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Simulation of shear wave elastography imaging using the toolbox “k-Wave”

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Elastography is used to map the local elasticity of tissue. It can detect areas inside the body with a different elasticity from that of surrounding tissue indicative of pathologies like tumours. Shear wave elastography imaging produces an elasticity map by analysing the propagation of shear waves. One source of shear wave is the acoustic radiation force produced by ultrasound. The prediction of the acoustic radiation force and of the shape and amplitude of the ensuing shear displacement is crucial. In this study we present simulations of the radiation force produced by an ultrasound transducer and of the shear displacement it produces using the software package “k-Wave”. Results from simulations in a homogeneous and isotropic medium are compared against analytical solutions and results from a finite element modelling software. The obtained shear displacements from k-Wave are very similar to the analytical solution with a root mean square error around the focal zone below 9% and 21% for short and large propagation times, respectively. K-Wave appears to be an accurate and efficient tool for simulation of acoustic radiation force and shear wave propagation. It combines the simplicity of finite time difference methods with the flexibility to simulate in any heterogeneous medium.
1 Introduction

Elastography is a technique aimed at imaging the elastic properties of a medium [1, 2]. In the context of medical imaging and diagnosis, it is used to detect tissues with varying elasticity. For example, it has been shown that cancerous tissue is often “stiffer” than healthy tissue [3, 4]. An elastography image of a liver, for instance, can therefore reveal the presence of a lump stiffer than the surrounding tissue. This lump could indicate a tumour that other imaging techniques or manual palpation could not detect.

Many techniques exist in elastography [1] but this article will deal with shear wave elastography imaging (SWEI) [5] where the shear wave is generated by the acoustic radiation force (ARF). It consists in applying a remote force deep into the tissue and imaging the resulting shear movement propagating in the medium. The propagation speed of the shear wave can then be recovered locally and directly linked to the local elasticity of the imaged tissue.

The remote force is created deep into the tissue by using the radiation force from a focused beam of ultrasound.

In that context, numerical simulation is an important tool to predict the ARF created by an ultrasound transducer and the shape and amplitude of the ensuing shear wave. Such simulations often involve finite element modelling. We looked at the possibility of using a simulation software with a pseudospectral time domain method and running under Matlab (Mathworks, Natick, MA): k-Wave [6]. This software package allows simulation of the propagation of compressional waves created by an ultrasound transducer as well as the propagation of shear waves.

We will first present a theoretical background behind the formulation of the acoustic radiation force and how it can be used in k-Wave for the simulation of shear waves. The simulation results for the acoustic radiation force are presented. We then compare the results for the shear wave displacement obtained by k-Wave, by a finite element modelling commercial software, and by an analytical solution in the case of propagation in a homogeneous and isotropic elastic medium.

2 Theoretical background

2.1 Formulations of the ARF

In modelling of the ARF, the assumption of an attenuated plane wave is often used [7, 8, 9]. Attenuation is also often considered to be mainly due to absorption and attenuation due to scattering is neglected.

Under these conditions, the ARF is expressed as

\[ F = \frac{2\alpha I}{c_0}, \]  

where \( \alpha \) is the attenuation coefficient in Np/m, \( I \) is the temporal average acoustic intensity in W/m\(^2\) at the spatial location, and \( c_0 \) is the sound speed in m/s.

These simplifications do not take into account the curvature of the wave front outside the focal zone of a focused transducer. In addition, the direction of the force is not given by this formula. The force is often taken parallel to the main propagation axis around the focus and along the Poynting vector outside the focal zone [9].

A more generic formulation of the ARF uses a second-order approximation and gives an expression for
the force applied to a chosen volume enclosed within a closed surface $S$ \[10, 11, 12\]

$$
\mathbf{F} = \int\int_{S} \left[ -\frac{1}{2\rho_0 c_0^2} \mathbf{n} + \left( \frac{1}{2}\rho_0 - |v_1|^2 \right) \mathbf{n} - \rho_0 \mathbf{v}_1 \mathbf{v}_1^T \cdot \mathbf{n} \right] dS,
$$
\[2\]

where $\rho$, $p$, $v$ are the fluid density, pressure, and particle velocity, respectively, $\mathbf{n}$ is the vector normal to $S$ and pointing outwards, and $<>$ designates the time average over a wave period.

In the above expression, a perturbation analysis was used and

$$
\rho = \rho_0 + \rho_1 + \rho_2 + \cdots \quad (3)
$$

$$
\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2 + \cdots \quad (4)
$$

$$
p = p_0 + p_1 + p_2 + \cdots \quad (5)
$$

where the subscripts, 0, 1, and 2 refer to undisturbed values, the first-order, and the second-order small quantities, respectively \[10, 13\].

The expression given in Eq. (2) is valid to second order but only involves first order quantities. It is obtained for an inviscid fluid. An equivalent expression for a viscous fluid was obtained by Danilov and Mironov \[14\] but its computation is more involved.

### 2.2 Shear wave propagation

To simulate the propagation of elastic waves, k-Wave solves two first-order differential equations in parallel \[15, 16\]:

$$
\frac{\partial \sigma_{ij}}{\partial t} = \lambda \delta_{ij} \frac{\partial v_k}{\partial x_k} + \mu \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad (6)
$$

$$
\rho \frac{\partial v_i}{\partial t} = \frac{\partial \sigma_{ik}}{\partial x_k} + F_i \quad (7)
$$

where $\sigma_{ij}$ designates the components of the stress tensor, $\lambda$ and $\mu$ are the Lamé parameters, $v_i$ are the components of the particle velocity vector, and $F_i$ is the component of the force applied to the unit volume.

In an orthogonal grid, as explained in Fig. 1, Eq. (2) can be re-written as

$$
F_i = - \frac{\partial \sigma_{ik}^{ARF}}{\partial x_k},
$$
\[8\]

where

$$
\sigma_{ij}^{ARF} = \frac{1}{2\rho_0 c_0^2} \mathbf{n} + \frac{1}{2}\rho_0 - |v_1|^2 \mathbf{n} - \rho_0 \mathbf{v}_1 \mathbf{v}_1^T \cdot \mathbf{n} > dS.
$$
\[9\]

The ARF can therefore be input as a volume force ($F_i$) or as a stress tensor ($\sigma_{ij}^{ARF}$).

In this study, we will only present results computed using the second-order approximation formulation for the ARF (Eqs. (8) and (9)).

### 2.3 Simulation method in k-Wave

Simulating the propagation of shear waves in k-Wave requires two stages. In the first stage, we simulate the propagation of the compressional wave emitted by the transducer using the function `kspaceFirstOrder3D()`. From the obtained pressure and velocity fields one computes the spatial distribution of the ARF.
Figure 1: Volume element in an orthogonal grid. The solid arrows show the outgoing normal vector $\mathbf{n}$ on each face. The stress components shown by the dashed arrows are drawn on two faces. The resulting force on the volume element can be expressed using the spatial derivatives of these stress components (Eq. (8)).

This is then used in a second stage as an input for computing the propagation of shear waves in the medium using the function `pstdElastic3D()`.

When computing the ARF we take a temporal average over a period of the transmitted pulse to compute $I$ when using Eq. (1), or $\sigma_{ij}^{ARF}$ when using Eqs. (8) and (9). We obtain three time varying components of the force on each point of a three-dimensional spatial grid.

However, only the force in the zero-elevation plane is used as an input for the simulation of shear wave propagation. This plane is where the force is maximum and the plane where shear wave propagation will be monitored for establishing the elasticity map in practice. Assuming symmetry on each side of this plane the out-of-plane component of the force can be neglected.

The two time-varying in-plane components of the ARF in the zero-elevation plane are then averaged over the propagation duration of the pulse (typically around 30 $\mu$s for 5 cm propagation depth) and applied as a constant input during the time corresponding to the chosen duration of the excitation pulse.

Since the volume force is applied only in one plane it can be expressed as a function of the spatial variables $x$ and $y$ multiplied by a delta function in the $z$ direction.

Note that in k-Wave there is no formal way to input a volume force. It only takes stress components or particle velocities as inputs. However, as explained in the k-Wave user manual [17] the input velocity is automatically scaled to a force per mass (m/s$^2$) by multiplying it with the factor $2c_0/\Delta x_i$ where $\Delta x_i$ is the spatial step in the chosen direction. To get a volume force $F_i$ as input, we can therefore use an input velocity equal to $\frac{F_i}{\rho_0 \Delta x_i}$.

Finally, this input should be divided by the spatial step $dz$ to ensure proper scaling of the delta function in the $z$ direction.
3 Numerical simulations

3.1 Setup for the ARF computation

The simulated transducer is a P4-2V phased array (Verasonics Inc., Redmond, WA). It is 20-mm wide and 14-mm high and is made of 64 elements. It has a center frequency around 3 MHz, a fixed focus in the elevation direction at 60 mm, and an adjustable electronic focus in the azimuth direction set to 30 mm in our simulations. The propagation medium is isotropic and homogeneous with a propagation speed, density, and attenuation coefficient of 1540 m/s, 980 kg/m$^3$, and 0.5 dB/MHz/cm, respectively.

The three-dimensional grid is made of 513, 133, and 93 points in the $x$ (depth), $y$ (azimuth), and $z$ (elevation) direction, respectively. The spatial step size is 0.16 mm in all three directions which gives the following dimensions for the grid: 82.1 mm (depth), by 21.3 mm (azimuth), by 14.7 mm (elevation).

The transducer is made of 128 points in the $y$ direction and 44 points in the $z$ direction. It is placed at $x = 0$ mm and centered in the corresponding $y, z$ plane. In addition, a perfectly matched layer (PML) is set around the computational domain. It is 20-point thick (3.2 mm) in the $x$ and $y$ direction, and 10-point thick (1.6 mm) in the $z$ direction. This PML ensures that no signal is reflected from the domain boundaries.

The signal transmitted by the transducer is a 5-cycle sine wave of frequency 3 MHz weighted by a Gaussian window. The simulation propagates the signal for 60 $\mu$s which allows it to travel through and beyond the specified domain.

Both the pressure $p$ and the non-staggered particle velocity $v$ are recorded over the whole domain and used to compute the ARF using Eq. (8) and (9).

It should be noted that k-Wave solves the pressure and particle velocity on a staggered grid. It is therefore crucial to use the non-staggered particle velocity (also available from k-Wave) when computing $\sigma_{ij}^{ARF}$ to make sure the pressure and particle velocity are evaluated at the same points.

3.2 Setup for the shear wave propagation

In that case, the domain dimensions in the $x$, $y$ and $z$ directions are 128, 128, and 32 points, respectively. The PML dimensions in the $x$, $y$, and $z$ directions are 10, 10, and 5 points, respectively. The spatial step size is 0.2 mm in all directions which gives the following dimensions for the grid: 25.6 mm ($x$), by 25.6 mm ($y$), by 6.4 mm ($z$).

This domain is centered around the focal point of the transducer. This means that in the coordinate system attached to the transducer the domain for simulation of elastic wave propagation extends from $30 \pm 12.8$ mm, $0 \pm 12.8$ mm, and $0 \pm 3.2$ mm in the $x$, $y$, and $z$ direction, respectively.

The medium is isotropic, homogeneous, and lossless. Its density is 980 kg/m$^3$ and the shear and compressional waves speeds are 1.7 m/s and 20 m/s. Note that such a compressional wave speed is not representative of relevant materials but it was taken artificially low to allow for larger time steps and reasonable computing time in the resolution of k-Wave. Using this speed, compressional waves have travelled 1 cm after 0.5 ms and are almost beyond the limits of the domain. The displacement they generate will therefore not interfere with the displacement created by the shear wave that travels much slower.

The ARF was applied first as a constant volume force for 200 $\mu$s from $t = 0$ and displacements were simulated for 5 ms.

Since k-Wave computes the particle velocity and not the displacement directly, the particle velocity has
to be integrated in time to get the final displacement. This was simply done using the recursion formula

\[ u(n) = u(n - 2) + 2\delta t \cdot v(n - 1) \]  

(10)

where \( u(n) \) and \( v(n) \) are the displacement and particle velocity at the \( n^{th} \) time step, respectively, and \( \delta t \) is the time step size.

### 3.3 Shear wave propagation using stress as input

As explained in the previous section, the ARF can be input as a volume force (Eq. (8)) or as an input stress (Eq. (9)). An alternative method to simulate the propagation of shear wave using k-Wave is therefore to use \(-\sigma_{ij}^{ARF}\) as an input stress.

In the zero elevation plane the force is contained in the plane due to symmetry (\( F_z = 0 \)) and only \( F_x \) and \( F_y \) need to be estimated. However, as shown in Eq. (8), the components of \( \sigma_{ij}^{ARF} \) need to be estimated in the entire space since spatial gradients in all three directions contribute to \( F_x \) and \( F_y \).

To limit the memory requirement and the computation time, we simulated the shear wave propagation with this method in two dimensions only (\( x \) and \( y \)) using the function \texttt{pstdElastic2D()}\(^4\). In this case, only three components of the symmetric tensor have to be computed: \( \sigma_{xx}^{ARF}, \sigma_{xy}^{ARF}, \) and \( \sigma_{yy}^{ARF} \).

When using stress as an input, it is the time derivative of \(-\sigma_{ij}^{ARF}\) that should actually be used as input to the function \texttt{pstdElastic2D()}\(^4\). Alternatively, one can use \(-\sigma_{ij}^{ARF}\) as an input to \texttt{pstdElastic2D()}\(^4\) and take the time derivative of the results. The time derivative of the displacement being the particle velocity k-Wave gives directly the displacement and the integration explained previously becomes unnecessary. This is the alternative we chose in our simulations.

The simulated domain is made of 200 and 128 points in the \( x \) and \( y \) direction, respectively. The spatial step is 0.2 mm which gives a grid of dimension 40 mm long by 25.6 mm wide and the PML thickness is 20 points (4 mm) in each direction.

### 3.4 Benchmark solutions

To benchmark the results for propagation of elastic waves from k-Wave, we compare them to an analytical solution using the Green’s functions [18], and to the finite element model (FEM) simulation software COMSOL Multiphysics\textsuperscript{\textregistered} v. 5.2 (COMSOL AB, Stockholm, Sweden). In both cases, the volume force computed from k-Wave is used as an input to the model. The medium parameters are identical to those used in the k-Wave simulations except for the compressional wave propagation speed that was set to 1500 m/s when solving using the analytical solution since it does not impact the computation requirements.

These comparisons are only done when computing the shear displacement using an input volume force not when we use an input stress.

### 3.5 Results

### 3.6 ARF computation

The pressure field obtained as well as the \( x \) and \( y \) components of the ARF are shown in Fig. 2.

Some “noise” appears on the edges of the domain both for \( F_x \) and \( F_y \). This comes from some ripple that appear when estimating the spatial gradient of \( \sigma_{ij}^{ARF} \) to compute the volume force components \( F_i \). Indeed, these gradients are evaluated via the frequency domain by applying a phase shift on the spatial Fourier

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3.7 Shear wave propagation

The displacement fields at three instants: 1, 2.5, and 4 ms obtained with k-Wave are shown in Fig. 3 together with those computed using the analytical solution and the FEM simulation.

Fig. 4 shows the displacement at the same three instants at three depths around focus: 25 mm, 30 mm, and 35 mm.

To quantify the comparison of the results, we compute the root mean square error (RMSE) which is the root mean square of the difference between the displacements shown in Fig. 4 obtained by the analytical solution and one of the two other solutions. This error is then normalized by the maximum displacement of the analytical solution at each time instants and depth.

The RMSE for the k-Wave solution is below 10% at all depths except at $t = 4$ ms and 25 mm depth where it is 15.3%. It is below 12% for the FEM solution except at $t = 4$ ms and 30 mm and 35 mm depth and where it is 16.5% and 19.3%, respectively.

3.8 Shear wave propagation with an input stress

The displacements resulting from the computation using stress as an input or the corresponding volume force are shown in Fig. 5. In our example, the medium around 25 mm depth is pulled towards the transducer while beyond 30 mm, it is pushed away from the transducer.

Fig. 6 shows the displacements at the same three instants at three depths: 25 mm, 30 mm, and 35 mm.
Figure 3: Displacement field computed at $t = 1$ ms (left column), $t = 2.5$ ms (middle column), and $t = 4$ ms (right column) using k-Wave (top row), the analytical solution (middle row), and FEM simulation (bottom row). The displacements are normalized by the maximum displacement at $t = 1$ ms.

The RMSE between the computed displacements is always below 11.3% apart from at $x = 25$ mm where it is 13.8%, 13.7%, and 21.1% at $t = 1$ ms, $t = 2.5$ ms, and $t = 4$ ms, respectively.

4 Discussions

Our simulations show that from the pressure and particle velocity fields created by a focused transducer, it is possible to compute the acoustic radiation force using the second-order approximation formulation (Eq. (2)). First the components of the tensor $\sigma^{ARF}$ are computed from the pressure and particle velocity and
then the force components are computed from spatial derivatives of the tensor elements.

Since the acoustic radiation force is a second-order quantity, it is crucial to carefully compute the spatial derivatives. Also, it is essential to use the values for the particle velocity on the non-staggered grid.

When compared to an analytical solution using the Green’s function, the displacement computed using k-Wave is very close with an RMSE below 11% except in one case. It actually performs better than the simulation run using COMSOL Multiphysics®. The FEM solution also seems to contain some oscillations visible at 30 mm depth for $t = 2.5$ ms and $t = 4.0$ ms which might be due to some other limitations of the method.

Two alternatives were tested to input the ARF into shear wave simulation using k-Wave: one using a volume force, the other a stress tensor. We showed using a simulation in two dimensions that the resulting displacements were very similar with an RMSE always below 12 % except at $t = 4$ ms and $x = 25$ mm where it was 15.3%.

Using the stress tensor no spatial derivatives need to be calculated to get the components of the ARF. The displacement is also directly obtained from k-Wave’s results and no integration of the particle velocity is necessary.

The drawback of this method is the need to input the stress tensor values in the whole space even if the ARF is contained in one plane (due to symmetry for example). This can render this method memory intensive.

With the chosen parameters, a simulation of shear wave propagation in two dimensions takes about 1.3 minutes on a PC equipped with an Intel i7 core (Intel, Santa Clara, CA) running at 2.6 GHz and 8 gigabytes of memory. For a simulation in three dimensions it takes about 2 hours and 3 minutes. All of the k-Wave code used was exclusively written for Matlab (Mathworks, Natick, MA), none of it was compiled.
Figure 5: Displacement field computed using stress as input (top row) and the corresponding volume force (bottom row) at \( t = 1 \) ms (left column), \( t = 2.5 \) ms (middle column), and \( t = 4 \) ms (right column). The displacements are normalized by the maximum displacement at \( t = 1 \) ms.

k-Wave offers therefore a good solution for simulating shear wave propagation used in SWEI. It combines the possibility to simulate heterogeneous medium and the simplicity of a finite time difference method.

5 Conclusion

k-Wave seems to be a good tool to both estimate the ARF created by a focused transducer and the ensuing shear wave displacement. The results show a very good agreement with an analytical solution in the case of propagation in an isotropic homogeneous medium.

It offers the flexibility to model propagation in heterogeneous media, which is crucial in applications such as SWEI, and the simplicity of a finite time differences method.

6 Acknowledgements

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Figure 6: Displacements at $t = 1$ ms (left column), $t = 2.5$ ms (middle column), and $t = 4$ ms (right column) at $x = 25$ mm (top row), $x = 30$ mm (middle row), and $x = 35$ mm (bottom row) using an input stress (solid), and an input volume force (dashed). All displacements have been normalized by the maximum absolute displacement at $t = 1$ ms.

References


