals the devices on the sled to start logging before the motors starts running.

4. **Stop motor.** The motor is turned off when this step is executed. The logging devices logs for 10 more seconds before stopping to include the deceleration phase of the sled. When the sled is at rest the power can be turned off.

While the sled is being reeled in, it is important to use the steering pin to guide the rope such that it is distributed evenly over the winch drum. If one fails to use this properly the rope may gather up in an unfortunate manner.

5.13 Error Analysis

- It was difficult to achieve identical vertical positions for every thermistor in the ski base. The thermistors measurements are highly dependent on their nearby surroundings, hence their respective positions will influence their measured temperatures.

- The velocity measurements were not optimal. A GPS with a higher sampling frequency would have been useful to achieve a more detailed velocity profile. The GPS that was used could have be configured to sample faster, but it would be at the cost of accuracy.

- The rope that was used had more stretch than anticipated. The acceleration phase had to be most careful in order to avoid too much tension being built up in the rope.

- The slope is never absolutely flat. Even if the surface beneath the snow is approximately flat, the snow may be packed in such a manner that the elevation profile becomes more uneven. To further improve the calculations regarding friction, one should include the height as parameter to determine the contribution from gravity.
CHAPTER 6

Field Experiment Measurements

This chapter will present the physical properties that will be measured and investigated in the Field Experiment and the parameters that these depends on. Some parameters can be adjusted manually in the setup and other parameters (such as weather) are less controllable.

6.1 Physical properties measured

The main physical property of interest is the kinetic friction between ski base and snow. This property, denoted $\mu_k$, is dependent on load, velocity, apparent contact area, surface topography and temperature. The obtained measurements will be compared and rationalized on the basis of hydrodynamic friction. In addition to the coefficient of friction which is calculated based on the measurements of the load cell (5.5), the temperature in the snow-ski interface and velocity of the sled will be measured and investigated.

6.1.1 Coefficient of friction

The S2M load cell measures the force, $F_P$, at which the winch pulls the sled during a run. The pull force $F_P$ is a combination of the frictional forces between the ski and the snow and aerodynamic drag of the sled. The product of the cross-sectional area $A$ and drag coefficient $C_D$ is smaller than a skier (0.09 vs $0.54 \text{ m}^2$ [2]). For the typical friction coefficients and velocities in this experiment, the air drag is less than 4% of the friction force and will therefore be neglected in the analysis. Hence the friction force, $F_F$, will be regarded as equal to the pull force, $F_P$, in section of constant velocity. Due to the acceleration phase, elasticity in the rope and the elevation profile, only a smaller part of test run can be used as a measurement section as shown in Figure 6.1. The coefficient of friction is determined by the measured force in this section and the normal load of the sled, $F_N$, and is given by

$$\mu_k = \frac{\bar{F}_F}{F_N}$$

where $\bar{F}_F$ is the arithmetic mean of the obtained values in the measurement section.
6. Field Experiment Measurements

6.1.2 Temperature in the snow-ski interface

Thermistors are attached in the ski base and will measure the temperature in the snow-ski interface. An increase in temperature is expected due to the frictional heating caused by sliding. The relevant data that will be presented is the maximum increase in temperature $\Delta T_{\text{max}}$ in addition to the temperature profile along the ski. $\Delta T_{\text{max}}$ is defined as the difference from the temperature $T_0$ before sliding and the maximum temperature $T_{\text{max}}$ during sliding (Figure 6.2). Furthermore, we will look at the temperature data for different conditions, i.e. variations in snow temperature, velocity and ski grind (wet or cold).

Heat considerations

In order to generate a water film the motion of skiing must produce enough energy to raise the snow temperature to its melting point as well as melting it. The maximum increase in temperature at the contact area between sliding bodies is given by [33] as

$$\Delta T = 2J \left( \frac{t}{\pi \lambda C_v \rho} \right)^{1/2}$$  \hspace{1cm} (6.2)

where $J$ is the heat current, $t$ is time of contact, and $\lambda$, $C_v$, $\rho$ are the thermal conductivity, the specific heat capacity and the density of the material, respectively. Equation 6.2 is deduced in Appendix C. If we assume that all of the heat generated goes into the lower solid we can calculate the maximum (flash) temperature in a junction. For the sake of simplicity, we assume that all asperities in the snow are ice particles when analyzing the thermal behavior. For ice at $-5^\circ C$ we have the following parameter values; $\rho = 917.5 \text{ kg/m}^3$, $\lambda =$
2.25 and \( C_v = 2027 \) J/kgK. The heat current \( J \) is given by \( J = \sigma_k v \), where \( \sigma_k = \sigma_0 \mu_k \). The yield stress of ice is \( \sigma_0 = 4 \times 10^7 \) N/m\(^2\). The coefficient of friction before a thin water film has been formed is \( \mu_k = 0.38 \) ([33]). \( t \) is given by \( t = D/v \), which is how long a junction will survive during steady sliding. If we use \( D = 100 \mu m \) (where \( D \) is the diameter of the junction) and a sliding velocity of \( v = 1 \) m/s we get \( t = 10^{-4} \). These values yield
\[
\Delta T = 84 \text{K}
\]
which would be the maximum possible increase in temperature of an ice grain during passing of a ski. This temperature will of course never occur since the asperity would start to melt at some point along the length of the slider. In addition, the friction would drop as soon as a layer of melt water forms, resulting in less frictional heat generated.

### 6.1.3 Velocity

The friction of the ski is dependent on the temperature beneath the ski, which in turn is dependent on velocity. Hence, velocity data will be obtained from GPS measurements. The velocity listed for the conducted runs, e.g. in tables, is the velocity obtained during steady sliding for that run. A dataset from a GPS measurement conducted March 14 is shown in Figure 6.3.

### 6.1.4 Ambient and snow temperature

The ambient and snow temperature during an experiment were measured by a Doric Trendindicator 412A (resolution 0.1°C, accuracy 0.6°C). The snow temperature was measured approximately 5 cm below the surface and the ambient temperature 30 cm above the surface. Both measurements were
6. Field Experiment Measurements

Figure 6.3: The sleds velocity profile during a run with motor power 1300. The obtained velocity during steady sliding is $v \approx 15$ km/h (indicated by the shaded area).

Conducted in the shade. In order to obtain lubrication by water film the friction caused by sliding must heat and melt the snow in the top layer. If the ambient and snow temperature is very low (typical less than -20°C), the frictional heating caused by the motion of skiing will not be sufficient to raise the temperature to a degree of which melting occurs. The friction will then be dominated by solid-solid interactions. At higher temperatures, close to the melting point, too much water may be produced which results in capillary drag effects [15]. The capillary bridges acts as bonds between the slider and the snow surface and results in a drag force on the slider. The increase in friction at higher temperatures due to capillarity is well mentioned in the literature, however, there is no existing physical or experimental model that describes the contribution of capillary bridges to the friction force [8].

6.2 Parameters

The kinetic friction between ski and snow is influenced by several parameters. Those that can be varied in the experimental setup will be presented in the following subsections.

6.2.1 Velocity

Velocity is varied from $v = 1$ m/s to $v = 9$ m/s. At velocities $v < 1$ m/s, the torque of the motor is insufficient to surmount the static friction. The upper velocity is limited by the acceleration phase and the stability of the sled. To get as accurate as possible results for the coefficient of friction it is desirable that the sled has a constant velocity for as long as possible. For higher velocities more of the track will be spent on acceleration and deceleration, and ultimately
A note on weather and temperature

will result in no part with constant acceleration. It was experimented with a faster acceleration to resolve this issue, but this lead to a new problem w.r.t. the rope. The rope is supposed to have very little stretch, but over a distance of 100-150 meter this was far from the case. At this distance it was possible to stretch the rope up till 1 to 2 meters. A too abrupt acceleration resulted in a high tension being built up in the rope before it catapulted the sled down the track. The sled achieved a velocity higher than the rate at which the winch pulled the rope, resulting in the sled sliding over the rope. This led to high noise in the load cell data and unreliable thermistor data.

6.2.2 Load

By adding weight plates the load of the sled could easily be varied from 12.75 kg to 100 kg. However, the skis are meant for an adult person, so too light or too heavy loads will result in a unrealistic pressure compared to that of the mean static pressure exerted by a skier on a snow surface.

6.2.3 Ski base

We had two pairs of ski at our disposal, one pair with cold grind and the other one with wet grind. The cold grind has the finest roughness and are used under cold and dry conditions. Its roughness yields a larger area of contact that should result in higher friction and more produced meltwater. The wet grind has a more coarse structure and is used under warmer and wet conditions. Moldestad (1999) [30] measured the ski base structure roughness (arithmetic mean roughness) to $R_a = 1-4 \, \mu m$ for skis with fine roughness and $R_a > 10 \, \mu m$ for very coarse roughness.

6.3 A note on weather and temperature

The ambient conditions yields a complex combination of parameters, none of them which can be controlled while conducting experiments in the field. Weather and snow parameters such as air temperature, snow temperature, snow density, relative humidity and cloudiness should ideally be characterized and measured when conducting in-situ sliding tests. The radiation of the sun will also contribute to melting along with the frictional heating. Due to the limitations of the measuring equipment at hand, only the snow and air temperature were measured. The field experiment was conducted during different kinds of weather. At the coldest the ambient temperature was -11 °C and the warmest conditions had an ambient temperature of 5°C. During all experiments it was mostly dry weather, it was only light snowfall at one occasion. The first tests were conducted in late February and the last in the start of April. The weather conditions for each experiment will be further characterized in the chapter presenting the results.
In this section the results from the ski experiments will be presented. A total of seven field experiments were conducted, the first on February 28 (2018) and the last on April 10 (2018). There was some trial and error during this period of testing. The making of the setup took longer than expected and spring was closing in, hence we had to initiate testing before all components were ready. Luckily, the winter was long and cold and the last experiments could be performed as planned. Since the number of field trips is relatively small, I will briefly present them one by one before I discuss the results. This will hopefully give the reader a better understanding of the challenges with such a setup and to see how the changes made during the period of testing impacts the measurements and the results. In the following section a brief summary of the conducted experiments will be presented.

7.1 Conducted Experiments - Notes in General

1. February 28 - Sognsvann Snøpark

<table>
<thead>
<tr>
<th>Time</th>
<th>Ambient temp. (°C)</th>
<th>Snow temp. (°C)</th>
<th>Wind (m/s)</th>
<th>Weather</th>
</tr>
</thead>
<tbody>
<tr>
<td>14:00-16:15</td>
<td>-11</td>
<td>*</td>
<td>4-5 m/s</td>
<td>Clouded</td>
</tr>
</tbody>
</table>

The first day of testing was conducted at Sognsvann Snøpark. Only the ambient temperature was measured at this field experiment. The thermistors were not attached to either of the ski pairs, but it was important to test the rest of the setup in the field to make sure everything else worked as planned in case adjustments had to be made. The main concern was the sleds stability and the motor controlling the winch. The motor had been tested in the workshop, but only with the winch attached (no load), so we were eager to see which velocities could be obtained when pulling a loaded sled on snow. After performing several tests the conclusion was that the setup worked as planned, only minor adjustments to the acceleration phase had to be made to achieve a more smooth acceleration. Further, it was experimented with different loads, from 20 to 60 kg. Seeing that the temperature measurements had not been conducted, it was the load cell data and GPS data that were of interest from this field trip.
7. Field Experiment - Results

2. March 7 - Sognsvann Snøpark

Table 7.2: Weather conditions March 07.

<table>
<thead>
<tr>
<th>Time</th>
<th>Ambient temp. (°C)</th>
<th>Snow temp. (°C)</th>
<th>Wind (m/s)</th>
<th>Weather</th>
</tr>
</thead>
<tbody>
<tr>
<td>11:30-13:00</td>
<td>−2</td>
<td>−4</td>
<td>2 m/s</td>
<td>Clouded, some light snow fall</td>
</tr>
</tbody>
</table>

The thermistors had only been attached to the left ski on the cold grinded pair, hence we only performed tests with this pair today. Two thermistors were attached, 25 cm and 40 cm from the ski tip, respectively. The data showed promising results.

3. March 14 - Sognsvann Snøpark

Table 7.3: Weather conditions March 14.

<table>
<thead>
<tr>
<th>Time</th>
<th>Ambient temp. (°C)</th>
<th>Snow temp. (°C)</th>
<th>Wind (m/s)</th>
<th>Weather</th>
</tr>
</thead>
<tbody>
<tr>
<td>10:00-14:00</td>
<td>0 to 4</td>
<td>−6 to −4</td>
<td>1 m/s</td>
<td>Sun</td>
</tr>
</tbody>
</table>

Both skis (cold and wet grind) were tested, this time both had 7 thermistors attached to the left ski. It was not possible to detect a difference between the skis w.r.t. $\mu_k$. This may be due to the fact that the conditions had changed. It passed nearly two hours between the first run with cold grind and the first run with wet grind, during this time the ambient temperature had increased with 2-3°C. In addition the slope was newly groomed while testing the cold grinded skis. When the runs with wet grind started the slope had experienced several passings which alters the surface.

4. March 15 - Sognsvann Snøpark

This was the fourth day of testing. At the end of the previous field trip it was discovered that thermistors located beneath the binding of the cold grinded ski were quite hidden inside the ski base. Hence these sensors had to be moved closer to interface. The goal was to see how the adjustment of the thermistors on the cold grinded ski effects the temperature measurements, thus only this pair was tested. The experiment conducted had a problem with the rope getting trapped beneath the ski which ruined both some load cell and thermistor measurements.

Table 7.4: Weather conditions March 15.

<table>
<thead>
<tr>
<th>Time</th>
<th>Ambient temp. (°C)</th>
<th>Snow temp. (°C)</th>
<th>Wind (m/s)</th>
<th>Weather</th>
</tr>
</thead>
<tbody>
<tr>
<td>13:00-15:00</td>
<td>−2 to −1</td>
<td>−4 to −3</td>
<td>2-3 m/s</td>
<td>Clouded/sun</td>
</tr>
</tbody>
</table>

5. March 16 - Sognsvann Snøpark

Table 7.5: Weather conditions March 16.

<table>
<thead>
<tr>
<th>Time</th>
<th>Ambient temp. (°C)</th>
<th>Snow temp. (°C)</th>
<th>Wind (m/s)</th>
<th>Weather</th>
</tr>
</thead>
<tbody>
<tr>
<td>12:00-14:30</td>
<td>−4 to −3</td>
<td>−8.8 to −7.3</td>
<td>1-2 m/s</td>
<td>Sun</td>
</tr>
</tbody>
</table>

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7.2. Tracks and elevation profile

The goal was to obtain a procedure to avoid trapping the rope beneath the ski. The thermistor that had showed the most promising temperature levels had moved back into the ski during the experiments, resulting in no measured change in temperature. However, the thermistor placed 40 cm from ski tip showed an increase of nearly 5°C. By plotting temperature and velocity in the same plot a high correlation was found.

6. March 27 - Sognsvann Snøpark

Table 7.6: Weather conditions March 27.

<table>
<thead>
<tr>
<th>Time</th>
<th>Ambient temp. (°C)</th>
<th>Snow temp. (°C)</th>
<th>Wind (m/s)</th>
<th>Weather</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:00-13:00</td>
<td>−4 to 1</td>
<td>−7.6 to −6.2</td>
<td>1 m/s</td>
<td>Sun</td>
</tr>
</tbody>
</table>

Experiments with cold and wet grinded skis at different velocities were conducted. A nice correlation between velocity and temperature was observed. Unfortunately the thermistor that was adjusted yesterday broke during transportation.

7. April 10 - Holmenkollen

Table 7.7: Weather conditions April 10.

<table>
<thead>
<tr>
<th>Time</th>
<th>Ambient temp. (°C)</th>
<th>Snow temp. (°C)</th>
<th>Wind (m/s)</th>
<th>Weather</th>
</tr>
</thead>
<tbody>
<tr>
<td>09:00-13:00</td>
<td>−1 to 5</td>
<td>−1.2 to −0.1</td>
<td>2-3 m/s</td>
<td>Sun</td>
</tr>
</tbody>
</table>

Due to the warm conditions the thermistors did not detect any change in temperature, all frictional heating contributed to melting since the snow temperature already was approximately 0°C. The track at Holmenkollen was far more straight than the track at Sognsvann, this gave much nicer load cell data.

7.2 Tracks and elevation profile

The measurement sections (as depicted in Figure 6.1) for each run is assumed to be flat. Thus the elevation profile of the track is not taken into consideration when calculation the coefficient of friction. The GPS was used to measure the elevation profile for the tracks where the experiments were conducted. The obtained data was promising, but the resolution is limited to 1 m. The GPS measured a constant value in both cases, \( h_S = 181 \text{ m} \) and \( h_H = 330 \text{ m} \) for Sognsvann Snøpark and Holmenkollen, respectively. Figure 7.1 shows the elevation data plotted against the displacement data for the two tracks. The track used at Sognsvann Snøpark was approximately 30 meters longer.

Due to the lack of equipment for snow and humidity measurements, the track conditions will described with simple keywords in order to provide some information beyond that of temperature.

- Prepared - The track has been prepared the same day and has experienced few passings before the field experiment was conducted.
7. Field Experiment - Results

- Glazed/Icy - The track has not been prepped the last 24 hours. Furthermore, the track has frozen overnight which results in a hard, icy surface.

- Transformed - The track has experienced ambient temperatures above 0°C.

![Elevation measurements for the track at Holmenkollen and the track at Sognsvann.](image)

Figure 7.1: Elevation measurements for the track at Holmenkollen and the track at Sognsvann.

7.3 Coefficient of Friction

Average coefficient of friction is reported Table 7.8 along with date, temperature, velocity and snow conditions. Each coefficient value is based on a sequence of runs, where each sequence consists of approximately 5 consecutive runs with the same grind and velocity. A sequence could occasionally have fewer or more runs due to time constraints, weather conditions or the desire to get replicate data. In the following, the effect of ambient temperature, surface roughness, velocity and normal load on snow friction are presented.

7.3.1 Effects of Ambient Temperature

The snow surface temperature is very difficult to measure due to radiational effects on sensors, hence the air temperature is often used as a parameter, although it is the temperature in the slider-snow interface that is of primary interest. The dependence of the friction coefficient on temperature has been confirmed by many researchers. It is a widely held view that the coefficient of friction decreases with increasing temperature due to enhanced lubrication at higher temperatures. Figure 7.2 shows the measured $\mu_k$ for six different temperature. The behavior is in accordance with previously published results.
7.3. Coefficient of Friction

Table 7.8: Table of coefficients of friction.

<table>
<thead>
<tr>
<th>Date</th>
<th>Temp. (°C)</th>
<th>Snow type</th>
<th>Velocity (km/h)</th>
<th>Coefficient of friction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Cold Grind</td>
</tr>
<tr>
<td>28-Feb</td>
<td>-11</td>
<td>Prepared</td>
<td>8</td>
<td>0.0326</td>
</tr>
<tr>
<td>07-Mar</td>
<td>-2</td>
<td>Prepared</td>
<td>8</td>
<td>0.0229</td>
</tr>
<tr>
<td>07-Mar</td>
<td>-2</td>
<td>Prepared</td>
<td>14</td>
<td>0.0265</td>
</tr>
<tr>
<td>14-Mar</td>
<td>0</td>
<td>Prepared</td>
<td>8</td>
<td>0.0182</td>
</tr>
<tr>
<td>14-Mar</td>
<td>1</td>
<td>Prepared and Transformed</td>
<td>14</td>
<td>0.0217</td>
</tr>
<tr>
<td>14-Mar</td>
<td>2</td>
<td>Prepared and Transformed</td>
<td>8</td>
<td>-</td>
</tr>
<tr>
<td>14-Mar</td>
<td>3</td>
<td>Prepared and Transformed</td>
<td>14</td>
<td>-</td>
</tr>
<tr>
<td>15-Mar</td>
<td>-2</td>
<td>Prepared</td>
<td>8</td>
<td>0.0251</td>
</tr>
<tr>
<td>16-Mar</td>
<td>-4</td>
<td>Prepared</td>
<td>8</td>
<td>0.0241</td>
</tr>
<tr>
<td>27-Mar</td>
<td>-3</td>
<td>Glazed/Icy</td>
<td>8</td>
<td>0.0214</td>
</tr>
<tr>
<td>10-Apr</td>
<td>0</td>
<td>Prepared and Transformed</td>
<td>14</td>
<td>0.0145</td>
</tr>
<tr>
<td>10-Apr</td>
<td>3</td>
<td>Prepared and Transformed</td>
<td>14</td>
<td>0.0179</td>
</tr>
<tr>
<td>10-Apr</td>
<td>5</td>
<td>Prepared and Transformed</td>
<td>14</td>
<td>-</td>
</tr>
</tbody>
</table>

[9, 26, 18, 11, 4, 31]. However, more data should be obtained in a larger temperature range to achieve a more detailed view of this behavior.

![Graph](image)

Figure 7.2: $\mu_k$ for cold grind and $v = 8$ km/h at different temperatures. Each $\mu_k$ is plotted with $2\sigma$. Dashed line are included as guide to the eye.

7.3.2 Effect of Surface Roughness - Cold vs. Wet Grind

The effect of the skis surface roughness w.r.t. friction will be investigated in this section. It is a commonly held view that a rougher surface should be used during warm and wet conditions and that a smoother surface should be used during cold and dry conditions. Shimbo (1971) [37] conducted experiments where he found that friction increases with roughness at sub-freezing temperatures and that friction decreased with roughness for $T = 3$°C. From our conducted experiments there were three valid datasets to analyzed for this dependency. All three experiments were conducted in ambient temperature $0^\circ$C $\leq T_{amb}$. 
7. Field Experiment - Results

hence we would expect the wet grind to yield lower $\mu_k$ in all cases. The result is shown in Figure 7.3 and none of these differences were statistically significant w.r.t. $2\sigma$. Since friction is generally low, it is difficult to perform field tests to show it varies w.r.t. roughness. There may be other properties that influence the friction more, e.g. change in ambient temperature, snow temperature and track conditions during testing, making it hard to differentiate the difference in $\mu_k$ due to different grind.

### 7.3.3 Effect of Velocity

The friction for objects sliding on snow or ice is dependent on velocity. Oksanen and Keinonen [32] conducted experiments with sliding friction on ice and showed that the coefficient of friction increases with velocity for snow temperatures near zero ($T_{\text{snow}} > -1^\circ\text{C}$, $0.5 \leq v \leq 3$ m/s). For colder snow temperatures ($T_{\text{snow}} = -15^\circ\text{C}$) the friction decreased with increasing velocity ($0.5 \leq v \leq 3$ m/s). These results are physically explained by the presence of the water film. At temperatures near zero, the water film may already be present, and further frictional heating will generate too much water and induce capillary drag. At colder temperatures the frictional regime is dominated by solid-to-solid contact and the water film generated by frictional heating will reduce friction. In our conducted experiments there were only one occasion where the temperature was low enough to explore the decrease in friction with increasing velocity. Unfortunately, some of the load cell data from this trip was useless due to disturbance from the rope, hence no comparison from this field trip could be made. In the remaining field experiments the snow temperature was close to zero and an investigation of the frictions dependency of velocity in the temperature regime could be made. For each date, all runs (both high and low velocity) were conducted within 30 minutes, hence the conditions should be equal. As
7.3. Coefficient of Friction

Figure 7.4: Mean and standard deviation (2 σ) for μ_k for two velocities at different dates. Three of the datasets are from cold grinded skis and one is from wet grinded skis.

can be seen in Figure 7.4, the friction is lower for the lowest velocity in all four cases. This result is also shown by [27, 23, 4].

7.3.4 Effect of Normal Load

Nachbauer et al. [31] conducted experiments (full-scale) with skis sliding on snow with a linear tribometer. For speeds of 5 and 15 m/s, snow temperature T_s = −4°C and F_N = 146N they measured the friction coefficient to be independent of the normal force. Buhl et al. [11] found no detectable difference between different loads for temperature above −6°C. Bäurle [4] performed experiments with different load in three different (ambient) temperature regimes. At low temperatures (T_amb = −10°C) no dependency was found. At intermediate temperatures (T_amb ≈ −5°C), the coefficient of friction increased slightly with load. At temperatures close to the melting point (−0.5°C < T_amb < 0.5°C) the friction coefficient decreased with increasing load. From our conducted field experiments there was only one valid dataset (March 15) that could be used for investigation of the dependency between friction and load. The conditions during this test sequence can be found in Table 7.4. In our case, as shown in Figure 7.5, the coefficient of friction is found to be independent w.r.t. to load. This is in compliance with [31, 11]. According to [4] one could might expect a difference in either direction, seeing that our experiment was conducted in an temperature higher than his intermediate regime and lower than his close-to-zero regime. However, our dataset on this subject is quite small. In order to provide a more thorough and accurate analysis of this dependency, further data collection is required from our setup.
7. Field Experiment - Results

Figure 7.5: Mean and standard deviation (2 $\sigma$) for $\mu_k$ for two different loads.

7.4 Temperature

The first tests with thermistors attached to the skis showed promising results. Only two sensors (25 cm and 40 cm from ski tip) were attached and the sensor closest to the center showed the highest increase in temperature. This result is in compliance with the effect of frictional heating: the sensor closest to the center experience a increase in temperature due to the heating generated by the ski base in front. In Figure 7.6, the temperature profile from four runs executed 07 March are shown. These have the same input motor power, which corresponds to approximately 8 km/h when sliding at constant velocity. Higher velocity led to increasing temperature, as shown in Figure. 7.7, which is consistent with increasing frictional power. The temperature increase in the ski-snow interface increased with decreasing snow temperature. At lower temperatures the solid-to-solid interactions are more present, this results in higher sliding friction and consequently more frictional heating. The highest rise in temperature was obtained at 16 March during runs with high velocity ($v_{max} \approx 27$ km/h). At this velocity a relative increase in temperature of $\Delta T \approx 5^\circ C$ was achieved, the snow temperature was $-8.1^\circ C$. The relative change in temperature for each thermistor is presented in Figure 7.8. This result fits with other reported values. Schindelwig et al. [36] measured an increase in snow temperature of $4^\circ C$ with infrared sensors placed in the bottom of the ski. The snow temperature in the laboratory setup was $-8.3^\circ C$. Colbeck [14] conducted field tests where thermocouples were installed in skating skis. He measured an increase from $-5^\circ C$ to an average temperature of about $-1.5^\circ C$ from standstill to steady skiing. Turning back to Figure 7.8, it is clear that the sensors closest to the binding that shows the highest increase in temperature. This is where the pressure is highest. At the rear end of the ski there is almost no pressure when the ski is loaded with 75 kg which explains why the thermistor positioned at 170 cm shows almost no change in temperature (except for higher velocities).
7.4. Temperature

Figure 7.6: Temperature profile of four consecutive runs with same motor power.

Table 7.9 shows the maximum relative increase in temperature - \( \Delta T_{\text{max}} \) (°C) - for the seven thermistors for different test sequences. In the following, the effect of surface roughness, velocity and normal load on the temperature in the ski-snow interface are presented.

7.4.1 Effect of Surface Roughness - Cold vs. Wet Grind

A highly relevant question is if the system is able to detect a difference in temperature between the cold and the wet grind. The coefficient of friction may be higher or lower for either depending on the circumstances, but the cold grind (due to its finer roughness which results in a larger area of contact) should yield a higher temperature increase in the ski-snow interface than the wet grind under the same conditions. The biggest concern regarding this analysis is the lack of comparable datasets. Of the seven conducted field experiments there were only three occasions where both wet and cold grind were tested. One of these three experiments was conducted at Holmenkollen on the 10th of April in warm weather \((T > 0^\circ \text{C})\), hence the temperature evolution beneath the ski is expected to be marginal (if not absent). In all cases, the three thermistors placed closest to the rear end of the skis showed little or no change in temperature. These thermistors had very little or no contact with the snow surface, hence their measurements will be omitted from this presentation. Moreover, some of the
7. Field Experiment - Results

(a) Run 1.
(b) Run 2
(c) Run 3
(d) Run 4

Figure 7.7: Temperature profile of four consecutive runs with same motor power.

Table 7.9: A table presenting maximum increase in temperature - $\Delta T_{\text{max}}$ for each thermistor for different test sequences. Temperature is given in kelvin.

<table>
<thead>
<tr>
<th>Date</th>
<th>Velocity (km/h)</th>
<th>Grind</th>
<th>Thermistor position (cm from ski tip.)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>25</td>
</tr>
<tr>
<td>07-Mar</td>
<td>8 km/h</td>
<td>Cold</td>
<td>0.53</td>
</tr>
<tr>
<td>07-Mar</td>
<td>14 km/h</td>
<td>Cold</td>
<td>1.35</td>
</tr>
<tr>
<td>14-Mar</td>
<td>8 km/h</td>
<td>Cold</td>
<td>0.53</td>
</tr>
<tr>
<td>14-Mar</td>
<td>8 km/h</td>
<td>Wet</td>
<td>0.29</td>
</tr>
<tr>
<td>14-Mar</td>
<td>14 km/h</td>
<td>Cold</td>
<td>1.19</td>
</tr>
<tr>
<td>14-Mar</td>
<td>14 km/h</td>
<td>Wet</td>
<td>0.05</td>
</tr>
<tr>
<td>14-Mar</td>
<td>27 km/h</td>
<td>Cold</td>
<td>1.90</td>
</tr>
<tr>
<td>14-Mar</td>
<td>27 km/h</td>
<td>Wet</td>
<td>0.10</td>
</tr>
<tr>
<td>15-Mar</td>
<td>8 km/h</td>
<td>Cold</td>
<td>0.14</td>
</tr>
<tr>
<td>15-Mar</td>
<td>14 km/h</td>
<td>Cold</td>
<td>0.70</td>
</tr>
<tr>
<td>16-Mar</td>
<td>8 km/h</td>
<td>Cold</td>
<td>0.11</td>
</tr>
<tr>
<td>16-Mar</td>
<td>14 km/h</td>
<td>Cold</td>
<td>1.79</td>
</tr>
<tr>
<td>16-Mar</td>
<td>27 km/h</td>
<td>Cold</td>
<td>2.13</td>
</tr>
<tr>
<td>27-Mar</td>
<td>8 km/h</td>
<td>Cold</td>
<td>1.55</td>
</tr>
<tr>
<td>27-Mar</td>
<td>14 km/h</td>
<td>Cold</td>
<td>2.25</td>
</tr>
</tbody>
</table>
thermistors (most likely due to their vertical position in the ski base) showed a temperature close to zero while the sled was at rest. In addition there was an unfortunate long period of time between the runs with cold grinded skis and wet grinded skis in which the ambient temperature increased. Thus these thermistors were unfit for comparison since they already measured temperatures close to 0°C. Due to these discrepancies the presentation will focus on the thermistor which was most exposed and closest to the ski-snow interface, namely T2, the thermistor placed 40 cm from the ski tip.

The first dataset that will be evaluated is from March 14. Two sequences with cold grinded skis will be compared with two sequences with wet grinded skis. All sequences have the same load, while the velocity differ. The details concerning the sequences can be viewed in Table 7.10 (low velocity) and Table 7.11 (moderate velocity). The results from these sequences are shown in

<table>
<thead>
<tr>
<th>Cold grind</th>
<th>Wet grind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snow temp. (°C)</td>
<td>-5.8</td>
</tr>
<tr>
<td>Ambient temp. (°C)</td>
<td>0</td>
</tr>
<tr>
<td>Time interval of sequence</td>
<td>10:25-10:37</td>
</tr>
</tbody>
</table>
7. Field Experiment - Results

Figure 7.9: Mean and standard deviation (2σ) of the maximum ΔT obtained from four consecutive runs (for wet and cold grinded skis for two different velocities).

Table 7.11: Temperatures and time stamp for test sequence for cold and wet grind conducted 14 March. Both with \( m = 75 \) kg and \( v = 14 \) km/h.

<table>
<thead>
<tr>
<th></th>
<th>Cold grind</th>
<th>Wet grind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snow temp. (°C)</td>
<td>-5.2</td>
<td>-3.5</td>
</tr>
<tr>
<td>Ambient temp. (°C)</td>
<td>1.2</td>
<td>3.1</td>
</tr>
<tr>
<td>Time interval of sequence</td>
<td>10:40-10:50</td>
<td>12:30-12:40</td>
</tr>
</tbody>
</table>

Figure 7.9. In both cases, the result looks promising. The ski pair with blue grind yield a higher \( \Delta T_{\text{max}} \) for \( T2 \) for both low and moderate velocity. These measurements are in compliance with theory, i.e. the cold grinded ski base yields higher frictional heating due its larger area of contact with snow compared to a wet grinded ski. A similar analysis can be done for the test sequences conducted at March 27. Due to time constraints, only one comparable set of test sequences was obtained, the details can be seen in Table 7.12. The ambient and snow temperature were lower at this experiment, thus the results from \( T1 \) (closest to ski tip) will be included. The results from Fig. 7.10 yields a similar result.

Table 7.12: Temperatures and time stamp for test sequence for cold and wet grind conducted 27 March. Both with \( m = 75 \) kg and \( v = 8 \) km/h.

<table>
<thead>
<tr>
<th></th>
<th>Cold grind</th>
<th>Wet grind</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snow temp. (°C)</td>
<td>-7</td>
<td>-5</td>
</tr>
<tr>
<td>Ambient temp. (°C)</td>
<td>-1</td>
<td>-0.1</td>
</tr>
<tr>
<td>Time interval of sequence</td>
<td>10:50-11:05</td>
<td>12:15-12:30</td>
</tr>
</tbody>
</table>

as those obtained at March 14, but they are statistical indistinguishable w.r.t. 2σ. The main error regarding these results is the non-negligible amount of time
passed between the respective sequences, in which the temperature, track and weather conditions had changed. This will effect the measurements, but given the information at hand it is hard to state at what extent. An obvious improvement would include more accurate snow surface temperature measurements (at several vertical positions) in addition to reduce the time between the sequences that are to be compared. The procedure involving changing skis on the sled had not been optimized at the time of these conducted experiments. Hence, at these field experiments, all runs with cold grinded skis were conducted before changing to wet grinded skis in order to be efficient. At the last experiment (in Holmenkollen) we found a procedure to quickly change the skis and re-connect the thermistor wires \((t < 60 \text{ seconds})\) such that this could be done after every five run. Unfortunately, the snow and ambient temperature were too high to investigate the frictional heating in the ski-snow interface.

### 7.4.2 Effect of Velocity

An increase in temperature is expected to occur as soon as reasonable cross-country velocities is achieved, hence it is of interest to investigate the correlation between velocity and temperature. From Equation 3.7 we have that the heat generation is proportional to \(v\). Figure 7.11. shows the high correlation between velocity and temperature. For lower velocities the temperature increase is smaller, but the high correlation is still clear. Notice that for the highest velocity it is the sensor in front of the binding which shows the highest \(\Delta T\), while for the lower velocity it is the sensor on the rear side of the binding. For higher velocities the temperature accumulate faster and high temperature peaks may occur closer to the ski tip. The runs conducted only have one acceleration phase which makes it difficult to detect at which velocity a sharp

Figure 7.10: Mean and standard deviation of the four maximum \(\Delta T\). obtained from the four runs (for each ski). The thermistor at 140 cm on the cold grinded ski is broken, hence no temperature was measured.
7. Field Experiment - Results

Figure 7.11: Velocity and $\Delta T$ for two runs with different velocities. The two sensors which shows the highest increase in temperature are shown. The left axis shows temperature and the right axis shows velocity.

(a) Run 5. Low velocity

(b) Run 16. High velocity

Figure 7.11: Velocity and $\Delta T$ for two runs with different velocities. The two sensors which shows the highest increase in temperature are shown. The left axis shows temperature and the right axis shows velocity.
increase in temperature occurs. Given a a flat and straight test track that is long enough, one could perform runs with several acceleration phases, i.e. stepwise acceleration every \( t \) seconds. Such an approach could help reveal the relation between velocity and temperature in the ski-snow interface, i.e. show the temperature as a function of velocity.

### 7.4.3 Effect of Normal Load

Regarding temperature and load, there was only one valid dataset that could be examined. These tests were conducted March 15 and the details are shown in Table 7.13. A higher load will produce more frictional heating (Equation 3.7), especially for lower temperatures where dry friction is dominant. The ambient temperature on March 15 was close to the melting point, hence the differences in \( \Delta T_{\text{max}} \) between these sequences are due to the change in pressure distribution. As the load changes, so does the apparent area of contact between ski and snow. Since the apparent area is larger for the lighter load, all thermistors will be presented in the result. The estimated apparent contact area for the different loads is depicted in Figure 7.12. The maximum temperature for the two sequences are presented in Figure 7.13. The thermistors in the rear end for the lighter loaded ski measures a larger \( \Delta T \) than that of the heavier loaded ski. For \( m = 75 \text{ kg} \), thermistors T6 and T7 have little or no pressure exerted to them which explains why \( \Delta T_{\text{max}} \) is quite low.

### 7.5 A note on hydrodynamic lubrication

The results obtained will be compared to theory concerning hydrodynamic lubrication. In this discussion the temperature and real contact area must also be taken into consideration. We can start off with Equation (4.3) which estimates the wet friction given that a thin film separates the bodies in motion. Since we already have obtained values for \( F_F \), we can solve the equation for \( h \) instead, i.e.

\[
    h = \frac{\mu_{\text{water}} U A}{F_F}
\]

Before we can solve this equation the most crucial parameter, the area of contact \( A \), must be elaborated. It is crucial that good estimates of the area of real contact and the contact spot size is obtained in order to provide an accurate result. In this thesis there has not been any measurements of the area of contact in the experimental setups. Hence, I will use the findings in relevant literature to present an estimated value of this parameter. Due to the shape

---

Table 7.13: Temperatures and time stamps for test sequences with different load conducted March 15. Both sequences were conducted with cold grinded skis and \( v = 8 \text{ km/h} \).

<table>
<thead>
<tr>
<th></th>
<th>75 kg</th>
<th>55 kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snow temp. (°C)</td>
<td>-3.1</td>
<td>-2.9</td>
</tr>
<tr>
<td>Ambient temp. (°C)</td>
<td>-1.0</td>
<td>-0.5</td>
</tr>
<tr>
<td>Time interval of sequence</td>
<td>13:20-13:40</td>
<td>14:20-14:50</td>
</tr>
</tbody>
</table>
Figure 7.12: Thermistor positions and area of apparent contact when loaded with 55 kg and 75 kg, respectively.
and camber of a cross-country ski, the apparent contact area between the ski and snow surface is much smaller than the area of the ski. Furthermore, the real area of contact between ski and snow is even smaller, depending on the texture of the ski base and the properties of the snow. The most recent and thorough work in the field of contact area between ski and snow is done by [4]. He used X-ray computer tomography and scanning electron microscopy in his experimental investigation of the contact area between polyethylene and snow (polyethylene is the principal component at the snow-contact face of the ski). Even though these measurements were static, they are still of great help to give a better understanding and insight regarding the contact characteristics. Let $A_{\text{app}}$ denote the apparent contact area. The real contact area is given by $A_{\text{real}} = a A_{\text{app}}$, where $a \in (0, 1]$. [6] measured the fraction of the real contact to be $a \in [0.01, 0.1]$, while [13] measured it to be in the range $a \in [0.001, 0.015]$. For the sake of the example, we choose $a = 0.01$. The ski has length 1.8 m and width 0.05 m, hence $A_{\text{real}} = 1.8 \times 0.05 \times 0.01 \text{ m}^2 = 0.009 \text{ m}^2$. The dynamic viscosity for water at 0.01 °C is approximately 0.0018 Pa s. From Table 7.8 we choose values from March 7, i.e. $U = 3.9 \text{ m/s}$ and $F_F = \mu k F_N \approx 20 \text{ N}$. This gives us the following film thickness 

$$ h = \frac{\mu_{\text{water}} U A_{\text{real}}}{F_F} \approx 3.2 \mu \text{m} $$

given that it is uniformly distributed over the area of real contact. Let us now consider this film thickness in relation to frictional heating and the energy necessary to generate a film of this order in the ski-snow interface. A given section of the track will experience an increase in heat due to the passing of a ski equal to $Q = F_F L$. We have $F_F = 20 \text{ N}$ and $L = 1.8 \text{ m}$ which yields $Q = 36 \text{ J}$. Though it is not realistic, we assume that all of the energy goes into the snow asperities. Furthermore, we assume that half of the energy goes to heating snow.
up to its melting point and the other half to melting the snow. Given the energy available we can calculate how much snow \( (m_{\text{snow}}) \) that can be heated from \(-2\) to \(0^\circ\text{C} \), i.e. \( \Delta T = 2 \text{ K} \). This can be calculated by the equation

\[
m_{\text{snow}} = \frac{Q/2}{C_v \Delta T}
\]

where \( C_v = 2090 \text{ J/kgK} \) is heat capacity for snow. From this it follows that \( m_{\text{snow}} \approx 4.3g \) can be heated up to its melting point. Melting snow takes a lot more energy that raising its temperature to the melting point. The latent heat fusion of snow is \( H_{\text{fusion}} = 334 \text{ 000 J/kg} \), i.e. it takes 160 times the energy to melt 1 g snow compared to increasing the temperature of 1 g snow by 1 \(^\circ\text{C}\). The amount of snow that can be melted is

\[
m_m = \frac{Q/2}{H_{\text{fusion}}} = 0.05g
\]

Given a snow density of \( \rho_{\text{snow}} = 500 \text{ kg/m}^3 \) and that this amount of melted water is distributed uniformly across the ski base, this would yield a film of thickness

\[ h = 0.6 \mu \text{m} \]

This film thickness is highly dependent of the real area of contact which is difficult to determine. Published results for this parameter w.r.t. solid-to-solid interactions differ by two orders of magnitude and a film separating the solids is believed to further complicate the estimation. Furthermore, the heating and melting of snow does not happen at once, one must solve to heat equation to investigate the propagation of heat. The overall friction regime will comprise both dry and wet friction. The front of the ski will experience solid-to-solid contact since this part always is in contact with snow that has not experienced any frictional heating. The frictional heating generated by the dry contact results in lubricated friction for the succeeding section of the ski. As a layer of meltwater forms and the friction drops, less frictional heat will be generated which in turn results in decreasing meltwater production and higher friction. This process demonstrates the difficulty in quantifying the effects in the ski-snow interface.

### 7.6 Phase Transition

With the current setup a phase transition could not be detected. During the field experiments where both ambient and snow temperature were below zero degrees the thermistors measured temperature in the interface remained well below \( 0^\circ\text{C} \) despite the relatively low friction values (0.022-0.033). Regarding velocity, the GPS had too low sampling rate to be able to detect sudden, minor changes in speed. Furthermore, a change in the load cell readings due to a drop in friction would most likely be much smaller than the combined effect caused by variations in the elevation profile, the stretch in the rope and the pull from the winch. Thus it would be difficult to differentiate between the different effects.
7.7 Limitations

The dominant causes for imprecision w.r.t. to the coefficient of friction are the elasticity of the rope and the lack of data regarding the elevation profiles of the slopes. For the temperature measurements it is the positioning of the thermistors. Potential improvements will be discussed in chapter 8.2.2.

7.8 Conclusion

The field experiment setup has been conducted and the overall results are in compliance with theory. The influence of different parameters on the coefficient of friction and the temperature evolution in the ski-snow interface has been investigated and the behavior is in accordance with previous published results. However, such a field experiment shows limited control over the different variables. The results are greatly dependent on the state of the track, air temperature, snow temperature, snow water content and radiation from the sun. Furthermore, the number of successfully completed runs and test sequences are relatively small. Several of the runs were used to find optimal settings for the execution of the experiment and did not result in any usable data. Additional experiments should be conducted to acquire a larger body of test data over a wider range of environmental conditions in order to more accurately quantify the effects of the different parameters.
CHAPTER 8

Conclusions and Future Work

8.1 Conclusion

Two different setups for investigating friction have been developed. The linear channel tribometer features the ability to conduct experiments with different kinds of frictional regimes. The setup has been validated against reported frictional properties. The frictional regimes investigated were dry sliding, wet friction and rolling friction. The following observations were made from the experiments conducted in the channel:

- The coefficient of friction during dry sliding is independent of load and sliding velocity. This is in agreement with the elementary properties of sliding friction.

- A wetted surface greatly reduces the sliding friction. In the case of high density polyethylene, the friction decreased by a factor of 10 (from 0.2 to 0.02). Furthermore, the measured friction is quite close to the analytical solution obtained from hydrodynamic lubrication theory.

- During rolling friction, i.e. sliding on a monolayer of spheres, the coefficient of rolling friction $\mu_r$ was found to decrease with sphere radius. This is in compliance with theory [20, 17].

The outdoor tribometer allows snow friction measurements in field under realistic loads and velocities w.r.t. cross-country skiing. This setup enables temperature measurements in the ski-snow interface. Measured coefficient of friction and temperature profile under a gliding skating ski are in compliance with reported values from similar setups. Some of the findings when investigating the influence of different parameters on the coefficient of friction and the temperature in the ski-snow interface are listed below.

- The coefficient of friction on snow decreased for increasing temperatures ($T_{amb} \leq 0^\circ$C). This is in agreement with several reported findings [9, 26, 18, 11, 4, 31]. The temperature increase in the ski-snow interface increases with decreasing temperature due to solid-to-solid interactions which results in higher frictional heating.

- The coefficient of friction increases with velocity for snow temperatures near zero ($T_{amb} > -1^\circ$C, $8 \leq v \leq 27$ km/h). Since a water film is expected to already be present during these temperatures, further frictional heating...
8. Conclusions and Future Work

will generate to much water and induce capillary drag. This is also shown by [27, 23, 4]. The temperature in the ski-snow interface increases with increasing velocity with due to a higher heat generation.

In addition to validating the setup, the goal with the field experiment was to investigate if it were possible to detect a phase transition with such a setup. The current study was unable to achieve this goal. Some of the most crucial limitations are the lack of obtained data and the coarse velocity measurements (w.r.t. sampling rate). If one are to detect when a phase transition occurs one must investigate several different velocities in the velocity range were the transition is expected to occur. Thus it is crucial that the velocity measurements have high accuracy and resolution in order to differentiate between the different velocities. Further improvements that could be done in order to increase the possibility of detecting a phase transition during experiments with the field setup is presented in the following section. Overall, the obtained results from both experimental setups provide continued support for improving cost-efficient, versatile instrumental design for investigation of ski-snow friction.

8.2 Future Work

8.2.1 Laboratory Experiment

There are several improvements regarding the channel tribometer setup that could be done in order to provide more accurate results. In addition to the mentioned errors in section 2.8 that should be rectified, the following adjustments are proposed

- The use of a rotary encoder attached to the winch would give more accurate velocity measurements. This would also give more detailed information for comparison between the duty cycle given by the user and the motors corresponding output power. In addition it would make the setup more mobile, seeing that the camera above the channel would no longer be necessary. One could then use this setup outside, or in a cold storage to do small-scale testing of ski-friction on snow.

- A procedure to measure the film thickness when performing experiments with fluid friction should be developed. This would greatly improve the analysis for comparing the obtained results with theory.

- The area of real contact is of high interest when studying frictional properties. In order to provide a more accurate analysis of the conducted experiments there should be developed a procedure to measure this parameter.

- The preliminary tests with thermistors showed no significant change in temperature in the slider-surface interface. However, by using a load cell with higher capacity one could conduct tests with heavier loads and thus increase the heat generation. Furthermore, by applying a longer slider, smoother surface and higher velocities one could obtain a heat generation of the same order as in the field experiment. These adjustments would allow investigation of the temperature increase in the slider-surface interface.
8.2. Future Work

8.2.2 Field Experiment

The test period that lasted from February 28th to April 10th revealed several aspects that could greatly improve the setup:

- A long, straight and flat test track is beneficial when conducting experiments with the ski-tribometer. Such a slope can be difficult to obtain, hence an appropriate device for measuring the elevation profile of the chosen track should be used. With such information one could estimate the contribution from gravity due to the inclination of the slope which would yield higher accuracy when calculating $\mu_k$ in the field.

- Seeing that $\mu_k$ in cross-country skiing is dependent on the velocity it would be an improvement to have more accurate velocity measurements. This could for example be a GPS with higher accuracy and sampling rate. A rotary encoder attached to the winch would also be able to provide more precise velocity measurements. Such a device would give a more detailed picture of how the winch operates given the motor power input from the user. This feature could provide useful information regarding how the winch and the motor would respond to varying friction during a test run. Velocity data with higher accuracy would be important w.r.t. comparison with the load cell data, i.e. to ensure that the velocity truly is constant in the selected measurement section. More importantly, if a phase transition was to occur, the friction would drop and an increase in velocity could be detected.

- The temperature measurements showed promising results, hence more thermistors should be placed under the ski. This would give an even more detailed picture of the temperature profile. Attaching sensors under both skis should be done for redundancy. This would make the setup less vulnerable if some of the thermistors were to break. It is important that all sensors have identical conditions, thus even more care should be taken when placing the thermistors in the holes. A setup which would allow a more precise adjustment of the thermistors should be made. Furthermore, both the ceramic tubes and the holes where they appear on the base of the skis should be even more polished to provide a smoother gliding surface.

- A more static rope should be used when pulling the slider to avoid building up tension during a run.

- Detailed measurements of snow structure and weather parameters were not in the scope of this thesis. This is something that should be considered if someone is to continue working with this setup. Such characteristics are crucial if one are to really understand the measured values from ski friction, i.e. $\mu_k$ and the temperature evolution in the ski-snow interface.
Appendices
APPENDIX A

Notation

Table A.1: List of symbols - physical properties

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>area</td>
<td>[m²]</td>
</tr>
<tr>
<td>A_{app}</td>
<td>area of apparent contact</td>
<td>[m²]</td>
</tr>
<tr>
<td>A_{real}</td>
<td>area of real contact</td>
<td>[m²]</td>
</tr>
<tr>
<td>ADCval</td>
<td>integer mapped from voltage value</td>
<td>[-]</td>
</tr>
<tr>
<td>C_v</td>
<td>specific heat capacity</td>
<td>[Jkg^{-1}K^{-1}]</td>
</tr>
<tr>
<td>d</td>
<td>sphere diameter</td>
<td>[m]</td>
</tr>
<tr>
<td>F_F</td>
<td>friction force</td>
<td>[N]</td>
</tr>
<tr>
<td>F_N</td>
<td>load, normal force</td>
<td>[N]</td>
</tr>
<tr>
<td>F_P</td>
<td>pull force</td>
<td>[N]</td>
</tr>
<tr>
<td>h</td>
<td>fluid film thickness</td>
<td>[m]</td>
</tr>
<tr>
<td>H_{fusion}</td>
<td>latent heat fusion</td>
<td>[Jkg^{-1}]</td>
</tr>
<tr>
<td>I</td>
<td>ampere</td>
<td>[A]</td>
</tr>
<tr>
<td>L</td>
<td>length of slider</td>
<td>[m]</td>
</tr>
<tr>
<td>m</td>
<td>mass</td>
<td>[kg]</td>
</tr>
<tr>
<td>P</td>
<td>power</td>
<td>[W]</td>
</tr>
<tr>
<td>q</td>
<td>heat generation per unit area</td>
<td>[Wm^{-2}]</td>
</tr>
<tr>
<td>Q</td>
<td>heat generation</td>
<td>[W]</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
<td>[K]</td>
</tr>
<tr>
<td>T_{amb}</td>
<td>ambient temperature</td>
<td>[K]</td>
</tr>
<tr>
<td>v_{s}</td>
<td>sliding velocity*</td>
<td>[ms^{-1}]</td>
</tr>
<tr>
<td>V</td>
<td>voltage</td>
<td>[volt]</td>
</tr>
<tr>
<td>W</td>
<td>Work</td>
<td>[J]</td>
</tr>
<tr>
<td>λ</td>
<td>thermal conductivity</td>
<td>[Wm^{-1}K^{-1}]</td>
</tr>
<tr>
<td>μ</td>
<td>dynamic viscosity</td>
<td>[m²s^{-1}]</td>
</tr>
<tr>
<td>μ_k</td>
<td>coefficient of kinetic friction</td>
<td>[-]</td>
</tr>
<tr>
<td>μ_r</td>
<td>coefficient of rolling friction</td>
<td>[-]</td>
</tr>
<tr>
<td>ρ</td>
<td>density</td>
<td>[kgm^{-3}]</td>
</tr>
<tr>
<td>θ</td>
<td>angle</td>
<td>[-]</td>
</tr>
<tr>
<td>σ_o</td>
<td>yield stress</td>
<td>[Pa]</td>
</tr>
</tbody>
</table>

*in the field experiment the velocity is given in km/h.
### A. Notation

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ADC</td>
<td>Analog-to-digital converter</td>
</tr>
<tr>
<td>PMMA</td>
<td>polymethyl methacrylate</td>
</tr>
<tr>
<td>HDPE1000</td>
<td>High Density Polyethylene</td>
</tr>
<tr>
<td>PETG</td>
<td>Polyethylene Terephthalate Glycol</td>
</tr>
<tr>
<td>UHMW</td>
<td>Ultra High Molecular Weight Polyethylene</td>
</tr>
<tr>
<td>SD</td>
<td>Secure Digital (non-volatile card memory)</td>
</tr>
</tbody>
</table>
Hydrodynamic Lubrication

To deduce the equations concerning hydrodynamic lubrication we start with the continuity and Navier Stokes equations [1], [33]:

\[ \nabla \cdot u = 0, \quad (B.1) \]
\[ \frac{\partial u}{\partial t} + u \cdot \nabla u = -\frac{1}{\rho} \nabla p + \nu \nabla^2 u \quad (B.2) \]

where \( \rho \) is the mass density and \( \nu = \mu / \rho \) is the kinematic viscosity of the fluid.

We will consider flow in a thin film, i.e. we have a viscous fluid in a steady flow between two rigid boundaries \( z = 0 \) and \( z = h(x,y) \). \( U \) is a horizontal flow speed and \( L \) is the horizontal length scale of the flow. Since this is a thin film, we have

\[ h \ll L \quad (B.3) \]

The no-slip conditions must be satisfied at \( z = 0 \) and \( z = h \), hence \( u \) will change by an amount of order \( U \) over a \( z \)-distance of order \( h \). This will be used to perform order of magnitude estimation of our equations.

\[ \frac{\partial u}{\partial z} \] will be of order \( U/h \), and \[ \frac{\partial^2 u}{\partial z^2} \] will be of order \( U/h^2 \). From (B.3) we see that the horizontal gradients of \( u \) must be much smaller: \( \frac{\partial^2 u}{\partial x^2} \sim \frac{\partial^2 u}{\partial y^2} \sim U/L^2 \). This leaves us with the following approximation of the viscous term:

\[ \nu \nabla^2 u \simeq \nabla \frac{\partial^2 u}{\partial z^2} \quad (B.4) \]

We expect that the viscous term greatly exceeds the term \( u \cdot \nabla u \). This can be shown by an order of magnitude estimation (where have have \( u = (u,v,w) \)):

\[ u \cdot \nabla u = \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z}, \frac{\partial v}{\partial x} + u \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z}, \frac{\partial w}{\partial x} + u \frac{\partial w}{\partial y} + v \frac{\partial w}{\partial z} \right) \quad (B.5) \]

From the continuity equation we have

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (B.6) \]

Since the two first terms in this equation are of order \( U/L \), it follows that \( \frac{\partial w}{\partial z} \) is of order \( U/L \), and hence \( w \sim U h / L \). If we combine this with what we have shown earlier, we get the following order of magnitude estimation for (B.5):

\[ u \cdot \nabla u \sim \frac{U^2}{L} \left( 1, h \frac{1}{L} \right) \quad (B.7) \]
B. Hydrodynamic Lubrication

Similar, for the viscous term, we have

\[ \nu \nabla^2 u \sim \frac{\nu U}{h^2} \left( 1, 1, \frac{h}{L} \right) \quad (B.8) \]

In order for the viscous term to be much greater than \( u \cdot \nabla u \) we must have

\[ \frac{U^2}{L} \ll \frac{\nu U}{h^2} \quad (B.9) \]

which can be rewritten to

\[ \frac{UL}{\nu} \left( \frac{h}{L} \right)^2 \ll 1. \quad (B.10) \]

where \( R = UL/\nu \). The Reynolds number does not need to be that small, as long as \( h/L \) is sufficiently small.

In relation to cross-country skiing.

- The transition from laminar to turbulent flow is given by the critical Reynolds number. If the flow is to remain laminar in a rectangular cross section the following has to hold [41].

\[ R = \frac{\rho hU}{\mu W} < R_{crit} = 1600 \quad (B.11) \]

or if (using \( \mu W = 1.79 - 0.054t, \) mPa s with \( t \)s being the snow temperature [31])

\[ hv < 2 \cdot 10^{-3} m^2/s \quad (B.12) \]

In cross-country skiing speeds are obviously below 40 m/s and the water film thickness is clearly less than 50 \( \mu m \) [3]. Hence, the flow in the water film is laminar and during steady sliding the term \( \partial u/\partial t \) vanishes.

Since the terms \( u \cdot \nabla u \) and \( \partial u/\partial t \) are negligible, we obtain the slow flow equations

\[ 0 = -\nabla p + \mu \nabla^2 u \]

\[ \nabla \cdot u = 0 \quad (B.13) \]

where body forces are absent. In addition, the viscous term is greatly simplified, this yields the following equations

\[ \frac{\partial p}{\partial x} = \mu \frac{\partial^2 u}{\partial z^2}, \]

\[ \frac{\partial p}{\partial y} = \mu \frac{\partial^2 v}{\partial z^2}, \]

\[ \frac{\partial p}{\partial z} = \mu \frac{\partial^2 w}{\partial z^2}, \]

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (B.14) \]

We have that \( w \ll u,v \) (by a factor of \( h/L \)), hence the vertical pressure gradient must be much smaller than the horizontal pressure gradients, i.e.
\[ \frac{\partial p}{\partial z} \ll \frac{\partial p}{\partial x} \sim \frac{\partial p}{\partial y}. \]

Further, we integrate the two top equations from (B.14) with respect to \( z \) to obtain

\[
\begin{align*}
    u &= \frac{1}{2\mu} \frac{\partial p}{\partial x} z^2 + Az + B, \\
    v &= \frac{1}{2\mu} \frac{\partial p}{\partial y} z^2 + Cz + D,
\end{align*}
\]

where \( \frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, A, B, C, D \) are functions of \( x \) and \( y \) only.

### B.1 Ski gliding on snow.

We'll consider a system where a ski glides over a snow surface and a thin film has been generated due to frictional heating. In more general terms we have a block of length \( L \) which moves with velocity \( U \) past a stationary, rigid lower boundary at \( z = 0 \). A viscous fluid is occupying the space between them and the pressure is \( p_0 \) at both sides of the ski. Given that the no-slip condition holds, we have the following boundary conditions for the velocity:

\[
    u = U \quad \text{at} \quad z = h, \quad u = 0 \quad \text{at} \quad z = 0. \tag{B.16}
\]

In addition, we neglect side leakage (in \( y \)-direction). By integrating \( u \) from equation B.15, the fluid velocity in \( x \)-direction will be as follows:

\[
    u = \frac{z}{2\mu} \frac{\partial p}{\partial x} (z - h) + \frac{Uz}{h} \tag{B.17}
\]

where we have \( p = p(x) \). The pressure distribution \( p(x) \) can be determined so that the same amount of fluid flows per unit time through each cross-sectional area (normal to the \( x \) direction) of the sliding junction. Hence volume flux \( Q \) must be independent of \( x \):

\[
    Q = \int_0^{h(x)} u \, dz = \frac{1}{2} U h - \frac{h^3}{12\mu} \frac{\partial p}{\partial x} \tag{B.18}
\]

We rewrite the equation to obtain an expression for \( \frac{\partial p}{\partial x} \):

\[
    \frac{\partial p}{\partial x} = 6U\mu \frac{Q}{h^2} - \frac{12\mu}{h^3} Q \tag{B.19}
\]

Further, we assume that \( h \) varies linearly between the values \( h_1 \) at \( x = 0 \) and \( h_2 \) at \( x = L \)

\[
    h = h_1 + ax = h_1 + \frac{h_2 - h_1}{L} x \tag{B.20}
\]

By integrating equation (B.19) we get

\[
    \frac{p - p_0}{6\mu} = U \int_0^x \frac{1}{h^2(s)} \, ds - 2Q \int_0^x \frac{1}{h^3(s)} \, ds. \tag{B.21}
\]

Since we have \( p = p_0 \) at \( x = 0 \) and \( x = L \) we get

\[
    Q = U \int_0^L \frac{1}{h^2(s)} \, ds \left/ 2 \int_0^L \frac{1}{h^3(s)} \, ds, \tag{B.22}
\]

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B. Hydrodynamic Lubrication

By integrating and inserting the values at the limits \((h = h_1 \text{ at } x = 0 \text{ and } h = h_2 \text{ at } x = L)\) we obtain

\[
Q = U \left( \frac{1}{h_2} - \frac{1}{h_1} \right) \left( \frac{1}{h_2^2} - \frac{1}{h_1^2} \right) = U \frac{h_1 h_2}{h_1 + h_2}
\]  
(B.23)

This expression can be used to calculate the pressure and we get

\[
p - p_0 = \frac{(h_1 - h)(h_2 - h)}{6 \mu U L} \left( \frac{h_2^2}{h_1^2} - \frac{h_2^2}{h_1^2} \right) h
\]  
(B.24)

Since \(h(x)\) lies between \(h_1\) and \(h_2\), it is clear that \(p\) will be greater than \(p_0\) throughout the film, hence there will be a net upward force on the block to support a load. For this to be true, we must have \(h_2 < h_1\), i.e. the width of the lubricating layer decreases in the direction of flow. If we instead had \(h_1 > h_2\), we will have an unstable sliding motion. The fluid pressure between the substrate and the block would be below the external pressure and the two surfaces would be sucked together leading to boundary lubrication and a large friction force.

In this case of stable sliding motion \((h_1 > h_2)\), the lift force sustaining the object is therefore given by the pressure difference \(p - p_0\) integrated over the contact area \(A = L \times B\):

\[
F_N = \int_A (p - p_0) \, dx \, dy = \int_A 6 \mu U L \frac{(h_1 - h)(h_2 - h)}{(h_2^2 - h_1^2) h^2} \, dx \, dy
\]  
(B.25)

we insert \(h = h_1 + ax\) and get

\[
F_N = 6 \mu U L \int_0^B \int_0^L \frac{-(ax(h_2 - h_1) - ax)}{(h_2^2 - h_1^2)(h_1 + ax)^2} \, dx \, dy
\]

\[
= 6 \mu U L \left[ \frac{2(h_2 - h_1) + (h_1 + h_2) \ln(h_2/h_1)}{(h_2 - h_1)(h_2^2 - h_1^2)} \right]
\]

\[
= \frac{\mu U A L}{h_1^2} \alpha
\]

where

\[
\alpha = \frac{6}{(\xi - 1)^2} \left[ \ln(\xi) - \frac{2(\xi - 1)}{(\xi + 1)} \right]
\]  
(B.26)

and \(\xi = h_1/h_2\).

Furthermore, the friction force \(F_F\) is obtained by integrating the tangential stress \(\sigma_{xz}\) exerted by the fluid on the surface \(z = h(x)\) over the contact area \(A\). We have \(w \approx 0\) which yields

\[
\sigma_{xz} = \frac{\partial u}{\partial z}
\]  
(B.27)

and

\[
F_F = \mu \int_0^B \int_0^L \left. \frac{\partial u}{\partial z} \right|_{z=h} \, dx \, dy
\]  
(B.28)
B.1. Ski gliding on snow.

This can be solved in a similar manner as (B.25), and we get the friction force

\[ F_F = \frac{\mu AU}{h_1} \beta \]  

(B.29)

where

\[ \beta = \frac{1}{\xi - 1} \left[ 4 \ln \xi - \frac{6(\xi - 1)}{\xi + 1} \right] \]  

(B.30)

In cases where \( h_1/h_2 \approx 1 \), i.e. \( h_1 \approx h_2 \), we get \( \beta \approx 1 \), which yields

\[ F_F = \frac{\mu AU}{h_2} \]  

(B.31)

Equation (B.31) can be used to estimate the friction force for a laminar flow when a complete water film exists between snow asperities and ski base.
APPENDIX C

Frictional Heating

The increase in temperature caused by frictional heating is not uniformly distributed across the base of the ski. The temperature increase occurs where the asperities are in contact, and the total area of these contact spots are only a fraction of the apparent contact area. Since all of the energy generated by the sliding is concentrated in quite a small area, the typical flash-temperature increase $\Delta T$ expected during sliding can be very high ([33]).

Persson ([33]) has deduced equations for calculating such flash-temperatures at the contact areas between sliding bodies. The same example as in appendix B.1 can be used. Consider a ski sliding on snow surface separated by a few monolayers of water and assume that the heat current $J$ flows into the snow. The heat diffusion law can be used to calculate the temperature $T(z,t)$ of the lower solid:

$$\frac{\partial T}{\partial t} - \kappa \frac{\partial^2 T}{\partial z^2} = 0; \quad (C.1)$$

with

$$T(z,0) = T_0, \quad -\lambda \frac{\partial T(0,t)}{\partial z} J.$$

where $T_0$ is the temperature of the snow at the beginning of sliding, $\kappa = \lambda / \rho C_v$ is the thermal diffusion coefficient, $\rho$ the mass density and $C_v$ the heat capacity. Given that $J$ is independent of time for $t > 0$, the solution is

$$T = T_0 + \frac{2J}{\lambda} \left( \frac{\kappa t}{\pi} \right)^{1/2} f(q),$$

where

$q = z/(4\kappa t)^{1/2}$

and

$$f = e^{-q^2} - 2q^2 \int_1^\infty d\eta e^{-\eta^2 q^2}.$$

Further, we have that $f(0) = 1$ so that the temperature at the snow surface $z = 0$ at time $t$ equals $T_0 + \Delta T$ where

$$\Delta T = \frac{2J}{\lambda} \left( \frac{\kappa t}{\pi} \right)^{1/2} \quad (C.2)$$
The data obtained in the measurements were smoothed by a Savitzky-Golay filter [35]. This is especially useful for the load cell data, which has a lot of noise due to the rope, the sliders internal motion and wobbling and the mechanisms of the motors rotary device and gear configuration. In general, such filters increases the signal-to-noise ratio without greatly distorting the signal.

Savitzky-Golay finds filter coefficients, $c_n$, which preserves higher moments. It approximates the underlying function within the moving window by a polynomial of higher order, e.g. quadratic of quartic, instead of a constant (whose estimate is the average). The filter performs a least squares fit of a small set of consecutive data points to a polynomial of desired order and takes the calculated central point of the fitted polynomial curve as the new smoothed data point.
Bibliography


