

Cosmic magnetization in three-form inflationTomi S. Koivisto¹ and Federico R. Urban²¹*Institute for Theoretical Astrophysics, University of Oslo, P.O. Box 1029 Blindern, N-0315 Oslo, Norway*²*Department of Physics and Astronomy, University of British Columbia, 6224 Agricultural Road, Vancouver, British Columbia V6T 1Z1, Canada*

(Received 12 December 2011; published 5 April 2012)

A completely new mechanism to generate the observed amount of large-scale cosmological magnetic fields is introduced in the context of three-form inflation. The amplification of the fields occurs via fourth order dynamics of the vector perturbations and avoids the backreaction problem that plagues most previously introduced mechanisms.

DOI: [10.1103/PhysRevD.85.083508](https://doi.org/10.1103/PhysRevD.85.083508)

PACS numbers: 98.80.Cq, 98.62.En

I. INTRODUCTION AND CONCLUSION

Large-scale magnetic fields exist in most observed structures in our Universe such as galaxies, clusters, filaments, and beyond, stretching to regions in the sky where little to next to no gas is present. Observational evidence is growing by the day, unveiling an unsettling uniform and ubiquitous presence; yet, their nature and origin remains draped by a hazy veil of mystery; see the reviews [1–9].

The property which is perhaps most intricate to frame within a consistent theoretical model for cosmic magnetic fields is their pervasively wide coherence length, which can stretch well beyond the Mpc mark; cosmological inflation might be the only viable candidate for automatic transformation of short wavelength fluctuations into beyond-the-horizon correlations [10].

Many simple options for inflationary magnetogenesis suffer from a backreaction issue; see [11–14]. In short, as inflation proceeds at amplifying electromagnetic waves, the overall energy density of the latter catches up with that of the former, leading to instability in the de Sitter background. When this occurs inflation stops, and if this happens too early then we are not able to generate an almost scale invariant spectrum of curvature perturbations, or expand the causal region of the Universe beyond the observed one today—a possible viable model is perhaps the simplest [10], but it appears to be successful only in the strong coupling (hence nonperturbatively trustable) regime [11,12] (but see the proposal of [15] and criticism in [16]).

The reasons behind this unhappy ending are twofold. In the first place, what is generally found is that the electric field, while strongly suppressed in a highly conductive plasma (such as the primordial soup during and after reheating), plays the part of the giant in vacuum; see [13]. Second, the electric field tends to be redder than the magnetic field [13], and since inflation generally blindly processes an extremely wide range of modes, the magnetic spectrum needs to be close to being red or all the allowed energy density will be snatched by small scales; but if this is so, the electric field will greatly dominate in the IR, leaving only crumbs of power to the magnetic field.

In this work we show how these general issues can be circumvented in a particular model of inflation, three-form driven inflation [17,18], thanks to the peculiar reshaping of the magnetic spectrum the coupling between the three-form and electromagnetism (EM) produces.

Three-form inflation is a generally viable alternative to scalar-field models. Details of the rich dynamics that three-form inflation allows can be found in various cases in Refs. [17–22]; for instance, the simplest models cannot produce phantom inflation without instabilities or a blue-tilted spectrum, but there are viable models generating a slight red tilt with or without a slow roll.

Once coupled to EM, three-form inflation provides us with the two key ingredients for a promising magnetization of the large-scale Universe. First, it entails the possibility of exponential growth in the EM sector, driven by an instability in the three-form rotational sector. Moreover, and more importantly, it provides an effective UV cutoff at which the instability dies off; it then automatically splits it from the scale of inflation, allowing for a careful selection of the modes that are amplified, thereby upsetting the democraticity of inflation.

This can be understood in the dual description of three-form inflation as a scalar-field inflation, since the duality maps kinetic into potential and vice versa. However, the duality can break down in the presence of nontrivial self-interactions [22] of the three-form or nonminimal couplings to geometry [17] or to matter [21]. Thus three-form inflation is equivalently described by a scalar-field model only in the simplest minimally coupled cases. The novel aspect here is the coupling to the electromagnetic field: this both promotes the model to a physically distinct alternative to scalar-field driven constructions by breaking the duality (as it is apparent when one sees that the three-form has propagating vector degrees of freedom), and resolves a generic shortcoming by enabling sufficient magnetic field production without the backreaction issue.

Our results are shown in Figs. 1 and 2: despite the electric field still being the one to watch out for, magnetic fields which today have strengths in the 10^{-15} G range can be easily produced without excessive fine-tuning of the

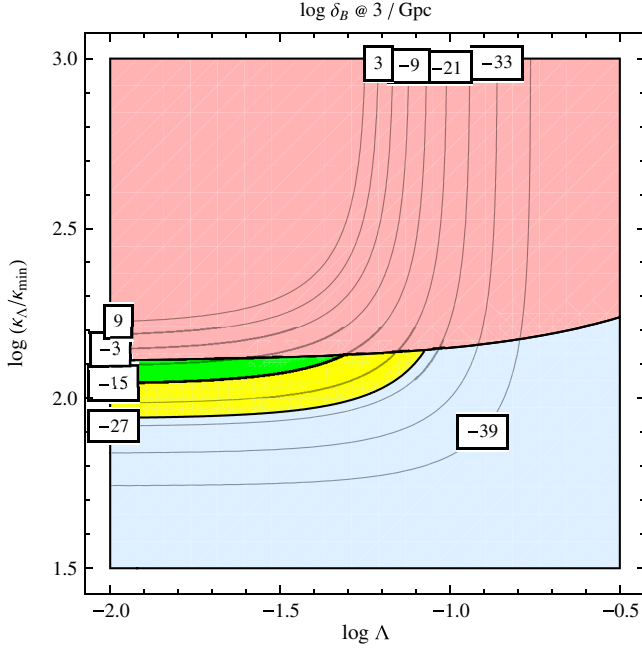


FIG. 1 (color online). Allowed (in light blue, lower shaded area) and disallowed (in light red, upper shaded area) regions in the $(\log\Lambda, \log(k_\Lambda/k_{\min}))$ parameter space. Superimposed are contours of constant $\log\delta_B^0$ today, at 3/Gpc (in Gauss units).

parameters, and without jeopardizing the development and completion of a long epoch of inflation.

II. THE COUPLING

Let A^μ be the photon vector potential and $B^{\mu\nu\rho}$ the three-form. The canonical Lagrangian including both fields is

$$\mathcal{L}_A + \mathcal{L}_B = -\frac{1}{4}F^2(A) - \frac{1}{48}F^2(B) - V(B^2), \quad (1)$$

where the Faraday forms are computed from an n -form potential N as $F(N)_{\mu_1\dots\mu_{n+1}} = (n+1)!\partial_{[\mu_1}N_{\mu_2\dots\mu_{n+1}]}$. The components of the dual of the three-form are [23]

$$\tilde{B}_\alpha \equiv \frac{1}{6}\epsilon_{\alpha\beta\gamma\delta}B^{\beta\gamma\delta}. \quad (2)$$

The most general Lorentz-invariant, quadratic, second order and $U(1)$ -invariant coupling of the two fields is of the form

$$\mathcal{L}_{AB} = -\frac{1}{2}\alpha F_{\mu\nu}(A)F^{\mu\nu}(\tilde{B}). \quad (3)$$

III. THE AMPLIFICATION MECHANISM

We decompose the spatial part of the vector potential $A_\mu = (A_0, A_i)$ in terms of its transverse and longitudinal components as $A_i = A_i^T + \partial_i\chi$, where $\nabla \cdot \mathbf{A}^T = 0$. The three-form is decomposed in a similar way, where $\tilde{B}_i = B_i^T + \partial_i\xi$. Since the physical photons correspond to the transverse degrees of freedom, we are particularly

interested in the vector perturbations. The vector part of the Einstein-Hilbert Lagrangian coupled to the three-form is [24]

$$\mathcal{L}_{\text{EH}+B}^{(v)} = \frac{1}{2}\left[\frac{M_{\text{p}}^2 k^2}{2}\mathbf{C}^{T2} - \frac{V_{,X}}{X}(\tilde{\mathbf{B}}^T - X\mathbf{C}^T)^2\right], \quad (4)$$

where we have used the background Friedmann equation and \mathbf{C}^T is the gauge-invariant rotational degree of freedom of the metric perturbations [25]; X is the background value of the three-form field $X = \sqrt{-B^2}$ in the notation of Ref. [22]. Eliminating the unphysical A_0 through $A_0 = \chi' - \alpha(\tilde{B}_0 - \xi')$ [26], the relevant part of the action (4) simplifies to

$$\mathcal{L}_{A+AB} = \frac{1}{2}[\mathbf{A}^T(-\partial_\eta^2 + \Delta)(\mathbf{A}^T + 2\alpha\tilde{\mathbf{B}}^T) + \alpha^2(\tilde{B}_0 - \xi')\Delta(\tilde{B}_0 - \xi')], \quad (5)$$

where η is conformal time and $\Delta \equiv \delta^{ij}\partial_i\partial_j$. The scalar polarizations of the photon are nondynamical, as expected. It is interesting to note though that the scalar part of the interaction decouples from the electromagnetic field; it contributes to the effective sound speed of the three-form.

The equations of motion for the vector degrees of freedom become (omitting, from now on, the superscripts T , since all vectors are considered to be transverse)

$$M_{\text{p}}^2 k^2 \mathbf{C} + 2V_{,X}(\tilde{\mathbf{B}} - X\mathbf{C}) = 0, \quad (6)$$

$$V_{,X}(\tilde{\mathbf{B}} - X\mathbf{C}) - \alpha X(-\partial_\eta^2 + \Delta)\mathbf{A} = 0, \quad (7)$$

$$(-\partial_\eta^2 + \Delta)(\mathbf{A} + 2\alpha\tilde{\mathbf{B}}) = 0. \quad (8)$$

We note that in the absence of the coupling, both the metric and three-form vector perturbations are nondynamical. We can eliminate \mathbf{C} and obtain a closed equation for the three-form perturbation in Fourier space:

$$\left[\partial_\eta^2 + \left(1 + \frac{1}{fk^2}\right)k^2\right]\tilde{\mathbf{B}} = 0, \quad (9)$$

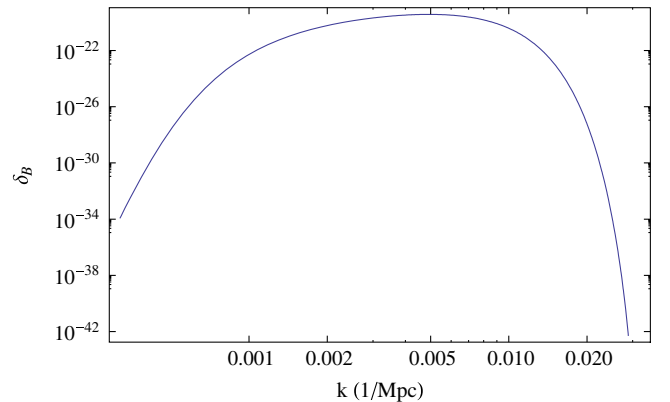


FIG. 2 (color online). The power spectrum today, δ_B^0 (Gauss), for $\Lambda = 10^{-2}$ and $k_\Lambda/k_{\min} = 10^2$.

where

$$f = 2\alpha^2 \frac{X}{V_{,X}} \left(\frac{2V_{,XX}}{M_{\text{p}}^2 k^2} - 1 \right), \quad (10)$$

and $\tilde{\mathcal{B}}$ is the Fourier transform of \tilde{B} . Thus the three-form rotational modes propagate with a nontrivial dispersion relation; they can therefore, in principle, be significant even at large scales. In particular, there is a divergence at $f \rightarrow 0$, that is, $2V_{,XX} \rightarrow M_{\text{p}}^2 k^2$. Depending on the sign of the sound speed squared, which in turn is determined by the shape of the potential, the vector perturbations over some range of scales can be drastically amplified (see below).

Plugging the solution into Eq. (8) then shows that the electromagnetic potential evolves as a harmonic oscillator driven by an external force, $\mathcal{A}'' + k^2 \mathcal{A} = \mathbf{F}(\tilde{\mathcal{B}})$, where $F(\tilde{\mathcal{B}}) = -2\alpha(\partial_\eta^2 + k^2)\tilde{\mathcal{B}}$ and \mathcal{A} is the Fourier transform of \mathbf{A} . One can also consider the autonomous evolution equation for \mathbf{A} ,

$$(-\partial_\eta^2 + \Delta)[1 - f(-\partial_\eta^2 + \Delta)]\mathbf{A} = 0. \quad (11)$$

From this it is obvious that the usual plane wave solutions always exist, but now there are two additional modes. At $f \sim 0$, where the three-form perturbation is divergent, Eq. (11) reduces to second order. Going to Fourier space, we have for each propagating degree of freedom

$$\begin{aligned} \mathcal{A}^{(4)} + 2\frac{f'}{f}\mathcal{A}^{(3)} + \frac{1}{f}(f'' - 1 + 2k^2 f)\mathcal{A}'' + 2\frac{f'}{f}k^2\mathcal{A}' \\ + k^2\left(\frac{f'' - 1}{f} + k^2\right)\mathcal{A} = 0. \end{aligned} \quad (12)$$

Notice the appearance of a fourth order time derivative in our equations, which might lead to the appearance of negative norm states. We are working here with an effective field theory which is only valid at low energies $k \ll V_0^{1/4}$, where V_0 sets the UV mass scale of the model, and we need to ensure that the dynamic effects are confined to such a range; we will see that this is indeed the case in our specific examples.

Hence, to summarize, the effective driving force for \mathbf{A} induced by the instability of $\tilde{\mathbf{B}}$ provides a mechanism to boost the magnetic fields. We are not aware of any previous amplification mechanism based on nontrivial dynamics of the primordial rotational perturbations, since most magnetogenesis in the literature can be effectively described by introducing either a mass term or a time-dependent coupling constant to the photon [11].

IV. EXAMPLES

For simplicity, we consider de Sitter solutions with constant comoving field X . There are two classes of such fixed points [22]: class B , which are the critical points corresponding to $X^2 = \pm \frac{2}{3}M_{\text{p}}^2$, and class C , which are

the critical points corresponding to the minima of the potential. The stability of these fixed points depends on the shape of the potential. In both cases the evolution equation (12) simplifies to

$$\mathcal{A}^{(4)} - \left(\frac{1}{f_0} - 2k^2\right)\mathcal{A}'' - k^2\left(\frac{1}{f_0} - k^2\right)\mathcal{A} = 0, \quad (13)$$

where f_0 is given by Eq. (10) evaluated at the fixed point. We can readily solve this equation to obtain

$$\begin{aligned} \mathcal{A}(\eta) = \mathcal{A}_1 \cos(k\eta) + \mathcal{A}_2 \sin(k\eta) + \mathcal{A}_3 e^{\Gamma k\eta} \\ + \mathcal{A}_4 e^{-\Gamma k\eta}, \end{aligned} \quad (14)$$

where \mathcal{A}_j are constant vectors and $\Gamma^2 + 1 \equiv 1/(f_0 k^2)$. Besides the usual plane waves, the two interesting and new solutions are also oscillatory if $\Gamma^2 < 0$, but if $\Gamma^2 > 0$, one of them is exponentially growing and the other exponentially decaying.

As an example, consider the exponential model $V = V_0 \exp(-\beta X^2/M_{\text{p}}^2)$. At the fixed point B , which is stable if β is positive, we have

$$\Gamma^2 = \frac{\kappa_\Lambda^2 - \kappa^2}{\Lambda^2 \kappa_\Lambda^2 + \kappa^2}, \quad (15)$$

where we have defined $\Lambda^2 \equiv 8\alpha^2/3/(1 - 8\alpha^2/3) \simeq 8\alpha^2/3$ and $\kappa_\Lambda^2 \equiv 8\beta V/(3M_{\text{p}}^2 \Lambda^2) \simeq \beta V/(\alpha^2 M_{\text{p}}^2)$. In this way it is immediate to see that the exponentially growing solution exists only for $k \leq k_\Lambda$; k_Λ is therefore the UV scale which defines the instability band. We will repeatedly be using the notation $\kappa \equiv k/\mathcal{H}_e$, which normalizes k to the highest mode accessible by inflation, \mathcal{H}_e —we denote $\mathcal{H} \equiv a'/a$, where a is the scale factor of the Universe and the prime stands for the conformal time derivative.

Therefore, the scale acting as a *de facto* UV cutoff is identified with k_Λ ; this is well below the actual UV cutoff, set by $V_0^{1/4}$, and our theory is trustable within this limit. In the more general and complete theory, alongside terms in the Lagrangian like $F(A)F(B)$, which would make the Hamiltonian unbounded from below, also higher order, stabilizing, terms such as, schematically, $(F(A)F(B))^2/V_0$ would appear; in the full theory, if constructed appropriately, there will be no ghosts (see, for instance, the Galileon case)—we believe that it goes beyond the scope of the present work to show explicitly that such a theory can be assembled, and we shall be content with working in the safe low energy limit.

The other fixed point C at the origin is an attractor when β is negative. There we have

$$\Gamma^2 k^2 = \frac{\beta V_0}{\alpha^2 M_{\text{p}}^2} - k^2. \quad (16)$$

Thus, when this point is an attractor, there is no instability of the vector modes. If we are inflating at a saddle point (i.e., $\beta > 0$) vector modes will be unstable at scales $k^2 < \frac{\beta V_0}{\alpha^2 M_{\text{p}}^2}$.

We summarize the properties of the fixed points for several classes of potentials and the corresponding expressions for $\Gamma^2 + 1$ in Table I.

In general, it is easy to see when the instability for the vector potential \mathcal{A} , parametrized by Γ^2 , is present, and at which scales. If large scales, $k \rightarrow 0$, are to be exponentially amplified, then it is necessary that $4\alpha^2 X^2/M_{\text{p}}^2 \leq 1$. Notice that at large momenta $\Gamma^2 \rightarrow -1$ and we recover plane wave solutions. Hence, there is, in general, a low energy band which is unstable given the above condition.

Beside the turning point where Γ^2 changes sign, typically there would be a singularity in Γ^2 for the mode k which nullifies its denominator. This can lie within the exponentially growing band resulting in an infinite energy density around such a scale. The question of where the singularity will occur for a specific potential depends on the sign of $V_{,X}X$: if it is positive the divergence will appear; in the opposite case, $V_{,X}X < 0$, the problematic scale is beyond the UV cutoff. In the specific examples we discuss this uncontrolled instability is avoided, whereas the exponential, controlled, growth is attained for a low energy band of modes.

V. THE SPECTRUM

The action (5) is canonical for the EM field, and as long as we treat the coupling to the three form as a perturbation, we can quantize canonically [27]. We are interested of course in solutions which grow with conformal time, so we will choose $\mathcal{A}_1 = \mathcal{A}_2 = \mathcal{A}_4 = 0$ —although this may appear artificial, it is immediate to convince oneself that, were these solutions kept, they would become very rapidly irrelevant in comparison to the growing \mathcal{A}_3 term (\mathcal{A}_1 and \mathcal{A}_2 are simply vacuum plane waves, and \mathcal{A}_4 decays exponentially).

We have two options in fixing the initial conditions (IC) for the EM vector potential \mathcal{A} in Fourier space: either we set them at some initial time, which can be taken to correspond to the beginning of inflation η_i , for all modes, or we can normalize the fields to their Bunch-Davies vacuum $\sqrt{2k}\mathcal{A} = e^{-ik\eta}$ at the time they become superhorizon. The first possibility is the simplest, but

perhaps not the most physical nor appealing one, for it introduces an explicit and very strong dependence on the IC of inflation, which is antithetic to what inflation is brought in for. Also, it poses the theoretical question of how to properly choose a vacuum state since the effective coupling is never turned off. Nonetheless, the latter issue can be resolved if we work in the small coupling regime: in this case it is consistent to choose the Bunch-Davies solution as the approximate IC.

The other choice corresponds to a coupling which dynamically becomes relevant at some time (e.g. horizon crossing), while being inefficient for smaller scales. This option makes it clear that for all modes the specific IC do not matter, while they can be consistently chosen to be in vacuum at any early time without affecting the final result. However, this demands a more elaborate theoretical motivation for the time dependence in α —if possible at all—and will be discussed in a subsequent publication [24].

One more comment is in order here. Since we work with the simplified and idealized toy model where X does not go anywhere, f is also constant, and the equation of motion for $\tilde{\mathcal{B}}$ (9) does not admit plane wave solutions at any time. However, it is easy to convince ourselves that in more realistic models, when $fk^2 \rightarrow \pm\infty$ as $k\eta \rightarrow -\infty$ (which depends on the shape of the potential), the rotational perturbation $\tilde{\mathcal{B}}$ is initially in its Bunch-Davies vacuum, and so is \mathcal{A} . That is, the exponent Γ in reality is time dependent as well, and flips from imaginary to real at a given time; we take the latter to be η_i for simplicity. This also means that in the full model the dependence on the initial conditions will be pronouncedly less severe.

The IC fix the value of \mathcal{A}_3 and give us the following solution to work with:

$$\mathcal{A} = \frac{1}{\sqrt{2k}} e^{\Gamma k(\eta_i - \eta)} \text{IC} \quad \text{at} \quad \eta = \eta_i. \quad (17)$$

We can immediately write down the corresponding expressions for the energy density associated with the EM field at the end of inflation as

$$\rho_{\text{EM}} = \frac{1}{4\pi^2 a_e^4} \int \frac{dk}{k} k^3 [|\mathcal{A}'|^2 + k^2 |\mathcal{A}|^2] \quad (18)$$

TABLE I. Stability of the de Sitter fixed points and the characteristic width of instability of the vector potential in four classes of models. U—unstable; S—stable.

$V(X)/V_0$	B : stability	B : $\Gamma^2 + 1$	C : stability	C : $\Gamma^2 + 1$
$\exp(-\beta X/M_{\text{p}})$	B_+ S for $\beta > 0$, B_- S for $\beta < 0$	$\frac{\lambda V_0}{2\alpha^2(\frac{2}{3}\lambda V_0 \pm \sqrt{2/3}M_{\text{p}}^2 e^{\pm\lambda\sqrt{2/3}})}$	S	$\frac{\lambda V_0}{2\alpha^2 X M_{\text{p}} k^2}$
$\exp(-\beta X^2/M_{\text{p}}^2)$	S for $\beta > 0$	$\frac{\beta V_0}{\alpha^2(\frac{2}{3}\beta V_0 + e^{-(2/3)\beta} M_{\text{p}}^2 k^2)}$	S for $\beta < 0$	$\frac{\beta V_0}{\alpha^2 M_{\text{p}}^2 k^2}$
$(X/M_{\text{p}})^2$	U	$\frac{V_0}{\alpha^2(\frac{2}{3}V_0 - M_{\text{p}}^2 k^2)}$	S	$-\frac{V_0}{\alpha^2 M_{\text{p}}^2 k^2}$
$(X/M_{\text{p}})^{2n}$, $n > 1$	U	$\frac{nV_0}{\alpha^2(\frac{2}{3}nV_0 - (\frac{2}{3})^{n-1}M_{\text{p}}^2 k^2)}$	S	0
$(X^2 - C^2)/M_{\text{p}}^4$	S for $C > \sqrt{2/3}M_{\text{p}}$	$\frac{2V_0(\frac{2}{3} - C^2/M_{\text{p}}^2)}{\alpha^2(\frac{16}{3}V_0(\frac{2}{3} - C^2/M_{\text{p}}^2) - M_{\text{p}}^2 k^2)}$	C_1 S, C_2 U	$\frac{2V_0 C^2/M_{\text{p}}^2}{\alpha^2 M_{\text{p}}^2 k^2}$

$$= \frac{\mathcal{H}_e^4}{8\pi^2 a_e^4} \int d\kappa \kappa^3 (\Gamma^2 + 1) e^{2\Gamma\kappa(1/\kappa_i - 1)}, \quad (19)$$

where of course $\kappa_e = 1$ by definition, and we can clearly ignore the second piece in the exponent, being much smaller than the first, positive, contribution. Also, we work with small α , which means $\Lambda \ll 1$ and $\Gamma \gg 1$; this implies that, in general, $E^2 \gg B^2$.

The integral (19) is not simply dealt with, and we must resort to numeric methods to solve it. The limits of integration are $\kappa = \kappa_i = \exp(-N)$, where N is the total number of e-folds inflation lasts for, and $\kappa = \kappa_\Lambda$, beyond which the solution (17) does not apply anymore.

In Fig. 1, we present the regions in the parameter space (Λ, k_Λ) for the well-behaved exponential potential $\exp(-\beta X^2)$, for which the ratio ρ_{EM}/ρ_X stays below 1 at the end of inflation (blue region) or goes beyond 1, thereby leading to an instability in the de Sitter background (red region). Superimposed are the contours for a given value of the magnetic power at 3/Gpc today, where the regions for which $\delta_B^0 \geq 10^{-15}$ G and $\delta_B^0 \geq 10^{-25}$ G are in evidence (green and yellow, respectively)—we define $(\delta_B^0)^2 = k^5 |\mathcal{A}|^2 / 4\pi^2$. These are values that correspond to the magnetic fields observed in the intergalactic medium, and a typical value for a successful seed to be fed the magnetohydrodynamic plasma at late times.

Figure 2 plots the power spectrum today, for $\Lambda = 10^{-2}$ and $k_\Lambda/k_{\min} = 10^2$; notice the knee at high energy and the following very rapid decay. Thanks to the UV cutoff, such a spectrum also nicely helps in avoiding the stringent constraints which late time cosmology poses on smaller scale fields [28]. These figures were obtained for around $N = 66$ e-foldings of inflation, $\mathcal{H}_e/a_e \approx 10^{13}$ GeV, and assuming the Universe was always dominated by radiation from the end of inflation onwards.

There is some degree of fine-tuning: as one realizes by looking at Eq. (17), there is a very strong sensitivity (doubly exponentially in terms of the total number of e-folds N) to the initial time, and only scales not too far from the first ones available to inflation can be amplified efficiently without leading to a catastrophic backreaction effect. In the figure we show the final power at $k = 3/\text{Gpc}$, where the largest mode is only around $1/\mathcal{H}_0 \approx 4$ Gpc. The plot of the spectrum makes it clear that only a couple of orders of magnitude in k will be amplified efficiently; for instance, the strength at 10/Gpc is 10^{-21} G, but drops to 10^{-47} G at just 30/Gpc. If we were to push the beginning of inflation further back, at much smaller \mathcal{H}_i , it would be impossible to amplify, without backreacting,

today's large scales. With the alternative prescription of a time-dependent coupling, the sensitivity to initial conditions can, in principle, be eliminated. Recall again also that this feature is partly due to the idealization of the model we work with.

Looking again at the allowed region in Fig. 1, it appears that one may push the coupling $\alpha \approx \Lambda$ to arbitrarily small values and still come out with enough power on a given scale. This is only partially true, since α also enters the UV cutoff, and as $\alpha \rightarrow 0$ one needs to keep $\alpha^2 k_\Lambda^2 \approx \beta V_0 e^{-2\beta/3} / M_{\text{p}}^2$ fixed; that is, the window of modes which can be boosted has to be kept narrow. Order-one slopes β would then demand extremely low bare V_0 , which is desirable since it corresponds to little kinetic energy in the dual scalar description. Furthermore, one can consider models with either very high or very low potential slope parameters β to reconcile suitable UV cutoffs with small couplings α .

ACKNOWLEDGMENTS

T. K. is grateful to Nelson Nunes for n-formal discussions. T. K. is supported by the Research Council of Norway. F. U. thanks Chiara Caprini for hospitality at the IPHT, Saclay, and valuable insight.

Note added.—After this work was completed, two preprints dealing with the anisotropies generated in inflationary magnetogenesis appeared [29,30]. Their conclusion is that such contributions to curvature perturbations become unacceptably large during the subsequent radiation era, thereby lapidarily coercing the largest classes of models to fail. Although we have not completed the full, and very laborious and lengthy, calculation, we believe that this result need not apply *as is* to our mechanism for three reasons.

The result of [29,30] strongly depends on the limitations on the generated power spectrum; our case presents a very peculiar spectrum, due to its low energy cutoff, which allows for almost any slope, thereby offering the possibility that the dangerous term in fact is not as big as we thought.

Second, our Bardeen equation is quite different due to the three-form perturbations [22]; the solutions to this equation during inflation can have different scaling with momentum k and defuse the too-large anisotropies.

Finally, the time evolution of \mathcal{A} is not a power law as in [29,30], but exponential, which in turn is strongly k dependent; this implies that the time evolution of perturbations could further differ from what is predicted in their case.

We elaborate on this critical issue in our upcoming publication [24].

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