The Role of Private Finance in Public-Private Partnerships

Torje Hegna

Master’s degree programme in Economics
Department of Economics
University of Oslo
May 2017
Preface

I would first and foremost like to thank my supervisor and former employer Bård Harstad for his excellent guidance and insightful comments. I am also very grateful to the Department of Economics for granting me the Scholarship in Advanced Methods and Research, supporting me through this semester. I would also like to thank Professor Tore Nilssen for the discussions resulting in the research idea for this thesis, as well as friends, family and colleagues for their various contributions, comments and patience. All remaining errors are, of course, my own.
Abstract

It seems to be a never-ending debate on whether the public or the private sector should deliver services to the public. Although the political divide seems to be ideologically motivated, the arguments made are often one of efficiency and optimal outcomes. The field of economics plays an important role in separating the facts from the values in this debate, highlighting the advantages and disadvantages of both public and private ownership in various sectors and organizational forms.

Public-Private Partnerships (PPPs) are no exception to this. PPPs have been heavily debated since its increasing popularity in the 1980s and 1990s. Its organizational features of providing incentives compatible with socially efficient outcomes are theoretically attractive. In practice, however, there are still much uncertainty as to whether these benefits actually materializes and whether they outweigh the potential costs of such arrangements. While much work has been done on the efficiency gains of PPP arrangements, surprisingly little has been done on the private finance aspect of PPP (Dewatripont and Legros, 2005). It is of course very likely that more research on this aspect of PPPs has been done since 2005. However, to my knowledge there is still quite limited literature on the effect from private finance on the PPP arrangement.

In this thesis I investigate the potential benefits of private finance. Assuming private finance is not necessary for realizing the benefits if PPP, does it still have a valuable contribution and if so, how much private finance is necessary to reap the benefits of private finance? Furthermore, does this private finance have any distorting effect on the efficiency gains from PPP?

In an influential paper on PPP, Iossa and Martimort (2012) uses an incomplete contract approach to show the costs and benefits of bundling the building and operation of infrastructure projects. Towards the end of the paper, the authors presents a scenario in which there exists a Private Financier with expertise in evaluating project risk. Together with the assumption of private finance being more costly than public finance, this sets the stage for a discussion on the costs and benefits of private finance. This thesis picks up on this discussion and explores when private finance is socially preferable. I investigate this in a setting with a simplified PPP structure but with a more complex private finance arrangement.

In this paper I first present a simple PPP model with and without a third party Private Financier, building on the incomplete contract approach by Iossa and Martimort (2012). Then I present the main model including a Private Financier with several changes to the assumption made by Iossa and Martimort (2012). I assume that the Private Financier contributes with some level of private finance to the infrastructure investment and is compensated by receiving a share of the variable income to the consortium of builder and operator in the PPP (from now on referred to as "PPP Consortium"). This Private Financier has the option to exert a level of effort to observe
an exogenous shock, to which it can insure the PPP Consortium should the shock be observed. Both the PPP Consortium and the Private Financier are assumed to be risk averse. The Private Financier receives the same share of the risk as the share of revenue it can claim. Thus, the share of risk and revenue needed to provide incentives for Private Financier to exert effort must be large enough such that the cost of exerting effort is lower than the disutility the Private Financier gets from the exogenous risk.

I show that under certain conditions, it is socially optimal to have partly private financing and partly public financing. When the infrastructure is only partly financed by a Private Financier, it entails that the Government has been able to extract all the surplus from these contracts. If the Private Finance investment are no more costly than public financing investments, then this solution must also involves no distortions of the PPP Consortium’s effort in creating infrastructure quality. However, if we assume that there is a higher financing cost from private finance, like Iossia and Martimort (2012) suggests, then even though the Government extracts all the surplus there is a distortion of reduced PPP Consortium infrastructure quality effort. Thus, whether the Private Finance is socially optimal depends on whether the cost of this distortion is lower than the lump-sum benefit of insuring the PPP Consortium.

If the Private Financier’s cost of making an effort in observing the shock is high enough relative to the share of risk and revenue it needs to exert this effort, then this model shows that the benefits of having private finance is outweighed by the costs. The reduced infrastructure quality effort made by the PPP Consortium costs more from the Governments point of view than the benefits it gets from insuring the PPP Consortium. Although this implies no private finance, in this model we might still consider including the Private Financier as a form of insurance company.

If the total cost of Private Financier effort and investment cost is low enough, then the optimal level of Private Finance is full Private Finance. This scenario might actually involve the Private Financier extracting some of the surplus from the arrangement, assuming that the share of the risk and revenue it needs to exert the effort is high enough relative to total effort and investment costs. This means that even though the Private Financier is able to make a positive profit from the PPP arrangement, it might still be the socially optimal case. This has some interesting real world consequences. It suggests that even though one observes the private financier profiting from PPPs above the competitive level, it might still be the socially optimal solution. This is, of course, not necessarily true in reality but the model does suggests that under certain restricting conditions it might be. Furthermore, this scenario would imply that a higher financing premium is a cost born by the Private Financier and thus the socially optimal case is not affected by changes in the private financing premium.

The model I propose in this thesis relies on several simplifying
assumptions. I have assumed a separable disutility function for the exogenous risk. This allows for the simple approach of treating the exogenous risk as a separable cost, which is a function of the risk. However, this does run the risk of oversimplifying the problem. Furthermore, the quite restricted financial contractual form I propose must be seen as one of many potential forms that could represent the real world. Therefore, the results and conclusions in this thesis should not be viewed in isolation but seen as a contribution that might provide some additional insights to the role of private finance in Public-Private Partnerships.
1 Introduction

Public-Private Partnerships (PPP) is an alternative to the traditional procurement of building and operating public infrastructure. It could be seen as a long term construction and service contract between a government and a consortium of private firms, often called a special purpose vehicle (SPV). In the most common form of PPP the public puts a PPP out to tender to which a consortium typically consisting of a construction firm, financiers and maintenance and operation provider together make a proposal for. The PPP consortium winning the project is then responsible for the construction, maintenance and operation of the infrastructure and earns a revenue based on either user fees or the public government payment for the service provided. This involves the infrastructure being owned by the private sector, but usually ownership is transferred back to the public sector after a period of 25-30 years.

The main argument for organising the construction and operation of an infrastructure through a PPP is efficiency gains. Välima (2005) suggests that there are three main sources of such efficiency gains; that is ownership, the bundling of construction and operation phases and the appropriate risk sharing. For much of this analysis we will assume a bundling effect as the main value contributor of the PPP Consortium.

While the traditional procurement involves the government put out to tender the construction and operating service contract separately, the PPP ensures that the builder and operator have joint incentives. It is easy to imagine how such joint incentives may lead to efficiency gains. One example could be that the builder can exert a non-contractible effort in the building phase. This may lead to lower operating cost or higher service quality in the operating phase. If this effort costs less than the cost reduction then a PPP consortium would make such an investment. On the other hand, in an unbundled case such as in traditional procurement the builder would not have incentives to make the same cost reducing investments since it is the operator who receives the benefit from it. In the literature on PPP there are several variants of this potential efficiency gain, with differing characteristics and hence differing social gains.

The flip side of the efficiency gain from PPP is the higher financing cost. Private finance is arguably more costly than public financing. Yescombe (2007) finds that PPP projects have historically had cost of capital around 200-300 basis points higher than cost of public funds. Furthermore, this has almost doubled since the credit crisis (Engel et al, 2014). Thus, when choosing between a traditional procurement and a typical PPP arrangement there exist a trade-off between an efficiency gain and higher financing cost.

There are varying levels of a PPP. From simple construction and operation bundling, to a more comprehensive design, development of project and construction and operation. Furthermore, in addition to the more
common privately financed PPP, one may have PPP with public financing. The existence of such PPP arrangements with public financing opens up for questioning whether private finance is necessary to obtain the benefits from PPPs, a question I will return to later in the introduction. But first, I will discuss another potential downside with private finance.

An important issue with PPP is the political motives for PPP that has no social benefits. By postponing the governments costs of investing in infrastructure, politicians can achieve their political goals while hiding the financial burden their governing incurs on the society. Some countries or municipalities may also have certain fiscal rules disallowing them from taking on too much investment cost, which may create incentives for politicians to invest through PPPs as a way to sidestep such restrictions. Levin and Tadelis (2006) shows that in U.S cities local political and budgetary motives has had significant impact in government service contracts in general. Thus, without a strong and sensible practices for recording PPP on the fiscal accounts PPPs can be a way for politicians to avoid such fiscal rules. According to IMF (2004) this is very much the case. IMF states that having PPPs in the private sectors balance sheet as oppose to the public sector’s balance sheet can indeed create unwarranted incentives favouring PPPs. Although may exist scenarios in which such an inter-temporal shift in the budget may be valuable, such as existence of liquidity constraints today but with strong growth prospects in the future, in the general case this budgetary motive does not hold from a social perspective (Engel et al 2014). Some important measures have been taken to reduce exploitation of accounting rule loopholes and political bookkeeping incentives (Eurostat, 2004 and European Commission, 2003), but it seems optimistic to assume that such guidelines and directives would remove unwarranted incentives completely.

The existence of unwarranted political incentives can make one question the social value of a PPP in general and private finance of a PPP in particular. If we distance ourselves from the political issues in a PPP and focus on the socially attractive features of a PPP, is there then still grounds for having a privately financed PPP? There are at least two ways that this may still be the case. First of all, this could be the case if having private ownership is necessary to ensure the bundling of construction and operation of the infrastructure. In other words, if private ownership is necessary in order to achieve the efficiency gains. This is a question without an easy answer and in the literature this have both been assumed and not assumed. Secondly, even if we can obtain a bundled PPP organization without private finance there still might be some potential benefits of having privately financed PPP. Private financier may have expertise and knowledge that a public sector do not have. This can be related to monitoring of the project, observing exogenous risk or choosing the right projects to invest in. Such valuable expertise might then outweigh the increased financing cost.

Brealey et.al. (1997) discusses how it might be easier for private firms to
deal with principal-agent issues. If this is the case, there might be reasons to prefer private financing as oppose to public financing. However, this does raise some interesting questions as to when the private sector have advantages in solving such principal-agents issue and whether this is particularly relevant in PPPs. One of my main hypothesis arises from this issue. Suppose that private finance is not necessary to obtain the benefits of bundling building and operation, is there still benefits with private finance? Furthermore, would the benefits of private finance necessarily need full private finance or might the advantages of private finance be obtained with only partly private financing? Finally, how does this effect the efficiency gains from the PPP Consortium?

Before moving on, we should make note of another important aspect in the PPP discussion; renegotiation issues. After the initial competitive PPP tender, the relationship between the Government and the private PPP Consortium that is awarded the project is transformed from a one-sided Government monopoly into a bilateral monopoly for the remainder of the long-term contract. The hold-up problems that can arise from such a relationship paves the way for renegotiations disfavouring the Government. Guasc (2004) finds that more than 50% of the PPPs studied in Latin-America was renegotiated. Engel et al (2009) finds that approximately 25% of investment in PPP arised from renegotiations in PPPs in Chile and Bitran et al (2013) found that this percentages was as high as 63% in Colombia. Needless to say, renegotiation is an important feature of a PPP and with private ownership this might constitute an even higher risk. Although this is an important issue and is closely related to the discussion of private ownership in PPP, for the scope of this thesis I disregard such renegotiation issues.

1.1 Literature overview

There are two main strands of theory on PPP (Dewatripont and Legros, 2005). One of which is generalized in Hart (2003), but springs out from several earlier papers including Grossman and Hart (1986), Hart and Moore (1990) and Hart et. al. (1997). Hart (2003) designs a general incomplete contracts approach to PPP which several important papers related to PPP is at least partly built upon, including Bennett and Iossa (2006), Dewatripont and Legros (2005) and Martimort and Iossa (2012). This strand also build upon the work of Williamson (1975, 1980) and utilizes the multi-task principal-agent model by Holmström and Milgrom (1991). The other strand is what Dewatripont and Legros (2005) calls “new economics of regulations” and is referred in particular to the book “A Theory of Incentives in Procurement and Regulation” (Laffont and Tirole, 1993). This strand focuses on the asymmetric nature of the relationship between firms and regulator. This involves both the possibility of hidden actions and hidden
information, which leads to a trade-off for the regulator between achieving efficient production and rent extraction.

PPP contracts are inherently incomplete. The environment of the long-term service contracts are necessarily uncertain, with future investment costs, technological development, service costs and demand for service all have varying degree of uncertainty. There may be uncertainty over future risk and over whether risk is exogenous or endogenous. The level of effort are to a varying degree unobservable at various organizational levels during both construction and operating phase. The general incomplete contract approach in Hart (2003) discusses the cost and benefits of PPP when the builder can make two type of investments in the building phase, both of which are cost saving for the operator. The contracts are incomplete in the sense that these type of investments are non-contractible and are decided upon by the agents based on their own objective. The investments are, however, affecting the government’s objective through effects on quality. One investment is what Hart calls productive and one is unproductive, meaning that the latter is a form of “quality-shading” investment. The government is only interested in productive investments and thus by bundling the builder and operator Hart shows that we get too much of the unproductive investment but the optimal amount (or closer to the optimal amount) of the productive investments. Thus, whether or not PPP is socially optimal would depend on the relative effects on and from these investments. This model serves as a good basis for this strand of PPP theory. However, in this model the only way to provide incentives for effort is through ownership and it ignores the ownership effect from either having private vs. public ownership.

Bennet and Iossa (2006) also study the positive effects from PFI (Private Finance Initiative), which is another variant for the PPP concept (and sometimes used interchangeably as the name for PPP, mostly in Britain and Australia). However, they go into more detail on the benefits of private ownership. Particularly, their contribution is when and whether control rights should be given to private firms and how the role of the projects residual value have on the optimal ownership model. The authors specify the original trade-off between unbundled and bundled building and construction before moving on to comparing the effects with public ownership and private ownership. The idea with private ownership is that it provides incentives for innovations and investment that are beneficial. The paper also describes a similar situation as in Hart (2003), where there could be positive or negative externalities on social benefit with bundling the project (which Hart refers to as productive and unproductive investments). One important deviations from the paper by Hart (2003) is that investments are assumed verifiable and there exists possibility of renegotiation ex post. The general result from Bennet and Iossa (2006) is that a PPP with private ownership is the optimal solution if the externality is mostly positive, if effect of innovations on social benefit is relatively small and effect on residual value is relatively large.
Iossa and Martimort (2012) builds on the incomplete contract approach using a dynamic multitask moral-hazard environment. The paper is a rather technical paper and it shows under which conditions a bundling of building and operations dominates a traditional procurement with separate builder and operator of a project. In this model, both builder and operator exert effort. Builder exert effort to increase quality, which is measured by a contractible, but noisy, quality index. The operator induce effort to increase revenues from user fees or to increase value to end users. Furthermore, the authors allow for productivity shocks after construction which alters the optimal level of effort from operator. After exploring several variants of this model, under which the productivity shock is observable by one, both or neither parts, the authors use this analysis as a basis for discussing cost and benefits of private financing in a PPP. They assume that a private financier potentially has advantages in observing productivity shocks. This can be seen as representing a form of due diligence and professional expertise. Thus, having private finance involves observing productivity shocks which increases the social value of the project. On the other hand, private financiers have a higher financing cost. Their analysis of this is rather limited and they simply acknowledge that there is a trade-off in which private finance creates both costs and benefits. One of the reasons for the limited discussion on this might be that the productivity shock affects operators effort and thus isolating the final trade-off becomes quite complicated. Exploring this trade-off further in a simplified setting is the main objective of this thesis.

In this thesis I will first present a very simple model using contract theory, consisting of a contract between a PPP consortium and a Government. Next I will expand this model by introducing a Private Financier which can take on the role of financing the infrastructure investment instead of the government. The benefit of this Private Financier could be that it can observe an exogenous shock in the future. If this shock is not observed, the government must compensate the risk averse PPP consortium for this risk. Thus, the private financier can insure the PPP consortium for this risk if it observed and this gain might outweigh the higher financing cost occurring when we have a Private Financier. This is the basic model's equivalent to the analysis made by Iossa and Martimort (2012). Then, I will expand the model further by restricting the financial contract between the PPP Consortium and the Private Financier and furthermore allow for partly private financing of the infrastructure. I show that the Government in most cases is still able to extract all surplus from the arrangement, but that the increased financing cost in most cases distorts the optimal efficiency level of the PPP Consortium's effort. Moreover, I show that under certain circumstances there might be optimal with partly private finance and partly public finance.
2 The Basic PPP Model

In this basic model we have a Government planning an infrastructure investment $I$. Instead of a traditional procurement of this investment, the Government puts out a tender for a Public-Private Partnership (PPP). We assume that there is sufficient competition for this project such that this can be represented by a two-stage model in which the government in the first-stage offers a contract to the PPP consortium who can then can accept or reject this contract. In the second stage, the PPP consortium then decides on the level of effort it will exert, which affects the quality of the infrastructure. In Iossa and Martimort (2012) there are two levels of effort made by the PPP consortium, one in an operating phase and one in building phase. Since the focus of this analysis is the role of the Private Financier, I simplify by setting only one level of effort exerted in building phase. The role of having a PPP as oppose to a traditional procurement would then be simply that the observed quality index will only be observed in operating phase and would thus not be contractible without a merged PPP consortium consisting of both a builder and operator. This simplification will later on allow us to focus on how introducing a Private Financier would affect the contract between the Government and PPP consortium.

The Government wishes to maximise the quality of the infrastructure, measured by a contractible quality index. Let’s first adopt the notation from Iossa and Martimort:

$$Q = a + \epsilon$$

where $Q$ is the quality index, $a$ is the level of effort exerted by the PPP consortium and $\epsilon$ is a random shock with the property that $E[\epsilon] = 0$. However, once again I would like to simplify further by assuming that this quality index is perfectly precise so that there is no random shock ($\epsilon = 0$). Thus, in this simple model the quality index becomes

$$Q = a$$

Next I introduce the linear transfer scheme offered by the Government, $t(Q)$, and the PPP consortium’s cost of exerting effort, $C(a)$, both which is adopted from Iossa and Martimort:

$$t(Q) = \gamma_0 + \gamma_1 Q$$

$$C(a) = \frac{a^2}{2}$$

Finally I will introduce the exogenous shock occurring in the future, $\theta$, which can take on both a positive and negative value, but with the property that $E[\theta] = 0$. In Iossa and Martimort, the shock $\theta$ was a productivity shock observable after the building phase but before operator decides on their
level of effort. Since there is no operating effort, however, this does not affect the choice made by the PPP consortium. Furthermore, I assume a risk averse PPP consortium which sets the stage for an efficiency loss when this uncertainty is borne by the PPP consortium. However, I will make another simplifying assumption; the model treats the costs and benefits of this contract for the PPP consortium as occurring in the short term while the exogenous shock occurring in a more distant future. By doing this I assume that the scenario may be represented by a separated utility function, one for the certain and contractible parameters and one for the disutility of the uncertain future shock. This will be a necessary assumption for our later expansions to the model. Although the assumption might be somewhat weak, it allow us to study an interesting trade-off later in the model. Thus, the disutility for the PPP consortium of this exogenous shock $\theta$ is represented by a separable function

$$-f(\theta)$$

which has the properties of $f(\theta) > 0$, $f'(\theta) > 0$, $f''(\theta) > 0$ and $f(0) = 0$. In other words, the disutility is increasing and convex, implying that the utility of this uncertainty shock is negative and concave. Now I present the following profit functions for the Government and the PPP consortium, respectively:

$$\Pi_{GOV} = Q - t(Q) - I = -\gamma_0 + (1 - \gamma_1)a - I$$

$$\Pi_{PPP} = t(Q) - C(a) - f(\theta) = \gamma_0 + \gamma_1a - \frac{a^2}{2} - f(\theta)$$

Note that for now I simply assume that the investment cost $I$ is incurred by the Government, as is done in Iossa and Martimort. However, an investment cost incurred by PPP consortium instead would not make any difference in this model since the Government simply extracts any surplus by setting the $\gamma_0$ accordingly. This will be made more clearly in the solution below. Thus, in the optimal solution, if we were to assume that investment was made by PPP consortium instead it would simply increase $\gamma_0$ by exactly $I$, resulting in the exact same profits and incentive structure.

Now that I have defined the basic components, we can solve this two stage model by backwards induction.

### 2.1 Stage 2 - PPP consortium’s maximisation problem

In the second stage, the PPP consortium maximises their profit function, $\Pi_{PPP}$, for a given transfer scheme, $t(Q)$, by setting their optimal level of $a$. I denote the optimal level of effort as $a$ and the choice variable as $\hat{a}$ and maximise the objective function with respect to $\hat{a}$:

$$\max_{\hat{a}} \quad \Pi_{PPP} = \gamma_0 + \gamma_1\hat{a} - \frac{\hat{a}^2}{2} - f(\theta)$$
which yields a first-order condition of

\[ FOC_a \quad \gamma_1 - \dot{a} = 0 \]

and a second-order condition

\[ SOC_a \quad -1 < 0 \]

which confirms the fairly simple conclusion that our first-order condition yields the optimal level of effort \( a \),

\[ a = \gamma_1 \]

2.2 Stage 1 - Government’s maximisation problem

The Government can choose any levels of \( \gamma_0 \) and \( \gamma_1 \) to maximise their objective function as long as the PPP consortium would accept the contract offered. In this simplified setting I have not set any outside option, thus Government’s maximisation problem including the participation constraint for the PPP consortium is

\[
\max_{\gamma_0, \gamma_1} \quad \Pi_{GOV} = -\gamma_0 + (1 - \gamma_1)a - I
\]

subject to

\[
PC_{PPP} : \quad \Pi_{PPP} = \gamma_0 + \gamma_1a - \frac{a^2}{2} - f(\theta) \geq 0
\]

By the result in the previous subsection the PPP consortium will set an effort \( a = \gamma_1 \) for any level of transfer scheme \( t(Q) \). Thus, we can insert for this optimal level \( a \):

\[
\max_{\gamma_0, \gamma_1} \quad \Pi_{GOV} = -\gamma_0 + (1 - \gamma_1)\gamma_1 - I
\]

subject to

\[
PC_{PPP} : \quad \Pi_{PPP} = \gamma_0 + \gamma_1^2 - \frac{\gamma_1^2}{2} - f(\theta) = \gamma_0 + \frac{\gamma_1^2}{2} - f(\theta) \geq 0
\]

To solve this, we can first note that the participation constraint must hold with equality. To see this, suppose by contradiction that the participation constraint did not hold with equality in the optimal solution:

\[
\gamma_0^* + \frac{\gamma_1^*}{2} - f(\theta) > 0
\]

Thus there must exist a positive factor \( \delta > 0 \) such that

\[
\gamma_0^* - \delta + \frac{\gamma_1^*}{2} - f(\theta) = 0
\]
which implies that there must exist some $\gamma_0 = \gamma_1^* - \delta$ such that the Government can be strictly better off by

$$\Pi_{GOV}(\gamma_0^*, \gamma_1^*) = -\gamma_0^* + (1 - \gamma_1^*)\gamma_1^* - I < -\gamma_0 + (1 - \gamma_1)\gamma_1 - I = \Pi_{GOV}(\gamma_0, \gamma_1)$$

since this $\gamma_0$ does not violate the participation constraint and strictly improves the objective function, $\gamma_0^*$ could not have been the optimal solution. Thus the participation constraint must hold with equality.

Since we know that the participation constraint is binding in optimum, we define the optimal $\gamma_0$ as the $\gamma_0^{FB}$ and rewrite the constraint as:

$$PC_{PPP} : -\gamma_0^{FB} = \frac{\gamma_1^2}{2} - f(\theta)$$

We can insert this into the governments objective function

$$\max_{\gamma_1} \Pi_{GOV} = \frac{\gamma_1^2}{2} - f(\theta) + (1 - \gamma_1)\gamma_1 - I$$

which yields our first-order condition

$$FOC_{GOV} : \gamma_1 + 1 - 2\gamma_1 = 0$$

and a second-order condition of

$$SOC_{GOV} : 1 - 2 < 0$$

Thus, we know that first-order condition yields the optimal $\gamma_1$, $\gamma_1^{FB} = 1$, which implies that the government fully incentivizes the PPP consortium to set the optimal level of effort and then extract all surplus by its:

$$\gamma_0^{FB} = -\frac{1}{2} + f(\theta)$$

The findings for this basic model is summarized in Proposition 1.

**Proposition 1.** In the basic model with no Private Finance the optimal contract offered by the Government is

$$t^{FB}(Q) = \gamma_0^{FB} + \gamma_1^{FB}a = -\frac{1}{2} + f(\theta) + a$$

which yields the following effort and Government profit

$$e^{FB} = 1$$

$$\Pi_{GOV}^{FB} = \frac{1}{2} - f(\theta) - I$$

where (2) refers to the efficient first-best effort made by the PPP Consortium and (3) shows the maximum Government surplus. The Government extracts all the surplus by the fixed transfer $\gamma_0^*$, but must compensate the PPP Consortium for the exogenous risk equal to the disutility of this risk, $f(\theta)$. 

13
Finally, we can note that this implies that the highest possible investment that is made, \( I \), is the investment yielding Government profit equal to zero:

\[
\Pi_{GB}^{Eb} = \frac{1}{2} - f(\theta) - I = 0 \quad \Rightarrow \quad I = \frac{1}{2} - f(\theta)
\]

Thus, any investment cost \( I \leq I \) results in the infrastructure being built.

3 Introducing Private Financier

3.1 Private Financier in Iossa and Martimort’s framework

Iossa and Martimort (2012) introduces a Private Financier (PF) that makes the investment \( I \) in the infrastructure and observes the shock \( \theta \) without making an effort. In their paper, the timing involves the Private Financier offering a linear state-contingent financial contract to the PPP. The Private Financier then provides insurance to the PPP consortium, extracting all the state-contingent surplus and avoids distorting the optimal incentive structure. This implies that there need not be any exogenous risk held by the risk averse PPP consortium and that the Government can still extract all the surplus with its linear transfer scheme. Thus, the authors show how a Private Financier can ensure the same result as when there are no exogenous shock. However, they comment that we observe empirically that private finance involves a higher financing cost (House of Lords, 2010). Thus, the trade-off is between compensating a risk averse PPP consortium from the exogenous risk and a higher investment cost \( (1 + \rho)I \), where \( \rho \) represents the higher finance cost from private finance.

We can take a quick look at how our model will look like if we made similar assumptions. The transfer scheme offered by the Private Financier to receive from PPP consortium, \( z(\theta, t(Q)) \), is defined as:

\[
z(\theta, t(Q)) = \beta_0(\theta) + \beta_1(t(Q))
\]

We have a three-stage model where the government offer a transfer scheme to PPP, then the Private Financier offers a financial contract to the PPP consortium before the PPP consortium in the final stage decides on its level of effort. Solving this by backwards induction we find that PPP consortium maximises

\[
\max_{\hat{a}} \quad \Pi_{PPP} = -\beta_0(\theta) + (1 - \beta_1)(\gamma_0 + \gamma_1\hat{a}) - \frac{\gamma_2^2}{2} - f(\theta)
\]

which yields the optimal level of effort from PPP consortium to be

\[
\hat{a} = (1 - \beta_1)\gamma_1
\]

This results in the Private Financier’s maximisation problem to be

\[
\max_{\beta_0, \beta_1} \Pi_{PF} = \beta_0(\theta) + \beta_1(t(Q)) - (1 + \rho)I = \beta_0(\theta) + \beta_1(\gamma_0 + \gamma_1^2(1 - \beta_1))
\]
subject to participation constraint of the PPP consortium

\[ PC_{PPP} : \Pi_{PPP} = -\beta_0(\theta) + (1 - \beta_1)(\gamma_0 + \gamma_1^2(1 - \beta_1)) - \frac{[\gamma_1(1 - \beta_1)]^2}{2} - f(\theta) \geq 0 \]

which must be binding by the same logic as in the basic model, since \( \beta_0(\theta) \) can be increased in the case of non-binding constraint and increasing \( \beta_0(\theta) \) strictly improves the objective of Private Financier it cannot be the case that it does not bind. Thus we can rewrite the PPP’s participation constraint:

\[ PC_{PPP} : \quad \beta_0(\theta) = (1 - \beta_1)(\gamma_0 + \gamma_1^2(1 - \beta_1)) - \frac{[\gamma_1(1 - \beta_1)]^2}{2} - f(\theta) \]

We can insert this expression for \( \beta_0(\theta) \) in the objective function to solve for the optimal level of \( \beta_1 \). The maximisation problem becomes:

\[ \max_{\beta_1} \quad \Pi_{PF} = \gamma_0 + \gamma_1^2(1 - \beta_1) - \frac{[\gamma_1(1 - \beta_1)]^2}{2} - f(\theta) - (1 + \rho)I \]

The first-order condition becomes

\[ FOC_{PF} : \quad -\gamma_1^2 + \gamma_1^2(1 - \beta_1) = 0 \]

and the second-order condition of

\[ SOC_{PF} : \quad -\gamma_1^2 < 0 \]

implies that the first-order condition yields the optimal \( \beta_1 \) which becomes \( \beta_1^* = 0 \). This gives the optimal \( \beta_0(\theta) \) as the one maximising:

\[ \max_{\beta_0(\theta)} \quad \Pi_{PF} = \gamma_0 + \gamma_1^2 - \frac{[\gamma_1(1 - \beta_1)]^2}{2} - f(\theta) - (1 + \rho)I \]

which is optimal when \( f(\theta) \) is minimised. In other words, the optimal state-contingent transfer fully insures the PPP consortium. This means including the \( \theta \) in the transfer to the Private Financier, resulting in the disutility being \( f(0) = 0 \). Finally, since Private Financier ensures the satisfaction of PPP’s participation constraint, the government can maximise its profits only with respect to the participation constraint of the Private Financier:

\[ \max_{\gamma_0,\gamma_1} \quad \Pi_{GOV} = -\gamma_0 + (1 - \gamma_1)\gamma_1 \]

subject to

\[ PC_{PF} : \quad \Pi_{PF} = \gamma_0 + \frac{\gamma_1^2}{2} - (1 + \rho)I \geq 0 \]

By the exact same argument as before we can claim equality in the constraint and insert for \(-\gamma_0\) rewritten from binding participation constraint:

\[ \max_{\gamma_1} \quad \frac{\gamma_1^2}{2} - (1 + \rho)I + (1 - \gamma_1)\gamma_1 \]
The solution to this must be the same as in our basic model since the only difference between the two problems are fixed values independent of $\gamma_1$.

Note, I denote the solution in this section, in the lack of better term, by a superscript IM. The optimal incentive structure is $\gamma_1^{IM} = 1$ which give the Government a profit

$$\Pi_{GOV}^{IM} = -\gamma_0^{IM} = \frac{1}{2} - (1 + \rho)I$$

When comparing this results to the basic model we see that including the Private Financier is profitable for the Government, which I assume is equivalent to being socially optimal, if

$$\Pi_{GOV}^{IM} = \frac{1}{2} - (1 + \rho)I \geq \frac{1}{2} - f(\theta) - I = \Pi_{GOV}^{FB}$$

$$\Rightarrow \rho I \leq f(\theta)$$

These findings are summarized in Proposition 2:

**Proposition 2.** Introducing a Private Financier in the same manner as Iossa and Martimort (2012) yields the following contract offered by the Government:

$$t^{IM}(Q) = \gamma_0^{IM} + \gamma_1^{IM} a = (1 + \rho)I - \frac{1}{2} + a$$  \hspace{1cm} (4)

and the contract offered by the Private Financier (from PPP Consortium to the Private Financier) is

$$z(\theta, t(Q)) = \beta_0^{IM}(\theta) + \beta_1^{IM} t(Q) = \gamma_0^{IM} + \gamma_1^{IM} \frac{a}{2} + \theta = (1 + \rho)I + \theta$$  \hspace{1cm} (5)

This results in the PPP Consortium exerting the First-Best effort, $e^{FB}$, implying no efficiency distortions. The Government extracts all the surplus from both the PPP Consortium and the Private Financier. The Private Financier provides insurance to the PPP Consortium, but it comes at a higher financing cost. Allowing for Private Finance in this model is socially optimal if

$$\rho I \leq f(\theta)$$  \hspace{1cm} (6)

That is if the reduced disutility from PPP Consortium being exposed to exogenous risk is higher than the increased financing cost from Private Finance.

Although this result is very intuitive and not particularly surprising, it provides a benchmark for what the results in this thesis’ model would be under the Private Financier assumptions that Iossa and Martimort (2012) makes.
3.2 Changing the Private Financier assumptions

In the analysis going forward I will relax some of the assumptions made by Iossa and Martimort (2012). First of all, I allow for a shared investment, meaning that the Private Financier can invests a share $q$ of the initial investment, where $q \in [0, 1]$. The remaining investment $(1 - q)I$ is either done by the Government or by the PPP consortium itself, consisting of the builder and operator. I will keep the assumption that there might be an increased financial cost (a private finance premium, $\rho$) as presented in Iossa and Martimort (2012). This can be seen as a form of outside option for the Private Financiers investment. As previously mentioned, whether I restrict the remaining investments, $(1 - q)I$, to be made by Government or PPP consortium does not make a difference as the Government in the optimal solution simply extract the surplus with its fixed transfer, $\gamma_0$.

Secondly, I will assume that the PPP consortium is the one offering a contract to the Private Financier. I would argue that it is more reasonable to assume that the PPP consortium is the one with market power to offer such a contract rather than vice versa. Should the PPP consortium offer such a contract after winning a tender, then it must be the case that the PPP consortium is the only company offering such a deal. Furthermore, I find it reasonable to assume that there exists sufficiently many banks and insurance companies to ensure competition between them. Should the PPP consortium and Private Financier enter a financial contract before the tender, any one Private Financier could enter an agreement with as many potential PPP consortium as they like, since there can be only one winning the tender. This implies that each PPP consortium has unique market power over their product. Thus, as long as there exists enough financiers, it seems to be a more reasonable assumption that the PPP consortium is the one offering a contract to the Private Financier. On the other hand, one may imagine some few financial companies becoming experts in this particular field with their competence in observing $\theta$ being superior to competitors. If so, one may have a scenario in which a Private Financier have market power as well. In any case, I will limit this analysis to the assumption of PPP consortium offer financial contract to the Private Financier.

I will also introduce an effort $e$ needed by the Private Financier to observe the $\theta$. This effort comes with a cost $ce$ and is non-contractible. Thus, there is need to provide incentives for the Private Financier to exert this effort. Introducing this costly effort can either represent the Private Financier performing a due diligence at a certain cost before the project, Private financier exert a costly effort during construction and operation phase in monitoring of the environment or some other forms of monitoring of external factors outside of the PPP consortium’s control. It is possible to view this effort as a binary variable in which if $e = 0$ then $\theta$ is not observed and if $e = 1$ then $\theta$ is observed. One might also introduce a continuous effort,
in which the probability of observing $\theta$ is $p(e)$, where $p'(e) > 0$, $p''(e) < 0$ and for example $p(0) = 0$ and $p(1) = 1$. However, for the remainder of this thesis I will focus on the binary case.

Furthermore, I will from this point on restrict the ways in which the financial contract is stipulated. I assume that the financial contract is limited in its nature such that the existence of this Private Financier might result in a distortion compared to the first best solution. The insurance made from the Private Financier is assumed to be non-contractible. Instead of a contractible effort, I provide incentives to the Private Financier by allowing a transfer of both revenue and risk to the Private Financier. Note that I have not yet made any assumption on how the Private Financier realistically would provide insurance to the PPP consortium. It could have been that the Private Financier was risk neutral and simply took on the risk themselves. Alternatively we could assume that the Private Financier is risk averse as well, but with the ability to hedge or diversify the infrastructure company from this exogenous shock risk, should it observe the shock $\theta$. If the latter is the case, then transferring risk to the the Private Financier provides incentives for making an effort in observing $\theta$. If it is the case with a risk neutral Private Financier, transferring as much risk to Private Financier is preferable as it reduces the risk compensation needed to ensure participation of the PPP consortium. In the main model below I will assume that the Private Financier is risk averse with the ability to insure the entire PPP from the exogenous risk.

In the main model below I have restricted the financial contract transfer scheme to one variable, $\beta_1$. That is, we do not allow for a state-contingent transfer $\beta_0(\theta)$ or even a lump-sum transfer $\beta_0$ from PPP Consortium to the Private Financier. The no state-contingent transfer is a result of $\theta$ being non-contractible. Instead, the incentives for Private Financier to exert costly effort, $e$, is through sharing both variable profit and the exogenous risk. By the discussion above, the risk averse Private Financier will thus have the ability to insure this risk if it observes it. Note that the insurance is attached to the entire PPP arrangement and thus both Private Financier and PPP Consortium is insured from the risk. Furthermore, I also assume that the $\beta_1$ share of the revenue is only attached to the variable revenue, $\gamma_1 e$. This could be interpreted as the share of the revenue made from operations while the fixed transfer $\gamma_0$ can be interpreted as an upfront transfer between the PPP Consortium and the Government. This assumption is of course debatable, but it seems to me just as reasonable as including $\gamma_0$ in the sharing agreement. Moreover, by restricting the financial contract this way I allow for a more interesting model. Contrarily, if we were to include $\gamma_0$ it would imply a completely overlapping objectives for the PPP Consortium and Private Financier once the financial contract is agreed upon, which would not result in any particular interesting analysis.
4 The Main Model

In the main model I have used the basic PPP model with a Private Financier. I have made several changes to the assumptions compared to the discussion by Iossa and Martinmort (2012). All the changed assumption are discussed in the previous section.

Assumption 1. The Government can demand a share $q \in [0,1]$ of private finance of the investment cost $I$, and it contributes with the remaining share $(1-q)I$. The share of private finance involves a private finance premium, $\rho$.

Assumption 2. The financial contract is a transfer from PPP Consortium to Private Financier on the form

$$z(\gamma_1, Q) = \beta_1 \gamma_1 Q + \beta_1 \theta$$

This implies no fixed transfer $\beta_0$ and the Private Financier only gets a share of the variable revenue. Furthermore, the Private Financier also receives a corresponding share of the exogenous risk.

Assumption 3. The financial contract is offered by the PPP Consortium, to which the Private Financier have the option to accept or decline.

Assumption 4. The Private Financier is risk averse with the same disutility from the exogenous risk as PPP Consortium, $-f(\cdot)$. The Private Financier and PPP Consortium’s disutilities are $f(\beta_1 \theta)$ and $f((1-\beta_1)\theta)$, respectively.

Assumption 5. The Private Financier has the option to exert a costly binary effort $e \in \{0, 1\}$ at a cost, $ce$. If $e = 1$, then $\theta$ is observed. If $e = 0$ then $\theta$ is not observed.

By use of Assumption 4 and 5, I redefine the disutilities of both Private Financier and PPP Consortium to $(1-e)f(\beta_1 \theta)$ and $(1-e)f((1-\beta_1)\theta)$, respectively. This represents that if effort is made by the Private Financier, then $\theta$ is observed and the Private Financier automatically insures the entire PPP arrangement from the exogenous risk resulting in no disutility from this shock.

When including these changes to the assumptions from the basic model the profit functions in the main models becomes:

$$\Pi_{PF} = \beta_1 \gamma_1 a - ce - (1-e)f(\beta_1 \theta) - q(1 + \rho)I$$

$$\Pi_{PPP} = \gamma_0 + (1-\beta_1)(\gamma_1 a) - \frac{a^2}{2} - (1-e)f((1-\beta_1)\theta)$$

$$\Pi_{GOV} = -\gamma_0 + (1-\gamma_1)a - (1-q)I$$

To solve this, we use backwards induction.
4.1 Stage 3 - Private Financier choose optimal effort

For a given level of transfer scheme, \( t(Q) \), and offered share of profit and risk, \( \beta_1 \), the Private Financier chooses the level of effort that maximises their profit. Here I define \( e \) as the optimal level of effort and \( \hat{e} \) as the choice variable. The optimal \( e \) is then

\[
e = \arg\max_{\hat{e}} \beta_1 \gamma_1 a - c\hat{e} - (1 - \hat{e}) f(\beta_1 \theta) - q(1 + \rho) I
\]

Since this model is restricted to a binary effort this implies that

\[
e = \begin{cases} 
1 & \text{if } f(\beta_1 \theta) \geq c \\
0 & \text{otherwise}
\end{cases}
\]

Note that the optimal effort from Private Financier is independent of the level of effort it expects from PPP Consortium, thus we do not have to take this into account in this subsection.

4.2 Stage 2 - PPP consortium’s maximisation problem

Let’s first simply assume that it is optimal for the PPP consortium to provide incentives for an effort from the Private Financier. We need to check the condition for this later, but we can already note that if this is not the case, then the Private Financier has no value contribution and thus both the optimal \( \beta_1 \) and optimal \( q \) must be zero. Now, the PPP consortium’s objective function does not need to include the disutility of the exogenous shock since I assume that it is optimal for the PPP consortium to provide incentives for Private Financier to exert effort and thus the PPP consortium will be insured. However, we must include an incentive compatibility constraint, ensuring the effort from Private Financier. PPP consortium’s maximisation problem can be written as:

\[
max_{\hat{a}, \beta_1} \Pi_{PPP} = \gamma_0 + (1 - \beta_1)(\gamma_1 \hat{a}) - \frac{\hat{a}^2}{2}
\]

subject to

\[
PC_{PF} : \beta_1 \gamma_1 a - c - q(1 + \rho) I \geq 0
\]

\[
ICC_{PF} : f(\beta_1 \theta) \geq c
\]

4.2.1 The optimal PPP Consortium effort

The PPP Consortium’s maximisation problem actually consists of two stages. First the contract is offered to the Private Financier. Then, after the Private Financier accepts or rejects the contract, the PPP Consortium decides on its effort \( a \). This means that the Private Financier do not know the level of effort the PPP consortium makes when it accepts or rejects the
contract. In other words, the Private Financier must deduce it from PPP Consortium’s optimal level of effort for the given $\beta_1$. By our backwards induction approach this implies that we first find the optimal level of effort, $a$, then solve for the optimal contract offered to the Private Financier. Since the contract is already agreed upon at the time where effort is made, the PPP Consortium maximises their effort $a$ irrespective of the participation and incentive compatibility constraints of the Private Financier. Thus, to find the optimal level of effort made by the Private Financier we simply maximise the PPP consortium’s objective function with respect to $a$, for a given level of $\beta_1$.

$$\max_a \Pi_{PPP} = \gamma_0 + (1 - \beta_1)(\gamma_1 a) - \frac{a^2}{2}$$

This yields the first-order and second-order conditions of

$$FOC_{a} : (1 - \beta_1) \gamma_1 - \dot{a} = 0$$

$$SOC_{a} : -1 < 0$$

By the negative second-order condition, the optimal effort must then be:

$$a = (1 - \beta_1) \gamma_1$$

Since this is the optimal level of effort made by the PPP Consortium for any level of $\beta_1$, both the PPP Consortium and Private Financier are aware of this at the time when the contract is signed. Thus, they both take this level of effort made by the PPP Consortium into consideration when PPP Consortium offers the $\beta_1$ and when the Private Financier decides whether to accept or reject the contract. Thus, we can simply insert for this effort as a function of the $\beta_1$ and the PPP Consortium’s maximisation problem thus becomes

$$\max_{\beta_1} \Pi_{PPP} = \gamma_0 + (1 - \beta_1)(1 - \beta_1)\gamma_1 - \frac{[(1 - \beta_1)\gamma_1]^2}{2} = \gamma_0 + \frac{[(1 - \beta_1)\gamma_1]^2}{2}$$

subject to

$$PC_{PF} : \beta_1 (1 - \beta_1) \gamma_1^2 - c - q(1 + \rho)\delta \geq 0$$

$$ICC_{PF} : f(\beta_1, \theta) \geq c$$

### 4.2.2 The optimal contract offered to the Private Financier

We observe that the PPP Consortium’s objective function is strictly decreasing in $\beta_1$. This implies that the PPP Consortium wishes to offer the lowest possible $\beta_1$. However, both the participation constraint and the incentive compatibility constraint for the Private Financier has a lower limit to how low the $\beta_1$ can be before the constraints are violated. Thus, the
PPP Consortium will set the lowest possible $\beta_1$ that does not violate any of the two constraints. In other words, there are two possible solutions to this problem; either the participation constraint is binding or the incentive compatibility constraint is binding (or both).

Suppose it is the participation constraint of the Private Financier that is binding. Then we can rewrite this participation constraint to:

$$PC_{PF} : \beta_1(1 - \beta_1)\gamma_1^2 = c + q(1 + \rho)I$$

$$\Rightarrow -\beta_1^2 \gamma_1^2 + \beta_1 \gamma_1^2 - (c + q(1 + \rho)I) = 0$$

I define the optimal level of $\beta_1$ in this scenario as $\hat{\beta}_1$. By the quadratic formula, the optimal level $\hat{\beta}_1$ is

$$\hat{\beta}_1 = \frac{-\gamma_1^2 \pm \sqrt{\gamma_1^4 - 4\gamma_1^2(c + q(1 + \rho)I)}}{-2\gamma_1^2}$$

$$\Rightarrow \hat{\beta}_1 = \frac{1}{2} \pm \frac{\sqrt{\gamma_1^2 - 4(c + q(1 + \rho)I)}}{2\gamma_1}$$

The solution to this must in fact be:

$$\hat{\beta}_1 = \frac{1}{2} - \frac{\sqrt{\gamma_1^2 - 4(c + q(1 + \rho)I)}}{2\gamma_1} \quad (7)$$

Proof. First, I will argue that the solution for $\hat{\beta}_1$ must be smaller than one half, that is $\hat{\beta}_1 \in [0, \frac{1}{2}]$. The Private Financier would not want a $\beta_1$ higher than one half, since the corresponding effort by PPP would be reduced. In fact, the highest possible revenue for the Private Financier is achieved when $\beta_1 = \frac{1}{2}$. The solution in (7) must be the lowest possible $\beta_1$ that satisfies the participation constraint of the Private Financier. However, it cannot be that the lowest possible $\beta_1$ yields a lower revenue to the Private Financier than another lower $\beta_1$. Thus, the optimal $\hat{\beta}_1$ cannot be larger than one half. In other words, since both $\gamma_1 > 0$ and the square root expression must be positive we know that the optimal $\beta_1$ in this scenario must in fact include a negative sign such that we have (7).

Now let’s consider the other possible solution for optimal $\beta_1$. That is supposing it is the incentive compatibility constraint of the Private Financier that binds first. Then we know that the optimal level of $\beta_1$ must be the $\beta_1$ that solves the binding incentive compatibility constraint for the Private
Financier. Thus, the optimal solution in this case, which I denote $\hat{\beta}_1$, is the one that satisfies:

$$f(\hat{\beta}; \theta) = c$$

I define this optimal $\hat{\beta}_1$ as a function of $c$ and $\theta$:

$$\hat{\beta}_1 \equiv g(c, \theta) \quad (8)$$

We can note that the function $g(\cdot)$ have the property that

$$\frac{dg(c, \theta)}{dc} > 0$$

Furthermore, although I have defined $E[\theta] = 0$, I have not said anything about the variance of exogenous shock, $Var(\theta)$. However, we can say that if this variance of the exogenous shock increases, the disutility function $f(\cdot)$ increases and thus the $\beta_1$ needed to provide incentives for Private Financier to make an effort decreases. In other words, $\hat{\beta}_1 = g(c, \theta)$ decreases as the variance of the exogenous shock increases:

$$\frac{dg(c, \theta)}{dVar(\theta)} < 0$$

Finally, the results can be summarized and we can see that the optimal transfer to Private Financier, $\beta^*_1$, is the highest of the two lower limits, $\hat{\beta}_1$ and $\tilde{\beta}_1$:

$$\beta^*_1 = max\{\hat{\beta}_1, \tilde{\beta}_1\} = max\left\{\frac{1}{2} - \frac{\sqrt{\gamma_1^2 - 4(c + q(1 + \rho)I)}}{2\gamma_1}, g(c, \theta)\right\} \quad (9)$$

4.3 Stage 1 - Government’s maximisation problem

The Government’s optimisation problem is to maximise its profit

$$max_{\gamma_0, \gamma_1, q} \quad \Pi_{GOV} = -\gamma_0 + (1 - \gamma_1)a - (1 - q)I$$

subject to

$$PC_{PF} : \quad \Pi_{PF} = \beta^*_1 \gamma_1 a - c - q(1 + \rho)I \geq 0$$

$$PC_{PPP} : \quad \Pi_{PPP} = \gamma_0 + (1 - \beta^*_1)\gamma_1 a - \alpha^2 \geq 0$$

The Government anticipates the optimal effort made by the PPP Consortium. Therefore, we can insert for $a = (1 - \beta^*_1)\gamma_1$:

$$max_{\gamma_0, \gamma_1, q} \quad \Pi_{GOV} = -\gamma_0 + (1 - \beta^*_1)(1 - \gamma_1)\gamma_1 - (1 - q)I$$

subject to

$$PC_{PF} : \quad \Pi_{PF} = \beta^*_1(1 - \beta^*_1)\gamma_1^2 - c - q(1 + \rho)I \geq 0$$
In addition, the Government anticipates the optimal contract between the PPP Consortium and the Private Financier. The next step is thus to insert for these optimal levels into their objective functions. To solve this I split the problem into three subsections, one for the scenario in which \( \hat{\beta}_1 > \beta_1 \), one for when \( \hat{\beta}_1 < \beta_1 \) and one where \( \hat{\beta}_1 = \beta_1 \). I will show that in these three scenarios we must have optimal share of Private Finance \( q = 0, q = 1 \) and \( q \in [0, 1] \), respectively.

### 4.3.1 Governments optimisation when \( \beta^*_1 = \hat{\beta}_1 > \beta_1 \)

In this subsection I will show that if \( \hat{\beta}_1 > \beta_1 \) then there must be a corner solution for \( q \) where it is optimal with no private finance (\( q = 0 \)). In this scenario, we know that \( \beta^*_1 = \hat{\beta}_1 \) is offered such that the Participation Constraint of the Private Financier holds with equality. Thus, if this is the optimal solution, the government need not worry about the Private Financiers participation. However, although I have assumed in this subsection that we are in a scenario in which \( \beta^*_1 = \hat{\beta}_1 \), the Government can affect which of the two solutions for \( \beta^*_1 \) is chosen by the PPP Consortium. The condition for whether \( \beta_1 \) or \( \hat{\beta}_1 \) is chosen is which of the two is the highest must then be included in the problem. Thus, the Government’s problem can be written as:

\[
\max_{\gamma_0, \gamma_1, q} \Pi_{GOV} = -\gamma_0 + (1 - \hat{\beta}_1)(1 - \gamma_1) - (1 - q)I
\]

subject to

\[
PC_{PPP} : \quad \Pi_{PPP} = \gamma_0 + \frac{[(1 - \hat{\beta}_1)\gamma_1]^2}{2} \geq 0
\]

and assuming

\[
\hat{\beta}_1 > \beta_1 \iff \frac{1}{2} - \frac{\sqrt{\gamma_1^2 - 4(c + q(1 + \rho)I)}}{2\gamma_1} > g(c, \theta)
\]

Just like before, I argue that the participation constraint of the PPP Consortium always will hold with equality, since if this was not the case, the objective function could be improved by lowering the \( \gamma_0 \) without violating the participation constraint. Thus we can rewrite the participation constraint as

\[
PC_{PPP} : \quad -\gamma_0 = \frac{[(1 - \hat{\beta}_1)\gamma_1]^2}{2}
\]

and this we can insert into our objective function:

\[
\max_{\gamma_1, q} \Pi_{GOV} = \frac{[(1 - \hat{\beta}_1)\gamma_1]^2}{2} + (1 - \hat{\beta}_1)(1 - \gamma_1)\gamma_1 - (1 - q)I
\]
assuming

\[ \hat{\beta}_1 > \beta_1 \iff \frac{1}{2} - \frac{\sqrt{\gamma_1^2 - 4(c + q(1 + \rho)I)}}{2\gamma_1} > g(c, \theta) \]

**Proposition 3.** When the optimal contract offered by the Government results in \( \hat{\beta}_1 > \beta_1 \) for the PPP Consortium in stage 2, then it must be the case that the Government offers no private finance, \( q = 0 \).

*Proof.* The Private Financier’s profit is by the definition of \( \hat{\beta}_1 \) zero. Furthermore, we have a binding participation constraint of the PPP Consortium. Thus, the Government captures the entire surplus in this scenario. Next, we observe from the optimal level of effort made by the PPP Consortium, \( a = (1 - \beta_1)\gamma_1 \), any \( \beta_1 > 0 \) results in either an efficiency distortion or a necessary increased \( \gamma_1 \) by the Government.

If there is an distortion then it is increasing in \( \beta_1 \). In other words, higher \( \beta_1 \) means a lower effort \( a \) which lowers the Governments profit, holding \( \gamma_1 \) constant. Furthermore, we know that the \( \hat{\beta}_1 \) is increasing in \( q \) which means that a higher \( q \) increases this distortion.

If the Government instead of allowing distortion increased the \( \gamma_1 \), then for any level of private finance premium, \( \rho > 0 \), this increasing cost to maintain incentives from to the PPP Consortium is higher than the gains of reducing its own investment, since the private finance investment comes at a higher financing cost.

Thus, the Government can gain from reducing \( q \) since the Government already captures the entire surplus and reducing \( q \) reduces the efficiency distortions and investment cost. I summarize this by stating that for any level of \( q > 0 \) in which \( \beta_1^* = \beta_1 > \beta_1 \), the Government could reduce \( q \) and be strictly better off. In order to maximise its objective function, the Government will therefore continue to reduce the \( q \) until it reaches either \( q = 0 \) or \( \hat{\beta}_1 = \beta_1 \). Thus, the only way in which we can have \( \hat{\beta}_1 > \beta_1 \) is when we have the corner solution \( q = 0 \).

In the scenario in which \( \hat{\beta}_1 > \beta_1 \) we can therefore conclude that the optimal level of Private Finance is zero, that is \( q = 0 \). However, it might still be the case that the PPP Consortium wishes to offer the Private Financier a contract in order to be insured. In other words, although this is not the main focus in this thesis, there exists a possibility that the optimal condition involves the Private Financier without any actual private finance. In this case, it would be more appropriate to think of the Private Financier as an insurance firm or a consulting firm of some sorts, but for the sake of consistency I maintain the Private Financier terminology. In the scenario with \( q = 0 \), the \( \hat{\beta}_1 \) becomes

\[ \beta_1^* = \hat{\beta}_1 = \frac{1}{2} - \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} > g(c, \theta) \]
We note that as the cost of making an effort is reduced, the $\hat{\beta}_1$ approaches zero.

The Government’s objective function is now:

$$\max_{\gamma_1} \quad \Pi_{GOV} = \left(\frac{1 - \hat{\beta}_1}{2}\right)^2 + (1 - \hat{\beta}_1)(1 - \gamma_1)\gamma_1 - I$$

given that

$$\hat{\beta}_1 = \frac{1}{2} - \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} > g(c, \theta) = \hat{\beta}_1$$

The solution to this is summarized in the following proposition.

**Proposition 4.** When the optimal contract offered by the Government to the PPP Consortium results in $\hat{\beta}_1 > \beta_1$ for the PPP Consortium in stage 2, then the optimal contract must be

$$\hat{t}(Q) = \gamma_0 + \hat{\gamma}_1 Q = -\frac{1}{2} + (1 + c)Q \quad (10)$$

**Proof.** As a first step to prove this proposition, note that the $\hat{\beta}_1$ is a function of $\gamma_1$. Thus, we should calculate the derivative of the $\hat{\beta}_1$ with respect to $\gamma_1$. We use implicit derivation from the definition of $\hat{\beta}_1$:

$$\hat{\beta}_1(1 - \hat{\beta}_1)\gamma_1^2 - c + q(1 + \rho)I = \hat{\beta}_1 \gamma_1^2 - \hat{\beta}_1^2 \gamma_1^2 - c = 0$$

$$\Rightarrow \quad \frac{d\hat{\beta}_1}{d\gamma_1} \gamma_1^2 + 2\gamma_1 \hat{\beta}_1 - 2\hat{\beta}_1 \frac{d\hat{\beta}_1}{d\gamma_1} \gamma_1^2 - 2\gamma_1 \hat{\beta}_1^2 = 0$$

$$\Rightarrow \quad \frac{d\hat{\beta}_1}{d\gamma_1} (\gamma_1^2 - 2\hat{\beta}_1 \gamma_1^2) = -2\gamma_1 \hat{\beta}_1 + 2\gamma_1 \hat{\beta}_1^2$$

$$\Rightarrow \quad \frac{d\hat{\beta}_1}{d\gamma_1} = \frac{-2\gamma_1 \hat{\beta}_1 + 2\gamma_1 \hat{\beta}_1^2}{\gamma_1^2 (1 - 2\hat{\beta}_1)} = \frac{-2\hat{\beta}_1 (1 - \hat{\beta}_1)}{\gamma_1 (1 - 2\hat{\beta}_1)} < 0$$

The inequality must hold for any $\hat{\beta}_1 < \frac{1}{2}$, for which we know that any $\hat{\beta}_1 > \frac{1}{2}$ would not be the efficient, since this is the $\beta_1$ that satisfies the participation constraint of the Private Financier (as argued when defining the $\beta_1$). The other possibility is if $\hat{\beta}_1 = \frac{1}{2}$, but then it must be the case that $\sqrt{\gamma_1^2 - 4c} = 0 \Rightarrow \gamma_1 = \sqrt{c}$. Thus we must keep in mind that our derivative is not defined for this case. However, as it turns out, this will not be an issue in the analysis going forward.

Now, we can maximise the objective function, assuming the condition holds (which I will check the conditions for afterwards).

$$\max_{\gamma_1} \quad \Pi_{GOV} = \left(\frac{1 - \hat{\beta}_1}{2}\right)^2 + (1 - \hat{\beta}_1)(1 - \gamma_1)\gamma_1 - I \quad (11)$$
\[ FOC_{\gamma_1} : (1 - \hat{\beta}_1)^2 \gamma_1 - \frac{d\hat{\beta}_1}{d\gamma_1} (1 - \hat{\beta}_1) \gamma_1^2 + (1 - 2 \gamma_1) (1 - \hat{\beta}_1) - \frac{d\hat{\beta}_1}{d\gamma_1} (1 - \gamma_1) \gamma_1 = 0 \]

In order for this first-order condition to yield a maximised Government profit, we require the second-order condition to be negative. I will provide the proof of this now, however, note that this is quite a lengthy proof. Thus, for any disinterested reader, feel free to skip to the middle of page 28.

To prove that the second-order condition is negative, I will first rewrite the expression for the first-order condition:

\[
(1 - \hat{\beta}_1)^2 \gamma_1 + \frac{2\hat{\beta}_1 (1 - \hat{\beta}_1)^2 \gamma_1}{(1 - 2\hat{\beta}_1)} + (1 - 2 \gamma_1) (1 - \hat{\beta}_1) + \frac{2\hat{\beta}_1 (1 - \hat{\beta}_1) (1 - \gamma_1)}{(1 - 2\hat{\beta}_1)}
\]

\[
\Rightarrow (1 - \hat{\beta}_1)^2 \gamma_1 + (1 - 2 \gamma_1)(1 - \hat{\beta}_1) + \frac{2\hat{\beta}_1(1 - \hat{\beta}_1)((1 - \hat{\beta}_1) \gamma_1 + 1 - \gamma_1)}{(1 - 2\hat{\beta}_1)}
\]

\[
\Rightarrow (1 - \hat{\beta}_1)^2 \gamma_1 + (1 - 2 \gamma_1)(1 - \hat{\beta}_1) + \frac{2\hat{\beta}_1 (1 - \hat{\beta}_1) (1 - \gamma_1 \hat{\beta}_1)}{(1 - 2\hat{\beta}_1)}
\]

If we insert for \( \hat{\beta}_1 \) this becomes

\[
\Rightarrow \left( 1 - \left[ \frac{1}{2} - \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} \right] \right)^2 \gamma_1 + (1 - 2 \gamma_1) \left( \frac{1}{2} - \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} \right)
\]

\[
+ \frac{2\left[ \frac{1}{2} - \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} \left( 1 - \frac{1}{2} - \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} \right) \right]}{(1 - 2\left[ \frac{1}{2} - \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} \right])} \left( 1 - \gamma_1 \frac{1}{2} - \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} \right)
\]

We can simplify this first-order derivative further:

\[
\Rightarrow \frac{d\Pi_{GOV}}{d\gamma_1} = \left( \frac{1}{2} + \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} \right)^2 \gamma_1 + (1 - 2 \gamma_1) \left( \frac{1}{2} + \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} \right)
\]

\[
+ \frac{\left( 1 - \frac{\sqrt{\gamma_1^2 - 4c}}{\gamma_1} \right)^2 \left( \frac{1}{2} + \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} \right)}{\sqrt{\gamma_1^2 - 4c}} \left( 1 - \frac{\gamma_1}{2} + \frac{\sqrt{\gamma_1^2 - 4c}}{2} \right)
\]

\[
= \left( \frac{1}{2} + \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} \right)^2 \gamma_1 + \frac{1}{2} + \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} - \gamma_1 - \frac{\gamma_1^2 - 4c}{\sqrt{\gamma_1^2 - 4c}}
\]

\[
+ \frac{(\gamma_1 - \frac{\gamma_1^2 - 4c}{\gamma_1} \left( \frac{1}{2} + \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} \right) \left( 1 - \frac{\gamma_1}{2} + \frac{\sqrt{\gamma_1^2 - 4c}}{2} \right)}{\sqrt{\gamma_1^2 - 4c}}
\]
\[
\begin{align*}
&= \left( \frac{1}{2} + \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} \right)^2 \gamma_1 + \frac{1}{2} + \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} - \gamma_1 - \sqrt{\gamma_1^2 - 4c} \\
&\quad + \frac{1}{2} \left( \gamma_1 + \sqrt{\gamma_1^2 - 4c} - \sqrt{\gamma_1^2 - 4c} - \frac{\gamma_1^2 - 4c}{\gamma_1} \right) \left( 1 - \frac{\gamma_1}{2} + \frac{\sqrt{\gamma_1^2 - 4c}}{2} \right) \\
&= \left( \frac{1}{2} + \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} \right)^2 \gamma_1 + \frac{1}{2} + \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} - \gamma_1 - \sqrt{\gamma_1^2 - 4c} \\
&\quad + \frac{c}{\sqrt{\gamma_1^2 - 4c}} + \frac{\gamma_1^2 - 4c}{\gamma_1} \\
&= \left( \frac{1}{4} + \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} + \frac{\gamma_1^2 - 4c}{4\gamma_1^2} \right) \gamma_1 + (1 - 2\gamma_1) \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} \\
&\quad + \frac{1}{2} - \gamma_1 + \frac{(2 - \gamma_1)c}{\gamma_1 \sqrt{\gamma_1^2 - 4c}} + \frac{c}{\gamma_1} \\
&= \frac{\gamma_1}{4} + \frac{\gamma_1 \sqrt{\gamma_1^2 - 4c}}{2\gamma_1} + \frac{\gamma_1^2 - 4c}{4\gamma_1^2} - \frac{c}{\gamma_1} + (1 - 2\gamma_1) \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} \\
&\quad + \frac{1}{2} - \gamma_1 + \frac{(2 - \gamma_1)c}{\gamma_1 \sqrt{\gamma_1^2 - 4c}} + \frac{c}{\gamma_1} \\
&= \frac{1}{2} - \frac{\gamma_1}{2} + (1 - \gamma_1) \frac{\sqrt{\gamma_1^2 - 4c}}{2\gamma_1} + \frac{(2 - \gamma_1)c}{\gamma_1 \sqrt{\gamma_1^2 - 4c}} \\
&= \frac{1}{2} \gamma_1 - \frac{1}{2} - \frac{\gamma_1}{2} + \frac{(1 - \gamma_1)(\gamma_1^2 - 4c)}{2\gamma_1 \sqrt{\gamma_1^2 - 4c}} + \frac{(2 - \gamma_1)2c}{2\gamma_1 \sqrt{\gamma_1^2 - 4c}} \\
&= \frac{1}{2} - \frac{\gamma_1}{2} + \frac{\gamma_1^2 - 3c + 4c\gamma_1 + 4c - 2c\gamma_1}{2\gamma_1 \sqrt{\gamma_1^2 - 4c}} \\
&\Rightarrow \frac{d^2 \Pi_{GOV}}{d\gamma_1^2} = \frac{1}{2} - \frac{\gamma_1}{2} + \frac{\gamma_1(1 - \gamma_1) + 2c}{2\sqrt{\gamma_1^2 - 4c}}
\end{align*}
\]

Now we are ready to take the second derivative of the Governments profit
function, which can be solve using the quotient rule:

$$\frac{d^2 \Pi_{GOV}}{d \gamma_1^2} = -\frac{1}{2} + \frac{(1 - 2 \gamma_1)2\sqrt{\gamma_1^2 - 4c} - ((1 - \gamma_1) \gamma_1 + 2c)\frac{2\gamma_1}{\sqrt{\gamma_1^2 - 4c}}}{4(\gamma_1^2 - 4c)}$$

Next we check the condition for when this is negative

$$\frac{d^2 \Pi_{GOV}}{d \gamma_1^2} = -\frac{1}{2} + \frac{(1 - 2 \gamma_1)2\sqrt{\gamma_1^2 - 4c} - ((1 - \gamma_1) \gamma_1 + 2c)\frac{2\gamma_1}{\sqrt{\gamma_1^2 - 4c}}}{4(\gamma_1^2 - 4c)} < 0$$

$$\Rightarrow -\frac{1}{2} < \frac{(2\gamma_1 - 1)2\sqrt{\gamma_1^2 - 4c} + ((1 - \gamma_1) \gamma_1 + 2c)\frac{2\gamma_1}{\sqrt{\gamma_1^2 - 4c}}}{4(\gamma_1^2 - 4c)}$$

(12)

First of all, we can immediately see that the expression on left-hand side is negative. Also, we can observe that the right-hand side of (12) must be positive for any $\gamma_1 \geq \frac{1}{2}$, which does include our solution for $\gamma_1$. However, I will also provide a proof that this is also the case if $\gamma_1 < \frac{1}{2}$. The right-hand side of (12) is positive if:

$$\frac{(2\gamma_1 - 1)2\sqrt{\gamma_1^2 - 4c} + ((1 - \gamma_1) \gamma_1 + 2c)\frac{2\gamma_1}{\sqrt{\gamma_1^2 - 4c}}}{4(\gamma_1^2 - 4c)} > 0$$

$$\Rightarrow (1 - 2 \gamma_1)2\sqrt{\gamma_1^2 - 4c} < ((1 - \gamma_1) \gamma_1 + 2c)\frac{2\gamma_1}{\sqrt{\gamma_1^2 - 4c}}$$

$$\Rightarrow (1 - 2 \gamma_1)(\gamma_1^2 - 4c) < (1 - \gamma_1) \gamma_1^2 + 2\gamma_1c$$

$$\Rightarrow \gamma_1^3 - 4c(1 - 2 \gamma_1) - 2\gamma_1c < 0$$

$$\Rightarrow \gamma_1^3 + 4c - 6\gamma_1c > 0$$

$$\Rightarrow \gamma_1(\gamma_1^2 - 4c - 2c) + 4c > 0$$

$$\Rightarrow \gamma_1(\gamma_1^2 - 4c) + 2c(2 - \gamma_1) > 0$$

The first part of this expression must be positive since any negative number would imply no solution for $\beta_1$ (it would imply a negative expression inside the square root in the expression for $\beta_1$). Furthermore, we see that for any value $\gamma_1 < 2$ the second part is also positive. In addition, as we have already observed, the right-hand side of (16) shows that if $\gamma_1 > \frac{1}{2}$ then the right-hand side of (12) must be positive. Together this must imply that for all values of $\gamma_1$ where $\beta_1$ is defined, the right-hand side of (12) is in fact positive. Since the left-hand side is negative this leads to a negative second-order condition:

$$\frac{d^2 \Pi_{GOV}}{d \gamma_1^2} < 0$$

Thus, we have confirmed that the first-order condition solves the maximisation problem.
Given the negative second-order condition, we can continue solving the first-order condition to find the optimal $\gamma_1$. We rewrite the first-order condition:

$$FOC_{\gamma_1}: \quad (1 - \hat{\beta}_1)^2 \gamma_1 + (1 - 2\gamma_1)(1 - \hat{\beta}_1) - \gamma_1 \frac{d\hat{\beta}_1}{d\gamma_1} [(1 - \hat{\beta}_1)\gamma_1 - (1 - \gamma_1)] = 0$$

Next, we can note that by the derivative of $\hat{\gamma}_1$ with respect to $\gamma_1$ we must have that

$$-\gamma_1 \frac{d\hat{\beta}_1}{d\gamma_1} = \frac{2\hat{\beta}_1(1 - \hat{\beta}_1)}{(1 - 2\hat{\beta}_1)}$$

which we can insert into our first-order condition and solve:

$$\Rightarrow (1 - \hat{\beta}_1)^2 \gamma_1 + (1 - 2\gamma_1)(1 - \hat{\beta}_1) - \frac{2\hat{\beta}_1(1 - \hat{\beta}_1)}{(1 - 2\hat{\beta}_1)} [(1 - \hat{\beta}_1)\gamma_1 - (1 - \gamma_1)] = 0$$

$$\Rightarrow (1 - \hat{\beta}_1)^2 \gamma_1(1 - 2\hat{\beta}_1) + 2\hat{\beta}_1(1 - \hat{\beta}_1)^2 \gamma_1 + (1 - 2\hat{\beta}_1)(1 - 2\gamma_1)(1 - \hat{\beta}_1)$$

$$+ 2\hat{\beta}_1(1 - \hat{\beta}_1)(1 - \gamma_1) = 0$$

$$(1 - \hat{\beta}_1)\gamma_1(1 - 2\hat{\beta}_1) + 2\hat{\beta}_1(1 - \hat{\beta}_1)\gamma_1 + (1 - 2\hat{\beta}_1)(1 - 2\gamma_1) + 2\hat{\beta}_1(1 - \gamma_1) = 0$$

$$(1 - \hat{\beta}_1)\gamma_1 + (1 - 2\hat{\beta}_1)(1 - 2\gamma_1) + 2\hat{\beta}_1(1 - \gamma_1) = 0$$

$$(1 - \hat{\beta}_1)\gamma_1 + (1 - 2\gamma_1) - 2\hat{\beta}_1(1 - 2\gamma_1 - 1 + \gamma_1) = 0$$

$$(1 - \hat{\beta}_1)\gamma_1(1 - 2\gamma_1) + 2\hat{\beta}_1 \gamma_1 = 0$$

$$\gamma_1 - \hat{\beta}_1 \gamma_1 + 1 - 2\gamma_1 + 2\hat{\beta}_1 \gamma_1 = 0$$

$$-\gamma_1 + \hat{\beta}_1 \gamma_1 + 1 = -\gamma_1(1 - \hat{\beta}_1) + 1 = 0$$

I define the optimal $\gamma_1$ that solves this as $\hat{\gamma}_1$:

$$\Rightarrow \hat{\gamma}_1 = \frac{1}{1 - \hat{\beta}_1}$$

We can solve this further by inserting for $\hat{\beta}_1$.

$$\Rightarrow 1 = \hat{\gamma}_1(1 - \hat{\beta}_1) = \hat{\gamma}_1 \left(1 - \left(1 - \frac{1}{2} - \frac{\sqrt{\hat{\gamma}_1^2 - 4c}}{2\hat{\gamma}_1}\right)\right) = \hat{\gamma}_1 \left(\frac{1}{2} + \frac{\sqrt{\hat{\gamma}_1^2 - 4c}}{2}\right)$$

$$\Rightarrow 2 = \hat{\gamma}_1 + \frac{\sqrt{\hat{\gamma}_1^2 - 4c}}{2}$$

$$2 - \hat{\gamma}_1 = \frac{\sqrt{\hat{\gamma}_1^2 - 4c}}{2}$$

$$4 - 4\hat{\gamma}_1 + \hat{\gamma}_1^2 = \hat{\gamma}_1^2 - 4c$$

$$4 - 4\hat{\gamma}_1 = -4c$$

$$\hat{\gamma}_1 = 1 + c$$
Next we need to show that $\gamma_0 = \frac{1}{2}$. However, to find $\gamma_0$ we must first find $\hat{\beta}_1$. This we can find either by using the fact that $\hat{\gamma}_1 = \frac{1}{1-\hat{\beta}_1} = 1 + c$, or by inserting $\hat{\gamma}_1$ into $\hat{\beta}_1$:

$$\hat{\beta}_1 = \frac{1}{2} - \frac{\sqrt{\hat{\gamma}_1^2 - 4c}}{2\hat{\gamma}_1} = \frac{1}{2} - \frac{\sqrt{(1 + c)^2 - 4c}}{2(1 + c)}$$

$$\Rightarrow \frac{1}{2} - \frac{\sqrt{1 + 2c + c^2 - 4c}}{2(1 + c)} = \frac{1}{2} - \frac{\sqrt{1 - 2c + c^2}}{2(1 + c)} = \frac{1}{2} - \frac{\sqrt{(1 - c)^2}}{2(1 + c)}$$

$$\Rightarrow \frac{1}{2} - \frac{1 - c}{2(1 + c)} = \frac{1 + c}{2(1 + c)} - \frac{1 - c}{2(1 + c)} = \frac{2c}{2(1 + c)}$$

$$\hat{\beta}_1 = \frac{c}{1 + c} \quad (13)$$

Before continuing, we make a small note that this implies the fairly intuitive result that a higher $c$ must involve a higher compensation to the Private Financier. In other words, the $\hat{\beta}_1$ is increasing in $c$, which can be confirmed by taking the derivative with respect to $c$:

$$\frac{d\hat{\beta}_1}{dc} = \frac{1 + c - c}{(1 + c)^2} = \frac{1}{(1 + c)^2} > 0$$

Finally, we insert this $\hat{\beta}_1$ for the optimal $\hat{\gamma}_0$ we get:

$$\hat{\gamma}_0 = -\frac{[(1 - \hat{\beta}_1)\hat{\gamma}_1]^2}{2} = -\frac{\left(1 - \frac{c}{1 + c}\right)(1 + c)}{2} = \frac{1}{2}$$

which completes the proof of Proposition 4.

**Proposition 5.** The solution $\hat{t}(Q)$ in Proposition 4 implies no distortions in the optimal infrastructure quality effort made by the PPP Consortium.

**Proof.** The first-best infrastructure quality effort made by the PPP Consortium in equation (2) shows that the most efficient effort made by PPP Consortium is $a = 1$. The effort in this scenario becomes:

$$a = (1 - \hat{\beta}_1)\hat{\gamma}_1 = \left(1 - \frac{c}{1 + c}\right)(1 + c) = 1$$

Thus, there is no distortion from the first-best case. 

Next, we take a closer look at the conditions for this solution.
Proposition 6. The condition for Propositions 3-5, $\beta_1 > \tilde{\beta}_1$, implies that

$$\frac{c}{1 + c} > g(c, \theta) \quad \text{or} \quad c > \frac{g(c, \theta)}{1 - g(c, \theta)} \quad (14)$$

Furthermore, for the solution in Proposition 4 we need the following condition

$$f(\theta) \geq c + \frac{c^2}{2} \quad \text{or} \quad c \leq \sqrt{1 + 2f(\theta)} - 1 \quad (15)$$

to ensure no deviation from the PPP Consortium when deciding on $\beta_1$.

Proof. The condition for $\beta_1 > \tilde{\beta}_1$ we find simply by noting that:

$$\tilde{\beta}_1 = \frac{c}{1 + c} > g(c, \theta) = \beta_1$$

Next we must check whether the PPP Consortium would deviate from the $\beta_1$ by not offering any contract to Private Financier. If PPP Consortium deviates and set $\beta_1 = 0$, their profit becomes:

$$\Pi_{PPP} = \gamma_0 + a\tilde{\beta}_1 - \frac{a^2}{2} - f(\theta)$$

$$= -\frac{1}{2} + a(1 + c) - \frac{a^2}{2} - f(\theta) = -\frac{1}{2} + (1 + c)^2 - \frac{(1 + c)^2}{2} - f(\theta)$$

$$= -\frac{1}{2} + \frac{(1 + c)^2}{2} - f(\theta) = -\frac{1}{2} + \frac{1 + 2c + c^2}{2} - f(\theta)$$

$$= \frac{2c + c^2}{2} - f(\theta) = c + \frac{c^2}{2} - f(\theta)$$

The condition for no deviation is that this must be non-positive since we know that in this solution with complying PPP Consortium their profit is zero. We have already assumed that $c < f(\theta)$ (if not it would never be efficient to have insurance from Private financier). However, if this is barely the case and we have the scenario in which

$$f(\theta) \in \left(c, c + \frac{c^2}{2}\right)$$

then the PPP Consortium could make a profit by not offering a contract to the Private Financier. Thus, we must restrict the assumption slightly more:

$$f(\theta) \geq c + \frac{c^2}{2} \iff c \leq \sqrt{1 + 2f(\theta)} - 1$$

Next we will take note of a few consequences of these conditions.
Proposition 7. The conditions proposed in Proposition 6 is easier to satisfy when \( c \) is very small and when the exogenous risk becomes very large.

Proof. To prove Proposition 7, we begin with latter part of this proposition. Note that the exogenous risk can be interpreted as the variance of \( \theta, \text{Var}(\theta) \). If this increases then the disutility of \( \theta \) increases as well. Now, it follows directly from condition (15) that a given \( c \), an increasing \( f(\theta) \) must make condition (15) easier to satisfy. Furthermore, we know that

\[
\frac{dg(c, \theta)}{d\text{Var}(\theta)} < 0
\]

Thus, increasing the exogenous risk must imply that the \( g(c, \theta) \) is reduced which lowers the right-hand side (of both versions) of condition (14), making it easier to satisfy. Thus, it is more likely that both condition (14) and (15) holds when \( f(\theta) \) becomes large.

To prove that Proposition 6 is easier to satisfy when \( c \) becomes very small we note that for a given level of \( f(\theta) \) and a decreasing \( c \), condition (15) is more likely to hold. To show the same result for condition (14) is a bit more complicated. Suppose there exists a \( c' \) and \( \tilde{\beta}_1' = g(c', \theta) \) such that

\[
c' = \frac{\tilde{\beta}_1'}{1 - \tilde{\beta}_1'} = \frac{g(c', \theta)}{1 - g(c', \theta)}
\]

By the definition of \( \tilde{\beta}_1 \) it must also imply that \( c' = f(\tilde{\beta}_1' \theta) = f(g(c', \theta) \theta) \). Now, let suppose that there is a shift in the cost, reducing it to \( c'' \) such that \( c' = \alpha c'' \), for \( \alpha \in (0, 1) \). Then, by definition we have a corresponding

\[
\tilde{\beta}_1'' = g(c'' \theta) < g(c' \theta) = \tilde{\beta}_1'
\]

where \( c'' = f(\tilde{\beta}_1'' \theta) \). By the convexity of the disutility function (concavity of the utility) the following inequality must hold

\[
f(\alpha \tilde{\beta}_1'' \theta) < \alpha f(\tilde{\beta}_1'' \theta) = \alpha c'' = c' = f(\tilde{\beta}_1' \theta)
\]

Since the function \( f(\cdot) \) is increasing in \( \beta_1 \), it must then be the case that

\[
f(\alpha \tilde{\beta}_1'' \theta) < f(\tilde{\beta}_1' \theta) \quad \Rightarrow \quad \alpha \tilde{\beta}_1'' < \tilde{\beta}_1'
\]

I use this fact and the fact that \( \tilde{\beta}_1' > \tilde{\beta}_1'' \) from the original assumption

\[
\alpha c'' = c' = \frac{\tilde{\beta}_1'}{1 - \tilde{\beta}_1'} > \alpha \frac{\tilde{\beta}_1''}{1 - \tilde{\beta}_1''} > \alpha \frac{\tilde{\beta}_1''}{1 - \tilde{\beta}_1''}
\]

which must imply that

\[
c'' > \frac{\tilde{\beta}_1''}{1 - \tilde{\beta}_1''}
\]
Thus, for any level of $c$ in which the lower limit of $c$ in the original condition was binding, any $c$ lower than this would strictly satisfy the condition. This means that it is more likely that both (14) and (15) is satisfied when $c$ becomes small.

Next, we can confirm that the Government prefers this solution over the solution without a Private Financier. We know that the Government extracts the entire surplus since by definition of $\hat{\beta}_1$ the Private Financier receives zero profit and by our $\hat{\gamma}_0$ the PPP Consortium also receives zero profit. Then since the Government extract all surplus and there are no distortions in PPP Consortiums effort it follows directly that this solution including the Private Financier must be preferred. However, we can also show it directly. The Government’s profit becomes:

$$\Pi_{GOV} = \frac{[(1 - \hat{\beta}_1)\hat{\gamma}_1]^2}{2} + (1 - \hat{\beta}_1)(1 - \hat{\gamma}_1)\hat{\gamma}_1 - (1 - q)I$$

$$\Rightarrow \quad \Pi_{GOV} = \frac{a^2}{2} + a(1 - \hat{\gamma}_1) - I = \frac{1}{2} - c - I$$

This will be preferred to a solution without a Private Financier if

$$\Pi_{GOV} = \frac{1}{2} - c - I \geq \frac{1}{2} - f(\theta) - I = \Pi_{GOV}$$

which holds for all $f(\theta) \geq c$, which is the minimum condition for Private Finance to be considered and for the sake of this thesis I assume that this always holds. Thus, we do not need to worry about any condition for when the Government deviates from using Private Finance

### 4.3.2 Governments optimisation when $\beta^*_1 = \hat{\beta}_1 > \tilde{\beta}_1$

In this subsection I will show that if $\hat{\beta}_1 > \tilde{\beta}_1$ we must have a corner solution for $q$, where it is optimal with full private finance ($q = 1$). But first, I will make the claim that when $\hat{\beta}_1 > \tilde{\beta}_1$ then it must be the case that the participation constraint of the Private Financier is not only satisfied, but also non-binding. This is a fairly intuitive result, as I assume here that the optimal $\beta^*_1$ offered by the PPP Consortium to the Private Financier is higher than the $\beta_1$ necessary for making the participation constraint of the Private Financier binding. However, this is not a sufficient proof, since the gross income of the Private Financier is not increasing in $\beta_1$ for all levels of $\beta_1$ (due to effort $a = (1 - \beta_1)\gamma_1$, the maximum revenue Private Financier can get is when $\beta_1 = \frac{1}{2}$).

To show that the participation constraint of the Private Financier is non-binding when $\hat{\beta}_1 > \tilde{\beta}_1$ we can simply start from this condition. In this...
scenario, we know that the optimal $\beta_1^* = \hat{\beta}_1$. Thus we rewrite

$$\beta_1^* = \hat{\beta}_1 > \hat{\beta}_1 \iff \hat{\beta}_1 > \frac{1}{2} - \frac{\sqrt{\gamma_1^2 - 4(c + q(1 + \rho)I)}}{2\gamma_1}$$

$$\Rightarrow \quad 2\gamma_1 \left[ \hat{\beta}_1 - \frac{1}{2} \right] > -\sqrt{\gamma_1^2 - 4(c + q(1 + \rho)I)}$$

$$2\gamma_1 \left[ \frac{1}{2} - \hat{\beta}_1 \right] < \sqrt{\gamma_1^2 - 4(c + q(1 + \rho)I)}$$

$$4\gamma_1^2 \left[ \hat{\beta}_1 - \frac{1}{2} \right]^2 < \gamma_1^2 - 4(c + q(1 + \rho)I)$$

$$4\gamma_1^2 \left[ \hat{\beta}_1^2 - \frac{1}{4} \right] < \gamma_1^2 - 4(c + q(1 + \rho)I)$$

$$\gamma_1^2 (4\hat{\beta}_1^2 - 4\hat{\beta}_1 + 1) < \gamma_1^2 - 4(c + q(1 + \rho)I)$$

$$\Rightarrow \quad \Pi_{PF} = \gamma_1^2 \hat{\beta}_1 (1 - \hat{\beta}_1) - c - q(1 + \rho)I > 0$$

Thus, the participation constraint for the Private Financier is satisfied and non-binding.

**Proposition 8.** When the optimal contract offered by the Government results in $\beta_1 > \hat{\beta}_1$ for the PPP Consortium in stage 2, then it must be the case that the Government offers full private finance, $q = 1$.

**Proof.** Suppose by contradiction that there exists an optimal $\hat{q} < 1$ in the scenario in which $\beta_1^* = \hat{\beta}_1 > \hat{\beta}_1$. Then by our proof above, we know that

$$\Pi_{PF} = \gamma_1^2 \hat{\beta}_1 (1 - \hat{\beta}_1) - c - \hat{q}(1 + \rho)I > 0$$

Thus, there must exists a $q^* = \hat{q} + \epsilon \leq 1$, where $\epsilon > 0$, such that

$$\Pi_{PF} = \gamma_1^2 \hat{\beta}_1 (1 - \hat{\beta}_1) - c - q^*(1 + \rho)I = 0$$

But, since our Government’s objective is strictly increasing in $q$, we must have

$$\Pi_{GOV}(q = q^*) = \Pi_{GOV}(q = \hat{q} + \epsilon) > \Pi_{GOV}(q = \hat{q})$$

which would mean that for the Government $q^*$ gives a higher profit than $\hat{q}$ and neither are violating any constraints in the optimisation problem. This means that $\hat{q}$ cannot be an optimal solution. Thus, any $q < 1$ cannot be the optimal solution when $\beta_1^* = \hat{\beta}_1$ which means that the optimal solution when $\beta_1 > \hat{\beta}_1$ must be $q = 1$. 

\[\square\]
By the fact that this participation constraint for the Private Financier must be satisfied and that \( q = 1 \), the Government’s maximisation problem is now reduced to

\[
\max_{\gamma_0, \gamma_1} \Pi_{GOV} = -\gamma_0 + (1 - \gamma_1)(1 - \hat{\beta}_1)
\]

subject to

\[
P_{PPP} : \quad \Pi_{PPP} = \gamma_0 + \frac{(1 - \hat{\beta}_1)\gamma_1^2}{2} \geq 0
\]

and assuming that

\[
\hat{\beta}_1 < \hat{\beta}_1 \iff \frac{1}{2} - \frac{\sqrt{\gamma_1^2 - 4(c + (1 + \rho)I)}}{2\gamma_1} < g(c, \theta)
\]

The solution to this is summarized in the following proposition.

**Proposition 9.** When the optimal contract offered by the Government to the PPP Consortium results in \( \hat{\beta}_1 > \hat{\beta}_1 \) for the PPP Consortium in stage 2, then the optimal contract must be

\[
i(Q) = \hat{\gamma}_0 + \hat{\gamma}_1 Q = -\frac{1}{2} \left[ \frac{1 - g(c, \theta)}{1 + g(c, \theta)} \right]^2 + \frac{1}{1 + \hat{\beta}_1} Q
\]  

(16)

**Proof.** In order to prove Proposition 9, we first note that there must be equality in the participation constraints of the PPP Consortium by the exact same argument as in the previous sections. We insert for the \( \gamma_0 \) solving this binding participation constraint for the PPP Consortium and rewrite the maximisation problem to:

\[
\max_{\gamma_1} \Pi_{GOV} = \frac{(1 - \hat{\beta}_1)\gamma_1^2}{2} + (1 - \gamma_1)(1 - \hat{\beta}_1)
\]

assuming that

\[
\hat{\beta}_1 < \hat{\beta}_1 \iff \frac{1}{2} - \frac{\sqrt{\gamma_1^2 - 4(c + (1 + \rho)I)}}{2\gamma_1} < g(c, \theta)
\]

We get the first-order and second-order conditions of

\[
FOC_{\gamma_1} : \quad (1 - \hat{\beta}_1)^2\gamma_1 + (1 - 2\gamma_1)(1 - \hat{\beta}_1) = 0
\]

\[
SOC_{\gamma_1} : \quad (1 - \hat{\beta}_1) - 2(1 - \hat{\beta}_1)(1 - \hat{\beta}_1) = 0
\]

Once again, the negative second-order condition confirms that the optimal \( \gamma_1 \) is the one satisfying the first-order condition. I define the optimal \( \gamma_1 \) in this scenario as \( \hat{\gamma}_1 \) and we can solve the first-order condition to find the \( \hat{\gamma}_1 \):

\[
FOC_{\hat{\gamma}_1} : \quad (1 - \hat{\beta}_1)^2\hat{\gamma}_1 + (1 - 2\hat{\gamma}_1)(1 - \hat{\beta}_1) = 0
\]

\[
(1 - \hat{\beta}_1)\hat{\gamma}_1 + (1 - 2\hat{\gamma}_1) = 0
\]

\[
(1 - \hat{\beta}_1 - 2)\hat{\gamma}_1 + 1 = 0
\]
\[ \Rightarrow \hat{\gamma}_1 = \frac{1}{1 + \beta_1} \]

By this solution for \( \hat{\gamma}_1 \), the optimal \( \tilde{\gamma}_0 \) becomes:

\[ \tilde{\gamma}_0 = -\frac{[(1 - \hat{\beta}_1)\hat{\gamma}_1]^2}{2} = -\frac{1}{2} \left[ 1 - \frac{1}{1 + \beta_1} \right]^2 = -\frac{1}{2} \left[ 1 - \frac{1 - g(c, \theta)}{1 + g(c, \theta)} \right]^2 \]

This, combined with \( \hat{\beta}_1 = g(c, \theta) \) yield the result (16) in Proposition 9. \( \square \)

**Proposition 10.** The solution in Proposition 9 involves an efficiency distortion in the optimal level of effort in infrastructure quality made by the PPP Consortium. This distortion is increasing in the level of revenue and risk sharing agreement needed to provide incentives to the Private Financier, but independent of the private finance premium.

**Proof.** We can show the efficiency distortion from the basic case \((e^{FB} = 1)\) by observing that the optimal level of \( a \) must be:

\[ a = (1 - \hat{\beta}_1)\gamma_1 = \frac{1 - \hat{\beta}_1}{1 + \beta_1} = \frac{1 - g(c, \theta)}{1 + g(c, \theta)} \]

which clearly shows that there is a lower effort than originally optimal effort and that the effort is further decreasing in \( g(c, \theta) \). However, we can also observe that the distortion is independent of \( \rho \). \( \square \)

Based on proposition 9 and 10, we can see that the Government’s profit becomes:

\[ \Pi_{GOV} = \frac{[(1 - \hat{\beta}_1)]^2}{2(1 + \hat{\beta}_1)^2} + \left( 1 - \frac{1}{1 + \beta_1} \right) \frac{1}{1 + \beta_1} (1 - \hat{\beta}_1) \]

\[ \Rightarrow \frac{1 - \hat{\beta}_1}{(1 + \hat{\beta}_1)^2} \left( \frac{1}{2} (1 - \hat{\beta}_1) + (1 + \hat{\beta}_1) - 1 \right) \]

\[ \Rightarrow \frac{1 - \hat{\beta}_1}{(1 + \hat{\beta}_1)^2} \left( \frac{1}{2} (1 + \hat{\beta}_1) \right) = \frac{1}{2} \left( \frac{1 - \hat{\beta}_1}{1 + \beta_1} \right) \]

\[ \Pi_{GOV} = \frac{1}{2} \left( \frac{1 - \hat{\beta}_1}{1 + \beta_1} \right) = \frac{1}{2} \left( \frac{1 - g(c, \theta)}{1 + g(c, \theta)} \right) \]

Furthermore, we remember that this solution implies that the Government does not extract the entire profit from the PPP arrangement and make note of the profit of the Private Financier being, quite unsurprisingly:

\[ \Pi_{PF} = \frac{\hat{\beta}_1(1 - \hat{\beta}_1)}{(1 + \beta_1)^2} - c - (1 + \rho)I > 0 \]
Going forward, there are two different assumptions one can make related to how the PPP Consortium must react to the Government inducing private finance \((q > 0)\). One is to assume that when the Government sets \(q = 1\) (or any \(q > 0\)), the PPP Consortium then must include Private Finance and does not have the opportunity to deviate by not including a Private Financier. However, if this is the case it makes our assumption that the Private Financier does not have any market power weaker. If the Private Financier knows that the PPP Consortium are in the need of finance, it might level the market power and negotiation power somewhat. On the other hand, one might assume that the PPP Consortium always have the opportunity to resort to being financed by the Government. If this is the scenario, one must make a slightly unrealistic assumption that the Government cannot claim any renegotiation of the contract. If the former is true, there is no risk of non-compliance from the PPP Consortium, if the latter is true we must check the conditions for when PPP Consortium does not deviate by not offering a financial contract to Private Financier. Although I would prefer the former assumption over the latter, I include in the analysis the condition for compliance from the PPP Consortium.

Next we take a closer look at the conditions for this solution all together.

**Proposition 11.** The conditions for Proposition 9 are

\[
\frac{g(c, \theta)(1 - g(c, \theta))}{(1 + g(c, \theta))^2} > c + (1 + \rho)I \quad (17)
\]

\[
\frac{g(c, \theta)(1 - g(c, \theta))}{(1 + g(c, \theta))^2} \leq 2f(\theta) \quad (18)
\]

\[
\frac{g(c, \theta)}{1 + g(c, \theta)} \leq f(\theta) + I \quad (19)
\]

Equation (17) ensures the condition with \(\hat{\beta}_1 > \tilde{\beta}_1\), i.e. that the Private Financier extracts some profit from this arrangement. Equation (18) is required for incentive compatibility with PPP Consortium’s optimal behaviour, while (19) refers to the Government preferring this solution over the basic model without a Private Financier.

**Proof.** First we simply note that the condition for our assumption of \(\hat{\beta}_1 > \tilde{\beta}_1\) implies that the participation constraint for the Private Financier is larger than zero:

\[
\hat{\beta}_1 < \tilde{\beta}_1 \iff \tilde{\beta}_1 (1 - \beta_1) - c - (1 + \rho)I > 0
\]

\[
\Rightarrow \frac{\hat{\beta}_1 (1 - \tilde{\beta}_1)}{(1 + \beta_1)^2} - c - (1 + \rho)I > 0
\]

\[
\Rightarrow \frac{\hat{\beta}_1 (1 - \tilde{\beta}_1)}{(1 + \beta_1)^2} > c + (1 + \rho)I
\]
or alternatively
\[
g(c, \theta)(1 - g(c, \theta)) \frac{1}{(1 + g(c, \theta))^2} > c + (1 + \rho)I
\]
This confirms (17).

Next we can check the condition for PPP Consortium not deviating. If
they deviate, their profit becomes:
\[
\Pi_{PPP} = \gamma_0 + \tilde{\gamma}_1 a - \frac{a^2}{2} - f(\theta) = \gamma_0 + \frac{\tilde{\gamma}_1^2}{2} - f(\theta)
\]
\[
\Rightarrow - \frac{[(1 - \tilde{\beta}_1)]^2}{2(1 + \beta_1)^2} + \frac{1}{2(1 + \beta_1)^2} - f(\theta) = \frac{1 - (1 - \tilde{\beta}_1 + \tilde{\beta}_1^2)}{2(1 + \beta_1)^2} - f(\theta)
\]
\[
\Pi_{PPP} = \frac{\tilde{\beta}_1(1 - \tilde{\beta}_1)}{2(1 + \beta_1)^2} - f(\theta)
\]
Thus, the condition for the solution in this subsection, given an assumption
of PPP Consortium actually having the possibility to deviate, is that
\[
f(\theta) \geq \frac{\tilde{\beta}_1(1 - \tilde{\beta}_1)}{2(1 + \beta_1)^2} = \frac{g(c, \theta)(1 - g(c, \theta))}{2(1 + g(c, \theta))^2}
\]
which confirms (18). Note that we can combine this condition with the
previous condition for being in this scenario, and we find the condition that
\[
c + (1 + \rho)I < \frac{g(c, \theta)(1 - g(c, \theta))}{(1 + g(c, \theta))^2} \leq 2f(\theta)
\]
Finally, we confirm that it is optimal for the Government to use Private
Finance as oppose to the First Best solution in the basic model. That is, we
need the following condition to hold
\[
\Pi_{GOV} = \frac{1}{2} \left( \frac{1 - g(c, \theta)}{1 + g(c, \theta)} \right) \geq \frac{1}{2} - f(\theta) - I = \Pi_{GOV}^1
\]
\[
\Rightarrow \frac{1}{2} \left( \frac{1 - g(c, \theta) - (1 + g(c, \theta))}{1 + g(c, \theta)} - 1 \right) \geq -f(\theta) - I
\]
\[
\frac{1}{2} \left( \frac{-2g(c, \theta)}{1 + g(c, \theta)} \right) \geq -f(\theta) - I
\]
\[
\frac{g(c, \theta)}{1 + g(c, \theta)} \leq f(\theta) + I
\]
This confirms condition (19).
Proposition 12. The conditions proposed in Proposition 11 requires an interval of the investment cost \( I \) satisfying both (17) and (19). The conditions for this solution are also easier to satisfy when the private finance premium, \( \rho \), is lower. Furthermore, a decreasing \( c \) makes (17) and (19) easier to satisfy but (18) harder to satisfy. Equation (17) and (19) is also easier to satisfy when the disutility of the exogenous shock is higher (or if the variance of the shock is large). Furthermore, (18) is also most likely easier to satisfy if this exogenous shock is large as well.

Proof. It is straightforward to see that a higher \( I \) makes the condition (19) easier to satisfy. Similarly it is easy to see that the right-hand side of (19) increases when the risk increases (that is when the \( \text{Var}(\theta) \) increases). To prove that it must be the case that it is easier to satisfy this entire condition when \( f(\theta) \) increases we must then show that the left-hand side decreases in \( \text{Var}(\theta) \).

\[
\frac{d}{d\text{Var}(\theta)} \left[ \frac{g(c,\theta)\left(1+g(c,\theta)\right)}{1+g(c,\theta)} \right] \left[ 1 + g(c,\theta) - g(c,\theta) \right] \frac{dg(c,\theta)}{d\text{Var}(\theta)} = \left[ \frac{1}{1+g(c,\theta)^2} \right] \frac{dg(c,\theta)}{d\text{Var}(\theta)}
\]

Since we know that the \( g(c,\theta) \) is decreasing in \( \text{Var}(\theta) \), this expression must be negative. Thus, (19) is easier to satisfy when the exogenous risk is increasing.

Furthermore, it is easy to see that expression (17) becomes easier to satisfy when \( \rho \) and \( I \) decreases. I can also show that the condition is also easier to satisfy when \( c \) becomes smaller and when the exogenous risk is increases. To prove that condition (17) is easier to satisfy when \( c \) is reduced and \( \text{Var}(\theta) \) is increased, we first note that the right-hand side is obviously decreasing when \( c \) is decreasing and it is independent of the risk. Thus, if we can prove that the left-hand side is increasing when \( c \) is decreased and \( \text{Var}(\theta) \) is increased then we can conclude that the condition is strictly easier to satisfy for a decreasing \( c \) and increasing \( \text{Var}(\theta) \). We rewrite the expression on the right hand side:

\[
g(c,\theta)(1-g(c,\theta)) \Rightarrow \frac{g(c,\theta)-g(c,\theta)^2}{(1+g(c,\theta)+g(c,\theta)^2)}
\]

\[
\Rightarrow \frac{[g(c,\theta)-g(c,\theta)^2]}{(1+g(c,\theta)+2g(c,\theta)^2)+[g(c,\theta)-g(c,\theta)^2]}
\]

This simple rewritten expression shows that when the expression \((1+g(c,\theta)+2g(c,\theta)^2)\) is decreasing, the entire expression (left-hand side of condition (17)) must increase. That is, when \( g(c,\theta) \) decreases, the left-hand side of condition (17) increases.

We know that \( g(c,\theta) \) is increasing in \( c \) which allows us to write that

\[
\frac{d}{dc} \left[ 1 + g(c,\theta) + 2g(c,\theta)^2 \right] < 0
\]
which implies that a decrease in $c$ must imply an increase of the right-hand side.

Likewise, we know that the $\beta_1$ needed to satisfy the the $c = f(\beta_1 \theta)$ is decreasing in $\text{Var}(\theta)$, that is:

$$\frac{d\beta_1}{d\text{Var}(\theta)} = \frac{dg(c, \theta)}{d\text{Var}(\theta)} < 0$$

which means that an increase in $\text{Var}(\theta)$ decreases $g(c, \theta)$ and which leads to an increase in the right-hand side of equation (17). Thus, (17) is easier to satisfy when $c$ decreases and the exogenous risk increases.

Note that the by the last proof, it must be the case that (18) is harder to satisfy when $c$ decreases. Since the right-hand side of equation (18) is independent of $c$, the only effect is through the left-hand side which by the previous proof must be increasing in a decreasing $c$. Interestingly, we cannot conclude that equation (18) is easier to satisfy when $f(\theta)$ is increasing. That is, when the disutility of the exogenous risk (or $\text{Var}(\theta)$) is increasing. Although this is a very natural and likely assumption to make, both arguments on each side is increasing in $\text{Var}(\theta)$. To see this, simply note that by the previous proof, the left-hand side of equation (18) must be increasing in $\text{Var}(\theta)$ and that the right-hand side of equation (18) is obviously increasing in the exogenous risk as well. Although I have not made any specification of the disutility function such that we can prove it, I do find it very probable that equation (18) is likely to hold when $f(\theta)$ or $\text{Var}(\theta)$ increases. I could explore this point further, with examples of utility functions and compare the results, however I settle for simply assuming that this holds for now.

Finally in this subsection, I will make a note on the conditions for both cases, $\hat{\beta}_1 > \beta_1$ (and $q = 0$) and $\hat{\beta}_1 < \beta_1$ (and $q = 1$). Although it is very natural conclusion that springs out from the assumption I started with I will now show that it will never be the case that both conditions hold. Suppose by contradiction that there exists a $c$ in which both conditions holds. Then from the two conditions we know that it must be the case that

$$\frac{g(c, \theta)}{1 - g(c, \theta)} < c < \frac{g(c, \theta)(1 - g(c, \theta))}{(1 + g(c, \theta))^2} - (1 + \rho)I$$

$$\Rightarrow \frac{g(c, \theta)}{(1 - g(c, \theta))^2} < \frac{c}{1 - g(c, \theta)} < \frac{g(c, \theta)}{(1 + g(c, \theta))^2} - \frac{(1 + \rho)I}{1 - g(c, \theta)}$$

However, this is a contradiction since for any positive valued $g(c, \theta)$ it must also be the case that

$$\frac{g(c, \theta)}{(1 - g(c, \theta))^2} > \frac{g(c, \theta)}{(1 + g(c, \theta))^2} - \frac{(1 + \rho)I}{1 - g(c, \theta)}$$
Thus, there does not exist any $c$ satisfying both conditions. Furthermore, this suggests that there exists a relationship between $c$, $f(\theta)$ and $(1 + \rho)I$ that does not satisfy either one of the conditions. This brings us over to the final scenario, where $\hat{\beta}_1 = \hat{\beta}_1$.

### 4.3.3 Governments optimisation when $\beta_1^* = \hat{\beta}_1 = \hat{\beta}_1$

In this scenario I allow for an interior solution of $q$, that is $q \in [0,1]$. The optimal $\beta_1^* = \hat{\beta}_1 = g(c, \theta)$ and the government can set $q$ such that the participation constraint of the Private Financier binds such that we also get $\beta_1^* = \beta_1 = \hat{\beta}_1$. The original maximisation problem, only inserted for optimal effort $a = (1 - \beta_1^*)\gamma_1$ is

$$\max_{\gamma_0, \gamma_1, q} \Pi_{GOV} = -\gamma_0 + (1 - \gamma_1)\gamma_1(1 - \beta_1^*) - (1 - q)I$$

subject to

$$PC_{PF} : \quad \Pi_{PF} = \beta_1^*\gamma_1^2(1 - \beta_1^*) - c - q(1 + \rho)I \geq 0$$

$$PC_{PPP} : \quad \Pi_{PPP} = \gamma_0 + \frac{[(1 - \beta_1^*)\gamma_1]^2}{2} \geq 0$$

We can summarize the result of this maximisation problem in the following proposition:

**Proposition 13.** There exists a level of private financing, $q^*$, defined as

$$q^* = \frac{g(c, \theta)(1 - g(c, \theta))(1 + \rho)}{I(1 - g(c, \theta) + (1 + g(c, \theta))\rho)^2} - \frac{c}{(1 + \rho)I}$$

which is the optimal level of private finance if it satisfies $q^* \in [0,1]$. Then the optimal contract offered by the Government is

$$t^*(Q) = -\frac{1}{2} \left[ \frac{(1 - g(c, \theta)) + (1 - g(c, \theta))\rho}{(1 - g(c, \theta) + (1 + g(c, \theta))\rho)^2} + \frac{Q}{1 - g(c, \theta) \frac{1 - \rho}{1 + \rho}} \right]$$

**Proof.** Under the assumption of $\beta_1^* = \hat{\beta}_1 = \hat{\beta}_1$ we can claim equality in both participation constraints, since any solution with inequality in any of the two constraint can be improved upon by either reducing $\gamma_0$ or increasing $q$, or both, without violating the constraints. Thus, we can rewrite the participation constraints as

$$PC_{PF} : \quad qI = \frac{\beta_1^*\gamma_1^2(1 - \beta_1^*) - c}{1 + \rho}$$

$$PC_{PPP} : \quad -\gamma_0 = \frac{[(1 - \beta_1^*)\gamma_1]^2}{2}$$
and insert both into the objective function

$$\max_{\gamma_1} \Pi_{GOV} = \frac{[(1 - \beta_1^*)\gamma_1]^2}{2} + (1 - \gamma_1)\gamma_1(1 - \beta_1^*) - I + \frac{\beta_1^*\gamma_1^2(1 - \beta_1^*)}{1 + \rho} - c$$

This leads to the following first-order and second-order conditions

$$\text{FOC}_{\gamma_1} : \quad (1 - \beta_1^*)^2\gamma_1 + (1 - 2\gamma_1)(1 - \beta_1^*) + \frac{2\gamma_1\beta_1^*(1 - \beta_1^*)}{1 + \rho} = 0$$

$$\text{SOC}_{\gamma_1} : \quad (1 - \beta_1^*)^2 - 2(1 - \beta_1^*) + \frac{2\beta_1^*(1 - \beta_1^*)}{1 + \rho} = 0$$

$$\Rightarrow \quad (1 - \beta_1^*)\left(1 - \beta_1^*\frac{1 + \rho}{1 + \rho} - 2 + \frac{2\beta_1^*}{1 + \rho}\right) = (1 - \beta_1^*)\left(-1 + \frac{2\beta_1^* - \beta_1^* - \rho\beta_1^*}{1 + \rho}\right)$$

$$\Rightarrow \quad - (1 - \beta_1^*)\left(1 - \beta_1^*\frac{1 - \rho}{1 + \rho}\right) < 0$$

The negative second-order condition holds for any non-negative $\rho$ which confirms that the first-order condition must yield the optimal $\gamma_1^*$. We solve for this optimal $\gamma_1^*$ from the first-order condition:

$$\text{FOC}_{\gamma_1^*} : \quad (1 - \beta_1^*)^2\gamma_1^* + (1 - 2\gamma_1^*)(1 - \beta_1^*) + \frac{2\gamma_1^*\beta_1^*(1 - \beta_1^*)}{1 + \rho} = 0$$

$$\Rightarrow \quad (1 - \beta_1^*)\gamma_1^* + 1 - 2\gamma_1^* + \frac{2\gamma_1^*\beta_1^*(1 - \beta_1^*)}{1 + \rho} = 0$$

$$\Rightarrow \quad 1 = \gamma_1^*\left(-1 + \beta_1^* + 2 - \frac{2\beta_1^*}{1 + \rho}\right) = \gamma_1^*\left(1 + \frac{\beta_1^*(1 + \rho)}{1 + \rho} - \frac{2\beta_1^*}{1 + \rho}\right)$$

$$\Rightarrow \quad 1 = \gamma_1^*\left(1 - \beta_1^*\frac{1 - \rho}{1 + \rho}\right) = \gamma_1^*\left(1 - \beta_1^*\frac{1 + \rho}{1 + \rho}\right)$$

$$\Rightarrow \quad \gamma_1^* = \frac{1}{1 - \beta_1^*\frac{1 - \rho}{1 + \rho}}$$

(22)

Immediately we can note that this has some interesting features. We see that a higher financing cost, $\rho$, yields lower $\gamma_1^*$. Furthermore, we can insert the $\gamma_1^*$ to find the optimal level $q^*$:

$$q^* = \frac{\beta_1^*\gamma_1^{*2}(1 - \beta_1^*)}{(1 + \rho)I} - \frac{c}{(1 + \rho)I}$$

$$\Rightarrow \quad \frac{\beta_1^*(1 - \beta_1^*)}{(1 + \rho)I\left(1 - \beta_1^*\frac{1 - \rho}{1 + \rho}\right)^2} - \frac{c}{(1 + \rho)I} = \frac{\beta_1^*(1 - \beta_1^*)}{(1 + \rho)I\left(1 - \beta_1^*\frac{1 + \rho}{1 + \rho}\right)^2} - \frac{c}{(1 + \rho)I}$$

43
\[ q^* = \frac{\beta_1(1 - \beta_1^*) (1 + \rho)}{I(1 + \rho - \beta_1^*(1 - \rho))^2} - \frac{c}{(1 + \rho)I} = \frac{\beta_1(1 - \beta_1^*) (1 + \rho)}{I(1 - \beta_1^* + (1 + \beta_1^*)\rho)^2} - \frac{c}{(1 + \rho)I} \]

To find the optimal \( \gamma_0 \) we simply insert the \( \gamma_1^* \) and solve:

\[
\gamma_0^* = -\frac{[(1 - \beta_1^*)\gamma_1^*]^2}{2} = -\frac{1}{2} \left[ \frac{1 - \beta_1^*}{1 - \beta_1^* \frac{1 - \beta_1^*}{1 + \rho}} \right]^2
\]

Inserting for \( \beta_1 = g(c; \theta) \) this yields the result in (21).

**Proposition 14.** The solution when \( q^* \in [0, 1] \) from Proposition 13 implies that there is a distortion in the optimal level of infrastructure quality effort made by the PPP Consortium only when \( \rho > 0 \). If \( \rho = 0 \) then there is no distortion. If \( \rho > 0 \) then the effort made by PPP Consortium is reduced and this distortion is increasing in both \( \rho \) and \( g(c; \theta) \).

**Proof.** The optimal effort from the PPP Consortium becomes:

\[
a = (1 - \beta_1^*) \gamma_1^* = \frac{1 - \beta_1^*}{(1 - \beta_1^* \frac{1 - \beta_1^*}{1 + \rho})} = \frac{(1 - \beta_1^*) (1 + \rho)}{1 + \rho - \beta_1^*(1 - \rho)}
\]

\[
\Rightarrow a = \frac{(1 - \beta_1^*) + (1 - \beta_1^*)\rho}{(1 - \beta_1^*) + (1 + \beta_1^*)\rho} = \frac{(1 - g(c; \theta)) + (1 - g(c; \theta))\rho}{(1 - g(c; \theta)) + (1 + g(c; \theta))\rho} = (24)
\]

Thus, we can see that if \( \rho = 0 \) then the optimal effort \( a = 1 \). We can also simply observe from equation (24) that this effort must be decreasing in both \( g(c; \theta) \) and \( \rho \).

The interpretation of this is actually fairly intuitive one, since when there is an interior solution for \( q \), the Government is able to extract all the surplus from the Private Financier by setting the optimal \( q^* \), just as it does with the PPP Consortium by setting the optimal \( \gamma_0 \). This means that the only source of distortions in PPP Consortium infrastructure quality effort is through a higher private financing cost, \( \rho \).

**Proposition 15.** The optimal level of private finance, \( q^* \), is decreasing in private finance premium (\( \rho \)) and investment cost (\( I \)). Furthermore, it is increasing in \( g(c; \theta) \) if \( g(c; \theta) > \frac{1}{2} \) and decreasing in \( g(c; \theta) \) if \( g(c; \theta) < \frac{1}{2} \). The Private Financiers cost of effort (\( c \)) have also a negative effect on \( q^* \) for all \( g(c; \theta) \) smaller than one half, but has an unclear effect on \( q^* \) when \( g(c; \theta) \) is larger than one half.
Proof. To prove this we can take a look at the effect and increasing \( g(c, \theta) \) by taking the derivative of \( q^* \) with respect to \( \beta_1^* \):

\[
\frac{dq^*}{d\beta_1^*} = \left( 1 - 2\beta_1^* \right) \left( 1 + \rho \right) 2\gamma_1^* \frac{d\gamma_1^*}{d\beta_1^*} = \left( 1 - 2\beta_1^* \right) \left( 1 + \rho \right) 2\gamma_1^* \left( 1 - \frac{1}{1 + \rho} \right) \left( 1 - \beta_1^* \frac{1 - \rho}{1 + \rho} \right)^2
\]

which must have the same sign as the expression \( 1 - 2\beta_1^* \) since all remaining factors are positive. Thus, the \( q^* \) is increasing in \( \beta_1^* \) if \( \beta_1^* < \frac{1}{2} \) and decreasing in \( \beta_1^* \) if \( \beta_1^* > \frac{1}{2} \).

In addition, it is easy to see that the optimal \( q^* \) must be decreasing in \( c \) if \( \beta_1^* < \frac{1}{2} \), since the direct effect from an increased \( c \) is to reduce \( q^* \) and since increased \( c \) also decreases \( g(c, \theta) \) which by the proof must also decrease \( q^* \). However, if we are in the scenario with \( \beta_1^* > \frac{1}{2} \), the effect on \( q^* \) from changes in \( c \) are unclear. It can be showed that when \( \beta_1^* \) is close to one half the negative direct effect from \( c \) dominates, while when \( \beta_1^* \) moves toward one, the negative indirect effect dominates. However, since this result is not particularly interesting, I will settle for this comment and not provide the proof.

To prove that \( q^* \) is decreasing in both \( \rho \) and \( I \), we can use the expressions (22) and (23). The entire expression for \( q^* \) in (23) is divided by \( (1 + \rho)I \), thus increasing either of these two must consequently have a direct effect of reducing \( q^* \). Additionally, the only remaining effect on \( q^* \) from such a change is through the effect on \( \gamma_1^* \) in (22), which we already have established is reduced when \( \rho \) is increased. Thus we can confirm that \( q^* \) is decreasing in both \( \rho \) and \( I \).

We remember that in this scenario the Government extracts all the surplus, thus the only interesting profit to determine is that of the Government:

\[
\Pi_{GOV} = \frac{a^2}{2} + (1 - \gamma_1^*)a - I + \frac{\beta_1^*\gamma_1^*a - c}{1 + \rho} = \frac{a^2}{2} + \left( 1 - \gamma_1^* + \frac{\beta_1^*\gamma_1^*}{1 + \rho} \right) a - \frac{c}{1 + \rho}
\]

Note that we can rewrite the expression \( (1 - \gamma_1^*) \) as:

\[
1 - \gamma_1^* = 1 - \frac{1}{1 - \beta_1^* \frac{1 - \rho}{1 + \rho}} = \frac{1 - \beta_1^* \frac{1 - \rho}{1 + \rho} - 1}{1 - \beta_1^* \frac{1 - \rho}{1 + \rho}} = \frac{-\beta_1^* \frac{1 - \rho}{1 + \rho}}{1 - \beta_1^* \frac{1 - \rho}{1 + \rho}}
\]

Furthermore, we have that:

\[
\frac{\beta_1^* \gamma_1^*}{1 + \rho} = \frac{\beta_1^*}{1 + \rho} \left( 1 - \beta_1^* \frac{1 - \rho}{1 + \rho} \right)
\]

45
thus we can rewrite the following expression

$$\left(1 - \gamma_i^* + \frac{\beta_i^* \gamma_i^*}{1 + \rho}\right)^a = \left(\frac{\beta_i^* \psi_i^* - \beta_i^* \phi_i^*}{1 - \beta_i^* \psi_i^* / (1 + \rho)}\right)^a = \frac{\beta_i^* \rho}{(1 - \beta_i^*) + (1 + \beta_i^*) \rho^a}$$

If we insert this for $a$ it becomes

$$\Rightarrow \frac{\beta_i^* \rho (1 - \beta_i^*) + (1 - \beta_i^*) \rho}{[(1 - \beta_i^*) + (1 + \beta_i^*) \rho]^2} = \frac{\beta_i^* \rho - \beta_i^* \rho^2 + \beta_i^* \rho^2 - \beta_i^* \rho^2}{[(1 - \beta_i^*) + (1 + \beta_i^*) \rho]^2}$$

Now we can insert for $a$ in the expression for $\gamma_0$ as well:

$$-\gamma_0 = \frac{a^2}{2} = \frac{1}{2} \left[\frac{1 + 2\rho + \rho^2 - 2\beta_i^* \rho - 2\beta_i^* \rho^2 - 2\beta_i^* \rho^2 + \beta_i^* \rho^2 + \beta_i^* \rho^2 + 2\beta_i^* \rho^2 + \beta_i^* \rho^2 \rho^2}{[(1 - \beta_i^*) + (1 + \beta_i^*) \rho]^2}\right]$$

Now we can insert this into the Governments profit:

$$\Pi_{GOV} = \frac{1}{2} \left[\frac{1 + 2\rho + \rho^2 - 2\beta_i^* \rho - 2\beta_i^* \rho^2 - 2\beta_i^* \rho^2 + \beta_i^* \rho^2 + 2\beta_i^* \rho^2 + \beta_i^* \rho^2 \rho^2}{[(1 - \beta_i^*) + (1 + \beta_i^*) \rho]^2}\right]$$

$$+ \frac{\beta_i^* \rho - \beta_i^* \rho^2 + \beta_i^* \rho^2 - \beta_i^* \rho^2}{[(1 - \beta_i^*) + (1 + \beta_i^*) \rho]^2} - I - \frac{c}{1 + \rho}$$

$$= \frac{1 + 2\rho + \rho^2 - 2\beta_i^* \rho - 2\beta_i^* \rho^2 - 2\beta_i^* \rho^2 + \beta_i^* \rho^2 + 2\beta_i^* \rho + \beta_i^* \rho^2}{2[(1 - \beta_i^*) + (1 + \beta_i^*) \rho]^2}$$

$$+ \frac{2\beta_i^* \rho - 2\beta_i^* \rho^2 + 2\beta_i^* \rho^2 - 2\beta_i^* \rho^2}{2[(1 - \beta_i^*) + (1 + \beta_i^*) \rho]^2} - I - \frac{c}{1 + \rho}$$
By bringing together the two first fraction in this expression we can simplify:

\[
\Rightarrow \quad \frac{1 + 2\rho + \rho^2 - 2\beta^*_2 \rho + \beta^*_2 - \beta^*_2 \rho^2}{2[(1 - \beta^*_1) + (1 + \beta^*_1)\rho]^2} - I = \frac{c}{1 + \rho}
\]

\[
= \frac{(1 + \rho)^2 - 2\beta^*_1 (1 + \rho) + \beta^*_1 (1 - \rho^2)}{2[(1 - \beta^*_1) + (1 + \beta^*_1)\rho]^2} - I = \frac{c}{1 + \rho}
\]

\[
= \frac{(1 + \rho)((1 + \rho) - 2\beta^*_1 + \beta^*_1 (1 - \rho))}{2[(1 - \beta^*_1) + (1 + \beta^*_1)\rho]^2} - I = \frac{c}{1 + \rho}
\]

\[
= \frac{(1 + \rho)(1 + \rho - 2\beta^*_1 + \beta^*_1 (1 - \rho))}{2[(1 - \beta^*_1) + (1 + \beta^*_1)\rho]^2} - I = \frac{c}{1 + \rho}
\]

\[
= \frac{(1 + \rho)((1 - \beta^*_1)^2 + \rho(1 - \beta^*_1)(1 + \beta^*_1))}{2[(1 - \beta^*_1) + (1 + \beta^*_1)\rho]^2} - I = \frac{c}{1 + \rho}
\]

\[
= \frac{(1 + \rho)(1 - \beta^*_1)(1 - \beta^*_1 + (1 + \beta^*_1)\rho)}{2[(1 - \beta^*_1) + (1 + \beta^*_1)\rho]^2} - I = \frac{c}{1 + \rho}
\]

\[
= \frac{(1 + \rho)(1 - \beta^*_1) + (1 - \beta^*_1)\rho}{2[(1 - \beta^*_1) + (1 + \beta^*_1)\rho]} - I = \frac{a}{2} - I - \frac{c}{1 + \rho}
\]

\[
\Pi_{GOV} = \frac{1}{2} \frac{(1 - \beta^*_1) + (1 - \beta^*_1)\rho}{(1 - \beta^*_1) + (1 + \beta^*_1)\rho} - I - \frac{c}{1 + \rho} = \frac{a}{2} - I - \frac{c}{1 + \rho}
\]

Thus, we can see that the Government extracts a revenue from this arrangement equal to half the effort made by the PPP Consortium.

Now we move over to discussing the conditions for this solution. As discussed in the previous section where \(q = 1\), regarding the conditions for when the PPP Consortium has the incentives to deviate from this inclusion of the Private Financier, we must then assume that it is in fact possible for the PPP Consortium to actually deviate. It might be somewhat unlikely since this would also imply that the Government will contribute with investment like in the basic model, but not demand a renegotiation of the contract between the Government and the PPP Consortium.

**Proposition 16.** In addition to \(q^* \in [0, 1]\), for the optimal solution with private finance in Proposition 13 we need the following conditions

\[
f(\theta) \geq \frac{g(c, \theta)\rho}{(1 - g(c, \theta)) + (1 + g(c, \theta))\rho} + \frac{c}{1 + \rho} \tag{25}
\]

\[
f(\theta) \geq \frac{g(c, \theta)(2 - g(c, \theta))}{2\left(1 - g(c, \theta)\frac{1 - \rho}{1 + \rho}\right)^2} \tag{26}
\]

Where (25) refers to the solution being preferable from a social point of view and (26) refers to it being incentive compatible with the PPP Consortium’s optimal response.
Proof. If the PPP Consortium deviates, it receives the same transfer \( t(Q) = \gamma_0 + \gamma_1 a \), but offers \( \beta_1 = 0 \) and adjust it’s effort accordingly to \( a = \gamma_1^* \) in order to capture more of the revenues. The downside is the lack of insurance and thus it gets the disutility of the exogenous risk. Their profit thus becomes

\[
\Pi_{PPP} = -\frac{[(1 - \beta_1^*)\gamma_1^*]^2}{2} + \gamma_1^* a - \frac{a^2}{2} - f(\theta) = -\frac{[(1 - \beta_1^*)^2\gamma_1^*]^2}{2} + \gamma_1^* a - \frac{a^2}{2} - f(\theta)
\]

\[
= \frac{\gamma_1^*}{2}(1 - (1 - \beta_1^*)^2) - f(\theta) = \frac{\gamma_1^*}{2}(1 - (1 - 2\beta_1^* + \beta_1^*)^2) - f(\theta)
\]

\[
= \frac{\gamma_1^*}{2}(\beta_1^*(2 - \beta_1^*)) - f(\theta) = \frac{\beta_1^*(2 - \beta_1^*)}{2(1 - \beta_1^*)} - f(\theta)
\]

Thus, for this solution to be incentive compatible with PPP Consortium offering a contract to the Private Financier, we need their profit to be non-positive if PPP Consortium deviates. That is, we must have

\[
f(\theta) \geq \frac{g(c, \theta)(2 - g(c, \theta))}{2 \left(1 - g(c, \theta)\frac{1 - \rho}{1 + \rho}\right)}
\]

Next we check the condition for when the Government prefers this over the no-private finance solution. We demand that this profit is higher than the profit made in the basic model.

\[
\Pi_{GOV} = \frac{1}{2} (1 - \beta_1^*) + \frac{(1 - \beta_1^*)\rho}{(1 - \beta_1^*) + (1 + \beta_1^*)\rho} - I - \frac{c}{1 + \rho} \geq \frac{1}{2} - I - f(\theta) = \Pi_{GOV}'
\]

\[
\Rightarrow \frac{1}{2} (1 - \beta_1^*) + \frac{(1 - \beta_1^*)\rho}{(1 - \beta_1^*) + (1 + \beta_1^*)\rho} - \frac{c}{1 + \rho} \geq \frac{1}{2} - f(\theta)
\]

Alternatively, we can rewrite it as:

\[
\Rightarrow \frac{1}{2} \left(1 - \frac{(1 - \beta_1^*) + (1 - \beta_1^*)\rho}{(1 - \beta_1^*) + (1 + \beta_1^*)\rho}\right) + \frac{c}{1 + \rho} \leq f(\theta)
\]

\[
\frac{1}{2} \left(\frac{(1 - \beta_1^*) + (1 + \beta_1^*)\rho - (1 - \beta_1^*) - (1 - \beta_1^*)\rho}{(1 - \beta_1^*) + (1 + \beta_1^*)\rho}\right) + \frac{\beta_1^*\rho}{(1 - \beta_1^*) + (1 + \beta_1^*)\rho} + \frac{c}{1 + \rho} \leq f(\theta)
\]

This confirms (25). Note that if \( \rho = 0 \), this becomes

\[
\Rightarrow c \leq f(\theta)
\]

which is the basic assumption I have made for the entire thesis. \( \square \)
Finally, we make some inference about when these condition are likely to hold. We can note that the right-hand side of (26) is increasing in $\beta_1^i$ for any value of $\beta_1^i$ less than one, which obviously is never violated. Thus, the condition becomes harder to satisfy when the $\beta_1^i$ (or $g(c, \theta)$) is higher. Furthermore, it is easy to see that it is easier to satisfy when $\rho$ is high and of course the same goes for a high disutility of exogenous risk, $f(\theta)$.

Next we take the derivative of the Government’s profit in this scenario to investigate how this changes when $\rho$ increases.

$$\frac{d\Pi_{GOV}}{d\rho} = -\beta_1^i[(1 - \beta_1^i) + (1 + \beta_1^i)\rho] - \beta_1^i[(1 - \beta_1^i) + (1 - \beta_1^i)\rho] + \frac{c}{(1 + \rho)^2}$$

$$= \frac{1}{2} \left[ \frac{-2\beta_1^i[1 - \beta_1^i] + \rho}{[(1 - \beta_1^i) + (1 + \beta_1^i)\rho]^2} \right] + \frac{c}{(1 + \rho)^2}$$

$$\frac{d\Pi_{GOV}}{d\rho} = \frac{c}{(1 + \rho)^2} - \frac{\beta_1^i{(1 - \beta_1^i)} + \beta_1^i\rho}{[(1 - \beta_1^i) + (1 + \beta_1^i)\rho]^2} < 0$$

In fact, this must be negative. To see this we can use the fact that $\Pi_{PF} = 0$:

$$\Pi_{PF} = \beta_1^i \gamma^i a - c - q^i(1 + \rho)I = \beta_1^i(1 - \beta_1^i)\gamma^i I^2 - c - q^i(1 + \rho)I = 0$$

$$\Rightarrow \frac{\beta_1^i(1 - \beta_1^i)}{(1 - \beta_1^i)^2 + \rho(1 + \rho)} - c - q^i(1 + \rho)I = 0$$

$$\Rightarrow \frac{\beta_1^i(1 - \beta_1^i)}{(1 + \rho - \beta_1^i(1 - \rho))^2} - c - q^i(1 + \rho)I = 0$$

$$\Rightarrow \frac{\beta_1^i(1 - \beta_1^i)(1 + \rho)^2}{(1 + \rho - \beta_1^i(1 - \rho))^2} - c - q^i(1 + \rho)I = 0$$

$$\Rightarrow \frac{\beta_1^i(1 - \beta_1^i)}{(1 + \rho - \beta_1^i(1 - \rho))^2} - \frac{c}{(1 + \rho)^2} - \frac{q^i I}{1 + \rho} = 0$$

For any $q^i > 0$, this must imply that

$$\Rightarrow \frac{c}{(1 + \rho)^2} < \frac{\beta_1^i(1 - \beta_1^i)}{(1 + \rho - \beta_1^i(1 - \rho))^2} < \frac{\beta_1^i(1 - \beta_1^i) + \beta_1^i\rho}{(1 + \rho - \beta_1^i(1 - \rho))^2}$$

And thus we can conclude that

$$\frac{d\Pi_{GOV}}{d\rho} = \frac{c}{(1 + \rho)^2} - \frac{\beta_1^i(1 - \beta_1^i) + \beta_1^i\rho}{[(1 - \beta_1^i) + (1 + \beta_1^i)\rho]^2} < 0$$

49
4.4 Summarizing the Government’s optimal contract

Although I have identified that when \( \hat{\beta}_1 > \hat{\beta}_1 \) then \( q = 0 \) and when \( \hat{\beta}_1 < \hat{\beta}_1 \) then \( q = 1 \), this is not the complete solution for when \( q = 0 \) and \( q = 1 \), respectively.

**Proposition 17.** The complete solution for the optimal share of private finance of the PPP investment, defined as \( q_{PF} \), is

\[
q_{PF} = \begin{cases} 
0 & \text{if } q^* \leq 0 \Leftrightarrow g(c, \theta)(1 - g(c, \theta))\gamma_1^* \leq c \\
1 & \text{if } q^* \geq 1 \Leftrightarrow g(c, \theta)(1 - g(c, \theta))\gamma_1^* \geq c + (1 + \rho)I \\
q^* & \text{otherwise}
\end{cases}
\]

(27)

**Proof.** To find the final condition for when \( q = 0 \) we must check when the mathematical expression for \( q^* \) becomes equal to or less than zero:

\[
q^* = g(c, \theta)(1 - g(c, \theta))(1 + \rho)\left(1 - g(c, \theta) + (1 + g(c, \theta))\rho\right)^2 - \frac{c}{(1 + \rho)I} \leq 0
\]

\[
\Rightarrow \frac{g(c, \theta)(1 - g(c, \theta))(1 + \rho)^2}{(1 - g(c, \theta) + (1 + g(c, \theta))\rho)^2} \leq c
\]

Likewise, to find when the optimal solution is \( q = 1 \) we check when the expression \( q^* \) is greater or equal to one:

\[
q^* = g(c, \theta)(1 - g(c, \theta))(1 + \rho)\left(1 - g(c, \theta) + (1 + g(c, \theta))\rho\right)^2 - \frac{c}{(1 + \rho)I} \geq 1
\]

\[
\Rightarrow \frac{g(c, \theta)(1 - g(c, \theta))(1 + \rho)^2}{(1 - g(c, \theta) + (1 + g(c, \theta))\rho)^2} \geq c + (1 + \rho)I
\]

\[\square\]

From Proposition 17 we see that is socially optimal to include some level of private finance if the cost of Private Financier’s effort is less than the optimal revenue the Private Financier receives from the profit sharing agreement with the PPP Consortium. Furthermore, it is socially optimal to have full private finance if the total cost of effort and financing is less or equal to the minimum revenue the Private Financier must receive to provide incentives for it to make an effort \( e = 1 \).

One important question one might ask by now in this thesis might be, what happens with optimal \( \gamma_1^* \) as \( q^* \) moves toward 1? We know that our scenario in which \( \hat{\beta}_1 = \hat{\beta}_1 = \beta_1 \), we get

\[
q^* \in (0,1) \quad \text{and} \quad \gamma_1^* = \frac{1}{1 - g(c, \theta)\frac{1 - \rho}{1 + \rho}}
\]
while in a scenario in which \( \beta_1 < \tilde{\beta}_1 \) we get

\[
q = 1 \quad \text{and} \quad \gamma_1 = \frac{1}{1 + g(c, \theta)}
\]

At first glance this might seem to be a contradicting solution for the \( \gamma_1 \) when \( q = 1 \). The reason for this is that as long as the Government can extract surplus from the Private Financier using \( q \), then this is preferable to decreasing the \( \gamma_1 \). However, once we reach the maximum amount \( q = 1 \), the Government prefers to reduce the \( \gamma_1 \) it pays for the quality \( Q = a \). It will continue to reduce \( \gamma_1 \) until it either binds the participation constraint \( \hat{\beta}_1 = \tilde{\beta}_1 = g(c, \theta) \) or until it reaches the optimal condition regardless of the Private Financiers profit, and we end up in the scenario in which \( \beta_1 < \tilde{\beta}_1 \) and the optimal \( \gamma_1 \) is \( \tilde{\gamma}_1 \). In other words, there is a gap between the two solutions, where the Government sets \( \gamma_1 \) between \( \gamma_1^* \) and \( \tilde{\gamma}_1 \) such that the participation constraint for the Private Financier is binding. If this is the case, this optimal \( \gamma_1 \), which we define as \( \tilde{\gamma}_1^* \), is set such that the Private Financier receives zero profit and can be written as:

\[
\tilde{\gamma}_1^* = \sqrt{\frac{c + (1 + \rho)}{g(c, \theta)(1 - g(c, \theta))}} \in \left( \frac{1}{1 + g(c, \theta)}, \frac{1}{1 - g(c, \theta)} \right) (28)
\]

By the same approach we can investigate what happens when the optimal \( q^* \leq 0 \). Once it becomes negative, the Government can of course not set a negative \( q \), so the original \( \gamma_1^* \) would be too low to ensure participation from the Private Financier. Thus, the Government must set a slightly higher \( \gamma_1 \), and will in fact set the \( \gamma_1 \) such that the Private Financiers participation constraint is just satisfied. Thus, as with the case for \( q = 1 \), they will set a \( \gamma_1 \) such that

\[
\tilde{\gamma}_1^* = \sqrt{\frac{c}{g(c, \theta)(1 - g(c, \theta))}} \in \left( \frac{1}{1 - g(c, \theta)}, \frac{1}{1 - g(c, \theta)} \right) = 1 + c (29)
\]

Once the relationship between \( c \) and \( g(c, \theta) \) is such that

\[
\frac{c}{1 + c} = \beta_1 > \tilde{\beta}_1 = g(c, \theta)
\]

the Government prefers to stop increasing \( \gamma_1 \) and rather let the PPP Consortium set a higher \( \beta_1 \) than necessary to induce effort (that is, \( \beta_1 > \beta_1 \)). We summarize the entire solution for the optimal contract offered from the Government to the PPP Consortium when including a Private Financier in the following proposition:

**Proposition 18.** When the Government offers a contract including the Private Financier, it offers the optimal contract \( t^{PF}(Q) = \gamma_0^{PF} + \gamma_1^{PF} a \).
It provides incentive for PPP Consortium effort by setting the optimal $\gamma_{1}^{PF}$ such that

$$
\gamma_{1}^{PF} = \begin{cases} 
\hat{\gamma}_1 & \text{if } \hat{\gamma}_1 \geq \hat{\gamma}_1^* \\
\hat{\gamma}_1^* & \text{if } \hat{\gamma}_1^* \in (\hat{\gamma}_1^*, \hat{\gamma}_1) \\
\hat{\gamma}_1 & \text{if } \hat{\gamma}_1 \in (\hat{\gamma}_1^*, \hat{\gamma}_1^*), \\
\hat{\gamma}_1 & \text{if } \hat{\gamma}_1 \geq \hat{\gamma}_1^* 
\end{cases}
$$

where $\hat{\gamma}_1 = 1 + c$

$$
\hat{\gamma}_1^* = \sqrt{\frac{c}{g(c, \theta)(1-g(c, \theta))}} \\
\hat{\gamma}_1^* = \frac{1}{1-g(c, \theta)(1+\rho)} \\
\hat{\gamma}_1^* = \sqrt{\frac{c+1+\rho}{g(c, \theta)(1-g(c, \theta))}} \\
\hat{\gamma}_1^* = \frac{1}{1+g(c, \theta)}
$$

(30)

The optimal $\gamma_0$, $\gamma_0^{PF}$, is defined as the one yielding zero profit to the PPP Consortium:

$$
\gamma_0^{PF} = -\frac{[(1-\beta^*)\gamma_0^{PF}]^2}{2}
$$

This optimal contract entails capturing the entire surplus from the PPP Consortium in all cases and from the Private Financier in all cases except for when $\gamma_1^{PF} = \gamma_1$. Furthermore, the higher financing cost $\rho$ only distorts the $\gamma_1^{PF}$ and thus optimal effort $\gamma_1^{PF}$ if $\gamma_1^{PF} = \gamma_1^*$ or $\gamma_1^{PF} = \gamma_1^*$, that is, when there is some level of private finance, but the Government is still able to extract the entire surplus from the Private Financier.

A final note, the conditions for when the Government prefers to include the Private Financier or not will depend on which of the scenarios that is optimal. Thus, for a given scenario the Government will find the optimal solution given a Private Financier and ensure that the PPP Consortium does not have incentives to deviate before comparing it with the first-best scenario with no private finance and with exogenous risk. The details of these conditions are specified in each subsection.

5 Beyond the Main Model

Going beyond the main model, there are several discussions about the results and further extensions to the model that could be interesting to look at. In this section I take a look at a few of them.

5.1 Uncertainty & signalling from Private Financier

So far in the analysis I have made a simplifying assumption that the PPP Consortium and the Private Financier gets the same disutility from the exogenous risk. Although the assumption is made, most of the analysis and results does not hinge on this. Most of the analysis in the main model has revolved around the assumption that it is beneficial to provide incentives for Private Financier effort and thus insurance. If one simply allows for this to be two different levels of risk aversion (or disutility), say $f^{PPP}(\cdot)$ and
If the Private Financier has a relatively lower level of disutility, such that \( f_{PP}^{PF}() > f_{PF}^{PF}() \), then we will in most cases need a higher share of risk transfer to ensure that the Private Financier exert effort to observe the shock (the only exception being when \( \gamma_{1}^{PF} = \tilde{\gamma}_{1} \)). In other words, this involves a higher distortion of the PPP Consortium’s effort since this distortion of effort \( a \) is increasing the share \( \beta_{1} \). It also involves a higher probability of the Private Financier gaining a surplus from the arrangement, that is higher probability that \( \gamma_{1}^{PF} = \tilde{\gamma}_{1} \). Alternatively, if we already were in the scenario in which \( \gamma_{1}^{PF} = \tilde{\gamma}_{1} \) then the higher \( \beta_{1} \) simply increases the profit for the Private Financier (if \( \beta_{1} \) is smaller than one half). In both cases, it is easy to see that the Private Financier would gain from having a lower disutility of exogenous risk. On the other hand, the same assumption implies that the PPP Consortium’s disutility of the risk is relatively higher which means that the PPP Consortium’s benefit of potentially being insured is higher. Thus there are more need for the Private Financier from a social point of view. From this there are some potentially interesting analysis that could be done.

The Private Financier in this case benefits from having a lower disutility of the exogenous risk. It is not a very unlikely assumption that there might be some uncertainty as to the level of disutility that this Private Financier has from exposure to the exogenous risk. Thus, suppose there are some way in which the Private Financier might signal or inform the Government and the PPP Consortium to their preference to risk. Then it must be the case that Private Financier has incentives to understate their level of risk aversion.

Suppose the Private Financier has this ability to signal before the first stage of Government offering a contract to the PPP Consortium and let us assume that this signal is in fact credible. The Private Financier will need to take into account the condition for when the Government prefers to include Private Finance over the basic model solution with no insurance. As long as this condition holds, however, the Private Financier would report a risk aversion such that it can maximise its revenue by setting the appropriate \( \beta_{1} \). In other words, the Government thus runs the risk of over-compensating the the Private Financier to provide incentives for making an effort in observing \( \theta \). In fact, it seems very likely that under some simplifying assumptions and under certain conditions, the Private Financier might be able to extract all the benefits from their own insurance, rendering the Government fairly indifferent to whether they use Private Finance or not.

If this signalling is unavoidable when using Private Finance, does the Private Finance benefit disappear? In this analysis I have focused on maximising the profit for the Government. An underlying assumption is thus that the Government represents the people’s preferences and utility by focusing on reducing their spending (which ultimately represent taxes) and
on increasing the quality of the infrastructure which benefits the consumers. Thus, it can be seen as a sort of consumer welfare approach to social welfare. If we have this particular scenario with a Private Financier contributing to a more efficient contract agreement, but extracts all the surplus themselves, it is still arguably a social improvement. As long as the Government would not have the opportunity to see through the reported disutility function for the Private Financier, then it is no worse off by still including a Private Financier. Meanwhile, the Private Financier makes profit and is thus strictly better off. As this is obviously an Pareto improvement it must still be preferable to no Private Finance at all from a social point of view.

This signalling discussion relies on two critical assumptions. First of all, there must exist a possibility to credibly signal their disutility of exogenous risk. As we have discussed, the Private Financier has incentives to understate their level of disutility which means that it is likely that the Government would anticipate this understatement. Thus, justifying an introduction of such a credible and realistic signalling setting seems to be difficult. Secondly, I have assumed in the analysis that there is a sufficient competitive situation for the Private Financier firms such that the scenario can be represented by a take-it-or-leave-it contract offered from the PPP Consortium to the Private Financier. If there is such a competitive situation, a signalling situation in which the Private Financier(s) could not be affected by the same competitive forces. One would therefore need some further assumption on the nature of this signalling. This could be justifiable if the Government regardless of the signals observed assumes (or knows) that all Private Financiers have the same level of disutility from risk. Then the Private Financier would not benefit from competing in the signalling and all Private Financiers have incentives to signal the same “false” risk aversion.

5.2 Possible further expansions

The main model in this thesis is a fairly simple introduction to expanding the analysis of private finance in Public-Private Partnerships. There are several potential interesting expansion and changes that can be made from this model, exploring this relationship between a PPP Consortium and a Private Financier further.

One way to expand this model would be to introduce an uncertainty in regards to whether the exogenous shock, \( \theta \), is observed by the Private Financier. This can be introduced by a simple probability of observing \( \theta \) as a function of the Private Financiers effort, \( e \), which could be denoted as \( p(e) \). This would expand the model by relaxing some of the simplifying assumption I have made in this thesis. Although this would contribute to a more complete and potentially more realistic version of the analysis in this thesis, there might not be much additional analysis and discussions arising from such an expansion to make up for the increased complexity of the model.
Another interesting expansion would be to change the assumption as to how the risk is transferred to the Private Financier. We could imagine that Private Financier does not in fact insure the PPP Consortium. Instead we could assume that the Private Financier is risk neutral to the exogenous risk \( \theta \). Thus, transferring a share \( \beta \) of the risk to the Private Financier would reduce the amount of risk the PPP Consortium would be left with. This would also imply that we do not need an effort made by the Private Financier, there is simply a trade-off between transferring risk from the PPP Consortium to the Private Financier at a cost of lowering the PPP Consortium’s transfer and thus reduces its incentives to provide quality-inducing effort, \( a \). The Government would then need to weigh these two factors against each other when providing incentives to the PPP Consortium through the contract \( t(Q) \). This trade-off seems to be potentially quite interesting and thus this might be a natural step to expand the analysis further.

A third way to expand this model is to allow for a more complicated PPP structure. I have for simplicity assumed a PPP arrangement in which there is only effort in building phase and I have made the very simplifying assumption of separable utilities between present day contractable parameters and the future exogenous risk. Although relaxing this assumption would naturally make the analysis quite a bit more complex, including an effort ex post (after the exogenous shock is observed) as is done in Iossa and Martimort (2012) seems to be worthy of some attention.

6 Concluding Remarks

Iossa and Martimort (2012) shows that when including a Private Financier who observes an exogenous shock to a PPP Consortium, the Private Financier fully insures the PPP Consortium for this potential shock. Furthermore, they state that even if this Private Financier has a higher financing cost the Government still manages to extract the entire surplus for this arrangement and the increased financing cost does not distort the optimal incentive solution. The trade-off in their analysis of the cost and benefit of Private Finance is whether the value of the insurance provided to the PPP Consortium outweighs the higher financing cost.

In this thesis’ main model I simplify the PPP Consortium’s arrangement and complicate the role of a Private Financier. I show that when we restrict the financial contract between the PPP Consortium and the Private Financier then we are likely to have a distortion in the optimal effort made by the PPP Consortium as a result of the higher private finance investment cost. This restriction of the financial contract involves no lump-sum transfer between the PPP Consortium and Private Financier and the exogenous shock being non-contractible. The incentives for insuring the PPP Consortium
instead comes from a revenue and risk sharing agreement between the PPP Consortium and Private Financier.

I also show that by the assumption of a necessary effort made by the Private Financier to observe the exogenous shock, we might have a scenario in which the Government is not able to extract the entire surplus. If the share of the risk needed to provide incentives for the Private financier to exert an effort ($\beta_1$) results in a higher Private Financier profit than their maximum amount of investment cost and effort cost, then the Private Financier profits from this arrangement. Interestingly, it might still be the case that this is profitable for the Government if we are in a scenario with high enough disutility from the exogenous risk with correspondingly high effort cost. In other words, even though the Private Financier profits from this arrangement it might still be the socially optimal solution. This suggests that observing Private Financiers in the real world profiting above competitive levels does not necessarily imply a socially inefficient solution. However, I would be the first to admit that any real world inference we can draw from this result is quite limited and I would be careful to avoid any conclusion suggesting that involving private finance profiting from PPPs does not imply a social inefficient solution.

In the main model I also introduce the possibility of partly private financing. That is, we allow for Private Financier to finance a share $q$ of the investments. I find that under certain conditions the optimal level of Private Finance is between zero and one, meaning that the optimal financing structure involves both the Government and the Private Financier financing the PPP arrangement. In this scenario, setting the optimal level of Private Finance serves as a way to extract a lump-sum transfer from the Private Financier and thus ensuring that the Government extracts all the surplus. Furthermore, the only efficiency distorting factor remaining is then the higher financing cost (or private finance premium, $\rho$). The private finance premium results in a lower effort from the PPP Consortium than the most efficient one and this effort distortion is increasing in the private finance premium.

One weakness in this model is the simplification of separable utilities. For simplicity of the model I have assumed that the disutility stemming from the exogenous risk can be separated out from the other financial transfers in the contract. In this way, I allow for this disutility to simply be a separate function of the amount of risk the PPP Consortium are exposed to. I have argued for this by viewing the exogenous risk as a risk in the far future (say 20 years from now) while the financial transfers are mostly relevant in the near future during construction and potentially through the first years of operation. Furthermore, related to this simplification I have assumed that the PPP Consortium does not make any decisions on effort ex post. That is, the exogenous shock does not have any influence on the construction or operation of the infrastructure as the efforts are made before the realisation of the shock. This is also a large simplification from the model by Iossa and
Martimort (2012) which avoids quite a few technical problems but might also lose some insight and realism on the way.

Another weakness in this model is that I have made quite a specific restriction on the financial contract. Whether this type of contract is a good representation of the real world is debatable. Thus, it is important to view this model and its conclusions in a bigger picture. Exploring how changing the restrictions in this financial contracts would relate to our findings could be an interesting way forward for further research. Moreover, it seems interesting to investigate how more advanced forms of PPPs with several potential efforts or investment, like in Iossa and Martimort (2012) and Hart (2003), could be affected by the same type of financial contract I propose in this model. Such an expansion would naturally be mathematically more complicated, but could nevertheless be an interesting way to further explore the role of private finance in Public-Private Partnerships.

References


