UNIVERSITY OF OSLO

MASTER THESIS

The impacts of water monitoring on water treatment and health

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Declaration of Authorship

I, Martha GJERMO, declare that this thesis titled, “The impacts of water monitoring on water treatment and health” and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

UNIVERSITY OF OSLO

Abstract

Faculty of Social Science
Department of Economics

Master in Economics

The impacts of water monitoring on water treatment and health

by Martha GJERMO

In this thesis, I investigate the relationship between two different water monitoring options, and their impact on optimal water treatment and health. To see whether one monitoring option has an advantage compared to the other, I build a theoretical model of the differences between real-time continuous monitoring and test-based monitoring of water quality. I use the model to investigate how the expected utility of the consumer is affected by the initial probability of contamination in drinking water, the effect from chlorine, the cost of chlorine, and the cost of falling sick relative to the cost of drinking bottled water. I further investigate how parameters affect the difference in expected utilities for the consumers between the two monitoring options. The result show that the monitoring option with the lowest cost of contamination in the drinking water yields the highest expected utility. The difference in utilities between the monitoring options decreases with a high efficiency of chlorine, and increases with a high cost of chlorine, a high initial level of contamination in the water or a high difference between the cost of falling sick and the cost of bottled water.

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Dedicated to my belowed mother

Chapter 1

Introduction

Water is one of the worlds most important resources. Without fresh water nothing can live. Almost 70% of the earth is covered with water, but only 3% is fresh water, and most of our fresh water resources is in the form of glacier and snow. Only 0.01% of the water on earth is considered available for human use on a regular basis (Hinrichsen and Tacio 2002). Humans needs water for much more than just to drink. Most of the worlds fresh water resources is used in agriculture and food production. We also use water to wash ourselves, our clothes, houses, water the garden and wash our cars. Only a little fraction of the 0.01% available fresh water in the world is used as drinking water. Lack of safe and clean drinking water is one of todays largest challenges.
Severe water stress affects 3 billion people today (Sachs 2013), and water scarcity issues are growing. By 2025 about half of the world's population is assumed to be living in water-stressed areas (WHO 2017). The demand for water is normally higher in areas with high population density, and water needs are quickly arising in Asia which account for 60% of the global population and only 36% of the worlds water (Zimmerman et al. 2008). Management of the water resources is more important than ever before. Our climate is changing, and we experience more and larger floods and longer droughts. Both causing poorer water quality and availability. Already 844 million people lack access even to a basic drinking water service (WHO 2017). Contaminated water together with poor hand hygiene and sanitation is estimated to cause 2.2-5 million deaths every year (Gleick et al. 2002). Most of the cases are diarrheal deaths, caused by drinking contaminated water or eating food made with contaminated water. There are also several other waterborne diseases that causes global deaths such as intestinal worms, trachoma and schistosomiasis which are all proven to be reduced prominently with safe water-supplies (WHO et al. 2000).
In the developing part of the world there is little doubt that securing safe drinking water to the growing populations of people who lives there is a challenge that must be resolved as quickly as possible. However the problem is complex. Also in the developed parts of the world, water stress and lack of clean water is a growing issue. This spring Cape Town faced a possible cut-off of the public water-supply in May 2018 as their reservoirs were drying up (Said-Moorhouse 2018). They have managed to postpone the cut-of date until an unknown date in 2019 (Cape town" 2018), by strict restrictions on water usage together with some much needed rain (Businesstech 2018) but the situation is critical. Today in the US, there is an estimated 4-12 million cases of acute gastrointestinal illness annually due to infections in the drinking water (Colford et al. 2006). There has been several drinking water crises in the U.S. and Baum, Bartram, and Hrudey 2016 states that risk management of drinking water needs to become more preventive. The estimated total number of waterborne illnesses in the U.S. is 19.5 million cases every year (Reynolds, Mena, and Gerba 2008). Most outbreaks of waterborne decease are never even identified

because identification requires at least 1% of the population in the affected area to become ill within a time frame of a few months (Regli et al. 1991). So how do we know whether the water we drink makes us sick? And how can we better manage and monitor this resource so that we manage to use the available drinking water as efficiently as possible? Regli et al. (1991) states that

> "The absence of a methodology for characterizing whether negligible microbial risk is being provided by a water treatment plant can lead, in the interest of public safety, to over design and excess disinfection and associated by-products."

We manage our water resources with the objective to provide safe water and to save lives. We also know that a too high use of disinfectants might be poisonous for the population and a too low level might cause disease outbreaks in a population. Modeling the water management decisions, taking the utilities of being healthy and the dis-utility of falling sick into account, is crucial to make an efficient and good decision about water management.

Montgomery and Elimelech (2007) address the importance of adding population health in the water management equations, especially in the developing part of the world. Access to a safe and reliable water resource is essential for good health (Hunter, MacDonald, and Carter 2010), and Majuru et al. (2011) found that implementing community water supplies reduced the self reported diarrhea by approximately 50%, showing that water quality and health is closely related.

In most high-income countries the monitoring and quality of water is generally high, but even in the countries with the highest income and most advanced monitoring systems today there is scope for reducing the number of waterborne disease outbreaks. In America there has been several water crises, and lead poisoning in the drinking water in Flint is a recent example of a recent large-scale water crises. The water crisis mostly affected children and pregnant mothers (Hanna-Attisha et al. 2016). Estimations of the costs and benefits of lead hazard control show that there are substantial returns to these investments (Gould 2009). Landrigan et al. (2002) estimated the total annual cost of lead poisoning in America to be $43.4 billion. Another example of waterborne disease outbreaks in the developed world is the Washington water crisis (Renner 2004), the Giardia outbreak in Bergen, Norway (Robertson et al. 2006), and three years later a water contamination incident in Oslo, Norway (Robertson et al. 2009).

In total, there is potential to prevent at least 9.1% of the global disease burden or 6.1% of all deaths from improving water, sanitation and hygiene (Pruss-Ustun, Organization, et al. 2008).

There is also a threat to water safety that someone willfully contaminates the water resources. The fact that there is a delay in the monitoring system as it works today from the time when the test of the water is taken to when the result is ready makes it possible to imagine that such a threat might be real. The effect from a willful contamination in water-sources that are not continuously monitored might be assumed very deadly. The effect might be causing fear also for those not affected as there is uncertainty about the quality of the drinking water.

In this thesis, I analyze how two different methods of monitoring water-quality affect water-treatment and health. The objective of the thesis is to create a framework to compare the optimal use of water treatment, and the expected utility for real-time continuous monitoring and delayed test based monitoring. The analysis focuses on the differences between the real-time continuous monitoring of water quality and the normal test-based monitoring of water quality. What are possible gains from real time continuous monitoring and how will it affect the levels of disinfectants used in the water? Does continuous real-time knowledge of the water-quality reduce the level of disinfection and by-products in the water? I have created a model to provide a theoretical contribution on how to compare new monitoring technologies and how they might affect the treatment of water and the health of the population. The treatment level affect health by changing the probability of contamination in the water and the health costs from chlorine in the water. For the continuous monitoring option it is assumed that no one falls sick, and the health benefit of not falling sick is balanced with the cost of drinking bottled water to avoid falling sick. My model takes the perspective of a planner. It can be modified to fit in different settings, both in high income and developing countries.

I have investigated how the monitoring of water can affect the health of the population, how the two types of monitoring affect the optimal use of treatment and protection of the water resources, and the expected utility for the two monitoring options. To compare the monitoring options I exploit the fact that there is a time-gap from when the test of the water is taken to when there is knowledge about the water quality in the delayed monitoring option. I then compare the utility in the delayed monitoring option to the utility in the continuous monitoring option when there is real-time knowledge of the water quality.

The model does not distinguish between high and low income countries, rural and urban areas, large and small cities, or access to water resources. I discuss different types of settings, and how the parameters and costs will vary between different settings and how these variations affects the expected utility for the two monitoring options. The accessibility of the two types of monitoring in the area of interest is anyhow necessary for the model to be usable. For my model to be relevant there must exist some technology, or interest in developing technology that provides continuous monitoring of water quality. At the moment there is rapid development of real-time remote monitoring systems that can detect and respond to different threats to the water such as toxicants and bacterias that represents a possible threat to human health (Glasgow et al. 2004). The interest for continuous monitoring is high, and the technology is under development. In my model I assume that there will enter new technology on the water monitoring market with the possibility of real-time continuous monitoring of water quality.

Many studies have been, and are being done on the management of water resources, on how to best use the water available, on how to preserve the resources, and on the effects of overuse. In this thesis, I investigate the relationship between water monitoring, water quality and health, and was prepared to find literature on economic analysis and comparisons of different water quality monitoring options. There exists a lot of engineering articles and management strategies on the topic, however very little economic literature on the comparison of water quality monitoring options. Examples of economic research on the impact of water scarcity on water management as are Draper et al. (2003) that build a model of optimization of Californian water management focusing on scarcity issues, and Pollock (1988) that build a pricing and evaluation model for water accounting for scarcity rents. (Carey and Zilberman 2002) investigate investment of water related technology under uncertainty, and created a model over farms willingness to invest in modern water-related technologies. There are no models on the comparison between monitoring options in terms of efficiency, population health or utility of the consumers. The scope of improving water management to avoid disease outbreaks is very large, and investigating how continuous real time monitoring technology might contribute to achieve better health of the population is important for the further development of such technologies. The model that I am presenting in this master thesis is my own, created under the supervision of my supervisor.

The main source of information about water management and treatment of water for this thesis is the WHO. (2011) guidelines for drinking water quality. The guidelines provide thorough and specific information on how to achieve safe water supplies, from microbial and chemical risks in the water to the surveillance system. The guidelines do not discuss continuous monitoring of the water quality or potential gains or costs of such a system. Throughout the thesis, other relevant articles for specific details about water quality, probabilities of contamination and effects of chlorination have been used to validate the setup of the model.

Chapter 2

Theory and model

2.1 Natural contamination

My model is a simple static model and consists of a policy-maker and consumers. I model the outcome for continuous monitoring and delayed monitoring separately. I further distinguish the model into two contamination cases, natural and willful contamination, and model the two contamination cases separately for both monitoring options.
In the natural contamination case, the contamination in the water is caused naturally, and water treatment has some effect on reducing the probability of contamination in the water. Treating the water induces a cost of treatment, a cost of the negative health effect from adding treatment to the drinking water, and a reduction in the probability of natural contamination in the water. The consumer choose between drinking tap water or bottled water, and the policy-makers decide whether to have continuous or delayed monitoring. Depending on their choice of monitoring they choose an optimal level of treatment which affects the utility of the consumer.

2.1.1 Setting

The consumers consume the water and choose between the consumption of bottled water and tap-water. I assume that the consumers will always and only drink bottled-water when they are aware of contamination in the tap-water. Depending on the quality of the water they consume, they fall sick $\underline{C}$ or stay healthy $\overline{C}$. Adding treatment, $l \geq 0$, to the water gives a negative health-effect depending on the level of treatment used in the water, $d(l)$, and reduces the probability of contamination in the water depending on the level of treatment used, $p(l)$.

The policy-makers maximizes the utility of the consumers by choosing a level of treatment depending on the chosen type of monitoring. The consumers choose whether to consume bottled or tap-water. They fall sick or stay healthy with some probability depending on the use of treatment. In the delayed monitoring option there is a delay in knowledge of water quality for as long as it takes from when the test is taken to when the result of the test is ready and known to the managers of the water resource. The model exploits this time-gap in when there is knowledge about the water quality for the two monitoring options, and in the model there is always uncertainty about the water quality for the delayed monitoring option and full certainty about the water quality for the continuous monitoring option. There is certainty about the probability of contamination in the water for both monitoring options, but the real time knowledge about the quality of the water is only known for the continuous monitoring option. The model is solved for both monitoring options separately. The outcomes of the model in terms of optimal use of treatment and expected utility is then compare between the two monitoring options.

Chlorine is the most commonly used disinfectant, so in the model chlorine represents the treatment. It is possible to exchange chlorine with any other preferred disinfectant, but the effectiveness, costs and negative health effects are of course not the same.

The utility of the consumer depends on whether they are sick, $\underline{C}$, or healthy, $\overline{C}$, the health impact of chlorine in the water and the price of chlorine ($d(l)$), and their expenses from consumption of bottled water b when they are aware of contamination in the tap-water.

Only some fraction of those drinking contaminated water falls sick. This fraction is represented by a constant, $\theta, \in [0,1]$ where 0 is the case where no one becomes sick from drinking contaminated water, and 1 is the case where everyone becomes sick. The utility in the two monitoring options is denoted U_c for the continuous monitoring option, and U_d for the delayed monitoring option. This notation, subscript c for the continuous monitoring option and subscript d for the delayed monitoring option, will separate the two monitoring options throughout the thesis.

$$U_c(l) = \begin{cases} \overline{C} - d(l) \\ \overline{C} - d(l) - b \end{cases} \qquad U_d(l) = \begin{cases} \overline{C} - d(l) & \text{if not contaminated} \\ (1-\theta)\overline{C} + \theta\underline{C} - d(l) & \text{if contaminated} \end{cases} \tag{2.1}$$

$p(l) \in [0,1]$ is the probability of contamination in the water. The probability function is dependent on the level of chlorine as a higher level of chlorine decreases the probability of infection (Regli et al. 1991). The expected utility for the continuous monitoring option and the delayed monitoring option is then

$$E(U_c(l)) = (1-p(l))\overline{C} + p(l)\left(\overline{C} - b\right) - d(l) \tag{2.2}$$

$$\begin{aligned} E(U_d(l)) &= (1-p(l))\overline{C} + p(l)\left((1-\theta)\overline{C} + \theta\underline{C}\right) - d(l) \\ &= (1-p(l))\overline{C} + p(l)\left(\overline{C} - \theta\left(\overline{C} - \underline{C}\right)\right) - d(l). \end{aligned} \tag{2.3}$$

I assume that the probability of contamination in the water, $p(l)$, decreases with an increase in chlorine, and that there is a falling convex relationship between the probability of contamination and the level of chlorine, so that

$$p_l' < 0 \qquad \text{and} \qquad p'' > 0. \tag{2.4}$$

I further assume that the health damage of chlorine increases with an increase in chlorine, and that there is an increasing convex relationship between the health damage and the level of chlorine, so that

$$d_l' > 0 \qquad \text{and} \qquad d'' > 0. \tag{2.5}$$

There can not be negative levels of chlorine in the water, so $l \geq 0$. In order to maximize the expected utility of the consumer, the policymaker must choose a level of chlorine that maximizes the expected utilities.

$$max_l E(U(l))\text{for} \qquad l \geq 0 \tag{2.6}$$

2.1.2 Calculations

Solving the maximization problem yields the first order conditions

$$
\begin{aligned}
\frac{\partial}{\partial l} E\left(U_c\right) = & \quad -p'\left(l\right)\overline{C} + p'\left(l\right)\left(\overline{C} - b\right) - d'\left(l\right) = 0 \\
\Rightarrow & \quad -p'\left(l\right) b = d'\left(l\right) && (2.7) \\
\Rightarrow & \quad b = \frac{d'\left(l^*\right)}{-p'\left(l^*\right)} && (2.8) \\
\frac{\partial}{\partial l} E\left(U_d\right) = & \quad -p'\left(l\right)\overline{C} + p'\left(l\right)\left(\left(1-\theta\right)\overline{C} + \theta\underline{C}\right) - d'\left(l\right) = 0 \\
\Rightarrow & \quad -p'\left(l\right)\theta\left(\overline{C} - \underline{C}\right) = d'\left(l\right) && (2.9) \\
\Rightarrow & \quad \theta\left(\overline{C} - \underline{C}\right) = \frac{d'\left(l^*\right)}{-p'\left(l^*\right)}. && (2.10)
\end{aligned}
$$

Interpretation

By assumption $d'\left(l\right) > 0$ and $p'\left(l\right) < 0$ as the costs and health effects from by-products are growing in l, and the probability of disease is decreasing in l. The size of the derivatives can be interpreted as the effectiveness from the next unit of chlorine on the probability of contamination in the water $p'\left(l\right)$ and the size of the negative health effect from the next unit of chlorine $d'\left(l\right)$.
Taking the total derivative of 2.7 gives the opportunity to see how the optimal level of chlorine, l^*, varies with the cost of bottled water, which is the consumers alternative to chlorine usage when there is contaminated water in the case of continuous monitoring. Since the function of the negative health impacts of chlorine is denoted $d(l)$, I use d as the total derivative symbol in the equations 2.11 and 2.12 to avoid confusion.

$$
\begin{aligned}
& -p''(l^*)\mathrm{d}l \cdot b - p'(l^*)\mathrm{d}b = d''(l^*)\mathrm{d}l \\
\Rightarrow & \mathrm{d}l\left(d''(l^*) + p''(l^*)b\right) = -p'(l^*)\mathrm{d}b \\
\Rightarrow & \frac{\mathrm{d}l}{\mathrm{d}b} = -\frac{p'(l^*)}{d''(l^*) + p''(l^*)b} \qquad (2.11)
\end{aligned}
$$

If the price of bottled water increases, the optimal level of chlorine, l^*, changes depending on the marginal probability of contamination $p'\left(l\right)$, over the change in marginal health damage from chlorine $d''\left(l\right)$ together with the change in marginal probability of contamination in the water added with the cost of bottled water $p''(l^*)b$. In the case of linear functions, the second derivative is always 0, and the marginal reduction of probability or the marginal negative health effect is the same for all levels of l. The fraction is undefined for the case where both $d(l)$ an $p(l)$ is linear.
Under the assumptions of 2.4 and 2.5 the optimal use of chlorine increases when b increases. A high marginal probability of contamination in the water imply that the use of chlorine is very efficient, and the change in optimal use of chlorine is large when the cost of bottled water increases. A high change in marginal damage from chlorine, the convexity of $d(l)$, reduces the increase in optimal use of chlorine when the cost of bottled water increases, as higher use of chlorine increases the health damage convexly. A high change in marginal probability of contamination in the water, convexity of $p(l)$, together with a high cost of bottled water reduces the size

of the increase in the optimal use of chlorine, as higher use of chlorine reduces the efficiency of chlorine convexly, and a high cost of bottled water yields a high initial level of chlorine.

Taking the total derivative of 2.9 gives the opportunity to see how the optimal level of chlorine, l^*, varies with the cost of falling sick. $\left(\overline{C} - \underline{C}\right)$ is the relative cost between being healthy and sick and equals the difference in utility when healthy and when sick. Increasing the utility when healthy increases the difference between being sick and healthy, and the relative cost from being sick increases. Decreasing utility when sick increases the difference between being sick and healthy, and the relative cost from being sick increases. The cost of falling sick, $\left(\overline{C} - \underline{C}\right)$, can increase either from a higher $\overline{C}$ or a lower $\underline{C}$. To simplify the derivation I define the cost of falling sick as C in this particular case, so $C \equiv \left(\overline{C} - \underline{C}\right)$

$$\begin{aligned} &- p''(l^*)\mathrm{d}l \cdot \theta C - \theta p'(l^*)\mathrm{d}C = d''(l^*)\mathrm{d}l \\ &\Rightarrow \mathrm{d}l\left(d''(l^*) + p''(l^*)\theta C\right) = -\theta p'(l^*)\mathrm{d}C \\ &\Rightarrow \frac{\mathrm{d}l}{\mathrm{d}C} = -\frac{\theta p'(l^*)}{d''(l^*) + p''(l^*)\theta C} \end{aligned} \tag{2.12}$$

Given the assumptions in 2.4 and 2.5 increasing the cost of falling sick increases the optimal use of chlorine. θ represents the fraction of people that falls sick from drinking contaminated water. A high marginal probability of contamination in the water imply that the use of chlorine is very efficient, and the change in optimal use of chlorine is large when the cost of falling sick increases. If θ is low it decreases the effect from the marginal probability of contamination, as very few actually falls sick, and the total cost of contamination in the water $\theta\left(\overline{C} - \underline{C}\right)$ is relatively low, and increasing slower than an increase in only the cost of falling sick $\left(\overline{C} - \underline{C}\right)$. A high change in marginal damage from chlorine, the convexity of $d(l)$, reduces the increase in optimal use of chlorine when the cost of falling sick increases, as higher use of chlorine increases the health damage convexly. A low change in marginal damage from chlorine yields a smaller reduction in the optimal use of chlorine than a large change in marginal damage from chlorine. A high change in the marginal probability of contamination in the water, convexity of $p(l)$, reduces the increase in optimal use of chlorine when the cost of falling sick increases, as higher use of chlorine decreases the probability of contamination in the water convexly and falling. A low change in marginal probability of contamination in the water yields a smaller reduction in the optimal use of chlorine than a large change in marginal probability of contamination in the water. The fraction that falls sick θ determines the size of the effect from the marginal probability of contamination in the water, and the change in marginal probability of contamination in the water. When very few of those drinking contaminated water actually falls sick, the change in probability of contamination in the water have a smaller effect on the cost of falling sick, compared to when almost everyone falls sick when drinking contaminated water.

together with a high cost of falling sick and a large fraction of people falling sick, reduces the size of the increase in optimal use of chlorine, as higher use of chlorine reduces the efficiency of chlorine convexly, and a high cost of falling sick together with a high fraction of people that falls sick yields a high initial level of chlorine.

The probability of contamination

The probability of contamination in the drinking water is decreasing in chlorine. The exact relationship between the levels of chlorine and the probability of contamination depend on the types of contaminants already existent in the water and the level of contamination (WHO. 2011). Council (2016) states that it only took less than one part per million of chlorine to virtually eliminate waterborne typhoid fever in the U.S. implying that chlorine is very efficient in the first levels of use. When the use of chlorine increases it is seen in chlorinated waters a higher concentration of chlorine resistant bacterias, and some bacterias are very resistant to chlorine, whereas others are not (Ridgway and Olson 1982). This implies a convex and falling function where the first unit of chlorine is extremely effective on eliminating contaminants in the water, and that the effect becomes smaller as there is higher levels of chlorine. After a certain amount of chlorine is added into the water it will no longer have any effect on reducing the probability of contamination in the water, so the effect is reduced when there is higher levels of chlorine. In my model the probability-function is the relationship between chlorination and risk of contamination, and I assume a falling convex relationship between the probability of contamination and the level of chlorine. I assume a static initial probability of contamination in the water prevailing without the use of chlorine, k.

$$p(l) = k * e^{-nl}. \tag{2.13}$$

$k \in [0, 1]$ is the assumed static probability of contamination in the water, where 0 is no probability of contamination and 1 is 100% probability of contamination, so that every drop of water contains some contaminant. l, is the level of chlorine and $n \geq 0$, is the effectiveness of chlorine, l. If $n = 0$, there is no effect from the chlorine and the probability of contamination is equal to the initial level k.

The health-impact and the cost of chlorine usage

Chlorination of water is widely used to disinfect water, and Hamidin, Yu, and Connell (2008) measure negative health effects from chlorination in chlorination by-products that are taken up by humans when drinking chlorinated water. They find that with small levels of chlorination by-products the health effects are insignificant, but they increase in the level of by-products, and WHO. (2011) operates with maximum levels of chlorine by-products in their guidelines to ensure the safety of the consumer. As the health damage is insignificant for low levels of by-products and larger for higher levels of by-products I assume a convex relationship. The actual cost of the use of chlorine is assumed linear and both costs are represented in the function

$$d(l) = e^{\alpha l} - 1 \tag{2.14}$$

where $\alpha > 0$ is the costs and the slope of l, both represented in α since I do not need to separate them in this model. The damage starts at 0 so that when there is no use of chlorine $d(l) = 0$.

Applying the functions to the model

In order to calculate the optimal use of chlorine given the functional forms of the probability of contamination in the water, $p(l)$ and the negative health impact from chlorine, $d(l)$ I insert the derivatives of the functions $p(l)$ and $d(l)$ in 2.8 for the continuous monitoring option and in 2.10 for the delayed monitoring option and solve for l for both monitoring options. First I calculate the marginal change in probability of contamination in the water from an increase in chlorine, and the marginal change in the cost and health damage from chlorine from an increase in chlorine

$$p'(l) = -nke^{-nl} \qquad d'(l) = \alpha e^{\alpha l}. \tag{2.15}$$

This derivatives shows that increasing the level of chlorine, l, reduces the probability of contamination in the water, $p(l)$, as the derivative is negative. The cost and health damage from chlorine, $d(l)$, increases with an increase in l, as the derivative is positive. As there can not be negative use of chlorine, I assume $l \geq 0$. Inserting 2.15 in 2.8 and 2.10 and solving for l yields the optimal use of chlorine

$$l_c^* = \frac{ln\left(\frac{bnk}{\alpha}\right)}{\alpha + n} \qquad l_d^* = \frac{ln\left(\frac{\theta\left(\overline{C}-\underline{C}\right)nk}{\alpha}\right)}{\alpha + n}. \tag{2.16}$$

The optimal level of chlorine, l^* is not defined when $bnk = 0$ for the continuous monitoring option, and when $\theta\left(\overline{C} - \underline{C}\right) nk = 0$ for the delayed monitoring option, as $ln(0)$ is not defined. To see if there exists negative values for l^* I take the limits as, $bk \rightarrow 0$, as $n \rightarrow 0$, and as $\alpha \rightarrow \infty$

$$\lim_{bk \to 0} \frac{ln\left(\frac{bnk}{\alpha}\right)}{\alpha + n} = -\infty \tag{2.17}$$

$$\lim_{n \to 0} \frac{ln\left(\frac{bnk}{\alpha}\right)}{\alpha + n} = -\infty \tag{2.18}$$

$$\lim_{\alpha \to \infty} \frac{ln\left(\frac{bnk}{\alpha}\right)}{\alpha + n} = \lim_{\alpha \to \infty} \frac{\frac{\ln\left(\frac{1}{\alpha}\right)}{\alpha}}{\frac{n}{\alpha} + \frac{\alpha}{\alpha}} = \lim_{\alpha \to \infty} \left(\frac{-\frac{\ln(\alpha)}{\alpha}}{\frac{n}{\alpha} + 1}\right) = \frac{0}{1} = 0 \tag{2.19}$$

The interpretation of this results is that the costs of falling sick, bk, and the effect from chlorine, n, is balanced with the cost of chlorine α, so that when there is no cost of falling sick, or the chlorine is completely ineffective there is no reason to use chlorine, in fact it would be more effective with a negative use from chlorine. When the cost of chlorine becomes infinitively high, the cost of using chlorine is much higher than not using chlorine, and the most efficient solution is not to use chlorine.

In this model there can not be negative uses of chlorine, and in the Kuhn Tucker 2.38 and 2.39 I found that there exists an interior solution for the optimal level of chlorine, l^*, and that the constraint on l^* is binding. A positive optimal level of chlorine, l^*, requires that $\frac{bnk}{\alpha} \geq 1$, and the optimal level of chlorine, l^*, will equal 0 for all cases where $\frac{bnk}{\alpha} < 1$. This implies that some dis-utility from contamination in the water is accepted before it is preferable to use chlorine, and that as long as the actual cost of contamination, bnk, is larger than the cost and damage from chlorine, α, it is preferable to use chlorine. The result is the same for the delayed monitoring case where b is exchanged with $\theta\left(\overline{C} - \underline{C}\right)$.

The only difference between the optimal use of chlorine in the two monitoring settings are the costs of bottled water b and the costs of falling sick $\theta\left(\overline{C} - \underline{C}\right)$. Together with k they represent the cost of contamination in the water when nothing is being done to reduce the contamination. Since the equations are otherwise similar I will calculate the main results for the continuous monitoring case and then explain how the results look for the delayed monitoring case when relevant, to avoid doing the same calculation twice throughout the whole thesis.

Assuming that in most cases, $b < \theta\left(\overline{C} - \underline{C}\right)$, and assuming that α, n and k are exogenous variables and assumed at constant levels, and that the values of the parameters are the same in the same setting for both monitoring options, the chlorine level l is in the optimal solution smaller in the case of continuous monitoring than in the case of delayed monitoring as an increase in b or $\theta\left(\overline{C} - \underline{C}\right)$ yields an increase in the optimal use of chlorine l^* as shown in proposition1.

proposition 1. *Increasing the alternative costs to chlorine usage in the parameters k (initial static level of infectious disease), b (cost of bottled water),* $\left(\overline{C} - \underline{C}\right)$ *(relative cost of falling sick) or* θ *(fraction of people who falls sick) will increase the optimal use of chlorine,* l^*.

Proof.

$$\frac{\partial l_c^*}{\partial k} = \frac{1}{k\left(\alpha+n\right)}, \quad \frac{\partial l_c^*}{\partial b} = \frac{1}{b\left(\alpha+n\right)}, \quad \frac{\partial l_d^*}{\partial\left(\overline{C} - \underline{C}\right)} = \frac{1}{\left(\overline{C} - \underline{C}\right)\left(\alpha+n\right)}, \quad \frac{\partial l_c^*}{\partial \theta} = \frac{1}{\theta\left(\alpha+n\right)}$$

All the derivatives are positive. □

When the costs of falling sick, b or $\left(\overline{C} - \underline{C}\right)$, increases, the relative cost of chlorine decreases so that it will be more effective to use more chlorine to decrease the possibility of contamination in the water, as it is now more expensive to fall sick. The same is true if the initial level of infection, k, and the fraction of people who falls sick, θ, increases, as more people will fall sick. Increasing the use of chlorine decrease the probability of contamination in the water, and the number of people falling sick decreases.

proposition 2. *Increasing the cost of using chlorine or the damage associated with chlorine,* α*, decreases the optimal use of chlorine,* l^*.

Proof. $\frac{\partial l_c^*}{\partial \alpha} = \frac{-n-\alpha-\alpha\ln\left(\frac{bnk}{\alpha}\right)}{\alpha(\alpha+n)^2} < 0$ □

Increasing the cost of chlorine, α, will increase the cost of using chlorine relative to the cost of falling sick. This makes it more effective to reduce the level of chlorine. This result is the same in the case of delayed monitoring where b is exchanged with $\theta\left(\overline{C} - \underline{C}\right)$.

proposition 3. *Increasing the effectiveness of chlorine, n, will increase the optimal use of chlorine,* l^**, if the effectiveness of chlorine, n, or the cost of contamination when there is no use of chlorine, bk, are sufficiently low or the cost of chlorine,* α*, is sufficiently large.*

Proof.

$$\frac{\partial l_c^*}{\partial n} = \frac{\alpha+n-n\ln\left(\frac{bkn}{\alpha}\right)}{n\left(\alpha+n\right)^2}$$

$$\text{if } \alpha+n-n\ln\left(\frac{bkn}{\alpha}\right) > 0 \qquad \text{the derivative is positive}$$

$$\text{if } \alpha+n-n\ln\left(\frac{bkn}{\alpha}\right) < 0 \qquad \text{the derivative is negative}$$

To see how the derivative depends on the size of the parameters I have taken the limits of the parameters when they approach, 0, and, ∞ I first prove that the proposition is true, and then prove that when the effectiveness of chlorine, n, or cost of contamination when there is no use of chlorine, bk, are sufficiently high or when the cost of chlorine, α, is sufficiently small the derivative is negative.

$$lim_{n\to 0}\alpha + n - n\ln\left(\frac{bkn}{\alpha}\right) = \alpha - 0 = \alpha \tag{2.20}$$

$$lim_{bk\to 0}\alpha + n - n\ln\left(\frac{bkn}{\alpha}\right) = \alpha + n - (-\infty) = \infty \tag{2.21}$$

$$lim_{\alpha\to\infty}\alpha + n - n\ln\left(\frac{bkn}{\alpha}\right) = \infty + n - (-\infty) = \infty \tag{2.22}$$

when n or bk approaches 0 and when α approaches ∞ the derivative is positive.

$$\lim_{n\to\infty:}\left(\alpha + n - n\ln\left(\frac{bkn}{\alpha}\right)\right) = \lim_{n\to\infty}\left(n\left(1 - \frac{n\ln\left(\frac{nbk}{\alpha}\right)}{n}\right)\right) = \infty(-\infty) = -\infty \tag{2.23}$$

$$\lim_{bk\to\infty}\left(\alpha + n - n\ln\left(\frac{bkn}{\alpha}\right)\right) = \alpha + n - \infty = -\infty \tag{2.24}$$

$$\lim_{\alpha\to 0}\left(\alpha + n - n\ln\left(\frac{bkn}{\alpha}\right)\right) = n - \infty = -\infty \tag{2.25}$$

when n or bk approaches ∞ and when α approaches 0 the derivative is negative.
□

If the effectiveness of chlorine, n, is low, then the optimal use of chlorine l^* is already very low. An increase in the effectiveness would also increase the use of chlorine, as the cost of using chlorine relative to the cost of chlorine decreases. Little use of chlorine imply that the costs of using chlorine are high relative to the cost of falling sick, and when the effectiveness of chlorine increases, since it is possible to achieve a larger effect of the little chlorine already in use, it is effective to increase the use of chlorine. If the effectiveness is already very high then the optimal use of chlorine would already be very high, and an increase in the effectiveness would decrease the need for chlorine, as it is possible to achieve the same effect with less use of chlorine. For this case the use of chlorine would be reduced.

If the cost of contamination, kb, is very low, the use of chlorine is already very low, and an increase in the effectiveness of chlorine reduces the relative cost of using chlorine as you gain more effect pr unit. For this case the use of chlorine increases. If the cost of contamination in the water is already very high, the optimal use of chlorine is already very high, and increasing the effectiveness of chlorine makes the chlorine in use more effective and it is possible to even reduce the optimal use of chlorine.

If the cost of using chlorine is already very high, the optimal use is very low and an increase in the effectiveness of chlorine reduces the cost of the wanted effect and the use increases. If the cost of using chlorine is already very low the optimal level of chlorine is already very high, and it will be more effective to reduce the levels of chlorine when the chlorine becomes more effective.

This results are the same in the case of delayed monitoring where b is exchanged with $\theta\left(\overline{C}-\underline{C}\right)$.

From proposition 1,2 and 3 it can be seen that if the alternative cost of using chlorine increases, the use of chlorine increases. But if the cost of using chlorine increases the use decreases. If the effectiveness of chlorine increases then if the use of chlorine is already very high the use decreases, but if the use is already very low the use increases.

To calculate the expected utilities, I insert 2.16 back into 2.2 and 2.3

$$E\left(U_c\right) = \overline{C} - bk\left(\frac{bkn}{\alpha}\right)^{-\frac{n}{n+\alpha}} - \left(\left(\frac{bkn}{\alpha}\right)^{\frac{\alpha}{n+\alpha}} - 1\right) \tag{2.26}$$

$$E\left(U_d\right) = \overline{C} - \theta\left(\overline{C}-\underline{C}\right)k\left(\frac{\theta\left(\overline{C}-\underline{C}\right)kn}{\alpha}\right)^{-\frac{n}{n+\alpha}} - \left(\left(\frac{\theta\left(\overline{C}-\underline{C}\right)kn}{\alpha}\right)^{\frac{\alpha}{n+\alpha}} - 1\right). \tag{2.27}$$

To interpret what happens if there is no use of chlorine I have used 2.17, 2.18 and 2.19. As the optimal level of chlorine, l^*, cannot take on negative values as proven in 2.38 and 2.39, whenever n approaches 0 or α approaches ∞, the optimal use of chlorine, $l^* = 0$. Inserting this results in the original expected utilities yields

$$E\left(U_c\right) = \overline{C} - bke^{-n*0} - \left(e^{\alpha*0} - 1\right) = \overline{C} - bk \tag{2.28}$$

$$E\left(U_d\right) = \overline{C} - \theta\left(\overline{C}-\underline{C}\right)ke^{-n*0} - \left(e^{\alpha*0} - 1\right) = \overline{C} - \theta\left(\overline{C}-\underline{C}\right)k. \tag{2.29}$$

This means that if the effect of chlorine is so low that it makes no sense to use chlorine or if the cost of using chlorine is so high that it will make more harm than good, there will be no use of chlorine.
The bk and $\theta\left(\overline{C}-\underline{C}\right)k$ can be interpreted as the actual cost of having infection in the water without treating the water. In the continuous monitoring case the cost of infectious water is the cost of the bottled water. In the delayed monitoring the cost of infectious water is the utility-loss from the fraction of people that actually falls sick. When k or $b/\theta\left(\overline{C}-\underline{C}\right)$ approaches 0 the expected utility will be

$$E\left(U_c\right) = \overline{C} \tag{2.30}$$

$$E\left(U_d\right) = \overline{C}. \tag{2.31}$$

If there is no infection in the water, $k = 0$, there is no point in using chlorine to remove infections, and there will be no cost from infections in the water. In this case, everyone will have the utility of drinking healthy and safe water.

If there is no cost from infectious water, bottled water is free or no one falls sick, $b = 0$ or $\theta\left(\overline{C}-\underline{C}\right) = 0$, there is no point in chlorinating the water. Everyone will be better off by just drinking the infected water and not fall sick or by drinking free bottled water. Their expected utility will in both cases be $\overline{C}$.

The expected utilities have the same form in both monitoring options. The values for all parameters are the same except for b and $\theta\left(\overline{C}-\underline{C}\right)$ which are different parameters that represent the cost of contamination in the water for the two different monitoring options. They therefore have the same function in the equations, e.g. evaluating the cost of the alternative to drinking clean water.

proposition 4. *An increase in the cost of contamination in the water, b (in the continuous monitoring case) and* $\theta\left(\overline{C}-\underline{C}\right)$ *(in the delayed monitoring case), will decrease the expected utility.*

Proof.

$$\frac{\partial E(U_c)}{\partial b} = -k\left(\frac{knb}{\alpha}\right)^{-\frac{n}{n+\alpha}} < 0 \tag{2.32}$$

The result is the same for the delayed monitoring option, exchanging b with $\theta\left(\overline{C}-\underline{C}\right)$. □

A higher cost of bottled water b or cost of falling sick $\left(\overline{C}-\underline{C}\right)$ will increase the use of chlorine as shown in proposition 1. From 2.15 it can be seen that the increase in use of chlorine will reduce the probability of falling sick, and that the costs of using chlorine will increase with a higher use of chlorine, so that the overall expected utility decreases. This result shows that the monitoring option with the lowest cost of the alternative to drinking clean water will yield the highest expected utility.

proposition 5. *An increase in the initial probability of contamination, k, will decrease the expected utility.*

Proof.

$$\frac{\partial}{\partial k}E\left(U_c\right) = -b\left(\frac{knb}{\alpha}\right)^{-\frac{n}{n+\alpha}} < 0 \tag{2.33}$$

□

When the initial probability of contamination increases, the use of chlorine increases as shown in proposition 1. The increase in the optimal use of chlorine, l^*, is a trade of between the costs of adding chlorine in the water, and the costs of not adding chlorine in the water. The probability that there is contamination in the water increase with a higher initial probability of contamination, and decrease with a higher use of chlorine. As the increase of chlorine is a trade of between the costs of adding chlorine and the costs of not adding chlorine, the increase in chlorine usage will not be sufficient to totally remove the increase in probability of contamination from a higher initial probability of contamination k. The probability of contamination will therefore increase, and the probability of falling sick or buying bottled water increases. The increased use of chlorine does also increase the negative health effects from chlorine usage, so that the expected utility decreases.
The result is the same for the delayed monitoring option exchanging b with $\theta\left(\overline{C}-\underline{C}\right)$.

proposition 6. *Increasing the damage and/or cost of chlorine, α, will reduce the expected utility.*

Proof.

$$\frac{\partial E(U_c)}{\partial \alpha} = -\frac{(knb)^{\frac{\alpha}{n+\alpha}}\ln\left(\frac{knb}{\alpha}\right)}{\alpha^{-\frac{n}{n+\alpha}}\left(n+\alpha\right)^2} - \frac{(kb)^{\frac{\alpha}{n+\alpha}}\,n^{\frac{2\alpha+n}{n+\alpha}}\ln\left(\frac{knb}{\alpha}\right)}{\alpha^{\frac{\alpha}{n+\alpha}}\left(n+\alpha\right)^2} < 0 \tag{2.34}$$

□

Increasing the damage from or cost of chlorine decreases the optimal level of chlorine, as shown in proposition 2. In 2.15 it is shown that reducing the use of chlorine increases the probability of contamination and reduces the cost of chlorine. This reduces the overall expected utility.

From the propositions 4, 5 and 6 it is shown that an increase in costs, either costs of contaminated water or the costs of chlorine, decreases the expected utility independently of the use of chlorine, as increased costs of contaminated water increases the use of chlorine while increased costs of chlorine decreases the use of chlorine.

proposition 7. *Increasing the effectiveness of chlorine, n, will increase the expected utility.*

Proof.

$$\frac{\partial E(U_c)}{\partial n} = \frac{(kb)^{\frac{\alpha}{n+\alpha}} n^{-\frac{n}{n+\alpha}} \alpha^{\frac{2n+\alpha}{n+\alpha}} \ln\left(\frac{knb}{\alpha}\right) + \alpha^{\frac{n}{n+\alpha}} \ln\left(\frac{knb}{\alpha}\right) (knb)^{\frac{\alpha}{n+\alpha}}}{(n+\alpha)^2} > 0 \tag{2.35}$$

□

Increasing the effectiveness of chlorine reduces the costs of using chlorine, and decreases the probability of contamination in the water as $\frac{\partial p(l)}{\partial n} = -lke^{-ln} < 0$. Decreasing the probability of contamination in the water, decrease the need to buy bottled water or the number of people that becomes sick, so that the costs are reduced and the expected utility increases. The result is the same for the delayed monitoring option exchanging b with $\theta\left(\overline{C} - \underline{C}\right)$.

Second order conditions

To make sure that I am actually maximizing the expected utility, I check the second order derivatives to see that they are negative.

$$\begin{aligned} \frac{\partial^2}{\partial l^2} E(U_c) &= -p''(l)\,\overline{C} + p''(l)\left(\overline{C} - b\right) - d''(l) \\ &= -p''(l)\,b - d''(l) \\ &= -bn^2ke^{-ln} - e^{\alpha l}\alpha^2 < 0 \end{aligned} \tag{2.36}$$

$$\begin{aligned} \frac{\partial^2}{\partial l^2} E(U_d) &= -p'(l)\,\overline{C} + p'(l)\left((1-\theta)\,\overline{C} + \theta\underline{C}\right) - d'(l) \\ &= -p''(l)\,\theta\left(\overline{C} - \underline{C}\right) - d''(l) \\ &= -\theta\left(\overline{C} - \underline{C}\right) n^2ke^{-ln} - e^{\alpha l}\alpha^2 < 0 \end{aligned} \tag{2.37}$$

The two results imply local maximums in both monitoring options.

Solving with Kuhn Tucker

To see if the constraint of positive use of chlorine is binding, and make sure I have an interior solution, I have solved the maximization problem with Kuhn Tucker.

$$max_l E(U) \qquad \text{Subject to} \qquad l \geq 0$$

$$\frac{\partial}{\partial l}E\left(U_c\right) = b - \frac{\alpha e^{\alpha l}}{nke^{-nl}} \leq 0 \quad \wedge \quad l \geq 0 \quad \wedge \quad l\left(b - \frac{\alpha e^{\alpha l}}{nke^{-nl}}\right) = 0 \tag{2.38}$$

Inserting for the optimal l^*, $b - \frac{\alpha e^{\alpha l}}{nke^{-nl}} = 0$, so the conditions are satisfied if $l > 0$ implying that there exists an interior solution, and that the constraint is binding.

$$\frac{\partial}{\partial l}E\left(U_d\right) = \theta\left(\overline{C} - \underline{C}\right) - \frac{\alpha e^{\alpha l}}{nke^{-nl}} \leq 0 \quad \wedge \quad l \geq 0 \quad \wedge \quad l\theta\left(\overline{C} - \underline{C}\right) - \frac{\alpha e^{\alpha l}}{nke^{-nl}} = 0 \tag{2.39}$$

Inserting for the optimal l^*, $\theta\left(\overline{C} - \underline{C}\right) - \frac{\alpha e^{\alpha l}}{-nke^{-nl}} = 0$, so the conditions are satisfied if $l > 0$ implying that there exists an interior solution, and that the constraint is binding.

2.2 Comparison of the two monitoring options under natural contamination

To compare the two models I subtract the expected utility of delayed monitoring from the expected utility of continuous monitoring. If Continuous monitoring yields the highest expected utility, the expression is positive. If delayed monitoring yields the highest expected utility, the expression is negative. The difference in the two expected utilities is the parameters $\theta\left(\overline{C} - \underline{C}\right)$ and b. They represent the costs of contamination in the water in the two different monitoring options. The other parameters are the same for both monitoring options. I assume that the parameters are the same for both monitoring options in the same setting, but that the parameters might take on different values in different settings.

$$E\left(U_c\right) - E\left(U_d\right) = \overline{C} - bk\left(\frac{bkn}{\alpha}\right)^{-\frac{n}{n+\alpha}} - \left(\left(\frac{bkn}{\alpha}\right)^{\frac{\alpha}{n+\alpha}} - 1\right) - \left(\overline{C} - \theta\left(\overline{C} - \underline{C}\right)k\left(\frac{\theta\left(\overline{C} - \underline{C}\right)kn}{\alpha}\right)^{-\frac{n}{n+\alpha}} - \left(\left(\frac{\theta\left(\overline{C} - \underline{C}\right)kn}{\alpha}\right)^{\frac{\alpha}{n+\alpha}} - 1\right)\right) \tag{2.40}$$

proposition 8. *Increasing the cost of bottled water, b, decreases the difference in expected utilities if* $b < \theta\left(\overline{C} - \underline{C}\right)$.

Proof. From 2.8 and 2.10 it can be seen that when $b = \theta\left(\overline{C} - \underline{C}\right)$, the optimal use of chlorine l^* is the same for both monitoring options. Inserting the same l^* in the two expected utilities 2.2 and 2.3 yields the same result, as the only difference in the two equations is the parameters b and $\theta\left(\overline{C} - \underline{C}\right)$, so that $E(U_c) = E(U_d)$ when

$b = \theta\left(\overline{C} - \underline{C}\right)$.
As proven in proposition 4, in equation 2.32, the expected utility decreases when b or $\theta\left(\overline{C} - \underline{C}\right)$ increases. $b < \theta\left(\overline{C} - \underline{C}\right) \Leftrightarrow E(U_c) > E(U_d)$. Increasing b will decrease the expected utility for the continuous monitoring option as $\frac{\partial E(U_c)}{\partial b} < 0$, so that the $E(U_c)$ decreases, and reduces the differences in expected utilities between the two monitoring options. □

The monitoring option with the lowest cost from contamination in the water $\theta\left(\overline{C} - \underline{C}\right)$ or b is the monitoring option that yields the highest expected utility. Increasing the difference in costs of contamination in the water will also increase the differences in expected utility, and the monitoring option with the lowest cost of contamination in the water is the one with the highest expected utility.

proposition 9. *Increasing the initial level of contamination in the water, k, will increase the absolute difference between the expected utilities for the two monitoring options.*

Proof. To see how the difference changes with an increase in k, I take the derivative of the difference in the utilities 2.40 with respect to k

$$\begin{aligned}
&\frac{\partial}{\partial k}\left(E\left(U_c\right) - E\left(U_d\right)\right) \\
&= -b\left(\left(\frac{bnk}{\alpha}\right)^{-\frac{n}{n+\alpha}} - \frac{b^{-\frac{n}{\alpha+n}} n^{\frac{\alpha}{\alpha+n}} k^{-\frac{n}{\alpha+n}}}{\alpha^{-\frac{n}{\alpha+n}}\left(n+\alpha\right)}\right) - \frac{b^{\frac{\alpha}{\alpha+n}} n^{\frac{\alpha}{\alpha+n}} k^{-\frac{n}{\alpha+n}}}{\alpha^{-\frac{n}{\alpha+n}}\left(\alpha+n\right)} \\
&- \theta\left(\overline{C} - \underline{C}\right)\left(\left(\frac{\theta\left(\overline{C} - \underline{C}\right) nk}{\alpha}\right)^{-\frac{n}{n+\alpha}} - \frac{\theta\left(\overline{C} - \underline{C}\right)^{-\frac{n}{\alpha+n}} n^{\frac{\alpha}{\alpha+n}} k^{-\frac{n}{\alpha+n}}}{\alpha^{-\frac{n}{\alpha+n}}\left(n+\alpha\right)}\right) \\
&- \frac{\theta\left(\overline{C} - \underline{C}\right)^{\frac{\alpha}{\alpha+n}} n^{\frac{\alpha}{\alpha+n}} k^{-\frac{n}{\alpha+n}}}{\alpha^{-\frac{n}{\alpha+n}}\left(\alpha+n\right)}.
\end{aligned}$$

Simplifying this expression yields the result

$$\begin{aligned}
&\frac{\partial}{\partial k}\left(E\left(U_c\right) - E\left(U_d\right)\right) \\
&= \left(-b\left(\frac{knb}{\alpha}\right)^{-\frac{n}{n+\alpha}}\right) - \left(-\theta\left(\overline{C} - \underline{C}\right)\left(\frac{kn\theta\left(\overline{C} - \underline{C}\right)}{\alpha}\right)^{-\frac{n}{n+\alpha}}\right).
\end{aligned} \tag{2.41}$$

When $b = \theta\left(\overline{C} - \underline{C}\right) \Leftrightarrow 2.41 = 0$, and the expected utilities will continue to be the same in both monitoring options. If $b < \theta\left(\overline{C} - \underline{C}\right)$, then $E\left(U_c\right) > E\left(U_d\right)$ as proven in proposition 8. So that for the difference between the expected utilities to increase, the derivative of the difference between the expected utilities with respect to k must be larger than 0 when $b < \theta\left(\overline{C} - \underline{C}\right)$

$$\frac{\partial}{\partial k}\left(E(U_c) - E(U_d)\right) > 0. \tag{2.42}$$

Applying 2.41 into 2.42 yields

$$-b\left(\frac{knb}{\alpha}\right)^{-\frac{n}{n+\alpha}} > -\theta\left(\overline{C} - \underline{C}\right)\left(\frac{kn\theta\left(\overline{C} - \underline{C}\right)}{\alpha}\right)^{-\frac{n}{n+\alpha}}. \tag{2.43}$$

taking the inverse and simplifying yields

$$
\begin{aligned}
& \frac{knb}{\alpha} > \left(\frac{\theta\left(\overline{C}-\underline{C}\right)}{b}\right)^{-\frac{n+\alpha}{n}} \frac{kn\theta\left(\overline{C}-\underline{C}\right)}{\alpha} \\
\Leftrightarrow \quad & \frac{b}{\theta\left(\overline{C}-\underline{C}\right)} > \left(\frac{\theta\left(\overline{C}-\underline{C}\right)}{b}\right)^{-\frac{n+\alpha}{n}} \\
\Leftrightarrow \quad & 1 > \left(\frac{\theta\left(\overline{C}-\underline{C}\right)}{b}\right)^{1-\frac{n+\alpha}{n}}. \qquad (2.44)
\end{aligned}
$$

Defining $\gamma \equiv \frac{n+\alpha}{n} - 1 > 0$ and inserting in 2.44 yields

$$
\begin{aligned}
& 1 > \left(\frac{b}{\theta\left(\overline{C}-\underline{C}\right)}\right)^{\gamma} \\
\Leftrightarrow \quad & \theta\left(\overline{C}-\underline{C}\right) > b \qquad (2.45) \\
\Leftrightarrow \quad & E(U_d) < E(U_c) \qquad (2.46) \\
\Leftrightarrow \quad & E(U_c) - E(U_d) > 0. \qquad (2.47)
\end{aligned}
$$

The derivative is positive when $E(U_c) > E(U_d)$, and the difference in absolute value in expected utilities will increase with a higher k.

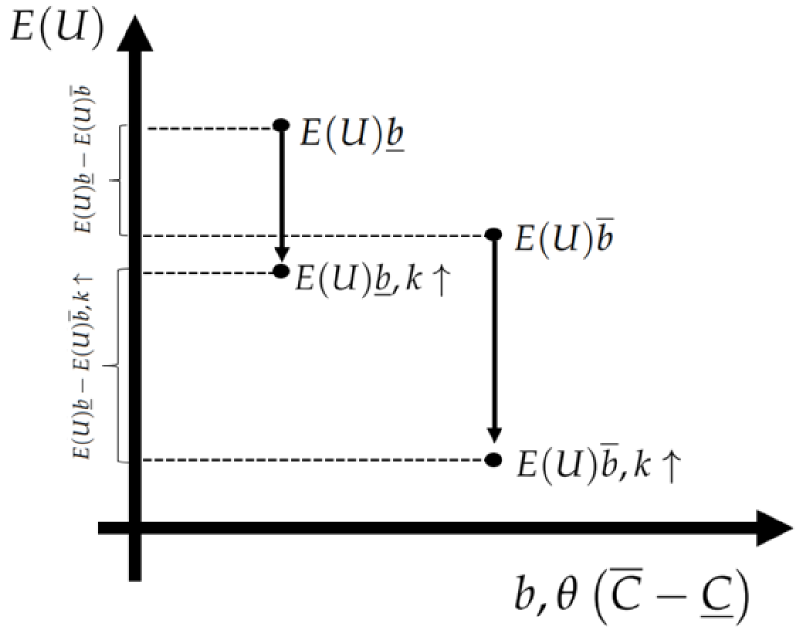

FIGURE 2.1: Graphical illustration of proposition 9.
As b and $\theta\left(\overline{C}-\underline{C}\right)$ expresses the cost of contamination in the water, the figure does not separate between them. $\underline{b}$ represents the lowest cost of contamination in the water, and $\overline{b}$ represents the highest cost of contamination in the water. For $\overline{b}$ (the highest cost of contamination in the water) an increase in k reduces the expected utility more than for $\underline{b}$ (the lower cost of contamination in the water). Thus, the difference between the expected utilities increases when k increases.

If $b < \theta\left(\overline{C} - \underline{C}\right)$ and $E\left(U_c\right) > E\left(U_d\right)$, the expected utility is already higher in the continuous monitoring case $\frac{\partial}{\partial k} E\left(U_c\right) - E\left(U_d\right) > 0$. This proofs that increasing k will increase the differences between the expected utilities.

If the other is true so that $b > \theta\left(\overline{C} - \underline{C}\right)$ and $E\left(U_c\right) < E\left(U_d\right)$, the expected utility is already smaller in the continuous monitoring case $\frac{\partial}{\partial k} E\left(U_c\right) - E\left(U_d\right) < 0$, and the difference between utilities will increase also in this case.

□

When the initial level of contamination in the water increases, the optimal level of chlorine increases as proven in proposition 1, and the expected utility decreases as proven in proposition 5. The expected utility decreases more with a higher cost of contamination in the water. Therefore the expected utility that already has the highest cost of contamination in the water, and the low expected utility, has a larger decrease in expected utility. The difference between the two expected utilities will then increase, as shown in figure 2.1 when the initial probability of contamination in the water increases. In the case with the highest cost of contamination in the water, the use of chlorine is already higher, and the costs from the use of chlorine is also higher. When increasing the initial level of contamination, the use of chlorine increases even more, and as the costs of the use of chlorine increases convexly, so that the cost will increase more for the case where the use is already higher, than for the case with lower levels of use. There is a falling convex relationship between the use of chlorine and the probability of contamination in the water. When the use of chlorine is high, the effect from the next unit of chlorine is smaller than when there is little use of chlorine. So the cost of using chlorine is increasing faster and the effect on the contamination in the water is increasing slower with a higher use of chlorine, and so the decrease in expected utility is higher when the use of chlorine is already high.

proposition 10. *Increasing the effectiveness of chlorine decreases the differences between the expected utilities.*

Proof. To show how the difference in expected utilities changes with an increase in n, I take the derivative of 2.40 with respect to n.

$$\frac{\partial}{\partial n}\left(E(U_C) - E(U_d)\right) = \frac{\overbrace{(kb)^{\frac{\alpha}{n+\alpha}} n^{-\frac{n}{n+\alpha}} \alpha^{\frac{2n+\alpha}{n+\alpha}} \ln\left(\frac{knb}{\alpha}\right) + \alpha^{\frac{n}{n+\alpha}} \ln\left(\frac{knb}{\alpha}\right)(knb)^{\frac{\alpha}{n+\alpha}}}^{\equiv f(b)}}{(n+\alpha)^2} -$$
$$\left(\frac{\overbrace{\left(k\theta\left(\overline{C} - \underline{C}\right)\right)^{\frac{\alpha}{n+\alpha}} n^{-\frac{n}{n+\alpha}} \alpha^{\frac{2n+\alpha}{n+\alpha}} \ln\left(\frac{kn\theta\left(\overline{C}-\underline{C}\right)}{\alpha}\right) + \alpha^{\frac{n}{n+\alpha}} \ln\left(\frac{kn\theta\left(\overline{C}-\underline{C}\right)}{\alpha}\right)\left(kn\theta\left(\overline{C} - \underline{C}\right)\right)^{\frac{\alpha}{n+\alpha}}}^{\equiv f\left(\theta\left(\overline{C}-\underline{C}\right)\right)}}{(n+\alpha)^2}\right) \tag{2.48}$$

The expected utilities increase when n increases, and $\frac{\partial}{\partial n}\left(E(U_C) - E(U_d)\right) = 0$ if $b = \theta\left(\overline{C} - \underline{C}\right) \Leftrightarrow f(b) = f\left(\theta\left(\overline{C} - \underline{C}\right)\right)$. The values of b and $\theta\left(\overline{C} - \underline{C}\right)$ are the only difference in the equations. If they take on the same values the expected utilities does not change and the expression will equal 0. To see if the difference in expected utilities is increasing or decreasing in b, when n increases, I take the derivative of 2.48 with respect to b

$$\frac{\partial}{\partial b}\left(f(b) - f\left(\theta\left(\overline{C}-\underline{C}\right)\right)\right) = \frac{b^{\frac{n}{n+\alpha}}\left(k^{\frac{\alpha}{n+\alpha}}(nb)^{-\frac{n}{n+\alpha}}\alpha^{\frac{3n+2\alpha}{n+\alpha}}\ln\left(\frac{knb}{\alpha}\right) + (kn)^{\frac{\alpha}{n+\alpha}} b^{-\frac{n}{n+\alpha}}\alpha^{\frac{2n+\alpha}{n+\alpha}}\ln\left(\frac{knb}{\alpha}\right)\right)}{b^{\frac{n}{n+\alpha}}(n+\alpha)^3} + \frac{\left(k^{\frac{\alpha}{n+\alpha}} n^{-\frac{n}{n+\alpha}}\alpha^{\frac{2n+\alpha}{n+\alpha}} + (kn)^{\frac{\alpha}{n+\alpha}}\alpha^{\frac{n}{n+\alpha}}\right)(n+\alpha)}{b^{\frac{n}{n+\alpha}}(n+\alpha)^3} > 0 \qquad (2.49)$$

This expression is larger than 0 so the increase in expected utility from an increase in n, is growing in b and as the result is the same for the delayed monitoring case if exchanging b with $\theta\left(\overline{C}-\underline{C}\right)$

$$f' > 0. \qquad (2.50)$$

If the expected utility is higher for continuous monitoring than for delayed monitoring, $E(U_c) > E(U_d)$, then $b < \theta\left(\overline{C}-\underline{C}\right)$. In this case it follows from 2.48 that

$$f(b) < f\left(\theta\left(\overline{C}-\underline{C}\right)\right) \Leftrightarrow \frac{\partial}{\partial n}\left(E(U_C) - E(U_d)\right) < 0. \qquad (2.51)$$

This shows that when n increases the difference between the utilities decreases. □

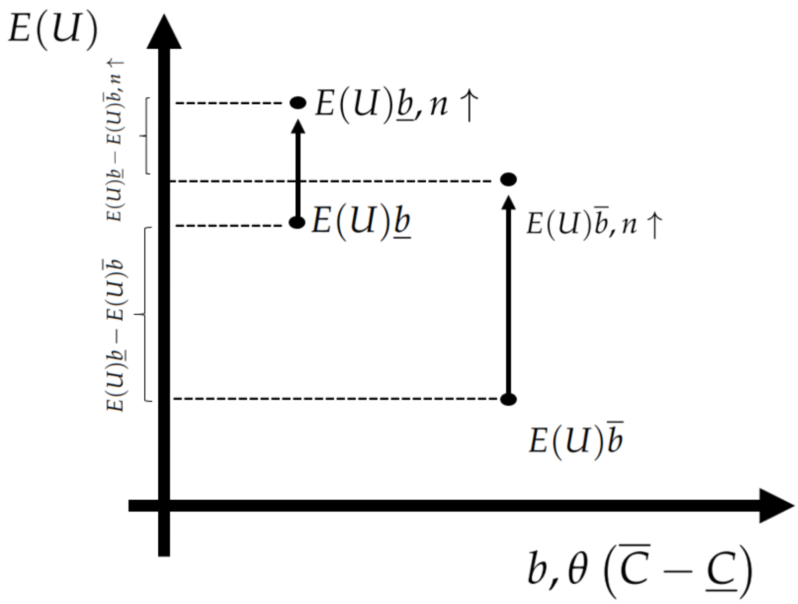

FIGURE 2.2: Graphical illustration of proposition 10.
As b and $\theta\left(\overline{C}-\underline{C}\right)$ expresses the cost of contamination in the water, the figure does not separate between them. $\underline{b}$ represents the lowest cost of contamination in the water, and $\overline{b}$ represents the highest cost of contamination in the water. For $\underline{b}$ (the lower cost of contamination in the water) an increase in n increases the expected utility more than for $\overline{b}$ (the higher cost of contamination in the water). Thus, the difference between the expected utilities decreases when n increases.

Increasing the effectiveness of chlorine, n, increases the expected utility as proven in proposition 7. The lowest expected utility with the highest cost of contamination in the water will increase the most when the effectiveness of chlorine increases. The difference between the expected utilities will therefore decrease, as the highest expected utility, with lower cost of contamination in the water, will have a smaller increase in expected utility when the effectiveness of chlorine increases, as shown in the figure 2.2. When the effect of chlorine increases the cost of using chlorine relative to the cost of not using chlorine decreases. This decreases the difference between the two monitoring options, as a higher cost of contamination in the water leads to a higher use of chlorine as proven in proposition 1. When the use of chlorine becomes more effective it decreases with high use. This means that the benefit of the increased efficiency of chlorine will be greatest where the chlorine is most needed, and there is more need for chlorine where the cost of contamination in the water is high. The monitoring option with the highest cost of contamination in the water is the one with the lowest expected utility, but also the one with the greatest benefit from an increase in the effect of chlorine. The difference between the expected utilities will therefore decrease when the effect of chlorine increases.

proposition 11. *Increasing the cost of chlorine will increase the difference between the expected utilities.*

Proof.

$$\frac{\partial}{\partial\alpha}E(U_c) - E(U_d) = -\overbrace{\frac{(knb)^{\frac{\alpha}{n+\alpha}}\ln\left(\frac{knb}{\alpha}\right)}{\alpha^{-\frac{n}{n+\alpha}}(n+\alpha)^2} - \frac{(kb)^{\frac{\alpha}{n+\alpha}}n^{\frac{2\alpha+n}{n+\alpha}}\ln\left(\frac{knb}{\alpha}\right)}{\alpha^{\frac{\alpha}{n+\alpha}}(n+\alpha)^2}}^{\equiv g(b)} - \overbrace{\left(-\frac{\left(kn\theta\left(\overline{C}-\underline{C}\right)\right)^{\frac{\alpha}{n+\alpha}}\ln\left(\frac{kn\theta\left(\overline{C}-\underline{C}\right)}{\alpha}\right)}{\alpha^{-\frac{n}{n+\alpha}}(n+\alpha)^2} - \frac{\left(k\theta\left(\overline{C}-\underline{C}\right)\right)^{\frac{\alpha}{n+\alpha}}n^{\frac{2\alpha+n}{n+\alpha}}\ln\left(\frac{kn\theta\left(\overline{C}-\underline{C}\right)}{\alpha}\right)}{\alpha^{\frac{\alpha}{n+\alpha}}(n+\alpha)^2}\right)}^{\equiv g\left(\theta\left(\overline{C}-\underline{C}\right)\right)} \tag{2.52}$$

Increasing the cost of chlorine reduces the expected utilities, and $\frac{\partial}{\partial\alpha}E(U_c) - E(U_d) = 0$ when $b = \theta\left(\overline{C}-\underline{C}\right) \Leftrightarrow g(b) = g\left(\theta\left(\overline{C}-\underline{C}\right)\right)$. To see if the difference between the expected utilities is increasing or decreasing in b, when α increases, I take the derivative of 2.52 with respect to b

$$\frac{\partial}{\partial b}\left(g(b) - g\left(\theta\left(\overline{C}-\underline{C}\right)\right)\right) = -\frac{\alpha(kn)^{\frac{\alpha}{\alpha+n}}\ln\left(\frac{knb}{\alpha}\right) + (kn)^{\frac{\alpha}{\alpha+n}}(\alpha+n)}{\alpha^{-\frac{n}{n+\alpha}}b^{\frac{n}{\alpha+n}}(\alpha+n)^3} - \frac{n^{\frac{2\alpha+n}{n+\alpha}}\left(\alpha k^{\frac{\alpha}{\alpha+n}}\ln\left(\frac{knb}{\alpha}\right) + k^{\frac{\alpha}{\alpha+n}}(\alpha+n)\right)}{\alpha^{\frac{\alpha}{n+\alpha}}b^{\frac{n}{\alpha+n}}(\alpha+n)^3} < 0. \tag{2.53}$$

This expression is smaller than 0, so the decrease in expected utility when α increases is decreasing in b, and as the result is the same for the delayed monitoring case if exchanging b with $\theta\left(\overline{C}-\underline{C}\right)$

$$g' < 0. \tag{2.54}$$

If the expected utility is higher for continuous monitoring than for delayed monitoring, $E(U_c) > E(U_d)$, then $b < \theta\left(\overline{C}-\underline{C}\right)$. In this case it follows from 2.52 that

$$g(b) > g\left(\theta\left(\overline{C}-\underline{C}\right)\right) \Leftrightarrow \frac{\partial}{\partial\alpha}E(U_c) - E(U_d) > 0 \tag{2.55}$$

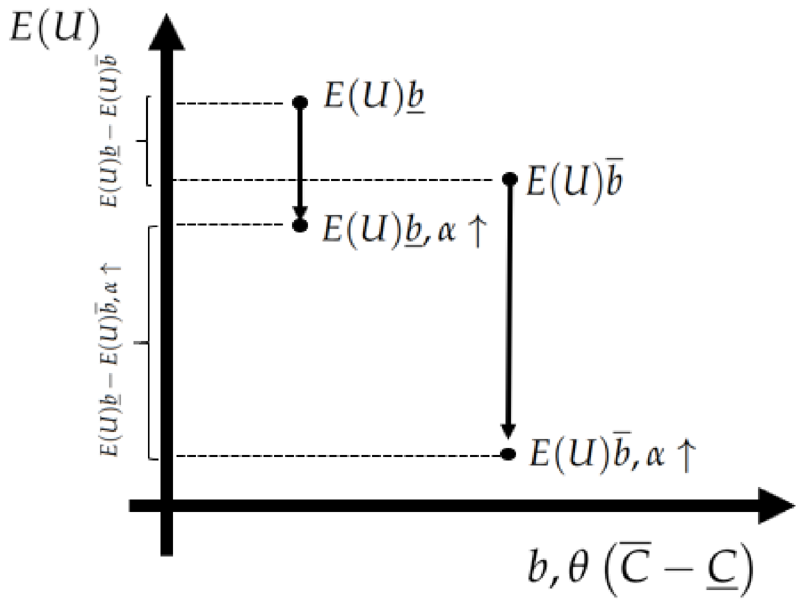

FIGURE 2.3: Graphical illustration of proposition 11.
As b and $\theta\left(\overline{C}-\underline{C}\right)$ expresses the cost of contamination in the water, the figure does not separate between them. $\underline{b}$ represents the lowest cost of contamination in the water, and $\overline{b}$ represents the highest cost of contamination in the water. For $\overline{b}$ (high cost of contamination in the water) an increase in α decreases the expected utility more than for $\underline{b}$ (low cost of contamination in the water). Thus, the difference between the expected utilities increases when α increases.

□

Increasing the cost of chlorine in the water decreases the optimal use of chlorine as proven in proposition 2, and decreases the expected utility as proven in proposition 6. The decrease in expected utility from an increase in the cost of chlorine is higher when the cost of contamination in the water is high, as shown in figure 2.3. The monitoring option with the highest cost of contamination in the water has the lowest expected utility as proven in proposition 1, and does also have the largest decrease in expected utility from an increase in the cost of chlorine as proven in 2.53. This shows that the difference between the expected utilities increase when the cost of chlorine increases. When the cost of chlorine increases, the use of chlorine decreases so that the probability of contamination in the water increases. The increase in probability of contamination in the water is much more expensive for the monitoring option where the cost of contamination in the water is high, so that a decrease in the use of chlorine decreases the expected utility much more in the case where the cost of contamination in the water is high.

2.2.1 Including the costs of the monitoring options

To fully compare the monitoring options I must also take into account the costs of the delayed monitoring (e.g. costs of water sample testing and quality testing), and the monitoring costs of the continuous monitoring (e.g. price of the monitoring option and costs of water sample testing). Assuming that the firm that produces the continuous monitoring solution knows that a large investment cost will make the product less attractive, I assume that they will sell the continuous monitoring system with long term contracts to some yearly average cost including system updates, replacement and maintenance of parts and other potential costs included in the yearly price. I assume that the costs of water sample testing are larger for the delayed response monitoring, as the tests needs to be performed more frequently than in the continuous monitoring case where they only carry out testing when there is contamination in the water. Depending on how often the water is contaminated, continuous monitoring will reduce the testing frequency and also the test costs. The costs of the two monitoring options are denoted m in the model, and are the sum of all average costs associated with each monitoring option. m_c is the yearly average cost of the continuous monitoring option and the water sample testing for the continuous monitoring option, and m_d is the average cost of the water sample testing and the quality control for the delayed monitoring option.

If a planner is to choose between the two types of monitoring he must compare the expected utilities over the planning period to the costs of the different monitoring options over the same planning period. As all costs are presented as yearly average costs I compare the expected utilities and the costs period by period. The comparison will now differ both in the costs of having contamination in the water b and $\theta\left(\overline{C}-\underline{C}\right)$ and in the average costs of the monitoring options m_c and m_d

$$E\left(U_c\right)+m_c-\left(E\left(U_d\right)+m_d\right). \tag{2.56}$$

if $E\left(U_c\right)+m_c-\left(E\left(U_d\right)+m_d\right)$ is positive then the continuous monitoring option yields a higher output than the delayed monitoring option, given the costs and the expected utilities of the two monitoring options. If the value is negative the delayed monitoring option yields the highest output. Even if a monitoring option yields a higher expected utility, the costs of the monitoring option might make it less effective than the other monitoring option. The monitoring option with the highest expected utility might also be more expensive than the other monitoring option and still be the most effective as long as the difference between the costs of the two monitoring options are not higher than the difference in expected utilities between the two monitoring options.

if the difference in expected utilities is larger than the difference in costs, $E(U_c)-E(U_d)\geq m_c-m_d$, or $E(U_d)-E(U_c)\geq m_d-m_c$, then the costs are not large enough to change which monitoring option that is the most effective. If the difference in costs are larger than the difference in expected utilities, and the monitoring option with the largest cost is the one with the lowest expected utility, the costs outweigh the higher expected utility, and the most effective monitoring option will be the one with the lowest monitoring costs. When making a decision, the planner should include the expected utilities from both contamination cases, as both monitoring options monitor both types of contamination.

2.3 Willful contamination

Water supply has historically been used as a political or military tool for more than 2500 years (Gleick 2006). Creating a model on water monitoring management and not taking into account the possibility of willful contamination of the water resources is unimaginable.

> "Water infrastructure can be targeted directly or water can be contaminated through the intentional introduction of poison or disease-causing

agents. The damage is done by hurting people, rendering water unusable, or destroying purification and supply infrastructure. Some important water facilities, such as dams, reservoirs and pipelines, are easily accessible to the public at various points and there are new worries that computer control systems may be accessible to hacking. Many large dams are tourist attractions and offer tours to the public, while many reservoirs are open to the public for recreational boating and swimming. Pipelines are often exposed for long distances. Water and wastewater treatment plants dot our urban and rural landscape" (Gleick 2006).

Cases of willful contamination of drinking water (e.g. acts of terrorism) is separately modeled. I assume that chlorination of water does not affect the probability of willful contamination in the water, as the willful contamination would be designed to not be affected by chlorination. The protection of the water plant and the supply chain affects the probability of an attack and hence the probability of willful contamination in the water, so that the protection level is now the variable of interest.

2.3.1 Review of the differences between the contamination cases

I model the willful contamination setting in the same way as the natural contamination using the same parameters. The variable of interest, l, is now the protection level of the water resources, which is assumed to reduce the probability of willful contamination in the drinking water resources.

θ is the fraction of those drinking contaminated water who actually falls sick. In the case of terror-based contamination in the water, θ is assumed to be higher than in the case of natural contamination. This assumption is based on the idea that anyone who would want to spread disease in the water would want to harm as many, and as much as possible. The cost of bottled water is the same for both types of contamination, but the cost of falling sick can be assumed higher in the willful contamination case, as the contamination is designed to harm, and there is assumed higher severity and mortality of the diseases that are willfully spread as contamination in the water.

The cost of protection is assumed to be higher than the cost of chlorination. Chlorine is a relatively cheap disinfectant and even large amounts of chlorine are not very costly. The costs of protection can be argued to be increasing convexly, as the cheapest and easiest protection opportunities will be applied first, and as higher levels of protection is required, the advancement and costs are increasing. The last level of protection is significantly more expensive than the first level of protection. The visibility of protection might also infer some indirect cost. High degrees of protection might spread some degree of fear in the population causing some indirect costs that might vary convexly with the amount of protection used. The model accounts for this effect in the function $d(l)$ that measures the negative effects from protection and the protection costs.

The probability of willful contamination in the drinking water $p(l)$ is much lower than the probability of natural contamination in the drinking water. The probability is assumed to decrease in the level of protection so that a higher level of protection l yields a lower probability of contamination $p(l)$. The function is assumed convex, so that the first level of protection will be the most effective, in the sense that it creates a barrier to contaminating the water and makes sufficiently difficult to do, so that no one will be able to willfully contaminate the water without planning it more thoroughly. Higher levels of protection creates new barriers, and fewer will have the capability to pose a threat to the water quality.

The main differences in the case of natural contamination of the drinking water and willful contamination in the drinking water is the cost of falling sick $(\overline{C} - \underline{C})$, the differences in probability of contamination in the water $p(l)$, the fraction of people who actually become sick from exposure, θ, the cost of protection, α, and the effectiveness of the protection, n.

2.3.2 The model

In the willful contamination case, the model differs in the variable of interest. I assume that chlorine will not have any effect on the probability of willful contamination as it would not make sense to try to contaminate water with something that is sensitive to the most commonly used disinfectant. Instead I introduce protection l of the water resources and distribution system as a variable, and assume that a higher level of protection reduces the probability of willful contamination of the water $p(l)$. A high level of physical protection might cause unwanted fear in the population, so there might be some negative effects on the population that increases with the level of protection $d(l)$. I separate the continuous and the delayed monitoring cases, and assume that people behave in the same way when the water is willfully contaminated as when the water is contaminated naturally. They buy bottled water only and always if they know that the water is contaminated. The model now looks the same as in the natural contamination case, but the interpretation of the parameters and the main variable is different.

The probability of willfully contaminated water

The probability of willfully contaminated water depends negatively on the level of protection, in the sense that the higher the level of protection, the lower the probability of willful contamination. In the case of chlorine it is possible to argue that the first drop of chlorine is much more effective than the last. In the case of protection it is not so obvious how the graph should look. Visible protection might scare people from trying to contaminate the water, but increasing the protection increases the difficulty of accessing the water. It is hard to draw the line where more protection is not needed, as it is also the level of threat that decides the necessary level of protection. If there is no threat, or if there is little power in the threat, a little protection might be very efficient. If there is sufficient threat with much power and capacity it requires a much higher level of protection. If the protection level is raised, the threatener's might raise their threat level which implies that a new rise in protection level is needed. The probability of an attack will no matter what protection level never be entirely 0. Anyhow, just a little protection might be able to scare of some possible less advanced threats so I assume that the first levels of protection are more efficient than the later ones. Under these assumptions, it makes sense to use the same functional form as in the natural contamination model, but the efficiency of the protection is assumed to be less efficient than the effect of chlorine.

The initial probability of willful contamination, k, is assumed very much lower than the initial probability of natural contamination.

The costs of protection

The costs of protection are assumed convex. The first levels of protection will be the cheapest ones, and as increased levels of protection requires more advanced technology, equipment and knowledge, the protection costs will increase convexly. The negative effect on the population from protection will be 0 until the protection is made visible. The effect is two-sided. If people worry about terrorism in their water, visible protection might calm them. If they initially did not worry, visible protection might make them realize that there is a threat that they should worry about, and they might become more scared. If there are very high levels of protection it can be assumed to cause some fear in the population. Connecting the costs and fear I then assume a convex function of the same form as in the natural contamination model. The cost of protection is assumed higher than the cost of chlorine.

The cost of contamination in the water

In the case of willful contamination $\left(\overline{C} - \underline{C}\right)$ is assumed to be significantly larger than in the natural contamination case. The effect of a willful contamination is assumed to be very damaging to the health of the affected population, and θ is assumed to be much higher than in the natural contamination case, as a willful contamination would be executed to harm as many as possible. The cost of bottled b water is assumed to be equal in the two contamination cases.

2.4 Discussing the two monitoring options for the willful contamination case

As the functions $p(l)$ and $d(l)$ are of the same form, and the parameters are the same in the natural and the willful contamination case, the calculations for the natural contamination case can be applied also for the willful contamination case.

2.4.1 Continuous monitoring

The main differences between the natural contamination case and the willful contamination case for the continuous monitoring is,

- The initial probability of contamination is much lower
- The effect from protection is lower
- The cost of protection is higher.

Decreasing the initial probability of contamination decreases the optimal level of protection, as proven in proposition 1, increases the expected utility, as proven in proposition 5, and decreases the difference between the expected utilities of the two monitoring options, as proven in 9.

Decreasing the effect from protection will decrease the expected utility, as proven in 7, and increase the differences between the expected utilities of the two monitoring options, as proven in 10. The level of protection is decreasing or increasing depending on the level of protection already in use.

Increasing the cost of protection decreases the optimal level of protection, as proven in proposition 2, decreases the expected utility, as proven in proposition 6, and increases the differences between the expected utilities of the two monitoring options, as proven in 11.

2.4.2 Delayed monitoring

The main differences between the natural contamination case and the willful contamination case for the delayed monitoring option are the same as for the continuous monitoring option, with the exception of the cost of falling sick. All the changes in the expected utilities presented for the continuous monitoring option will apply also for the delayed monitoring option, and in addition there is a change in the cost of falling sick.

Increasing the cost of falling sick, both θ, and $\left(\overline{C} - \underline{C}\right)$, increases the use of protection, as proven in proposition 1, and decreases the expected utility, as proven in proposition 4. It follows from proposition 8 that increasing the differences in costs of contamination in the water increases the differences in expected utilities. As the cost of bottled does not change, the difference between the costs of contamination in the water increases when the cost of falling sick increases.

2.4.3 Comparing the expected utilities of the two monitoring options

Decreased efficiency of protection, increased costs of protection and increased differences between the costs of falling sick, increases the differences between the expected utilities for the two monitoring options in the willful contamination case relative to the natural contamination case. Anyhow the reduction in initial probability of contamination in the water decreases the difference in expected utilities between the two monitoring options. If the reduction in initial probability of contamination is sufficiently large, the difference in expected utilities will decrease. If the reduction in initial probability of infection is not sufficiently large, the difference in expected utilities will increase.

2.4.4 Comparison of the expected utilities and costs of both monitoring options under willful and natural contamination

The planner decide on a preferred monitoring option that covers both natural contamination and willful contamination. Once you monitor the water, both types of contamination are being monitored I assume that the cost of monitoring one contamination case is the same as the cost of monitoring both contamination cases for both monitoring options. To make the decision of the preferred monitoring option, the planner must compare the expected utilities from both monitoring options in both contamination cases with the costs of the monitoring option

$$E\left(U_c\right)_{natural} + E\left(U_c\right)_{willful} + m_c - \left(E\left(U_d\right)_{natural} + E\left(U_d\right)_{willful} + m_d\right). \qquad (2.57)$$

If 2.57 is positive, the continuous monitoring option is more effective than the delayed monitoring option, and yields the highest payoff. If 2.57 is negative, then the delayed monitoring option is more effective and yields the highest payoff.

Chapter 3

Qualitative discussion of application of the model

3.1 Qualitative discussion of the parameters in the model

in this section I have gone through each parameter, how they are assumed to be applied in the two cases, and how differences in application affects the expected utility in the two cases. Since most of the parameters will not have one specific value, but different values for different settings, I will not add one specific value to the parameters, but exemplify how the variables might look in some imagined places and settings in the end of this chapter.

3.1.1 Initial level of contamination/threat of contamination

The initial level of contamination in the natural contamination case will vary greatly between different water plants. The different types of water, groundwater, river, lake, have different types of initial contamination levels. Therefore adding an exact value for k will not make sense unless it is for a specific case and one specific water plant. Anyhow the initial level of contamination is never at 0 as there will always be some level of bacteria and parasites in the water. Climate changes also affect the initial water quality through droughts (Mosley 2015) and floods, and can increase the initial level of contamination in the water, where floods usually cause a larger increase over a shorter time-period whereas droughts cause a lower increase in initial contamination levels over a longer time-period (Hrdinka et al. 2012).

The initial level of threat of willful contamination is not easy to estimate, but the fact that there are several historical examples of planned and executed attacks on water systems suggests that there is some level of threat, and that the risk is real(Gleick 2006). The US government claims that Russian hackers have attacked the United States water management systems (US-CERT 2018). Such an attack can be argued to increase the level of risk in the United States as the Russian hackers might have access important information on how the water is distributed, weaknesses and strengths of the water supply management and other types of classified and critical information. Countries that are currently in conflict can also be assumed to have a higher initial level of threat than countries who are not in conflict especially if they have limited access to freshwater.

A higher k will increase the differences in expected utilities as proven in proposition 9. If all other parameters are equal between the natural and the willful contamination cases, and $b \neq \left(\overline{C} - \underline{C}\right)$, the differences in expected utilities with the two monitoring options will be larger in the contamination case with the highest initial probability of contamination.

It is plausible to assume a higher k in the natural contamination case than in the willful contamination case as there is still a very low threat level of willful contamination in almost every country in the world.

3.1.2 Cost of disinfectant/protection

There are numerous possible disinfectants to use, and each type of disinfectant has different prices and possible negative health effects. Disinfection practices vary by the source of water that is being treated, the objective of the treatment and the size of population that are being served (WHO. 2011), (Council and Association 2012). Since chlorine is a commonly used and well known disinfectant I use chlorine as an example disinfectant in this model, but it is possible to adjust the costs and negative health effects to fit any other type of disinfectant. The cost of chlorine is very low for most types of chlorine. The negative health effects from by-products are also insignificant and very low for small amounts of chlorine by-products. For high levels of chlorine the negative health effects are large and potentially very deadly. The levels of disinfectants normally used in water does not give high levels of by-product, and the costs and damage from the use of chlorine is therefore assumed to be fairly low for low levels of use, and increasing with higher levels of use. Other disinfectants might have other cost-structures.

The cost of protection is assumed to be higher than the cost of chlorine, both for low and high levels of use. The costs of protection is assumed to grow convexly and become really high for high levels of protection, as protection becomes more and more complex and advanced as the levels of protection increases. The harm from protection is assumed much smaller than the actual costs, but also increasing in the levels of protection.

A high cost of disinfectants/protection, α, decrease the expected utility as proven in proposition 6, and increase the differences in the expected utilities as proven in proposition 11. If all other parameters are equal between the natural and the willful contamination case, and $b \neq \theta\left(\overline{C} - \underline{C}\right)$, the differences in expected utilities with the two monitoring options will be larger in the contamination case with the highest initial cost of disinfectant/protection.

It is plausible to assume a higher cost of protection than cost of disinfection, so that the difference in expected utilities will be larger in the willful contamination case than in the natural contamination case if all other parameters are the same in the two cases.

3.1.3 Effectiveness of disinfectant/protection

The effectiveness of the disinfectant is depending on the disinfectant of choice, the level of disinfectant used in the water, the characteristics of the water and the type of contamination (WHO. 2011). It is assumed that the first levels of disinfection are more effective than the last levels of disinfection, and that the effectiveness is reduced convexly with the amount of disinfection that is used in the water.

In the willful contamination case the effectiveness of protection is fairly hard to estimate, but also here it is assumed that the first levels of protection are very effective, and that the effectiveness of protection is smaller when the levels of protection increases.

An increase in effectiveness increases the expected utility as proven in proposition 7, and decreases the difference between the expected utilities as proven in proposition 10. Assuming that the effectiveness is higher for disinfection in the natural contamination case than for protection in the willful contamination case, that all other parameters are of the same value in the two cases, and that $b \neq \theta \left(\overline{C} - \underline{C}\right)$, the difference between the expected utilities is smaller in the natural contamination case than in the willful contamination case.

3.1.4 Cost of falling sick/ the cost of bottled water

The cost of falling sick depends on how many that actually falls sick and what kind of disease they get from the contaminated water. The cost is estimated to be the difference in utility when healthy and when sick, including the loss of income, the production loss, and the treatment costs for the society. As a person that falls sick when drinking water might stay sick for several days, the estimated cost of falling sick must be the utility loss for all those days that the person is sick, as drinking clean water the next day will not remove the illness. In the natural contamination case only some fraction falls sick, and most waterborne diseases, with many exceptions, are easily treatable gastrointestinal diseases that will cure over a short period of time. It is reasonable to assume that the cost of falling sick normally is higher than the cost of bottled water, assuming that the fraction that falls sick are unfit to work and needs some medical attention that together with the reduced utility when sick, represents some costs to the society and the persons health, when this person is sick. θ is the fraction of those that drink contaminated water that will actually fall sick, as some people will have a higher tolerance to bacterias and parasites than others, and stay healthy even if they drink contaminated water. If the fraction of people that falls sick from drinking contaminated water is sufficiently small, the cost of falling sick might actually be smaller than the cost of bottled water, as everyone that has contamination in their water will buy the bottled water when they are aware that the water is contaminated. As there is an estimated 19.5 million cases of waterborne illnesses every year in the U.S. (Reynolds, Mena, and Gerba 2008), this imply that the fraction of people who falls sick when drinking contaminated water is relatively high, if the probability of contamination in the drinking water in the U.S is relatively low. It is plausible to assume a low probability of contamination in the drinking water in the U.S, and most of the developed countries in the world. In developing countries 2.2 - 5 million people die every year from waterborne diseases (Gleick et al. 2002). This indicates a much higher probability of contamination in the water in the developing part of the world.

In the willful contamination case the fraction of people that falls sick is assumed to be larger than in the natural contamination case, as a willful contamination would be executed to harm as many as possible, or cause as much fear as possible, in the population of interest. It is also plausible to assume that the disease spread would be of a more damaging kind, and harder to treat, than most of the diseases that are naturally present in the untreated water. Therefore I assume that the cost of falling sick is much larger in the willful contamination setting than in the natural contamination setting. The cost of bottled water will be the same in both settings, as everyone in the area where the water is contaminated will buy bottled water, and the price of bottled water will not change according to the type of contamination in the water.

If the cost of falling sick is higher than the cost of bottled water in the natural contamination case, it will also be the case that the cost of falling sick is higher in the willful contamination case, as the cost of falling sick in the willful contamination case is assumed higher than in the natural contamination case. This imply that the expected utility will be higher for continuous monitoring in both cases, as long as the cost of falling sick is higher than the cost of bottled water in the natural contamination case. The difference in the expected utilities will depend on the other parameters in the model, but it is the difference in the cost of falling sick that determines which monitoring option yields the highest expected utility.

3.2 Qualitative application of the parameters in the model

In this section, I qualitatively evaluate the parameters and apply the values to the model for one imagined setting. Then I will discuss how the values might change between different settings, and the possible effects of such changes.

3.2.1 Calculating the expected utilities for both monitoring options

For this calculations I will qualitatively evaluate the parameters in the imagined setting of a larger city in a high income country, using Los Angeles and the U.S. as inspiration, and using data such as average income, average price of bottled water and population measures from Los Angeles and U.S. The model calculates the daily utility from the water monitoring option per person.

Natural contamination

To find a number for the initial probability of contamination in the water, I have used the information about how many people that falls sick from contaminated water in the U.S. every year. This number is an estimated 19.5 million cases if illness each year. Assuming that the fraction of people that falls sick is at 0.5, so that half of those drinking contaminated water actually falls sick, then 39 million people drink water containing contamination each year. The American population is almost 328 million people (Bureau 2018). This means that 11.89% of the population some time during a year will drink water containing contamination. This yields a daily average probability of 0.033%. This is after the water is treated, so that the initial level of contamination in the water must be a lot higher, as the first levels of treatment has a high effect. In many developing countries there is little treatment of water, and Montgomery and Elimelech (2007) states that point- of- use household water treatment lead to a reduction in diarrheal decease of 35%. This indicates that the initial probability of contamination is much higher than the probability of contamination after treatment. I will therefore assume a initial probability of contamination of 40%.

The effectiveness of chlorine and levels of chlorine can now be set so that the probability of contaminated water equals 0.033% as seen from the numbers.

$$0.4e^{-nl} = 0.00033 \Rightarrow nl = 7.1 \tag{3.1}$$

Assuming that the level of chlorine equals one, the effect of chlorine then equals 7.1.

The cost of chlorine is very small, and will be set to 0.1$ per unit of chlorine in this example. The cost of bottled water is on average in the U.S 1.2$ per gallon of water (Statista 2018). Assuming that a person uses about a gallon of clean drinking water per day, the average cost of bottled water per person per day is 1.2$.

The average cost of falling sick is hard to estimate. Since all my other parameters are money metric, the utility must also be money metric. The estimated annual cost of lead poisoning in the U.S. is $43.4 billion. The cost of gastrointestinal disease can be assumed to be much lower, other diseases might be deadly and then be even more costly. Assuming that a person falls sick for an average of ten days, and that the person is unable to work for those days, the cost of falling sick will equal the loss of income for those ten days, the cost of health-care for the person and the sosciety, and the utility-loss from being sick rather than healthy. The average income in the U.S is 4700$ per month (Worlddata.info 2018), this yields an average of 157$ in income loss per person per day. Including health care costs, an estimate of 200$ in loss per day per person is a plausible estimate. This estimate can be interpreted as a minimum estimate as it is based on income loss and health costs, and the loss in utility from being sick is not taken into account. The person that becomes sick stays sick for as long as it takes to become healthy independently of the quality of the water, so that the loss of utility per person that becomes sick equals 200$*10=2000$. Since only half of those exposed to the contaminated water falls sick, the cost of contamination in the water equals $0.5 * 2000 = 1000$.

The utility of staying healthy represents is not money metric, and can not easily be evaluated in terms of dollars. I will set the utility of being healthy to 500$ per day per person as it is approximately three times what that person earns that day. The number is mainly used for comparison of the two models, so that the exact value of the initial expected utility is not a critical value.

Connecting all the numbers into the optimal consumption of chlorine yields

$$l_c* = \frac{ln\left(\frac{1.2*7.1*0.4}{0.1}\right)}{0.1+7.1} = 0.49 \tag{3.2}$$

$$l_c* = \frac{ln\left(\frac{1000*7.1*0.4}{0.1}\right)}{0.1+7.1} = 1.42. \tag{3.3}$$

This shows that there will be a positive use of chlorine for both monitoring options, and that the use of chlorine is significantly larger in for the delayed monitoring option than for the continuous monitoring option. As the estimated cost of falling sick is a minimum estimate, including the utility loss from falling sick, so that the cost of falling sick increases, will increase the use of chlorine for the delayed monitoring option even further, as proven in proposition 1, and further increase the difference in use of chlorine between the two monitoring options.

Inserting the values of the parameters in the expected utilities yields

$$E(U_c) = 500 - 1.2*0.4\left(\frac{1.2*7.1*0.4}{0.1}\right)^{-\frac{7.1}{7.1+0.1}} - \left(\left(\frac{1.2*7.1*0.4}{0.1}\right)^{\frac{0.1}{7.1+0.1}} - 1\right) = 499.93 \tag{3.4}$$

$$E(U_d) = 500 - 1000*0.4\left(\frac{1000*7.1*0.4}{0.1}\right)^{-\frac{7.1}{7.1+0.1}} - \left(\left(\frac{1000*7.1*0.4}{0.1}\right)^{\frac{0.1}{7.1+0.1}} - 1\right) = 499.83. \tag{3.5}$$

This result is the expected utility per person per day. The continuous monitoring option yields a benefit of 0.1$ per person per day. Scaling the result up to a city like Los Angeles in the U.S with a population of 3 million people, the difference in utilities in dollars per day equals

$$E(U_c) - E(U_d) = (499.93\$ - 499.83\$) * 3,000,000 = 300,000\$. \tag{3.6}$$

This yields a yearly pre-cost benefit from continuous monitoring of

$$300,000\$ * 365 = 109,500,000\$. \tag{3.7}$$

For the total of the U.S the result is

$$E(U_c) - E(U_d) = (499.93\$ - 499.83\$) * 328,000,000 = 32,800,000\$. \tag{3.8}$$

This yields a yearly pre-cost benefit from continuous monitoring of

$$32800,000\$ * 365 = 11,972,000,000\$. \tag{3.9}$$

This results show that there might be substantial pre-cost benefits from continuous monitoring for this specific case.

Willful contamination

In the willful contamination case, the parameters will take on different values than in the natural contamination case.

The initial probability of willful contamination is assumed very low. Although there are some examples of threats and attempts to willfully contaminate water, also in the U.S. (Maiolo and Pantusa 2018), I have not been able to find any recent examples of willful contamination where any part of the population in the U.S. have fallen sick. Since there is a threat, but that there are no recent examples, it is plausible to assume a very low initial probability of willful contamination. I will therefore assume it to be 0.01% per year. The daily average probability of willful contamination is then 0.00003%.

I assume that the fraction of people that falls sick from drinking the contaminated water is much higher than in the natural contamination case, and at 0.9.

The effect of protection will for simplicity be assumed at 1.

If the level of protection is also assumed to be 1, the probability of willful contamination is

$$0.00003e^{-1} = 0.00004 = 0.00001\%. \tag{3.10}$$

The cost of bottled water is the same in both cases e.g. 1,2$.

The cost of falling sick is assumed much higher, as the diseases spread are assumed to be more harmful in the willful contamination case than in the natural contamination case, so that people stay sick longer on average. Assuming that the average period of disease is three weeks, using the same calculation per day as i the natural contamination case it yields an average cost per person of falling sick on 200$*21=4200$. Assuming that the fraction of those drinking contaminated water that actually falls sick is 0.9, the cost of contamination in the water is $0.9 * 4200 = 3780$

The cost of protection is assumed much higher than the cost of chlorine, I assume it to be 100$ per unit.

The optimal use of protection in the two cases is

$$l_c^* = \frac{ln\left(\frac{1.2*0.00001*1}{100}\right)}{100+1} = -0.158 \tag{3.11}$$

$$l_d^* = \frac{ln\left(\frac{3780*0.00001*1}{100}\right)}{100+1} = -0.078. \tag{3.12}$$

As the optimal level of protection is negative for both monitoring options, the optimal use of protection in this case is 0. The fact that the water is already protected might indicate that my estimated initial probability of willful contamination, or the estimated cost of falling sick is too low. The estimated cost of falling sick is also in this example a minimum cost, so that the actual cost of falling sick might be perceived to be higher than in this example. The expected utilities for this case is

$$E(U_c) = 500\$ - 0.00001 * 1.2\$ = 499.999988\$ \tag{3.13}$$

$$E(U_d) = 500\$ - 0.00001 * 3780\$ = 499.9622\$. \tag{3.14}$$

The continuous monitoring option yields a pre-cost benefit of 0.0377$per person per day.

For a city of the size of Los Angeles it equals a daily pre-cost benefit of

$$0.0377\$ * 3,000,000 = 113,364\$. \tag{3.15}$$

This yields a yearly pre-cost benefit of

$$113,364\$ * 365 = 4,137,786\$. \tag{3.16}$$

For U.S. as a whole the daily pre-cost benefit equals

$$0.0377\$ * 328,000,000 = 12,365,600\$. \tag{3.17}$$

This yields a yearly pre-cost benefit of

$$12,365,600\$ * 365 = 4,513,444,000\$. \tag{3.18}$$

As the probability of willful contamination is very low, the difference between the monitoring options is also much lower than in the natural contamination case where the probability is significantly higher. The increased costs of falling sick does not outweigh the decrease in probability of contamination, and the difference in expected utility is, for this numbers, smaller in the willful contamination case than in the natural contamination case. The continuous monitoring option yields a pre-cost benefit in both contamination cases.

Evaluating the net benefit of the continuous monitoring option

To see the total benefit of the continuous monitoring option, the sum of the benefit in both contamination cases must be balanced with the difference in costs between the two monitoring options. The continuous monitoring option will be the most effective as long as the difference in cost between the two monitoring options does not exceed the total potential benefit from continuous monitoring

$$E\left(U_c\right)_{natural} + E\left(U_c\right)_{willful} - \left(E\left(U_d\right)_{natural} + E\left(U_d\right)_{willful}\right) > m_c - m_d.$$

To see how large the difference in costs can be until the delayed monitoring is more effective, I summarize the benefits of continuous monitoring from the expected utilities in the two contamination cases and compare them to the difference in costs between the two monitoring options

$$\begin{aligned} & 11,972,000,000\$ + 4,137,786\$ > m_C - m_d \\ \Rightarrow \quad & 11,976,137,786\$ > m_C - m_d. \end{aligned} \quad (3.19)$$

The costs of continuous monitoring must for this example be significantly larger, 11,976,137,786\$, than the costs of delayed monitoring in order to make the delayed monitoring option more effective than continuous monitoring. In the example, the cost of falling sick is calculated as a minimum cost, and a growth in the cost of falling sick, in either of the two contamination cases, increases the differences between the expected utilities of the two monitoring options even further, increasing the benefit of continuous monitoring.

3.2.2 Discussing differences in evaluation of the model between developing countries and high income countries

In the developing part of the world, approximately 2.2-5 million people die each year from water-related diseases, and many more falls sick from drinking contaminated water. This indicates that the probability of contamination in the water is much higher here, and/or that there is less use of chlorine or other disinfectants in the water to reduce the probability of contamination. This would increase the differences in expected utilities in areas where the initial probability of contamination in the water is very high.

Relative to income, the price of bottled water is also much higher in low income countries than in high income countries. For example, in Bangladesh, the average monthly salary is 111\$ and the cost of a 1.5 liters of bottled water is equal to 2.9% of the average income per person, while in the U.S the cost of 1.5 liters of bottled water is equal to 0.4% of the average income per person. When using loss of income as a means in calculating loss of utility from falling sick, the distance between the price of bottled water, and the cost of falling sick decreases for Bangladesh relative to U.S. so that the difference expected utilities for the two options is much smaller. Anyhow, as waterborne diseases lead to many deaths every year in developing countries, the diseases will be more severe, and therefore the cost of falling sick might be assumed to be much higher than in high income countries where very few die from water-related disease. The severeness of the water-related diseases in developing countries overweights the higher cost of bottled water, so that the differences in expected utilities will be higher when the severeness of the diseases is higher.

Treating water on the spot with chlorine is less effective than when treatment is applied under optimal conditions. In many developing countries, treatment of water is not as efficient as in many developed countries, reducing the efficiency of treatment and increasing the difference in expected utilities.

The damage from chlorine or other disinfectants vary with the disinfectant in use, as different types of disinfectants have different cost-structures and health-damage structures. There is also a lack of knowledge about the actual damage from chlorine, so that the damage function might change over time as researchers provides new knowledge about the health damage from chlorination of drinking water. Many households in developing countries treat their water in their homes, applying chlorine to their drinking water. This might increase the damage from chlorine, as there chance of overuse is very high, and higher levels of use increases the negative health effects and costs of chlorine. Increasing the cost of chlorine, increases the differences in expected utilities between the two monitoring options.

Different settings yield different expected utilities, and scaling up a result to the U.S. as a whole will simplify important differences between the different states and water supply systems throughout the country and the equation can at best be seen as an exemplification of how it might look.

3.3 Interpretation of the model

The model can be interpreted as a tool to calculate possible outcomes and gains from changing the monitoring strategy from delayed monitoring, that is most commonly used today to monitor the drinking water quality, to continuous monitoring with real-time knowledge about the water quality. I have tried to design a model that is applicable in as many different settings as possible, as there are large differences in available technology, access to water resources, quality of the water resource available et cetera. I explain how this differences will affect the outcome of the model through calculations and interpretation of each parameter and the effect of a change in optimal use of treatment or protection, the expected utility, and the difference between the expected utilities. The cost of bottled water in the model represents the alternative cost of contamination in the water to falling sick. There might be other alternatives than buying bottled water as for example to have more than one initial source of water supply, and simply change the initial source if one becomes contaminated until the contamination is resolved.

For most cases it seems that without taking the costs of monitoring into account the continuous monitoring option will yield a higher expected utility with a lower use of disinfectants. This higher utility comes from a lower cost of contamination in the water as no one will fall sick, but everyone buys bottled water whenever the tap-water is contaminated instead.

Chapter 4

Conclusion

4.1 Conclusion

In this thesis, I have created a simple theoretical model to investigate the relationship between delayed and continuous monitoring, how the monitoring options affect the level of treatment, and the expected utility of the consumer for the two monitoring options. From the model it can be seen that the monitoring option with the lowest cost of contamination in the water yields the highest expected utility for the consumer. It can also be seen that the delay in knowledge about the quality of the water yields a higher optimal use of treatment than real-time knowledge about the water quality, as long as the cost of bottled water is smaller than the cost of falling sick. Regli et al. (1991) stated that absence of methodology lead to over design and excess disinfection of the water, which is the tendency that is shown in the model. The higher the possible cost of falling sick, the higher is the use of treatment and protection in the water. This result is the same for both willful and natural contamination. It is hard to imagine a situation where it would be true that the cost of bottled water is higher than the loss in utility from falling sick. Therefore, in every setting where the cost of bottled water is smaller than the cost of falling sick, the continuous monitoring option yields a higher expected utility than the delayed monitoring option for both contamination cases.

The possible gain from continuous monitoring ,without taking the costs of the monitoring options into account, equals the difference in expected utilities between the two monitoring options. The model shows that the possible gain is depending on the cost and health damage from chlorine, the effectiveness of chlorine, the initial probability of contamination and the size of the difference in costs of contamination in the water between the two monitoring options.

High initial contamination, large difference in costs of contamination in the water and high costs of chlorine yield a large gain from continuous monitoring. High efficiency of chlorine decreases the gain from continuous monitoring.

As I highlighted in the motivation, scarcity issues are growing, and scarce water has a higher probability of contamination. As scarcity issues are growing, the probability of contamination might be perceived to be growing as well. If this is true, then the possible gain from continuous monitoring will continue to grow in the future, if all else is equal. The initial probability of contamination is also higher in areas with high population density, especially in combination with poor sanitation. This is the case in several developing countries, and the gains from continuous monitoring is larger in the developing part of the world than in the developed world, if all else is equal.

The difference in costs of contamination in the water between the two monitoring options increases the possible gain of the system with the lowest cost of contamination in the water. For most thinkable cases the cost of bottled water which is the cost of contamination in the water in the continuous monitoring option is smaller than the cast of falling sick, which is the cost of contamination in the water in the delayed monitoring option. The cost of bottled water is fairly low in most parts of the world, and the cost of falling sick in terms of income loss and health care expenses is fairly high relative to the cost of bottled water. In high income countries with high average wages, the cost of falling sick is very high relative to the cost of bottled water. In developing countries, the average wage is often fairly low, and even when the cost of bottled water is lower, the cost of bottled water relative to the average wage is much higher. The cost of falling sick is also depending on how long you are sick on average, and the severity of the disease. In developing countries the severity of waterborne diseases is much higher than in high income countries, and the mortality of waterborne diseases is very much higher. This indicates that the disease will last longer and be more harmful in developing countries, increasing the average cost of falling sick relative to the cost of bottled water, and therefore possibly increasing the gain from continuous monitoring.

The threat of willful contamination to the water resources is fairly low, but as I have discussed there are examples of attempts and threats of willful contamination. Historically there are examples of countries contaminating the enemies water resources, or trying to control them. As the initial probability of willful contamination is very low, the possible gains from continuous monitoring are much lower than if the probability of willful contamination was higher. The cost of bottled water is the same in both contamination cases, as the cost will not vary with the type of contamination in the water, but the cost of falling sick is perceived to be much higher in the willful contamination case since the contamination is willfully spread to harm as many, and as much as possible. Therefore the fraction that falls sick, and the severity of the disease is expected to be much higher, and the cost of falling sick is higher. This increases the gain from continuous monitoring.

The costs of lower levels of protection is also much smaller than the costs and negative health effects for chlorine, until a certain threshold where the health effects from by-products from chlorine becomes very large, further increasing the possible gains from continuous monitoring.

The gains from continuous monitoring must be balanced against the costs of the continuous monitoring options, subtracting the costs for the delayed monitoring option. If the difference in costs is smaller than the gain from continuous monitoring, then continuous monitoring is more effective than delayed monitoring.

Improving water, sanitation and hygiene might prevent 9.1% of the global disease burden. Developing cheap and accessible continuous monitoring systems that can provide information about the water quality, not only to high income countries, but that can also be implemented in developing areas has large potential to contribute in improving access to clean and safe drinking water and to reduce waterborne disease outbreaks.

The model is based only upon theory, and should be complemented by careful qualitative analysis in the future. I hope that this study can inspire further investigation on the relationship between water monitoring options and the health effect for a population. As we know that several million people falls sick from drinking contaminated water each year, and hundreds of thousands die from water-related diseases, the scope of improvement in the drinking water system is very large.

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