Forecasting the Exchange Rate

How Norway’s External Budget Constraint Predicts Changes in the Exchange Rate

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Abstract

Norway’s large increase in its net foreign asset position has increased the potential for large capital gains and losses due to exchange rate movements. By incorporating these valuation effects in a measure of external imbalance, Gourinchas and Rey (2007) manage to successfully forecast the multilateral U.S. dollar and beat the benchmark model; the random walk. Using the same methodology on Norwegian data, I want to investigate whether the same result holds for Norway. The measure of external imbalance is created from a country’s external budget constraint which is log approximated around its trend. Regressions indicate that the measure of external balance contains predictive information on future cyclical returns on net foreign assets up to 5 years ahead, and about 2-4 years ahead for cyclical net export growth. It seems that parts of this predictability may come from predicted changes in the exchange rate. When comparing the out-of-sample forecasts against the random walk, statistical tests cannot reject that the models perform equally well at the investigated horizons, except at 10 quarters ahead. At this horizon, the random walk performs significantly better.
Preface

I want to thank my supervisor Marcus Hagedorn for the guidance throughout the whole process of writing this thesis. I also want to thank the Professorship in Macro and Monetary Policy Issues held by Steinar Holden for awarding me the student scholarship in Macro and Monetary Policy Issues. The financial grant and office space have been very much appreciated. I also want to thank friends at the master program in economics for helpful discussions related to the thesis.

The dataset can be made available upon request.

Fartein Valen Slåttemrekk
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1 Introduction

Since the 90’s, Norway has transitioned from being a net debtor to a sizable net creditor, at least relative to its GDP level (see figure 1). Norway’s foreign asset to GDP ratio has almost increased by factor of ten since 1996, and the ratio of foreign liabilities to GDP has more than quadrupled over the same period according to my calculations. But Norway is not the only country that has experienced an increase in its foreign asset and liability positions. The financial globalization that has taken place over the last decades has resulted in a general increase in gross external asset positions (Devereux, Senay, and Sutherland, 2014). This increase allows for potentially large capital gains and losses through exchange rate movements (Gourinchas and Rey, 2007). In the influential paper by Gourinchas and Rey (2007), exchange rate movements are linked to the dynamics of a country’s external balance, and they use this connection to successfully predict future movements in the multilateral USD rate. Following the same methodology, I want to find out whether the same results holds for the multilateral NOK exchange rate, or in other words; how Norway’s external budget constraint predicts changes in the exchange rate. It’s interesting to see whether the Gourinchas and Rey’s (2007) model is applicable for a small open economy such as Norway, especially since Norway has run a current account surplus for many years and thus increased its net foreign asset position.

Using data on GDP, exports, imports, foreign assets and liabilities, I start by constructing Norway’s external budget constraint. In order to evaluate it relative to wealth, it’s divided by GDP as a proxy for households’ wealth. The aim is to evaluate the budget constraint in deviations from trend, and this is accomplished by performing a first order Taylor approximation of the budget constraint around its trend. The trends are estimated by applying Hodrick Prescott filters on each of the variables in the budget constraint such that cycles that last more than 50 years are filtered out. The detrending procedure makes it possible to abstract from structural changes that may cause the trends, and rather focus on the deviations from them. These deviations are the cyclical components of the variables. In order to create the measure of external imbalances (the external budget constraint in deviations from trend) as Gourinchas and Rey (2007) do, each variable’s cyclical component is weighted by its relative importance over the sample period. These weights are calculated using the sample average of each variable’s trend. The empirical validity of the approximated measure of cyclical external imbalances is confirmed when tested on the data, where the reported approximation error is small.
To test if the measure of external imbalances contains predictive information about future cyclical real returns on net foreign assets and cyclical export growth, these variables are regressed on lagged values of the measure of external imbalances. The regressions report significant reductions in cyclical real returns on net foreign assets that are associated with increases in cyclical imbalances up to 20 quarters ahead. The variation in cyclical real returns on net foreign assets that can be explained by change in the measure of cyclical imbalances peaks at 18.7 percent, and the annualized predicted decrease in cyclical returns on net foreign assets one quarter ahead in response to a one standard-deviation increase in the measure of cyclical imbalances is almost 7 percent. For cyclical net export growth, the associated decrease in response to a one standard-deviation increase in cyclical imbalances 10 quarters ahead is about 12 percent. The share of variation in cyclical net export growth that can be explained by changes in cyclical imbalances peaks at 4.4 percent, and significant estimates are found about 2-4 years ahead. The trade and import-weighted exchange rate are then regressed on the measure of cyclical imbalances too see if some of the predicted decrease in cyclical returns and net export growth may be due to exchange rate movements. Significant estimates are found at 6-10 quarters ahead for the trade-weighted exchange rate, and 7-9 quarters ahead for the import-weighted exchange rate. The variation in exchange rates that can be explained by changes in the measure of cyclical imbalances vary between 11-15 percent at the horizons with significant imbalances.

Note: The figure shows foreign assets, liabilities, and foreign net assets to GDP. Sample: 1996q1-2017q3. Source: Statistics Norway (2017c)
estimates. The dataset is then divided in two, and rolling recursive regressions are run over the first half of the dataset, and out-of-sample forecasts are made over the second half of the dataset. The mean squared forecast error (MSFE) of the forecasts are computed and adjusted according to Clark and West (2006) in order to account for “noise” that comes from the estimation procedure (Clark and West, 2006). Simple t-tests are performed in order to test if the MSFE for the forecasts are significantly different from the MSFE from the random walk. For the trade-weighted exchange rate, the random walk outperforms the model using cyclical imbalances significantly at 10 quarters ahead at a five percent significance level, but the null that the forecasting models perform equally well cannot be rejected at the other horizons for either the trade or import-weighted exchange rate. The statistical software STATA is used in all calculations and estimations.

The rest of the thesis is as follows: The next section gives a brief overview over the literature related to exchange rate forecasting. Section 3 provides a thorough walkthrough of the theoretical framework which relates the modern theory of the dynamics of the current account to the methodology of Gourinchas and Rey (2007). Section 4 describes the data and the construction of variables that are used in estimations. A description of the construction of the measure of external imbalances is provided in section 5 along with descriptions of the type of regressions and the method of comparing the out-of-sample forecasts against the random walk. Discussions on the validity of assumptions and method is also provided in this section. In section 6, the empirical results are presented which is followed by a discussion of the results in section 7. The last section contains concluding remarks.
2 Literature Overview

Predicting the exchange rate has preoccupied many economists over the years, and it has resulted in many different methods of predicting the exchange rate. The famous paper of Meese and Rogoff (1983) showed that the random walk outperformed every exchange rate model at that time on performing out-of-sample forecasts, and this result became known as the Meese and Rogoff puzzle (Rossi, 2013). For more than 20 years, no one managed to beat the random walk on short to medium horizons until the paper of Gourinchas and Rey (2007) (Gourinchas and Rey, 2007). It is, however, worth noting that they only beat the random walk on forecasting the multilateral dollar exchange rate, and that they did not apply their method to other exchange rates. Rogoff (2009) describes the problem of beating the random walk as not only outperforming it on predicting the dollar rate, but being able to predict the “cross-currency rates between any two currencies with relatively low inflation rates, relatively open capital markets, and where policymakers do not narrowly target the exchange rate” (Rogoff, 2009, p. 2). In that sense, one should be careful considering the random walk forecast of exchange rates as fully beaten by Gourinchas and Rey (2007).

The use of macroeconomic fundamentals in forecasting the exchange rate has changed over the years from traditional fundamentals such as money, output, interest rates and prices to Taylor fundamentals\(^1\) and net foreign asset positions (Rossi, 2013). The exchange rate models Meese and Rogoff (1983) tested against the random walk were based on realized values of traditional fundamentals, and the models using traditional fundamentals are generally not viewed as providing accurate exchange rate forecasts (Engel and West, 2005) (Rossi, 2013). An important contribution to the literature was the paper of Engel and West (2005) who provided evidence that exchange rate movements may provide information about future macro fundamentals. They used an asset-pricing model based on the work of Campbell and Shiller (1988) who forecasted dividend-price ratios. Their forecast was based on rational expectations about the present value of future discount rates and growth in dividend price ratios (Campbell and Shiller, 1988). Applied on exchange rates, it means that exchange rates represent the present value of future changes in observed macro fundamentals and unobserved shocks (Engel and West, 2005). Gourinchas and Rey’s (2007) measure of external imbalances is also the result of rational

\(^1\) Taylor fundamentals are interest rates, inflation and inflation target, output and output gap. If two countries follow a Taylor rule when setting their interest rates, the Taylor fundamentals can be used to predict the bilateral exchange rate movements between the two countries (Rossi, 2013).
expectations about the present values of future values. These values are the real returns on net foreign assets and net export growth (Gourinchas and Rey, 2007). Their measure of external imbalances have shown predictive properties in other studies such as Alquist, Chinn, McNelis, and Valente (2008), Corte, Sarno, and Sestieri (2012) and Grisse and Nitschka (2016).

Rossi (2013) has made a critical review of the recent literature on forecasting exchange rates and he pinpoints and evaluates five factors that determine the ability to forecast the exchange rate. That is (1) the choice of model, (2) evaluation method and choice of benchmark model, (3) sample period and forecast horizon, (4) the use of filters for detrending and (5) the choice of predicting variables. The results are the following: First, the overall empirics are in favor of using a linear model, although there is no consensus on this matter (Rossi, 2013). Second, the choice of evaluation method is important in determining whether the forecast performs well or not. But there is no clear evidence on which test that should be used, although the two tests that are typically used for out-of-sample forecasts are the “DMW” developed by Diebold and Mariano (1995) and West (1996) and the “CW” developed by Clark and West (2006). The choice of the benchmark model used to evaluate the predictive power of the forecast is also of great importance (Rossi, 2013). The literature agrees on that the random walk without drift is the toughest one to beat, and thus the most appropriate benchmark model. Third, the forecast sample is generally very important, and a specific forecast may perform very differently at different forecast samples. Rossi (2013) doesn’t mention how forecasts based on net foreign assets are affected by the choice of forecast sample, but he states that net foreign assets has predictive powers at both short and long horizons. Fourth, the use of filters and seasonally adjustments of the variables should only be applied up until the forecasting sample; that is, those adjustments should only be applied over the periods that are not forecasted. If not, variables that have been adjusted over the whole sample will contain information on the future, and the out-of-sample forecasts will have a bias toward predictability. Fifth, net foreign assets is considered to have the best out-of-sample predictive power together with Taylor fundamentals (Rossi, 2013). It should also be noted that success of the different approaches to forecasting the exchange rate is to a large extent country and time specific (Rossi, 2013).

Keeping Rossi’s (2013) results in mind, Gourinchas and Rey’s (2007) approach on forecasting the exchange seems to be one of best as its specifications are in line with all five results, with a possible exception on the choice of sample period (3). Neither Gourinchas and Rey (2007) nor Rossi (2013) have explored the choice of sample period in relation to the approach that is used
in Gourinchas and Rey (2007). This makes it difficult to judge whether the approach is sensitive to the choice of forecasting sample. What is certain, however, is that Gourinchas and Rey (2007) find predictability over their whole forecasting sample at both short and long horizons for the multilateral dollar rate (Gourinchas and Rey, 2007).
3 Theoretical Framework

This section includes an overview over the theory of the dynamics of the current account which is linked to exchange rate movements. A presentation of Gourinchas and Rey’s (2007) methodology for forecasting the multilateral dollar rate is also included. Their work relates to the “intertemporal approach to the current account” which is discussed in this section. They use a weighted sum of the deviations from trends in exports, imports, foreign assets and liabilities to create a measure of external imbalances, \( nxa \). It is this that measure will be used to predict future changes in the multilateral NOK exchange rate.

3.1 The Current Account

One can define the current account as the balance of net exports including the net of all cross-borderer asset income and payouts like dividends, interest payments, insurance payments and payments which also includes the net capital gains on foreign assets (Obstfeld and Rogoff, 1995, p. 1733). The dynamics of the current account has been studied by many economists and is in modern theory analyzed using “the intertemporal approach to the current account” (Gourinchas and Rey, 2007). This approach assumes that economic agents are forward-looking in their investment decisions in the sense that they have expectations on future economic factors such as productivity growth, interest rates and government spending (Obstfeld and Rogoff, 1995). The result is that people, and thus the country wants to smooth consumption over time (Obstfeld and Rogoff, 1996). It means that an expected future reduction in output and thus consumption due to i.e. temporary lower government spending, will increase the current account surplus today. This increase will reduce the negative effect on consumption when the government cuts its spending in the future as the country will sell some of its claims on foreign incomes and outputs. The implication is that the country’s consumption today will decrease due to increased savings (a current account surplus) and that the expected reduction in future consumption is dampened by running a current account deficit when the government cuts its spending. Hence the expression “consumption smoothing”. No country can forever have a current account deficit or surplus, so each deficit must be offset by a future surplus, and each surplus must be offset by a future deficit (Obstfeld and Rogoff, 1996, p. 77). Gourinchas and Rey (2007) define two channels through which the current account change. That is through net exports (the trade channel) or the return on net foreign assets (valuation channel). An increase in exports or a decrease in imports will improve a country’s current account through the trade
channel by improving the trade balance, while an increase in the returns on foreign assets or a decrease in the returns on foreign liabilities will improve the current account through the valuation channel by increasing the returns on net foreign assets. From now on and throughout the paper, when I write assets or liabilities I refer to foreign assets and foreign liabilities.

### 3.2 The Current Account, External Imbalances and Exchange Rates

It’s important to make the distinction between the current account and the measure external imbalances that Gourinchas and Rey (2007) construct and use in their predictions of the exchange rate. The current account contains information about cross-border capital flows as in the definition above, whereas the measure of external imbalances as Gourinchas and Rey (2007) define it, contains information on both capital flows and foreign asset and liability positions. Their measure of external imbalances can then be interpreted as a wider definition of the current account. Gourinchas and Rey’s (2007) work is thus closely related to the current account, and the dynamics of the current account can still be associated to the dynamics of Gourinchas and Rey’s (2007) measure. A current account surplus leads to an increase external imbalances by accumulating foreign assets, and a current account deficit leads to a decrease in the measure of external imbalances. The measure of external imbalances is a country’s external budget constraint in deviations from trend, which is also referred to as cyclical imbalances throughout the thesis.

One of Gourinchas and Rey’s (2007) findings is that the share of adjustments in the U.S. measure of external imbalances that come from changes in the return on net foreign assets when including exchange rate effects, is estimated to 27 percent over the period 1952-2004 for the U.S. The result emphasizes the importance of the return on net foreign assets for the dynamics of a country’s cyclical imbalances. An important component of the returns on foreign assets or liabilities is capital gains and losses. These come from fluctuations in the market values of assets and liabilities. Fluctuations in the market value may come from changes in the fundamental value of an asset or liability, but for assets and liabilities denoted in foreign currencies, there is another source of risk, and that is the risk of exchange rate movements. Such movements will affect the market value of an asset or liability (denoted in foreign currency) when measured in domestic currency. It’s this connection between capital gains and losses due to exchange rate movements that is essential when predicting future realizations of the
exchange rate using Gourinchas and Rey’s (2007) methodology. A further explanation of the exchange rate effect on returns is provided in the next paragraph.

If Norway were to experience an appreciation of its currency, it would reduce the value of Norway’s foreign assets in NOK. This happens as the price of NOK increases relatively to the prices of foreign currencies. As a result, the returns on foreign assets decrease (due to a capital loss) with an appreciation of the NOK which tightens the external budget constraint. The effect of an appreciation of the NOK on foreign liabilities is the same, except that the reduction of the value of foreign liabilities (through a capital loss) will expand Norway’s external budget constraint. Whether the net effect is positive or negative depends on the two factors; first, the relative size of foreign assets to foreign liabilities, and second, the share of foreign assets and liabilities that is denoted in foreign currency. As figure 1 shows, Norway’s net foreign asset position turned positive at the end of 1997 and has grown to be around 8 times the size of GDP in 2017. That’s an asset to liability ratio of about two (see figure 2). If the share of foreign assets and liabilities denoted in foreign currency were the same, it would imply a worsening of Norway’s external budget constraint if the NOK appreciated since Norway’s asset position is larger than its liability position. For countries with large ratios of assets and liabilities to GDP and large shares denoted in foreign currencies, exchange rate movements have the potential to cause large capital gains and losses. Combining Norway’s asset to liability ratio with having a net foreign asset to GDP ratio of about 8 makes Norway especially exposed to large capital gains and losses through exchange rates movements.

Gourinchas and Rey (2007, p. 666) argue that the “valuation effects are absent not only from theory by also from official statistics”. Whether the valuations effects are absent from theory can be debated as Obstfeld and Rogoff (1995) includes capital gains and losses in their definition of the current account, which is also the definition I use (Obstfeld and Rogoff, 1995, p. 1733). But the failure to implement capital gains and losses due to data availability is acknowledged by Obstfeld and Rogoff (1995, p. 1735). Gourinchas and Rey’s (2007) have thus, due to the lack of official statistics on valuation effects, spent time constructing a dataset to estimate the capital gains and losses such that they get a more precise picture of the dynamics of the U.S. external budget constraint. Fortunately, does Statistics Norway provide data on valuation effects, which is more discussed in section 4.

The exchange rate’s role in the adjustments of a country’s external budget constraint also extends to the trade channel through changes in future exports and imports (Gourinchas and
If the NOK were to appreciate, it would have an effect through the trade channel by increasing the price of exported goods and lowering the price of imported goods. Higher export prices are likely to reduce exports as it becomes relative more expensive to buy Norwegian goods than other foreign goods. This will in turn have a negative effect on the external budget constraint. Equally, will lower import prices most likely increase imports of foreign goods as they become relatively cheaper. This will also affect the external budget constraint by reducing a current account surplus or cause a deficit. Since a current account deficit today predicts a future current account surplus, and exchange rate movements affects this adjustment, it may be that the dynamics of the external budget constraint also contains predictive information about future exchange rate movements. This is what Gourinchas and Rey (2007) examines, and they find predictability of the multilateral dollar exchange rate at 1 to 24 quarter horizon using their self-constructed measure of cyclical external imbalances.

### 3.3 Creating a Measure of External Imbalances

Following Gourinchas and Rey (2007), I start with the accumulation identity for net foreign assets, which also is a country’s external budget constraint:

\[
NA_{t+1} \equiv R_{t+1}(NA_t + NX_t)
\]  

\[
A_{t+1} - L_{t+1} = R_{t+1}(A_t - L_t + X_t - M_t)
\]
Equation (2) is the expanded version of equation (1). In equation (1), $NA_t$ is net foreign assets and consists of foreign assets $A_t$ subtracted by foreign liabilities $L_t$. These positions are measured at the beginning of period $t$. $NX_t$ represents net exports and is defined as exports $X_t$ less imports $M_t$, and $R_{t+1}$ expresses the gross return on net foreign assets between $t$ and $t+1$. According to this budget constraint, a country’s net foreign asset position increases with the return on net foreign assets or by increases in net exports. Asset and liability positions are reported at the beginning of the period as in Gourinchas and Rey (2007). Gourinchas and Rey (2007) then divide equation (2) by a measure of wealth $W_t$ and define $Z_t \in [X_t,M_t,A_t,L_t]$ such that $\hat{Z}_t = Z_t/W_t$. Equation (2) can then be rewritten as:

$$\hat{A}_{t+1} - \hat{L}_{t+1} = \frac{R_{t+1}}{I_{t+1}} (\hat{A}_t - \hat{L}_t + \hat{X}_t - \hat{M}_t)$$  \hspace{1cm} (3)$$

The derivation of equation (3) is shown in the appendix. The growth rate of wealth is captured by the term $\Gamma_{t+1} = W_{t+1}/W_t$ in equation (3). Seasonally adjusted mainland GDP is used as a proxy of wealth. A discussion on the use of data is provided in section 4. For Norway, the ratios of foreign assets and liabilities to GDP contain a clear positive trend over the last twenty years (see figure 3). The trends of the ratios of exports and imports to GDP, however, are less obvious. The observed trends, especially in assets and liabilities to GDP ratios, may be a result of the financial globalization that has taken place over the years. Norway’s foreign asset position, however, is special due to the large revenues from oil sector. Instead of focusing on the trends and the reasons for the development of the trends, Gourinchas and Rey (2007) focus on the deviations from trends such that they can abstract from the structural changes in the economy that are reflected in the trends. A further discussion on this is provided in section 5.2. In order to focus on the deviations from trends, Gourinchas and Rey (2007) rewrite their accumulation identity for net foreign assets (from now on referred to as the external budget constraint). This is done by constructing an external budget constraint (4) that represents an economy that follows deterministic trends. In equation (4), $\bar{Z}_t$ is defined as the equilibrium value of the ratio $Z_t/W_t$ where $Z_t \in (X_t,M_t,A_t,L_t)$ in the deterministic economy. $\bar{Z}_t$ also represents the trends in the deterministic economy.

$$\bar{A}_{t+1} - \bar{L}_{t+1} = \frac{\bar{R}_{t+1}}{\bar{I}_{t+1}} (\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t)$$  \hspace{1cm} (4)$$

By taking the difference between the realized values (3) and deterministic economy (4), one gets the external budget constraint in deviations from deterministic trends. A variable in (log)
deviation from its trend is defined as following: $\epsilon_t = (\hat{Z}_t / \bar{Z}_t)$ and is also referred to as the cyclical component of $Z_t$. Both terms will be used interchangeably throughout the paper. Additionally, it’s assumed that $|\epsilon_t| \ll 1$. When deviations from trends is mentioned in the thesis, it’s the log deviation from trend that it refers to. The deviation from trend for returns and wealth are defined as $\hat{r}_{t+1} = \ln \left( \frac{R_{t+1}}{\hat{R}_{t+1}} \right)$ and $\epsilon_{t+1} = \ln \left( \frac{\Gamma_{t+1}/\hat{\Gamma}_{t+1}}{1} \right)$ respectively.

Gourinchas and Rey (2007) then do a log approximation of the realized budget constraint (3) around its trend (4) and get the result:

$$n a_{t+1} \approx \frac{1}{\rho_t} n a_t + (\hat{r}_{t+1} - \epsilon_{t+1} \Delta w) - \left( \frac{1}{\rho_t} - 1 \right) n x_t$$

where

$$n x_t \equiv \mu_t^x \epsilon_t^x - \mu_t^m \epsilon_t^m$$

$$n a_t \equiv \mu_t^a \epsilon_t^a - \mu_t^l \epsilon_t^l$$

$$\mu_t^a = \frac{\hat{A}_t}{\hat{A}_t - \hat{L}_t}; \quad \mu_t^l = \mu_t^a - 1$$

$$\mu_t^x = \frac{\hat{X}_t}{\hat{X}_t - \hat{M}_t}; \quad \mu_t^m = \mu_t^x - 1$$

$$\rho_t \equiv 1 + \frac{\bar{X}_t - \bar{M}_t}{\hat{A}_t - \hat{L}_t}$$

The derivation of equation (5) is shown in section A2 in the appendix. The term $\mu_t^a$ represents the trend share of assets in the foreign asset position, and $\mu_t^l$ represents the trend share of liabilities in the net foreign asset position. The similar interpretation of the trend shares in net exports can be made; $\mu_t^x$ is the trend share of exports in net exports, and $\mu_t^m$ is the trend share of imports. These trend shares may vary over time as the trends themselves also may vary over time. If $\mu_t^a > 0$, it means that a country is a net creditor in period $t$, and if $\mu_t^x > 0$ it means that a country is a net exporter in period $t$. In the expressions for net exports ($n x_t$) and net foreign assets ($n a_t$) the trend shares function as weights. They will be referred to as both trend shares and weights interchangeably throughout the paper. Each weight is multiplied by its corresponding variable’s deviation from trend. This makes $n x_t$ and $n a_t$ linear combinations of stationary (trend shares/weights) and cyclical (log deviations from trends) components. $\rho_t$ is the discount rate which is less than one when either the trend net exports or the trend net foreign assets is negative. The discount factor should over time be less than one such that the budget
constraint in deviations from trend can be solved forward to obtain the intertemporal budget constrain in deviations from trend. This could be a problem for Norway who has experienced both positive net assets and net exports over time. Grisse and Nitschka (2016) have replicated Gourinchas and Rey’s (2007) study on Swiss data, and they also observe positive net foreign assets and net exports over time. They argue to adjust the trend shares in order to keep the discount rate lower than one at all times (Grisse and Nitschka, 2016). This is discussed in further detail in section 5.2. The last variables that the approximation of the external constrain consists of, are the cyclical return on net foreign assets $\hat{r}_{t+1}$ and the cyclical growth rate of wealth $\Delta w_t$.

Norway’s estimated trends and cyclical components for exports, imports, assets and liabilities to GDP are shown in figure 3.
Equation (5) states that \( n\alpha_{t+1} \) grows if there is a positive log deviation from trend in exports or in the return on net foreign assets. Equally, will \( n\alpha_{t+1} \) decrease if there are positive deviations from trend in imports or negative deviations in the return on net foreign assets. This will only happen if: \( \mu^a > 0 \) and \( \mu^x < 0 \), or \( \mu^a > 0 \) and \( \mu^x > 0 \). Equation (5) is not robust to changes in trend shares resulting from a country transitioning from being a net creditor and having a trade surplus (\( \mu^a > 0 \) and \( \mu^x > 0 \)) to being a net debtor and having a trade deficit (\( \mu^a < 0 \) and \( \mu^x < 0 \)). This can be fixed by using the absolute values of the weights, which is done in the next section. There, I also add the assumption of a common growth rate for the trend components.

### 3.4 Adding a Common Growth Rate

Gourinchas and Rey (2007) argues in favor of assuming a common and possibly time-varying growth rate \( \mu_t \) for the deterministic economy that follows deterministic trends. That indicates a common growth rate for the variables \( \overline{Z}_t \). Keeping in mind that \( \overline{Z}_t \) is the trend and equilibrium ratios of \( Z_t/W_t \) in the deterministic economy, \( \mu_t \) represents the growth rate of exports, imports, assets and liabilities relative to the growth rate of wealth. The term \( \mu_t \) should not be interchanged by \( \mu^Z_t \) which is the trend shares/weights of the ratios exports, imports, assets and liabilities to GDP that are defined in equation (8) and (9). A common growth rate (\( \mu_t \)) for the trends will result in constant weights (trend shares \( \mu^Z_t \)) in equation (8) and (9). This is because the trends in exports, imports, assets and liabilities relative to wealth remain the same. It also means that no single component in \( Z_t \) can forever grow at a rate higher than any of the others, since they all face a common growth rate in their trends. Exports, for example, cannot forever grow faster than imports, assets or liabilities in the deterministic economy. This will be discussed more in detail in section 5. With constant weights resulting from the common growth rate assumption, Gourinchas and Rey (2007) are able to obtain more accurate approximations. This is especially so because the weights become very large when \( \overline{A}_t - \overline{L}_t \approx 0 \) or \( \overline{X}_t - \overline{M}_t \approx 0 \) (see equation (8) and (9)). Gourinchas and Rey (2007) use the sample average of the trend components to estimate the constant weights that are used in their measure of external imbalance. Another implication of a common growth rate is that the discount rate \( \rho_t \) becomes constant. It is a function of the trends, and when the trends grow at the same speed, \( \rho_t \) remains unchanged (see equation (10)). Furthermore, the consequence of the common growth rate for the trends is that equation (5) simplifies to:
\[ nxa_{t+1} \approx \frac{1}{\rho} nxa_t + r_{t+1} + \Delta nxa_{t+1} \]  

(11)

where

\[ nxa_t \equiv |\mu^a|\varepsilon_t^a - |\mu^i|\varepsilon_t^i + |\mu^x|\varepsilon_t^x - |\mu^m|\varepsilon_t^m \]  

(12)

\[ \Delta nxa_{t+1} \equiv |\mu^x|\Delta\varepsilon_{t+1}^x - |\mu^m|\Delta\varepsilon_{t+1}^m - \varepsilon_{t+1}^\omega \]  

(13)

\[ r_{t+1} \equiv \frac{\mu^a}{|\mu^a|} \hat{r}_{t+1} \]  

(14)

\[ \rho \equiv 1 + \frac{\overline{X} - \overline{M}}{\overline{A} - \overline{L}} \]  

(15)

The derivation of equation (11) is shown in the appendix. Notice the new term \( nxa_t \) which is defined in equation (12) and is incorporated in equation (11). This is the measure of external imbalances that Gourinchas and Rey (2007) construct and use in their regressions and forecasts of the exchange rate. Equation (12) carries out the interpretation that it increases with cyclical exports and assets, and decreases with cyclical imports and liabilities. It also increases with the return on net foreign assets. In contrast to the previous log approximation of the budget constraint around its trend (equation (5)), equation (11) is robust to different combinations of positive and negative values of the trend shares. This because the absolute value of the trend shares is used in the definition of \( nxa_t \). Also, notice that the weights in equation (12), and \( \rho \) in equation (15) do no longer have subscripts that denote time. They are now constants because of the common growth rate-assumption discussed above. The \( \Delta \) represents first differences in a variable between period \( t \) and \( t + 1 \).

### 3.5 The Intertemporal Approach to “The External Budget Constraint”

In order to relate the measure of external imbalances to “the intertemporal approach to the current account” it must be solved forward such that the intertemporal budget constraint is obtained. Gourinchas and Rey (2007) impose three assumptions before doing this.

1. Asymptotically, \( \lim_{t \to \infty} \mu_t = 1 \).

2. The trend return \( \overline{R}_{t+1} \) and growth rate \( \overline{\Gamma}_{t+1} \) converge to \( R \) and \( \Gamma \) such that \( R > \Gamma \).
3. \( \lim_{j \to \infty} \rho^j nxa_{t+j} = 0 \)

One should keep in mind that the trends \( \overline{Z}_t \) are defined as the equilibrium ratios of \( Z_t/W_t \) at time \( t \), and that the common and possibly time-varying growth rate \( \mu_t \) is the growth rate of \( \overline{Z}_t \). The term \( \mu_t \) is thus the relative growth rate of the trend ratios \( Z_t \in (X_t, M_t, A_t, L_t) \) to wealth. Assumption 1 implies that this ratio is equal one in the long run steady-state. Letting \( \Gamma \) represent the long run steady-state growth rate of wealth as defined in assumption 2, the implication of assumption 1 is that the trend components grow at speed \( \Gamma \) in the long run steady-state. It rules out the possibility of either of the variables in \( Z_t \) to forever grow at a rate higher than the growth of wealth. Assumption 1 does, however, allow for the ratios \( Z_t/W_t \) to be different; they need only to grow at the same rate in the long run steady-state. Gourinchas and Rey (2007) describes assumption 1 as the economy settling into a balanced growth path. Assumption 2 states that the long run steady-state rate of return is larger than the growth of wealth. This assumption is used in many growth models and is considered a mild assumption along with the first one (Gourinchas and Rey, 2007). Assumption 3 is a no-Ponzi-condition which states that the discount factor \( \rho \) must be less than 1 such that the present value of \( nxa \) in infinity is zero and not infinitely large. With these assumptions fulfilled it’s possible to solve equation (11) forward to obtain the intertemporal budget constraint in log deviations from trend:

\[
nxa_t \approx - \sum_{j=1}^{+\infty} \rho^j E_t [r_{t+j} + \Delta nxa_{t+j}] \tag{16}\]

The intertemporal budget constraint in deviations from trend (16) states that the measure of external cyclical imbalances at period \( t \) is equal to the negative discounted sum of all deviations from trends in the return on net foreign assets and net exports into infinity. In other words, does \( nxa_t \) contain information on all future deviations from trends in net foreign assets and net exports. If there is a positive external imbalance in period \( t \), equation (16) states that is must be offset by future returns on net foreign assets that are below trend or it can be offset by changes in net exports that are below trend. This is in line with “the intertemporal approach to the current account”. An important difference is that this measure is in log deviations from trend, in contrast to the traditional approach to the current account.
4 Data

This section provides a description of the data used in the thesis. It also provides explanations of how I construct the different variables that are used for the regressions and forecasts. All variables in the dataset are available from 1996 at the minimum.

4.1.1 Exchange rates

I use monthly data on the trade-weighted NOK exchange rate which is weighted against Norway’s 25 major trade partners (Norges Bank, 2018). The monthly rates are calculated by taking the average of the daily rates within each month. In order to get quarterly data, I calculate the average of the months within each quarter over the sample period. The same is done for the import-weighted NOK exchange rate which is weighted against the 44 countries that Norway imports the most from (Norges Bank, 2018). For both the trade-weighted and import-weighted exchange rate, an increase in the index represents a depreciation of the NOK (Norges Bank, 2018).

4.1.2 CPI

Data one the Norwegian consumer price index (CPI) is used (Statistics Norway, 2018a). This data is not available on a quarterly basis, but it is available on a monthly and yearly basis. By extracting data on the first day of March, June, September and December for each year, I will be able to construct a dataset on a quarterly basis.

4.1.3 Wealth

Gourinchas and Rey (2007) used household net worth as a proxy for wealth. The data on Norwegian households’ net worth which includes the gross worth of households’ primary dwellings is available for just a limited time period on an annual basis (Statistics Norway, 2017d). There is, however, data available on households’ net financial assets available for the relevant time period, but households’ estimated real capital which includes property holdings is not included in financial assets (Statistics Norway, 2018c). In 2015, 66.7 percent of households’ estimated gross worth was due to privately owned dwellings (Statistics Norway,

2 The estimated gross wealth of household was 8435 billion and the privately owned dwellings amounted to 5629 billion NOK (Statistics Norway, 2016)
Using an estimate of households’ net worth that doesn’t include factors that may contribute to more than half of what’s estimated is not ideal, to put it mildly. Due to these problems, I follow Grisse and Nitschka (2016) and use seasonally adjusted GDP as a measure for wealth. This isn’t ideal either, but it might be better than using net financial wealth as an estimate for households’ wealth. When using GDP as a measure of wealth, it may be preferable to use seasonally adjusted GDP levels. It provides a more accurate description of changes in the quarterly GDP levels as seasonal effects are filtered out. This means that an increase in the seasonally adjusted GDP levels can to a larger extent be interpreted as an increase in the wealth of Norway. At the same time will the seasonally adjusted measures lead to less volatility in the measure of wealth. This may be beneficial when performing regressions later on. Since Norway is a special case of having a large oil and gas industry relative to it’s GDP level, it could be beneficial to use seasonally adjusted mainland GDP\(^3\) as a measure of wealth. Seasonally adjusted mainland GDP levels are less exposed to changes in the oil and gas price, which reduces the volatility in the measure of wealth. Data on seasonally adjusted GDP are gathered from Statistics Norway (2017a) and are denoted in NOK million.

### 4.1.4 Asset and Liability Positions

Foreign asset and liability positions are in NOK million current prices and are available on a quarterly basis from Statistics Norway (2017c). The data are measured at the end of the quarter, but in order to adjust it to fit the framework of Gourinchas and Rey (2007), I want to use data that report positions at the beginning of each period. All observations on asset and liability positions, including variables that denote changes in the positions that are not due to transactions are thus moved one quarter ahead, such that the original end-of-period positions are equal to the beginning-of-the-next period positions.

### 4.1.5 Returns, Exports and Imports

Quarterly data on exports and imports are also used. They are denoted in million NOK current prices and are not seasonally adjusted. The data is collected from Statistics Norway (2017c). Data on capital transfers from abroad and to abroad are available from the same statistics, and the transfers are subcategorized into interest received, dividends and reinvested earnings.

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\(^3\)Mainland GDP consists of all domestic production activity except exploration of crude oil and natural gas, transport via pipelines and ocean transport” (Statistics Norway, 2018e)
Data on exports and imports are collected from 1981 since it is beneficial to have data available for a longer period when estimating their trends. Export value from mainland Norway and exports of natural gas and crude oil are gathered from Statistics Norway (2018b). Production volume of natural gas and crude oil are found at The Norwegian Petroleum Directorate (2018). These data, including those on export values are available at a monthly basis. To convert them into quarterly data, the three months that make up the first, second, third and fourth quarter separately, are summarized over the whole sample. In order to construct the measure real returns on net foreign assets, I start by constructing measures of the percentage real return on foreign assets and foreign liabilities. Nominal returns on foreign assets are calculated by summarizing capital transfers from abroad that can be subcategorized into interest received, dividends and reinvested earnings. A variable that represents “other changes” in foreign assets is also included the measure of return on foreign assets. By including “other changes”, adjustments in the asset and liability positions that come from exchange rate movements or changes in the price of assets or liabilities are accounted for in the measure of nominal returns. Note that all measures are in NOK. In order to express the nominal returns in percentage terms, I divide the summarized returns by the foreign asset position. Finally, to obtain real returns, I subtract the inflation rate between period \( t \) and \( t + 1 \). The percentage real return on foreign liabilities are calculated in the same fashion. The net real return on the net foreign asset position is then constructed according to

\[
r_t = |\mu^a| r_t^a - |\mu^l| r_t^l\tag{17}
\]

, where \( r_t^a \) is the percentage real returns on foreign assets and \( r_t^l \) is the percentage real return on foreign liabilities. The percentage real return on foreign assets and liabilities are weighted by their respective trend shares in the net foreign asset position, whose construction was discussed in section 3.3. This is important in order to adjust for the relative size of assets and liabilities in the net foreign asset position. It’s worth noting that I compute the real returns on net foreign assets in a different way than Gourinchas and Rey (2007), although they weight the returns on assets and liabilities according to equation (17). They don’t have data on “other changes” readily available as I do, so they calculate the returns on all foreign assets and

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4 “Other changes in assets and liabilities record changes in stocks that are not the result of transactions. They are either: changes in assets and liabilities due to exceptional and unforeseen events, changes in classification and structure, changes in exchange rates or changes in the price of the assets/liabilities” (Statistics Norway, 2018d).
liabilities with the subcategories: equity, foreign direct investments (FDI), debt, and other assets before they are adjusted for inflation and converted into USD (Gourinchas and Rey, 2007).
5 Methodology

In this section, I will go through the methodology that is used for the forecasts of the exchange rate. My approach differs in some ways from the approach of Gourinchas and Rey (2007), and these differences will be highlighted. Comparisons to the study of Grisse and Nitschka (2016) who have replicated Gourinchas and Rey (2007) on Swiss data will also be made. The section also includes discussions on the different assumptions that is made in section 3.

5.1 The Measure of External Imbalances, Regressions and Predictions

The first part of this subsection will on the construction of the measure of external imbalances. Then, a description on the estimation of the relationships between external imbalances, cyclical real returns on net foreign assets, cyclical net export growth and exchange rates will be made. Finally, the method of behind the out-of-sample forecasts and the tests of differences in the performance between the out-of-sample forecasts and the random walk are explained.

5.1.1 Constructing the Measure of External Imbalances

In line with Gourinchas and Rey (2007) I use ratios of exports, imports, assets and liabilities to wealth to create the measure of external imbalances. I apply a Hodrick-Prescott filter (HP-filter) on the ratios to find the trends and the log deviation from trends. By using a smoothing parameter $\lambda = 2 400 000$, cycles that last more than 50 years are filtered out as in Gourinchas and Rey (2007). To more accurately estimate the trends in exports and imports, I apply the HP-filter on them between 1981q1-2017q3. But only values (trends and cyclical components) between 1996q1-2017q3 are used in the actual construction of $\pi x a$ and regressions. The trend components that are created from applying the HP-filter work as estimates of the equilibrium ratios $Z_t$ in equation (4) for the deterministic economy. I then construct the trend shares for exports, imports, assets and liabilities relative to wealth which function as weights in the measure of external imbalances. In line with Gourinchas and Rey (2007), the sample averages of the trend components are used to construct the trend shares for the different variables. It results in a constant discount factor and constant weights in line with the consequence of a common growth rate that was discussed in section 3.4. The estimated weights are shown in equation (18) below. Gourinchas and Rey (2007) report that they get more accurate predictions
by using constant weights. My own regressions of the NOK depreciation rate on lagged values of the measure of external imbalances report both lower p-values and higher r-squareds with constant trend shares, so this is also the approach I choose. The measure of external imbalances is then created according to the equation below, which is equation (12) in section 3.

\[ nxat \equiv |\mu^a|\varepsilon^a_t - |\mu^l|\varepsilon^l_t + |\mu^x|\varepsilon^x_t - |\mu^m|\varepsilon^m_t \]

The estimated weights from equation (18) are then used to obtain equation (19).

\[ \mu^a = 3.14, \quad \mu^l = 2.14, \quad \mu^x = 4.69, \quad \mu^m = 3.69 \] (18)

\[ nxat = 3.14\varepsilon^a_t - 2.14\varepsilon^l_t + 4.69\varepsilon^x_t - 3.69\varepsilon^m_t \] (19)

By dividing equation (19) by \( \mu^x = 4.69 \), the measure of external imbalances will be in “export units”. The interpretation of equation (20) is that a one percent increase in cyclical imbalances would be the same as a one percent increase in cyclical exports.

\[ nxat = 0.67\varepsilon^a_t - 0.46\varepsilon^l_t + \varepsilon^x_t - 0.79\varepsilon^m_t \] (20)

Equation (20) gives a picture of the relative importance of assets, liabilities, exports and imports in the measure of external imbalances.

5.1.2 Future Returns and net Export Growth

The measure of external imbalances should contain predictive information on future real returns on net foreign assets and net export growth according to the theory. This is investigated by regressing cyclical returns and cyclical net export growth on the measure of external imbalances. These are ordinary least square regressions with Newey-West standard errors where the maximum number of lags that are to be controlled for serial correlation is \( k - 1 \) as in (Gourinchas and Rey, 2007). It’s worth noting that the cyclical returns on foreign assets and liabilities are very close to the actual returns (see figure 4). This implies that the regression results on cyclical real returns on net foreign assets presented in section 6 can to a large extent be interpreted as changes in the actual variables.
5.1.3 Rolling Recursive Regressions and Forecasts

The regressions used for the out-of-sample forecasts are rolling recursive regressions. Such regressions are recognized by a continuing expansion of the sample-size, which will be explained in more detail below. Both the trade-weighted and import-weighted NOK are regressed on lagged values of \( nxa \) from the first to the 20\textsuperscript{th} lagged value of \( nxa \). Let \( y_{t,t+k} \) represent the depreciation rate between \( t \) and \( t+k \), where \( y_{t,t+k} = 1/k \sum_{i=1}^{k} y_{t+i} \). The variable \( y_{t,t+k} \) should be interpreted as the average depreciation rate between \( t \) and \( t+k \) when the horizon exceeds one. \( y_{t,t+k} \) is then regressed on the measure of external imbalances \( nxa_t \). Newey West standard errors are reported, and the maximum number of lags that are to be controlled for serial correlation is set to \( k - 1 \) as in Gourinchas and Rey (2007). Significant estimates are then used to forecast the exchange rate over the out-of-sample.

To begin with, I divide the sample in half at 2006q4, where the first half is the in-sample and the second half is the out-of-sample. Regressions are then run over the in-sample to estimate relationships between the variation in cyclical imbalances (\( nxa \)) and depreciation rates. Based on these regressions, forecasts are made over the out-of-sample. Since all estimated trends used in constructing \( nxa \) are estimated over the whole sample period, \( nxa \) must be re-estimated over
the in-sample before it’s regressed on the depreciations rates. In that way, no variables over the in-sample contain any information about future realizations of any variables. This prevents any bias toward future realized values. Then, based on the regressions over the in-sample, a forecast is made for the first period in the out-of-sample that contains no information from the in-sample. For example, a forecast of the depreciation rate six quarters ahead must be performed for the sixth period in the out-of-sample. This is because the forecast is a prediction of the average depreciation rate over a six-quarter period. A forecast of the first five (or less) quarters in the out-of-sample will then contain information from the in-sample. After the forecast is made for the first period that contains no information from the in-sample, the in-sample size is expanded by one period and \( p \times \alpha \) is re-estimated before regressions are run, and a forecast is made for the first period that contains no information from the in-sample. This continues until there is no out-of-sample depreciation rates to forecast.

### 5.1.4 Testing the Difference in Mean Squared Forecast Errors

In order to compare the accuracy of the forecast model that uses a measure of external imbalances against the random walk, I compare the mean squared forecast errors (MSFEs) for the different models. For the random walk, the MSFE is calculated as described by equation (21)

\[
MSFE = \frac{1}{F - k} \sum_{t=T-F+k}^{T} (\Delta e_{t,t+k})^2
\]

, where \( T \) is total observations, \( F \) is the number of out-of-sample forecasts, \( k \) is the forecast horizon and \( \Delta e_{t,t+k} = \frac{1}{k} \sum_{i=1}^{k} \Delta e_{t+i} \). The interpretation of variable \( \Delta e_{t,t+k} \) is the average depreciation rate over the period between \( t \) and \( t + k \). Since the random walk forecasts no change in the exchange rate (no depreciation), any change in the exchange rate will result in an increased MSFE. For the alternative model (the one that uses the measure of external imbalances), the MSFE is calculated by equation (22),

\[
MSFE = \frac{1}{F - k} \sum_{t=T-F+k}^{T} (\Delta e_{t,t+k} - \Delta e_{t,t+k}^f)^2
\]

where \( f \) denotes the forecasted exchange rate. The MSFE for the alternative model is constructed in a slightly different way in order to adjust for “noise” that increases the size of
the MSFE relative to the one for the random walk (Clark and West, 2006). Following Clark and West (2006), this noise is equal to:

\[
\text{noise} = \frac{1}{F-k} \sum_{t=T-F+k}^{T} (\Delta e_{t+k}^{f})^2
\]  

(23)

By subtracting equation (23) from (22), the noise term is adjusted for, and the MSFE-adjusted is obtained (Clark and West, 2006). One can then perform t-tests to test the statistical significance of the differences in the MSFE for the random walk and the adjusted MSFEs for the alternative model (Clark and West, 2006). For each rolling forecast made by the alternative model, the MSFE-adjusted is calculated. This is the same procedure as Gourinchas and Rey (2007) use to compare the accuracy of the different models.

5.2 Discussing the Methodology

In this subsection I discuss to what extent the methods used by Gourinchas and Rey (2007) and Grisse and Nitschka (2016) can be applied to Norway and the validity of the assumptions they make.

5.2.1 Abstracting from Trends, and Choice of Smoothing Parameter

Gourinchas and Rey (2007) argue that they abstract from trends in their study simply because they are not relevant for investigating the cyclical adjustments, and they provide a discussion on whether the development of trend imbalances that have been building up for the U.S affects the trend exchange rate. This is a relevant discussion for Norway since the trend imbalances for Norway have been building up over the last 20 years (see figure 3), although they are of the opposite character as of the U.S. If there exist trends in the exchange rate that are the results of trend imbalances that are building up, predictive information on future exchange rates is lost when abstracting from trends (Gourinchas and Rey, 2007). It should thus be discussed to what extent one can expect that trends in the exchange rate are results of trending imbalances. Norway’s trend imbalances have been building up since the late 90’s, which could lead one to expect a trending appreciation of the NOK over the last years or at some time in the future. An appreciation of the NOK would then reduce the value of foreign assets and most likely reduce future net exports. This would contribute to a reduction in the trend imbalances. Figure 5, however, does not show a trending appreciation of the NOK over the last 20 years. The slope
of the trend is slightly positive, which indicates a trending depreciation of the trade-weighted NOK. If anything, the estimated trend in the exchange rate would imply an increase in Norway’s trend imbalances. But the estimated trend in the exchange rate is not far from horizontal, which may indicate that the trend exchange rate has a small or no impact on trend imbalances. This is especially so when one compares it to the sharp increase in trend imbalances in net foreign assets that has taken place over the last 20 years (see figure 1). This indicates that abstracting from trends does not mean that important information about future realizations of liabilities, exports, imports, real returns and exchange rate movements is lost.

Another aspect of the discussion on abstracting from trends, is the choice of smoothing parameter used in the HP-filters. It decides the length of cycles that are filtered out of the variables. A smoothing parameter \( \lambda = 1600 \) is most commonly used for detrending quarterly data. According to Maravall and Del Río (2007, p. 981) a smoothing parameter of that size filters out cycles that are “very approximately” 10 years or longer. This is also the size that Hodrick and Prescott (1997) recommended in their presentation of what is known as the HP-filter. But their justification of the size of smoothing parameter was not directly related to length of cycles that should be filtered out: “Our prior view is that a 5 percent cyclical component is moderately large, as is a one-eighth of 1 percent change in the growth rate in a quarter. This led us to select \( \sqrt{\lambda} = 5/(1/8) = 40 \) or \( \lambda = 1600 \) as a value for the smoothing parameter.” (Hodrick and Prescott, 1997, p. 5). Gourinchas and Rey (2007), however, use a smoothing

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**Figure 5. The Trade-Weighted Exchange Rate**

![Graph showing the trade-weighted exchange rate]

Note: The trade-weighted depreciation rate and its estimated trend from applying a HP-filter with a smoothing parameter \( \lambda = 2400000 \). The index’s reference point is in 1990 where is equals 100.
parameter $\lambda = 2\ 400\ 000$ which filters out cycles of 50 years or more. This makes the trend components more linear and the deviations from trends more volatile. The advantage of this is that less information from the cyclical components is filtered out, such that they may contain more predictive information on future exchange rates, returns or net exports. Since only cycles longer than 50 years are filtered out, abstracting from trends is only a problem if these trends play a role in the adjustments of trend imbalances. As argued above, it does not seem that the estimated trend in the exchange rate has had any sizeable impact on Norway’s net foreign asset position. This is also in line with what Gourinchas and Rey (2007) argue. They find it hard to believe that cycles longer than 50 years contribute to any adjustments through the “trend valuation channel” because it would imply predictable returns more than 50 years ahead (Gourinchas and Rey, 2007). They also argue that cycles of more than 50 years in the real exchange rate do not necessarily have an impact on net exports. The example they use is Japan between the 50s and 90s. During that time, Japan experienced a real appreciation of its currency, but its trade with the U.S. remained unchanged over the same period (Gourinchas and Rey, 2007). Abstracting from trends longer than 50 years should thus not be a problem for the predictability of exports, imports, returns or exchange rate movements. But if there are any trend adjustments in Norway’s net foreign asset position coming from trends in the exchange rate, abstracting from trends will in that case decrease the predictable properties of the measure of external imbalances.

5.2.2 A Discount Factor Larger than one

In Grisse and Nitschka’s (2016) replication of Gourinchas and Rey (2007) on Swiss data, they get a discount factor that is larger than one ($\rho = 1.01$) when they use the average of the trends in exports, imports, assets and liabilities to construct the trend shares. This is because Switzerland on average has both been a net creditor and a net exporter over the sample period 1970-2012 (Grisse and Nitschka, 2016). Using the same method to create constant trend shares for Norway, I get $\rho = 1.03$. Grisse and Nitschka (2016) see this as a problem because a discount rate above one violates the no-Ponzi assumption which was discussed in section 3.5. A violation implies that the external constraint (11) cannot be solved forward to obtain the intertemporal external budget constraint (16) because $n_x a_t$ grows infinitely large. Grisse and Nitschka (2016) argue to make some adjustments in their measure of external imbalances to get a discount rate less than one. These adjustments are made by merging imports and exports into a single variable: net exports. An adjustment term is then subtracted from net exports to ensure that net
exports always is negative. The same adjustment term is then added to foreign assets. Hodrick-Prescott filters are then applied and their measure of external imbalances is then a sum of three linear combinations of the cyclical and stationary components of assets, liabilities and net exports, as opposed to Gourinchas and Rey (2007) whose measure is a sum of four linear combinations. The advantage of using net exports instead of exports and imports separately is that there is no need to estimate the weights of exports and imports. Only the weights of assets and liabilities are estimated by the sample averages of the trends in constructing $\pi x a$, which means that two sources of measurement errors in the approximated $\pi x a$ is avoided (Grisse and Nitschka, 2016). Since Grisse and Nitschka’s (2016) adjusted measure of net exports always is negative, they get a discount rate smaller than one when using the sample averages of the trends in adjusted assets, liabilities and net exports in their calculation of the discount rate (Grisse and Nitschka, 2016).

Grisse and Nitschka (2016) argue that the adjustments only will affect the level of the trends, and that the deviations from trends still provide robust approximations of $\pi x a$. When they increase the size of the adjustment terms, however, their predictions of the Swiss franc become less significant and they obtain lower adjusted r-squareds, but their results remain robust (Grisse and Nitschka, 2016). One can thus argue that having lower adjustment terms is more desirable, and if possible, it’s best to avoid adjusting the variables at all. The size of the adjustments term for Grisse and Nitschka (2016) is about 12 percent of quarterly GDP, whereas an adjustment term of about 21 percent of quarterly GDP would be necessary for Norway to ensure a discount factor smaller than one, according to my calculations. It’s thus more likely that the adjustments for Norway could affect the explanatory power of the predictions even more than in the Swiss study. With this in mind, I want to argue in favor of allowing a discount factor above one.

A discount factor above one is in principle a violation of the no-Ponzi assumption given in section 3.5. But keeping in mind that the discount factor was calculated over a sample period covering 43 years for Switzerland and 22 years for Norway, one can argue that the calculated discount factors do not represent the long term discount factors for the countries. It could very well be that both Norway and Switzerland experience large deficits on their trade balance somewhere in future, which could have corrected their discount factors. This we cannot know for sure, but it illustrates the point that using a 43 or 23-year sample period to estimate a discount rate that is supposed to be constant into infinity, might not be a solid estimate because it is subject to a high degree of uncertainty. Followingly is a discount rate above one over a 20-40-
year sample period perhaps not a good enough reason to adjust variables that will be used in estimations and predictions. I will thus not adjust any variables in my replication of Gourinchas and Rey (2007), even though it implies a discount rate above one over the sample period. It violates the no-Ponzi condition over the sample period, but the formal no-Ponzi assumption made in section 3.5 covers all period into infinity. That assumption, I do not consider as violated.

The validity of the approximated budget constraint around its trend (equation 11, section 3.4), can be tested on the data by taking the difference between the right-hand and left-hand side of equation (11), which is shown in figure 6. The approximation error has a standard deviation of 1.06 percent and is 16 times less volatile than $nxa$ and 7 times less volatile than the flow component. That’s a smaller approximation error than Gourinchas and Rey (2007) obtain, which has a standard deviation of 1.67 percent and is seven times less volatile than $nxa$ and 2.5 times less volatile than their flow component (Gourinchas and Rey, 2007). Also the correlation (0.01) between the flow component and the residual is smaller than in Gourinchas and Rey (2007) who report a correlation of 0.05. This indicates that my way of constructing the cyclical real returns on net foreign assets provides accurate approximations of $nxa$. The only downside is that the approximation error based on Norwegian data has a mean of 0.2 and not zero as in Gourinchas and Rey (2007). Given that the approximation error is very small, this mean should not be a problem. The important implication of the small approximation error is that it provides an empirical verification of the log approximation of the budget constraint around its trend. It also indicates that using a discount factor above one yields accurate approximations.
Figure 6. The Approximation of $nxa$

Note: The top left panel shows the measure of external imbalances, $nxa$, constructed according to equation (12). The top right panel is the approximated flow term ($r_{t+1} + \Delta nx_{t+1}$) from equation (11), which is the approximated budget constraint in deviations from trend. The bottom panel is the approximation error from equation (11) defined as: $\epsilon = nxa_{t+1} - nx_{t}/\rho - r_{t+1} - \Delta nx_{t+1}$.
6 Empirical Results

The empirical results are presented in this section. The predictive properties of the measure of external imbalances are investigated for cyclical real returns on net foreign assets, cyclical net export growth and exchange rates. Finally, the statistical tests of the relative performance of the forecasting model using \( nxa \) and the random walk are provided. When referring to statistical significant results, I refer to those that are significant at a five percent level unless I explicitly refer to those that are significant at a 10 percent level. Cyclical real returns on net foreign assets and cyclical net export growth are interchangeably referred to as returns and export growth.

6.1 Real Returns on net Foreign Assets

Table 1 shows the results from regressing the average real returns on net foreign assets between \( t \) and \( t + k \), defined as: \( r_{t,t+k} = 1/k \sum_{i=1}^{k} r_{t+i} \) on the measure of external imbalances \( nxa_t \). The table shows significant results at all horizons from 1-20 quarters ahead. The adjusted r-squared peaks at the eight quarter where 18.7 percent of the variation in the average cyclical change in real returns is explained by movements in the measure of external imbalances. It’s at its lowest for the first and 16th quarter. The coefficient sizes generally decrease over time, which indicates a reduced predictive power of the measure of external imbalances over time. The interpretation of the coefficient for the first quarter is that a one-percent increase in the measure of external imbalances is associated with a reduction in the real return on net foreign assets in the next quarter of about 0.11 percent, which at an annual rate is about 0.42 percent. The results are in line with the theoretical framework discussed in section 3, where the intertemporal budget constraint in deviations from trend (measure of external imbalances) was constructed. It stated that any positive deviations in the measure of external imbalances today, must be offset by future negative deviations which would either come from future reductions in net exports or in the returns on the net foreign assets. The associated reduction in future returns on net foreign assets in response to an increase in the measure of external imbalances today, is significant 5 years ahead. This indicates that international financial adjustments in response to cyclical imbalances are at work at relatively long horizons. Even though the adjusted r-squareds are relatively high, they are smaller than those Gourinchas and Rey (2007) report. Their regressions are able to explain 14-38 percent of the variation in returns 1-24 quarters ahead (Gourinchas and Rey, 2007).
With an established relationship between cyclical imbalances and future returns, I next explore the empirical relationship between cyclical imbalances and the future changes in net exports. Regression results that are easily comparable to those on cyclical returns are shown in table 1, but a more detailed overview of the significant results at a 1-10 percent level is provided in table 2 below. The regressions report significant estimates at a one or five percent level 9-10 and 13-14 quarters ahead. The adjusted r-squared peaks at the 10th quarter, connecting 4.4 percent of the variation in cyclical net exports to changes in the measure of cyclical imbalances. This is a lot smaller than for real returns, but the coefficient sizes are larger for net exports than for real returns. Compared to Gourinchas and Rey (2007) who obtain r-squareds up to 0.58 for the 24th quarter, the adjusted r-squareds in table 2 are remarkably lower. The largest effect (at a five or one percent significance level) of a change in cyclical imbalances is found at the 10th quarter with a coefficient of -0.074. The interpretation is that a one-percent increase in cyclical imbalances is associated with decrease in cyclical net exports over the next 10 quarters of about 0.74 percent. It supports the suggested dynamics of the intertemporal budget constraint in deviations from trend, since an increase in cyclical imbalances is associated with future reductions in cyclical net exports. It seems, however, that the adjustment horizon for cyclical net exports in response to an increase in cyclical imbalances is shorter than for returns, and that it takes longer time before cyclical net exports reacts to changes in cyclical imbalances. There

### Table 1

<table>
<thead>
<tr>
<th>FORECAST HORIZON (QUARTERS)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>10</th>
<th>16</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Real Returns on net Foreign Assets ( r_{t+k} )</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( nxa )</td>
<td>-0.106**</td>
<td>-0.111***</td>
<td>-0.054***</td>
<td>-0.032***</td>
<td>-0.021***</td>
<td>-0.010***</td>
<td>-0.011***</td>
</tr>
<tr>
<td>(adj. R²)</td>
<td>0.062</td>
<td>0.135</td>
<td>0.104</td>
<td>0.187</td>
<td>0.113</td>
<td>0.047</td>
<td>0.079</td>
</tr>
<tr>
<td><strong>Net Export Growth ( \Delta nxa_{t+k} )</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( nxa )</td>
<td>-0.294</td>
<td>-0.349</td>
<td>-0.107</td>
<td>-0.041</td>
<td>-0.074***</td>
<td>-0.015</td>
<td>-0.009</td>
</tr>
<tr>
<td>(adj. R²)</td>
<td>0.011</td>
<td>0.044</td>
<td>0.009</td>
<td>-0.002</td>
<td>0.044</td>
<td>-0.009</td>
<td>-0.013</td>
</tr>
</tbody>
</table>
| Standard OLS regressions of the form \( y_{t+k} = \alpha + \beta nxa_t + \epsilon_{t+k} \) are run, where \( y_{t+k} \) is cyclical real returns on net foreign assets \( (r_{t+k}) \) and cyclical net export growth \( (\Delta nxa_{t+k}) \) between \( t \) and \( t + k \) in the top and lower panel respectively. The estimated coefficient on \( nxa \) is reported. Newey-West standard errors are in parenthesis and are calculated by controlling for serial correlations up to lag number \( k - 1 \). Significant levels at a ten, five and one percent level is marked with one, two and three stars respectively.

### 6.2 Cyclical Net Exports

With an established relationship between cyclical imbalances and future returns, I next explore the empirical relationship between cyclical imbalances and the future changes in net exports. Regression results that are easily comparable to those on cyclical returns are shown in table 1, but a more detailed overview of the significant results at a 1-10 percent level is provided in table 2 below. The regressions report significant estimates at a one or five percent level 9-10 and 13-14 quarters ahead. The adjusted r-squared peaks at the 10th quarter, connecting 4.4 percent of the variation in cyclical net exports to changes in the measure of cyclical imbalances. This is a lot smaller than for real returns, but the coefficient sizes are larger for net exports than for real returns. Compared to Gourinchas and Rey (2007) who obtain r-squareds up to 0.58 for the 24th quarter, the adjusted r-squareds in table 2 are remarkably lower. The largest effect (at a five or one percent significance level) of a change in cyclical imbalances is found at the 10th quarter with a coefficient of -0.074. The interpretation is that a one-percent increase in cyclical imbalances is associated with decrease in cyclical net exports over the next 10 quarters of about 0.74 percent. It supports the suggested dynamics of the intertemporal budget constraint in deviations from trend, since an increase in cyclical imbalances is associated with future reductions in cyclical net exports. It seems, however, that the adjustment horizon for cyclical net exports in response to an increase in cyclical imbalances is shorter than for returns, and that it takes longer time before cyclical net exports reacts to changes in cyclical imbalances. There
are, however, estimates at a 10 percent significance level which could indicate that net export growth may also be affected at both longer and shorter horizons than what’s indicated in table 1. But one should be careful at interpreting coefficients at a 10 percent level due to their higher probability of falsely rejecting the null hypothesis of the coefficients being equal to zero.

The empirics speak in favor of the dynamics of the intertemporal budget constraint in deviations from trend presented in section 3. This is promising evidence for the potential empirical connection between the intertemporal budget constraint in deviations from trend (the measure of cyclical imbalances) and future exchange rates. As argued in section 3.2, do movements in the exchange rate affect both future net exports and returns on net foreign assets. The question is whether the observed negative relationship between the measure of external imbalances, future returns and cyclical net exports can be explained by changes in the exchange rate, such that the measure of external imbalances contains predictive powers for the exchange rate.

Table 2

<table>
<thead>
<tr>
<th>FORECAST HORIZON (QUARTERS)</th>
<th>2</th>
<th>5</th>
<th>6</th>
<th>9</th>
<th>10</th>
<th>13</th>
<th>14</th>
<th>18</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Delta n x_{t+k} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n x a )</td>
<td>-0.349*</td>
<td>-0.128*</td>
<td>-0.108*</td>
<td>-0.0867**</td>
<td>-0.0738**</td>
<td>-0.0507**</td>
<td>-0.0459**</td>
<td>-0.0310*</td>
</tr>
<tr>
<td>(0.054)</td>
<td>(0.087)</td>
<td>(0.056)</td>
<td>(0.004)</td>
<td>(0.004)</td>
<td>(0.045)</td>
<td>(0.050)</td>
<td>(0.092)</td>
<td></td>
</tr>
<tr>
<td>adj. ( R^2 )</td>
<td>0.044</td>
<td>0.021</td>
<td>0.036</td>
<td>0.029</td>
<td>0.044</td>
<td>0.016</td>
<td>0.027</td>
<td>0.020</td>
</tr>
</tbody>
</table>

Standard OLS regressions of the form \( y_{t,t+k} = \alpha + \beta n x a_t + \epsilon_{t,t+k} \) are run, where \( y_{t,t+k} \) is cyclical net export growth (\( \Delta n x_{t,t+k} \)) between \( t \) and \( t + k \). The estimated coefficient on \( n x a \) is reported. Newey-West standard errors are in parenthesis and are calculated by controlling for serial correlations up to lag number \( k - 1 \). Significant levels at a ten, five and one percent level is marked with one, two and three stars respectively.

6.3 The Exchange Rate

The estimated relationship between the measure of external imbalances and future depreciation rates are shown in table 3. For the trade-weighted exchange rate, there is a significant relationship at six to ten quarters ahead. The adjusted r-squared is between 0.11 – 0.14 for quarters six to nine, but for the tenth quarter it’s only at 0.08, which also has a lower coefficient size than the other quarters. The coefficient for quarters six to nine are remarkably equal of size, but most of the variation in the exchange rate that can be explained by changes in \( n x a \) is found at quarter eight and nine. Note that all estimated coefficients of the relationship between \( n x a \) and future changes in the exchange rate, should be interpreted as the associated average change.
in the exchange rate up until that quarter. The interpretation of the coefficient on the eighth lagged \( n.x.a \), is that a one-percent increase in cyclical imbalances today is associated with an average appreciation over the next eight quarters that is equal to 0.025 percent. The total associated appreciation over the next eight quarters would be approximately 0.2 percent. The estimated coefficients for the trade-weighted depreciation rate thus support the dynamics of the intertemporal budget constraint in deviations from trend. This is because an appreciation of the NOK would lead to capital losses on net foreign assets and thus help to reduce cyclical imbalances.

Table 3

<table>
<thead>
<tr>
<th>FORCAST HORIZON (QUARTERS)</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trade-Weighted Depreciation rate ( \Delta \varepsilon_{t,t+k} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( n.x.a )</td>
<td>-0.022*</td>
<td>-0.025*</td>
<td>-0.025*</td>
<td>-0.025*</td>
<td>-0.023*</td>
<td>-0.017*</td>
<td>-0.013</td>
</tr>
<tr>
<td>(0.091)</td>
<td>(0.043)</td>
<td>(0.022)</td>
<td>(0.013)</td>
<td>(0.015)</td>
<td>(0.044)</td>
<td>(0.121)</td>
<td></td>
</tr>
<tr>
<td>adj. ( R^2 )</td>
<td>0.072</td>
<td>0.113</td>
<td>0.127</td>
<td>0.144</td>
<td>0.142</td>
<td>0.080</td>
<td>0.048</td>
</tr>
</tbody>
</table>

| Import-Weighted Depreciation rate \( \Delta \varepsilon_{i,t+k} \) |
| \( n.x.a \) | -0.021 | -0.025* | -0.025* | -0.025* | -0.023* | -0.020* | -0.015 |
| (0.120) | (0.061) | (0.038) | (0.027) | (0.029) | (0.054) | (0.128) |
| adj. \( R^2 \) | 0.068 | 0.115 | 0.132 | 0.150 | 0.147 | 0.114 | 0.070 |

Note: Standard OLS regressions of the form \( y_{t,t+k} = \alpha + \beta n.x.a_t + \epsilon_{t,t+k} \) are run, where \( y_{t,t+k} \) is cyclical real returns on net foreign assets (\( \Delta \varepsilon^T_{t,t+k} \)) and cyclical net export growth (\( \Delta \varepsilon^I_{t,t+k} \)) in the top and lower panel respectively. The estimated coefficient on \( n.x.a \) is reported. Newey-West standard errors are in parenthesis and are calculated by controlling for serial correlations up to lag number \( k - 1 \). Significant levels at a ten, five and one percent level is marked with one, two and three stars respectively.

The estimated coefficients for the import-weighted exchange rate are about the same size as for the trade-weighted, but they are not significant over the same horizon. They are only significant 7-9 quarters ahead compared to a 6-10 quarters horizon for the trade-weighted exchange rate. The adjusted r-squareds for the regressions on the import-weighted exchange rate are about the same size as for the trade-weighted at seven to nine quarters ahead. An increase in cyclical imbalances is associated with an appreciation of the import-weighted exchange rate as for the trade-weighted exchange rate. This supports the theory of the dynamics of the intertemporal current account. The reported adjusted r-squareds are a lot smaller than those in Gourinchas and Rey (2007) which vary between 0.28 and 0.41 3-12 quarters ahead. But compared to Grisse and Nitschka (2016), the reported adjusted r-squareds are not low. Grisse and Nitschka’s (2016)
adjusted r-squareds are between 0.01 and 0.14 for their significant estimates, where it peaks at the 11th quarter.

Given that the regressions of $nxa$ on real returns reported significant relationships up to about 20 quarters ahead, it’s interesting that the regressions on the trade-weighted and import-weighted exchange rate only are significant 6-10 quarters ahead. If indeed the exchange rate is an important determinant for future returns, one might expect significant estimates of the relationship between $nxa$ and future changes in the exchange rate at a one to five quarter horizon where the estimated effects on real returns were strongest. It could, however, be that the exchange rate is an important determinant for future returns, but that the relationship between cyclical imbalances and future returns is not explained by exchange rate movements, at least for the first five quarters. This is discussed in more detail in section 7. In relation to the results on cyclical net export growth, it could make sense that the significant estimates on future exchange rates are found at 6-10 quarters ahead whereas the significant results on net export growth were at 9-10 and 13-14 quarters ahead. If an increase in cyclical imbalances leads to an appreciation of the NOK 6-10 quarters ahead, it could be that the appreciation leads to lower import prices and higher export prices such that imports increase, and exports decrease 9-14 quarters ahead of the initial increase in cyclical imbalances. The lag between an appreciation of the NOK and a decrease in net export growth may be due to price stickiness such that it takes time before imports and exports change.

It seems that the significant relationships between cyclical imbalances and future exchange rate movements coupled with the significant relationships between $nxa$, future returns and net exports, could indicate that some of the predicted reductions in future returns and net export growth are due to predicted changes in the exchange rate. The out-of-sample predictability of the measure of external imbalances is tested in the next section.

### 6.4 Out-of-Sample Forecasts

The estimated relationships between cyclical imbalances and depreciating rates for the trade-weighted and import-weighted exchange rates are used to perform out-of-sample forecasts of future depreciation rates as described in section 5.1.3. The MSFE-adjusted for the out-of-sample forecasts are then compared to the MSFE for the random walk forecasts by performing t-tests. The null hypothesis is that that the difference between the MSFE for the random walk and the MSFE-adjusted is zero against the alternative that it is negative. The interpretation of
the null is that the two forecasting models perform equally well, and the alternative is that the model using cyclical imbalances performs poorer than the random walk. The results are presented in table 4. It states that the null hypothesis is rejected at a five percent significance level for the trade-weighted depreciation rate 10 quarters ahead, which means that there is no statistical evidence supporting that the out-of-sample forecasts are at least as good as the random walk at forecasting the trade-weighted exchange rate 10 quarters ahead. The same test rejects the null hypothesis for the trade-weighted exchange rate at 9 quarters ahead and at 8-9 quarters ahead for the import-weighted exchange rate, which indicates that the random walk may perform better than the model using \( n x a \) as a predictor. The other forecasts are not rejected at a five percent significance level, which means that we cannot reject that the two forecasting models perform equally well. The MSFE-ratios in the table provide a good indication of the relative forecast accuracy of the different models. A ratio above one means that the random walk has a lower MSFE than the mean of the MSFEs-adjusted for the model using \( n x a \) as an explanatory variable. The lowest ratios are found at the sixth and seventh quarter for the trade-weighted exchange rate, and at the sixth quarter for the import-weighted exchange rate. All the ratios are above one.

**Table 4**

<table>
<thead>
<tr>
<th>FORECAST HORIZON (QUARTERS)</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trade-Weighted Depreciation Rate ( \Delta \epsilon_{t,t+k} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSFE-ratio</td>
<td>1.20</td>
<td>1.27</td>
<td>1.40</td>
<td>1.54</td>
<td>1.71</td>
</tr>
<tr>
<td>ΔMSFE-adjusted</td>
<td>-0.21</td>
<td>-0.25</td>
<td>-0.30</td>
<td>-0.38*</td>
<td>-0.40**</td>
</tr>
<tr>
<td>p-value</td>
<td>0.18</td>
<td>0.15</td>
<td>0.11</td>
<td>0.07</td>
<td>0.045</td>
</tr>
<tr>
<td>Import-Weighted Depreciation Rate ( \Delta \epsilon_{t,t+k} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MSFE-ratio</td>
<td>1.30</td>
<td>1.44</td>
<td>1.58</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔMSFE-adjusted</td>
<td>-0.25</td>
<td>-0.32*</td>
<td>-0.37*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>p-value</td>
<td>0.14</td>
<td>0.09</td>
<td>0.06</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: The MSFE-ratio is the ratio of the MFPE from the model using \( n x a \) as a predictor variable divided by the MSFE from the random walk. A ratio above one indicates that random walk performs better than the model using \( n x a \). ΔMSFE-adjusted is the difference between the MSFE for the random walk and the MSFE-adjusted for the model using \( n x a \). The adjustments are made according to Clark and West (2006). P-values from a one-side t-test of the null that the two models perform equally well is tested against the alternative that the random walk performs better. Significant levels at a ten, five and one percent level is marked with one, two and three stars respectively.
7 Discussing the Results

In this section, I will discuss potential mechanisms behind the results presented in tables 1-4. Discussions on why the results may differ from Gourinchas and Rey (2007) and Grisse and Nitschka (2016) are also provided.

7.1.1 Data

A potential reason for why the out-of-sample forecasts of the multilateral NOK exchange rate are beaten by the random walk is the limited dataset size used in the regressions. The whole dataset covers only 22 years compared to Gourinchas and Rey (2007) whose data set covers 52 years. Grisse and Nitschka’s (2016) dataset is also larger covering 41 years. Since the dataset is divided in half when performing the out-of-sample forecasts, the in-sample only covers at the minimum 11 years which amounts to 44 observations. The first out-of-sample forecasts will then be based on regressions on about half the number of observations compared to Gourinchas and Rey (2007) and Grisse and Nitschka (2016). This has two potential consequences. One that $n\times\alpha$ may be more imprecisely estimated since the weights are estimated over a shorter period. The weights are calculated using the sample average of the estimated trends, and the trends are re-estimated over the in-sample. As the in-sample size increases, the estimated trends also differ, such that the re-estimated $n\times\alpha$ over the in-sample is different from the $n\times\alpha$ that is created over the whole sample. The larger the sample size is, the more accurate the estimated trends become such that the approximated $n\times\alpha$ also become more accurate. If the relationships I try to estimate are true, the more accurate $n\times\alpha$ is, the more likely it is that the regressions report significant estimates. Another potential consequence of a small sample is that the depreciation rates may be regressed on too few observations to produce significant estimates. This may happen since the standard error of an estimated coefficient is a decreasing function of the number of observations. The t-statistic used to test the null that the estimated coefficient is zero against the alternative that it’s negative, is calculated by the ratio of the estimated coefficient divided by its standard error. By reducing the standard error and keeping the coefficient size constant, larger t-statistics are computed which is equivalent to more significant results. It does, however, not mean that a larger sample size always leads to more significant results, but the potential of finding significant results increases with the sample size, given that there indeed is a true link between cyclical imbalances and future depreciation rates at 1-20 quarters ahead.
Another possible explanation for the no so strong out-of-sample forecasting properties of $n \times a$, is the time period the dataset covers. Since Gourinchas and Rey’s (2007) dataset ends in 2004, it doesn’t include the financial crisis that emerged in 2007. This could be an important difference since the crisis caused a lot of turbulence in international financial markets. Rossi (2013) mentions that the predictability of future exchange rates seems to be both time and country specific, which could explain why the forecast for the NOK exchange rate is less successful than the forecast of the multilateral USD rate in Gourinchas and Rey’s (2007) paper. In Grisse and Nitschka’s (2016) replication on Swiss data, they also find less predictability of future exchange rates compared to what Gourinchas and Rey (2007) do. But their findings are more favorable towards the model using cyclical external imbalances than those presented here, and they are able to beat the random walk at 4, 8 and 16 quarters ahead (Grisse and Nitschka, 2016).

It could also be that the lack of data on households’ net worth is contributing to less successful results compared to Gourinchas and Rey (2007). Mainland seasonally adjusted GDP is not an accurate measure for household’s net worth, and it could contribute to poorer estimations and predictions of future exchange rates. The measure of real returns on net foreign assets is also constructed differently compared to Gourinchas and Rey (2007). But this is not likely a big problem since the approximation error in figure 6 is smaller than in Gourinchas and Rey (2007).

### 7.1.2 The Currency Composition of Foreign Assets and Liabilities

The limited success in forecasting the import and trade-weighted exchange rate could also be related to the currency composition of Norway’s foreign assets and liabilities. It could be that the import-weighted and trade-weighted NOK rate do not reflect the currency composition of Norway’s foreign assets and liabilities. If a depreciation of the trade-weighted NOK doesn’t imply a depreciation of the “foreign asset and liability-weighted” exchange rate, there is less reason to expect significant estimates between $n \times a$ and the trade-weighted exchange rate. The same goes for the import-weighted exchange rate. That could explain why there was no significant relationship between cyclical external imbalances and depreciation rates one to five quarters ahead. But even if both the trade and import-weighted exchange rate don’t reflect the currency composition of Norway’s foreign assets and liabilities, one could expect a positive correlation between the trade or import-weighted exchange rate and the “foreign asset and liability-weighted” exchange rate. The reason is that Norway’s Government Pension Fund Global (known as The Oil Fund) which made up more than half of Norway’s foreign assets in
2016\(^5\) has invested in 72 different countries (Norges Bank Investment Management, 2018). This indicates that more than half of Norway’s foreign assets are diversified into a relatively high number of currencies. It’s therefore unlikely that the trade-weighted NOK (weighted against 25 countries) or the import-weighted NOK (weighted against 44 countries) which have a correlation of 0.988 can depreciate at the same time as the “foreign asset and liability-weighted” exchange rate (weighted against at least 72 countries) appreciates. Since the “foreign asset and liability-weighted” exchange rate should be positively correlated with the trade and import-weighted exchange rate, it’s limited how much the difference in the currency composition of Norway’s foreign assets and liabilities and the import and trade-weighted exchange rate could affect the results.

### 7.1.3 Consumption Smoothing

Another potential explanation of why the out-of-sample forecasts are not as successful in Gourinchas and Rey (2007), is that the rebalancing of cyclical imbalances may to a larger extent work independently of exchange rate movements for Norway than the U.S. Rebalancing independently from exchange rate movements may be the result of consumption smoothing which was discussed in section 3.1. Table 2 showed that the there was an associated negative growth in future cyclical net exports in response to an increase in cyclical imbalances. It is this change that may be explained by consumption smoothing. If economic agents are forward-looking in their investment decisions such that they form expectations about future realizations of economic factors such as interest rates and productivity growth as discussed in section 3.1, it could make sense to reduce exports in response to an increase in cyclical imbalances. The logic is that increases in cyclical imbalances could create optimistic expectations about the future. When there are optimistic expectations about the future in the sense that there are expectations about higher output and thus higher consumption, forward-looking agents would save less (or take up debt) today in order to consume more now at the cost of a smaller increase in consumption in the future as described in section 3.1. If cyclical imbalances indeed are associated with optimistic expectations about the future, consumption smoothing could explain why there is a negative relationship between cyclical imbalances and net exports. A decrease in net exports implies that Norway saves less, which could take form through an increase in

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\(^5\) The Government Pension Fund Global had a market value of 7.510 billions NOK at the end of 2016 (Norges Bank Investment Management, 2018), whereas Norway’s total foreign assets was worth 13.249 billions NOK (Statistics Norway, 2017c).
imports or decrease in exports. One should, however, be careful on interpreting the associated decrease in net exports as a result of consumption smoothing. This is because Norway is a large exporter of both crude oil and natural gas, see figure 7. As the figure shows, the export value of both oil and gas have been quite volatile, and they account for a large share of Norway’s total exports. This makes Norway’s exports very exposed to changes in the price of crude oil and natural gas. It could thus be that the movements in cyclical net exports are driven by oil (and recently also natural gas) price changes, although changes in production has changed over the last years according to figure 7. Oil production is clearly reduced in terms of volume, whereas production of natural gas has increased over the same period. By comparing oil and gas production with their respective export values, one can infer that there have been fluctuations in both the oil and gas price. This shows that one should be careful on interpreting the reduction in net exports as consumption smoothing.

Figure 7. Export Value and Production Volume of Crude Oil and Natural Gas

![Figure 7](image)

Note: The left figure shows the export value of crude oil, natural gas and mainland exports in current NOK million. Source: Statistics Norway (2018b). The right figure shows production volume of natural gas and crude oil. They are measured in million square meter oil equivalents. Source: The Norwegian Petroleum Directorate (2018)

### 7.1.4 Real Returns on Net Foreign Assets

Negative returns on net foreign assets would also contribute to a reduction in cyclical imbalances but returns on foreign assets should be independent from domestic consumption preferences, especially for a small country as Norway. But it does not mean that an increase in the return on foreign assets is independent from consumption smoothing. From the perspective of domestic residents will a decrease in the returns on foreign assets create incentives to save less and consume relatively more today. This may take form through reduced exports or
increased imports. Since an increase in cyclical imbalances was associated with both negative returns on net foreign assets and negative net export growth, it may be that a change in the return on foreign assets contribute to lower net exports through consumption smoothing.

Going back to table 1, the coefficients were largest at short horizons for real returns, but at these horizons there were no significant relationships between cyclical imbalances and future depreciation rates (table 3). This indicates that there are other factors than exchange rate movements that cause the significant relationship between cyclical imbalances and future returns on net foreign assets. In other words, there may be factors that cause lower returns in foreign countries that are likely to be correlated with Norway’s cyclical imbalances. Such factors could be the oil or gas price. Since Norway’s exports to a large degree depend on the price of gas and oil, it’s likely that an increase those prices will cause an increase in Norway’s cyclical imbalances. The increase in exports implies that other countries have increased their imports. These countries are likely to experience negative cyclical imbalances due to their increase in imports. Given that most countries aren’t net exporters of oil\(^6\), high oil prices are likely to cause negative cyclical imbalances in those countries because their imports increase. This could in turn contribute to higher production costs for firms in those countries as oil is involved in most production chains, either directly or indirectly in the production of the good itself or in transportation of the good. If production costs increase, it could reduce those firms’ profits and thus reduce shareholders’ dividend received and the value of their stocks. For Norway, this would imply reduced or negative returns on foreign assets. Additionally, if Norway’s positive increase in cyclical imbalances causes more economic activity in Norway, one could expect an increase in Norway’s stock market prices. That would increase the returns on foreign liabilities. The effect on the returns on net foreign assets would then be negative both due to lower returns on foreign assets and higher returns on foreign liabilities. In other words, the reduction in returns on net foreign assets may be due to a shift in the relative return on foreign liabilities and assets. This shift could be due to shifts in the oil or gas price and to a certain degree be independent from exchange rate effects.

If there is an effect on real returns on foreign assets that are due to shifts in the relative return on foreign liabilities and assets, it could explain why there were significant relationships between cyclical imbalances and future returns 1-20 quarters ahead, whereas estimates of the

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\(^6\) Only three out of 30 members of The International Energy Agency were net exports of oil in 2017. Norway was one of them (International Energy Agency, 2018)
relationship between \( nxα \) and future depreciation rates only where significant 6-10 quarters ahead. Additionally, could a decrease in returns on foreign assets create incentives to smooth consumption in the sense of increasing consumption today relative to the future.

### 7.1.5 Exchange rate Volatility

Exchange rate volatility could also be a factor that determines the successfulness of forecasting the exchange rate. If the multilateral NOK exchange rate is more volatile than the multilateral USD exchange rate, it could make the NOK more difficult to forecast. Using Gourinchas and Rey’s (2007) dataset, the calculated standard deviation of the foreign direct investment (FDI)-weighted USD exchange rate between 1952-2004 is 2.78 and the trade-weighted USD exchange rate has a standard deviation of 3.44 over the period 1973-2004. The trade-weighted NOK exchange rate, however, is less volatile with a standard deviation of 2.39 over the time period 1995-2017. This indicates that the multilateral USD rate is more volatile and more predictable than the multilateral NOK. Thus is high volatility in the exchange rate probably not the reason why the out-of-sample forecasts are not as good for the NOK as for the USD in Gourinchas and Rey (2007).
8 Conclusion

In this thesis, I’ve investigated the predictive properties of Norway’s external budget constraint for future returns, net exports and exchange rates. The budget constraint was evaluated around its trend which gave rise to a measure of cyclical imbalances. The measure contained predictive properties for cyclical real returns on net foreign assets up to 5 years ahead, and around 2-4 years ahead for cyclical net export growth. This link was then connected to exchange rate movements. The result was that the cyclical imbalances could be associated with future exchange rate movements 6-10 quarters ahead. This was tested more thoroughly by performing out-of-sample forecasts and comparing the predictability of the model against the random walk. The model was significantly outperformed by the random walk at 10 quarters ahead at a five percent level, but the possibility of the two models performing equally well at the same significance level could not be rejected at the other horizons that were investigated. The forecasting properties of the measure of external imbalances is a lot weaker than in Gourinchas and Rey (2007), which may be due to factors such as a dataset of less than half the size as Gourinchas and Rey (2007) and that the dataset covers the financial crisis. Despite less accurate forecasts compared to Gourinchas and Rey (2007), there is promising evidence in favor of the measure of cyclical imbalances containing predictive information about future NOK exchange rate movements. It would be interesting to see how the results would differ if the dataset covered a longer horizon as in Gourinchas and Rey (2007).
References


Appendix

A.1 Deriving equation (3):

Start with the accumulation identity:

$$A_{t+1} - L_{t+1} = R_{t+1} (A_t - L_t + X_t - M_t)$$  \hfill (A1)

Dividing (A1) by $W_t$ and $W_{t+1}$

$$\frac{1}{W_t} \frac{1}{W_{t+1}} (A_{t+1} - L_{t+1}) = \frac{R_{t+1}}{W_{t+1}} \frac{1}{W_t} (A_t - L_t + X_t - M_t)$$

Defining $\hat{Z}_t = Z_t/W_t$

$$\frac{1}{W_t} (\hat{A}_{t+1} - \hat{L}_{t+1}) = \frac{R_{t+1}}{W_{t+1}} (\hat{A}_t - \hat{L}_t + \hat{X}_t - \hat{M}_t)$$

Rearranging and defining $\Gamma_{t+1} = \frac{W_{t+1}}{W_t}$:

$$\hat{A}_{t+1} - \hat{L}_{t+1} = \frac{R_{t+1}}{\Gamma_{t+1}} (\hat{A}_t - \hat{L}_t + \hat{X}_t - \hat{M}_t)$$  \hfill (A2)

A.2 Deriving equation (5).

I start with the accumulation identity for net foreign asset relative to wealth (the external budget constraint):

$$\Gamma_{t+1} (\hat{A}_{t+1} - \hat{L}_{t+1}) = \hat{R}_{t+1} (\hat{A}_t - \hat{L}_t + \hat{X}_t - \hat{M}_t)$$  \hfill (A3)

I then do first order Taylor expansions of the variables in $\hat{Z}_t \epsilon(\hat{A}_t, \hat{L}_t, \hat{X}_t, \hat{M}_t)$, $\hat{R}_{t+1}$ and $\Gamma_{t+1}$ around their respective trends: $\bar{Z}_t$, $\bar{R}_{t+1}$ and $\bar{\Gamma}_{t+1}$:

$$\hat{A}_t \approx \bar{A}_t (1 + \epsilon^a_t)$$  \hfill (A4)

$$\hat{L}_t \approx \bar{L}_t (1 + \epsilon^l_t)$$  \hfill (A5)

$$\hat{X}_t \approx \bar{X}_t (1 + \epsilon^x_t)$$  \hfill (A6)
\[ \hat{M}_t \approx \hat{X}_t (1 + \epsilon_t^m) \]  
(A7)

\[ \bar{\rho}_{t+1} \approx \bar{\rho}_{t+1} (1 + \hat{\rho}_{t+1}) \]  
(A8)

\[ \Gamma_{t+1} \approx \Gamma_{t+1} (1 + \epsilon_{t+1}^\Delta) \]  
(A9)

I then use the approximated values given in equation (A4) – (A9) and insert them into equation (A3), which is the budget constraint. The left hand side of equation (A3) becomes:

\[ \Gamma_{t+1} (1 + \epsilon_{t+1}^A) (\bar{A}_{t+1} (1 + \epsilon_{t+1}^a) - \bar{L}_{t+1} (1 + \epsilon_{t+1}^l)) \]  
(A10)

Which can be expanded:

\[ \Gamma_{t+1} (1 + \epsilon_{t+1}^A) (\bar{A}_{t+1} + \bar{A}_{t+1} \epsilon_{t+1}^a - \bar{L}_{t+1} + \bar{L}_{t+1} \epsilon_{t+1}^l) \]

And expanded:

\[ \Gamma_{t+1} \left( (\bar{A}_{t+1} + \bar{A}_{t+1} \epsilon_{t+1}^a - \bar{L}_{t+1} + \bar{L}_{t+1} \epsilon_{t+1}^l) + \epsilon_{t+1}^A (\bar{A}_{t+1} + \bar{A}_{t+1} \epsilon_{t+1}^a - \bar{L}_{t+1} + \bar{L}_{t+1} \epsilon_{t+1}^l) \right) \]

Then rearranged:

\[ \Gamma_{t+1} \left( (\bar{A}_{t+1} - \bar{L}_{t+1} + \bar{A}_{t+1} \epsilon_{t+1}^a - \bar{L}_{t+1} \epsilon_{t+1}^l) + \epsilon_{t+1}^A (\bar{A}_{t+1} - \bar{L}_{t+1} + \bar{A}_{t+1} \epsilon_{t+1}^a - \bar{L}_{t+1} \epsilon_{t+1}^l) \right) \]

One can then multiply the expression above by \( \frac{\bar{A}_{t+1} - \bar{L}_{t+1}}{\bar{A}_{t+1} - \bar{L}_{t+1}} \):

\[ \Gamma_{t+1} (\bar{A}_{t+1} - \bar{L}_{t+1}) \left( \frac{\bar{A}_{t+1} - \bar{L}_{t+1}}{\bar{A}_{t+1} - \bar{L}_{t+1}} + \frac{\bar{A}_{t+1} \epsilon_{t+1}^a - \bar{L}_{t+1} \epsilon_{t+1}^l}{\bar{A}_{t+1} - \bar{L}_{t+1}} \right) \]

\[ + \epsilon_{t+1}^\Delta \left( \frac{\bar{A}_{t+1} - \bar{L}_{t+1}}{\bar{A}_{t+1} - \bar{L}_{t+1}} + \frac{\bar{A}_{t+1} \epsilon_{t+1}^a - \bar{L}_{t+1} \epsilon_{t+1}^l}{\bar{A}_{t+1} - \bar{L}_{t+1}} \right) \]

Simplifying:

\[ \Gamma_{t+1} (\bar{A}_{t+1} - \bar{L}_{t+1}) \left( 1 + \frac{\bar{A}_{t+1} \epsilon_{t+1}^a - \bar{L}_{t+1} \epsilon_{t+1}^l}{\bar{A}_{t+1} - \bar{L}_{t+1}} + \epsilon_{t+1}^\Delta + \epsilon_{t+1}^\Delta \right) \]

(A11)

Using that \(|\epsilon_t^l|, |\hat{\rho}_{t+1}|, |\epsilon_{t+1}^\Delta| \ll 1\). The implication is that:
Because the product of two factors that are close to zero are approximately equal to zero. (A11) can then be rewritten as:

\[
(\bar{A}_{t+1} - \bar{L}_{t+1})C_{t+1} \left(1 + \frac{\bar{A}_{t+1}e_{t+1}^a - \bar{L}_{t+1}e_{t+1}^l}{\bar{A}_{t+1} - \bar{L}_{t+1}} + \epsilon_{t+1}^{\Delta w} \right)
\]

(A12) is the left hand side of the approximated budget constraint. By using the definitions for the approximations given above, one can write the right hand side of equation (A3) as:

\[
\overline{R}_{t+1}(1 + \bar{r}_{t+1}) \left(\bar{A}_t + \bar{A}_t e_{t}^a - \bar{L}_t - \bar{L}_t e_{t}^l + \bar{X}_t + \bar{X}_t e_{t}^x - \bar{M} - \bar{M} e_{t}^m \right)
\]

Which can be expanded:

\[
\overline{R}_{t+1}(1 + \bar{r}_{t+1}) \left(\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t + \bar{A}_t e_{t}^a - \bar{L}_t e_{t}^l + \bar{X}_t e_{t}^x - \bar{M} e_{t}^m \right)
\]

and rearranged:

\[
\overline{R}_{t+1}(1 + \bar{r}_{t+1}) \left(\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t + \bar{A}_t e_{t}^a - \bar{L}_t e_{t}^l + \bar{X}_t e_{t}^x - \bar{M} e_{t}^m \right)
\]

One can then multiply the expression by \(\frac{\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t}{\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t}\):

\[
\overline{R}_{t+1}(1 + \bar{r}_{t+1}) \left(\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t \right) \left(\frac{\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t}{\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t} + \frac{\bar{A}_t e_{t}^a - \bar{L}_t e_{t}^l + \bar{X}_t e_{t}^x - \bar{M} e_{t}^m}{\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t} \right)
\]

and then simplify:

\[
\overline{R}_{t+1}(1 + \bar{r}_{t+1}) \left(\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t \right) \left(1 + \frac{\bar{A}_t e_{t}^a - \bar{L}_t e_{t}^l + \bar{X}_t e_{t}^x - \bar{M} e_{t}^m}{\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t} \right)
\]

(A13) is then the approximated right-hand side of the accumulation identity. In the next step, I set the left-hand side (A12) equal the right-hand side (A13):
where

One can then multiply the parenthes
which can be simplified to:

By substituting the accumulation identity in trends (A3) into the expression above, one gets:

which can be simplified to:

One can then multiply the parentheses to get:

where

The last term in equation (A14) above is approximately equal to zero since it’s the product of two factors that are almost equal to zero. Equation (A14) can then be rewritten as:
\[
\frac{\bar{A}_{t+1} \epsilon^a_{t+1} - \bar{L}_{t+1} \epsilon^l_{t+1}}{\bar{A}_{t+1} - \bar{L}_{t+1}} + \epsilon^w_{t+1} = +\bar{r}_{t+1} + \frac{\bar{A}_t \epsilon^a_t - \bar{L}_t \epsilon^l_t + \bar{X}_t \epsilon^x_t - \bar{M} \epsilon^m_t}{\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t}
\] (A15)

Rearranging:

\[
\frac{\bar{A}_{t+1}}{\bar{A}_{t+1} - \bar{L}_{t+1}} \epsilon^a_{t+1} - \frac{\bar{L}_{t+1}}{\bar{A}_{t+1} - \bar{L}_{t+1}} \epsilon^l_{t+1} + \epsilon^w_{t+1} = +\bar{r}_{t+1} + \frac{\bar{A}_t \epsilon^a_t - \bar{L}_t \epsilon^l_t}{\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t} + \frac{\bar{X}_t \epsilon^x_t - \bar{M} \epsilon^m_t}{\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t}
\]

and manipulating:

\[
\frac{\bar{A}_{t+1}}{\bar{A}_{t+1} - \bar{L}_{t+1}} \epsilon^a_{t+1} - \frac{\bar{L}_{t+1}}{\bar{A}_{t+1} - \bar{L}_{t+1}} \epsilon^l_{t+1} + \epsilon^w_{t+1} = +\bar{r}_{t+1} + \frac{\bar{A}_t \epsilon^a_t - \bar{L}_t \epsilon^l_t}{\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t} + \frac{\bar{X}_t \epsilon^x_t - \bar{M} \epsilon^m_t}{\bar{A}_t - \bar{L}_t + \bar{X}_t - \bar{M}_t}(A16)
\]

Now, one can use the definitions:

\[
n_a_t = \mu^a_t \epsilon^a_t - \mu^l_t \epsilon^l_t
\]

\[
n_x_t = \mu^x_t \epsilon^x_t - \mu^m_t \epsilon^m_t
\]

\[
\mu^a_t = \frac{\bar{A}_t}{\bar{A}_t - \bar{L}_t}, \quad \mu^l_t = \frac{\bar{L}_t}{\bar{A}_t - \bar{L}_t}
\]

\[
\mu^x_t = \frac{\bar{X}_t}{\bar{X}_t - \bar{M}_t}, \quad \mu^m_t = \frac{\bar{M}_t}{\bar{X}_t - \bar{M}_t}
\]

\[
\rho_t = 1 + \frac{\bar{X}_t - \bar{M}_t}{\bar{A}_t - \bar{M}_t}
\]

in equation (A16):

\[
\mu^a_{t+1} \epsilon^a_{t+1} - \mu^l_{t+1} \epsilon^l_{t+1} + \epsilon^w_{t+1} = +\bar{r}_{t+1} + \frac{\mu^a_t \epsilon^a_t - \mu^l_t \epsilon^l_t}{1 + \frac{\bar{X}_t - \bar{M}_t}{\bar{A}_t - \bar{M}_t}} + \frac{\mu^x_t \epsilon^x_t - \mu^m_t \epsilon^m_t}{\frac{\bar{X}_t - \bar{M}_t}{\bar{X}_t - \bar{M}_t} + 1}
\]

Using the definitions above again yields:
\[ n_{a_{t+1}} + \epsilon_{t+1}^A = +\tilde{\gamma}_{t+1} + \frac{n_{a_t}}{\rho_t} + \frac{n x_t}{\bar{L_t} - \bar{L_t}} \]

The expression can be manipulated more:

\[ n_{a_{t+1}} + \epsilon_{t+1}^A = +\tilde{\gamma}_{t+1} + \frac{n_{a_t}}{\rho_t} + \frac{n x_t}{\bar{A_t} - \bar{L_t} + \frac{(\bar{A_t} - \bar{L_t})(\bar{X_t} - \bar{M_t})}{\bar{X_t} - \bar{M_t}}(\bar{A_t} - \bar{L_t})} \]

Further manipulating:

\[ n_{a_{t+1}} + \epsilon_{t+1}^A = +\tilde{\gamma}_{t+1} + \frac{n_{a_t}}{\rho_t} + \frac{n x_t}{\rho_t} \left( \frac{1}{1 + \frac{\bar{A_t} - \bar{L_t}}{\bar{X_t} - \bar{M_t}} \left( \frac{\bar{A_t} - \bar{L_t}}{\bar{X_t} - \bar{M_t}} \right)} \right) \]

One can then use the definition of \( \rho_t \) in the expression above:

\[ n_{a_{t+1}} + \epsilon_{t+1}^A = +\tilde{\gamma}_{t+1} + \frac{n_{a_t}}{\rho_t} + \frac{n x_t}{\rho_t} \left( \frac{1}{\rho_t} - \frac{\bar{X_t} - \bar{M_t}}{\bar{A_t} - \bar{L_t}} \right) \]

Then manipulate:

\[ n_{a_{t+1}} + \epsilon_{t+1}^A = +\tilde{\gamma}_{t+1} + \frac{n_{a_t}}{\rho_t} + \frac{n x_t}{\rho_t} \left( 1 + \frac{\bar{X_t} - \bar{M_t}}{\bar{A_t} - \bar{L_t}} - \frac{1}{\rho_t} \right) \]

One can then add and subtract \( 1/\rho_t \) in the last term:

\[ n_{a_{t+1}} + \epsilon_{t+1}^A = +\tilde{\gamma}_{t+1} + \frac{n_{a_t}}{\rho_t} + \frac{n x_t}{\rho_t} \left( \frac{1}{\rho_t} \right) \]

Then sum two of the terms in the parenthesis:

\[ n_{a_{t+1}} + \epsilon_{t+1}^A = +\tilde{\gamma}_{t+1} + \frac{n_{a_t}}{\rho_t} + \frac{n x_t}{\rho_t} \left( 1 + \frac{\bar{X_t} - \bar{M_t}}{\bar{A_t} - \bar{L_t}} - \frac{1}{\rho_t} \right) \]
Now use the definition of $\rho_t$ again:

$$na_{t+1} + \epsilon_{t+1} = +\tilde{r}_{t+1} + \frac{na_t}{\rho_t} + nx_t \left( \frac{\rho_t - 1}{\rho_t} \right)$$

Then rewrite:

$$na_{t+1} = \frac{1}{\rho_t} na_t + (\tilde{r}_{t+1} - \epsilon_{t+1}^{\Delta w}) + nx_t \left( 1 - \frac{1}{\rho_t} \right)$$  \hspace{1cm} (A17)

Equation (A17) is then the log approximated budget constraint around its trend and equal to equation (5).

### A.3 Deriving equation (11)

A common growth rate for the trends results in constant trend shares ($\mu^x$) and a constant discount factor ($\rho$). Therefor they do not have any subscripts denoting time.

By adding and subtracting $nx_{t+1}$, equation (A17) becomes:

$$na_{t+1} - nx_{t+1} = \frac{1}{\rho_t} na_t + (\tilde{r}_{t+1} - \epsilon_{t+1}^{\Delta w}) + nx_t \left( 1 - \frac{1}{\rho_t} \right) - nx_{t+1}$$

One can open the parenthesis:

$$na_{t+1} - nx_{t+1} = \frac{1}{\rho_t} na_t + \tilde{r}_{t+1} - \epsilon_{t+1}^{\Delta w} + nx_t - \frac{nx_t}{\rho} - nx_{t+1}$$  \hspace{1cm} (A18)

Now using the definitions:

$$nx a_t = na_t - nx_t$$  \hspace{1cm} (A19)

$$\Delta nx_{t+1} = nx_t - nx_{t+1} - \epsilon_{t+1}^{\Delta w}$$  \hspace{1cm} (A20)

$$r_{t+1} = \tilde{r}_{t+1}$$  \hspace{1cm} (A21)

to rewrite (A18)

$$nx a_{t+1} = \frac{1}{\rho_t} nx a_t + r_{t+1} + \Delta nx_{t+1}$$  \hspace{1cm} (A22)

Equation (A22) is equal to equation (11) in the text.