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IMPACT OF NONLINEARITY UPON WAVES TRAVELLING OVER A SUBMERGED CYLINDER

by

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ABSTRACT

Incoming Stokes waves passing over a submerged cylinder situated close to the free surface will give rise to higher order harmoninc waves behind the cylinder. In this paper measurements of the second order wave behind the cylinder are given. Also the amplitude and phase of the first order transmitted wave are measured. The measured quantities are compared with second order potential theory computations by Vada (1987). Good agreement between experiments and theory is found for small incoming wave amplitudes. However, the measurements do not confirm the results from the second order potential theory for moderate incoming wave amplitude. It is found that the amplitude of the second order wave and the phase of the transmitted first order wave may be overpredicted more than 100% by the theory.

extensive measurements of the nonlinear forces acting on a restrained circular cylinder are given. Also a second order in the wave amplitude steady circulation around the cylinder being set up due to viscosity is investigated. Chaplin concludes that the coupling between the wave field and this circulation significantly reduces the oscillating force on the cylinder for large incoming wave amplitude.

Chaplin also reports measurements of the reflection and the transmission of the incoming waves. His measurements confirm the well known result form linear potential theory that there is no reflection of the incoming waves due to a circular cylinder (Dean 1948, Ursell 1950, Grue and Palm 1984). In Chaplin's most severe case, with the amplitude of the incoming waves being one third of the cylinder diameter, and the cylinder submergence being D/R = 1, there was only 4% wave reflection of the basic mode (D is the distance between the uppermost point of the cylinder and the undisturbed free surface, R is the radius of the cylinder, see figure 1). Chaplin also concludes that there are no higher order reflected waves at all. The theoretical predictions by Vada (1987) agree that there is no second order reflected wave due to the circular cylinder.

Chaplin concludes that a second order wave of considerable amplitude exists behind the cylinder, however without quantifying it. Also, Longuet-Higgins' measurements are too sparse for relevant comparisons with the available theoretical results. The present paper is therefore devoted to the measurements of the amplitude of the second order wave behind the cylinder for comparison with the

The result, that second order theory may be insufficient for the description of nonlinear waves generated by the presence of a submerged body close to the free surface, agrees with the results by Tuck (1965) and Salvesen (1969) who consider nonlinear free surface effects on a submerged cylinder and a thick submerged hydrofoil moving with a constant forward speed in calm water close to the free surface.

2. THE NONLINEAR FREE SURFACE WAVES

In the cases when the waves are not breaking and there is no separation at the cylinder, the flow outside the boundary layer of the cylinder may be modelled by potential flow. The nonlinear free surface condition obtained by potential theory in Euler coordinates, given e.g. by Newman (1977), reads

$$\Phi_{tt} + g\Phi_y + 2\nabla\Phi\bullet\nabla\Phi_t + \frac{1}{2}\nabla\Phi\bullet\nabla(\nabla\Phi\bullet\nabla\Phi) = 0 \qquad (2.1a)$$

applied on

$$\eta(x,t) = -\frac{1}{g}(\Phi_t + \frac{1}{2}\nabla\Phi\bullet\nabla\Phi)_{y=\eta} \qquad (2.1b)$$

Here x, y are space coordinates with the x-axis coinciding with the undisturbed free surface, and the y-axis being positive upwards. Furthermore, t denotes time, η the free surface elevation, g acceleration of gravity and Φ the velocity potential. The velocity in the fluid is then given by $\mathbf{v} = \nabla\Phi$.

The goal is now to measure the following quantities: a_1, δ_1 and a_2. In non-dimensional terms we will measure the amplitude of the second order wave

$$T_2 = a_2 \frac{R}{a^2} \tag{2.5}$$

and the amplitude of the basic mode

$$T_1 = \frac{a_1}{a} \tag{2.6}$$

In the consistent second order theory (Vada 1987) T_2 depends only on two parameters, namely the dimensionless wave number kR and the submergence of the cylinder D/R, and T_1 equals unity. The measurements reveal, however, that T_2 is a function of the incoming wave amplitude also, i.e

$$T_2 = T_2(kR, D/R, a/R) \tag{2.7}$$

This is also true for T_1, which means that T_1 is not equal to unity except in the limit as $a/R \rightarrow 0$.

In the present small scale experiment the amplitudes of the third and higher order free waves behind the cylinder are too small to be measured within reasonable accuracy.

The voltmeter has a resolution of the surface elevation corresponding to 0.3mm. For very slow variations of the surface elevation there is no meniscus at the wave probe, and we obtain a 1% relative accuracy of the measured surface elevation. For long periodic waves the effect of the meniscus is very small and can be disregarded. Hence, for incoming wave amplitudes smaller than 20mm, the absolute accuracy is better than 0.5mm. Repeated tests with fixed incoming wave characteristics reveal, on the other hand, a much better absolute resolution of the wave elevation than 0.5mm.

The data of the surface elevation at the different wave probes are recorded over three periods of the incoming waves, before any reflected wave from the shore has reached the probes. Then Fast Fourier Transform (FFT) is applied to the time series to obtain the amplitudes of the waves behind the cylinder.

4. WAVE MEASUREMENTS

The wave generator is programmed such that the incident waves upon the cylinder are pure Stokes waves, and there is no free second order harmonic wave due to the motion of the wave generator. Chaplin's extensive measurements conclude that there is no reflection of the incoming waves, even to the higher order. In our measurements it is therefore sufficient to apply only one wave gauge between the paddle and the cylinder to obtain the exact amplitude of the incoming waves for each run.

To examine the effect of the nonlinearity more closely we have in figures 4 displayed T_2 for fixed values of the wave length of the incoming wave and submergence of the cylinder, and then varied the amplitude of the incoming wave. As seen from figures 4a - b there is good agreement between theory and experiment for $a/R < 0.1$. For larger values of a/R, T_2 shows a strong monotonous decay until the waves break as they pass over the cylinder for $a \simeq 0.22$ (for D/R=0.5). We note that the experimental value of T_2 (and hence a_2) is approximately the half of the theoretical for a/R = 0.15. This discrepancy is significant since the accuracy of T_2 is better than 20% in this case.

For the deeper submerged cylinder (D/R=1) the decay of T_2 is weaker. However, we note that the theory overpredicts T_2 with almost 100% for a/R = 0.3, as seen in figure 4c. In this case breaking occurs for $a/R \simeq 0.44$. It is found that breaking of the waves as they pass over the cylinder occurs when

$$a > \min(0.44D,\ 0.44/k) \qquad \text{for all } k \qquad (4.1)$$

(ak = 0.44 is the breaking limit for the incoming Stokes waves.)

flume, which displays a considerable steeper wave at the cylinder. For an incoming wave with still larger amplitude (figure 9c), we notice the striking difference between the very steep wave observed in the wave flume at the cylinder, and the smooth surface elevation predicted by the second order theory.

These photographs suggest a need for a higher order theory in order to model the deformation of Stokes waves passing over a submerged cylinder being situated close to the free surface. Steep waves and smooth waves at the cylinder will generally give rise to free second order waves of different amplitudes. A higher order theory which may apply to the present problem is the high-order spectral method outlined by Dommermuth and Yue (1987).

The strong decay of the phase lag δ_1 for increasing value of a/R may be explained on the basis of the very steep wave being formed as the incoming wave passes over the cylinder. Since the top of the cylinder acts as a local shore, and thereby introduces shallow water properties of the flow straight above the cylinder, it its expected that waves of higher amplitude propagate over the cylinder with a larger phase velocity than a wave with smaller amplitude. Hence, the phase lag will decrease with increasing amplitude of the incoming wave.

where

$$\epsilon' = \epsilon \,/\, \frac{1}{4}\, \rho g a^2 (g/k)^{\frac{1}{2}} \left(1 + \frac{3}{2}\,(ak)^2 \right) \qquad (5.4)$$

From the measured values of T_1 and T_2 the energy loss may then be quantified. In table 1 ϵ' is obtained on the basis of T_1 and T_2 displayed in figures 5b and 4b, respectively, for D/R = 0.5 and kR = 0.7.

a/R	T_1	$T_2 \frac{a}{R}$	$T_1^2(1+\frac{3}{2}(T_1^2-1)(ak)^2)$	ϵ'
0.05	0.95	0.18	0.92	0.08
0.10	0.90	0.28	0.85	0.15
0.15	0.85	0.32	0.77	0.23
0.20	0.81	0.17	0.66	0.34
0.25 (breaking)	0.75	0.11	0.56	0.44
0.3 (breaking)	0.70	0.09	0.48	0.52

Table 1. Values of energy loss ϵ' vs. a/R for D/R=0.5, kR=0.7. Values of T_1 and T_2 from figures 5b and 4b, respectively. (Best curve fit.)

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Figure 6. a. Experimental values of phase lag δ_1 vs. kR. Small incoming wave amplitude (a≃9mm). + D/R=0.5, D/R=0.75, Δ D/R=1. Solid line: Best curve fit to measurements.

b. Theoretical values of δ_1 vs. kR computed by Vada's theory (1987). D/R=0.5, 1.

Figure 7. Experimental values of δ_1 vs. kR obtained by measurements. + Small wave amplitude of incoming wave (a≃9mm), medium wave amplitude (a≃12mm), Δ large wave amplitude (a≃15mm). Solid line: Best curve fit. a. D/R=0.5. b. D/R=1.

Figure 8. Experimental values of δ_1 vs. a/R for fixed values of kR and D/R. Horizontal arrow indicates theoretical prediction. Vertical arrow indicates when breaking occurs.

a. D/R=0.5, kR=0.4. b. D/R=0.5, kR=0.7. c. D/R=1, kR=0.4.

Figure 9 a-c. Photographs of the surface elevation in the wave flume for incoming wave passing over the cylinder. D/R=0.5, kR=0.6.

a. a=9mm (a/R=0.09). b. a=13mm (a/R=0.13). c. a=20mm (a/R=0.2).

d. Computation of surface elevation from Vada's theory (Vada 1984). a/R=0.13, D/R=0.5, kR=0.6.

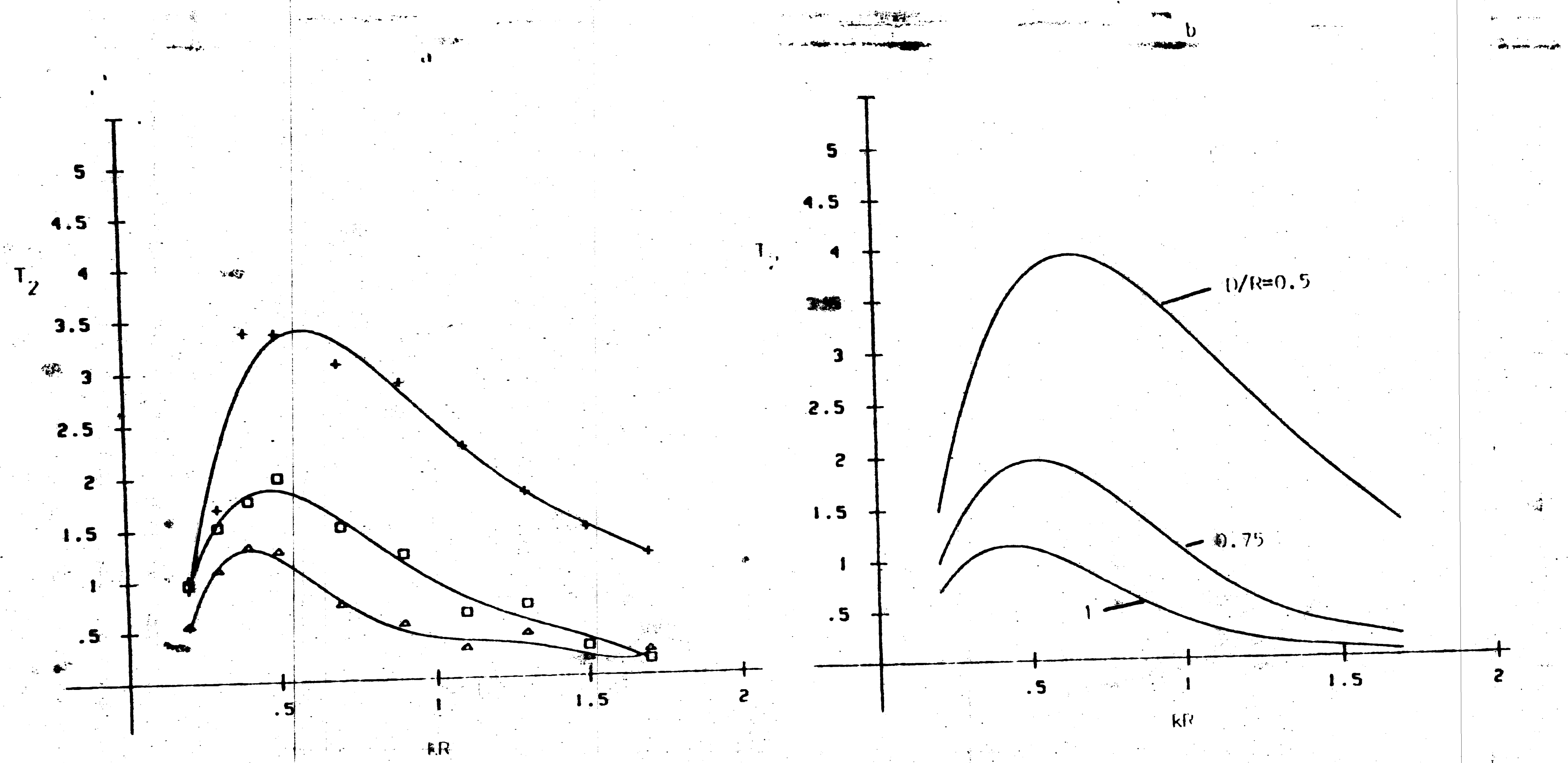

Figure 2. a. Experimental values of T_2 vs. kR. Small incoming wave amplitude (a≃9mm). + D/R=0.5, □ D/R=0.75, Δ D/R=1. Solid line: Best curve fit to measurements.

b. Theoretical values of T_2 vs. kR for D/R=0.5, 0.75, 1. (Vada 1987,

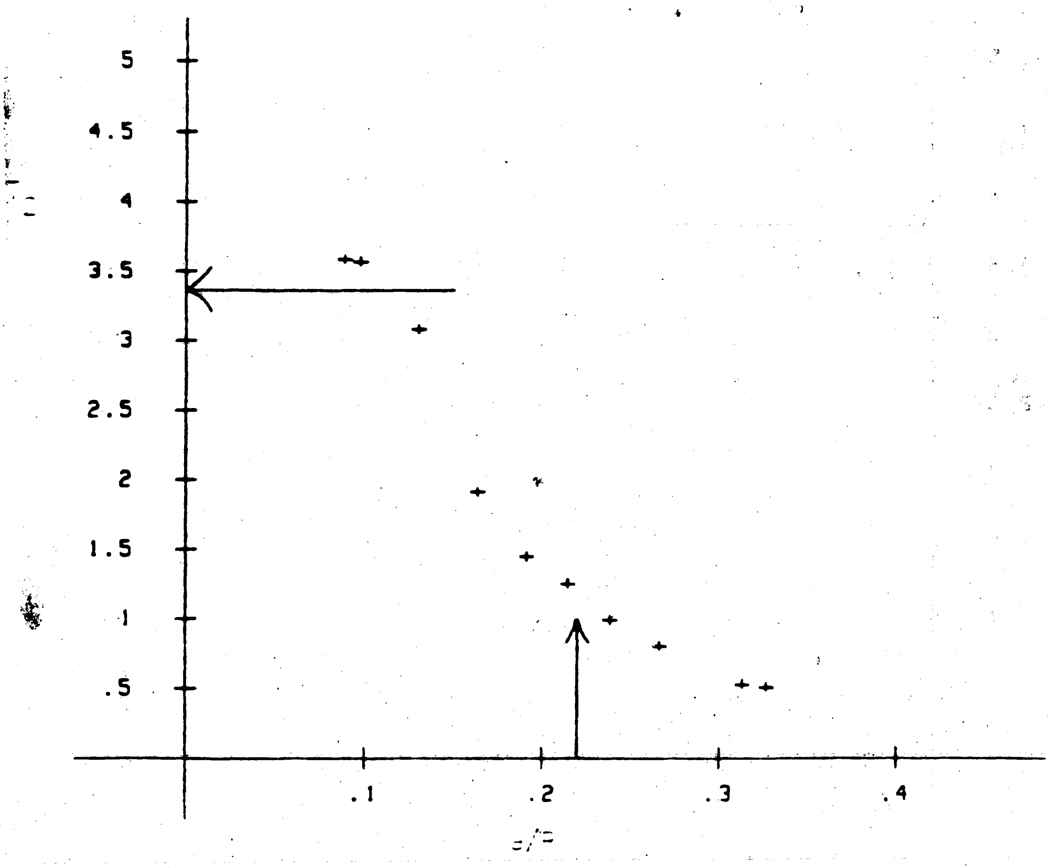

Figure 4. Experimental values of T_2 vs. a/R for fixed values of kR and D/R. Horizontal arrow indicates theoretical prediction (Vada 1987). Vertical arrow indicates breaking of the wave when passing over the cylinder.

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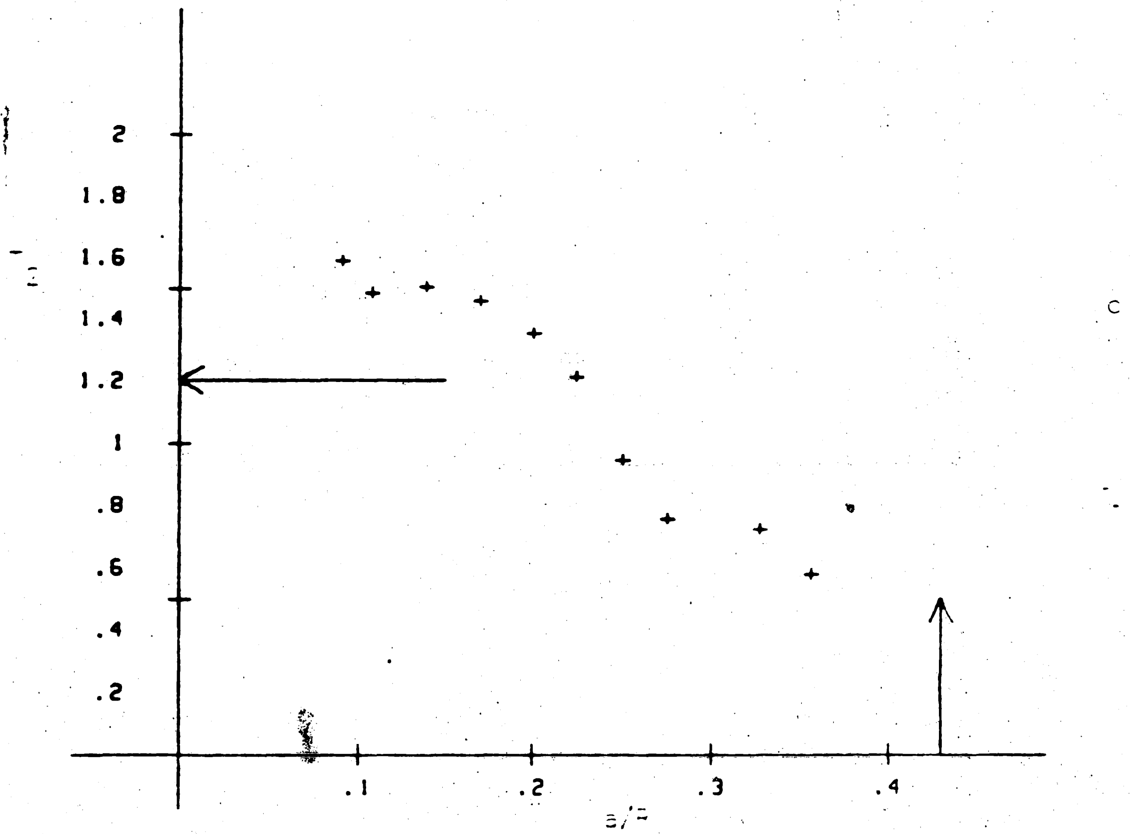

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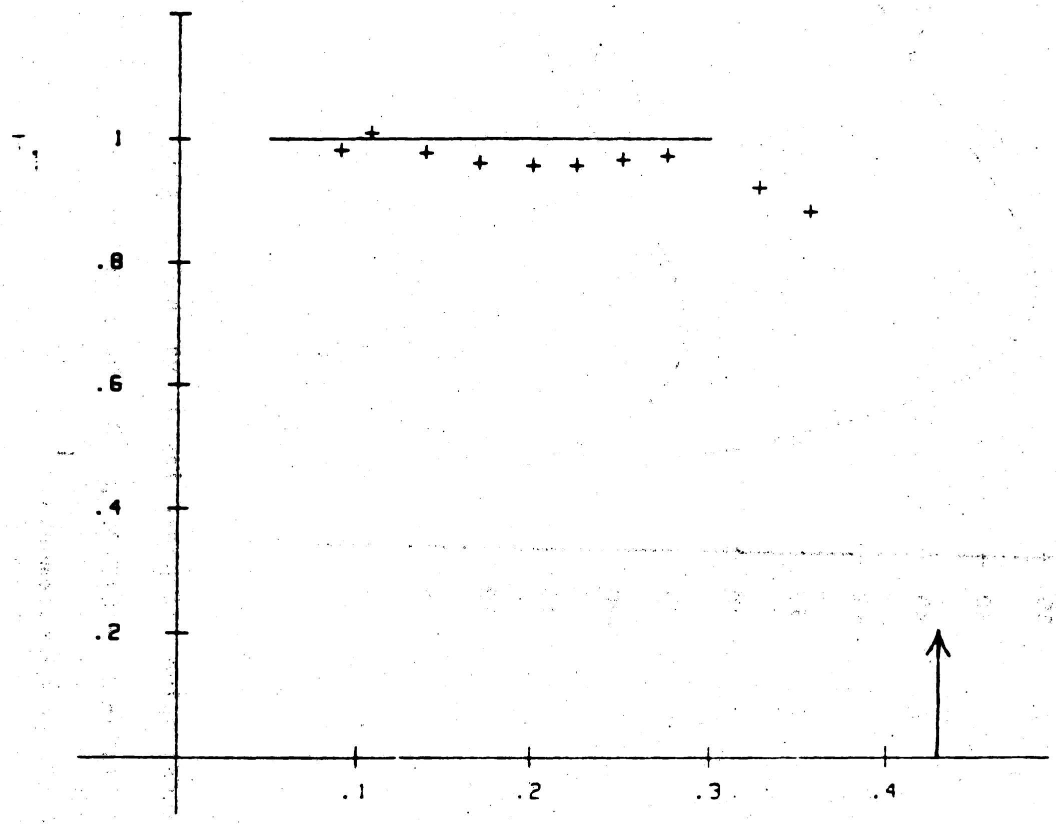
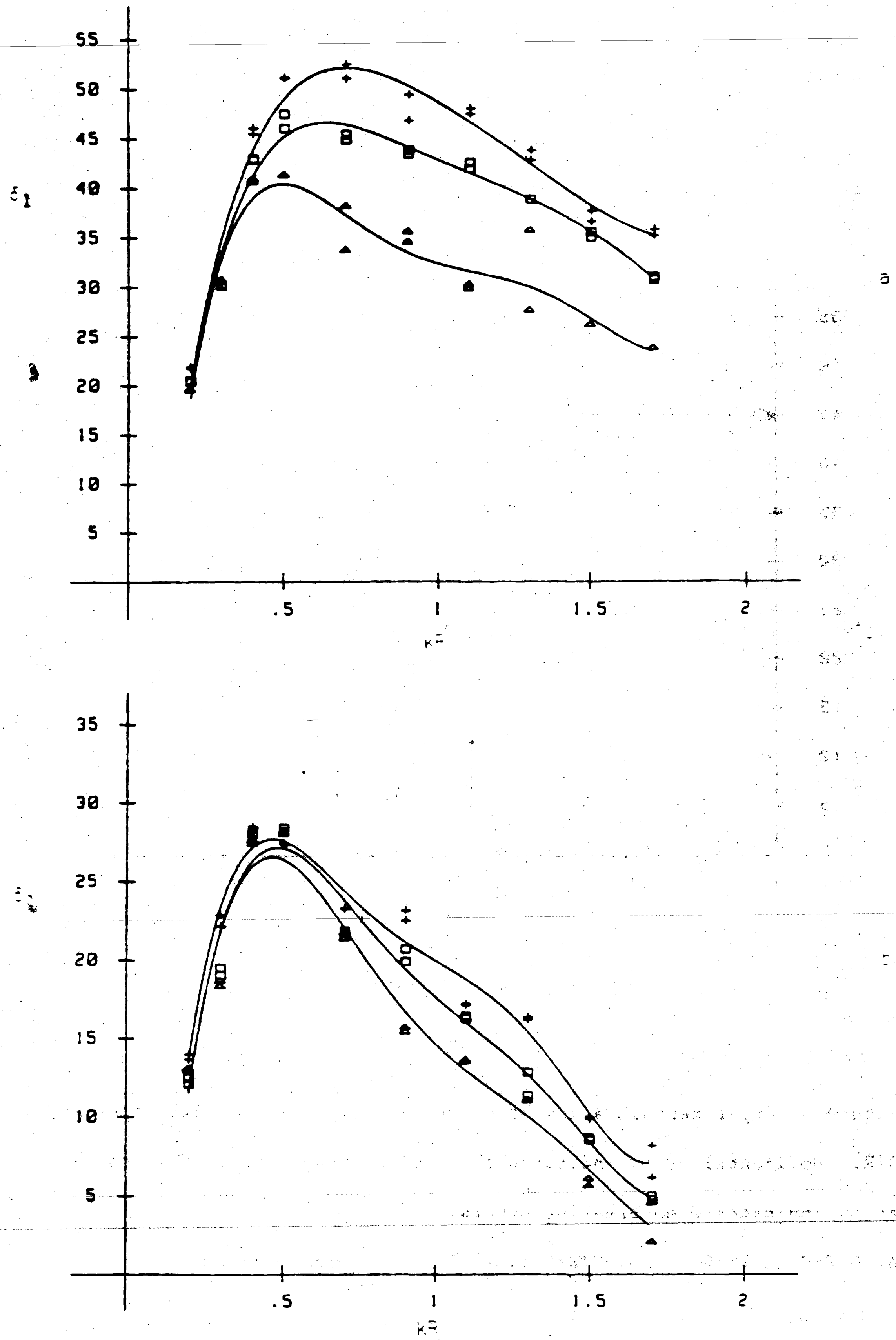


Figure 5. Experimental values of T_1 vs. a/R. Vertical arrow indicates when breaking occurs.

a. D/R=0.5, kR=0.4. b. D/R=0.5, kR=0.7. c. D/R=1, kR=0.4.



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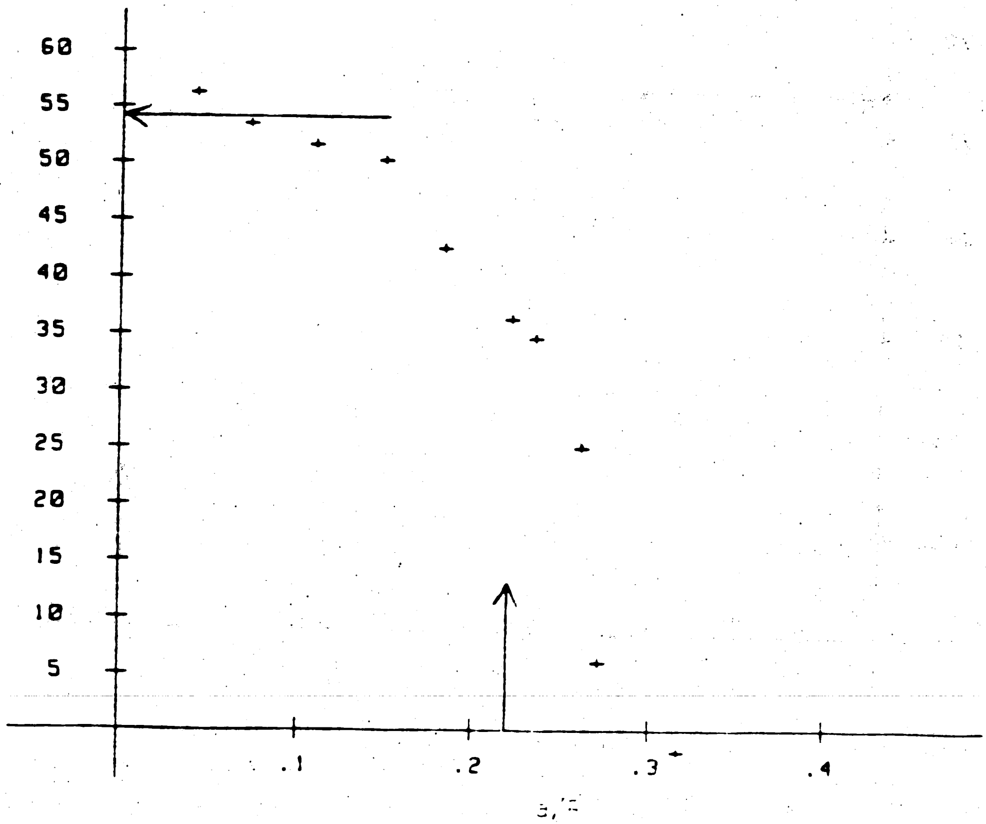

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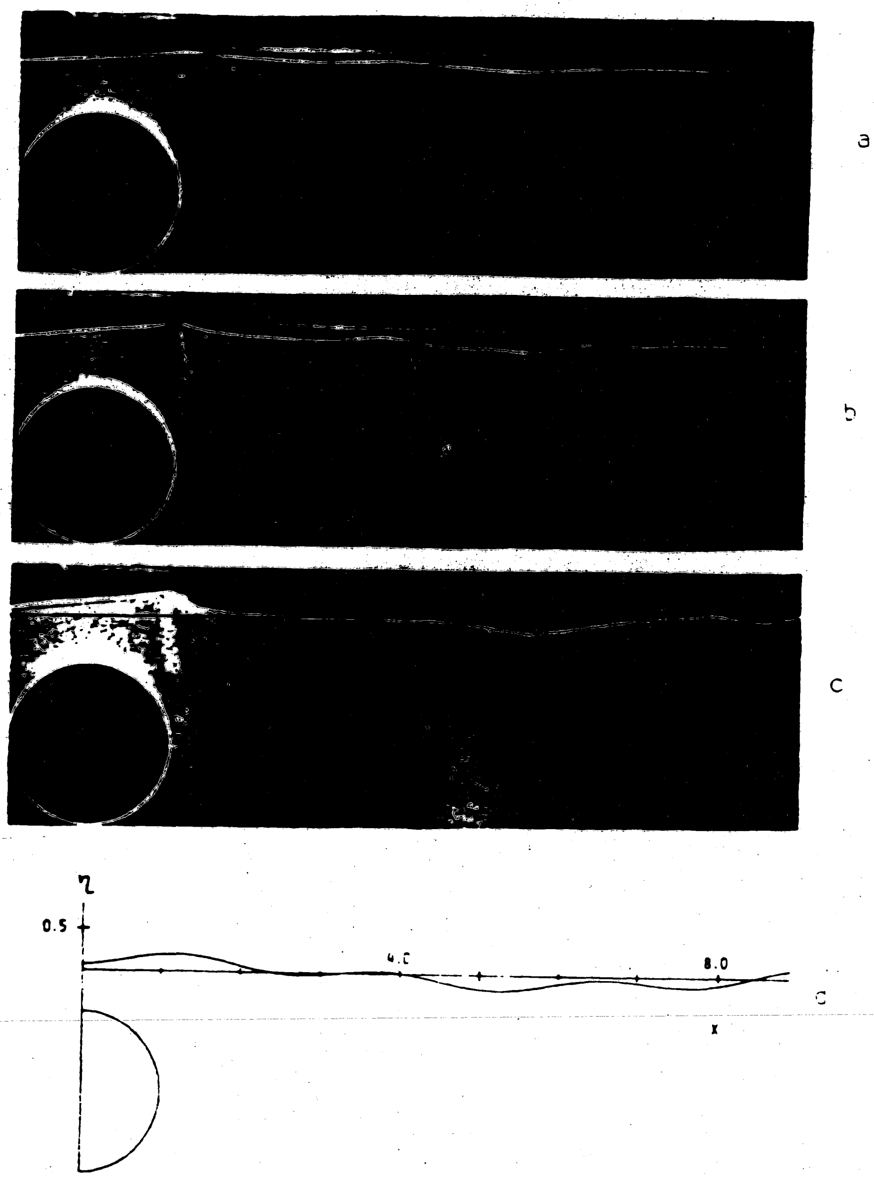

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