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Key Points:

- The Stokes drift at the equator in the Rossby wave mode (1,1) is basically westward
- The total zonal Stokes volume transport in the Rossby wave mode (1,1) is identically zero
- A Kelvin wave caused by reflection must at least be a 2. mode component in the vertical

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Equatorial Stokes drift and Rossby rip currents

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Abstract The Stokes drift in long baroclinic equatorial Rossby waves is investigated theoretically by using eigenfunction expansions in the vertical. These waves are nondispersive and propagate westward along the equator. Particular attention is paid to the first baroclinic, first meridional Rossby wave mode which has been observed in the equatorial Pacific. It is demonstrated that the Stokes drift depends very much on the depth-variation of the Brunt-Väisälä frequency. Even more importantly, it is found that, for arbitrary stable stratification, the total zonal Stokes volume transport induced by the Rossby wave mode (1,1) is identically zero. The eastward drift due reflected wave energy in the form of internal equatorial Kelvin waves is also addressed. Due to the very long period of the incident Rossby wave mode (1,1), the reflected equatorial Kelvin wave must at least be a 2. mode component in the vertical. The corresponding Stokes drift only induces a minor change near the surface of the total westward drift velocity at the equator. The implication for the existence of compensating Rossby rip currents along the equator is discussed.

1. Introduction

The region near the western oceanic boundary is known to be a significant sink of energy for westward propagating Rossby waves and eddies; see e.g., *Zhai et al.* [2010]. Therefore, this region is often referred to as the Rossby graveyard. By analogy with surface waves breaking at the beach, and the associated generation of rip currents, *Marshall et al.* [2013] discuss a similar effect for baroclinic Rossby waves at mid-latitudes. They argue that to upheld mass balance, their calculated westward Stokes mass transport in baroclinic Rossby waves and eddies, vanishing at the Rossby graveyard, must be compensated by eastward Eulerian mean (rip) currents.

In their analysis, *Marshall et al.* [2013] apply a reduced gravity model. In such models, the major part of the Stokes drift occurs in the upper layer and is independent of depth. If we turn to Kelvin waves, it is known that reduced gravity models for internal Kelvin waves, which only reproduce the first baroclinic mode (the interfacial wave), fail to yield the backward Stokes drift found in the case of a continuous stratification [*Wunsch*, 1973; *Weber et al.*, 2014]. The reduced gravity model also fails to yield the vanishing Stokes volume transport in each baroclinic mode, independent of the vertical variation of the Brunt-Väisälä frequency N [*Weber et al.*, 2014; *Weber and Ghaffari*, 2014].

No such investigation has been undertaken for long Rossby waves when N varies with depth, and this has motivated the present study. However, baroclinic Rossby waves are unstable at mid-latitudes and high latitudes [*LaCasce and Pedlosky*, 2004; *Isachsen et al.*, 2007], while at low latitudes they are able to cross the ocean basin before they succumb to unstable perturbations. Therefore, within the equatorial band (latitudes $|\theta| \leq 5°$), it appears reasonable to consider constant amplitude Rossby wave modes as a basis for nonlinear drift calculations. This has been done for example by *Thompson and Kawase* [1993] and *Li et al.* [1996], applying reduced gravity models. But since the two-layer model produces erroneous results for the Stokes drift in baroclinic Kelvin waves, it is also likely to do so for long equatorial Rossby waves. Indeed, in the present paper, we demonstrate that a 2-D study of Rossby waves is not adequate for calculating the total zonal Stokes volume transport in such waves.

The source of long baroclinic equatorial Rossby waves is located at the oceanic east coasts, where they can be generated by local wind-induced upwelling, or as a reflection of wave energy from an incoming Kelvin wave [*Busalacchi and O'Brien*, 1980]. The sink is at the west coasts, earlier referred to as the Rossby graveyard. However, in the equatorial region, nondissipated wave energy in the west can be reflected eastward as equatorial Kelvin waves [*McCreary*, 1983].

The Rossby wave signal emanating from the eastern boundary will in fact be composed of several meridional components, but since the first baroclinic, first meridional mode has the fastest group velocity, and suffers the least dissipation, it is the most likely candidate to be observed in the ocean basin away from the boundaries. In fact, the first baroclinic, first meridional equatorial Rossby wave mode has been verified from hydrographic measurements in the central equatorial Pacific [*Yu and McPhaden*, 1999], and by satellite observations [*Delcroix et al*., 1991; *Polito and Cornillon*, 1997; *Chelton et al*., 2003] (the early observation by *Harvey and Patzert* [1976] in the Pacific had probably a too short period (25 days) for qualifying as a long Rossby (1,1) mode).

The present analysis neglects the effect of background currents. Although this is a simplification anywhere, it is perhaps easier to defend at mid-latitudes than in the equatorial wave guide. Here zonal currents may distort Rossby wave eigenfunctions and dispersion relations [*Chelton et al*., 2003; *Durland et al*., 2011]. Furthermore, it is known that various meridional Rossby modes, in the presence of a background current, may interact to produce tropical instability waves [*Lyman et al*., 2005].

The rest of this paper is organized as follows: In section 2, we derive a general expression for the Stokes drift in baroclinic equatorial Rossby waves, and in section 3, we find the explicit Stokes drift for the first baroclinic, first meridional component. In section 4, we discuss the effect of stratification and calculate the eigenfunctions and the Stokes drift for a peaked Brunt-Väisälä frequency typical of the equatorial Pacific. This is compared to the result for a constant *N*. In section 5, we demonstrate that the total Stokes volume transport vanishes for arbitrary (stable) vertical density distributions. Section 6 discusses the possibility that some of the Rossby wave energy may be reflected at the western boundary and propagate eastward along the equator as a Kelvin wave. Finally, section 7 contains some concluding remarks. For didactic reasons, we give a brief review of baroclinic equatorially trapped waves in Appendix A.

2. The Stokes Drift

We consider the drift in freely propagating baroclinic equatorial Rossby waves and choose a Cartesian coordinate system (x, y, z) such that the origin is situated at the undisturbed surface. The *x* axis is directed eastward along the equator, the *y* axis points northward, and the *z* axis is directed vertically upward. The respective unit vectors are $(\mathbf{i}, \mathbf{j}, \mathbf{k})$. The reference system rotates about the vertical axis with angular velocity $f/2$, where *f* is the Coriolis parameter. We discuss motion close to the equator and apply the beta-plane approximation, i.e., $f=\beta y$, where $\beta=2.3\times10^{-11}$ m^{-1} s^{-1}. The description of motion is Eulerian, which means that all dependent variables are functions of x, y, z and time *t*. We take that the horizontal scale of the motion is so large compared to the depth that we can make the hydrostatic approximation in the vertical. Furthermore, we apply the Boussinesq approximation for the density ρ. In the analysis, $\mathbf{v}=(u, v, w)$ is the velocity vector and *p* denotes pressure. The effect of eddy diffusion on momentum and density is entirely omitted.

The waves result from small perturbations from a state of rest characterized by a horizontally uniform stable stratification $\rho_0(z)$ in the gravity field. Details of this problem can be found in text books like *LeBlond and Mysak* [1978], or *Gill* [1982]. We here give a very brief account. In principle, we expand our solutions in series after the wave steepness as a small parameter (but we retain our dimensional variables). The first-order (linear) equations for the conservation of momentum are, marking the linear periodic wave variables by a tilde:

$$\begin{aligned}\frac{\partial\tilde{u}}{\partial t}-\beta y\tilde{v}&=-\frac{\partial\tilde{P}}{\partial x},\\ \frac{\partial\tilde{v}}{\partial t}+\beta y\tilde{u}&=-\frac{\partial\tilde{P}}{\partial y},\\ 0&=-\frac{\partial\tilde{P}}{\partial z}-\frac{\tilde{\rho}}{\rho_r}g.\end{aligned}\tag{1}$$

Here $\tilde{P}=\tilde{p}/\rho_r$ is the pressure per unit reference density ρ_r and *g* the acceleration due to gravity. The conservation of density for an incompressible fluid implies

$$\frac{\partial\tilde{\rho}}{\partial t}+\tilde{w}\frac{d\rho_0}{dz}=0.\tag{2}$$

$$\frac{\partial \tilde{u}}{\partial x}+\frac{\partial \tilde{v}}{\partial y}+\frac{\partial \tilde{w}}{\partial z}=0. \tag{3}$$

Introducing the Brunt-Väisälä frequency $N(z)$ defined by

$$N^2=-\frac{g}{\rho_r}\frac{d\rho_0}{dz}, \tag{4}$$

it follows straight away from (1) and (2) that

$$\tilde{w}=-\frac{1}{N^2}\frac{\partial^2 \tilde{P}}{\partial z \partial t}. \tag{5}$$

The expression for the Stokes drift [*Stokes*, 1847] along the x axis, valid to second order in the wave steepness, can be written [*Longuet-Higgins*, 1953]:

$$\bar{u}_S=\overline{\left(\int \tilde{u}dt\right)\frac{\partial \tilde{u}}{\partial x}}+\overline{\left(\int \tilde{v}dt\right)\frac{\partial \tilde{u}}{\partial y}}+\overline{\left(\int \tilde{w}dt\right)\frac{\partial \tilde{u}}{\partial z}}, \tag{6}$$

where the over-bar denotes averaging over the wave cycle. The waves in our study are periodic and propagate in the x direction with constant phase speed c without changing shape. For such waves, we must have that

$$\frac{\partial}{\partial x}=-\frac{1}{c}\frac{\partial}{\partial t}. \tag{7}$$

Utilizing (1), (4), and (7), we arrive at

$$\bar{u}_S=\frac{1}{c}\overline{\tilde{u}^2}+\frac{1}{\beta y}\overline{\tilde{u}\frac{\partial \tilde{u}}{\partial y}}-\frac{1}{c\beta y}\overline{\tilde{P}\frac{\partial \tilde{u}}{\partial y}}-\frac{1}{N^2}\overline{\frac{\partial \tilde{P}}{\partial z}\frac{\partial \tilde{u}}{\partial z}}. \tag{8}$$

The Stokes drift in the meridional direction can be written

$$\bar{v}_S=\overline{\left(\int \tilde{u}dt\right)\frac{\partial \tilde{v}}{\partial x}}+\overline{\left(\int \tilde{v}dt\right)\frac{\partial \tilde{v}}{\partial y}}+\overline{\left(\int \tilde{w}dt\right)\frac{\partial \tilde{v}}{\partial z}}. \tag{9}$$

From the continuity equation (3), we note that for a progressive periodic wave in the x direction, $\tilde{u}$ is 90° out of phase with $\tilde{v}$, while $\tilde{v}$ and $\tilde{w}$ are in phase. Accordingly, all three terms in (9) are zero when averaged over the period (or the wavelength), so $\bar{v}_S=0$.

What now remains is to insert for the linear solutions of this problem into (8). The equatorial wave problem has been thoroughly reviewed and discussed in various textbooks. We give a short overview in Appendix A for didactic reasons.

3. Calculations for the First Baroclinic, First Meridional Mode

Long baroclinic equatorial Rossby waves are approximatively nondispersive and propagate westward along the equator. They consist of n baroclinic modes in the vertical, and each mode has m independent horizontal modes in the meridional direction. The calculation of the complete Stokes drift for baroclinic equatorial Rossby waves is a formidable task and will not be done here. However, some modes are more interesting in the sense that they have considerably more energy than others. This is particularly so for the first baroclinic mode in the vertical [*Lighthill*, 1969]. This mode is also relevant for the comparison with 1.5-layer models with a discontinuity in density between the upper (active) and lower (passive) layer [*McCreary*, 1976; *Busalacchi and O'Brien*, 1980]. Hence, we take $n=1$. For the modes in the meridional direction, $m=1$ is particular interesting, since this mode has been verified from hydrographic measurements in the equatorial Pacific [*Yu and McPhaden*, 1999], and by satellite observations [*Delcroix et al.*, 1991; *Polito and Cornillon*, 1997; *Chelton et al.*, 2003].

Inserting from (A12) and (A13) into (8), we can write the Stokes drift for this mode:

$$\bar{u}_{S11}=\frac{1}{2c_{11}}\left[\left(U_1^2-\frac{2U_1U'_1}{3Y}+\frac{2P_1U'_1}{3c_{11}Y}\right)Q_1^2-\frac{c_{11}(dQ_1/dz)^2}{N^2}P_1U_1\right]. \tag{10}$$

Here we have introduced the nondimensional length scale $Y=y/a_1$, where a_1 is given by (A10), and defined $' = d/dY$. For the zonal and meridional velocity components of the $n=1, m=1$ mode, we have from (A11) to (A13):

$$U_1=\frac{A_1}{3}(3-Y^2)\exp(-Y^2/4),$$
$$V_1=-\frac{8ka_1A_1}{9}Y\exp(-Y^2/4). \tag{11}$$

Here A_1 is the dimensional amplitude of the zonal velocity. This is a natural choice, since for long Rossby waves $ka_1 \ll 1$, and hence the zonal component is the most energetic one (and the easiest one to measure). Finally, we obtain for the pressure per unit density:

$$P_1=-\frac{c_1A_1}{3}(1+Y^2)\exp(-Y^2/4). \tag{12}$$

By inserting into (10), we finally obtain

$$\bar{u}_{S11}=u_0\left[-\frac{(27-46Y^2+7Y^4)}{27}Q_1^2+\frac{(3+2Y^2-Y^4)c_1^2}{27}\frac{(dQ_1/dz)^2}{N^2}\right]\exp(-Y^2/2). \tag{13}$$

Here we have defined the dimensional scaling factor u_0 by

$$u_0=\frac{3A_1^2}{2c_1}. \tag{14}$$

In Figure 1, we have plotted the nondimensional Stokes drift from (13) at the surface, where $Q_1=1,\ dQ_1/dZ=0$; see (A6). We note that the meridional distribution of the Stokes drift at the surface is similar to that that obtained in the upper layer from a reduced gravity model; see e.g., *Thompson and Kawase* [1993] and *Li et al.* [1996]. However, a two-layer model gives a false picture of the variation with depth of the Stokes drift in a stratified ocean, and it also fails to yield the correct total volume transport by the Rossby wave mode (1,1). These points will be discussed in detail in the next sections.

4. The Effect of Stratification

When N is constant, one readily finds [see e.g., *LeBlond and Mysak*, 1978], that $c_n=NH/(n\pi)$ and $Q_n=\cos(n\pi z/H)$ which can be inserted into (13). One easily obtains for mode (1,1) at the equator:

$$\bar{u}_{S11}(Y=0)=u_0\left[-\cos^2\left(\frac{\pi z}{H}\right)+\frac{1}{9}\sin^2\left(\frac{\pi z}{H}\right)\right]. \tag{15}$$

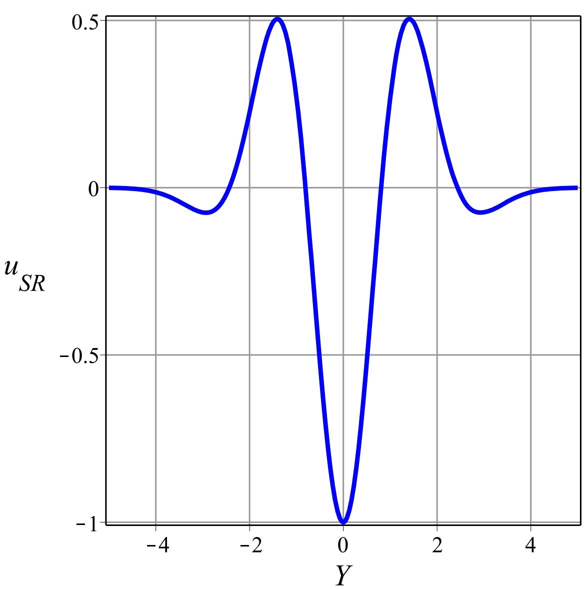

Figure 1. The meridional variation of the nondimensional drift $u_{SR}=\bar{u}_{S11}(z=0)/u_0$ at the surface from (13). Here $Y=y/a_1$.

We not that the Stokes drift for constant N at the equator that is symmetric about the mid-depth $z=-H/2$. Here the drift is eastward and small, while at the top and bottom there are much larger westward drift velocities.

However, a constant Brunt-Väisälä frequency is not a good approximation for the equatorial region. Here we find a pronounced thermocline; see e.g., *Colin et al.*, [1971] for the Pacific. In *Kessler* [2005], the depth at which N attains its maximum is estimated to about 250 m. In *Hayes et al.* [1985] and *Tang et al.* [1988], the peak of N is located much closer to the surface. We can approximate $N^2(z)$ in a very simple way by taking:

$$N^2(z)=N_d^2-\frac{z}{D}N_0^2\exp\left(\frac{z}{D}\right). \tag{16}$$

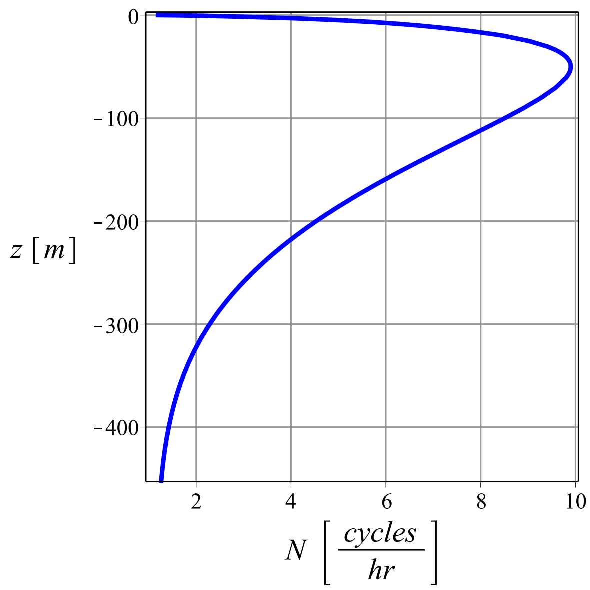

Figure 2. The Brunt-Väisälä frequency N from (16) as function of depth.

By assuming $N_0^2=8\times10^{-4}$ s^{-2}, $N_d^2=4\times10^{-6}$ s^{-2}, and $D=50$ m, we have plotted $N(z)$ in Figure 2. The distribution of N with depth in Figure 2 reproduces quite well a smoothed plot of $N(z)$ in *Tang et al.* [1988, their Figure 7], based on observations at 0°W, 110°W in the Pacific. Applying (16), a simple shooting procedure yields $c_1=(gh_1)^{1/2}=2.36\,\mathrm{m\,s^{-1}}$ from (A4)–(A6) for the first baroclinic mode in the vertical. This value fits well with that given by *Tang et al*. The eigenfunction Q_1 is proportional to the first mode horizontal velocity; see e.g., (A2). From (5), we note that the first mode vertical velocity is proportional to $W_1=(N_0^2/N^2)dQ_1/dZ$. The vertical variation of these modes is plotted in Figure 3. We note from the figure that these vertical distribution as fairly similar to those plotted by *Tang et al*. [1988, their Figure 8]. In particular, the zero crossing of the first mode horizontal velocity occurs at a depth of about 1500 m, where we also find the maximum of the first mode vertical velocity. It should be noted that these distributions, as well as the eigenvalue c_1, are extremely sensitive to the variation of N with depth, and we must look on (16) only as a simplified and qualitative representation of real ocean data. For the first baroclinic, first meridional equatorial Rossby wave mode, we now obtain from (A9):

$$c_{11}=-\frac{c_1}{3}=-0.8\,\mathrm{m\,s^{-1}}. \tag{17}$$

This fits well with earlier estimates by *Yu and McPhaden* [1999] of -0.9 and -0.81 m s^{-1} for the (1,1) mode.

With the modal structure displayed in Figure 3, we may now proceed to calculate the vertical variation of the Stokes drift. In Figure 4, we have plotted the nondimensional Stokes drift $u_{SR}=\bar{u}_{S11}(y=0)/u_0$ from (13) for the Rossby wave mode (1,1) at the equator as function of nondimensional depth. For comparison, we have also displayed the result (15) for constant N. We note that the Stokes drift at the equator is basically directed westward (in the same direction as the waves), with a small eastward drift near middepth, while for constant Brunt-Väisälä frequency there is a slightly larger eastward drift at middepth. For a variable N, the largest drift velocities are confined to the upper 1000 m. Although there is no symmetry about middepth anymore, as for constant N, there is still a nonnegligible westward Stokes drift in the bottom layer.

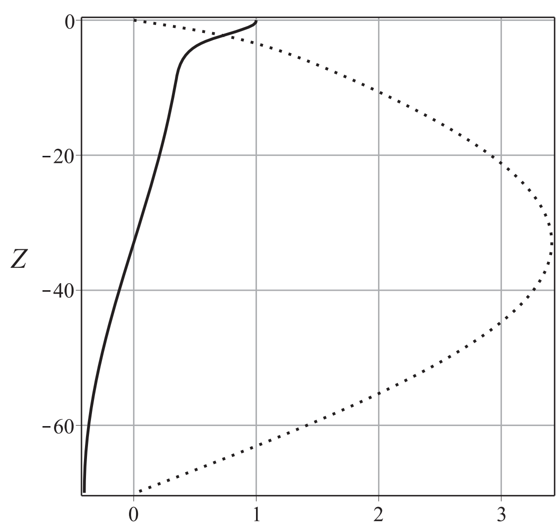

Figure 3. Vertical variation of the normalized eigenfunction Q_1 for the first mode horizontal velocity (solid line) and the first mode vertical velocity W_1 (dotted line) when N is given by (16). Here $Z=z/D$.

5. The Stokes Volume Transport

Applying the rigid lid approximation at the surface, it has been demonstrated that the Stokes volume transport in baroclinic equatorial Kelvin waves is zero [*Weber et al*., 2014]. This is due to the fact that the direction of the Stokes drift alternates in the vertical direction, independent of the y coordinate. For the Rossby mode (1,1), the situation is more complicated since the Stokes drift changes sign in the vertical as well as in meridional direction. By integrating in the y direction, using the well-known results

$$\left.\begin{aligned}&\int_{-\infty}^{\infty}\exp(-\xi^2/2)d\xi=(2\pi)^{1/2}\\&\int_{-\infty}^{\infty}\xi^2\exp(-\xi^2/2)d\xi=(2\pi)^{1/2}\\&\int_{-\infty}^{\infty}\xi^4\exp(-\xi^2/2)d\xi=3(2\pi)^{1/2}\end{aligned}\right\}, \quad (18)$$

we find for the various terms in (13) that

$$I_1=-\int_{-\infty}^{\infty}\exp(-Y^2/2)(27-46Y^2+7Y^4)dY=-2(2\pi)^{1/2}, \quad (19)$$

$$I_2=\int_{-\infty}^{\infty}\exp(-Y^2/2)(3+2Y^2-Y^4)dY=2(2\pi)^{1/2}=-I_1. \quad (20)$$

Hence, the zonal transport $F_M(z)$ per unit depth becomes

$$F_M(z)=\int_{-\infty}^{\infty}\bar{u}_{S11}dy=-F_0\left(Q_1^2-\frac{c_1^2}{N^2}\left(\frac{dQ_1}{dz}\right)^2\right), \quad (21)$$

where

$$F_0=\frac{2(2\pi)^{1/2}a_1u_0}{27}. \quad (22)$$

In Figure 5, we have plotted $F=F_M/F_0$ versus nondimensional depth. We note that the 1.5-layer model yields a net westward zonal flux. Actually, since the linear wave fluxes are equal and oppositely directed in each layer in this case, and the Stokes drift scales as the square of the wave velocity divided by the (negative) phase speed, the Stokes drift is westward in each layer. Moreover, if the thickness of the deep lower layer is H_2, the ratio between the Stokes fluxes in the upper layer and the lower layer is of the order $H_1/H_2 \ll 1$. The small westward Stokes flux in the lower layer for the 1.5-layer model has been neglected in Figure 5.

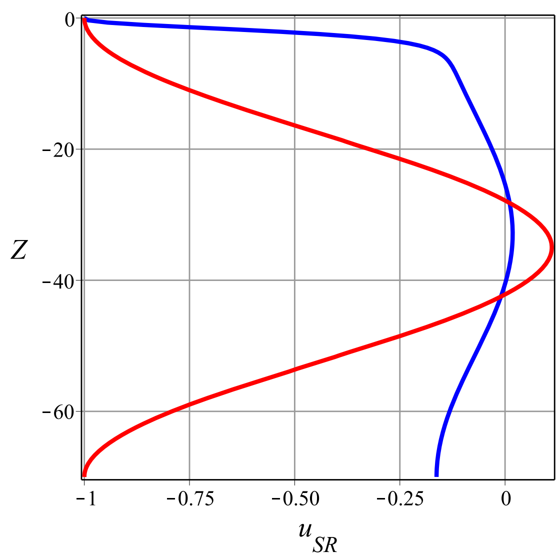

Figure 4. The nondimensional Stokes drift $u_{SR}=\bar{u}_{S11}(y=0)/u_0$ at the equator from (13) as function of the nondimensional depth $Z=z/D$, when $N(z)$ is given by (16) (blue line), and when N is constant (red line).

Our model with a continuous equatorial pycnocline yields a completely different picture (Figure 5, blue line). We now have a considerable eastward flux near middepth that compensates exactly the westward fluxes in the surface and bottom layers. We can in fact prove that vanishing total zonal Stokes transport in the Rossby mode (1,1) is not limited to the pycnocline model (16) used here but is generally true for any (stable) continuous stratification. This is seen by integrating (21) and using (A4). We then obtain for the total Stokes zonal volume transport:

$$F_{tot}=\int_{-H}^{0}F_M(z)dz=F_0\left[\frac{c_1^2}{N^2}Q_1\frac{dQ_1}{dz}\right]_{z=-H}^{z=0}. \quad (23)$$

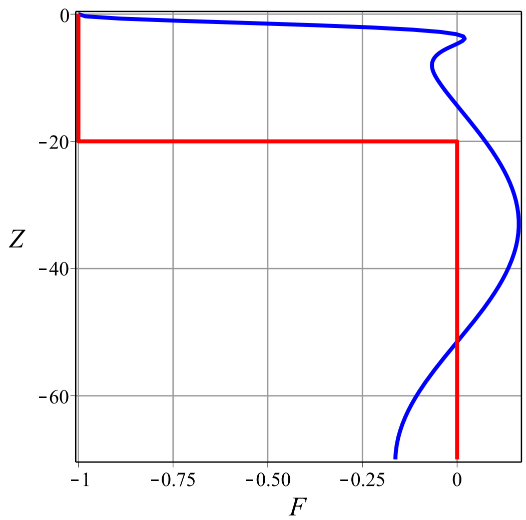

Figure 5. The nondimensional zonal transport $F=F_M/F_0$ per unit depth from (21) as function of $Z=z/D$ when $N(z)$ is given by (16) (blue line). The red line depicts the result from applying a 1.5-layer reduced gravity model with upper layer depth $H_1=1000$ m; see e.g., *Thompson and Kawase* [1993] and *Li et al.* [1996].

Now by applying (A5) and (A6), we find that

$$F_{tot}=\int_{-H}^{0}\int_{-\infty}^{\infty}\bar{u}_{S11}dydz=0, \tag{24}$$

which proves our point.

6. Reflection of Wave Energy

Unlike the western oceanic boundaries in general, the western region close to the equator need not be a complete Rossby graveyard. As pointed out by *McCreary* [1983], some of the Rossby wave energy could be reflected as an eastward propagating equatorial Kelvin wave. In this process, the wave period must be conserved. A typical period T for a long Rossby wave in the eastern equatorial Pacific could be about 35 days [*Farrar and Durland*, 2012]. In the present problem, the phase speed for the first baroclinic mode, when the Brunt-Väisälä is given by (16), is found to be $c_1=2.36$ m s^{-1}. With $T=35$ days, the equivalent wavelength for the first Kelvin mode becomes about 7140 km, which appears to be too long for an equatorial wave in the ocean. It is then more realistic that the reflected signal could be represented by the second baroclinic Kelvin mode (or even higher modes). Again, with N^2 given by (16), a shooting procedure yields $c_2=1.61$ m s^{-1} for the second mode. Incidentally, this value fits well with that reported by *Kessler and McPhaden* [1995] from the Hawaii-Tahiti Shuttle Experiment ($c_2=1.74$ m s^{-1}). This corresponds to a second mode wavelength of 4876 km, which could occur in the equatorial Pacific. To make things simple, we assume that the reflected wave (the second mode Kelvin wave) has a horizontal amplitude given by RA_1, where R is a reflection coefficient ($R\in[0,1]$), and A_1 is the amplitude of the zonal Rossby (1,1) component; see (11). From *Weber et al.* [2014], it is easy to show that the Stokes drift for the second baroclinic Kelvin component in this case can be written as

$$\bar{u}_{SK2}=\frac{c_1R^2u_0}{3c_2}\left[\left(\frac{d\phi_2}{dZ}\right)^2-\frac{N^2D^2}{c_2^2}\phi_2^2\right]\exp\left(-y^2/(2a_2^2)\right), \tag{25}$$

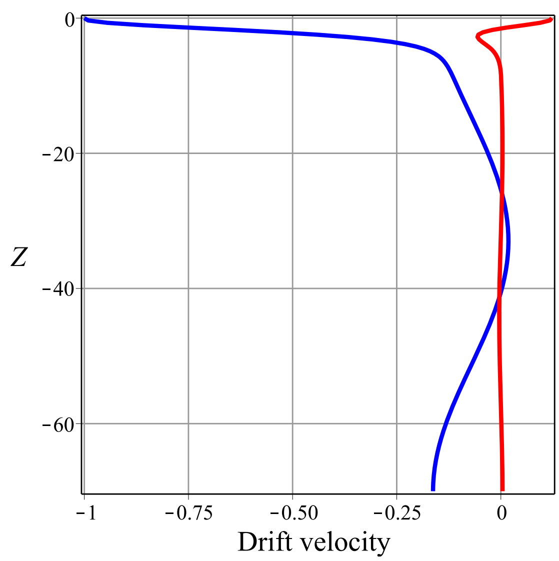

Figure 6. The nondimensional Stokes drift u_{SR} in the Rossby mode (1,1) (blue line), and u_{SK} in the Kelvin wave mode 2 (red line) at the equator as function of nondimensional depth when $R=1/2$.

where $a_2=(c_2/(2\beta))^{1/2}$ is the Rossby radius for the second mode and $\phi_n(z)$ is the eigenfunctions for the vertical isopycnal displacement. Furthermore, the scaling factor u_0 is given by (14).

Due to irregular bottom and coastal conditions near the western boundary, it seems unlikely that the entire wave amplitude should be preserved. Even in that case, with $R=1$, we find from (25) that the total westward drift at the surface is reduced by a about factor ½. Since in this case, the Rossby radii are $a_1=226$ km and $a_2=187$ km, this reduction at the surface occurs only within a distance a_1 at both sides of the equator; see Figure 1. Beneath the surface layer, the reduction is negligible.

A more reasonable value for the reflection coefficient could be $R=1/2$. In Figure 6, we have depicted $u_{SR}=\bar{u}_{S11}/u_0$ from (13) and $u_{SK}=\bar{u}_{SK2}/u_0$ from (25) when $y=0$. The total drift along the equator is just the sum of the two curves in Figure 6. We note that the contribution from the Kelvin wave only gives a small correction near the surface.

7. Concluding Remarks

The present investigation of the Stokes drift in the first baroclinic, first meridional Rossby wave mode with a realistic Brunt-Väisälä frequency shows that drift at the equator is basically westward. The largest westward drift occurs in the surface layer, with a maximum at the surface and extending down to about 1000 m. The drift velocity is slightly eastward at middepth, while in the bottom layer there is a nonnegligible westward Stokes drift. At the bottom, it is nearly 1/4 of the surface value. At any depth level, the mean drift varies in the meridional direction. At the surface the drift is westward along the equator in a meridional section of about $2a_1$. In the region between $y \sim a_1$ and $y \sim 2a_1$ on both sides of the equator, the drift is eastward, which is similar to the findings of *Thompson and Kawase* [1993] and *Li et al.* [1996] from reduced gravity models. But such modes produce a net westward zonal Stokes transport. The inclusion of a continuous stratification in the present paper corrects this result and shows that the total zonal volume transport must be zero. We think that this is a novel and important result. It means that if the west coast acts as a sink for this wave component, no compensating eastward Eulerian mean currents (e.g., equatorial Rossby rip currents) are needed for the zonal mass balance. Even if some of the Rossby wave energy should be reflected at the west coast as an eastward propagating internal equatorial Kelvin wave [see e.g., *McCreary*, 1983], this would not matter, since also the total Stokes mass transport in internal Kelvin waves is zero [*Weber et al.*, 2014]. For the vertical variation of the total drift velocity, the presence of a 2. mode Kelvin wave as a result of reflection only yields a minor correction near the surface, as seen from Figure 6.

Appendix A: Equatorial Baroclinic Waves

Assuming separation of variables, we can write for a Fourier component of an equatorially trapped internal wave:

$$\tilde{P} = Q_n(z)P_n(y)\sin(kx-\omega t), \quad \text{(A1)}$$

$$\tilde{u} = Q_n(z)U_n(y)\sin(kx-\omega t), \quad \text{(A2)}$$

$$\tilde{v} = Q_n(z)V_n(y)\cos(kx-\omega t), \quad \text{(A3)}$$

where ω is the wave frequency and k is the wave number. We here take that Q_n is dimensionless. Utilizing the Boussinesq approximation, one finds [*LeBlond and Mysak*, 1978]

$$\frac{d}{dz}\left(\frac{dQ_n/dz}{N^2}\right) + \frac{1}{gh_n}Q_n = 0. \quad \text{(A4)}$$

Here $c_n = (gh_n)^{1/2}$ is the eigenvalue in this problem, where h_n is the equivalent depth [*Lighthill*, 1969]. The boundary conditions are

$$\begin{aligned} \frac{dQ_n}{dz} + \frac{N^2}{g}Q_n &= 0, \quad z=0, \\ \frac{dQ_n}{dz} &= 0, \quad z=-H. \end{aligned} \quad \text{(A5)}$$

In the present problem, we have that $|N^2H/g| \ll 1$, so the boundary condition at the surface can be written approximately as [*LeBlond and Mysak*, 1978]

$$\frac{dQ_n}{dz} = 0, \quad z=0. \quad \text{(A6)}$$

The meridional variation $V_n(y)$ is given by [*LeBlond and Mysak*, 1978, their equation (21.3)]

$$\frac{d^2V_n}{dy^2} + \left(\frac{\omega^2}{c_n^2} - k^2 - \frac{\beta k}{\omega} - \frac{\beta^2 y^2}{c_n^2}\right)V_n = 0, \quad \text{(A7)}$$

subject to

$$V_n \to 0, \quad y \to \pm\infty. \tag{A8}$$

Trapping at the equator of very long, low-frequency Rossby waves requires from (A7) that

$$c_{nm} = \frac{\omega}{k} = -\frac{c_n}{2m+1}, \; m=1,2,3,\ldots \tag{A9}$$

Within the equatorial band, the appropriate baroclinic Rossby radius for this problem is [*Gill*, 1982]

$$a_n = \frac{c_n^{1/2}}{(2\beta)^{1/2}}. \tag{A10}$$

The trapped solution of (A7) can be written in terms of Hermite functions, which are Hermite polynomials multiplied by a Gaussian. The simplest expression is obtained in terms of the nondimensional coordinate $\sigma = y/(\sqrt{2}a_n)$. The solution then becomes

$$V_n(\sigma) = \sum_m C_{nm}(-1)^m \exp\left(\frac{\sigma^2}{2}\right)\frac{d^m}{d\sigma^m}\left(\exp(-\sigma^2)\right). \tag{A11}$$

Finally, in order to calculate the various terms in the Stokes drift (8), we must express $P_n(y)$ and $U_n(y)$ in (A1) and (A2) as functions of $V_n(y)$. We find from (1) to (3) after some algebra that

$$P_n(y) = \frac{(2m+1)^2}{4m(m+1)k}\left[\beta y V_n - c_{nm}\frac{dV_n}{dy}\right], \tag{A12}$$

and

$$U_n(y) = -\frac{1}{kc_{nm}}[\beta y V_n - kP_n]. \tag{A13}$$

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