Immigration, borrowing constraints and housing market volatility in general equilibrium

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Preface

There are two people who have been of immense help during the work on this thesis. Fist of all my supervisor, Steinar Holden, who provided helpful discussions, comments and ideas. Secondly, thanks to Kjetil Stiansen for valuable discussions, coffee breaks, proof reading and comments. The thesis would have been poorer without them.

All remaining errors are my own.

Helene Onshuus
10. may 2016
Abstract

I present a simple two-period general equilibrium model with heterogeneous households, durable and non-durable consumption and loan-to-value collateral constraints. Some agents are simultaneously less wealthy and more mobile than others in the sense that they may relocate as response to an exogenous shock. The idea is to see how housing market volatility can arise as a result of instability in demand caused by having a mobile subgroup in the population. I show how volatility can be reduced if a loan-to-value borrowing constraint is imposed to reduce agents’ access to credit.
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1 Introduction

A loan-to-value borrowing constraint is a significant restriction on household behavior. All policy intervention seeks to steer agents’ behavior away from what individual agents would consider optimal in the absence of restrictions. In the wake of the financial crisis in 2007-09 many regulators have turned to macroprudential tools to manage excessive risk-taking on an economy-wide level. In Norway a strict loan-to-value policy was imposed in June 2015, restricting the amount a household can borrow to finance a house to 85% of the collateral value. To justify this kind of intervention there needs to be some kind of market inefficiency present.

A housing investment often constitutes a large part of the households’ total wealth. Price movements in the housing market therefore cause movements in home-owners’ wealth level. Large and unpredictable variations in household wealth can cause great problems for the individual household as well as have important macroeconomic consequences. The problem for the household arise first and foremost if a housing market downturn coincides with income loss. If the market value of a house falls, an indebted household may loose all its equity or even go underwater on its mortgage. If households run into payment trouble they may then be forced to default and sell their home at a loss. Even if they are not forced to default they may become unable to relocate to take new employment because they cannot afford to sell the house when the market value is below the value of their mortgage. Both income losses and house price movements may be highly correlated within the region, and widespread defaults and labor market rigidity may both have macroeconomic consequences.

In this thesis I will consider one source of housing market volatility, namely instability in the demand for housing. The housing market will normally at a given point in time have a limited number of buyers and sellers, and a shift in demand may therefore cause a large price response. I will present a model with heterogeneous households where one group is more mobile and more likely to relocate at some point in time. These agents can be thought of as either immigrants, young households or students who have not quite settled down yet. Because of this they may be more likely to relocate as a response to changes in the labor market or other exogenous forces. To the best of my knowledge there are no other studies that do anything similar.
There are several other sources of housing market volatility such as changing expectations, changing regulations, frictions, changes in the interest rate or household income, changing demographics, labor market fluctuations or changes in the local environment. I do not consider any of these sources of volatility in this model, even though they may in many cases be more important than instability in demand due to mobility of a subgroup of the population. The reason is primarily that I want to focus the analysis on the effect of heterogeneity and mobility, and secondly that the other topics are to a larger degree covered in the literature already.

I find that imposing a loan-to-value borrowing constraint does indeed dampen the volatility in the housing market. Housing investments are in most cases loan-financed, and access to credit is therefore crucial to households’ options in the housing decision. Throughout the analysis I assume that agents who have trouble meeting the initial down payment requirement when buying a house coincide exactly with those that are likely to suddenly relocate. This is to capture the idea that groups with little or no wealth, like immigrants or students, often are more mobile than other groups. The reduction in volatility arise because the constraint forces agents who are not able to pay the initial down payment requirement into the rental market, thus reducing demand for housing.

An important result of my analysis is that the constraint affects those who are already relatively poorer. Households who can afford the initial down payment requirement are unaffected, while those who cannot are forced into the rental market where they have to pay a higher price for their consumption of housing each period.

I will construct a simple two-period general equilibrium model where agents consume both housing and non-durable consumption goods. Volatility is caused by a shock to the population size in the second period. Due to the simple structure of the model I will define volatility as the difference between the price that agents expect to be realized in period 2 and the price that actually clears the market when the shock occurs.

I will present three versions of the model. First I construct a simple version with a market for non-durable consumption goods and a housing market and let the shock be completely unexpected. Second, I include a rental market for housing. Finally, I let agents have rational expectations and beliefs about the probability of the shock. For all three versions of the models I will consider both an unconstrained economy and an
economy where I impose a borrowing constraint. I want to discuss whether or not a borrowing constraint may dampen housing market volatility.

Throughout the thesis I will use the expression *price of housing services* as something slightly different than the house price. The price of housing services will be what the household has to pay to be able to consume one unit of housing for one period of time, while the house price is the amount the household needs to pay to buy and keep one unit of housing.

The thesis proceeds in the following way: Chapter 2 contains a brief survey of existing literature on household debt and macroprudential policies. Chapter 3 presents the baseline model including results from numerical simulation. Chapter 4 extends the model to include a rental market and chapter 5 extends the model further to feature uncertainty and rational expectations. Chapter 6 discusses the results and concludes.

2 Existing Literature

There are several strands of literature that discuss market inefficiencies, externalities and the case for macroprudential regulation. Borchgrevink, Ellingsrud and Hansen (2014) identify six categories of macro-level externalities that may call for macroprudential regulation (*pecuniary externalities*, *interconnectedness externalities*, *strategic complementarities*, *aggregate demand externalities*, *market for lemons* and *deviations from full rationality*). Of these there are three that concern the decisions made by the household sector, namely pecuniary externalities, aggregate demand externalities and deviations from full rationality. I will restrict attention to pecuniary externalities and aggregate demand externalities, as deviations from full rationality open up a very different range of research questions.

Following Holcombe and Sobel (2001), pecuniary externalities can be defined as external effects on a third-party through relative prices or asset prices. The price mechanism is necessary for market efficiency, and pecuniary externalities does not necessarily cause any inefficiency that calls for regulation. Aggregate demand externalities, on the other hand, arise when the price mechanism is distorted such that a fall in demand by one agent is not picked up by increased demand from other agents. The literature presented here makes a case for macroprudential policies to reduce household debt accumulation on the
basis of both pecuniary and aggregate demand externalities. None of these externalities are central features in my model, instead my analysis should be considered an addition to the discussions already covered in the literature.

2.1 Pecuniary externalities and overborrowing

Households typically face a borrowing constraint that restricts the amount the household is allowed to borrow to the value of the collateral it can raise. Miles (2015) argue that the fact that housing often is loan-financed has caused the observed volatility in the United States’ housing market over the last decade. Debt levels may rise in a boom because expectations of high and increasing future house prices increase demand. Increased demand in turn leads to higher house prices, allowing agents to take on even higher levels of debt. If expectations turn and house prices start to fall, agents may be left with unsustainably high debt levels. If one household is forced to default on their debt and sell the house, this may contribute to the fall in house prices and may cause more households to default. Miles further argues that the high volatility observed should not be interpreted as efficient price adjustments, and suggests that policy measures to limit debt accumulation are introduced.

The mechanism described by Miles (2015) is not necessarily inefficient, unless individual agents take on debt above the socially optimal level. Bianchi (2010) and Bianchi and Mendoza (2010) suggest that pecuniary externalities, if sufficiently severe, can cause excessive credit expansions above what is socially optimal. Inefficiently high debt-levels increase the risk of a financial crisis and can therefore have severe consequences. Bianchi (2010) constructs a non-linear dynamic stochastic general equilibrium model where he considers whether agents generate too much debt relative to the social optimum in the presence of a collateral constraint. He finds that when collateral constraints are binding, individual agents do not internalize their contribution to the debt-deflation mechanism as described by Fisher (1933). Debt levels therefore rise above the social optimum, increasing financial fragility.

In the housing boom leading up to the financial crisis in the United States, household credit increased in pace with house prices. There is a substantial body of mortgage default literature linking the numerous defaults during the crisis to the increase in leverage (Adelino, Schoar, and Severino 2015; Ferreira and Gyourko 2015; Mian and
Sufi 2010). This suggests a case for policy to try to restrict overborrowing.

2.2 Aggregate demand externalities and debt-deleveraging

Earlier models have implemented borrowing constraints to analyze how households will self-insure when the availability of debt is restricted and future income is stochastic (e.g. Aiyagari 1994; Bewley 1977). To better understand how leverage may create aggregate demand externalities there are several studies that have introduced new frictions into their models, such as the zero lower bound. Eggertson and Krugman (2012), Midrigan and Philippon (2011), Guerreri and Lorenzoni (2015) and Hall (2011) are all recognized as central to what has been known as debt-deleveraging theory.

These studies all analyze the mechanism at work when too much debt creates or amplifies a recession. Hall (2011) presents a general equilibrium model with debt and investment overhang at the beginning of the first period, meant to capture some of the observed features in the United States economy at the beginning of the housing market downturn in 2006/07. Rognlie, Shleifer and Simsek (2014) extend the discussion of an investment hangover to show how an ex-ante reduction of investment can reduce the subsequent economic downturn. Guerreri and Lorenzoni (2011) use a Bewley-model with debt constraint, but include durable goods to show how investment in housing increases household borrowing. When they introduce a shock to the debt-limit, the high level of accumulated debt leads to a greater consumption response than in the model without durables.

Eggertson and Krugman (2012) creates a simple framework for debt-deleveraging and aggregate demand externalities which has later been extended by studies such as Korink and Simsek (2014) and Fahri and Werning (2013). Eggertson and Krugman let agents have different discount factors, with the result that the more patient agents become savers while the more impatient are borrowers. Introducing a shock to the borrowing limit, borrowers have to reduce their debt instantly and are thus forced to cut back on consumption. When they do that, the equilibrium interest rate has to fall in order to induce patient agents to reduce their savings accordingly. If the interest cannot fall below zero the savers will not reduce their savings enough. Then there will be excess savings causing consumption levels below the social optimum. The insufficient consumption demand of one agent reduces aggregate output and thus lowers other agents’ income.
This way an aggregate demand externality arises in the liquidity trap and causes a recession.

Korinek and Simsek (2014) show that the credit boom arises even though agents expect the shock. In other words, agents do not internalize how their borrowing or saving decision affects other agents' income in the liquidity trap, causing aggregate demand to fall. This further lowers the total demand of consumption goods, creating a recession. Both Korinek and Simsek (2014) and Fahri and Werning (2013) show that macroprudential tools that restrict borrowing ex-ante are Pareto improvements to the unregulated market equilibrium.

3 The stylized model

The housing decision is intertemporal in nature because housing is a durable good. Most households borrow funds to be able to buy a house, pledging their future income to debt repayment and the house as collateral. If the household is constrained from borrowing more than a fraction of the collateral value, purchasing a house will demand a significant amount of equity up front. That may severely reduce low-equity households’ ability to invest in housing.

I consider a two-period model of a small open economy that produces both tradable and a non-tradable good. The non-tradable sector produces housing and the tradable sector produces consumption goods. The economy has a heterogeneous population with two types of households. The groups are similar, but differ in their attachment to the society they live in, some being more mobile then others.

The two population groups are labeled "aliens" and "natives". The natives are well-integrated into the society and are thought of as the majority population in any economy or society. The aliens on the other hand, are individuals that for some reason are not fully integrated into society. The low degree of integration makes them more mobile and more likely to leave the economy if circumstances change. The aliens can be thought of as immigrants who have recently arrived or young households or students who are still in the process of establishing themselves, all with prospects of being fully integrated into society in the future.
In all other respects, the natives and aliens are identical except for a difference in initial endowments level. All households are given an endowment in units of the consumption good in the first period. The endowment of natives is assumed to be significantly higher than the endowment of the aliens. If we think of aliens as being young, low-equity households, students or newly arrived immigrants it is realistic to assume that they are relatively poor compared to the majority of the population.

I focus the analysis on how the two types of households are affected differently by the borrowing constraint when all that separates them is their level of available resources in the first period. An alternative approach could be to assume that the two types are perceived differently by financial institutions. Natives might be considered to be safe borrowers while aliens are risky. In the model I have not included any consideration of risk by the financial institutions, and the only difference between the groups is their endowments. I assume that the difference in endowments is such that aliens will always be affected if the borrowing constraint is imposed, while natives always are unconstrained.

Because of the simple set-up with only two periods volatility will be defined as the difference between the price that agents expect and the price that is realized when the shock occurs. This is because I want to see how the market equilibrium is affected when the price that is realized after the shock differs from the price that agents expect, and have found no better word than volatility to use.

This chapter will be structured in the following way: First I define the economic environment with timing assumptions, production, agents and constraints. Then I define and solve for the equilibrium house prices that agents expect before they are aware of the shock. Then I introduce the shock, find the optimal response by the households and solve for the new housing market equilibrium. Finally I present the results of numerical simulation of the model and discuss the results.

3.1 Timing

Time in this model is divided into two periods. There are essentially two features that characterize the different periods. First of all, the availability of new housing capacity is elastic only in period 1. In this period, supply will equal demand to clear the market.
In period 2, the supply of housing is fixed and equal to what it was in period 1. Second, there is a shock that hits the economy when it enters the second period.

The fixed housing supply in period 2 is meant to capture the naturally slow adjustment of housing supply in the short term. Natural boundaries such as rivers, mountains and seasides restricts the availability of new land for housing constructions. Further, it is expensive and unpopular to demolish existing housing stock to construct new. Political and bureaucratic processes in construction cases may be slow. Home owners are often attached to their neighborhood, creating resistance to new construction. Lastly, the construction of a house is in itself time consuming.

The shock that is introduced to this economy is a demand shock. For some reason, a fraction \((1 - \theta)\) of the alien population, with \(\theta \in (0, 1)\) is forced to leave the economy. I specify no source of the shock because it could come from a broad range of exogenous or endogenous factors. First I will let the agents perceive the future as certain. They do not expect the shock to happen or even consider the possibility of it. Later I will extend the model to include rational expectations of the possibility of the demand shock, but for now the agents are unaware of it.

### 3.2 Production

#### 3.2.1 The tradable sector

The tradable sector produces consumption goods for sale at the world market at an exogenous price \(p_c\). The production uses labor as input and has constant returns to scale:

\[
c = f_c(l) \quad \quad \quad f'_c > 0 \quad \quad \quad f''_c = 0
\]

Because of the CRS production function and the constant price, wages are determined by the profit maximization of the firm in the tradable sector.

\[
\max_l \quad p_c f_c(l) - wl
\]

which has first order condition

\[
w = p_c f'_c(l)
\]

Because \(f'_c(l)\) is a constant, the wage and the agents labor income is fixed.
3.2.2 The non-tradable sector

The non-tradable sector consists of a representative firm that produces housing capacity with a decreasing returns to scale technology, using labor as input. Let \( h = f_h(l) \), with \( f_h' > 0 \) and \( 0 < \alpha < 1 \). The profit maximization of the representative firm is

\[
\max p_{H,1} f_h(l) - w l
\]

with first order condition

\[
f_h'(l) = \frac{w}{p_{H,1}}
\]

Let the production function be given by \( f_h(l) = l^\alpha \). Then the supply of housing is given by

\[
h_{\text{supply}} = \left( \frac{\alpha p_{H,1}}{w} \right)^{\frac{1}{1-\alpha}} = \left( \frac{\alpha p_{H,1}}{f_h'(l) p_c} \right)^{\frac{\alpha}{1-\alpha}}
\]

All firms are owned by the native households, and profits are part of their endowments \( e^N \).

3.3 Agents

The economy consists of a large number of households divided into two groups, natives and aliens. The agents type is denoted by \( i \in \{N, A\} \). Total population is normalized to unity, with a fraction \( \delta \in (0, 1) \) being aliens.

The agents are assumed to have identical preferences over housing and consumption goods. Their utility over the two periods is given by

\[
\max \sum_{t=1}^{2} \beta^{t-1} U(u(c_i^t) + v(h_i^t))
\]
where the outer function $U(\cdot)$ has $U' > 0$, instantaneous utility of consumption has $u' > 0$ and $u'' < 0$ and the instantaneous utility of housing $\nu(h)$ has $\nu' > 0$ and $\nu'' < 0$. Both instantaneous utility functions will be specified as log utility.

Labor supply is fixed and all receive the same wage. Their labor income in each period is denoted by $\omega$. Agents face the period-by-period budget constraints

$$\omega + e^i + b^i_2 \geq p_c c^1_i + p_{H,1} h^1_i$$
$$\omega + p_{H,2} h^1_i \geq p_c c^2_i + p_{H,2} h^2_i + (1 + r)b^2_2$$

where $e^i$ is the initial endowment and $b^i_2$ is the agents debt in period 1 to be repaid in period 2. Debt is available at the world financial market at interest rate $(1 + r)$, assuming that $r$ is such that $\beta(1 + r) = 1$. Housing is a durable good, and agents buy housing in period 1 both because they derive utility from it in that period and because it becomes part of their wealth in period 2. Because a household can sell its housing capacity at the market price in period 2, $p_{H,1} - p_{H,2}$ is the price of the services a house delivers in period 1. That is, in period 1 the house price will reflect the price of consuming housing services in both period 1 and period 2. This means that to buy a house in period 1 the household will have to pay for the housing it will consume in both period 1 and 2. In period 2 there are no more future periods and the price will only reflect the consumption of housing in that period. In period 2 they have the opportunity to change their consumption of consumer goods and housing if they receive new information.

I will analyze both an unconstrained economy, and one where I impose a debt constraint of the form

$$b^i_2 \leq \phi p_{H,1} h^i$$

This is a loan-to-value borrowing constraint where $\phi \in [0, 1]$ determines how large a fraction of the housing value the agent can borrow when the house is pledged as collateral for the loan. If $\phi = 0$ the household cannot borrow at all and if $\phi = 1$ agents can borrow the full value of the house. However, the households can never borrow to finance consumption of non-durable goods.

Let $\lambda_1$, $\lambda_2$ and $\mu$ and be the Lagrange multipliers of the optimization problem, then the
first order conditions of the utility maximization are

\[ U'(\cdot)u'(c_1^i) = \lambda_1 p_c \]
\[ \beta U'(\cdot)u'(c_2^i) = \lambda_2 p_c \]
\[ U'(\cdot)u'(h_1^i) = \lambda_1 p_{H,1} - \lambda_2 p_{H,2} - \mu \phi p_{H,1} \]
\[ \beta U'(\cdot)u'(h_2^i) = \lambda_2 p_{H,2} \]
\[ \lambda_1 = (1 + r) \lambda_2 + \mu \]
\[ \mu(b_2^i - \phi p_{H,1} h_1^i) = 0, \quad \mu \geq 0 \]

in addition to the budget constraints.

### 3.3.1 Unconstrained agents

If the household has sufficient funds available in period 1, it will not be affected by the borrowing constraint yielding \( \mu = 0 \) in the first order conditions. Imposing log utility and assuming \( \beta = 1, \ r = 0 \) for computational ease and clarity, the unconstrained agent has the following demand functions

\[ c_1^i = \frac{2\omega + e^i}{4p_c} \]
\[ c_2^i = \frac{2\omega + e^i}{4p_c} \]
\[ h_1^i = \frac{2\omega + e_i}{4(p_{H,1} - p_{H,2})} \]
\[ h_2^i = \frac{2\omega + e_i}{4p_{H,2}} \]

With log utility it is optimal for the agent to spend a constant fraction of total wealth on each good. If the price structure over time is flat that implies a perfectly smooth consumption path.

### 3.3.2 Constrained agents

If the household does not have sufficient funds available in the first period it will have to reduce consumption of both housing and non-durable goods. The distortion arise because the amount the household can borrow depends on the value of the house at the
time it is bought and because the house price in period 1 reflects the price of consuming housing services in both period 1 and 2, meaning that the household has to pay for its housing consumption in both periods up front.

Imposing log utility and the assumptions on $\beta$ and $r$, the first order conditions with $\mu > 0$ yield the optimality condition

$$\frac{1}{h^*_1} = (1 - \phi) \frac{p_{H,1}}{\omega + e^i - (1 - \phi)p_{H,1}h^*_1} - \frac{p_{H,2} - \phi p_{H,1}}{\omega + (p_{H,2} - \phi p_{H,1})h^*_1}$$

This condition determines the household’s optimal consumption of housing in the first period taking prices as given. It is nonlinear and needs to be solved numerically. The solution is unique \(^1\) and is denoted $h^*_1(p_{H,1}, \phi)$. The demand functions for the constrained household are thus

$$c^1_i = \frac{\omega + e^i - (1 - \phi)p_{H,1}h^*_1}{p_c}$$

$$c^2_i = \frac{\omega + (p_{H,2} - \phi p_{H,1})h^*_1}{2p_c}$$

$$h^*_1 = h^*_1(p_{H,1}, \phi)$$

$$h^*_2 = \frac{\omega + (p_{H,2} - \phi p_{H,1})h^*_1}{2p_{H,2}}$$

The constrained agent is prevented from spending as much as he would like on both consumption goods and housing in the first period. To maximize utility the constrained agent reduces consumption of both housing and consumption goods in the first period and will enter the second period with excess wealth relative to the case where he is unconstrained.

3.4 Equilibrium

Equilibrium in the unconstrained economy where no shock occurs is given by a set of prices $\{p_{H,1}, p_{H,2}\}$, where $e$ denote expected to distinguish it from the price that is realized after the shock hits, and a set of optimal allocations $\{c^1_i, c^2_i, h^*_1, h^*_2\}$ for $i = 1, 2$ such that all agents maximize utility over the two periods subject to the binding budget constraints.

\(^1\) See appendix A
In the constrained economy the borrowing constraint $b_i^2 \geq \phi p_{H,1} h_i^1$ also needs to be satisfied. Further, all firms maximize profits and markets clear. Due to the assumption of a small open economy both the goods market and the capital market will clear trivially, while the market clearing conditions for housing are given by

$$(1 - \delta) h_i^N + \delta h_i^A = h_{supply}$$

$$(1 - \delta) h_i^N + \delta h_i^A = (1 - \delta) h_i^N + \delta h_i^A$$

This is the equilibrium that agents expect before they become aware of the shock. The equilibrium is fully determined by the demand functions of both types of agents, eight in total, and the two market clearing conditions in the housing market.

### 3.4.1 The unconstrained economy

If no borrowing constraint is imposed, the housing market equilibrium in period 1 is given by the market clearing condition

$$(1 - \delta) \frac{2\omega + e^N}{4(p_{H,1} - p_{H,2}^H)} + \frac{\omega + e^A}{4(p_{H,1} - p_{H,2}^H)} = \left( \frac{\alpha p_{H,1}}{f_c'(l) p_c} \right) \frac{\alpha}{1 - \alpha}$$

and in period 2 by

$$(1 - \delta) \frac{2\omega + e^N}{4p_{H,2}^H} + \frac{\omega + e^A}{4p_{H,2}^H} = \left( \frac{\alpha p_{H,1}}{f_c'(l) p_c} \right) \frac{\alpha}{1 - \alpha}$$

These two market clearing conditions define two non-linear equations in two unknown variables and have the unique solution

$$p_{H,1} = \left( \omega + \frac{(1 - \delta) e^N + \delta e^A}{2} \right) \frac{1 - \alpha}{\alpha} \left( \frac{f_c'(l) p_c}{\alpha} \right)^\alpha$$

$$p_{H,2}^H = \frac{1}{2} \left( \omega + \frac{(1 - \delta) e^N + \delta e^A}{2} \right) \frac{1 - \alpha}{\alpha} \left( \frac{f_c'(l) p_c}{\alpha} \right)^\alpha$$

In the unconstrained equilibrium households want to have the same amount of housing in both periods to smooth consumption across time. This consumption smoothing will
yields the price structure above where the price of one unit of services provided by housing is the same in both periods, \( p_{H,1} - p_{H,2} = p_{H,2} \). There will no trade of housing in the second period with this price structure, and in this sense there is perfect stability in the housing market.

3.4.2 The constrained economy

When imposing the borrowing constraint, the equilibrium prices cannot be found analytically. Instead the equilibrium is given by the three-equation system consisting of the optimality condition for alien housing demand and the market clearing conditions for period 1 and 2:

\[
(1 - \delta) \frac{2 \omega + e^N}{4(p_{H,1} - p_{H,2})} + \delta h_{A*}^1 = \left( \frac{\alpha p_{H,1}}{f_c(l)p_c} \right) \frac{\alpha}{\omega}
\]

\[
(1 - \delta) \frac{2 \omega + e^N}{4p_{H,2}^*} + \delta \frac{\omega + (p_{H,2}^* - \phi p_{H,1})h_{A*}^1}{2p_{H,2}^*} = \left( \frac{\alpha p_{H,1}}{f_c(l)p_c} \right) \frac{\alpha}{\omega}
\]

\[
\frac{1}{h_{A*}^1} = (1 - \phi) \frac{p_{H,1}}{\omega + e^t - (1 - \phi) p_{H,1}h_{A*}^1} - \frac{p_{H,2}^* - \phi p_{H,1}}{\omega + (p_{H,2}^* - \phi p_{H,1})h_{A*}^1}
\]

Numerical simulation suggest that there is a unique solution for the three endogenous variables \( h_{A*}^1 \), \( p_{H,1} \) and \( p_{H,2}^* \) within what are reasonable values\(^{2}\).

The borrowing constraint reduces alien demand for housing. The sharp fall in demand results in a house price in period 1 strictly lower than the price in the unconstrained economy. Due to the borrowing constraint, the aliens are unable to transfer as much funds from period 2 to period 1 as they would like and are in a way forced to save. They will have excess wealth in period 2 relative to the unconstrained case, increasing their demand for both housing and consumption goods in that period. The increase in demand leads to a higher price of housing in period 2 relative to the unconstrained economy. Further, in contrast to the unconstrained economy where there were no trade in housing in the second period, there will be a transfer of housing from the natives to the aliens in the second period in the constrained economy.

\(^{2}\)See appendix A
3.5 Housing market volatility

In this section I introduce an intergalactic shock that forces a fraction \((1 - \theta)\) of the aliens to leave the economy. This is a major demand shock, with \(\delta(1 - \theta)\) of the total population abruptly vanishing. Housing supply is fixed in the second period and the price of consumption goods is given at the world market. The entire effect of the demand shock is therefore reflected in a sharp fall in the house price. The effect differs between the unconstrained and the constrained economy.

The shock is unexpected. When agents receive new information they immediately reconsider their consumption decision and face a new optimization problem taking into account that a change in house prices affects their wealth. The household’s wealth in period 2 depends on the choices it made in period 1, particularly on the housing decision because the house enters the budget constraint as part of total wealth in period 2. Housing enters the budget constraints both as an expense and as a way of saving from the first to the second period.

Let \(W^i_2\) denote household \(i\)'s wealth when entering period 2. The maximization problem of household \(i\) is now

\[
\max \quad U(u(c^i_2) + v(h^i_2))
\]

\[
s.t. \quad W^i_2 \geq p_c c^i_2 + p_{H,2} h^i_2
\]

with first order condition

\[
 u'(c^i_2) = v'(h^i_2) \frac{p_c}{p_{H,2}}
\]

Optimization with log utility yields the demand functions

\[
c^i_2 = \frac{W^i_2}{2p_c}
\]

\[
h^i_2 = \frac{W^i_2}{2p_{H,2}}
\]

If the household was able to purchase its desired amount of housing in period 1 it will in period 2 have a wealth of

\[
W^i_2 = \frac{1}{4} (2\omega + e^i) (1 + \frac{p_{H,2}}{p^i_{H,2}})
\]
where the superscript $s$ denote shock. The last parenthesis makes clear how wealth depends on the difference between the expected price and the price that is realized after the shock hits. If the price of housing suddenly falls, household wealth will fall accordingly. A household that was constrained from purchasing its desired amount of housing in the first period will have a financial wealth in period 2 of

$$W_i^2 = \omega + (p_{H,2}^s - \phi p_{H,1})h_i^{1*}$$

3.5.1 Equilibrium after the shock

After the shock occurs and agents become aware of the change in circumstances, the equilibrium change. The new equilibrium is given by the house price $\{p_{H,2}^s\}$ and the set of allocations $\{c_i^2, h_i^2\}$ for $i = 1, 2$ such that all agents maximize utility in period 2 subject to the binding budget constraint

$$2\omega + c^i - p_c c_1^i - p_{H,1} h_1^i + p_{H,2}^s h_1^i = p_c c_2^i + p_{H,2}^s h_2^i$$

if the agent was unconstrained in the first period, and subject to the binding budget constraint

$$\omega - \phi p_{H,1} h_1^{1*} + p_{H,2}^s h_1^{1*} = p_c c_2^i + p_{H,2}^s h_2^i$$

if the agent was constrained in period 1. Further, markets clear. As before, the goods market clear trivially due to the assumption of a small open economy, while housing market clearing is given by the condition

$$(1 - \delta)h_2^N + \delta h_2^A = (1 - \delta)h_1^N + \delta h_1^A$$

The equilibrium is fully determined by the demand functions of both natives and aliens, four in total, and the market clearing condition for the housing market.
3.5.2 Market clearing in the unconstrained economy

Imposing the shock in the second period, the market clearing condition is

\[
(1 - \delta) \frac{\frac{1}{2}(2\omega + e^N)(1 + \frac{p^f_{H,2}}{p_{H,2}})}{2p^s_{H,2}} + \delta \theta \frac{\frac{1}{2}(2\omega + e^A)(1 + \frac{p^f_{H,2}}{p_{H,2}})}{2p^s_{H,2}} \\
= (1 - \delta) \frac{2\omega + e^N}{4(p_{H,1} - p^f_{H,2})} + \delta \frac{2\omega + e^A}{4(p_{H,1} - p^f_{H,2})}
\]

The left hand side is demand for housing in period 2. The right hand side is supply, which equals the amount of housing that the native and alien households take with them into period 2. There is no new production of housing in period 2.

The price that clears the market is

\[
p^s_{H,2} = p^f_{H,2} \frac{(1 - \delta)(2\omega + e^N) + \delta \theta (2\omega + e^A)}{(1 - \delta)(2\omega + e^N) + \delta(2 - \theta)(2\omega + e^A)}
\] (1)

The market clearing price after the shock is strictly lower than the expected price, the price that would have been if there were no shock. From equation (1) it is clear that it is the magnitude of the shock, $1 - \theta$, that drives the fall in the house price after the shock.

Volatility is given by the difference from the expected price to the realized price when the shock occurs, $p^f_{H,2} - p^s_{H,2}$. This difference is shown in the figure below as the difference between the intersections of the two demand curves with the supply curve.
Figure 1: Market clearing in the unconstrained economy. The plot on the left shows the equilibrium in period 1, while the plot on the right shows the difference between the expected equilibrium and the equilibrium that is realized after the shock occurs in period 2.

Figure 1 depicts the market equilibrium in the unconstrained economy in both periods and the difference between the expected equilibrium and the realized equilibrium after the shock in period 2. The upper demand curve in the right hand graph is the expected demand in period 2, while the lower curve is demand after the shock.

3.5.3 Market clearing in the constrained economy

The market clearing condition after the shock in the second period in the constrained economy is

\[
(1 - \delta) \frac{1}{2p_{H,2}^*} \left( \frac{2\omega + e^N}{1 + \frac{p_{H,2}^*}{p_{H,2}}} \right) + \delta \theta \left( \frac{p_{H,2}^* - \phi p_{H,1}}{2p_{H,2}^*} \right) = (1 - \delta) \frac{2\omega + e^N}{4(p_{H,1} - p_{H,2}^*)} + \delta h_1^A
\]

The price that clears the market is

\[
p_{H,2}^* = p_{H,2}^* \left( 1 - \delta \right) \left( \omega + \frac{e^N}{2} \right) + 2\delta \theta \left( \omega - \phi p_{H,1} h_1^A \right)
\]
With volatility again given by the difference between $p_{H,2}^e$ and $p_{H,2}^r$.

**Figure 2**: Market clearing in the constrained economy. The plot on the right shows the equilibrium in period 1, while the plot on the left shows the difference between the expected equilibrium and the equilibrium that is realized after the shock occurs in period 2.

Market equilibrium is depicted by figure 2. Again, volatility is given by the difference between the intersections with the inelastic supply of the expected demand curve and the demand curve after the shock.

### 3.5.4 Results from numerical simulation

With volatility defined as the difference between the expected price and the realized price, numerical simulation yields the non-intuitive result of higher volatility in the constrained economy. The expectation was to find that imposing the borrowing constraint reduces volatility because of the reduction in demand reduces both housing capacity and price in period 1. But the model may be unsuited to give a precise analysis of volatility because the only alternative for constrained agents is to buy a smaller amount of housing. The borrowing constraint reduces alien demand for *both* housing and consumption goods in period 1, in a way forcing them to save more than what is otherwise optimal. This forced saving leaves aliens with excess funds in period 2 relative to the unconstrained case, which pushes the house price in period 2 higher than in the unconstrained economy.
irrespective of the shock. A more realistic approach will be to include a rental market to allow agents to choose between renting and buying their desired amount of housing services. That would remove or at least reduce the effects that arise from this excess saving.

The house price in period 1 is strictly lower in the constrained economy, yielding a lower total amount of housing. This ex-ante reduction in supply reduce the effect of the shock in the second period. Although volatility is not reduced, the borrowing constraint is effective in preventing the price after the shock from falling as low as in the unconstrained economy. This dampens the wealth loss for home-owners.

The constrained economy is Pareto inefficient in the sense that there is lower total construction of housing. The borrowing constraint also affects the distribution of housing in both periods. In the unconstrained economy, the shock has the exact same effect on aliens and natives, with both types of households adjusting their consumption of housing by the same degree. When aliens are restricted from using their future income to consume housing in period 1, they naturally have lower consumption of housing in the first period. Natives are not borrowing constrained and due to the reduced house price in period 1 they increase their consumption of housing relative to in the unconstrained economy. But in the second period in the constrained economy, the aliens have higher wealth than they would have had if they were able to transfer funds freely. As a result, aliens increase their consumption of housing period 2 through buying housing from the natives, both irrespective of whether the shock takes place. Thus, the borrowing constraint distorts the smooth consumption path that households plan for in the unconstrained economy.

4 Extension: Rental market

Introducing a rental market gives agents the opportunity to rent housing period-by-period instead of investing in a house. This allows borrowing constrained households to consume more housing in the first period and introduces a more realistic possibility set for the household. Several groups of the population such as students, immigrants and young households are likely to rent housing for at least parts of their life, often while saving to meet the downpayment requirements.

I assume that renting has a transaction cost. This cost can be interpreted in several
ways. It can be a mark-up of a rental firm with some market power, it can be a mark-up the firm takes because the rental market is uncertain or it could be a cost associated with the transaction between owner and tenant. I have modeled it as the latter option, with a constant transaction cost $\tau$.

The rental sector is assumed to consist of a representative firm that buys housing at the market price and rents it out in a competitive market. There is therefore a precise relationship between the rental price and the housing price in equilibrium. An alternative could be to let households buy spare housing capacity and rent it out on the market, but that would not make much of a difference for the questions asked here as long as agents are not aware of the possible shock.

Other than the possibility to rent housing, nothing has changed from the stylized model. The chapter will follow the same structure as the above. First I present the rental sector and the households optimization problem when renting is an option. Then I solve the equilibrium that agents expect before they are made aware of the shock, and then I solve the new equilibrium that arise when they are made aware. Finally I will present some comparative statics and discuss.

4.1 Competitive rental sector

The rental sector consists of a profit maximizing representative firm that buys housing in quantity $h_{RS}$ in the first period and rents it out in the first and the second period. There is a transaction cost $\tau$ to renting that is taken by the rental firm. However, because of the linearity in the profit function of the rental firm, the full amount of the transaction cost is paid by consumers. In the second period the firm may choose to sell a fraction $\varphi \in [0, 1]$ of the housing stock it buys in period 1 instead of renting it out. The profit maximization problem of the firm is

$$\max \quad (p_{R,1} - \tau)h_{RS} + \varphi(p_{R,2} - \tau)h_{RS} + (1 - \varphi)p_{H,2}h_{RS} - p_{H,1}h_{RS}$$

with first order condition

$$p_{R,1} - \tau + \varphi(p_{R,2} - \tau) + (1 - \varphi)p_{H,2} = p_{H,1}$$
Consider the problem in the second period, when the firm chooses how much of its housing capacity to sell or rent out.

$$\max_{\varphi \in [0,1]} \varphi(p_{R,2} - \tau)h^{RS} + (1 - \varphi)p_{H,2}h^{RS}$$

The profit maximization yields the condition $p_{R,2} = p_{H,2} + \tau$, and it follows that $p_{R,1} = p_{H,1} - p_{H,2} + \tau$. In both periods, the price of renting will be exactly equal to the price of consuming housing services in that period plus the transaction cost. Because of the transaction cost, renting will be more expensive than buying and only agents who are borrowing constrained will choose to rent instead of buying.

4.2 Agents

If we allow households to choose between renting or buying their desired housing capacity, the maximization problem can be formulated as

$$\max \sum_{t=1}^{2} \beta^{t-1}U(u(c^t_i) + v(h^t_i + R^t_i))$$

s.t. $$\omega + e^t + b^t_2 \geq p_c c^t_1 + p_{H,1} h^t_1 + p_{R,1} R^t_1$$
$$\omega + p_{H,2} h^t_1 \geq p_c c^t_1 + p_{H,2} h^t_2 + p_{R,2} R^t_2 + (1 + r)b^t_2$$
$$b^t_1 \leq \varphi p_{H,1} h^t_1$$

Agents are assumed to gain no extra utility from owning over renting. This may not be true in all cases, but I choose to abstract from any variable that may influence the renting-buying decision other than relative prices.

Let $\lambda_1$, $\lambda_2$ and $\mu$ be the Lagrange multipliers of the optimization problem, then the first
order conditions of the utility maximization are

\[ U'(\cdot)u'(c_1^i) = \lambda_1 p_c \]
\[ \beta U'(\cdot)u'(c_2^i) = \lambda_2 p_c \]
\[ U'(\cdot)v_h(h_1^i + R_1^i) = \lambda_1 p_{H,1} - \lambda_2 p_{H,2} - \mu \phi p_{H,1} \]
\[ \beta U'(\cdot)v_h(h_2^i + R_2^i) = \lambda_2 p_{H,2} \]
\[ U'(\cdot)v_R(h_1^i + R_1^i) = \lambda_1 p_{R,1} \]
\[ \beta U'(\cdot)v_R(h_2^i + R_2^i) = \lambda_2 p_{R,2} \]
\[ \lambda_1 = (1 + r) \lambda_2 + \mu \]
\[ \mu (b_1^i - \phi p_{H,1} h_1^i) = 0, \quad \mu \geq 0 \]
in addition to the budget constraints.

4.2.1 Unconstrained agents

Because of the tight relationship between rental prices and housing prices found above, unconstrained agents will choose to buy all of its housing and rent nothing. They will never choose to both own and rent housing, because the utility derived from one unit of housing is the same whether the agent owns or rents it and owning has a strictly lower price. These agents will have the demand functions

\[ h_1^i \frac{2\omega + e^i}{4(p_{H,1} - p_{H,2})} \]
\[ h_2^i \frac{2\omega + e^i}{4p_{H,2}} \]
\[ R_1^i = 0 \]
\[ R_2^i = 0 \]
\[ c_1^i \frac{2\omega + e^i}{4p_c} \]
\[ c_2^i \frac{2\omega + e^i}{4p_c} \]

4.2.2 Constrained agents

Agents who will be affected by the borrowing constraint if they buy may choose to rent housing capacity instead. The transaction cost makes renting more expensive, but
the alternative is buying a much smaller amount of housing than otherwise optimal. A borrowing constrained household that chooses to buy will have to consume much less of both housing and the consumption good in period 1, but can in return consume more of both in period 2. What the household gains from renting is a smoother consumption path, because renting allows it to consume more of both consumption goods and housing in period 1. However, this consumption smoothing has the cost of having to pay a little more to cover the transaction cost. The decision of whether to buy or rent therefore comes down to a trade-off between total consumption and consumption smoothing.

Due to the concavity of the utility function households have a preference for consumption smoothing, but not at any cost. For a given debt constraint, there will therefore exist a threshold level of the transaction cost that determines whether the household prefers to rent or buy. If the transaction cost is higher, the constrained household chooses to buy and the equilibrium reduces to the constrained equilibrium in the economy with no rental market. I will assume that the transaction cost is sufficiently low such that alien households choose to rent.

In the second period all households will choose to buy to avoid the transaction cost. In a sense, constrained agents postpone buying a house if they are constrained early in life. The demand functions of the households will therefore be

\[
\begin{align*}
    h^i_1 &= 0 \\
    h^i_2 &= \frac{2\omega + e^i}{4p_{H,2}} \\
    R^i_1 &= \frac{2\omega + e^i}{4p_{R,1}} \\
    R^i_2 &= 0 \\
    c^i_1 &= \frac{2\omega + e^i}{4p_c} \\
    c^i_2 &= \frac{2\omega + e^i}{4p_c}
\end{align*}
\]
4.3 Equilibrium

Equilibrium in the unconstrained economy where no shock occurs is given by a set of prices \( \{p_{H,1}, p_{H,2}, p_{R,1}, p_{R,2}\} \) and a set of optimal allocations \( \{c^i_1, c^i_2, h^i_1, h^i_2, R^i_1, R^i_2\} \) for \( i = 1, 2 \) such that all agents maximize utility over the two periods subject to the binding budget constraints

\[
\begin{align*}
\omega + e^i + b^i_2 &= p_c c^i_1 + p_{H,1} h^i_1 + p_{R,1} R^i_1 \\
\omega + p_{H,2} h^i_1 &= p_c c^i_2 + p_{H,2} h^i_2 + p_{R,2} R^i_2 + (1 + r)b^i_2
\end{align*}
\]

Further, the rental prices needs to satisfy

\[
\begin{align*}
p_{R,1} &= p_{H,1} - p_{H,2} + \tau \\
p_{R,2}^e &= p_{H,2}^e + \tau
\end{align*}
\]

Again, \( e \) denotes expected. In the constrained economy the borrowing constraint \( b^i_2 \geq \phi p_{H,1} h^i_1 \) also needs to be satisfied. Further, all firms maximize profits and markets clear. Again, the goods market will clear trivially as long as the budget constraints are satisfied, while the market clearing conditions for housing are given by

\[
\begin{align*}
(1 - \delta)(h^N_1 + R^N_1) + \delta(h^A_1 + R^A_1) &= h^{\text{supply}} \\
(1 - \delta)(h^N_2 + R^N_2) + \delta(h^A_2 + R^A_2) &= (1 - \delta)(h^N_1 + R^N_1) + \delta(h^A_1 + R^A_1)
\end{align*}
\]

The equilibrium is fully determined by the demand functions of natives and aliens, twelve in total, the supply function of the firms that produce housing, the market clearing conditions for the housing market and the price setting by the representative rental firm.

4.3.1 The constrained economy

If no agents are constrained, the problem reduces to the unconstrained economy with no rental market as discussed above. If the aliens are constrained, but the natives are not, the house price in period 1 and 2 and the rental price in period 1 is given by the set of
Numerical simulation suggests that this system has a unique solution for the three endogenous variables $p_{H,1}$, $p_{H,2}$ and $p_{R,1}$\(^3\). Compared to the unconstrained economy, the house price in period 1 is strictly lower and the expected price in period 2 is slightly higher, in line with the expected results.

### 4.4 Housing market volatility

Introducing the shock changes the optimal behavior of the agents. Home-owners have invested part of their wealth in the home and when demand suddenly falls, they will see their housing wealth fall. Taking the new information into account, they face the utility maximization problem

$$
\max \quad U(u(c_{i2}) + v(h_{i2} + R_{i2})) \\
\text{s.t.} \quad W_{i2} \geq p_{c}c_{i2} + p_{H,2}h_{i1} + p_{R,2}R_{i2}
$$

Where $W_{i2} = 2\omega + e - p_{c}c_{i1} - p_{H,1}h_{i1} - p_{R,1}R_{i2} + p_{H,2}h_{i1}$ is the total wealth available to the agent at the beginning of period 2. Only those who were unconstrained and bought housing in the first period see their total wealth affected by the shock. Let $\lambda$ be the Lagrange multiplier, then the first order conditions of this maximization problem are

$$
U'(\cdot)u'(c_{i2}) = \lambda p_{c} \\
U'(\cdot)v'_h(h_{i2} + R_{i2}) = \lambda p_{H,2} \\
U'(\cdot)v'_R(h_{i2} + R_{i2}) = \lambda p_{R,2}
$$

Optimal behavior by those who were constrained and chose to rent in the first period will not change after the shock hits, because they do not see their total wealth change.

\(^3\)See appendix A
when house prices fall. In the second period, all agents will choose to buy housing in
the market to avoid the transaction cost in the rental market. Imposing log utility, the
optimal behavior of agents who were unconstrained in the first period is given by the
demand functions

\[
\begin{align*}
    c_i^2 &= \frac{(2\omega + e^i)(1 + \frac{p^s_{H,2}}{p^H_{H,2}})}{8p_c} \\
    h_i^2 &= \frac{(2\omega + e^i)(1 + \frac{p^s_{H,2}}{p^H_{H,2}})}{8p^s_{H,2}} \\
    R^t &= 0
\end{align*}
\]

4.4.1 Equilibrium after the shock

After the shock occurs and agents become aware of the change in circumstances, the
equilibrium change. As before, the equilibrium is given by a set of prices \( \{p^s_{H,2}, p^s_{R,2}\} \)
and allocations \( \{c_i^2, h_i^2, R^t_i\} \) for \( i = 1, 2 \) such that agents maximize utility in period 2
subject to the binding budget constraint

\[
2\omega + e^i - p_c c_1^i - p_{H,1} h_1^i - p_{R,1} R_2^i + p^s_{H,2}^i h_1^i = p_c c_2^i + p^s_{H,2}^i h_2^i + p^s_{R,2} R_2^i
\]

Further, markets clear and the rental price is given by the condition from the rental
firm’s profit maximization, \( p^s_{H,2} = p^s_{H,2} + \tau \). Housing market clearing is given by the
condition

\[
(1 - \delta)(h_2^N + R_2^N) + \delta \theta (h_2^A + R_2^A) = (1 - \delta)(h_1^N + R_1^N) + \delta (h_1^A + R_1^A)
\]

The new equilibrium is fully determined by the demand functions of natives and aliens,
six in total, the condition on the rental price and the market clearing condition for the
housing market.

4.4.2 Market clearing

If the constraint is not imposed the rental market will drop out, because no agent will
choose to rent. The housing market volatility in the unconstrained economy is the exact
same as before introducing the rental market, and depicted in figure 1 above.
In the constrained economy aliens are forced into the rental market. When the shock hits the economy in the second period, the house price that is realized is the one given by the market clearing condition

\[
(1 - \delta) \frac{(2\omega + e^N)(1 + \frac{p_{H,2}}{p_{H,2}^e})}{8p_{H,2}^e} + \delta \theta \frac{2\omega + e^A}{4p_{H,2}^e} = (1 - \delta) \frac{2\omega + e^N}{4(p_{H,1} - p_{H,2})} + \delta \frac{2\omega + e^A}{4p_{R,1}}
\]

The price that solves the market clearing condition is unique and strictly smaller than the expected price in the constrained economy. The market equilibrium in the constrained economy with a rental market is depicted in figure 3.

**Figure 3:** Market clearing in the constrained economy. The plot on the left shows the equilibrium in period 1, while the plot on the right shows the difference between the expected equilibrium in period 2 and the equilibrium that is realized after the shock occurs in period 2.

When there is a rental market, volatility is strictly lower in the constrained economy. The difference from the model with no rental market can be explained by the simple feature that agents now have an alternative to buy a smaller amount of housing. When they rent their housing they only pay the price of housing services \( p_{H,1} - p_{H,2}^e \) in the first period instead of the full purchasing price of the house. This means that they do not have to borrow to finance their consumption of housing and goods in the first period and are able to spend the desired constant fraction of lifetime wealth on each good. Thus they will not have excess wealth when entering the second period and we do not observe the same upwards push to the housing prices in period 2 as was the case in the model with no rental market.
4.5 Comparative statics

There are three parameters that are interesting when discussing the magnitudes of and differences in volatility: $\delta$, $\theta$ and $\tau$.

The fraction of households that are aliens, $\delta$, affects the equilibrium in several ways. Aliens are assumed to have lower endowments, and they will therefore demand less housing both in the constrained and in the unconstrained economy. Further, they are always affected by the borrowing constraint when it is imposed, reducing demand for housing in the constrained economy even more. Due to these two assumptions, a larger population of aliens will reduce total demand and thus reduce both the housing stock and the house prices in both periods. Further, the higher the population of aliens, the larger the demand shock and the higher the volatility.

![Figure 4: The partial effect of $\delta$ on the expected price and the price that is realized after the shock occurs in period 2, in the unconstrained economy. ($\theta = .5, \tau = .05$)](image-url)
Figure 5: The partial effect of δ on the expected price and the price that is realized after the shock occurs in period 2, in the constrained economy. (θ = .5, τ = .05)

Figure 4 shows how the expected housing price in period 2 and the price after the shock both fall as δ increase in the unconstrained economy. Figure 5 show the same for the constrained economy. Volatility is given by the difference between the curves, and volatility is lower in the constrained economy for all values of δ, although it is only barely visible in the figure. This is not surprising because volatility is reduced in the constrained economy by restricting the ability of the aliens to invest in housing in period 1.

1 − θ determines how large the fraction of the alien population that leaves if the shock occurs is, making it the second component in determining the magnitude of the demand shock, together with the size of the alien population. θ does not affect the economy in any other way, and in particular it does not affect the expected price in period 2. This means that the value of θ has a more direct effect on volatility than δ has, because the value of δ also affects both the price in period 1 and the expected price in period 2.
Figure 6: Partial effect of $\theta$ in the unconstrained economy. The expected price in period 2 is always unaffected, but the price realized after the shock occurs is strictly increasing in $\theta$. ($\delta = .5, \tau = .05$)

Figure 7: Partial effect of $\theta$ in the constrained economy. The expected price in period 2 is always unaffected, but the price realized after the shock occurs is strictly increasing in $\theta$. ($\delta = .5, \tau = .05$)

Figure 6 shows how the house price after the shock in period 2 increase with $\theta$ in the unconstrained economy. As $\theta$ approaches 1 the magnitude of the shock decreases since $\theta$ determines the fraction of the alien population that remains in the economy after the shock hits. Figure 7 shows how the value of $\theta$ affects the price after the shock in the
constrained economy. As in the plot for $\delta$ we can see that volatility is greater in the unconstrained economy for all values of $\theta$ except 0 and 1.

In contrast to the simple model, the borrowing constraint is not the direct cause of the reduced demand for housing in period 1. Here, the borrowing constraint only pushes low-equity households over to the rental market, away from the housing market. It is the transaction cost that causes the reduction in demand and the subsequent fall in volatility. This implies that the tightness of the borrowing constraint in itself is unimportant with regards to housing prices as long as it is tight enough that low-equity households prefer to rent.

The transaction cost is an indicator of rental market frictions. A low value of $\tau$ implies a rather well-functioning rental market while a high value would imply that there are substantial frictions. The frictions hurt those who cannot buy, while the home owners who are already relatively well off benefit because the reduction in demand from the renters yields lower prices in the first period.

If $\tau \to 0$ the constrained equilibrium approach the unconstrained. As $\tau$ increase the amount of housing that constrained agents can afford to rent decrease and demand in period 1 falls. The effect is depicted in figure 8.

![Figure 8](image.png)

**Figure 8:** The partial effect of $\tau$ in the constrained economy, given that aliens rent. The plot on the left shows how the price in period 1 vary with $\tau$, while the plot on the right shows how that affects the price in period 2. ($\delta = \theta = .5$)
As the transaction cost increases, demand for housing in period 1 falls and construction falls accordingly. In period 2 all agents will buy housing in the market and the transaction cost has no other effect on the housing price in period two than through the reduction in total housing stock. Both the expected price and the price after the shock in period 2 are strictly increasing in the transaction cost because of the reduction in supply.

5 Extension: Uncertainty

The model is easily extended to let agents expect the shock to occur with some given probability and give all agents have rational expectations over the two possible future states. The only uncertain variable in this model is the size of the population in period two. All agents know that a fraction \(1 - \theta\) of the aliens will have to leave if the shock hits, but which alien households are affected is unknown to all. The aliens’ choices in period one have no effect on their situation if they are forced to leave, and they will therefore act as if they were staying.

If the shock hits, the remaining agents will see housing prices fall because demand is suddenly reduced while supply is fixed. This fall in prices affect them in two ways. First, they will be able to afford more housing because prices are lower. Secondly, the households who are home-owners will see their total wealth fall because the housing they bring with them from period 1 looses value when the house price falls. The decision whether to buy or rent housing will therefore include an additional component in the model with uncertainty and rational expectations.

Because the scope of the thesis is limited, I restrict the analysis to the case where natives always buy and aliens switch to renting if the borrowing constraint is imposed. This is the most comparable to the situation discussed so far and the most interesting when it comes to the question of how the borrowing constraint affects house price volatility in an economy with rapid changes in the population size.

When the agents know that the shock may occur with a given probability, I will define volatility in this version of the model as the difference in the house price between the state where the shock does occur and the state where it does not. To find the volatility in this model, the problem needs to be solved recursively. This makes it easier to compare it with the previous versions of the model First I define the economic environment,
although I leave out those parts that are identical to the model where the shock was completely unexpected. Then I solve the optimization problem of the agents. This is done by solving the market outcomes in the two states, taking as given the choices made in the first period, then determining expected utility in the second period which determines the choices that will actually be made in the first period. Finally I will present some comparative statics and discuss the results.

5.1 The rental sector

The representative rental firm takes into account the uncertainty in period 2. Again, the representative firm buy housing in quantity $h^{rs}$ in period 1 at the market price and has a transaction cost when renting housing out to the household sector. In the second period the firm may either sell a fraction $\varphi$ of its housing stock in the competitive market or again rent it out to the household sector, at a transaction cost. The representative firm maximizes expected profit, and the maximization problem takes the form

$$\max_{h^{rs}} \mathbb{E}\{(p_{R,1} - \tau)h^{rs} + \varphi(p_{R,2} - \tau)h^{rs} + (1 - \varphi)p_{H,2}h^{rs} - p_{H,1}h^{rs}\}$$

with first order condition

$$\mathbb{E}\{(p_{R,1} - \tau) + \varphi(p_{R,2} - \tau) + (1 - \varphi)p_{H,2} - p_{H,1}\}$$

Consider the problem in the second problem, when the representative firm chooses how much of its housing stock to sell and how much to keep for renting out,

$$\max_{\varphi} \varphi(p_{R,2} - \tau)h^{rs} + (1 - \varphi)p_{H,2}h^{rs}$$

with first order condition

$$(p_{R,2} - \tau)h^{rs} = p_{H,2}h^{rs}$$

There is a tight relationship between the rental price and the house price, as before. The first order conditions determines the rental prices as the price of housing services in that period plus the transaction cost. If the shock occurs in the second period the rental price will be $p^{s}_{R,2} = p^{s}_{H,2} + \tau$ and if there is no shock $p^{ns}_{R,2} = p^{ns}_{H,2} + \tau$, where the superscripts denote shock and no shock, respectively. Profit maximization determines the rental price in period 1 as $p_{R,1} = p_{H,1} - \mathbb{E}(p_{H,2}) + \tau$. What is worth noting is that the relationship
between the housing prices and the rental prices has not changed qualitatively, although
the magnitude of the difference is somewhat different due to the fact that rental firms
now take into account the probability of the shock.

5.2 Agents

Agents expect the shock to occur with probability $q$. In the second period there will
therefore be two possible states of the world, one where nothing has changed and one
where the population is reduced by $\delta (1 - \theta)$. Agents now maximize expected utility,
subject to the budget and borrowing constraints:

$$\max E \left\{ \sum_{t=1}^{2} \beta^{t-1} U(u(c_i^t) + v(h_i^t + R_i^t)) \right\}$$

s.t. $p_c c_i^1 + p_{H,1} h_i^1 + p_{R,1} R_i^1 \leq \omega + e^i + b^i$
$$p_c c_i^2 + p_{H,2} h_i^2 + p_{R,2} R_i^2 + (1 + r)b_i^2 \leq \omega + p_{H,2} h_i^1$$
$$b_i^2 \leq \phi p_{H,1} h_i^1$$

As before, the borrowing constraint will only be imposed in the constrained economy.
Let $\lambda_1$, $\lambda_2$ and $\mu$ be the Lagrange multipliers of the optimization problem. Then the
first order conditions are

$$U'(\cdot) u'(c_i^1) = \lambda_1 p_c$$
$$E \{ U'(\cdot) u'(c_i^2) \} = \lambda_2 p_c$$
$$U'(\cdot) v'(h_i^1 + R_i^1) = \lambda_1 p_{H,1} - \lambda_2 p_{H,2} - \phi p_{H,1}$$
$$E \{ U'(\cdot) v'(h_i^2 + R_i^2) \} = \lambda_2 p_{H,2}$$
$$U'(\cdot) v'(h_i^1 + R_i^1) = \lambda_1 p_{R,1}$$
$$E \{ U'(\cdot) v'(h_i^2 + R_i^2) \} = \lambda_2 p_{R,2}$$

$$\lambda_1 = (1 + r)\lambda_2 + \mu$$
$$\mu (b_i^2 - \phi p_{H,1} h_i^1) = 0, \quad \mu \geq 0$$

5.2.1 Constrained agents

If the borrowing constraint is imposed and the household does not have sufficient equity
in the first period to buy the optimal amount of housing, it may choose to rent housing
in the first period instead.

Agents who rent have no uncertainty about their future financial wealth. Their lifetime income is given and because the form of the utility function makes it optimal to spend a constant fraction of their wealth on each good there are no changes in their demand functions. As in the model with no uncertainty, constrained agents will not continue to rent housing in the second period because of the transaction cost. Instead they will buy housing in the market.

Because agents who rent have no uncertainty about future housing wealth their demand functions will be identical to the demand functions in the model with no uncertainty

\[
\begin{align*}
  h_i^1 &= 0 \\
  h_i^2 &= \frac{2\omega + e^i}{4p_{H,2}} \\
  R_i^1 &= \frac{2\omega + e^i}{4p_{R,1}} \\
  R_i^2 &= 0 \\
  c_i^1 &= \frac{2\omega + e^i}{4p_c} \\
  c_i^2 &= \frac{2\omega + e^i}{4p_c}
\end{align*}
\]

5.2.2 Unconstrained agents

Agents unaffected by the borrowing constraint can choose freely whether to buy or rent their optimal amount of housing. As before, they will choose to buy all the housing they want in both periods, because of the transaction cost associated with renting. Thus \( R_i^1 = R_i^2 = 0 \). As seen above, uncertainty has not changed the relationship between the purchasing price and the rental price. The intratemporal and intertemporal optimality
conditions of an unconstrained household are

\[
\begin{align*}
\frac{1}{c_i^1} &= E\left\{ \frac{1}{c_i^2} \right\} \\
\frac{1}{h_i^1} &= E\left\{ \frac{1}{h_i^1} \right\} \frac{p_{H,1}}{p_{H,2}} \\
\frac{1}{h_i^1} &= \frac{1}{c_i^1} \frac{p_{H,1}}{p_e} \\
\frac{1}{h_i^2} &= \frac{1}{c_i^2} \frac{p_{H,2}}{p_e}
\end{align*}
\]

Jensen’s inequality implies precautionary savings due to the concavity of the utility function. Income is not stochastic, but because housing bought in period 1 becomes part of the households’ wealth, uncertainty about future house prices yields uncertainty of future total wealth for home-owners. This uncertainty arises because homeowners may sell some of the housing they bring with them from period 1 in period 2. Let \(c_i^{1*}\) and \(h_i^{1*}\) be the optimal amounts of consumption and housing bought in period 1, then the stochastic wealth \(\tilde{W}_2\) of unconstrained agents when entering period 2 is given by

\[
\tilde{W}_2 = 2\omega + e^i - p_e c_i^{1*} - (p_{H,1} - \tilde{p}_{H,2}) h_i^{1*}
\]

where \(\tilde{p}_{H,2}\) takes one out of two possible values after the shock is realized. The problem of the unconstrained households must be solved recursively, first determining demand and equilibrium prices in each state of the world in the second period taking as given consumption of housing and goods in period 2, then using this allocation to determine expected marginal utility in period 2, and finally using this to determine optimal consumption in period 1.

The maximization problem in period 2 for an agent that was unconstrained in period 1 has the form

\[
\max \quad U(u(c^2_i) + v(h^1_i + R^2_i))
\]

s.t. \(p_e c^2_i + \tilde{p}_{H,2} h^1_i + p_{\tilde{R},2} R^2_i \leq \tilde{W}_2^i\)

As before, agents will never choose to rent in the second period because of the transaction
cost. Imposing log utility, the demand functions will therefore be
\[ c_i^j = \frac{\tilde{W}_i^j}{2p_c} \]
\[ h_i^j = \frac{\tilde{W}_i^j}{2p_{H,2}} \]
\[ R_i^j = 0 \]

Using the first order conditions, demand for housing and consumption goods are implicitly given by the equations
\[ \frac{1}{h_1^i} = q \frac{2(p_{H,1} - p_{s,1}^H)}{2\omega + e^i - p_c c_1^i - (p_{H,1} - p_{s,1}^H) h_1^i} + (1 - q) \frac{2(p_{H,1} - p_{s,1}^H)}{2\omega + e^i - p_c c_1^i - (p_{H,1} - p_{s,1}^H) h_1^i} \]  
\[ \frac{1}{c_1^i} = q \frac{2p_c}{2\omega + e^i - p_c c_1^i - (p_{H,1} - p_{s,1}^H) h_1^i} + (1 - q) \frac{2p_c}{2\omega + e^i - p_c c_1^i - (p_{H,1} - p_{s,1}^H) h_1^i} \]  

These equations determine the demand for goods and housing by unconstrained agents as functions of wealth, probabilities and prices. Together with the market clearing conditions they will constitute the set of equations that can be solved numerically for the equilibrium prices.

5.3 Equilibrium

Equilibrium in the model with uncertainty is given by a set of prices \( \{p_{H,1}, p_{s,1}^H, p_{s,1}^{n,s}, p_{R,1}, p_{R,2}, p_{s,2}^R, p_{n,s}^R\} \) and a set of allocations \( \{c_1^i, c_2^i, h_1^i, h_2^i, s, h_2^{n,s}, R_1^i, R_2^i, R_2^{n,s}\} \) for \( i = 1, 2 \) where \( s \) denote shock and \( ns \) denote no shock, such that all agents maximize utility over the first period and the two possible future states subject to the binding budget constraints
\[ \omega + e^i + b_2^i = p_c c_1^i + p_{H,1} h_1^i + p_{R,1} R_1^i \]
\[ \omega + p_{H,2} h_1^i = p_c c_1^i + p_{H,2} h_2^i + p_{R,2} R_2^i + (1 + r) b_2^i \]

In the constrained economy the borrowing constraint \( b_2^i \geq \phi p_{H,1} h_1^i \) also needs to be satisfied. Further, all firms maximize profits and markets clear. The following conditions on the rental prices from the rental firm’s profit maximization must be satisfied
\[ p_{R,1} = p_{H,1} - E(p_{R,2}) + \tau \]
\[ p_{s,2}^R = p_{s,2}^H + \tau \]
\[ p_{n,s}^R = p_{n,s}^H + \tau \]
Due to the assumption of a small open economy the goods and capital markets will clear trivially, while the market clearing conditions for housing are given by

\[(1 - \delta)(h^N_1 + R^N_1) + \delta(h^A_1 + R^A_1) = h^{supply}\]
\[(1 - \delta)(h^{N,ns}_{2,ns} + R^{N,ns}_{2,ns}) + \delta(h^{A,ns}_{2,ns} + R^{A,ns}_{2,ns}) = (1 - \delta)(h^N_1 + R^N_1) + \delta(h^A_1 + R^A_1)\]
\[(1 - \delta)(h^N_{2,s} + R^N_{2,s}) + \delta(h^{A,s}_{2,s} + R^{A,s}_{2,s}) = (1 - \delta)(h^N_1 + R^N_1) + \delta(h^A_1 + R^A_1)\]

The equilibrium is fully determined by the demand functions of the natives and aliens, twelve in total, together with the supply function of the firm that produces housing, the conditions on the rental prices and the housing market clearing conditions for all periods and states.

5.3.1 The unconstrained economy

If no borrowing constraint is imposed, both aliens and natives will buy housing in the first period. Let \(W^{i,high}_2\) be the financial wealth in period 2 of an unconstrained agent of type \(i\) if the shock does not occur and \(W^{i,low}_2\) be the financial wealth of that agent if the shock does occur. In the state with no shock, the housing bought in period 1 will have higher value than in the state where the shock occurs because the house price will be higher.

In the unconstrained economy both aliens and natives will buy housing in period 1. The market clearing condition in the state with no shock is therefore

\[(1 - \delta)\frac{W^{N,high}_2}{p^{ns}_{H,2}} + \delta\frac{W^{A,high}_2}{p^{ns}_{H,2}} = (1 - \delta)h^N_1 + \delta h^A_1\]

yielding the unique equilibrium price

\[p^{ns}_{H,2} = \frac{(1 - \delta)(2\omega + e^N - p_c c^N - p_{H,1} h^N_1) + \delta(2\omega + e^A - p_c c^A - p_{H,1} h^A_1)}{(1 - \delta)h^N_1 + \delta h^A_1}\]  (4)

In the state where the shock is realized, the market clearing condition becomes

\[(1 - \delta)\frac{W^{N,low}_2}{2p^{s}_{H,2}} + \delta \theta \frac{W^{A,low}_2}{p^{s}_{H,2}} = (1 - \delta)h^N_1 + \delta h^A_1\]

with the unique market clearing price

\[p^{s}_{H,2} = \frac{(1 - \delta)(2\omega + e^N - p_c c^N - p_{H,1} h^N_1) + \delta(2\omega + e^A - p_c c^A - p_{H,1} h^A_1)}{(1 - \delta)h^N_1 + \delta(2 - \theta)h^A_1}\]  (5)

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It is the magnitude of the shock given by $\theta$ that drives the difference between the price in the two states, as can be seen from the equations (4) and (5). The equilibrium prices are determined simultaneously by a system of equations consisting of these two market clearing conditions, optimality conditions (2) and (3) for both aliens and natives and the market clearing condition in the first period:

$$(1 - \delta)h_1^{N*} + \delta h_2^A = \left( \frac{\alpha pH_1}{p_c f_c(l)} \right)^{\frac{\alpha}{1-\alpha}}$$

5.3.2 The constrained economy

When the borrowing constraint is imposed aliens choose to rent all the housing services they consume in period 1. Natives will always buy in period 1. In the state with no shock the market clearing condition in period 2 will be

$$(1 - \delta) \frac{W_2^{N, high}}{2p_{H,2}^n} + \delta \frac{2\omega + e^A}{4p_{H,2}^n} = (1 - \delta) h_1^{N*} + \delta \frac{2\omega + e^A}{4p_{R,1}}$$

with the unique equilibrium price

$$p_{H,2} = \frac{(1 - \delta)(2\omega + e^N - p_c c_4^{N*} - pH_1 h_1^{N*}) + \delta \frac{2\omega + e^A}{2}}{(1 - \delta) h_1^{N*} + \delta \frac{2\omega + e^A}{2p_{R,1}}}$$ (6)

In the state where the shock occurs the market clearing condition is instead

$$(1 - \delta) \frac{W_2^{N, low}}{2p_{H,2}^n} + \delta \theta \frac{2\omega + e^A}{4p_{H,2}^n} = (1 - \delta) h_1^{N*} + \delta \frac{2\omega + e^A}{4p_{R,1}}$$

and the unique market clearing price is

$$p_{H,2} = \frac{(1 - \delta)(2\omega + e^N - p_c c_4^{N*} - pH_1 h_1^{N*}) + \delta \theta \frac{2\omega + e^A}{2}}{(1 - \delta) h_1^{N*} + \delta \frac{2\omega + e^A}{2p_{R,1}}}$$ (7)

Again it is the magnitude of the shock given by $1 - \theta$ that drives the difference in housing price between the two states. But in the constrained case the aliens who remain in the economy have not borrowed to finance buying a house in period 1. They will therefore not face any wealth loss from falling house prices in period two if the shock occurs. This, along with the reduction in supply that follows from reducing alien demand in the first period, yields lower volatility in the constrained economy.
The equilibrium prices are determined by these two market clearing conditions, the optimality conditions for consumption and housing in (2) and (3) for natives and the market clearing condition in period 1:

\[(1 - \delta)h_1^{N*} + \delta \frac{2\omega + e^A}{4(p_{H,1} - E(p_{H,2}) + \tau)} = \left( \frac{\alpha p_{H,1}}{p_c f'_c(l)} \right)^{\frac{\tau}{\delta}}\]

5.4 Housing market volatility

Numerical simulation suggest that the equation systems that define the equilibrium in the constrained and the unconstrained economy both have unique solutions. Further, the qualitative results from the model with no uncertainty still apply and is even strengthened by the presence of precautionary savings. When imposing the borrowing constraint volatility is reduced because aliens cannot buy as much housing as they would like. They will instead choose to rent housing in the competitive rental market, but due to the transaction cost they will rent less than what would otherwise be their optimal amount of housing. Because demand is reduced in the first period there is less volatility in the second period.

Increased savings yields an upward push to house prices in period 2 because increased availability of funds increase demand for both housing and consumption goods. Because constrained agents do not risk any wealth loss they will not save precautionary, reducing the upward push the the house price irrespective of the realized state.

Figure 9 show the market equilibria in the unconstrained economy. Demand for housing in the state where the shock occurs is visibly reduced relative to the state where there is no shock. Volatility is given by the difference in the demand curves’ intersections with the inelastic supply curve.

Figure 10 show the equilibria in the constrained economy. Demand for housing in the state with no shock is reduced compared to the unconstrained economy. This has changed from the model with no uncertainty. The reason is that unconstrained agents will self-insure against the possibility of the shock and save precautionary. Precautionary savings by unconstrained agents will increase demand for both housing and consumption in period 2.

4See appendix A
Figure 9: Market clearing in the unconstrained economy. The plot on the right shows the equilibrium in period 1. The plot on the left shows the two possible equilibria in period two, depending on which state is realized.

Figure 10: Market clearing in the constrained economy. The plot on the left shows the equilibrium in the first period. The plot on the right shows the two possible equilibria in the second period depending on which state is realized.

When aliens are constrained their relative demand for housing will increase in period 2 because of the transaction cost they have to pay to rent in the first period. In the model without uncertainty this effect pushed the expected price in the constrained economy slightly higher than the price in the unconstrained economy. However, when there is uncertainty it seems that the effect that precautionary savings has on demand is stronger,
and the price in the state with no shock in the constrained economy is strictly lower than in the unconstrained economy.

In the state where the shock occurs the fall in demand is reduced compared to the unconstrained economy because aliens rent when constrained and will therefore not face any wealth loss in that state. The volatility in the constrained economy is strictly reduced compared to the unconstrained economy.

5.5 Comparative statics

The most interesting parameter in the model with uncertainty is the probability of the shock occurring, $q$. The qualitative results from the comparative statics analysis in the model where the shock was completely unexpected should still apply.

Figure 11: The effect of $q$ in the unconstrained economy. The plot on the left shows how the price in period 1 varies with the probability of the shock, while the plot on the right shows how this affects the price in period 2 in both possible states.

Figures 11 and 12 show how house prices vary with the probability of the shock. Housing is a risky investment in this model. When agents know about the risk of the shock in the second period they take this into account in their housing decision in the first period. A high probability $q$ means that the expected value of the house is low in period 2, decreasing demand for housing in period 1 and the price in period 1 falls. Low demand in period 1 decrease construction of housing, leaving a lower supply in period 2 in both
Figure 12: The effect of \( q \) in the constrained economy. The plot on the left shows how the house price in period 1 varies with the transaction cost, while the plot on the right shows how this affects the house price in period 2 in both possible states.

states. This increase the price of in period 2 in both states.

6 Conclusion

The model I have constructed in this thesis provides some intuition about how a loan-to-value borrowing constraint can be used to dampen one source of housing market volatility. There are numerous other sources of volatility such as search frictions, changing expectations of future prices, macroeconomic fluctuations, households responding to changes in the interest rate and so on that are not treated in the analysis. The importance of immigration on total volatility should not be overstated.

In this analysis I have found that instability in demand caused by sudden changes to the population size cause volatility in the house price. This instability can arise from immigration or from having a large population of young households or students who are still in the process of settling down. These agents are more mobile and more likely to relocate as a response to some exogenous shock. In the model presented above the shock to the population size is negative, with a fraction of the population suddenly leaving. The opposite could just as easily be the case, but I have no reason to believe that the
Limited access to credit reduces demand for housing in the first period because agents are unable to buy as much as they would like. The fall in demand lowers the price and the construction of housing. In the models where the shock is completely unexpected this leads to an increase in the expected price in the second period, because constrained agents in a way are forced to save and thus will have more funds available in the second period. Combined with a reduction in supply this relative increase in demand pushes the expected price in the second period up. But even though the increased expected house price in period 2 in a way contradicts the expectation of dampened volatility, due to the reduction in supply in the first period the price in the second period will not fall as low as in the unconstrained model.

In the model with rational expectation over the possibility of the shock, the result is exactly in line with expectations. Due to agents’ risk aversion all homeowners will save precautionary to minimize the wealth loss they might experience if the shock occurs. This precautionary saving will irrespective of the shock lead to an increase in agents available funds and thus in demand for both non-durable consumption goods and housing in period 2. The house price if there is no shock will increase accordingly. If the borrowing constraint is imposed the agents who are forced into the rental market will not risk losing any wealth in the second period and will not save precautionary. They will however have lower demand in period 1 due to the transaction cost. This dampens volatility because the reduction in saving reduce the upward push to the house price in period 2, while the reduction in demand leads to a reduction in construction of housing which again prevents the price from falling as low as in the unconstrained economy if the shock occurs.

I show that the constraint reduces volatility only though placing the burden on those who are already worse off, young households or immigrants who do not have sufficient wealth to buy housing as they wish. By restricting access to credit and demanding that a fraction of the value of the house is paid up front, total housing demand falls because these agents are forced into the rental market. Depending on how well-functioning the rental market is, they will then have to reduce their consumption of housing due to the transaction cost.

A strength of the model is that the results from the model where the shock is unexpected still holds when agents are given rational expectations and beliefs about the probability of
The shock. The analysis show that the households will save precautionary when housing becomes a risky investment. When future house prices are uncertain, renting becomes a safer alternative because agents who rent do not risk any future purchasing power if the house price falls.

The model I have proposed here has some significant simplifications. The division of time into two periods provides simplicity and flexibility of the model, but at the expense of both realism and more interesting dynamic analysis. An interesting extension to the model would be to use an environment with longer horizon and immigration-driven population growth to analyze housing price dynamics following a single shock or the housing investment decision when the household expects a sequence of stochastic shocks.

A discussion I have not covered in this analysis is how the decision whether to own or rent change with the introduction of stochastic shocks and rational expectations. Immigration shocks affect household wealth through the effect it has on housing prices, making housing a risky investment. The decision whether to buy or rent hinges on the tightness of the borrowing constraint and the magnitude of the transaction cost in the rental market. I leave this discussion as well as a topic for future research.
References


A Simulation

All numerical simulation is done by the Matlab routine *fsolve* that solves systems of equations. All systems of equations presented here seem to have a unique solution within what can be considered reasonable boundaries given the chosen parameter values. The following specifications are used in the numerical simulation of all three versions of the model:

\[
\begin{array}{cccccccc}
\phi & \omega & e^N & e^A & f'(l) & p_c & \alpha & \delta & \theta & \tau & q \\
0.1 & 1 & 2 & 0 & 1 & 1 & 0.4 & 0.5 & 0.5 & 0.05 & .5 \\
\end{array}
\]

Table 1: Parameter values

The parameters have no empiric justification, but are chosen to make the results easier to interpret. The main focus is the qualitative results and the simple structure of the model does not allow for realistic calibration.

The endowments of aliens and natives are chosen to ensure that the natives always are able to buy their optimal amount of housing while the aliens will be constrained and choose to rent if there is a rental market. Due to the functional form it is optimal for agents to spend \( \frac{1}{4} \) of their lifetime financial wealth on each good, except in the model with uncertainty where the decisions of unconstrained agents will feature some degree of precautionary savings. With this endowment and income structure natives will have \( \frac{3}{4} \) of their total lifetime financial wealth available in period 1 even if they cannot borrow. This allows them to buy optimal amounts of both housing and consumption goods even when the borrowing constraint is imposed. Aliens on the other hand have only \( \frac{1}{2} \) of their total lifetime financial wealth available in period 1 if they cannot borrow, and will not be able to buy their optimal amount of housing and consume optimal amount of goods in the first period in the constrained economy.

The prices in each version of the model are given in the following table:

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Table 2: Housing prices and volatility in the unconstrained (above) and the constrained (below) models

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<th>Housing market only</th>
<th>Rental market</th>
<th>Rental market and uncertainty</th>
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<td>1.8401</td>
<td>1.7937</td>
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