Essays on strategic considerations
in environmental economics

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Kristoffer Midttømme
Introduction
This thesis is about economic externalities and the problem of how to correct for them. As has been understood by economists since Arthur Pigou, an economic externality is present when there is a divergence between the private and the social consequences of an economic transaction or activity. In order for society to figure out how much of an activity it is desirable to carry out, the social marginal consequences are the ones that matter.

It might happen, for example, [...] that costs are thrown upon people not directly concerned, through, say, uncompensated damage done to surrounding woods by sparks from railway engines. All such effects must be included—some of them will be positive, others negative elements—in reckoning up the social net product of the marginal increment of any volume of resources turned into any use or place. (Pigou, 1932, Part II, Chapter II).

The desirable efficiency properties of markets, as expressed in the first fundamental theorem of welfare economics, rest on an assumption that no such externalities are present: in the absence of any market failures, and given convex technologies and preferences, any decentralized market equilibrium is Pareto efficient, meaning that no individual can be made better off without simultaneously making at least one other individual worse off. The welfare theorem no longer holds when there are discrepancies between the private and the social marginal consequences of actions; individual economic actors will be lead astray. This qualifies the infamous invisible-hand statement of Adam Smith.

Self-interest will tend to bring about equality in the values of the marginal private net products of resources invested in different ways. But it will not tend to bring about equality in the values of the marginal social net products except when marginal private net product and marginal social net product are identical. When there is a divergence between these two sorts of marginal net products, self-interest will not, therefore, tend to make the national dividend a maximum [...]. (Ibid, Chapter IX.)

The core of the problem is that each economic actor has no incentive to take into account the consequences of his actions for others. In deciding how many trains to run on his tracks, the railway operator does not take into account how his trains could cause economic damage to the foresters. Consequently, he can be expected to run too many trains. See Figure 1 for an analysis of such a situation. The problem can be thought of as one of missing markets: there
is no market in which the railway operator can buy the right to potentially set fire to the forest, and there is no market in which the foresters can pay the railway operator not to run too many trains. In addressing this problem, there are two opposing views: the so-called Coase theorem (Coase, 1960) tells us that absent transaction costs, and given a set of well-defined property rights, individual economic actors will transact or bargain such that the final outcome is Pareto efficient; whereas Arthur Pigou proposed government interference in the workings of the free market. “Consequently, certain specific acts of interference with normal economic processes may be expected, not to diminish, but to increase the dividend. It thus becomes important to inquire in what conditions the values of the social net product and the private net product [...] are liable to diverge from one another in either direction.” (Pigou, 1932, Part II, Chapter IX). Indeed, the most-studied such policy tool – the correcting tax – today carries Pigou’s name.

The economic problems treated in this thesis all have applications (although not exclusively) within environmental economics. When studying matters such as clean air, property rights are usually ill-defined. Furthermore, transaction costs are often high, as the people affected by environmental harm, and thus the people who have to come together to bargain, could be dispersed over large geographic areas (the same could also, of course, apply to the polluters). Neither in this thesis, therefore, nor in the broader environmental economics literature, are Coasian “laissez-faire” solutions considered feasible between individual consumers. The thesis instead follows in the tradition of modern environmental economics and public finance to study optimal instruments to tackle problems of externalities, and what incentives, if any, the governments of different countries have to coordinate their measures.

A common initial question is how a benevolent government could solve an economic problem when externalities are present. Economists usually favor one of two market-based mechanisms: either the Pigouvian tax that corrects for the – positive or negative – divergence between the marginal and social consequences of an action, or the issuance of tradeable permits or allowances. Both are studied in this thesis. In the aforementioned case where forests are exposed to railway sparks – such that the externality is negative – an unregulated outcome would lead to too many trains being run on the tracks. If the train operator instead had run fewer trains, the train operator and the foresters would together save costs that exceed the train operator’s foregone revenue. A benevolent government could implement this efficient outcome by levying a Pigouvian tax on the railway operator. When the tax equals the difference between the private and the social marginal costs, the railway
operator would find it privately optimal to run trains to the socially efficient extent.\footnote{This example is also considered by Coase (1960). He stresses the reciprocal nature of the problem: should the railway operator be allowed to harm the foresters (by running more trains), or should the foresters be allowed to harm the railway operator (by restricting the number of trains on the tracks).} An analysis of this is illustrated in Figure 1.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure1.png}
\caption{Illustrates the socially optimal amount of an economic activity, $q$, with a negative externality. The social marginal cost (SMC) exceeds the private marginal cost (PMC). The unregulated outcome is where the (constant) marginal benefit (MB) equals PMC, in $q^{UR}$. In this point the SMC exceeds the MB – the outcome is inefficient. A Pigouvian tax $t^*$ levied on the producer, increases the private marginal cost so that it coincides with the social marginal cost. The regulated outcome, $q^*$, is efficient.}
\end{figure}

The task left for the economist is to figure out “in what conditions the values of the social net product and the private net product [...] are liable to diverge from one another” (Pigou, 1932, Part II, Chapter IX). This is precisely what Chapter 1 of this thesis is about. Chapter 1 is written jointly with my supervisor, researcher Mads Greaker at Statistics Norway. In the paper we study the optimal taxation of environmentally dirty technologies when the consumers’ utilities from using the technology are characterized by network effects. A network effect is present whenever the utility one consumer derives from consuming a good depends on who others consume that same good.
There could then be consequences of a consumer’s purchasing decision that are not reflected in the price he pays for the good – such as the fact that he increases the utility of others. What we study in the paper is the simultaneous presence of a network effect and an environmental externality. A case study is the introduction of electric vehicles (EVs) in Norway. The more EVs that are driving on the roads, the more fuel stations, repairshops and supporting infrastructure there will be, and the more attractive it will be to be an EV owner. We show that the network effect and the environmental externality will interact, such that it is not sufficient to solve each of the two problems in isolation; in order to solve the problem, a benevolent government would also have to take into account the interaction between the two.

In principle, every environmental problem could be solved according to the two-step procedure outlined above: first identify and quantify the relevant economic externality; then impose a (possibly negative) Pigouvian tax on the relevant parties. Implicit in this analysis, unfortunately, is the assumption that all relevant economic effects occur within the same jurisdiction. As Mäler (1990) put it, “man-made borders are completely arbitrary from the point of view of the biosphere. There is no reason that environmental disturbances should be confined by human definitions of areas of jurisdiction.” (Mäler, 1990, p. 80). In terms of the normative, this poses no problem. There is no difference between a domestic and a cross-border externality – Pareto efficiency still requires equating social marginal costs and social marginal benefits. Whether these costs or benefits accrue in country A or country B does not matter for efficiency considerations.

The new dimension that arises in transboundary pollution problems, as has been analyzed since the early works of D’Arge (1975) and others, is due to the lack of a clearly defined government or authority. Consider a polluting factory in country A that is located close to the country’s border with country B. Even if the government of country A manages to capture all the harmful effects of the factory on its own citizens in a tax, the government of country A has no compelling reason to take into account the harm its factories inflict on the citizens of country B. This is especially true if, as is most likely the case, it would be costly for country A to take such external harm into account. The problem I first discussed, that consumers within a country have little incentive to take into account the external costs they inflict on others, now reappears as the problem that the different countries have little incentive to take into account the external costs they (or their citizens) inflict on the citizens of other countries. As above, an efficient solution would come about if a supranational authority imposed taxes that corrected for the externalities between countries. The key challenge is that no such authority exists. Any solution to such an
externality problem must instead be worked out by the individual countries involved. But even though it might be clear who is actually harmed, it is unclear whether country A has the right to pollute or whether country B has the right to clean air – these property rights are ill-defined. Furthermore, there exists no international court of law to which you can bring your case for processing and enforcement of an agreement.

Environmental resources, such as clean air and a healthy global climate, can also be seen as examples of public goods. A public good is characterized by its consumption being non-rivalrous, for instance in that one person’s enjoyment of a healthy climate does not diminish other people’s enjoyment of that same climate. The final three chapters of the thesis all contribute to the understanding of various aspects of such international public-goods provision. Efficient provision of public goods requires that the cost of providing the marginal unit of the good equals the sum total of utilities experienced by those who enjoy that marginal unit (Samuelson, 1954). In the case of carbon emissions, it means that the cost of abating a marginal unit of carbon (producing a marginal unit of ‘healthy climate’) should equal the sum of the utility that all people (the present and all future generations) experience when the climate improves. Obviously, in failing to abate a ton of carbon one inflicts harm on third-parties, i.e. one is causing an externality. Public goods (or bads) and externalities are therefore intimately linked: in diminishing a public good, one inflicts a negative externality on the people who could have enjoyed that public good to a larger extent. The Samuelson solution is therefore just a special case of Pigou’s prescription to equate social marginal products.

This view on international environmental problems raises two issues. The first is the normative one of designing efficient solutions to such problems. As already mentioned, the Samuelson solution for the efficient provision of public goods outlined here, and the logic of the analysis of Figure 1, should also hold when the public good can be produced at different locations, and enjoyed by people all over the world, as is the case when discussing abatement of carbon emissions. For the sake of the climate, it is irrelevant whether carbon is emitted at one location or the other, the various emission points are perfect substitutes. Efficient abatement then requires the marginal abatement costs to be equal across emission points, and equal to the sum of the marginal benefits of abatement. For efficiency purposes, when distributing abatement efforts across countries, the distribution of damages from climate change is therefore irrelevant. It could perfectly well be that it is globally efficient that a country that would experience no damages from climate change, undertakes substantial and costly abatement.
This normative prescription leads directly to the second issue raised by this view. This issue is more descriptive in nature, and concerns how countries will behave under different international negotiation protocols. This is also relevant for the normative question of how to design negotiation protocols in order to implement an efficient solution. As such, there is a thin line between the descriptive and the normative.

Damages from climate change are generally thought to be convex (Tol, 2009) in the amount of carbon emitted – for each unit of carbon we emit, the costs of emitting another unit increases. This is the same as saying that the marginal benefit of abatement is decreasing. Furthermore, since carbon emissions from different emission points are perfect substitutes, each country will find it privately optimal to abate less, the more that other countries abate. This means that countries have strong incentives to free ride on each other’s efforts – when one country cuts back on its abatement efforts, it rationally expects other countries to pick up (at least some of) the slack. This further means that it is very hard to achieve broad cooperation and substantial abatement efforts in environmental agreements. These agreements are predicted to be either “narrow” or “shallow”, meaning that either few countries participate, or they will not be able to agree to substantial abatement efforts. Barrett (2005) provides a comprehensive guide to these issues. An illustration of the incentive to free ride is provided in Figure 2.

Chapter 2 of this thesis, written jointly with fellow PhD student Katinka Holtsmark, investigates how these incentives to free ride are affected by the ability that countries have to strategically select their delegates to international negotiations over provision of public goods. It is an inescapable fact that some delegation has to occur: every citizen of Earth cannot get together in order to undertake Coasian bargaining over every international aspect. The citizens of countries are represented by governments, and governments are represented by delegates. Who these delegates are, or what they stand for, could potentially have an impact on the final agreement that is reached. We study a group of countries facing a public-good problem, and ask two questions. The first is to whom each country will want to, or should, strategically delegate their decision-making power. The second is what outcome one could expect from such negotiations when all countries strategically delegate, and all countries take into account that every other country also strategically delegates. The answers to both of these questions depend on whether the contributions the countries bargain over are strategic substitutes or strategic complements. A strategic substitute (complement) is something you want to do more (less) of when others do less. In the example in Figure 2, abatement levels are seen to be strategic substitutes. In the paper, we show that when public-
good contributions are strategic substitutes, countries have an incentive to choose delegates who care less about the public good (for instance the climate) than when contributions are strategic complements. We therefore expect a loss from strategic delegation that is worse under strategic substitutes than under strategic complements. The structure of the problem thus determines whether strategic delegation exacerbates or alleviates the free rider problem. We should, therefore, to the extent possible, design negotiation protocols such that the contributions the countries bargain over are strategic complements rather than substitutes.

The final two chapters consider situations where countries are unable to come together to bargain over a solution to the public-good problem they face. Chapter 3, also joint work with fellow PhD student Katinka Holtsmark, studies a group of countries facing an environmental problem. We consider a different mechanism by which global emissions are determined: the linking of the domestic emission permit markets of the various countries. In the model,
countries regulate their domestic emissions by means of a cap, and decide unilaterally (i.e., non-cooperatively) on the size of this cap. They then issue permits towards this cap, and the permits are tradeable among consumers in all the linked countries. These linked permit markets have been briefly studied previously, but have recently received increased attention as the number of real-world linkages increases, while the global climate negotiations are struggling (Ranson and Stavins, 2012). Our contribution is to show how such linkages can lead to reduced emissions and increased welfare, as the linked market provides the countries with incentives to issue fewer permits than they would absent the links.

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Table 1: The prisoners’ dilemma. Each player can choose either to contribute or not. The social optimum is where both players contribute. Each player has as a dominant strategy not to, however. The unique Nash equilibrium of this one-shot game is (Not, Not).

If one instead of considering the continuous choice of how much to abate – as was done above – rather considers the contributions to a public good as being binary decisions (i.e., either to contribute or not to contribute), public-goods contributions can be modeled by means of a prisoners’ dilemma. The prisoners’ dilemma, as illustrated in Table 1, is a game that captures the tension between the socially efficient and the individually rational thing to do: all players would be better off if everyone cooperated, but each player has a private incentive not to cooperate. Although a very simple game, the insight is powerful – the example in Figure 2 can for instance be seen as a prisoners’ dilemma. When played only once, the unique Nash equilibrium of the game is that no player cooperates. If players interact repeatedly (and indefinitely) and are very patient, however, they can sustain cooperation in equilibrium if they resort to history-dependent strategies. Each player can then credibly threaten to punish the other(s) in the future if they do not cooperate today. These equilibria are called subgame-perfect equilibria. If
the players instead play Markov-perfect strategies, strategies that are history-independent, the unique equilibrium involves no cooperation even when the game is played repeatedly. In the final chapter of the thesis, Chapter 4, which is single-authored, I offer a slight twist on the repeated prisoners’ dilemma. Instead of repeating the game over and over at discrete points in time, I rather consider two players playing the game in continuous time. My contribution is to show that when viewed this way, there exist equilibria in Markov-perfect strategies in which the two players are able to cooperate, even though they, with similar strategies, are unable to cooperate in the repeated discrete-time game. Furthermore, cooperation can come about regardless of the magnitude of the discount rate of the two players. I also argue that this continuous-time game is the more appropriate modeling choice in situations in which the two players are countries, as in the environmental examples above.

As I have argued throughout, environmental economics is both about how to correct for a economic externality given that all relevant effects occur within the same jurisdiction, and it is also about how different countries can – or cannot – cooperate in order to solve transboundary problems. The chapters of this thesis contribute to both of these strands of the environmental economics literature. In the remainder of the introduction I present a more in-depth summary of the four chapters of the thesis.

Chapter summaries

Chapter 1: Network effects and environmental externalities: do clean technologies suffer from excess inertia?

Written jointly with Mads Greake, researcher at Statistics Norway, supervisor.

In this paper we study two existing, competing technologies – one clean, the other dirty. Users of the dirty good inflict externalities on all consumers. What sets this paper apart is that both technologies inhibit network effects – the feature that the utility each consumer derives when consuming a good is increasing in the total number of consumers of that same good. We ask how the presence of these network effects should affect the optimal environmental taxes, and whether the network effects could lead to a socially too slow adoption of the clean good.

The literature on network effects has thus far not been able to agree on whether network effects constitute a proper market failure. The argument in favor is that each consumer in his purchasing decision fails to internalize the impact his choice has on the utility of other consumers. These network effects
could therefore go uninternalized. The key argument opposed is a Coasian one: when large gains from switching from one technology to another are left unexploited, enterprising entrepreneurs can find a way to reap the benefits while inducing a switch. An emerging consensus in the literature appears to be that although network effects are not always properly internalized in the marketplace, the market will tend to choose the right technology, but perhaps not choose it to the right extent (Liebowitz and Margolis, 2010).

In this paper we sidestep this debate. In general, when there are two externalities, two instruments are needed to correct the problem. The main result in our paper is that when these two externalities are present in the marketplace, they interact – they give rise to an interaction effect that we call an externality multiplier. Specifically, we show that even if the network effect was fully internalized – either by a “network tax” or by an enterprising agent – and the environmental externality was corrected for by means of a traditional Pigouvian tax, when these two are put together, the two separate instruments are insufficient to produce the first-best allocation. The reason is that neither instrument takes into account that every purchase of the dirty good not only leads directly to increased emissions, but also indirectly, as it induces other consumers to choose the dirty good in the future. Each dirty consumer is therefore in some way responsible for pollution in excess of just his own. A “network tax” would only take into account the consumption part of the network effect.

This result holds, irrespective of whether the two goods are supplied competitively at marginal cost, or are supplied by monopolist patent holders – so-called technology sponsors. Such a sponsor could subsidize early adopters and reap higher profit margins later on, thereby internalizing most of the network effects. In our model, we find that the social planner and the technology sponsor have such differing interests that the sponsor’s role is modest.

In line with the recent literature on network effects (for instance Cabral (2011)), we develop a dynamic model with an infinite horizon, and study Markov-perfect equilibria in the game between the government, two potential network sponsors and a group of consumers. Some qualitative results are provided in closed form, but to get at any quantitative results we develop a crude numerical simulation of the model using Norwegian electric vehicle adoption as a case study.

Chapter 2: Strategic delegation to negotiations over public goods

Written jointly with Katinka Holtsmark, fellow UiO PhD student.
We study a group of countries that face a public-good problem, like combating climate change. Since Schelling (1956), social scientists have been aware that delegating one’s decision-making power can serve as a useful commitment device. We recognize that countries will delegate their decision-making power to delegates when public-good policies are to be determined – citizens are represented by governments, and governments are represented by delegates. We ask how such delegation affects the collective outcome. We study two polar cases – either emissions are determined non-cooperatively by each delegate in a simultaneous Nash equilibrium, or the delegates come together and determine their emissions per the Nash bargaining protocol. Absent any strategic delegation, the first process leads to the tragedy of the commons, while the latter leads to the first-best emissions profile.

We study the subgame-perfect equilibria of a two-stage game in which the countries first choose their delegate, and then the delegate(s) determine(s) the emissions according to either the non-cooperative or the Nash bargaining protocol. When public-good contributions are determined non-cooperatively, and contributions are strategic substitutes, countries delegate to less committed delegates. A less committed delegate is one with weaker preferences for the public good, one who prefers to contribute less. The delegating authorities rationally expect the other countries to pick up some of the slack. When every country delegates in this way, the outcome is worse than under self-representation. If contributions are strategic complements, however, this logic is turned on its head: if a country sends a more committed delegate, they credibly signal that they will contribute more. Due to the strategic complementarity, other countries realize that it is in their own best interest to also contribute more. Every country will delegate in this way, and the outcome Pareto dominates the outcome under self-representation.

If contributions are determined by Nash bargaining, there is always a loss when countries can strategically delegate. The Nash bargaining protocol implements the first-best contribution profile relative to the delegates who show up at the negotiations, and a set of transfers is determined to ensure that every delegate experiences the same gain when comparing the agreement to a situation in which bargaining breaks down. If bargaining breaks down, we assume that the delegates are no longer in charge, so that the counties themselves determine contributions. The transfers that compensate the delegates ensure that each country has an incentive to delegate to someone who is less committed to the public good, as this increases the transfers they are granted. Given some restrictions on the model, however, we are able to demonstrate that the welfare loss from delegation is smaller under strategic complementarity than under strategic substitutability.
The paper connects the findings of several papers in various branches of economics, and may shed light on why some international public-good problems are harder to solve than others. It also provides some insight into how processes to determine public-goods contributions should be designed. In many cases, we argue, it is possible to choose the strategic property of the problem at hand. The paper argues that one should design the contribution process such that the contributions of countries are strategic complements rather than substitutes. One example is the case of linked emission permit markets discussed in The dynamics of linking permit markets. Winkler (2013) wonders why relatively few countries so far have linked their emission permit markets. He extends the static model of Helm (2003) and finds that electorates have an incentive to elect less green leaders when they expect that their leaders will link their emission permit market to those of other countries. This undermines the scope for gains from trade. What we show in The dynamics of linking permit markets is that this incentive is overturned in a dynamic model of permit market linkages, as the dynamics turns permit issuance decisions from strategic substitutes to strategic complements. Furthermore, as the static model is equivalent to a model in which emission permits are everlasting, this means that a permit-market designer should make sure that permits are shorter-lived, in order to reap the benefits from strategic delegation. By the logic above, the voters would then have an incentive to elect greener political leaders, as this will spur stricter environmental policies from all countries.

Chapter 3: The dynamics of linking permit markets

Written jointly with Katinka Holtsmark, fellow UiO PhD student.

In this paper, as in Strategic delegation to negotiations over public goods, we study a group of countries that face a common-good problem, like climate change. If each country regulates their own emissions in autarky, emissions will be suboptimally high – the tragedy of the commons. A first-best solution requires cooperation from all countries. But global cooperation is hard to establish and sustain. As for instance Newell et al. (2013, p. 123) note, the “[...] late-1990s dream of a top-down global [permit market] design now seems far away, if not impossible. Instead, we see a multiplicity of regional, national, and even subnational markets emerging.” Many of these subglobal markets are already linked to one another, such that permits issued in one jurisdiction can be used for compliance with permit requirements in other jurisdictions. Many more such linkages are expected in the future (Jaffe et al., 2009).

With this development in mind, we ask what effects one could expect from such permit market linkages. We assume that the permit markets of a group
of countries are linked, but that the countries are unable to coordinate and cooperate on the aggregate cap. We therefore let each country issue permits in order to maximize their own welfare, taking the permit issuance of the other countries as given. In each country there is a representative producer of renewable energy. We assume that such energy is produced without variable costs, from a durable stock of production capacity. The renewables producer is a price taker in the energy market and has perfect foresight. Each period he invests in production capacity in order to maximize discounted future profits. We assume that to the consumers, renewable energy is a perfect substitute to fossil energy. Fossil energy is abundantly available at zero cost, but one emission permit must be bought for each unit of fossil energy consumed. We study Markov-perfect equilibria of an infinite-horizon game played between the governments of the linked countries.

The main result of the paper is that such a linked permit market will lead to reduced emissions and increased aggregate welfare, compared to a situation with only autarkic regulations of emissions. This holds even in a situation where all countries are identical, a situation in which there are no traditional gains from trade. The mechanism through which this is achieved works via the renewable energy market. When a country withholds an emission permit, the permit price increases. Since permits are traded throughout the linked market, the permit price will increase in all countries. This provides the renewable energy producers in every country with incentives to increase their investments. Emission permits have a finite lifetime and do expire at some point. When new permits are to be issued, every country will have a higher stock of renewable energy capacity, compared to a situation in which the first country hadn’t withheld the initial permit. When all countries have higher domestic renewable energy capacities, they want to issue fewer permits – permit issuance decisions are now (intertemporal) strategic complements. The linked permit market thus provides each country with a mechanism through which they can reduce the externalities other countries inflict on them.

The outcome will not come about unless there is trade in either emission permits or renewable energy. As the argument above suggested, neither will it come about in a one-shot game or if permits are everlasting. Such a game was analyzed by Helm (2003), and he found that “on average” no gains can be expected from permit market linkages. We therefore show that this result does not hold when the dynamics are taken into account.
Chapter 4: Markov-perfect cooperation in continuous-time prisoners’ dilemmas

*Single-authored.*

In this paper I provide a small twist on the iterated prisoners’ dilemma. I recast the game in continuous time and derive Markov-perfect equilibria in which two players are able to cooperate.

The prisoners’ dilemma offers the canonical trade-off between what is privately beneficial and what is socially optimal. If both players cooperate, they achieve the socially optimal outcome. From that situation, however, each player has a private incentive to defect. The unique Nash equilibrium of a static prisoners’ dilemma is unfortunately that both players defect – the social good is lost. One way to achieve cooperation is to let players repeatedly play the game. They can then employ history-dependent strategies in order to punish non-cooperators, thereby sustaining cooperation through threats. One problem with such subgame-perfect equilibria is that there are many of them, the theory does not yield a unique prediction. Another problem is that they can depend on history in arbitrary ways. In contrast, Markov-perfect strategies are allowed to depend on only on the payoff-relevant parts of history. This means that they are simple. In fact, they prescribe “the simplest form of behavior that is consistent with rationality” (Maskin and Tirole, 2001, p. 193). And there are fewer of them. Indeed, in the repeated (or iterated) prisoners’ dilemma, there is a unique Markov-perfect equilibrium. In this equilibrium, however, the two players always defect.

As stated, the main result of the paper is that in a natural continuous-time analogue to the iterated prisoners’ dilemma, I find Markov-perfect equilibria in which the two players are able to cooperate. Indeed, there are equilibria in which cooperation is swiftly established and is long-lasting. This comes about even if players are unable to credibly promise future cooperation or threaten to punish defectors. Although many symmetric equilibria potentially exist in the game, generally speaking they belong to the same family. They all involve players randomizing to some extent, and the equilibrium separates the game into three distinct phases. In the attrition phase neither player is cooperating, and both players are waiting for the other to take the lead. When that happens, they move to the leadership phase. Here one player is currently leading (at a cost), while the other free rides on his efforts. If the leader gives up, they move back to the attrition phase. If instead the follower follows suit, the players move to the cooperation phase. From there, one player could defect again, in which case they move back to the leadership phase, and so on.
Furthermore, cooperation can be established in equilibrium regardless of the magnitude of the discount rate of the two players. This is in contrast to the literature on folk theorems, which rely on the players to be patient in order to sustain cooperation.

The reason there can be continuous-time Markov equilibria in which cooperation is sustained is that the current actions played by the players become payoff relevant. Instead of both players simultaneously and repeatedly choosing an action, players are now currently playing an action, and considering whether to switch. As such, there is a payoff-relevant difference between defecting when your opponent is currently cooperating and when he is already defecting. Recent economic experiments indicate that lab subjects are more able to cooperate in prisoners’ dilemmas played in continuous time, than in the same game played in discrete time (Friedman and Oprea, 2012; Bigoni et al., 2014). If players are indeed playing Markov-perfect strategies, as suggested by a lab experiment in another setting (Battaglini et al., 2012), my findings could help explain this outcome, as players will not cooperate in discrete-time Markov-perfect equilibria.

References


REFERENCES


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Chapter 1

Network effects and environmental externalities: do clean technologies suffer from excess inertia?*

Mads Greaker and Kristoffer Midttømme

Abstract

We study the diffusion of a clean substitute to a dirty good in a dynamic model. Consumer utility of both goods increase in their respective market shares due to network effects. We then find that excess inertia may happen with Pigovian taxation. The optimal emission tax rate has three components: an environmental damage part, a network effect part and an externality multiplier part, which to our knowledge has not been described before. A Pigovian tax only internalizes the first part, and a failure to account for the two other parts can block the diffusion of the clean good even if it is socially desirable that the clean good over-takes the market. We also find that excess inertia may happen even if the clean technology has a sponsor that partly internalizes the network effect, since the externality multiplier is still not accounted for with Pigovian taxation.

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1.1 Introduction

The solution to an environmental problem often involves replacing an old, dirty technology with a new, clean technology. For example, the depletion of the ozone layer was avoided by mandating a clean substitute to the ozone depleting substances (Barrett, 1999). In other instances, such as combating climate change, the clean technology alternatives are not so obvious. Thus, governments will try to avoid picking winners, and rather rely on setting a price on emissions. The question then arises; can we always rely on a standard, Pigovian emissions tax to induce market diffusion of the clean technology alternatives?

According to several authors the answer could be “no”. The private sector may be reluctant to switch from the dirty technology to the socially more desirable clean technology even if the negative externalities connected to the use of the dirty technology are internalized. So far the literature has focused on potential market failures in technological development that may slow or block the shift from dirty to clean technologies.¹ In this paper we study whether network effects may obstruct the diffusion of clean technologies.

Positive network effects arise if one agent’s adoption of a good (a) benefits other adopters of the good; and (b) increases others’ incentives to adopt it (Farrell and Klemperer, 2007). We have excess inertia when a new good would increase welfare, but does not successfully diffuse into the market (Farrell and Saloner, 1986). Network effects may be the case for video meetings, alternative fuel vehicles, public transport, carbon capture requiring pipeline transport services etc. Furthermore, it is generally acknowledged that clean technology diffusion is needed to solve many pressing environmental problems such as climate change. Our research question is therefore: Could a failure to account for network effects in emission taxes lead to excess inertia?

First, we characterize the optimal emission tax when there are network effects. The tax has three components: an environmental damage part, a network effect part and an externality multiplier part, which has not been described before in the literature. Second, we study whether excess inertia can occur if the emission tax is not optimally set. We then find that a tax that only accounts for the environmental damage part may lead to excess inertia. Excess inertia could happen even if the clean technology is proprietary, and the technology owner has incentives to sponsor the initial market diffusion of the technology.

¹See for instance Acemoglu et al. (2012) and Chakravorty et al. (2010).
1.1. INTRODUCTION

Very few papers study public intervention when there are network effects, in particular, network effects is only briefly covered in the environmental economics literature. Greaker and Heggedal (2010) build an explicit model of the relationship between the market share of hydrogen cars and the density of hydrogen filling stations, and show that this could lead to multiple equilibria. However, they do not look at public policies to induce a shift from an inferior equilibrium to the efficient equilibrium. The transportation market is also treated by Sartzetakis and Tsigaris (2005). However, as they do not characterize the optimal emission tax, their model is not well-suited to explain the reasons for excess inertia.

Moreover, our result that excess inertia may occur is contrary to much of the general literature on network effects. Farrell and Saloner (1986) find that when players have complete information about each other’s payoffs, and none of the technologies enjoy the advantage of an existing base of users, an uncoordinated adoption process will lead to the efficient outcome.

In a later paper Farrell and Saloner (1986) introduced players with private information about their own payoffs, and an installed base of users of the inferior technology. They then got under-adoption, or too slow adoption of the superior technology. However, Ochs and Park (2010) extend the analysis in Farrell and Saloner (1986), and find that if the most eager consumers move first and entry decisions are irreversible, then in a limiting case, any coordination problem found by Farrell and Saloner (1986) vanishes, and the equilibrium becomes efficient.

Katz and Shapiro (1986) introduced technology sponsors that have proprietary rights to the network technologies. They then found that as long as the superior network technology had a sole owner, it would dominate the market and excess inertia would not occur. This is in line with the argument put forward by Liebowitz and Margolis (1994). Liebowitz and Margolis (1994) doubt that excess inertia is likely to be a significant problem in a market economy. They argue that the definition of inefficiency is that the benefits of an unrealized outcome must exceed its costs. If so, these benefits can be exploited by private agents with profit motives.

One more example from this literature is Segal (1999). This paper studies contracting under network externalities, and outline sufficient conditions for when a network sponsor may contract with the adopters to achieve the efficient adoption. He finds that if the sponsor makes public offers, and can commit to them, then as long as there are only network effects present for this current good, then the network sponsor can achieve efficient adoption.

In line with the recent literature on network effects, we study a dynamic model with infinitely many periods based on Cabral (2011), but unlike Cabral
(2011), we introduce a government that seeks to regulate the market outcome. We solve the extended game, and find an expression for the Markov perfect emission tax rule. The emission tax rule should take into account the network effect by making the tax rate dependent on the market share of the clean technology. Thus, even if marginal environmental damage from the use of the dirty good is constant, the dirty good tax should depend on the market diffusion of the clean substitute.

In order to explain this result, we decompose the optimal emission tax into three components: an environmental damage part, a network effect part and an externality multiplier part. A Pigovian tax internalizes the environmental damage part. However, both the network effect part and an externality multiplier part may warrant a tax that departs from the Pigovian tax. In our simulations we find that the optimal dirty good tax should be higher when the dirty good has a high market share, and lower when the dirty good has a low market share. The simulations also show that a failure to account for the network effect part and an externality multiplier part in the taxation could lead to excess inertia.

The externality multiplier is an interaction effect between the network effect and the environmental externality. That is, a consumer who adopts a dirty good today, will make the dirty good more attractive to others in the future, and ceteris paribus, cause more future pollution than just his own use of the dirty good. With the standard Pigovian tax, the consumer would only face a tax equal to the social costs of his own emissions, and not of those he might induce through future consumers having a higher probability of adopting the dirty good.

The externality multiplier is not internalized by a potential clean technology sponsor. This implies that even if a clean technology sponsor internalizes the network effect part by setting low prices in an introductory phase, the sponsor may settle with a too low market share for the clean good. The government should therefore not lean back on a Pigovian tax, and let the clean technology sponsor decide the supply of clean goods. Contrary to the argument of Liebowitz and Margolis (1994), we find that excess inertia may occur with a Pigovian tax also when there is a clean technology sponsor.

The paper proceeds as follows: in Section 2 we lay out the model, while in Section 3 we derive the main results. In Section 4 we use the model to look at a particular case numerically. In Section 5 we introduce a clean technology sponsor, and in Section 6 we conclude.
1.2 Model primitives

The consumption side of our model is based on Cabral (2011) apart from a tax on one of the goods. We have discrete timing with two competing networks and a fixed number, $N$, of consumers. The market is fully covered, and all $N$ consumers enter one of the networks. Moreover, as in Cabral, the networks are differentiated, e.g. consumers have a preference for accessing either of the networks independent of network size. We will index the networks by $k = c$ for clean and $d$ for dirty.

For each network there is an access price denoted $p_k$ the consumer has to pay to join the network. These prices are set by the firms, and can be thought of as prices for some durable goods that grant the consumer access to the network in question. Cabral (2011) only has one producer for each network durable, while we in our base case allow for more producers of each network durable. When more firms are selling a durable good of the same type $k$, we assume that these durables are perfect substitutes, and that firms price at marginal cost which we normalize to zero.

After the consumer has decided which network to access, we assume that her use of the durable is given. Hence, the government can control pollution by setting a tax $t$ on the purchase of the dirty durable, and we look for the markov-perfect dirty network entry tax. The setup will be time-homogeneous, hence we suppress all time subscripts. The only payoff-relevant variables will be the network sizes, denoted $n_c$ and $n_d$.

1.2.1 The consumers

At the beginning of each period, there are $N − 1$ consumers present in the market. One consumer arrives, and is confronted with the prices and the tax. Subject to these and his private preferences, he has to choose which network he wants to enter. Then follows the aftermarket stage, in which the durable goods are being put to use. At this stage all consumers in network $k$ each enjoy the benefits $\lambda(n_k)$, common to all consumers, and weakly increasing in the network size $n_k$. At the end of the period, with uniform probability, one random consumer is chosen to exit the market.

In addition to $\lambda(n_k)$ that are common to all consumers, each consumer has two idiosyncratic, private utility components. The components, $\zeta_c, \zeta_d$,
determine the technology-specific utility he enjoys from joining either of the networks.

An entering consumer neither knows for how many periods he will enjoy the aftermarket benefits, nor how large the network is going to be in the future. We therefore introduce the function \( u_k(n_k) \) which is the expected present value (EPV) of entering network \( k \) at size \( n_k \).

The total expected net benefit of joining network \( k \) at size \( n_k \) today, \( B_k \), is thus given by:

\[
B_k = \begin{cases} 
\zeta_c + u_c(n_c + 1) - p_c(n_c), & \text{if clean network} \\
\zeta_d + u_d(n_d + 1) - p_d(n_d) - t(n_d), & \text{if dirty network}.
\end{cases}
\]

We assume that the values of \( \zeta_k \) are sufficiently high such that the consumer always chooses one of the networks. Since the market is then completely covered, we can restrict our attention to the distribution of the difference between the two utility parameters \( \xi_c \equiv \zeta_c - \zeta_d \). As we assume that the \( \zeta_k \) are i.i.d, \( \xi_c \) has expected value equal to zero. Further, we assume that \( \xi_c \) is normally distributed with cdf \( \Phi(\cdot) \) and density \( \phi(\cdot) \).

The consumer who is indifferent between the two networks will have: \( B_c = B_d \), or \( \xi_c = x(n_c) \), where the latter is given by:

\[
x(n_c) = p_c(n_c) - p_d(n_d) - t(n_d) - u_c(n_c + 1) + u_d(n_d + 1).
\]

That is, \( x(n_c) \) indicates the position along the real line of the consumer who is indifferent between the two goods when the clean network has size \( n_c \), and prices and taxes are as given. The probability that a newborn consumer chooses the dirty network is given by:

\[
q_d(n_d) = \Pr[\xi_c < x(n_c)] = \Phi[x(n_c)],
\]

and the probability of choosing the clean network is then:

\[
q_c(n_c) = 1 - \Phi[x(n_c)].
\]

The probabilities are related through \( q_c(a) + q_d(N - 1 - a) \equiv 1 \). Furthermore, we can see that the probability that firm \( k \) makes the next sale is, ceteris paribus, continuously and monotonically decreasing in \( p_k \), \( \partial q_k(n_k)/\partial p_k < 0 \). The probabilities can hence be interpreted as demand functions.

Given a sequence of taxes and prices, we now have the law of motion for the network shares. Given that every consumer has the same probability of
being chosen to leave the market, the EPV of future network benefits does not depend on how long a consumer has been present. We can therefore define $u_k(n_k)$ recursively in the following way:

$$
u_k(n_k) = \lambda(n_k) + \frac{1}{N} \cdot 0 + \delta \frac{n_k}{N} q_k(n_k) u(n_k + 1)$$

$$+ \delta \left[ \frac{n_k}{N} q_k(n_k - 1) + \frac{n_k}{N} q_k(n_k - 1) \right] u_k(n_k)$$

$$+ \delta \frac{n_k}{N} q_k(n_k - 1) u(n_k - 1)$$

(1.4)

where $-k = c$ if $k = d$, and vice versa.

Each period you enjoy the aftermarket benefit as a function of the market share. At the end of each period, there is a probability $1/N$ that you are the one that exits, after which you get zero by assumption. If you are not chosen to exit, there are three possibilities: your network increases, decreases or remains at the same size. There is only one possible way your network can increase in size: with a probability of $n_k/N$ someone in the clean network exits, and with probability $q_k(n_k)$ the arriving consumer opts for your network, and the network size increases one step. There are two events that may reproduce the current state the next period; that is when one of the networks experiences exit and the arriving consumer chooses to join that same network. And finally your network may decrease by one step if someone other than you exits, and the next consumer chooses the other network.

To gain some intuition on (1.4), we can consider the case with constant network benefits e.g. $\lambda(\cdot) = \Gamma$. Equation (1.4) then collapses to $u_k = \Gamma (1 - \delta \frac{N-1}{N})^{-1}$, i.e. the expected net present value of the future network benefits. Note that the discount factor is augmented with the factor $\frac{N-1}{N}$, that is the probability that the consumer will not exit. Define then $\delta = \delta \frac{N-1}{N}$, which is the effective discount rate of the consumers.

### 1.2.2 The government

Environmental damages from the polluting network accrues according to $\gamma n_d$, where $\gamma$ is a parameter and $n_d$ is the number of consumers present in the polluting network today. In addition to the environmental damage function, the public welfare function is assumed to be utilitarian, it is the unweighted sum of profits and consumer utility. Let the value function of the government be denoted by $g(n_c)$. Before the consumer chooses a network, $g(n_c)$ is given by:
The expected idiosyncratic utility of the newborn consumer conditional on the network size can be written $\mu + \sigma^2 \phi(x(n_c))$.\(^5\) The rest of the welfare measure is the expected value of two scenarios: Either the newborn consumer chooses the clean network with with probability $q_c(n_c)$, or he chooses the dirty network with probability $(1 - q_c(n_c))$. Inside the two large brackets we add the consumers’ network benefits in the coming period, subtract the environmental damages in the coming period, and add the expected continuation value $g(\cdot)$, conditional on the choice the newborn consumer made today.

Note that the tax and the price on the dirty durable, and the price on the clean durable all cancel out from the value function; these are costs to the consumer, but at the same time revenue for the government and the firms. Due to our assumption that the market is fully covered, neither the entry tax nor mark-up pricing by the firms will induce any dead weight loss.

### 1.3 Solving the model

#### 1.3.1 First best

What is the first-best allocation, that is, which network should the newborn consumer join, given the currently observed market shares? Since consumers are born with stochastic taste parameters, this amounts to choosing the state-dependent cut-off value $x(n_c)$ for the taste parameter, which divides the pool of potential newborn consumers into clean adopters and dirty adopters. The first-best allocation is thus the policy rule $x^*(n_c)$ defined by

$$x^*(n_c) = \arg \max_x g(n_c).$$

---

\(^5\)That is, $q_c(n_c) \mathbb{E}[\zeta_c | \xi_c > x(n_c)] + (1 - q_c(n_c)) \mathbb{E}[\zeta_c | \xi_c < x(n_c)] = \mu + \sigma^2 \phi(x)$. This expression is derived in the Appendix. The parameter $\mu$ is the expected value of the individual shocks $\zeta_k$, while $\sigma^2$ is the variance of the distribution of $\xi_c$.\(\)
1.3. SOLVING THE MODEL

To solve it, we differentiate (1.5) wrt. \( x \), and get:

\[
\frac{\partial g(n_c)}{\partial x(n_c)} = \sigma^2 \phi'(x) - \phi(x) \left[ n_d \lambda(n_d) + (n_c + 1)\lambda(n_c + 1) - \gamma n_d \right] - \phi(x) \delta \left[ \frac{n_c + 1}{N} g(n_c) + \frac{n_d}{N} g(n_c + 1) \right] + \phi(x) \left[ (n_d + 1)\lambda(n_d + 1) + n_c\lambda(n_c) - \gamma(n_d + 1) \right] + \phi(x) \delta \left[ \frac{n_c}{N} g(n_c - 1) + \frac{n_c - 1}{N} g(n_c) \right]
\]

To simplify the expression let \( \Delta_k(n_k) = (n_k + 1)\lambda(n_k + 1) - n_k\lambda(n_k), \ k = c, d \). That is, \( \Delta_k \) is the gain in total network benefits in network \( k \) with one more member in network \( k \). Then, setting \( \frac{\partial g(n_c)}{\partial x(n_c)} = 0 \), using that \( \sigma^2 \phi'(x) = -x(n_c)\phi(x) \) and that \( \phi(x) \) is bounded away from zero, and rearranging, we obtain:

\[
x^*(n_c) = \frac{\Delta_d(n_d) - \gamma}{\Delta_c(n_c) - \delta} \left[ \frac{n_c}{N} g(n_c) - 1 + \frac{n_d}{N} g(n_c + 1) \right] = \frac{\Delta_d(n_d)}{1 - [1 + q']\delta} \frac{\gamma}{1 - \delta} - \frac{q'\gamma}{1 - [1 + q']\delta} \frac{\delta}{1 - \delta}
\]

In the Appendix we show that the second-order condition is satisfied.

Like in other models with horizontal differentiation, there exists an optimal division of the market between the two goods. In our case this division depends on the state \( n_c \). In order to obtain an optimal division of the market, (1.6) assigns a value on \( \xi_c (= x^*(n_c)) \) for the indifferent consumer for every state. Remember that a consumer will only choose the dirty good if he has sufficiently strong preferences for it e.g. if \( \xi_c = \zeta_c - \zeta_d < x(n_c) \). Thus, if for instance the optimal \( x^*(n_c) \) is negative, only consumers with \( \zeta_d > \zeta_c \) should choose the dirty good.

To gain more intuition we proceed by simplifying (1.6) in the following manner: Assume that \( g(\cdot), \Delta_k \) and \( q_c \) are locally linear, which seems reasonable for \( N \) large.\(^6\) Implementing our assumptions in (1.6), and using that \( q_d(n_d) = 1 - q_c(n_c) \), we obtain:

\[
x^*(n_c) = \frac{\Delta_d(n_d) - \Delta_c(n_c)}{1 - [1 + q']\delta} - \frac{\gamma}{1 - \delta}
\]

\[
\frac{q'\gamma}{1 - [1 + q']\delta} \frac{\delta}{1 - \delta}
\]

\(^6\)This implies \( g(n_c - 1) - g(n_c - 2) = g(n_c) - g(n_c - 1) = g(n_c + 1) - g(n_c), \ \Delta_d(n_d + 1) = \Delta_d(n_d), \ \Delta_c(n_c) = \Delta_c(n_c - 1), \) and \( q_c(n_c + 1) - q_c(n_c) = q_c(n_c) - q_c(n_c - 1) \).
where \( \delta = \frac{N-1}{N} \) and \( q' = q_d(n_d + 1) - q_d(n_d) \), that is, the increase in the probability of a consumer choosing dirty when one more consumer joins the dirty network. We must have \( 1 - [1 + q'] \delta > 0 \).

Equation (1.7) tells us that the optimal value on \( \xi_c \) for the indifferent consumer depends on three terms. The first term in (1.7) is the present value of the loss/gain in the flow of network benefits from a dirty choice today. If \( n_c \leq n_d \) and the \( \lambda(\cdot) \) function is concave, the term will be negative. The second term in (1.7) is the present value of the environmental costs from a dirty choice today (stemming from the emissions of just that consumer who entered the market). This is always negative, and we will later refer to it as the **Pigovian entry tax**.7

The last term in (1.7) is the effect we coin "the externality multiplier". This term is negative as long as \( q' \) is positive e.g. the probability that the next consumer chooses dirty increases when the current consumer chooses dirty. In other words, a consumer who adopts a dirty good today, may make the dirty good more attractive to others in the future, and indirectly cause more future pollution than just from his own use of the dirty good.

We have the following proposition:

**Proposition 1.1.** The first best optimal division of the market between the dirty and the clean good depends on the sum of three effects i) a network effect part, ii) a Pigovian part and iii) an externality multiplier part.

As long as the sum of the terms is negative, some consumers should be "persuaded" to choose clean even if their idiosyncratic preferences tells them to choose dirty. We now proceed to look at the optimal tax in the case when \( p_c = p_d = 0 \) for all states.

### 1.3.2 Setting the optimal dirty good tax

The timing of the game in every period is as follows: First, the government sets the emission tax. Second, the consumer makes his choice, observing the current tax. As we do not find it reasonable that the government can commit to future tax rates, we will only allow a stagewise leadership. We write the dynamic programming problem of the government as follows:

\[
g^*(n_c) = \max_{\xi(n_c)} g(n_c),
\]

where \( g(n_c) \) is defined in (1.5).

---

7In the appendix, we solve for the optimal entry tax without network effects and without market power, and show that this entry tax indeed equals \( \gamma/(1 - \delta) \).
Note from (1.15) that neither the prices nor the tax enter the value function directly. They only enter indirectly through \( x(n_c) \). Hence, we have:

**Proposition 1.2.** The government can implement the first-best allocation.

Since the prices and taxes only enter the welfare expression through the indifference parameter \( x(n_c) \), we find a first-order condition for the purchase tax, \( t \), of the following form:

\[
0 = \frac{dx}{dt} \cdot \phi(x) \cdot \left[ \Delta_d(n_d) - \Delta_c(n_c) \right]
+ \delta \frac{n_c}{N} \left[ g(n_c - 1) - g(n_c) \right]
+ \delta \frac{n_d}{N} \left[ g(n_c) - g(n_c + 1) \right] - x^*(n_c).
\]  

We can see that (1.6) is a solution to this.\(^8\) This means that even in this dynamic taxation game, when we have restricted the government to choose from only time-consistent policies, the first best can be implemented. Since taxes are non-distortionary, the government can credibly promise to set whatever future taxes are needed to implement the first best. If high taxes were very costly (for instance if consumers then would choose some outside good and disappear from the market in question), one could imagine that the government’s threat to set high taxes in the future was not credible, and that it therefore might struggle to implement the first best today. This does not happen here.

The expression for the optimal tax rate \( t(\cdot) \) can, unfortunately, not be derived in closed form since it depends on solving for the fixed point on the form \( x(n_c) = G(x(n_c)) \). The function \( G(\cdot) \) involves the cumulative distribution function of the taste parameters, and the implicitly defined value function. However, by using the same simplifications as in the former section, we can expand the expression for \( x \) from (1.1), and hence decompose the optimal entry tax in the following manner:

\[
t(n_c) = \frac{\Delta_c(n_c) - \Delta_d(n_d)}{1 - [1 + q'] \delta}
+ \frac{\gamma}{1 - \delta}
+ \frac{q' \gamma}{1 - [1 + q'] \delta}
+ \frac{\delta}{1 - \delta}
- \left[ u_c(n_c + 1) - u_d(n_d + 1) \right] 
\]  

The three first terms are all known from the discussion of (1.7). The second term is the Pigovian entry tax, and since both the first and the third term might be positive, the optimal tax could exceed the Pigovian tax.

The last term tells us that the tax should be adjusted since the consumer himself internalizes a part of the network externality e.g. chooses clean if the

\(^8\)See the Appendix for a complete derivation of this result.
clean good yields him higher utility. With many consumers, this is likely a
minor effect.

It is obvious from (1.10) that the optimal tax cannot be constant when
there are network effects. Summing up, we have:

**Proposition 1.3.** The optimal entry tax departs from the Pigovian entry tax,
and in general depends on the market shares of the two durables, $n_c$ and $n_d$.

We have shown that the optimal tax should depart from the Pigovian tax.
What happens if the government sets the Pigovian tax, and leaves the market
to itself? May we for instance experience excess inertia? In general, the answer
to this question will depend on the functional forms. In the next section we
develop a numerical illustration of the model inspired by the introduction of
electric vehicles (EVs) in Norway.

### 1.4 The introduction of EVs in Norway

Cars with different kinds of engines may be examples of network goods.\(^9\) EVs
have been marketed in Norway for some time, but only recently have they
started to take off. In parallel with their take off the number of both normal
and fast charging stations has surged (see Figure 1.1). Clearly, the more
dense the network of charging station becomes, the more practical the EV will
appear to the consumer.\(^10\) As an illustration of the potential importance of
network effects for environmental policy, we have calibrated our model to the
development of the stock of EVs in Norway. A word of caution, our model is
a very abstract representation of the car market. The number of cars is given
independent of the fossil car tax, each car has a certain per period mileage
independent of type, there is no technological development and fossil cars
have no cost advantage.\(^11\)

Even if there is now around 20.000 EVs in Norway, this is still less than
1 % of the total car stock. Current sales is however around 8% of total sales.
Norway has large subsides for EVs in the form of tax rebates on EV sales,
exemption from road tolls, opportunity to drive in the bus lanes and free

---

\(^9\)Nicholas and Ogden (2009) report from a survey, demonstrating a strong relationship
between the willingness to pay for a hydrogen car and the availability of hydrogen filling
stations. The same type of interdependency would most likely also be the case for electric
cars and the network of fast-charging stations.

\(^10\)There are likely more kinds of complimentary services which would make the EV more
appealing such as a viable market for used EVs, a system for recycling of batteries, service
cars that can provide fast re-charging if stuck somewhere on the road etc.

\(^11\)EVs with the same performance as gasoline cars are more costly to make today, but this
may change, see for instance a study of battery costs by Hensley et al. (2012).
1.4. THE INTRODUCTION OF EVS IN NORWAY

![Graph showing EV market development in Norway with years on the x-axis and numbers on the y-axis, with data points for Charging Stations and Stock of EVs from 2002 to 2014.]

**Figure 1.1:** The stock of EVs and charging stations in Norway

Parking in some cities. Moreover, fossil cars in Norway are heavily taxed, and so is also gasoline. We have calculated the net present value of the EV use subsidies and the fossil fuel taxes, and added these to the tax rebate for EVs versus fossil cars. In our model this figure corresponds to the entry tax \( t_d \).\(^{12}\)

Furthermore, we represent the network effect \( \lambda(n_k) \) by a logistic curve:

\[
\lambda(n_k) = \frac{\psi}{1 + e^{\alpha_k - \beta_k \frac{n_k}{N}}}.
\]

(1.11)

Such a curve fits well with the network benefits we derive from the explicit network model in Greaker and Heggedal (2010). We then choose the parameters \( \psi, \alpha_k \) and \( \beta_k \) for fossil cars and EVs, respectively, such that we are able to replicate the development of EV sales in Norway given the current policy \( t_d \). With the representations of the \( \lambda \)-functions given in Figure 1.2, we can replicate the development in Figure 1.1 within our model.\(^{13}\)

---

\(^{12}\)Note that we have not included the option to use the bus line since it was hard to put a value on this benefit.

\(^{13}\)See the Appendix for the actual parameter values, and the calculation of \( t_d \). We cannot rule out that also other representations of the logistic curve, can replicate the development of the EV market in Norway. Moreover, that these other representations would yield other results.
In Figure 1.2, the market share of the EV is on the x-axis, while the use benefit of the car per consumer is on the y-axis. Note that the market share of fossil cars is just the inverse. In the figure we see that fossil cars and EVs give the same utility to the consumer if they both have a 50% market share. However, since we start out with fossil cars dominating the market, EVs yield lower use benefits. Note also that fossil cars are not so vulnerable to network effects e.g. it is easy to extend their driving range in response to fewer filling stations.

Since there are many providers of both fossil cars and EVs in Norway, we assume that the market for both types of cars is competitive. Having calibrated the $\lambda$-functions, the model can be solved numerically for the optimal entry tax. We have a fixed point problem for all $n_c$, and we can simply iterate to fix the optimal $x(n_c)$. The optimal entry tax is then determined as the tax that implements the pre-determined $x(n_c)$, given the choice rule of the consumers.

The currently observed taxes in Norway, together with the calculated optimal taxes are given in Figure 1.3. Again we have the market share of EVs at the X-axis. On the y-axis we have the entry tax measured in thousand USDs per car. First, note that the optimal tax depends on the market share of EVs, and that it starts high, increases further and then falls as the EVs pick-up. In our illustration, the current tax is too low, that is, the current tax falls short...
1.4. THE INTRODUCTION OF EVS IN NORWAY

The introduction of EVs in Norway has been a significant focus, with the government aiming to increase the market share of EVs. The optimal tax, as determined by the model, is effective as long as the market share of EVs is below approximately 35%. The government has stated that they will review the EV subsidies when the EV market share is about 2.5%. Interestingly, our model suggests that subsidies should rather be increased than decreased at this point.

Second, note that the Pigovian tax in our example is about half the size as the current tax ($t_d$), and even further below the optimal tax. The Pigovian tax is based on Norway’s national carbon emission reduction target. That is, it is equal to the net present value of all the expected emissions from a fossil car given the costs of reducing other carbon emissions in Norway. In our model, the current tax leads to a too slow market development, and the Pigovian tax leads to excess inertia, which can be seen from Figure 1.4.

Here we have the clean market share on the y-axis, while the x-axis shows time. The three graphs show the market development with three different tax rules. Note that even with high current tax, it takes some time to convert the market. The reason is that in our model some consumers strongly prefer fossil cars, and as long as other consumers have fossil cars these consumers will by a fossil car even if the tax is high.

With the optimal tax, the market converts to EVs faster which is good due to the high costs of carbon emission to Norway (given Norway’s emission reduction goal). Finally, note that setting the Pigovian tax instead of the

![Figure 1.3: The entry tax](image)

Current and optimal taxes

- Optimal taxes
- Current taxes
- Pigovian taxes

Tax on dirty good. Thousand USDs

Clean market share

of the optimal tax as long as the market share of EVs is below $\sim 35\%$. The government has stated that they will review the EV subsidies when the EV market share is about $2.5\%$. Interestingly, our model suggests that subsidies should rather be increased than decreased at this point.

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With the optimal tax, the market converts to EVs faster which is good due to the high costs of carbon emission to Norway (given Norway’s emission reduction goal). Finally, note that setting the Pigovian tax instead of the
CHAPTER 1. NETWORK EFFECTS

Figure 1.4: Market development

Current tax would likely not lead to any EV sales at all provided that the EV starts out with zero market share.

What drives the high, initial optimal tax in Figure 1.3? In Equation (1.10) we decompose the optimal entry tax, and identified the externality multiplier effect. This effect can be numerically calculated by the following formula:

$$\text{Externality multiplier effect} = \text{Fulltax} - \text{Network tax (for } \gamma = 0) - \text{Pigovian tax}$$

The network tax for $\gamma = 0$ has to be found by simulating the model. We then obtain the externality multiplier tax depicted in Figure 1.5.

As we can see from the figure, the externality multiplier effect partly explains the initial high tax rates. Thus, the government should not only set a high initial tax rate on fossil cars due to the network effect, but also since every fossil car sold will increase the probability that more fossil cars are sold in the future.

1.5 Clean technology sponsor

As mentioned in the introduction, Liebowitz and Margolis (1994) doubt that excess inertia is likely to be a significant problem in a market economy. Their
argument is that any potential for Pareto improvements inherent in an excess inertia situation can be exploited by private agents with profit motives. In this section we inquire further into their argument by introducing a clean technology sponsor.

### 1.5.1 Pricing rule

We now assume that the clean durable is proprietary to a single firm, which we coin the clean sponsor. The value function $v_c(n_c)$ of the clean sponsor is evaluated before the firm sets its price and the arriving consumer makes his choice. The total number of consumers who currently are in the market is therefore $N - 1$. Recall from (1.3) that the probability that the clean firm makes a sale is continuously and monotonically decreasing in $p_c$. The clean sponsor thus solves:

$$
v_c(n_c) = \max_{p_c} \left\{ q_c(n_c) \left( p_c + \delta \frac{n_d}{N} v_c(n_c + 1) + \delta \frac{n_c + 1}{N} v_c(n_c) \right) \right\} \quad (1.12)
+(1 - q_c(n_c)) \left( \delta \frac{n_d + 1}{N} v_c(n_c) + \delta \frac{n_c}{N} v_c(n_c - 1) \right)
$$
The first line above is the event that the newborn consumer chooses the clean good. It happens with probability \( q_c(n_c) \), decreasing in \( p_c \). Then, the clean sponsor sells a unit at value \( p_c \) and its network size increases to \( n_c + 1 \). In the next period there are two possibilities: either the dirty network experiences exit (with probability \( n_d/N \)); or someone in the clean network leaves (with probability \( (n_c + 1)/N \)). The network size at the beginning of the next period is updated accordingly. The second line is the event that the arriving consumer chooses the dirty network. In that case there is a higher probability that the dirty network experiences an exit in the next period.

We seek an expression for the optimal pricing rule \( p_c(n_c) \). The firm takes the sequence of taxes \( t(n_d) \), as given. The first-order conditions with respect to price is:

\[
q_c(n_c) + \frac{\partial q_c(n_c)}{\partial p_c(n_c)} p_c(n_c) + \frac{\partial q_c(n_c)}{\partial p_c(n_c)} \delta \left[ \frac{n_d}{N} v_c(n_c + 1) + \frac{n_c - n_d}{N} v_c(n_c) - \frac{n_c}{N} v_c(n_c - 1) \right] = 0,
\]

which, following Cabral (2011), can be rewritten to

\[
\frac{p_c(n_c) + w_c(n_c)}{p_c(n_c)} = \frac{1}{|\varepsilon_c|}.
\]

where \( w_c(n_c) \equiv \delta \left[ \frac{n_d}{N} v_c(n_c + 1) + \frac{n_c - n_d}{N} v_c(n_c) - \frac{n_c}{N} v_c(n_c - 1) \right] \) and where \( \varepsilon_c \) is the price elasticity of demand.

The function \( w_c(n_c) \) is the present value of the future excess revenue that stems from a current sale. It is thus very similar to the change in continuation value experienced by the social planner in (1.6). By rewriting the expression, we see that it is positive if \( v_c(n_c + 1) > v_c(n_c) > v_c(n_c - 1) \). As can be seen from (1.13), if \( w_c(n_c) > 0 \), the discounted prize of the sale plays the role of a negative marginal cost in the firm’s pricing problem. In other words, instead of experiencing a marginal cost associated with a sale, the firm experiences a more favorable distribution over the continuation values \( v_c(\cdot) \). This implies that the clean technology sponsor might be willing to forego profits today by setting a negative price. In our opinion, this is the mechanism Liebowitz and Margolis (1994) refer to when they argue against excess inertia. If maintaining the dirty durable is in-efficient due to the environmental externality, a clean technology sponsor should want to exploit this (given that the externality is internalized by a Pigovian tax).
1.5. CLEAN TECHNOLOGY SPONSOR

1.5.2 Optimal taxes

The timing of the game in every period is now as follows: First, the government sets the entry tax. Second, the clean sponsor observes the tax, and then she sets an entry price, $p_c$. Finally, the consumer makes his choice, observing the prices and the current tax. We are thus considering markov-perfect equilibria in which the government acts as a stagewise Stackelberg leader vis-a-vis the clean technology sponsor.

We already know the choice rule of the consumers from Section 2.1. Moreover, the best response of the clean sponsor is given from (1.13). As already noted, neither the prices nor the tax enter the value function directly (see (1.15)). Thus, the optimal tax rule is still given from (1.8), but this time solved subject to (1.13).

Moreover, since Proposition 2 still holds, the government will implement the same $x(n_c)$ as in the case in which both prices of the durables were zero for all states. The government can do this by implementing the following optimal tax rule:

$$
t(n_d) = \frac{\gamma}{1-\delta} + \frac{q'\gamma}{1-\delta} \frac{\delta}{1-\delta}
+ \frac{\Delta_c(n_c) - \Delta_d(n_d)}{1-\delta} - [u_c(n_c + 1) - u_d(n_d + 1)]
+ p_c(n_c)
$$

where we have used the same simplifications as when discussing the first best.\(^{14}\)

The expression is identical to (1.10) except from the final term. This term corrects the entry tax for the pricing of the clean technology sponsor. For instance, if the sponsor uses a negative price to attract the newborn consumer to her network, the tax should be lower. The expression (1.14) can, however, not by itself give an answer to the question: "Can Pigovian taxation leads to excess inertia? We have therefore also simulated the model with a clean technology sponsor.

1.5.3 Numerical simulation with clean sponsor

We represent the network effect $\lambda(n_k)$ by a logistic curve as above (see (1.11)), but we do not calibrate the model to the Norwegian EV experience.\(^{15}\) Instead

\(^{14}\)E.g. we assume that $g(\cdot), \Delta_k$ and $q_k$ are locally linear.

\(^{15}\)There are many companies competing over the EV market in Norway.
we use the same set of parameters $\psi$, $\alpha$ and $\beta$ for both the dirty and the clean durable. Hence, apart from the environmental externality, the two technologies are symmetric. With $\psi = 0.6$, $\alpha = 6$ and $\beta = 32$ the network benefit $\lambda(n_k)$ increases slowly for small markets shares, then increases markedly for market shares above 10%, and levels off at a 30% market share. We compare the optimal entry tax with the Pigovian entry tax, i.e. $t^{\text{pig}} = \frac{\gamma}{(1 - \delta \frac{N-1}{N})}$. We have used $\delta = 0.85$, and $\gamma = 0.2$. Simulating the model we get the entry tax rule of Figure 1.6 with a clean sponsor.

![Optimal taxes, clean sponsor, log-scale](image)

**Figure 1.6:** Optimal entry tax with a clean sponsor

The optimal tax is on the $y$-axis. Note that the variation in the tax is much higher compared to the competitive situation, which makes it necessary to use a log-scale on the $y$-axis. This is a result of the price responses by the clean technology sponsor; as the government raises its tax, the clean technology sponsor sometimes responds by increasing her price etc. Note further, the dip in the entry tax at a clean market share of about 10%. The clean technology sponsor at this point sets a very low price in order to get the market for the clean technology to take off. On the other hand, at once the clean technology has taken off, the clean sponsor starts to skim the market. Consequently, the entry tax on the dirty good has to be higher than the Pigovian tax as long as the market share of the clean good is between about 20% and about 80%. The clean technology sponsor and the government do not agree with respect to the optimal diffusion of the clean technology as can be seen from Figure 1.7.
In Figure 1.7, we have the clean market share on the y-axis, while the x-axis shows time. From the figure we see that the optimal tax induces a switch to the clean technology after some time. However, with a Pigovian tax the sales of the clean technology may stay low for up to $5 \times N$ periods. This is clearly a case of excess inertia.

Katz and Shapiro (1986) found that as long as the superior network technology had a sole owner, it would dominate the market and excess inertia would not occur. We get another result, which partly can be explained by the externality multiplier. That is, the clean technology sponsor does not internalize the environmental damages from more future dirty good adopters caused by one less clean good adopter today. Furthermore, in Katz and Shapiro (1986) the two technologies are perfect substitutes, while in our model they are horizontally differentiated. This likely exacerbates the probability of excess inertia by making it possible for the clean good provider to harvest the market.

### 1.6 Conclusion

As far as we know this is the first paper that treats optimal environmental policy when there are network effects. We find that governments should in-
tervene with a tax that depends on the market share of the clean technology independently of whether the clean technology has a sponsor or not. Maybe not so surprising, the optimal tax takes into account both the network effect, and the mark-up pricing of the potential technology sponsors. However, even if the government succeeds to internalize the network effect and deals with the monopolistic pricing of the potential technology sponsors by separate measures, the optimal tax rate should still be state dependent and depart from the Pigovian tax rate. The reason is that there exists an externality multiplier effect which makes the network effect either increase or reduce the actual environmental externality as the future choices of the consumers depend on current adoption decisions.

Our main finding is that sticking to the Pigovian tax may lead to excess inertia. This conclusion is based on our numerical simulations. In one of the simulations we have used the Norwegian EV experience as our point of departure, and in our model a Pigovian tax might not be enough to move a way from fossil cars even if it is optimal to convert the market to EVs. Note, however, that we look at an hypothetical situation in which EVs have the same cost and quality as a fossil cars. This seems not to be the case today, but could be the case in the future, see e.g. Hensley et al. (2012).

In the other simulation we show that excess inertia may also happen with a clean technology sponsor. This runs counter to much of the more general analysis of network effects.

Finally, we stress that governments ideally should have exact knowledge about the function $\lambda(n_k)$ (the network effect), before they introduce financial support to clean networks. Other simulations, not presented in this paper due to space limitations, show that excess momentum might happen with a Pigovian tax.$^{16}$

1. A Appendix - the conditional expected private utility

We want an expression for $\mathbb{E}(X|X - Z > a)$, where $X, Z : i.i.d. \sim N(\mu_X, \sigma_X)$. The distribution of $(X|X - Z > a)$ is called skew normal The expectation is derived in Birnbaum (1950). Relabel $Y \equiv X - Z$, and we have

$$\mathbb{E}(X|Y > a) = \mu_X + \rho_{X,Y} \sigma_X \frac{f_Y(a - \mu_Y)}{1 - F_Y(a - \mu_Y)}$$

$^{16}$See Greaker and Midttømme (2014) where we also include cases with excess momentum.
where \( f_Y(\cdot) \) is standard normal. Then we replace the standard normal with the \( \phi(\cdot) \)-distribution \( Y \sim N(0, \sigma_Y) \), which gives us

\[
\mathbb{E}(X|Y > a) = \mu_X + \mu_{X,Y} \sigma_X \sigma_Y \frac{\phi(a)}{1 - \Phi(a)}
\]

Now

\[
\rho_{X,Y} = \frac{\text{cov}(X, Y)}{\sigma_X \sigma_Y}
\]

which means that \( \rho_{X,Y} \cdot \sigma_X \cdot \sigma_Y = \text{cov}(X, Y) \), and we have that \( \text{cov}(X, Y) = \text{cov}(X, X - Z) = \text{var}(X) - \text{cov}(X, Z) = \text{var}(X) = \sigma_X^2 \). Thus we get

\[
\mathbb{E}(X|Y > a) = \mu_X + \sigma_X^2 \frac{\phi(a)}{1 - \Phi(a)}
\]

Our in our notation:

\[
\mathbb{E}(\zeta_c|\xi_c > x(n_c)) = \mu_\zeta + \sigma_\zeta^2 \frac{\phi(x(n_c))}{1 - \Phi(x(n_c))}
\]

Similarly

\[
\mathbb{E}(\zeta_d|\xi_c < x(n_c)) = \mu_\zeta + \sigma_\zeta^2 \frac{\phi(x(n_c))}{\Phi(x(n_c))}
\]

In our government value function, we want so sum

\[
q_c(n_c) \mathbb{E}(\zeta_c|\xi_c > x(n_c)) + [1 - q_c(n_c)] \mathbb{E}(\zeta_d|\xi_c < x(n_c))
\]

\[
= [1 - \Phi(x(n_c))] \left[ \mu_\zeta + \sigma_\zeta^2 \frac{\phi(x(n_c))}{1 - \Phi(x(n_c))} \right] + \Phi(x(n_c)) \left[ \mu_\zeta + \sigma_\zeta^2 \frac{\phi(x(n_c))}{\Phi(x(n_c))} \right]
\]

\[
= \mu_\zeta + 2\sigma_\zeta^2 \phi(x(n_c))
\]

which is what we use in (1.15).

1.B Appendix - deriving the first-order conditions

We start out by simplifying the expression for social welfare. As shown above, the conditional, expected idiosyncratic utility of the consumer for the two possible outcomes can be written \( \mu + \sigma^2 \phi(x) \). Second, note that all taxes and
prices paid are just transfers, and do not affect welfare directly under the additive welfare measure. This reduces eq. (1.5) to

\[ g(n_c) = \mu + \sigma^2 \phi(x) + q_c(n_c) \left[ n_d \lambda(n_d) + (n_c + 1)\lambda(n_c + 1) - \gamma n_d \right] + (1 - q_c(n_c)) [(n_d + 1)\lambda(n_d + 1) + n_c\lambda(n_c) - \gamma(n_d + 1)] + q_c(n_c)\delta \left[ \frac{n_c + 1}{N} g(n_c) + \frac{d}{N} g(n_c + 1) \right] + (1 - q_c(n_c))\delta \left[ \frac{n_c}{N} g(n_c - 1) + \frac{n_d + 1}{N} g(n_c) \right]. \]  

(1.15)

The government’s dynamic programming problem gives the following first-order conditions:

\[
\text{FOC } t(n_d): 0 = \sigma^2 \phi'(x(n_c)) \frac{dx(n_c)}{dt(n_d)} + \phi(x(n_c)) \frac{dx(n_c)}{dt(n_d)} \left[ \Lambda^d(n_d) - \Lambda^c(n_c) \right]
\]

\[
\text{FOC } \tau(n_d + 1): 0 = \sigma^2 \phi'(x(n_c)) \frac{dx(n_c)}{d\tau(n_d + 1)} + \phi(x(n_c)) \frac{dx(n_c)}{d\tau(n_d + 1)} \left[ \Lambda^d(n_d) - \Lambda^c(n_c) \right]
\]

where \( \Lambda^c(n_c) = n_d \lambda(n_d) + (n_c + 1)\lambda(n_c + 1) - \gamma n_d + \delta \frac{n_c + 1}{N} g(n_c) + \delta \frac{n_d}{N} g(n_c + 1) \) and \( \Lambda^d(n_d) = (n_d + 1)\lambda(n_d + 1) + n_c\lambda(n_c) - \gamma(n_d + 1) + \delta \frac{n_c}{N} g(n_c - 1) + \delta \frac{n_d + 1}{N} g(n_c) \). Thus, the functions \( \Lambda^k(n_k) \) is the current networks benefits subtracted the environmental costs, and the continuation values, given that the consumer chooses technology \( k \).

If we restrict our attention to the entry tax \( t(n_d) \), we get

\[ \left[ \sigma^2 \phi'(x(n_c)) + \phi(x(n_c)) (\Lambda^d(n_d) - \Lambda^c(n_c)) \right] \frac{dx(n_c)}{dt(n_d)} = 0 \]  

(1.16)

and when \( \phi(\cdot) \) is the normal density, we have that

\[ \phi'(x(n_c)) = - \frac{x(n_c)}{\sigma^2} \phi(x(n_c)) \]  

(1.17)

Rewriting (1.16) we obtain

\[ \left[ (\Lambda^d(n_d) - \Lambda^c(n_c)) - x(n_c) \right] \cdot \frac{dx(n_c)}{dt(n_d)} \cdot \phi(x(n_c)) = 0 \]

We must then show that \( \frac{dx(n_c)}{dt(n_d)} \neq 0 \). Using the the definition of \( x(n_c) \) (1.1) and the optimal price (response function) of the firms (1.13), we have that
\begin{equation}
\frac{dx(n_c)}{dt(n_d)} = \frac{\partial p_c(n_c)}{\partial t(n_d)} - 1 - \frac{\partial p_d(n_d)}{\partial t(n_d)} = 1 - \frac{q_c(n_c)}{\sigma^2} - \frac{q_d(n_d)}{\sigma^2} - 1 = 1 - q_c(n_c) \frac{x(n_c)}{\sigma^2} - \phi(x(n_c)) - 2\Phi(x(n_c)) - \frac{q_d(n_d)}{\sigma^2} - \phi(x(n_c)) - 1.
\end{equation}

We first look at the case with two sponsors. Assume \( \frac{dx(n_c)}{dt(n_d)} = 0 \). This implies:

\[ x(n_c) = \sigma^2 \frac{\phi(x(n_c))}{1 - 2\Phi(x(n_c))} \quad (1.19) \]

If we take the derivative of the right hand side (RHS), we get

\[ \frac{\partial \text{RHS}}{\partial x} = \frac{\phi'(x) [1 - 2\Phi(x)] + 2\phi(x)^2}{[1 - 2\Phi(x)]^2} \begin{cases} > 0 & \text{if } x < 0 \\ \text{not defined} & \text{if } x = 0 \\ > 0 & \text{if } x > 0 \end{cases} \]

and as we have that \( \text{RHS}(-\infty) = 0 \), \( \text{RHS}(x \nearrow 0) = +\infty \), while \( \text{RHS}(x \searrow 0) = -\infty \) and \( \text{RHS}(+\infty) = 0 \), we can see that (1.19) can never be satisfied.

In the competitive case we have \( \frac{\partial p_c(n_c)}{\partial t(n_d)} = \frac{\partial p_d(n_d)}{\partial t(n_d)} = 0 \). Hence, \( \frac{dx(n_c)}{dt(n_d)} = -1 \). In the case with only a clean sponsor, we have

\[ \frac{dx(n_c)}{dt(n_d)} = \frac{\partial p_c(n_c)}{\partial t(n_d)} - 1 = \frac{1 - \Phi(x(n_c)) x(n_c)}{-\phi(x(n_c)) \sigma^2} \]

which is zero only if \( x(n_c) = 0 \). This implies that the arriving consumer will choose either of the networks with equal probability for any network size and for any entry price the clean producer might set. This solution cannot be an optimum given that the dirty network pollutes. Hence, we conclude that the first-order condition requires:

\[ x(n_c) = \Lambda^d(n_d) - \Lambda^c(n_c). \]
We can now turn to the second-order condition. We differentiate (1.16) to get

\[
\sigma^2 \frac{\phi''(x)}{dx} \left( \frac{dx}{dt} \right)^2 + \sigma^2 \phi'(x) \frac{d^2 x}{dt^2} \\
+ \left( \Lambda^d(n_d) - \Lambda^c(n_c) \right) \cdot \left[ \phi'(x) \left( \frac{dx}{dt} \right)^2 + \phi(x) \frac{d^2 x}{dt^2} \right] \\
= \sigma^2 \left( \frac{dx}{dt} \right)^2 \left( -\phi(x) - \frac{x}{\sigma^2} \phi'(x) \right) - x \phi(x) \frac{d^2 x}{dt^2} \\
+ \left( \Lambda^d(n_d) - \Lambda^c(n_c) \right) \cdot \left[ -\frac{x}{\sigma^2} \phi'(x) \left( \frac{dx}{dt} \right)^2 + \phi(x) \frac{d^2 x}{dt^2} \right] \\
= \sigma^2 \left( \frac{dx}{dt} \right)^2 \left( -\phi(x) + \frac{\sigma^2}{\sigma^2} \phi'(x) \left( \frac{dx}{dt} \right) + \left( \Lambda^d(n_d) - \Lambda^c(n_c) \right) \phi(x) \frac{dx}{dt} \right) \\
- \phi(x) \left( \frac{dx}{dt} \right)^2 + \phi(x) \frac{d^2 x}{dt^2} \left( \Lambda^d(n_d) - \Lambda^c(n_c) - x \right) = 0 \\
= \phi(x) \left( \frac{dx}{dt} \right)^2 < 0
\]

We have a globally defined function, everywhere differentiable in \( x \) with only one stationary point, and this point is a local max. Hence it is also a global max. The problem might be that \( g(\cdot) \) is not continuous in \( t \) if firms respond very non-linearly to taxes, even if it is continuous in \( x \). Luckily, we have that \( x(n_c) \) is differentiable in \( t \), hence it is also continuous.

1.C Appendix - the Pigovian entry tax

In the numerical simulations, we use the Pigovian entry tax rate as a benchmark. This rate can be found by looking at the outcome of the model when all network effects are absent. The choice probabilities of the consumers will then only depend on the current prices and taxes, and not on the future expected sizes of the networks. Assume \( p_c = p_d = 0 \) for all states (as in the main part of the paper). For \( x(n_c) \) we then have:

\[
x(n_c) = -t
\]

(1.20)

When the firm prices are constant and the environmental damage is linear, we can ‘guess and verify’ a linear government value function:

\[
g(n_c) = \frac{1}{1 - \delta} \left[ \sigma^2 \phi(x) - N \cdot \gamma + q_c(x) \frac{\gamma}{1 - \delta \frac{N - 1}{N}} \right] + \frac{\gamma}{1 - \delta \frac{N - 1}{N}} n_c.
\]
Solving for the optimal $x(n_c)$, we find that the government implements the following:

$$x^*(n_c) = \frac{-\gamma}{1 - \delta \frac{N - 1}{N}}, \quad \forall n_c$$

where the right-hand side is the present value of expected environmental damages of joining the dirty network. We then have for the optimal entry tax:

$$t = -x^*(n_c) = \frac{\gamma}{1 - \delta \frac{N - 1}{N}},$$

which we use as our Pigovian tax rate. Note that this tax rate does not adjust for market power in the clean technology sponsor case.

1.D Appendix - the Norwegian EV case

There are approximately 2 million cars in Norway with a turnover of 0.1 million cars per year. We assume 200 "consumers" in the model, thus, each consumer is 0.01 million cars. The Poisson rate is then 10, and each period is 1/10th of a year. We apply a common discount factor of 0.995.

Norway has set a national target for greenhouse gas emissions. In order to reach this target, the government estimates that all non-EU ETS sectors, such as the transport sector, must be subject to a carbon tax of $533 per tones. For an average car in Norway, this amounts to approximately $10000 at the time of purchase. This is our Pigovian tax.

EVs in Norway do not pay sales taxes. On average this constitutes a subsidy to EVs at the time of purchase of $10000. Fossil fuel in Norway is heavily taxed, and together the road use tax and carbon taxes approximately constitutes $10000 at the time of purchase (average fuel consumption and yearly driving range). For more on these numbers see Holtsmark and Skonhoft (2014).

EVs also pay a special electricity tax when charging, which amounts to $500 at the time of purchase, however, they also have other benefits like no road tolls etc. Thus, we have set the current entry tax for fossil cars at $20000, which is twice the Pigovian rate. Note that in our model a subsidy to EVs is equivalent with a tax on fossil cars.

For $\lambda$-function we have $\psi = 0.62$ for both types of cars, while we have $\alpha = 2.4$ and $\beta = 11$ for fossil cars and $\alpha = -1.5$ and $\beta = 4$ for EVs. Finally, the variance of $\xi_c$ is set to 10.
References


Chapter 2

Strategic delegation to negotiations over public goods†

Katinka Holtsmark and Kristoffer Midttømme

Abstract

We study strategic delegation of decision-making power in a game where countries contribute to a public good. We consider both cooperative and non-cooperative behavior and show that the outcome of the delegation process strongly depends on whether contribution levels are strategic complements or substitutes. Countries delegate to delegates who care less about the public good, the higher the degree of strategic substitutability. Due to this strategic delegation, the outcome under strategic complementarity Pareto-dominates the equilibrium under self-representation if the contribution levels are determined non-cooperatively. If contributions are determined by Nash bargaining, there is always a loss from strategic delegation. Given some restrictions on the model, the resulting welfare loss is smaller under strategic complementarity than it is under strategic substitutability. Our findings may contribute to the explanation of why some international public-good problems seem much more difficult to solve than others. The results also connect the findings of several papers in the economics literature.

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2.1 Introduction

2.1.1 Strategic delegation

It is well-known how global public goods, such as a good climate, should be managed. However, efficient management requires cooperation between countries. Today, strong incentives to free ride on other countries’ efforts seem to overpower most attempts at cooperatively tackling the climate problem. On the other hand, there are examples of other international public-good problems that have been solved through international cooperation. Important questions are then; what characteristics of a public-good problem determine whether the international community is able to cooperate? And through which channels do these characteristics affect the final outcome? We argue in this paper that specific strategic properties of such problems are crucial in determining the success of international negotiations, by affecting who the different countries will send to represent them in negotiations. Specifically, we study how the degree of strategic substitutability or strategic complementarity of the countries’ contributions affect the countries’ incentives to strategically delegate their decision-making power to delegates who differ from themselves.

The characteristics of the delegates representing countries in negotiations over international agreements, will affect the outcome of the negotiations. By choosing a specific delegate, or more precisely, choosing a delegate with specific preferences, a country can to some extent commit to a stance in the negotiations. Hence, strategic considerations will be taken in the delegation process. It is these strategic considerations, and the incentives countries face in the delegation process, that we study in this paper. We show that these incentives are strongly dependent on whether contributions to the public good are strategic complements or strategic substitutes. Our results may contribute to the explanation of why some international public-good problems foster cooperation between countries, while others — such as the climate problem — do not.

We model a group of countries who face a public-good problem. The countries meet for some form of negotiations over the levels of contribution to this good, and each country is represented by a delegate in these negotiations. The delegates have their own preferences that might possibly differ from the preferences of their country in general. More specifically, each country may choose to send a delegate who has stronger or weaker preferences towards the public good, than the country itself. We say that the delegate can be more or less committed to the public good. In turn, the commitment level of the chosen delegates will be important for the final outcome of the negotiations.
Therefore, the strategic properties of the public-good problem affect this final outcome, since it affects the incentives in the delegation process.

The main contribution of this paper is to characterize how strategic properties of the public-good problem, through strategic delegation, translates into better or worse outcomes. We do this for different negotiation protocols.

First, we define whether a problem is characterized by strategic substitutability (STS), neutrality (STN) or complementarity (STC). This depends on whether the marginal gain from increased contributions in one country falls, remains the same, or increases as contributions from other countries increase.

Second, we model the outcome for the public good for two different negotiation protocols. These are meant to represent two extremes regarding the degree of cooperation that can be achieved between the delegates. In the first case, we consider a completely non-cooperative setting, where contracting on contribution levels is impossible. The delegate from each country will in this case set his own country’s contribution in order to maximize his own utility. In the second case, we consider the opposite extreme, where there is full cooperation between the delegates in determining contribution levels, and they sign and enforce binding contracts on these contributions. This protocol is modeled as a Nash Bargaining process.

Finally, we show that the strategic properties of the public-good problem are decisive for the outcome, since they affect the delegation process independently of the negotiation protocol. In particular, we show that strategic complementarity will in most cases lead to a better outcome than strategic substitutability as it provides each country with incentives to send a delegate who has stronger preferences for preservation of the public good.

The intuition behind our results depends on the degree of cooperation that is assumed in the negotiations. In the non-cooperative case, delegation is used as a commitment device by the countries. Sending a delegate who is committed to the public good serves as a commitment to higher contribution levels, and the opposite commitment is made by sending a delegate who is less committed. With strategic complementarity, commitment to a high level of contribution is beneficial, as other countries will respond by contributing more, and all countries will choose delegates that have stronger preferences for the public good. This way, the delegation process contributes to solving the problem. However, if the problem exhibits strategic substitutability the opposite will be true, and the delegation process will amplify to the tragedy of the commons.

In the case of Nash bargaining, a delegate who is less committed to the public good stands to gain less from reaching agreement, and is thus compensated for his contribution by a higher transfer from the other delegates. One
delegate being less committed also means that the aggregate contribution level will be lower. The magnitude of the change in aggregate contributions for a given change in a delegate’s preferences, depends on the strategic properties of the problem. Given some restrictions on the model, we show that the higher the degree of strategic substitutability, the stronger the incentives for each country to send a delegate who is less committed to the public good.

Different international public-good problems vary with regards to their strategic properties. The climate problem is generally believed to exhibit strategic substitutability, as the current state of science tells us that damages from further greenhouse gas (GHG) emissions are convex in the stock of GHGs currently in the atmosphere. This means that higher emissions of GHGs in one country — which increases the atmospheric stock — will increase the marginal damage from emissions in other countries (Tol, 2009). Our results hence contribute to the explanation of why there is so little progress in negotiations between countries over emission reductions that are necessary for solving the climate problem. Finally, our results suggest that if we could manage to change the problem into one characterized by strategic complementarity, the problem could be easier to solve. In Section 2.5, we discuss in depth how our findings can also shed light on the mechanisms in play for several other international public-good problems.

2.1.2 Related literature

There is an extensive literature discussing delegation and bargaining in various settings. The groundbreaking work was done by Schelling (1956, 1960), who showed that an advantage in bargaining situations can be achieved by credibly committing to a specific stand. The fact that delegation of decision-making power to a third party can constitute such a commitment device was formalized for instance by Jones (1989). In our paper, we consider negotiations over the levels of contribution to a public good, and how to share the surplus that is created. Many of these issues were anticipated informally by Johansen (1977). When considering delegation to a Nash bargaining process, our results relate closely to those of Jones (1989): the gains a player can reap when delegating to a delegate with weaker preferences for preservation of the public good, are due to the increased bargaining power this gives rise to.

We contribute to the general literature on strategic delegation by underlining the importance of such strategic considerations in a public-good setting, and by showing that different public-good problems provide different incentives to strategically delegate the decision-making power.
Segendorff (1998) studies two delegators who delegate their seats in a Nash bargaining protocol over contributions to a public good. He assumes that contributions are strategically neutral. He compares the case where the delegates themselves determine the BAU contribution levels (strong delegation) to the case where the delegators determine the BAU contribution levels (weak delegation). The main finding, as in our paper, is that self-representation never takes place in equilibrium under weak delegation, and thus at least one player always suffers a loss from strategic delegation. As we show in this paper, the size of the welfare loss from delegation depends on the strategic properties of the public-good problem.

In addition to this relatively general literature, the effects of strategic delegation have been studied in various specific fields within economics. In the literature on environmental economics, Eckert (2003) studies two countries who bargain over an environmental agreement. Her key question is whether countries should let their individual regions represent themselves in international climate negotiations, or whether the country should bargain as one collective unit. Emissions are assumed to be strategic substitutes. Her main result is that a country can benefit from delegating its seat at the bargaining table to one of its regions, when that region is responsible for most of the domestic pollution.

Buchholz et al. (2005) also study delegation of bargaining power in climate negotiations when emissions are strategic substitutes. They find that delegation introduces a selection effect, such that the delegates of the countries that choose to participate in the climate negotiations in equilibrium are less green than the delegates of the countries who choose to act alone outside of the bargained protocol. We believe that our results may contribute to further understanding of these results.

Furthermore, Roelfsema (2007) studies environmental tax competition between two countries when emissions stem from firms that engage in Cournot competition on the international market. Roelfsema finds that the delegator will delegate to a less green delegate if he himself cares little about the environment, while if the degree of spillovers between countries is sufficiently small and the delegator cares enough about the environment, he will delegate to a greener delegate. The first case corresponds to a situation in which the tax rates are strategic substitutes while the latter corresponds to the case in which they are strategic complements. Hattori (2010) extends the model of Roelfsema. He finds that when firms compete in prices, the tax rates of the two countries are strategic complements. The same is also true if the firms set emission standards instead of tax rates. In both cases he demonstrates that
this leads to equilibrium delegation to delegates that are greener than their respective delegators.

While the climate is a pure public good, there are also other kinds of problems facing the international community, where the same mechanisms apply. Persson and Tabellini (1992) study how capital mobility affects tax competition between two countries. In their set up, capital taxes are strategic complements, as a higher tax rate in one country reduces the marginal distortion from a higher tax rate in the other country. And in accordance with our results, they find that voters tend to elect politicians who favor higher tax rates than themselves, in order to soften the tax competition.

Somewhat similarly, Dolado et al. (1994) study inflationary spillovers between countries in a game of monetary policy. When one country increases its monetary base, they suffer from higher inflation but enjoy increased economic activity. In addition, there is a spillover to the neighboring country. The authors show that countries have an incentive to delegate to central bankers who are more averse to inflation than the median voter. We contribute to these different strands of literature by emphasizing the important role that strategic substitutability and complementarity play, and by demonstrating that these strategic properties can be decisive for the results.

In the literature discussed so far, the policies are either set non-cooperatively by the delegates, or determined in a Nash bargaining protocol with side transfers. There is also a strand of literature within political economy that studies issues of centralization and federalism, where understanding the delegation process is important. The main question posed in this literature is whether public-good spending in a federation should be determined at the federal or at the state level. Pioneered by Besley and Coate (2003), this literature investigates issues like majority requirements less than unanimity (Harstad, 2010), and the role of side payments (Harstad, 2008). When the majority requirement is less than unanimity, the winning majority coalition can expropriate the surplus of the minority. States then trade off a strengthened bargaining position (by delegating to delegates who care less about public goods) to an increased chance of participating in the minimum winning coalition (by delegating to delegates who care more about public goods). When the majority requirement is small, the latter effect dominates, while the former dominates when majority close to unanimity is required. When side payments are not part of the bargained protocol, the incentives to strategically delegate in order to strengthen one’s bargaining position are reduced. An example of this latter effect can be seen in Siqueira (2003) who studies two countries facing a climate problem. He finds that when emissions are determined in order to maximize the joint welfare of the delegates but there are no side transfers,
2.2. THE MODEL SETUP

then the outcome under strategic delegation is equal to the outcome under self-representation.

We attempt to provide a general model of delegation and public-goods provision, clearly stating the taxonomy of strategic substitutes, complements and strategically neutral actions. As this short review of the literature indicates, most papers assume that the public-good contributions fit within one of these categories; they seldom study more than one case. We solve for all three cases, in order to demonstrate that the strategic properties are important for the outcome. Furthermore, we consider not only non-cooperative policies, but attempt to compare the outcome also under a Nash bargaining protocol. Our results connect the findings of several of these papers, and contribute to the understanding of the mechanisms that are in play.

2.2 The model setup

We study an international public-good problem where \( N \) countries can make costly contributions to a public good. For each country \( i \), there will be a delegator, \( M_i \), who will delegate the country’s decision-making authority to a delegate, \( D_i \). We do not explicitly model the political process behind this delegation process, but the delegator can for example be thought of as a median voter or an elected parliament or government. We consider \( M_i \) as representing the preferences of the country as a whole, and country \( i \)’s welfare is measured according to the preferences of \( M_i \). The chosen delegate may, however, have preferences that differ from those of \( M_i \). Two real-world features of many international negotiation processes are not explicitly modeled here; ex-post ratification of the agreement and the fact that a delegate may be replaced in the midst of the negotiations. As both of these features would limit the extent to which countries are able to commit by strategically choosing a delegate, they could both weaken our results to some extent.

Each country \( i \) contributes an amount \( g_i \) to the public good. The cost of this contribution is increasing and convex in \( g_i \), and given by \( c(g_i) \), with \( c'(g_i) > 0 \) and \( c''(g_i) > 0 \). Each country also derives benefits from the aggregate contributions, and we allow for the possibility of imperfect spillovers between countries in parts of the paper. The benefits of country \( i \) are given by the function \( b(g_i + \alpha \sum_{j \neq i} g_j; \theta_i^M) \), where \( b_1 \equiv \partial b / \partial \tilde{G}_i > 0 \) when \( \tilde{G}_i \equiv g_i + \alpha \sum_{j \neq i} g_j \). \( \alpha \in [0,1] \) and \( \theta_i \) is a type-parameter in the benefit function determining the marginal utility from the public good. We define this parameter such that \( b_{12} > 0 \), meaning that a higher \( \theta_i \) gives a higher marginal utility of the public good.
The preferences of the delegator $M_i$ are then given by

$$M_i : U^M_i = b \left( g_i + \alpha \sum_{j \neq i} g_j; \theta^M_i \right) - c(g_i),$$

while the delegator’s preferences are given by

$$D_i : U^D_i = b \left( g_i + \alpha \sum_{j \neq i} g_j; \theta^D_i \right) - c(g_i).$$

The potential difference between the delegator and the delegate is only along one dimension, represented by the parameter $\theta_i$. $\theta^D_i$ may thus differ from $\theta^M_i$, meaning that the delegate may value contributions to the public good more ($\theta^D_i > \theta^M_i$) or less ($\theta^D_i < \theta^M_i$) than the delegator. A delegate — or a delegator — with a high $\theta_i$ is considered from here on as a more committed player. The $\theta^M_i$ is exogenous for all countries while $\theta^D_i$ will be chosen by $M_i$ in the delegation stage. That is, choosing the delegate is equivalent to choosing $\theta^D_i$.

We study two different protocols for determining global contributions:

- A non-cooperative protocol, in which the delegators delegate their decision-making power to a delegate (for instance an environmental protection agency), and these delegates then determine national contributions non-cooperatively, taking the contributions of the other countries as given. This determines a global contribution profile as a Nash equilibrium between the delegates, given by $NC_g = (NC_1, \ldots, NC_N)$.

- A Nash bargaining protocol, in which the delegators delegate their seat at the bargaining table to a delegate. The delegates negotiate over the global contribution profile $g$, and a set of transfers $t = (t_1, t_2, \ldots, t_N)$. In case an agreement is reached, we label the resulting contribution profile $NB_g = (NB_1, \ldots, NB_N)$. If the delegates fail to reach agreement, the delegators will take over and contributions will be determined non-cooperatively in a Nash equilibrium between the delegators. The equilibrium contributions in this case are labelled $BAU_g = (BAU_1, \ldots, BAU_N)$.

In both cases, the game we study has two stages:

- Stage 1: All the delegators $M_i$ simultaneously choose the type of their delegate $\theta^D_i$. There is full information across all delegates and delegators regarding the types that are chosen.

- Stage 2: The contribution profile $g$ will be determined, either according to the NC protocol, or the NB protocol (taking the BAU contributions as given).
We study Subgame-perfect equilibria (SPE) of this game.

We are interested in investigating how strategic properties of the public-good problem affect the outcome, by affecting the incentives countries face in the delegation process. More specifically, we are interested in whether contributions are strategic substitutes or complements, or are strategically neutral. This strategic property of the public-good problem is in this framework determined by the sign of the cross-derivative $\partial^2 U_i^M / \partial g_i \partial g_j = \alpha b_{11}$, as outlined in the following lemma:

**Lemma 2.1.** Individual contributions, $g_i$ and $g_j$, $j \neq i$, are strategic substitutes (STS) if $b_{11} < 0$, strategically neutral (STN) if $b_{11} = 0$, and strategic complements (STC) if $b_{11} > 0$.

**Proof.** This follows directly from the definition of the strategic property. □

The strategic properties of the problem thus depend on the benefit function: the marginal utility of the public good may increase or decrease in the contribution of others. In particular, contributions are STS (STC) if increased contributions from one country decreases (increases) other countries’ marginal utility from contributions. Contributions are STN if the incentive to contribute remains unaffected as others’ contributions change.

For each of these three cases, we determine how the aggregate welfare — as evaluated by the delegators — is affected by the countries’ ability to strategically delegate their public-good provision policy. We show that the welfare loss from delegation is higher for STS than for STC, due to the way these strategic properties affect the delegation process. Under some restrictions, this result holds under both protocols. The reason is that the equilibrium delegates are more committed under STC than under STS, leading to a better outcome in the former case. Indeed, under the non-cooperative protocol, delegation leads to a welfare gain when there is STC (compared to a non-cooperative outcome between the delegators), because the delegators face incentives to choose delegates who are more committed than they are themselves.

### 2.2.1 The benchmark cases

We establish two benchmarks: the first-best outcome where aggregate welfare is maximized given the delegators preferences $\Theta^M \equiv (\theta_1^M, \ldots, \theta_N^M)$; and the BAU outcome, which is the Nash equilibrium in a simultaneous contribution game played by the delegators. The first-best contribution levels must solve the
following problem:

$$\max_b \sum_j b \left( g_j + \sum_{h\neq j} \theta_h^M \right) - c(g_j)$$

The N first-order conditions for this problem are given by

$$0 = b_i \left( \tilde{G}_i; \theta_i^M \right) - c'(g_i) + \sum_{j \neq i} \alpha b_1 \left( \tilde{G}_j; \theta_j^M \right) \quad \forall i. \quad \text{(FB foc)}$$

These define the first best contribution levels $FB_g(\Theta^M)$, and the maximized aggregate welfare

$$FBW = b \left( FB_g(\Theta^M) + \sum_{h\neq j} FB_g(\Theta^M); \theta_j^M \right) - c \left( FB_g(\Theta^M) \right). \quad (2.1)$$

The second-order conditions hold as long as the Hessian matrix of $\sum_j U_j^M$ is negative definite. We consider only cases where this condition is satisfied. The condition is a trivial assumption when the problem exhibits STS and STN. However, the second order conditions restrict the degree of STC we allow for in our model. See Appendix 2.A.1 for implications of this assumption.

In the BAU the delegator in country $i$ solves the following problem:

$$\max_{g_i} \left\{ b \left( g_i + \sum_{j \neq i} g_j; \theta_i^M \right) - c(g_i) \right\},$$

giving the first-order condition

$$0 = b_1 \left( BAU g_i + \sum_{j \neq i} g_j; \theta_i^M \right) - c' \left( BAU g_i \right) \quad \forall i. \quad \text{(BAU foc)}$$

In equilibrium, the BAU contribution levels are hence determined by $\Theta^M; BAU g(\Theta^M)$.

Under the Nash bargaining protocol, the BAU constitutes the outside option for the delegates. Hence, the valuation of the BAU allocation by each delegate will be important. The utility of delegate $D_i$ under BAU is given by:

$$BAU V_i^D = b \left( BAU \tilde{G}_i(\Theta^M); \theta_i^D \right) - c \left( BAU g_i(\Theta) \right),$$

while $BAU W^D = \sum_j BAU V_i^D$ defines the sum of the delegates’ utilities.

The individual BAU utility will affect each delegate’s bargaining power, and therefore the transfer assigned to country $i$ in the NB protocol.
2.3 Delegation and non-cooperative behavior

This section studies the non-cooperative protocol, in which the delegators first delegate to their respective delegates and then these delegates subsequently determine national contributions non-cooperatively. The contributions are determined simultaneously, after each delegate has observed the types of all the other delegates. First, we determine the Nash equilibrium in the second stage, given a profile of delegate types, \( \Theta^D \equiv (\theta^D_1, \ldots, \theta^D_N) \). Then, we solve the problem of the delegator in each country when he anticipates the last-stage equilibrium outcome.

The delegate in country \( i \) solves the following problem:

\[
\max_{g_i} \left\{ b \left( g_i + \alpha \sum_{j \neq i} g_j; \theta^D_i \right) - c \left( g_i \right) \right\}.
\]

The first-order condition is given by:

\[
b_1 \left( \frac{NC g_i}{NC g_i + \alpha \sum_{j \neq i} g_j; \theta^D_i} \right) = c' \left( \frac{NC g_i}{NC g_i + \alpha \sum_{j \neq i} g_j; \theta^D_i} \right), \quad \forall i.
\]

From these \( N \) first-order conditions, we can derive the reaction functions, \( NC g_i(g_{-i}; \theta^D_i) \forall i \), and the equilibrium contributions, \( NC G(\Theta^D) \).

The second-order condition is that \( b_{11} - c'' < 0 \), which is satisfied whenever the second-order condition of the social welfare maximization problem is satisfied. We can now determine the slope of the reaction functions, and see how the equilibrium contributions depend on the type profile, \( \Theta^D \).

Define the aggregate contributions; \( NC G \equiv \sum_j NC g_j \).

Lemma 2.2.

- The non-cooperative reaction functions of the delegates satisfy

\[
NC g_i(g_{-i}; \theta^D_i) = \frac{\alpha b_{11}}{c'' - b_{11}} \begin{cases} < 0 & \text{if } STS \Leftrightarrow b_{11} < 0, \\ = 0 & \text{if } STN \Leftrightarrow b_{11} = 0, \\ > 0 & \text{if } STC \Leftrightarrow b_{11} > 0. \end{cases}
\]
• In response to an increase in the type of delegate \( i \), the aggregate equilibrium contributions go up, and satisfy

\[
\begin{align*}
\text{STS} : \quad & \frac{\partial NC_g_i}{\partial \theta_i} > \frac{\partial NC_G}{\partial \theta_i} > 0, \quad \forall i, \\
\text{STN} : \quad & \frac{\partial NC_g_i}{\partial \theta_i} = \frac{\partial NC_G}{\partial \theta_i} > 0, \quad \forall i, \\
\text{STC} : \quad & \frac{\partial NC_G}{\partial \theta_i} > \frac{\partial NC_g_i}{\partial \theta_i} > 0, \quad \forall i.
\end{align*}
\]

**Proof.** See Appendix 2.A.2.

In all three cases, a more committed delegate (increased \( \theta^D_i \)) will contribute more, because his utility from contributions is higher. Under STS, the other delegates will react to this by reducing their contributions, since their marginal utility of contributing goes down. An increase in \( \theta^D_i \) therefore leads to an increase in total contributions, but the change is smaller than the initial increase by the more committed delegate. Under STC, other delegates will respond to the (expected) initial impulse by further increasing their contributions. Aggregate contributions then increase more than the initial increase by the more committed delegate. Finally, under STN, each delegate has a dominant strategy, and the other delegates will not respond to the (expected) increase in the contribution from the more committed delegate.

We can now turn to stage 1. In this stage, the delegator in each country chooses his optimal delegate, taking the choice of delegates in other countries as given, and anticipating the resulting stage 2-equilibrium (SPE). The delegator in country \( i \) solves:

\[
\max_{\theta^D_i} \left\{ b \left( NC \tilde{G}_i(\Theta^D); \theta^M_i \right) - c \left( NC g_i(\Theta^D) \right) \right\}.
\]

The delegator’s first-order condition is given by:

\[
0 = \frac{d NC g_i(\Theta^D)}{d \theta^D_i} \cdot \left[ b_1 \left( NC \tilde{G}_i(\Theta^D); \theta^M_i \right) - c' \left( NC g_i(\Theta^D) \right) \right] + \sum_{j \neq i} \frac{d NC g_j(\Theta^D)}{d \theta^D_i} \cdot ab_1 \left( NC \tilde{G}_i(\Theta^D); \theta^M_i \right).
\]

(2.3)

When choosing his delegate, the delegator takes into account the second-stage behavior of his own delegate and the other delegates’ equilibrium reactions to his delegate’s type. The \( N \) first-order conditions determine the equilibrium type profile of the delegates: \( NC \Theta^D = (NC \theta_1, \ldots, NC \theta_N) \). The following proposition summarizes the equilibrium:
Proposition 2.1.

- Under STN ($b_{11} = 0$), the equilibrium features self-representation, $NC\theta_D^i = \theta_M^i$, and there is no welfare loss resulting from strategic delegation when contributions are determined non-cooperatively, $W^{NCg;\Theta_M} = W^{BAUg;\Theta_M}$.

- Under STS ($b_{11} < 0$), the equilibrium features representation by delegates who are less committed than their corresponding delegators, $NC\theta_D^i < \theta_M^i$, and there is a welfare loss resulting from strategic delegation when contributions are determined non-cooperatively, $W^{NCg;\Theta_M} < W^{BAUg;\Theta_M}$.

- Under STC ($b_{11} > 0$), the equilibrium features representation by delegates who are more committed than their corresponding delegators, $NC\theta_D^i > \theta_M^i$, and there is a welfare gain resulting from strategic delegation when contributions are determined non-cooperatively, $W^{NCg;\Theta_M} > W^{BAUg;\Theta_M}$.

Proof. See Appendix 2.A.3

The fact that commitment to specific policies or types of policies can be important in strategic interaction, was understood already by Schelling (1956). Delegation of decision-making power constitutes such a commitment device. Anticipating the stage 2-equilibrium, the delegator in each country commits to a certain policy stand by delegating his decision-making power to a delegate with certain characteristics. Here, we underline how certain characteristics of the public-good problem have an important effect on the final outcome when such a commitment device is available.

The incentives faced by the delegators depend on the extent to which other delegates’ decisions can be influenced. Under STN, each delegate has a dominant strategy of contributing $NCg_i(\theta_D^i)$, and cannot be influenced by a change in contributions from other countries. Thus, there is no reason to delegate strategically, and the equilibrium features self-representation.

Under STS, other countries will respond by contributing less if one country contributes more. By delegating to a less committed delegate, a country commits to lower national contributions, and this commitment is observed by all delegates before they determine their contributions. The cost of being represented by someone different from $M_i$ himself is smaller than the gain from this opportunity to free ride on increased contributions from others. As pointed out by Buchholz et al. (2005) and Graziosi (2009), the possibility to delegate thus gives rise to an additional free-rider problem when there is STS: since
each country has an incentive to delegate to a less committed delegate, aggregate contributions go down, and every country would be better off under forced self-representation.

Finally, under STC, all the other delegates will respond by contributing more if one delegate is more committed. Each delegator therefore faces an incentive to delegate to a more committed delegate in order to commit to a generous contribution. In equilibrium, all countries are represented by more committed delegates and the outcome is increased total contributions and increased welfare compared to the BAU.

In Section 2.5 we discuss several findings in the literature and how our findings can be made relevant to several public-good problems. We argue that the non-cooperative protocol resembles what we observe when countries for instance set their capital tax rates, their monetary policies and policies affecting several environmental problems. For other problems it is more natural to imagine that countries cooperate to some extent. In the following section we therefore analyze a situation where the delegates bargain cooperatively over their contribution levels.

\section{The Nash bargaining process}

Under the Nash bargaining protocol each delegator elects a delegate to represent the country at the bargaining table. We assume that the axioms of Nash bargaining are satisfied, such that if agreement is reached, the delegates agree to implement the contribution profile $^{NB}g$, maximizing the aggregate utility over all delegates, given the types of the delegates present. We assume identical bargaining weights. As transfers are allowed and bargaining weights are equal, the transfers are determined such that every delegate has the same gain from reaching agreement relative to the BAU outcome. Formally, the NB protocol solves

$$^{NB}g \leftarrow \arg \max \{g_i\}_{i=1}^{N} \prod_{i=1}^{N} \left( b \left( g_i + \alpha \sum_{j \neq i} g_j; \theta_{ij}^D \right) - c(g_i) \right).$$

Throughout this section, we further simplify the framework a little bit, by considering only problems concerning pure public goods. We do this in order to remain focused on the main point of this paper; the effect of strategic characteristics of the public-good problem on the negotiated outcome. This means that we disregard situations in which there are imperfect spillovers from contributions across countries, or more specifically, we set $\alpha = 1$, so that the
utility from contributions is now given by $b\left(\sum_j g_j; \theta^M_i\right)$ and $b\left(\sum_j g_j; \theta^D_i\right)$ for the delegator/country, and for the delegate, respectively.

Again, we start by solving for the contribution profile resulting from the Nash bargaining process in the second stage. As the NB profile by assumption is efficient relative to the delegate’s preferences, it solves:

$$\max_{\mathbf{g}} \sum_k \left\{b \left(\sum_j g_j; \theta^K_k\right) - c(g_k) \right\}$$

**foc** $g_i$: $0 = \sum_k b_1 \left(\sum_j NB g_j; \theta^K_k\right) - c' \left(NB g_i\right), \forall i. \quad (2.4)$

These $N$ first-order conditions determine the contributions $NB g(\Theta^D)$, and the resulting utility gross of transfers for each delegate:

$$NB V^D_i = b \left(\sum_j NB g_j(\Theta^D); \theta^K_i\right) - c' \left(NB g_i(\Theta^D)\right)$$

$$NB W^D = \sum_j V^D_i.$$ 

Next, the transfers will be determined such that each delegate experiences the same gain from reaching agreement,

$$NB V^D_i - BAU V^D_i + t_i = \frac{1}{N} (NB W^D - BAU W^D), \forall i. \quad (2.5)$$

The resulting transfer profile $t(\Theta^D) = (t_1(\Theta^D), \ldots, t_N(\Theta^D))$ of course depends on the type profile of the delegates. The transfer to country $i$ enters the utility of both $D_i$ and $M_i$ linearly.

**Lemma 2.3.** Under the Nash bargaining protocol, the contributions go up in all countries when one delegate becomes more committed:

$$\frac{\partial NB g_j}{\partial \theta^K_i} > 0, \forall i, j.$$

**Proof.** See Appendix 2.A.4

When one delegate becomes more committed, total contributions go up, because the Nash bargaining protocol maximizes the aggregate welfare of the delegates. Increased utility from the public good then leads to increased contributions. The cost efficient way to increase total contributions is to smooth
out the increase in contributions across all countries, such that marginal contribution costs are equalized.

Each delegator in stage 1 anticipates this Nash bargaining outcome, and solves the following problem:

\[
\max_{\theta_i^D} \left\{ b \left( \sum_j N^B g_j(\Theta^D); \theta_i^M \right) - c \left( N^B g_i(\Theta^D) + t_i(\Theta^D) \right) \right\}.
\]

Note that the delegator evaluates the equilibrium contributions according to his own type, \( \theta_i^M \), whereas the bargaining protocol operates relative to the delegates’ types \( \Theta^D \). The problem gives the following first-order condition:

\[
0 = \frac{d N^B g_i(\Theta^D)}{d\theta_i^D} (b_1 - c') + b_1 \sum_{j \neq i} \frac{d N^B g_j(\Theta^D)}{d\theta_i^D} + \frac{dt_i(\Theta^D)}{d\theta_i^D}.
\]

The \( N \) first-order conditions determine the equilibrium type profile of the delegates \( N^B \Theta^D = (N^B \theta_1, \ldots, N^B \theta_N) \) under the Nash bargaining protocol.

Under Nash bargaining, the cost and benefit from being represented by a delegate with a type different from \( M_i \) are slightly different from those arising when contributions are determined non-cooperatively. Under Nash bargaining, a change in \( \theta_i^D \) will change the aggregate contribution level, but only a small part of the change is borne by country \( i \). The reason is that the efficient way to alter contributions is to change the contribution level of all countries by a little amount. However, a change in \( \theta_i^D \) also changes how valuable the bargained contribution levels are to delegate \( i \). This leads to a change in the transfer to delegate \( i \) in addition to the change in the bargained contribution levels. The delegator in country \( i \) thus has to weigh the costs and benefits of changes in both the aggregate contributions and in the transfer he receives against each other. As in the non-cooperative case, the size of these costs and benefits will depend on the strategic properties of the public-good problem.

**Proposition 2.2.** Under Nash bargaining, the equilibrium always features delegation to less committed delegates, \( N^B \theta_i^D < \theta_i^M \), \( \forall i \). Hence, there is always a welfare loss from strategic delegation when contributions are determined by Nash bargaining: \( W( N^B g; \Theta^M) < FBW \).

**Proof.** See Appendix 2.A.5. \( \square \)

The key to understanding the intuition behind the following results is to understand the different costs and benefits, and how the transfer compensates the delegate. When the delegator delegates to a less committed type, the implemented contribution profile \( N^B g(\Theta^D) \) changes, and the delegate’s valuation
of that profile changes. The transfer is designed to compensate the delegate for both of these effects. When the delegate and the delegator agree (\(|\theta^D_i - \theta^M_i|\) small), they agree on the utility cost of the first effect, while only the delegate experiences the latter effect. As the transfer compensates the delegate for both effects, parts of the transfer is a pure gain to the delegator. The delegator will thus always delegate to a less committed type. When the delegate is further away from the delegator (\(|\theta^D_i - \theta^M_i|\) large), it is more and more costly to the delegator that aggregate contributions are lower. This latter effect will eventually dominate, thus there is an interior optimal degree of misrepresentation.

**Proposition 2.3.** There is a greater welfare loss from strategic delegation under STS than under STN, and a smaller welfare loss under STC than under STN,

\[
\left( FBW - W\left( NBg; \Theta^M \right) \right)|_{STS} > \left( FBW - W\left( NBg; \Theta^M \right) \right)|_{STN} > \left( FBW - W\left( NBg; \Theta^M \right) \right)|_{STC},
\]

as long as the difference \(NBG - BAUG\) does not change too much between STS and STC.

**Proof.** See Appendix 2.A.6. \(\square\)

The marginal cost of decreasing \(\theta^D_i\) does depend on the extent to which total contributions go down. The strength of this reaction is (as in the non-cooperative protocol) determined by whether we have STS, STN or STC. Under STS, total contributions do not go down much. When one delegate becomes less committed, there is initially an impulse to reduce total contributions. But as contributions go down, each delegate’s marginal utility of contributions increases, and the initial impulse is dampened. Under STC, however, total contributions go down a lot when \(\theta^D_i\) is reduced: in response to the initial impulse, each country instead finds itself with a lower marginal utility from the public good. Thus the Nash bargaining protocol will, in response to a reduced \(\theta^D_i\), implement a larger reduction in total contributions under STC than under STS, and the marginal cost of such misrepresentation for the delegator in country \(i\) is accordingly higher under STC than under STS. The welfare that arises when the delegators choose less committed delegates will therefore be larger under STS. This is what is stated in the proposition.

If \(NBG - BAUG\) changes a lot between STC and STS, however, there is an opposing effect that could dominate the effect above. When there is a high degree of STC, the overall gains to reaching agreement are very high: as the
bargained outcome internalized the public aspect of the contributions, the marginal utilities from further contributions go up. The bargained contributions will increase further. This means that there are large transfers that each delegator can capture if his delegate differs from the other delegates. Correspondingly, if we consider a situation in which there is a high degree of STS, the bargained outcome might not be too different from the BAU outcome. Thus the gains from reaching agreement are not that high, and the possible transfers to capture are small. If the difference $N_{BG} - BAU$ changes too much between STC and STS, this latter effect could dominate the former effect, thus the caveat in the proposition.

Although certain policies seem to be determined non-cooperatively between countries, and therefore best can be understood in terms of the analysis in section 2.3, other policies have to be understood as being determined more or less in cooperation between countries. There are examples of well-functioning environmental and trade agreements, and the European Union must also be considered an institution in which countries to some extent cooperate on public-good provision. These differences and how to leverage the outcome gap between STS and STC are further elaborated upon in the next section.

2.5 Discussion

In this section, we elaborate on the lessons we can learn from the stylized model we present. We also underline how we contribute to some strands of literature within economics.

Our main result is that when players can delegate their decision-making power in a public-good game to a delegate potentially different from themselves, the strategic properties of the public-good problem is important in determining which delegates will be chosen. More specifically, we show that the delegates chosen care more about the public good when public-good contributions are strategic complements, than they do when contributions are strategic substitutes. This finding is relevant in order to understand several results in the existing literature. Furthermore, it is also an important finding in terms of designing bargaining protocols, which will become clear in the following.

First, consider the literature mentioned in the introduction, focusing on tax competition between countries. As argued by Persson and Tabellini (1992), we can easily imagine that capital tax rates are strategic complements. If a country’s neighbor has a high capital tax rate, then the marginal distortion that arises when the first country increases its tax rate is smaller. Thus the benefit from increasing one’s capital tax rate is higher when the tax rate of
2.5. DISCUSSION

one’s neighbor is higher. This will lead the electorate to elect politicians who prefer higher tax rates than the median voter, as this serves as a commitment to soften the harmful tax competition. As Persson and Tabellini (1992) argue, following a tighter European integration, which is presumed to increase capital mobility, one should expect lowered tax rates as the tax competition would be intensified. But as they show, the tax rates will not be as low as they would have been absent the ability to strategically delegate, because when capital becomes more mobile, the electorate will respond by electing politicians with even stronger preferences for higher tax rates. The results we present here underline the importance of strategic complementarity for these results. For different tax schemes, the degree of complementarity will differ, which in turn will affect the process of delegation. Ultimately this therefore also affects the strength of the resulting tax competition.

Another area in which our results may shed light on existing findings, is within the literature on monetary competition between countries, as for instance studied by Dolado et al. (1994). It is less clear whether inflationary policies are strategic substitutes or complements than what is the case for capital tax rates. When one country expands its monetary base, it will suffer from increased inflation. At the same time, it will also experience increased economic activity. This activity has spillover effects on the neighboring countries, but the direction of the spillover is not obvious. Depending on the specific kind of activity that reacts the most to this monetary policy change, increased activity in one country may increase or decrease economic activity in the neighboring country. According to the framework presented in this paper, if the result is an increase in the neighboring country’s economic activity this means that inflationary policies are strategic substitutes. In this case, a country’s marginal benefit from increasing its inflation rate is lower when it’s neighbor has already increased its inflation rate. A monetary expansion in one country thus leads to a reduction in the monetary base of its neighbor. In the opposite case, when increased activity in one country decreases activity in the other country, inflationary policies are strategic complements. The framework we present in this paper hence predicts that the effect on the neighbors activity of domestic inflationary policies will determine the incentives the country has to delegate the policy-making authority to a central banker. Interestingly, Dolado et al. (1994) find that governments will delegate to more conservative central bankers in both cases, a result that initially seems to contradict our intuition. Further inspection reveals, however, that the results are in line with our intuition. When the inflationary spillover is positive, each country is providing the other with a public good when it enacts inflationary policies, and these policies are in turn strategic substitutes. Thus we expect the outcome to
be that countries commit to supplying less of the public good, i.e. delegating to more conservative central bankers: they rationally expect their neighbor to pick up the slack. When the inflationary spillover is negative, however, inflationary policies are strategic complements. Still, Dolado et al. (1994) find that countries will delegate to more conservative central bankers. The reason is that when inflationary spillovers are negative, inflationary policies are also public bads. Our results then tell us to expect that countries commit to less inflationary policies in order to soften the harmful monetary competition, in line with the results of Persson and Tabellini (1992).

In line with the above, our results can furthermore help shed light on the current European fiscal turmoil. As of this writing, Greece has just elected a left-wing, anti-austerity government.\textsuperscript{2} If one considers a stable common European currency as a public good, one could argue that contributions to this public good are strategic substitutes: the gain to each single Euro member of contributing towards a stable Euro is greater when other countries neglect to contribute than when every other country also contributes. Perfectly in line with our theoretical predictions, electing a left-wing government can therefore be seen as a commitment not to contribute towards a stable Euro. Greek voters are rationally expecting that other European countries will pick up the slack.

More generally, understanding what effects the different kinds of monetary and fiscal policy changes have on the incentives of policy makers in other countries, may turn out to be important in order to understand to whom the decision-making authority will be delegated. This is potentially the case for all kinds of policies where countries are affected by each other. While climate change is an example of a pure public-good problem, there exists a range of environmental challenges affecting more than one country. The costs incurred from climate change are thought to be convex in the concentration of carbon in the atmosphere (Tol, 2009). This means that the emission levels of different countries are strategic substitutes. This is thus the normal assumption applied when papers in environmental economics study the effects of strategic delegation (Eckert (2003); Siqueira (2003); Buchholz et al. (2005) and Buchholz et al. (2013)). But for other environmental problems this is not necessarily the case. Furthermore, even when considering the climate problem, if countries do not control emission levels directly but rather control instruments that indirectly affect emission levels, it will no longer necessarily be the case that policies are strategic substitutes.

\textsuperscript{2} Reuters, 2015. “Greek leftist Tsipras sworn in as PM to fight bailout terms.” January 26th. http://www.reuters.com/assets/print?aid=USKBN0KZ0D620150126
Roelfsema (2007) studies a model in which firms generate emissions and compete in quantities on the international market, and in which there are imperfect cross-border environmental spillovers. He then demonstrates that when countries control domestic emission tax rates and the median voter cares little about the environment, the median voter will delegate to politicians who are less green than himself. While if the median voter cares sufficiently much about the environment, and if the degree of cross-border pollution spillover is sufficiently low, the median voter will delegate to politicians who are greener than himself. These results seem quite particular, and the intuition behind them is not obvious. However, in light of our results, they can perhaps be understood a little clearer. The former case corresponds to the standard case in which tax rates are strategic substitutes; a higher tax rate in one country, and accordingly lower global emissions, reduces the other country’s gain to increasing their taxes, even though parts of the production sector shifts from the country with the higher tax rate to the country with the lower tax rate. Hence, strategic delegation worsens the outcome, in terms of the characteristics of the chosen delegates. While the latter case corresponds to a situation in which emissions are very local and very harmful. A higher tax rate in one country shifts production to the other country, which now experiences increased local pollution. Their damages from pollution are now higher, and their benefit from increasing the tax is accordingly higher; tax rates are strategic complements. Voters will therefore elect greener politicians in order to avoid that the polluting industry locates itself within their borders.

Hattori (2010) extends this analysis and demonstrates that if the polluting firms compete in prices rather than quantities, then tax rates are strategic complements, and the median voter will delegate political power to politicians who are more green than herself. He also studies pollution standards in addition to tax rates and demonstrates that these are strategic substitutes, thus providing the median voter with incentives to delegate to politicians who are less green. The decision of whether to regulate emissions by means of a tax or by means of pollution standards therefore impacts the incentives to strategically delegate, and may thus affect the countries’ ability to solve the environmental problem they face.

Furthermore, Helm (2003) studies a situation in which a group of countries participate in an international emission permit market. Given the assumptions of his model, emission permit issuance in each country are strategic substitutes. By including a political process where voters first elect politicians who then determine issuance, Winkler (2013) finds that voters have an incentive to elect less green politicians in this situation. He argues that this could then undermine the legitimacy and the efficacy of the permit market. However,
as is now clear, this conclusion strongly depends on the assumptions turning permit issuance into strategic substitutes. In another paper (Holtsmark and Midttømme, 2015), we show that when dynamics and endogenous technology levels are taken into account, permit allocations instead become (interetempo-ral) strategic complements. By the results in this paper, that distinction will lead voters to instead elect politicians who are greener than themselves. This matters for permit market design, as Holtsmark and Midttømme (2015) demonstrate that the model and results of Helm (2003) are limits of the corresponding dynamic model in which permits are everlasting. The choice of the duration period for emission permits therefore affects the incentives countries have to strategically elect politicians, and therefore also strongly affects the ability of the permit market to limit carbon emissions.

So far, we have discussed the issues brought up here without considering the exact process behind the decision of the delegates. In the main body of this paper, we look at two cases; completely non-cooperative determination of contributions to the public good on the one hand, and Nash bargaining between the delegates on the other hand. Though the results in the two cases were similar, there where also important differences.

Therefore, it is clearly important to take into account the protocol by which the delegates decide on contributions when attempting to predict the outcome in a specific setting. Though that is not the main goal here, it is worth noting that in most of the settings discussed above, the decisions are likely to be made by delegates in a rather non-cooperative manner. Tax rates in different countries are not determined in cooperation between governments, and neither are inflationary policies. For most environmental problems international cooperation is weak, which is also the case for the climate problem. This means that the strategic characteristics of the problem in question — whether strategies exhibit complementarity or substitutability — may determine even the direction in which strategic delegation affects the outcome. In some cases, there is more scope for strategic delegation than in others, and if institutions or political processes may affect the process, the welfare effect of strategic delegation is obviously important to understand.

On the other hand, there are cases in which policies such as tax rates or environmental protection are determined in a more cooperative manner. Within countries, such policies may differ across regions, and then the political system will determine the degree of cooperation between the delegates. Furthermore, within unions such as the European Union, even though each country in principle has the opportunity to leave, many policies will in reality be determined in a process that is perhaps closer to a Nash bargaining scheme than a non-cooperative scheme.
As a final note to this discussion, we want to mention that there are several international public-good problems where our stylized model may provide some insights, that have not been mentioned so far.

One example is the development of drugs and vaccines, or more generally, eradication of diseases. For a disease to be completely eradicated, it is not only necessary to have access to the right drugs, but effort is needed on a global scale. The extent to which countries contribute to the common good of a healthier humanity is partly determined by each country non-cooperatively, and partly by politicians and representatives getting together in international meetings and forums. And there may obviously be strategic considerations taken into account when the decision makers from each country are chosen. Several severe and dangerous diseases have been completely or almost completely eradicated due to international efforts and the availability of drugs. Building on the findings of this paper, we would argue that this may partly be due to the fact that in many of these cases, efforts are strategic complements: costly efforts to reduce the prevalence of a given disease are much more likely to pay off if other countries contribute to the same good. Closely related to this is the extensive use of antibiotics around the world, which contributes to the development of new and resistant diseases. For antibiotics, however, as far as we can understand it seems likely that prescriptions are strategic substitutes: it is not a severe problem if only a few countries excessively prescribe it, whereas the costs of additional prescriptions increase in the amount already prescribed. If so, this might have contributed to the severe situation of over-prescription and resistant diseases that we observe today.

When designing protocols for international cooperation and competition, there are strong reasons to take into account the effect on and of strategic delegation. When framed in different ways, some problems provide voters with incentives to delegate to politicians who care more for the public good, while others provide voters with incentives to delegate to politicians who care less for the public good.

2.6 Conclusion

This paper studies the strategic incentives countries face when they delegate their decision-making power to a delegate, before negotiating with other countries over contributions to a global public good. We investigate these incentives under two different protocols for negotiation, and we look at how they are affected by different characteristics of the public good. We show that the public-good problem is generally more difficult to solve when the problem is
characterized by strategic substitutability, than it is under strategic complementarity.

Delegation is an inescapable feature of collective decision making. Every inhabitant of our planet cannot meet up to perform Coasian bargaining over every public-good problem. We are organized into sovereign national states, many of which feature representative democracies. Hence, delegation of decision-making power is in most countries in place through elections. Furthermore, at international bargaining tables, each government is always represented by some delegate(s). As there is no way to observe, and thus no way to enforce, that such representation is “true”, in the sense that the delegate actually represents the true preferences of his delegators, strategic delegation is something we have to acknowledge, and deal with as best we can.

An important implication of our results is that we should do more research to figure out how we can transform public-good problems from the domain of strategic substitutability to the domain of strategic complementarity, as this would make them easier to solve. In the paper we provide important examples within the domain of environmental economics, where emissions can be regulated by instruments that are either strategic substitutes or strategic complements.
2.A Appendix

2.A.1 Second-order conditions for the first-best solution

First, define
\[
\rho \equiv c'' - b_{11}(1 + (N - 1)\alpha^2) \\
\mu \equiv -b_{11}\alpha(2 + (N - 2)\alpha).
\]

The Hessian matrix of \(\sum_j U_j\) is given by
\[
H = \begin{pmatrix}
-\rho & -\mu & \ldots & -\mu \\
-\mu & -\rho & \ldots & -\mu \\
\vdots & \ddots & \ddots & \ddots \\
-\mu & \ldots & -\mu & -\rho \\
\end{pmatrix}.
\]

As \(H\) is negative definite, all eigenvalues are negative, and we can derive some relevant implications. The trace of \(H\) is the sum of the eigenvalues, thus the trace must be negative, which implies that \(\rho > 0\). Furthermore, the determinant of \(H\) is \((\mu - \rho)^{(N-1)(-\rho - (N - 1)\mu)},\) and as the determinant is the product of the eigenvalues, it must be negative for odd \(N\), and positive for even \(N\). This implies that \((\mu - \rho) = -(c'' - b_{11}(1 - \alpha)^2) < 0,\) and that \((\rho + (N - 1)\mu) = c'' - b_{11}(1 + \alpha(N - 1))^2 > 0.\) It then follows that \(c'' - b_{11}(1 + \alpha(N - K)) > 0\) for any \(K \in [1, N],\) and finally that \(c'' - b_{11}(1 - \alpha) > 0.\)

The determinant condition must also hold for all leading principal minors (\(i.e.\) the matrices that result if you delete the \(k\) last rows and columns). In particular, delete the last row and column, and the condition then implies that \((\rho + (N - 2)\mu) > 0.\)

2.A.2 Proof of Lemma 2.2

The problem the delegate from country \(i\) solves is
\[
\max_{g_i} \left\{ b(g_i + \alpha \sum_{j \neq i} g_j; \theta^D_i) - c(g_i) \right\}
\]

f.o.c: \[
0 = b_1 (NCG_i + \alpha \sum_{j \neq i} NC g_j; \theta^D_i) - c'(NC g_i) \forall i \quad (2.6)
\]

\[
\Rightarrow NC g_i (g_{-i}; \theta^D_i) \\
\Rightarrow NC g_i (\Theta^D) \\
\Rightarrow NC V_i^D (\Theta^D) = b(NCG_i (\Theta^D); \theta^D_i) - c(NCG_i (\Theta^D)) \quad (2.7)
\]
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From the f.o.c. for $i$, we have that

$$NC g_i'(g_i; \theta^D_i) = \frac{\alpha b_{11}}{c'' - b_{11}} \begin{cases} > 0 & \text{if } b_{11} > 0, \\ = 0 & \text{if } b_{11} = 0, \\ < 0 & \text{if } b_{11} < 0, \end{cases}$$

as $c'' - b_{11} > 0$ by the second-order condition.

To see how the contributions change when $\theta^D_k$ increases, we differentiate through the first-order condition (2.6) to get

$$\frac{b_{12}}{\partial \theta^D_k} \frac{d\theta^D_k}{d\theta^D_k} + b_{11} \frac{d NC g_i}{d\theta^D_k} + \alpha b_{11} \sum_{j \neq i} \frac{d NC g_j}{d\theta^D_k} = c'' \frac{d NC g_i}{d\theta^D_k}, \quad \forall i.$$

$$\Rightarrow b_{12} \frac{d\theta^D_k}{d\theta^D_k} = (c'' - b_{11}) \frac{d NC g_i}{d\theta^D_k} - \alpha b_{11} \sum_{j \neq i} \frac{d NC g_j}{d\theta^D_k}, \quad \forall i.$$

For one $k$, these are $N$ equations to determine the $N$ responses $d NC g_i/d\theta^D_k$. Together, they form the following system:

$$\begin{pmatrix} 0 \\ \vdots \\ 0 \\ b_{12} \\ 0 \\ \vdots \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} c'' - b_{11} & -\alpha b_{11} & \ldots & -\alpha b_{11} \\ -\alpha b_{11} & c'' - b_{11} & \ldots & -\alpha b_{11} \\ \vdots & \ddots & \ddots & \vdots \\ -\alpha b_{11} & \ldots & -\alpha b_{11} & c'' - b_{11} \end{pmatrix} \begin{pmatrix} d NC g_1 \\ \vdots \\ d NC g_N \end{pmatrix}.$$

This system is solved by:

$$\frac{d NC g_j}{d\theta^D_k} = \frac{\alpha b_{11} b_{12}}{(c'' - (1 - \alpha)b_{11})(c'' - b_{11}(1 + (N - 1)\alpha))}, \quad j \neq k, \quad (2.8)$$

$$\frac{d NC g_k}{d\theta^D_k} = \frac{(c'' - b_{11}(1 + (N - K)\alpha))b_{12}}{(c'' - (1 - \alpha)b_{11})(c'' - b_{11}(1 + (N - 1)\alpha))}, \quad j = k, \quad (2.9)$$

$$\Rightarrow \frac{d NC G}{d\theta^D_k} = \frac{b_{12}}{(c'' - b_{11}(1 + (N - 1)\alpha))}, \quad (2.10)$$

From Appendix 2.A.1, we know that we must have $(c'' - (1 - \alpha)b_{11}) > 0$, and $(c'' - b_{11}(1 + (N - K)\alpha)) > 0$ for $K \in [1, N]$. The denominator is hence positive.
for all three expressions. The numerator has the same sign as $b_{11}$ in Equation (2.8), while the numerators are always positive in Equations (2.9) and (2.10). Hence, it follows that:

$$STS \iff b_{11} < 0 : \frac{\partial NC g_i}{\partial \theta_i^D} > \frac{\partial NC G_i}{\partial \theta_i^D} > 0, \ \forall i,$$

$$STN \iff b_{11} = 0 : \frac{\partial NC g_i}{\partial \theta_i^D} = \frac{\partial NC G_i}{\partial \theta_i^D} > 0, \ \forall i,$$

$$STC \iff b_{11} > 0 : \frac{\partial NC G_i}{\partial \theta_i^D} > \frac{\partial NC g_i}{\partial \theta_i^D} > 0, \ \forall i.$$

2.A.3 Proof of Proposition 2.1

The first-order condition for delegator $i$ states that

$$0 = \frac{d^{NC} g_i}{d \theta_i^D} \cdot \left( b_1(\frac{NC}{NC} g_i + \alpha \sum_{j \neq i}^{NC} g_j; \theta_i^M) - c'(\frac{NC}{NC} g_i) \right)$$

$$+ \left( \alpha b_1(\frac{NC}{NC} g_i + \sum_{j \neq i}^{NC} g_j; \theta_i^M) - c_2(\frac{NC}{NC} g_i) \right) \sum_{j \neq i}^{NC} \frac{d^{NC} g_j}{d \theta_i^D}.$$

From the optimal choice of the delegate, we know from eq. (2.2) that:

$$c'(\frac{NC}{NC} g_i) = b_1(\frac{NC}{NC} \tilde{G}_i; \theta_i^D),$$

which inserted into the delegator’s first-order condition gives

$$0 = \frac{d^{NC} g_i}{d \theta_i^D} \cdot \left( b_1(\frac{NC}{NC} \tilde{G}_i; \theta_i^M) - b_1(\frac{NC}{NC} \tilde{G}_i; \theta_i^D) \right)$$

$$+ \left( \alpha b_1(\frac{NC}{NC} \tilde{G}_i; \theta_i^M) \right) \sum_{j \neq i}^{NC} \frac{d^{NC} g_j}{d \theta_i^D},$$

$$\Rightarrow \int_{\theta_i^D}^{\theta_i^M} b_{12} d\theta = -\frac{\alpha b_1(\frac{NC}{NC} \tilde{G}_i; \theta_i^M)}{d^{NC} g_i} \sum_{j \neq i}^{NC} \frac{d^{NC} g_j}{d \theta_i^D}.$$

Here we use the fact that $\left( b_1(\frac{NC}{NC} \tilde{G}_i; \theta_i^M) - b_1(\frac{NC}{NC} \tilde{G}_i; \theta_i^D) \right) \equiv \int_{\theta_i^D}^{\theta_i^M} b_{12} d\theta$. As $b_{12} > 0$, it follows that the sign of $(\theta_M^i - \theta_i^D)$ equals the opposite of the sign of $\sum_{j \neq i}^{NC} d^{NC} g_j/d \theta_i^D$, and the proposition follows, given Lemma 2.2.
CHAPTER 2. STRATEGIC DELEGATION

2.A.4 Proof of lemma 2.3

Differentiate through the Nash bargaining first-order condition (eq. (2.4)) with respect to $\theta^D_k$, to get

$$0 = \sum_j b_{11} \frac{d^{NB} D_k}{d \theta^D_k} + \sum_j b_{12} \frac{d^{D_j} D_k}{d \theta^D_k} - c' \frac{d^{NB} g_k}{d \theta^D_k}, \quad \forall i$$

$$\Rightarrow b_{12} = \frac{d^{NB} g_k}{d \theta^D_k} (c'' - Nb_{11}) - \sum_{j \neq i} \frac{d^{NB} g_j}{d \theta^D_k} Nb_{11}, \quad \forall i.$$ 

These $N$ equations together determine $\frac{d^{NB} g_i}{d \theta^D_k}$ for every $i$. The system can be written as follows:

$$\begin{pmatrix} b_{12} \\ b_{12} \\ \vdots \\ b_{12} \end{pmatrix} = \begin{pmatrix} c'' - Nb_{11} & -Nb_{11} & \cdots & -Nb_{11} \\ -Nb_{11} & c'' - Nb_{11} & \cdots & -Nb_{11} \\ \vdots & \vdots & \ddots & \vdots \\ -Nb_{11} & \cdots & -Nb_{11} & c'' - Nb_{11} \end{pmatrix} \begin{pmatrix} \frac{d^{NB} g_1}{d \theta^D_k} \\ \frac{d^{NB} g_2}{d \theta^D_k} \\ \vdots \\ \frac{d^{NB} g_N}{d \theta^D_k} \end{pmatrix}$$

This system is solved by:

$$\frac{d^{NB} g_j}{d \theta^D_k} = \frac{b_{12}}{c'' - N^2 b_{11}}, \quad j \neq k,$$

$$\frac{d^{NB} g_k}{d \theta^D_k} = \frac{b_{12}}{c'' - N^2 b_{11}},$$

$$\Rightarrow \frac{d^{NB} G}{d \theta^D_k} = \frac{Nb_{12}}{c'' - N^2 b_{11}}. \quad (2.11)$$

From Appendix 2.A.1, it follows directly that the denominator of these three expressions is positive, while the numerator is positive by assumption. The result follows.

\[\square\]

2.A.5 Proof of Proposition 2.2

The Nash surplus is given by $^{NB} W - ^{BAU} W$, and the transfers are determined by

$$^{NB} V^D_i - ^{BAU} V^D_i + t_i \equiv \frac{1}{N} (^{NB} W^D - ^{BAU} W^D) \quad \forall i$$

$$\Rightarrow t(\theta^D)$$.
where

\[ \begin{align*}
    N_{B} V_{i}^{D} &= b(N_{B} G_{i}; \theta_{i}^{D}) - c(N_{B} g_{i}), \\
    B_{A} U V_{i}^{D} &= b(B_{A} U G_{i}; \theta_{i}^{D}) - c(B_{A} U g_{i}), \\
    N_{B} W^{D} &= \sum_{j} N_{B} V_{j}^{D}, \\
    B_{A} U W^{D} &= \sum_{j} B_{A} U V_{j}^{D}.
\end{align*} \]

In the following, note that:

\[ \begin{align*}
    \frac{d B_{A} U g_{j}}{d \theta_{i}^{D}} &= 0 \quad \forall i, j \\
    \sum_{j} \frac{d B_{A} U V_{j}^{D}}{d \theta_{i}^{D}} &= b_{2}(B_{A} U G_{i}, \theta_{i}^{D}),
\end{align*} \]

given the definition of the BAU: contributions are determined non-cooperatively by the delegators. Furthermore, remember that the first-order condition for the NB-contribution profile, given by Equation (2.4), and for each country in the BAU, given by Equation (BAU foc), must hold.

We can now find \( \frac{d t_{i}}{d \theta_{i}^{D}} \):

\[ \begin{align*}
    \frac{d t_{i}}{d \theta_{i}^{D}} &= \frac{1}{N} \left( \sum_{j} \frac{d N_{B} V_{j}^{D}}{d \theta_{i}^{D}} - \sum_{j} \frac{d B_{A} U V_{j}^{D}}{d \theta_{i}^{D}} \right) + \frac{d B_{A} U V_{i}^{D}}{d \theta_{i}^{D}} - \frac{d N_{B} V_{i}^{D}}{d \theta_{i}^{D}} \\
    &= \left( \frac{1}{N} - 1 \right) b_{2}(N_{B} G_{i}; \theta_{i}^{D}) + \left( 1 - \frac{1}{N} \right) b_{2}(B_{A} U G_{i}; \theta_{i}^{D}) - \frac{d N_{B} g_{i}}{d \theta_{i}^{D}} \left( b_{1}(N_{B} G_{i}; \theta_{i}^{D}) - c'(N_{B} g_{i}) \right) \\
    &\quad - b_{1}(N_{B} G_{i}; \theta_{i}^{D}) \sum_{j \neq i} \frac{d N_{B} g_{j}}{d \theta_{i}^{D}}.
\end{align*} \]

The first-order condition of the delegator is

\[ \begin{align*}
    0 &= \frac{d t_{i}}{d \theta_{i}^{D}} + \frac{d N_{B} g_{i}}{d \theta_{i}^{D}} \left( b_{1}(N_{B} G_{i}; \theta_{i}^{M}) - c'(N_{B} g_{i}) \right) \\
    &\quad + b_{1}(N_{B} G_{i}; \theta_{i}^{M}) \sum_{j \neq i} \frac{d N_{B} g_{j}}{d \theta_{i}^{D}}.
\end{align*} \]

Insert for \( \frac{d t_{i}}{d \theta_{i}^{D}} \) and rearrange, to get

\[ \begin{align*}
    0 &= \left(1 - \frac{1}{N} \right) [b_{2}(B_{A} U G_{i}; \theta_{i}^{D}) - b_{2}(N_{B} G_{i}; \theta_{i}^{D})] + \frac{d N_{B} G}{d \theta_{i}^{D}} \left[ b_{1}(N_{B} G; \theta_{i}^{M}) - b_{1}(N_{B} G; \theta_{i}^{D}) \right].
\end{align*} \]
Integrate out the two derivatives to arrive at

\[
\left(1 - \frac{1}{N}\right) \int_{BAUG}^{NBG} \frac{\partial^2 b(x; \theta^D)}{\partial x \partial \theta^D} dx = \frac{d^{NBG}}{\partial \theta^D} \int_{\theta^D}^{\theta^M} \frac{\partial^2 b(NBG; y)}{\partial NBG \partial y} dy.
\] (2.12)

As \(b_{12} > 0\), and \(d^{NBG}/d\theta^D > 0\), it follows that \(\theta^M > \theta^D\), since \(NBG > BAUG\).

\section*{2.A.6 Proof of Proposition 2.3}

From Equation (2.12), we have seen that we always have \(\theta^M > \theta^D\) in equilibrium, when the delegates determine contributions in a Nash bargaining process. The left-hand side of the expression is positive, as long as \(NBG > BAUG\). Furthermore, it is clear that the larger the change in contributions when \(\theta^D\) changes, \(d^{NBG}/d\theta^D\), the smaller is the difference \(\theta^M - \theta^D\) needed for the equality to hold. This is because the cost for \(M_i\) of being misrepresented depends on the strength of this reaction. From Equation (2.11), we see directly that \(d^{NBG}/d\theta^D\) is increasing in \(b_{11}\). It then follows from Equation (2.12) that \(\theta^M - \theta^D > 0\) is larger under STC \((b_{11} > 0)\), than it is under STN \((b_{11} = 0)\), and under STS \((b_{11} < 0)\). The amount of misrepresentation is then as claimed, as long as the left-hand side does not change too much between STS and STC.

The welfare result in the proposition follows as the welfare loss due to suboptimally low contributions is higher when the delegators delegate to less committed delegates.

\section*{References}


Chapter 3

The dynamics of linking permit markets‡

Katinka Holtsmark and Kristoffer Midttømme

Abstract

We present a novel benefit of linking emission permit markets. We consider a dynamic setting, and let the countries issue permits non-cooperatively. With exogenous technology levels, there are only gains from permit trade if countries are different. With endogenous technology, however, we show that there are gains from trade even if countries are identical. In this case, linking the permit markets of different countries will turn permit issuance into intertemporal strategic complements: if one country issues fewer permits today, other countries will respond by issuing fewer permits in the future. This happens because issuing fewer permits today increases current investments in green energy capacity in all permit market countries, and countries with a higher green energy capacity will respond by issuing fewer permits in the future. Hence, each country faces incentives to withhold emission permits. The outcome is reduced emissions, higher investments, and increased welfare, compared to a benchmark with only domestic permit trade. The more frequently participating countries reset their caps, the higher the gain from linking permit markets.

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CHAPTER 3. THE DYNAMICS OF LINKING PERMIT MARKETS

3.1 Introduction

3.1.1 International permit trade

There is currently little hope for a global climate change treaty. However, various regions have planned or negotiated linkages between their domestic emission permit markets. We show that even in a situation where nations non-cooperatively set their caps on emissions, a simple linkage between such markets can dramatically reduce emissions and raise investments in green technology. This is the case even if countries are identical and no international permit trade takes place in equilibrium.

The failure of free markets to provide efficient levels of public goods, such as a stable climate, is well known. Without intervention from policy makers, public goods will suffer from under-provision. The converse of this problem is the tragedy of the commons (Hardin, 1968): common goods are generally over-exploited. However, efficient management of common goods can be achieved by a price on access to the good. This common price should equal the aggregate marginal damage from exploitation (Samuelson, 1954). For common goods that are international, no super-national authority that can introduce such a price exists. Hence, efficient management of such goods requires international cooperation. Though there are examples of well-managed international common goods, such cooperation is in many cases difficult to achieve. Countries typically face incentives to free-ride on other countries’ efforts to reduce exploitation. The resulting over-exploitation is inefficient, but difficult for any single country to prevent. We identify a mechanism that will lead countries to voluntarily reduce their emissions when permit markets are linked.

The number of existing emission permit markets is high and increasing. Such permit markets exist on all regulatory levels. National permit markets are currently operated in for instance Kazakhstan, New Zealand, Norway, South Korea and all the EU member countries. Regional within-country markets exist in two Chinese provinces, in the Canadian province of Quebec and in several US states. Furthermore, both Tokyo, Rio, and five Chinese pilot cities currently operate their own city-wide emission permit markets. Additional national markets are planned/under development in China, Indonesia, Thailand, and Vietnam.

Many of these markets are linked. On the regional level California and Quebec are linked, as is a group of nine eastern US states in the Regional

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Greenhouse Gas Initiative. At the national level the EU Emissions Trading System (EU ETS) itself constitutes a set of linked countries. Iceland, Liechtenstein and Norway are linked to the EU ETS, and both the EU ETS and New Zealand are linked to the United Nation’s Clean Development Mechanism (CDM). For further discussion of existing permit markets, linkages, and various permit market features, see e.g. Liski and Montero (2011), Grubb (2012), Ranson and Stavins (2012), Newell et al. (2013) or Goulder (2013).

The large number of existing markets and the existing linkages mentioned above indicate a large potential for further permit market linkages. The lack of international cooperation suggests that such linkages could provide an important path towards global coordination in fighting climate change. Indeed, Newell, Pizer, and Raimi (2013, p. 123) argue that the “[...] late-1990s dream of a top-down global design now seems far away, if not impossible. Instead, we see a multiplicity of regional, national, and even subnational markets emerging.” However, the theoretical predictions regarding the effects on emissions of such linkages are mainly negative (see e.g. Helm (2003)). In contrast to this, we find that linkages can produce substantial emission reductions.

We consider introducing permit market linkages between countries, without assuming that the countries enter into an agreement on the aggregate cap on emissions. Instead, each country is free to issue as many emission permits as it wants. Energy consumers in each country can then buy or sell such permits from consumers in any other country participating in the market. We show that such non-cooperative international trade in permits can result in substantial emission reductions compared to a non-cooperative benchmark without international trade in permits.

We construct a dynamic model where a group of countries face damages from climate change. In each country, there are energy consumers and producers who invest in durable renewable energy production capacity. The government in each country non-cooperatively determines a domestic cap on emissions. When there is international trade, emission permits can be traded across borders. We find potentially large welfare gains from such trade, arising from a mechanism that turns the permit issuance of different countries into intertemporal strategic complements. This strategic complementarity leads to lower emissions and higher welfare, without requiring any country to commit to reducing its own emissions or punish other countries that do not. The main contribution of this paper is to show that permit market linkages will lead countries to voluntarily restrict emissions. This conclusion does not depend on countries being different, or permit trade taking place in equilibrium.

The mechanism we identify can be explained in terms of the three following steps. Firstly, more emission permits available in the market in any given time
period gives a lower equilibrium permit price. This means that if one country
withholds an emission permit today, the permit price will be higher. When
there is international trade in permits, the permit price will increase in all
countries. Secondly, an increase in the permit price will increase the demand
for green energy. This will lead green energy producers in every country to
increase their investments. These increased investments will in turn increase
the available production capacity in the future since capacity is durable. When
the current emission permits expire and countries issue new permits, they
will all have higher production capacity for green energy. Thirdly, countries
with more green energy capacity will issue fewer permits. In total, these
steps imply that lower permit issuance in one country in a given time period
leads to lower issuance in all countries in future periods. There is thus an
intertemporal strategic complementarity in permit issuance: if one country
withholds a permit today, investments in green energy will increase in every
country, and all countries will respond by issuing fewer permits in the future.
Countries will exploit this complementarity in order to reduce the costs of
climate change imposed on them by other countries.

The mechanism explained here leads to an outcome under international
permit trade that is better than the outcome under autarky. The welfare
gains from linking permit markets are due to emission reductions and are
independent of any trade taking place in equilibrium.

3.1.2 Related Literature

This paper contributes to a literature on linkages between markets for emission
permits. Linking permit markets will lead to gains as marginal abatement costs
will be equalized across markets (see e.g. Flachsland et al. (2009)). However,
linking permit markets may also affect the incentives and behavior of policy
makers. Several authors discuss the effects of permit market linkages and how
the effects of linkages depend on the exact institutional framework (see e.g.
Newell et al. (2013), Mehling and Haites (2009), Jaffe et al. (2009), Fischer
(2003), or Green et al. (2014)).

Helm (2003) and Rehdanz and Tol (2005) are the first authors to explicitly
model the incentives to alter the emission cap when national permit markets
are linked. Both find that some countries will increase their permit issuance,
while others will reduce it. Therefore, there is no ex ante reason to expect
emissions to go down when permit markets are linked across countries. The
exact welfare results will depend on the model parameters. Following these pa-
pers, there is a literature investigating these effects in numerical models, with
mixed conclusions (e.g. Carbone et al. (2009) and Holtsmark and Sommer-
3.1. INTRODUCTION

However, these papers only analyze countries engaging in static games. We show that when dynamics are allowed to play out in a very similar model framework, the effect of introducing permit market linkages changes substantially. Specifically, we identify a mechanism that results in emission reductions and positive welfare effects when permit markets are linked.

Permit market linkages have recently gained increased attention from researchers and policy makers, partly due to the lack of results from global climate negotiations. The observed failure to reach agreement is very much in line with theoretical predictions from the economics literature. Barrett (1994) shows that the number of countries willing to participate in climate coalitions is very small when emission levels are set in order to maximize the joint welfare of the coalition members. The incentives to free-ride that all countries face, prevents an efficient global solution to the climate problem (see also Barrett (2005)). Hoel (1992), Carraro and Siniscalco (1993) and Carraro et al. (2006) show that the predictions are the same when different institutional frameworks are considered. Endogenous technology investments will change the workings and optimal design of climate treaties, but generally not solve the free-rider problem (see e.g. Barrett (2006), Hoel and Jensen (2010), and Calvo and Rubio (2013)). Dixit and Olson (2000) demonstrate the failure of Coasian bargaining when many countries face a public good problem such as climate change. Taken together, this literature demonstrates the need to find mechanisms that do not rely on countries determining emission levels cooperatively. We show that when national permit markets exist, linking these markets will provide countries with incentives to voluntarily reduce their emissions.

In addition to the literature on permit market linkages, this paper also contributes to a more general literature on dynamic games of public goods provision. A general insight from this literature is that free-rider problems are more severe when dynamics are taken into account.

More specifically, several papers show that when technology investments are non-contractible, problems arise that increase the inefficiencies resulting from free-riding. One such problem is the hold-up problem: countries know that they will enter into (re)negotiations over emission levels in the future; thus, when they make their green technology investment decisions up front, they take into account that their bargaining position will be weakened if they invest a lot. The result is that all countries invest less in equilibrium. Both Buchholz and Konrad (1994) and Beccherle and Tirole (2011) show that this problem can lead to severe negative welfare consequences. In contrast to these findings, we show that the non-contractibility of green technology investments can contribute to a mechanism that results in emission reductions when permit markets are linked.
Harstad (2015) demonstrates that, because of the hold-up problem, treaties should be long-lasting. When the next renegotiation will take place far in the future, the perverse incentives to underinvest are much weaker. In our model we show that the emission caps should be reset often, in order to reap larger welfare gains from permit trade. This demonstrates how strongly the implications of including non-contractible green investments depend on the setting.

Finally, there are also other contributions to this literature that find a positive effect of the non-contractibility of green investments. These authors demonstrate how the hold-up problem can be leveraged to produce better outcomes, by specifically allowing for renegotiation of the treaties (Harstad, 2012), and exploiting the hold-up problem when punishing defecting countries (Battaglini and Harstad, 2015).

Introducing dynamics in climate change models may produce strategic spillovers that make the free-rider problem more severe. For instance, Hoel (1991) identifies a spillover due to the damages from emissions being convex. He shows that this spillover undermines any single country’s incentive to unilaterally reduce emissions, because reduced emissions in one country reduces the marginal damage other countries face. Thus, countries respond by increasing their emissions: emissions are strategic substitutes. Fershtman and Nitzan (1991) take the spillovers identified by Hoel (1991) to a dynamic setting, where countries have convex damage functions from a stock of carbon in the atmosphere. In this setting, emissions become intertemporal strategic substitutes, and countries can free ride on both the current and future effort of others. Contrasting this, we show that introducing dynamics can also reduce the free-rider problem.

Positive dynamic strategic spillovers are also found by both Ploeg and de Zeeuw (1992) and Golombek and Hoel (2004). Ploeg and de Zeeuw (1992) assume that technologies are pure public goods, and show how countries will overinvest in green technologies, in order to induce other countries to emit less in the future. Similar results are also found by Golombek and Hoel (2004), who study imperfect green technology spillovers. We show, however, that strategic links between emission levels in different countries can arise even if there are no technical spillovers.

The literature on climate coalitions discussed above, assumes that countries, once inside a coalition, can contract on emission levels. There is another strand of literature that excludes this type of exogenous compliance. This literature studies how cooperative behavior can be enforced by the threat of Nash reversion when it is taken into account that countries interact repeatedly. This can be thought of as endogenous enforcement of compliance with
3.2. THE MODEL

the agreement in a dynamic framework. Though the conclusions differ somewhat depending on the specific assumptions, this literature shows that low emission levels can be sustained in equilibrium in repeated games when countries are allowed to employ trigger strategies to punish defectors. See Barrett (1994), Asheim and Holtsmark (2008), Dutta and Radner (2004), and Dutta and Radner (2009). The basic assumptions in these models are close to the assumptions we make in this paper. However, by restricting our attention to Markov perfect equilibria, we show that such punishment schemes are not the only way to obtain higher welfare when policies are set non-cooperatively.

The paper proceeds as follows: in Section 3.2 we introduce the model setup and the benchmark outcomes. We solve for the Markov perfect equilibrium of the dynamic game and present our main results in Section 3.3. In Section 3.4, international trade in the substitute technology, rather than international permit trade, is discussed. In Section 3.5 we relax some of the assumptions and discuss extensions of the model framework, while we conclude in Section 3.6.

3.2 The model

In this section we present the model setup. We look at a group of \( N \) countries, who all incur some damages from climate change. The model spans an infinite number of discrete time periods, and each country makes policy decisions within each period. In each country there are price-taking energy consumers and renewable energy producers who also make decisions in every period. In each country there are price-taking energy consumers and renewable energy producers who also make decisions in every period.

We first introduce the problems solved by consumers and producers, and solve for their demand and supply. Then, we derive a first-best benchmark for consumption, investments, and emission levels. Finally, we derive the outcome under autarky, when governments set their optimal policy, but there is no trade among countries. When we later investigate the equilibrium under international permit trade, we compare the emission levels under such trade to the first-best levels and the levels under autarky.

In a given country \( i \), the representative consumer is a price taker and derives utility \( u_i(e_{it}) \) from consuming \( e_{it} \) units of energy in period \( t \). We assume \( u_i(\cdot) \) is increasing and concave in the relevant region. Energy is available from two sources, one fossil and the other renewable. For simplicity, assume that there is an abundant supply of fossil energy available for all to consume at zero price. In Section 3.5.1, we discuss this assumption, and argue that it does not drive our results. Consumption of fossil energy by country \( i \) in period \( t \) is
CHAPTER 3. THE DYNAMICS OF LINKING PERMIT MARKETS

denoted \( f_{it} \). The consumption of fossil energy drives climate change and hence determines the damages inflicted on all countries.

Given the damage from climate change, countries will want to use a policy instrument to reduce emissions, both under autarky and trade. We assume that each country under autarky sets a cap on domestic emissions and issues emission permits that grant the holder the right to consume fossil energy. The permits can either be auctioned off or distributed for free—for the purpose of this paper, this does not matter. The permits can then subsequently be traded among the country’s energy consumers. When discussing the outcome under autarky, the implemented allocation would be equivalent to that under regulation of emissions by use of a tax, or even direct regulation, as there is no uncertainty or asymmetric information in the model. Let \( \omega_{it} \) denote the number of permits issued in country \( i \). For every unit of fossil energy consumed, the consumer has to buy one emission permit, traded in the market at price \( p_{it} \). Domestic consumption of fossil energy must equal the number of permits issued domestically when permit markets are not linked. When permit markets are linked, the permits can be traded among consumers not only in the same country, but also among consumers in different countries. Domestic fossil energy consumption must then no longer equal domestic permit issuance, but total issuance of permits determines the total cap on emissions in the system. When there is international permit trade, the permit price, \( p_{it} \), will be equalized across countries.

In addition to fossil energy, consumers can consume renewable energy, denoted by \( z_{it} \). Total consumption, \( c_{it} \), is then \( f_{it} + z_{it} \), hence we assume the two types of energy to be perfect substitutes. The fact that they are perfect substitutes means that in equilibrium, we will have \( p_{it} = q_{it} \), where \( q_{it} \) denotes the price of renewable energy in country \( i \) in period \( t \). This means that international trade in either permits, renewables, or both, will equalize the price of both permits and renewables across countries, as long as there is positive demand for both types of energy in all countries. Renewable energy is not freely available, but produced by private firms, who take the price \( q_{it} \) to be exogenous. In Section 3.5, we discuss the implications for our model if investments in renewables were determined politically. We argue that this would not change our main results.

Each period, the representative renewables producer in country \( i \) can undertake a (non-negative) investment, \( r_{it} \), at a cost \( c_i(r_{it}) \). We assume this investment cost function to be increasing and convex in the current investments, and independent of the level of the stock. These investments contribute to a stock of renewables production capacity in country \( i \), denoted \( R_{it} \). We assume that the investments undertaken in period \( t \) are immediately available, and
3.2. **THE MODEL**

that there are no variable costs in supplying renewable energy from the stock. All actors in the model share the same discount factor, $\beta$. For each country, $i$, the renewables stock develops according to

$$R_{it+1} = \delta (R_{it} + r_{it}),$$

(3.1)

so that $\delta \in (0, 1)$ is the survival rate, and $(1 - \delta)$ the depreciation rate of the renewables stocks.

Define $f_t = \sum_j f_{jt}$ to be aggregate emissions in period $t$, equal to the aggregate consumption of fossil energy. Further, let $S_t$ be the stock of GHGs in the atmosphere in period $t$, which develops according to $S_{t+1} = \gamma (S_t + f_t)$, such that $(1 - \gamma)$ is the decay rate of the stock of GHGs. Each country incurs a damage in period $t$ from the stock of GHGs in the atmosphere, represented by the linear damage function $D_i(S_t + f_t)$.

This allows us to represent the present value of damages to country $i$, from emissions in period $t$, by a constant marginal damage $D_i$. We can then disregard the GHG stock as a state variable in our model. With this simplification, we focus attention on the strategic incentives created by establishing permit trade among countries, rather than the general effects arising from convex damage costs. This damage function has been extensively applied in the literature. See for instance Dutta and Radner (2009) for a thorough discussion of the implications. In Section 3.5, we also discuss this damage function and argue that our results are not driven by this assumption.

The welfare of country $i$ in period $t$ will consist of utility from consumption, renewables investment costs, damages from emissions, and, if there is international permit trade, the net cost or revenue from trading permits:

$$U_{it} = u_i(f_{it} + z_{it}) - c_i(r_{it}) + p_t \cdot (\omega_{it} - f_{it}) - D_i f_t,$$

(3.2)

where domestic consumption of renewable energy, $z_{it}$, equals the supply from the available domestic production capacity, $r_{it} + R_{it}$. If there is no international trade in permits, the net revenue from trade would of course be zero for all countries, as $\omega_{it} = f_{it}$. Under autarky, the prices $p_t$ and $q_t$ will differ across countries.

The timing of decisions within each time period in each country is demonstrated in Figure 3.1. The assumptions reflect how quickly we anticipate that

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3Given $\hat{D}_i(S_t + f_t)$, the increase in the present value of future damages by a marginal increase in emissions in period $t$, would be $D_i = \sum_{\tau=t}^{\infty} (\beta \gamma)^{\tau-t} \hat{D}'_i(S_\tau)$, which for constant $\hat{D}'_i(S) = \hat{D}_i$, is equivalent to $D_i = \frac{\hat{D}_i}{1 - \beta \gamma}$. 
each group of decision makers can react. Consider, for instance, the EU today. There, the current cap on emissions is determined through 2020, and it seems realistic that producers of renewables consider the current policy environment a given. Consumption takes place continuously and reacts to clear the market. Hence, we assume that each government issues permits at the beginning of every time period. Then, the renewables producers decide how much to invest, and finally consumption is determined and prices clear the markets. The renewables producers and the consumers of energy are all price takers, while governments realize that they will affect the permit price when issuing permits both under autarky and international permit trade.

3.2.1 Equilibrium consumption and investments

Consumers and producers are price takers, and behave in the same way, independent of whether or not permits and renewables are traded among countries. The consumer is endowed with a fixed per-period budget, which he allocates between consuming energy and all other goods, which we take to be the numeraire good. We assume that the budget constraint is such that there will always be an interior solution to the consumer’s maximization problem, and we therefore disregard this constraint in the following. Generally, we disregard any other possible market failure except for the climate problem.
3.2. THE MODEL

Observing prices $p_{it}$ and $q_{it}$, the representative consumer in country $i$ then solves a static problem in each period:

$$\max_{f_{it}, z_{it}} u_i(f_{it} + z_{it}) - p_{it} f_{it} - q_{it} z_{it},$$

with the solution

$$u'_i(f_{it} + z_{it}) = p_{it}$$
$$u'_i(f_{it} + z_{it}) = q_{it}. \quad (3.3)$$

Since the two energy sources are perfect substitutes, the price of renewables and permits must be equal in equilibrium, provided that both are consumed. We denote the common price $p_{it}$.

We then have $u'_i(e_{it}) = p_{it}$, which defines an energy demand function for country $i$, $e_{it}(p_{it}) = (u'_i)^{-1}(\cdot)$, with a derivative of

$$e'_i(p_{it}) = \frac{1}{u''_i(e_{it})} < 0. \quad (3.4)$$

The representative renewables producers in each country solve an intertemporal problem. In each period, they own a stock of renewables, they take the stream of prices as given, and decide to what extent they want to invest to increase their stock capacity. There is no rental or second-hand market for the stock capacity, and produced energy cannot be stored across periods. Each period, they sell the energy produced from the existing stock plus current investments. They pay investment costs, and take into account how investments today affect the future stock. The problem is solved by

$$c'_i(r_{it}) = p_{it} + \beta \delta p_{it+1} + (\beta \delta)^2 p_{it+2} + \ldots = \sum_{\tau=t}^{\infty} (\beta \delta)^{\tau-t} p_{\tau} \equiv \hat{p}_{it}. \quad (3.5)$$

As they are price takers, the renewables producers pay no attention to the current stock, and equate the current marginal investment cost with the discounted sum of all future prices, from now on denoted $\hat{p}_{it}$. The inverse of
the marginal cost curve defines current investments as a function of this price sequence, denoted \( r_i(\hat{p}_{it}) \), with

\[
r'_{i}(\hat{p}_{it}) = \frac{1}{c''_{i}(r_{it})} > 0. \tag{3.7}
\]

From the problems solved by consumers and producers it follows that consumption is lower, and investments are higher, when the price is high. Emissions from any given country will therefore be lower when the permit price is higher. The decrease in emissions resulting from a price increase will depend positively on the slope of the supply/investment curve and the demand curve.

From the derived investment behavior, it follows that the stock of renewable energy capacity available at the beginning of period \( t \) in country \( i \) is given by

\[
\delta^t R_{i0} + \delta^{t-1} r_i(p_{i1}) + \ldots + \delta r_i(p_{it-1}) = \delta^t R_{i0} + \sum_{s=1}^{t-1} \delta^{t-s} r_i(p_{is}).
\]

Furthermore, in a steady state where the price is constant (given by \( p_{iSS} \), for instance) and the stocks are at their steady-state levels, consumption will be given by

\[
e_{iSS} = e_i(p_{iSS}), \tag{3.8}
\]

and the stock in country \( i \) will be given by

\[
R_{iSS} = \frac{\delta}{1 - \delta} r_i \left( \frac{p_{iSS}}{1 - \beta \delta} \right). \tag{3.9}
\]

Throughout the paper, as we solve for the benchmark cases and the Markov perfect equilibrium under international permit trade, we assume that, in every country, there is not enough renewable energy to completely saturate energy demand. Thus, consumers in every country will consume both renewable energy and fossil energy, which must be accompanied by emission permits. The resulting positive permit demand is sufficient for international permit trade to equalize the price of both permits and renewable energy across countries. If instead, energy demand in country \( i \) was saturated by renewables alone, the domestic renewables price in country \( i \) would be decoupled from the international permit price. The following assumptions are sufficient to ensure that no country is completely saturated by renewables in any time period:
3.2. THE MODEL

\[ e_i \left( \sum_{j=1}^{N} D_j \right) > \frac{1}{1 - \delta} r_i \left( \frac{\sum_{j=1}^{N} D_j}{1 - \beta \delta} \right) \forall i, \]  
\[ (3.10) \]

\[ e_i \left( \sum_{j=1}^{N} D_j \right) > R_i + r_i \left( \frac{\sum_{j=1}^{N} D_j}{1 - \beta \delta} \right) \forall i. \]  
\[ (3.11) \]

In Section 3.5, we discuss the implications of relaxing this assumption.

Both these inequalities depend on the parameters of the utility and investment cost functions. Equation (3.10) states that steady state consumption must exceed the steady state stock of renewable capacity in every country, while Equation (3.11) also takes into account that demand must exceed the initial stock in every country. Given the other model parameters, this determines an implicit upper bound on \( \beta \) and \( \delta \). As \( \beta \) goes to 1, investors do not discount the future and are willing to undertake infinite investments in every period, as long as they expect a positive price in every future period. Furthermore, as \( \delta \) goes to 1, the stock never depreciates, so positive investments every period means that the stock will explode.

Note that there always exist parameters such that any pair \((\beta, \delta)\) satisfies (3.10) and (3.11), as long as \( \beta, \delta < 1 \).

Before we solve the individual optimization problems of the governments under international permit trade, we will present the first-best solution of the model and the governments’ solution under autarky. These two cases make up the benchmarks we use for comparison when we solve the model with international trade.

3.2.2 First best

Define aggregate welfare by the discounted sum of utility from consumption, subtracted the costs of investing in renewable energy capacity and the damages from climate change, for every country.

\[ W = \sum_{i} \sum_{t=0}^{\infty} \beta^t \left( u_i(f_{it} + z_{it}) - c_i(r_{it}) - D_if_t \right) \]
The first-best consumption levels and renewables investments in each period will be given by the solution to the following problem:

\[ W^{FB} \equiv \max \left\{ W_t \mid \{ (f_{it}, z_{it}, r_{it}) \}_{t=0}^{\infty} \right\} \]

subject to \( z_{jt} = (R_{jt} + r_{jt}) \quad \forall j, t \)

and \( R_{jt+1} = \delta (R_{jt} + r_{jt}) \quad \forall j, t \).

Given an interior solution, the first-best allocation is characterized by the following: first, marginal utility of consumption for each country and in every period, must equal the sum of the marginal damage of emissions across all countries:

\[ u_i'(f_{it} + z_{it}) = \sum_j D_j \quad \forall i, t. \]

Hence, total consumption in the first-best solution is constant over time. Secondly, since renewables investments today will also bear fruit in future periods, the marginal cost of producing renewables should equal the sum of the damages that can be avoided in all countries, discounted over all future periods:

\[ c_i'(r_{it}) = \sum_j D_j \left( 1 + \beta \delta + (\beta \delta)^2 + \cdots \right) = \frac{\sum_j D_j}{1 - \beta \delta} \quad \forall i, t. \]

First-best emission levels are thus determined by the development of the renewables stock over time. If all countries start out with renewables stocks below their steady state, each stock will increase until it reaches this stable steady state. Emissions will thus decrease over time as the economy’s renewables capacity increases. In the steady state, emissions are constant.

Given the behavior of consumers and renewables producers, given by (3.4) and (3.6), the first-best allocation can be implemented by a common price on emissions, equal to the sum of the marginal damages in every time period:

\[ p_t^{FB} = \sum_j D_j \quad \forall t. \]  

(3.12)

Note that this price only depends on the marginal damage of the countries. In particular, it does not depend on the survival rate of the renewables stock. This will be particularly relevant when we consider the outcome when permit markets are linked.

### 3.2.3 Autarky

We now introduce governments as decision makers in the case where there is no international permit trade. The permits that each government issues
are either auctioned or distributed for free among the domestic consumers and producers. This distinction is irrelevant to our results. These permits can then be freely traded domestically. Autarky thus designates a case where there is domestic permit trade, but no international trade. Under autarky, the per-period welfare of country $i$ is given by:

$$U_{it} = u_i(e_{it}) - c_i(r_{it}) - D_if_t.$$ 

Governments are assumed to maximize the discounted sum of future welfare. We set up the problem recursively and let different governments determine domestic emissions simultaneously.

As the model is time-independent, we suppress time indices from now on, unless clearly needed. Next-period stocks are denoted by $\plus$.

Under autarky, setting a tax $p_i$ on emissions is equivalent to setting a domestic cap. Emissions in each country will be given by the residual between total consumption and available renewables:

$$f_i = e_i(p_i) - R_i - r_i(\hat{p}_i),$$

where $\hat{p}_i$ is the discounted sum of future prices in country $i$.

Each country then solves

$$V_i^{aut}(\{R_j\}_{j=1}^N) = \max_{p_i} \left\{ u_i(e_i(p_i)) - c_i(r_i(\hat{p}_i)) - D_i \sum_j (e_j(p_j) - r_j(\hat{p}_j) - R_j) + \beta V_i^{aut}(\{\delta(R_j + r_j(\hat{p}_j))\}_{j=1}^N) \right\},$$

(3.13)

taking into account the response functions of producers and consumers. Existence of an interior solution follows from the assumption that Equations (3.10) (3.11) hold. Emissions in all other countries are taken as given and, due to the linear damage function, they do not affect the optimal policy of country $i$, even if known ex ante.

Each country then solves

$$p_i^{aut} = D_i, \ \forall t.$$ 

The government sets a price on emissions equal to the domestic marginal damage of emissions in every period, independent of the current renewables stock. This is the standard tragedy of the commons: each country will grant its consumers access to the commons until the private marginal utility equals the private marginal damage from depletion of the commons, and fail to take into account the damage incurred by other countries.
Given that the two policy instruments are equivalent, the same welfare level could of course also be implemented by setting a domestic cap on emissions, such that:

\[
\omega_i^{aut} = e_i(p_i^{aut}) - r_i(\hat{p}_i^{aut}) - R_i.
\]

The constant autarky price results in constant consumption and investments in each country, and the stock of renewables converges to some steady-state level. If the stock of renewables starts below this level, it will move along an increasing path and extraction from the commons will, over time, decrease along with it. From a social point of view however, emissions will forever remain suboptimally high.

As in the first-best solution, the carbon price in each country is constant over time, and is independent of the survival rate of the renewables stocks and the discount factors. Furthermore, as emissions are strategically neutral when the damage function is linear, there is no possibility for any country to affect current or future emission levels in other countries. Hence, no strategic considerations are taken and each country’s action affects only its own emissions.

### 3.3 International permit trade

#### 3.3.1 Setup

In this section, we present our results on the effect of introducing permit market linkages between countries. We show that the common price arising when there is international trade in emission permits changes the incentives countries face in the permit issuance stage.

Introducing international permit trade means allowing emission permits to be traded not only between energy consumers in the same country, but between consumers in all \( N \) countries. Our main focus is on how such trade affects total permit issuance, and hence emissions and welfare. In Section 3.5, we discuss international trade in renewable energy and show that emissions are affected in the same way by either trade in renewables or emission permits.

International permit trade is organized as follows. Countries are free to issue as many permits as they want. In the consumption stage, these permits are traded among consumers in all countries at a common price. One unit of fossil energy consumption requires one emission permit. We assume that countries honor this requirement, and that they do not exempt their consumers from this requirement ex post.
The introduction of international permit trade changes the dynamic game. As opposed to the situation under autarky, countries no longer have dominant strategies. We study Markov perfect equilibria (MPEs), in which the renewables stocks are the only payoff-relevant state variables. The government in each country understands the response functions of all consumers and renewables producers, as derived in Section 3.2.1.

Recalling that demand and investments depend on prices, market clearing requires

\[
\begin{align*}
\sum_j f_j(p) &= \sum_j \omega_j, \\
\Rightarrow \sum_j (f_j(p) + z_j(p)) &= \sum_j e_j(p) = \sum_j (R_j + r_j(\hat{p}) + \omega_j).
\end{align*}
\]

A key figure in the following analysis is the initial supply of energy, i.e. the supply before the renewables producers make their investments. At this stage, the \( N \) stocks of renewable capacity are known, and the governments have issued their permits, \( \omega_i \). Define the initial supply by \( s \equiv \sum_j R_j + \sum_j \omega_j \), and substitute it into the market clearing condition, to get

\[
\sum_j e_j(p) - \sum_j r_j(\hat{p}) = s.
\]

Given the behavior of consumers and investors, the price prevailing in the market is only a function of \( s \), \( p = p(s) \). An increase in this initial supply will lead to a price decrease. If this were not the case, there would be no change in either consumption or investments, and an increase in \( s \) would lead to excess supply. When \( s \) increases, demand must increase and/or investments must decrease in order for the market to remain in equilibrium. This happens only if \( p'(s) \) is negative. As the renewables producers are rational and farsighted, we might fear that an increase in \( s \) today would affect the behavior of the renewables producers in non-obvious ways. However, in Appendix 3.A.2, we prove that this is not the case, and further show that \( p'(s) = 1/(\sum_j (e'_j(p) - r'_j(\hat{p}))) < 0 \).
3.3.2 Markov perfect equilibrium

Under international permit trade, the government in country $i$ solves the dynamic problem:

$$V^\text{trade}_i(R_1, \ldots, R_N) = \max_{\omega_i} \left\{ u_i(e_i(p(s))) - c_i(r_i(\hat{p})) - D_i \sum_j \omega_j 
+ p(s) \cdot (\omega_i + R_i + r_i(\hat{p}) - e_i(p(s))) 
+ \beta V^\text{trade}_i(\delta(R_1 + r_1(\hat{p})), \ldots, \delta(R_N + r_N(\hat{p}))) \right\}. \quad (3.15)$$

In a given time period, the number of permits issued by all other countries are taken as given by country $i$, so permits are issued in a simultaneous Nash equilibrium. However, the country takes into account that the permit price today, and possibly also in the future, will be affected by the total number of permits issued through the initial supply, $s$. They are not, therefore, price takers. They also realize that the total number of permits issued in the future might depend on their current actions through the state variables $R^+_j$.

The solution to this problem will give us the permit price and the cap on emissions in each time period. We derive the following $N$ first-order conditions:

$$0 = p(s) + p'(s) \cdot (\omega_i + R_i + r_i(\hat{p}) - e_i(p(s))) + p'(s)e'_i(p(u'_i(e_i(p)) - p(s)) + \frac{d\hat{p}}{d\omega_i} r'_i(\hat{p}) \cdot (p(s) - c'_i(r_i(\hat{p}))) - D_i + \beta \frac{d\hat{p}}{d\omega_i} \sum_j r'_j(\hat{p}) \frac{\partial V^{\text{trade}+}_i}{\partial R^+_j}, \quad \forall i. \quad (3.16)$$

Together these conditions define our MPE. When an additional permit is issued by the government in country $i$, the country experiences a marginal cost and a marginal benefit. Since permits are issued simultaneously, the game resembles a Cournot game: each country acts as a monopolist on the residual demand, given the permit issuance of the other $N-1$ countries. When country $i$ issues one additional permit, its marginal revenue is given by two terms. Firstly, country $i$ has a direct gain from selling the new permit, given by the market price. Secondly, the price will decrease as a result of the increased permit supply. The country will benefit from the price decrease if it is a net importer of permits. The higher the net imports of country $i$, the higher the benefits. If the country is a net exporter of permits, the price decrease is costly, and again, the cost of a decrease in price increases in the net exports of country $i$. (The reduced price will also increase the utility of consumption through increased domestic demand for energy, but this cancels out against the price consumers have to pay for that energy.) Furthermore, the lower price will decrease domestic investments in renewables, which costs the country the price
3.3. INTERNATIONAL PERMIT TRADE

$p$ today, although the marginal cost $c'$ is saved. Since the renewables producers are forward-looking, this does not cancel in the same way as the change in consumption. Finally, there are two ways in which country $i$ incurs a cost from issuing an additional permit. Firstly, an increase in the permit supply directly increases emissions, and country $i$ incurs a cost $D_i$. Secondly, the actions today have an effect in the future: the price change today will lower investments in renewables, not only in country $i$ itself, but in all countries participating in the permit market. This results in a lower future stock of renewables, which may affect the country through the continuation value, $\partial V^\text{trade}_i/\partial R^+_j$. As in a Cournot oligopoly model, permits are strategic substitutes within each period: in the stage Nash equilibrium, the number of permits issued by country $i$ is decreasing in the number of permits issued by countries other than $i$. This is the case because increased issuance by other countries depresses the permit price.

The solution to country $i$'s maximization problem is given by the permit issuance $\omega_i$ which equalizes these marginal costs and benefits. The marginal benefit to country $i$ of issuing a permit is strictly decreasing in the number of permits issued: more permits will also lower the price of the inframarginal permits, and the marginal benefit will eventually become negative. Thus no country will ever want to issue an infinite number of permits.

The equilibrium will be given by the $N$ first-order conditions stated in Equation (3.16). Fudenberg and Tirole (1990) suggest the following selection criterion for MPEs: the infinite-horizon MPE should be the limit of the finite-horizon MPE of the truncated game as the horizon goes to infinity. In Appendix 3.B, we prove that our equilibrium is indeed the limit of the unique finite-horizon SPE of the game (and thus also the limit of the finite-horizon MPE).

While the bulk of the calculations are relegated to the appendices, we provide some important results here before explicitly discussing the equilibrium outcome.

**Lemma 3.1.**

1. The equilibrium policy functions satisfy

\[
\frac{\partial \omega^e_i}{\partial R_j} = \begin{cases} 
-1 & \text{if } j = i, \\
0 & \text{if } j \neq i.
\end{cases}
\]

2. The value function is linear in the stocks, with $\partial V^\text{trade}_i/\partial R_j = D_i/(1 - \beta \delta)$, $\forall i, j$.

**Proof.** See Appendix 3.A.2.
Lemma 3.1.1 states that an increase in the stock of renewables in country \( i \), will lead to fewer permits issued by country \( i \), one for one. Given the price, \( p \), there is only one level of net supply that is compatible with country \( i \)'s best response. To see why this has to be, consider the alternative reactions to an increase in \( R_i \). If country \( i \) decreased its issuance by less than one unit per unit increase in the stock, then country \( i \)'s net supply, and the initial supply, \( s \), would increase. This would decrease the price \( p \). This cannot be equilibrium behavior for country \( i \), since every country would want to issue fewer permits when the price is lower. Similarly, we immediately see that if country \( i \) reduces its own initial supply of permits in response to a higher stock, then \( s \) would decrease, and the price would increase. This cannot be an equilibrium either. The only response that is compatible with equilibrium from country \( i \)'s perspective is to leave the initial supply, \( s \), unaltered by reducing the number of permits issued one for one. Then the price remains unchanged and no other country would react to the increased stock of renewables in country \( i \).

Given this equilibrium behavior, the only effect of an increased stock of renewables is to replace fossil energy consumption by renewable energy consumption. Reduced emissions benefits country \( i \) by avoiding damages \( D_i \), no matter where the emissions were supposed to take place. This turns the domestic stocks of renewable energy into public goods. Lemma 3.1.2 then follows from the fact that a unit increase in the stock today gives an increase in the stock in the next period of \( \beta \delta \), and the period after of \( (\beta \delta)^2 \) and so forth.

The reactions outlined in Lemma 3.1.1 form the basis for the mechanism that will produce welfare gains when the countries' permit markets are linked. The next proposition demonstrates that countries can induce each other to issue fewer permits in the future by issuing fewer permits themselves today.

**Proposition 3.1. When the permit markets of \( N \) countries are linked, countries can induce increased investments in other countries by withholding permits today: \( \frac{\partial r_i}{\partial p} \frac{\partial p}{\partial \omega_i} < 0 \), for all \( i, j \). As a result, emission allowances in the different countries become intertemporal strategic complements: \( \frac{\partial \omega_j^+}{\partial R_j^+} \frac{\partial R_j^+}{\partial \omega_j^+} > 0 \), for all \( i, j \).**

**Proof.** In Appendix 3.A.2, we prove that \( p'(\cdot) \) is negative, and that \( \frac{dp_i}{ds_t} = p'(\cdot) \). Thus, one less permit issued today will increase the current equilibrium price. By Equation (3.7), this will further increase investments in every country, \( r_i'(\cdot) > 0 \), and by Lemma 3.1.1, future permit issuance will go down in every country, \( \partial \omega_j / \partial R_j = -1 \), for all \( j \).

---

\(^4\)Our definition of intertemporal strategic complementarity corresponds to definition in both Jun and Vives (2004) and Baldursson and Fehr (2007).
This means that introducing permit market linkages enables countries to induce each other to issue fewer permits in the future, by issuing fewer permits today. This is the case, even though permit issuance decisions are intra-period strategic substitutes. The intertemporal link between issuance in different countries consists of two steps. Firstly, the permit price increases when fewer permits are issued today. Renewable energy producers in every country respond to the increase in permit prices by increasing their investments. Secondly, when countries experience increased renewable energy stocks in the next period, by Lemma 3.1.1, they will respond by issuing fewer permits.

In the absence of international trade in permits, countries cannot act strategically in this way. When permits are only traded domestically, countries are unable to affect the permit price in other countries. Under international permit trade, the price is common across countries, and depends on the total number of permits issued. The following lemma solves for the equilibrium permit price.

**Lemma 3.2.** The equilibrium permit price is independent of time and the stocks of renewable energy capacity, and satisfies

\[
p_{\text{trade}}^{*} = \bar{D} \frac{1 + \Omega}{1 + \bar{\Omega}} > \bar{D}, \tag{3.17}
\]

where \( \bar{D} = \sum_j D_j / N \) is the average marginal damage of emissions across countries, and where we have defined, for notational purposes, \( \Omega_j = -\frac{\beta \delta}{1 - \beta \delta} r_j' (\tilde{p}) \cdot (\sum_j (e_j' (p) - r_j' (\tilde{p})))^{-1} > 0 \), \( \Omega = \sum_j \Omega_j \) and \( \bar{\Omega} = \Omega / N \).

**Proof.** See Appendix 3.A.2.

From the reactions outlined in Lemma 3.1.1, we know that the initial supply, \( s_t \), will be time-independent: whenever a country experiences an increase in their stock of renewable energy capacity, it will respond by issuing fewer permits, one for one. We also know that when \( s_t \) is determined so is the price \( p_t \). This means that the equilibrium permit price will be independent of time and the stocks of renewable energy capacity.

Furthermore, Proposition 3.1 states that a country can induce other countries to issue fewer permits in the future by withholding permits itself today. Its incentive to withhold permits depends on the extent to which it is able to induce increased investments in other countries today. In Equation 3.17, \( \Omega_j \) measures the present value of the discounted additional future investments in country \( j \), when one permit is withheld today: investments in country \( j \) will increase by \( r_j' (\tilde{p}) \) per unit of price increase. Each unit increase in \( r_j \) translates into a \( \delta \) unit increase in the stock in the next period, a \( \delta^2 \) increase two periods ahead, and so on ad infinitum. For each unit increase in the renewables.
stock, country \(j\) will issue one less emission permit, by Lemma 3.1.1. Thus, an increased stock of \(\delta\) units tomorrow translates into \(\delta\) fewer units of emissions tomorrow. Including \(\beta\) accounts for the discounting of the value of these emission reductions in terms of welfare. The sum of all \(\Omega_j\)'s is denoted \(\Omega\); the present value of the aggregate increase in renewables investments following a one unit decrease in the permit supply. By Lemma 3.1, \(\Omega\) thus also measures the present discounted value of the total future emissions avoided when one permit is withheld today. \(D \cdot (1 + \Omega)\) thus measures the present discounted value in terms of welfare to the average country, of the total emissions avoided when one permit is withheld; it is the value of one unit of emission reduction today plus the present value of the future emission reductions, given by \(\Omega\). Finally, \(\bar{\Omega}\) is the average \(\Omega_j\).

Lemma 3.2 hence shows that, due to the strategic complementarity outlined in Proposition 3.1, the equilibrium price under international permit trade is higher than the average marginal damage. The average price under autarky is equal to the average marginal damage (Equation (3.14)). Hence, the international permit market will increase the average price paid for emission permits relative to the situation under autarky. Countries withhold permits every period to induce other countries to withhold permits in the future, which leads to a higher equilibrium price. Furthermore, the lemma demonstrates that the more able countries are to affect each other in this manner, the higher the permit price is.

The fact that the equilibrium price under international permit trade, \(p_{trade}\), as well as each of the autarky permit prices, \(p_{auti}\), and the first-best price, \(p_{FB}\), are time- and stock-independent means that consumption, investment, emissions, and welfare can easily be compared in all time periods, not only in a steady state. The following results thus hold independently of whether renewables stocks are at their steady state levels or not.

### 3.3.3 Welfare implications

We now present the welfare implications of the equilibrium presented above. Intuitively, the introduction of international trade in permits might affect welfare in two ways. Firstly, by equalizing the price on emissions across countries, trade will lead to a cost-efficient distribution of abatement, regardless of the aggregate abatement level. Secondly, we have seen that international permit trade affects the incentives faced by countries when issuing permits and the equilibrium price. Therefore, international permit trade may also affect the prevailing aggregate emission level. The first effect is well understood, and in this paper we are mainly interested in the second. Therefore, when presenting...
the following result, we first assume that all countries are identical in order to remove the scope for pure cost-efficiency gains.

**Proposition 3.2.** Linking the permit markets of $N$ identical countries reduces emissions and increases welfare in every country by increasing investments and reducing consumption: $r^{\text{trade}}_i > r^{\text{aut}}_i$, $e^{\text{trade}}_i < e^{\text{aut}}_i$, $f^{\text{trade}}_i < f^{\text{aut}}_i$, and $V^{\text{trade}}_i > V^{\text{aut}}_i$, $\forall i$. There is no trade taking place in equilibrium.

**Proof.** The proof follows from Lemma 3.2. When all $N$ countries are identical, they share the same autarky price, $\overline{D}$. As $p^{\text{trade}} > \overline{D}$, it follows that every consumer and every producer experiences a price increase when international trade is introduced. This results in reduced consumption and increased investments in every country, and thus reduced emissions. As emissions in each country are inefficiently high under autarky, these emission reductions increase aggregate welfare. For identical countries, this means that welfare is increased in every country. I□

We see that welfare is increased when international permit trade is introduced even if all countries are identical. In this setting, there is no scope for traditional gains from trade since abatement efforts are also distributed cost-efficiently across countries under autarky. The welfare improvement is thus due to reduced emissions. Countries will voluntarily withhold permits when permit markets are linked following the intuition provided in the discussion after Lemma 3.1.

The current literature on non-cooperative international permit trade has not taken into account investments in a durable substitute technology. The typical finding in this literature is that there, a priori, is no reason to expect lower emissions as a result of linking permit markets. Indeed, if all countries face the same marginal damage of emissions, Helm (2003) shows that, in a static model, the price prevailing in the permit market will be the common marginal damage. In Helm’s static model, countries have no means to induce other countries to abate more, and they have no incentive to abate in excess of the autarky level. As we have shown, this is no longer the case when the dynamics of the substitute technology stock is taken into account. The durability of the renewables stock creates a mechanism through which future emissions can be affected by current governments.

The incentive to withhold permits leading to the result presented in Proposition 3.2 is not dependent on the countries being identical. Thus, welfare should increase under international permit trade, even if countries differ along some dimensions. However, the distribution of welfare effects across countries will depend on the characteristics of each country. Our next result deals
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with the effect of linking permit markets on emissions and welfare when countries differ along all dimensions, except their marginal damages from climate change.

**Proposition 3.3.** Consider a group of $N$ countries with identical marginal damages, $D_i = D, \forall i$. Linking the permit markets of these countries reduces emissions in every country and increases aggregate welfare by increasing investments and reducing consumption: $r_i^{\text{trade}} > r_i^{\text{aut}}, \epsilon_i^{\text{trade}} < \epsilon_i^{\text{aut}}, f_i^{\text{trade}} < f_i^{\text{aut}}, \forall i$, $\sum_i V_i^{\text{trade}} > \sum_i V_i^{\text{aut}}$.

**Proof.** Follows from Lemma 3.2. With a common marginal damage, countries share the same autarky price, $\bar{D}$. As $p^{\text{trade}} > \bar{D}$, it follows that every consumer and every producer experiences a price increase when international trade is introduced. This leads to reduced consumption and increased investments in every country, and hence reduced emissions. As the marginal costs and utilities equal $\bar{D}$ under autarky, and $\bar{D} < \sum_j D_j$, it follows that aggregate welfare increases when emissions decrease. \hfill $\Box$

When all countries face the same marginal damage from emissions, introducing international permit trade increases the permit price faced by consumers and investors in every country. With emissions above their first-best level, the resulting emission reductions will increase aggregate welfare. However, if utility from consumption and costs of investments differ greatly among countries, the welfare gain will not be evenly distributed. The gain from reduced emissions is the same in all countries whereas the cost of decreased consumption and increased investments will differ. Though aggregate welfare increases, some countries may incur a net loss due to the introduction of international trade in this case. We further explore such cross-country differences in the gains from linking markets in Section 3.3.4.

So far, the key to the welfare results has been the fact that all consumers and producers experience an increased price on emission permits. This price increase leads to emissions reductions in every country. The next proposition concerns the welfare effects that arise when countries differ in their marginal damage, $D_i$. When this is the case, it is no longer clear that all consumers and producers will face a price increase when international permit trade is introduced.

**Proposition 3.4.** Consider a group of $N$ countries, who all have identical quadratic utility and cost functions. Linking the permit markets of these countries reduces aggregate emissions and increases aggregate welfare by increasing aggregate investments and reducing aggregate consumption: $\sum_i r_i^{\text{trade}} > \sum_i r_i^{\text{aut}}, \sum_i \epsilon_i^{\text{trade}} < \sum_i \epsilon_i^{\text{aut}}, \sum_i f_i^{\text{trade}} < \sum_i f_i^{\text{aut}}$ and $\sum_i V_i^{\text{trade}} > \sum_i V_i^{\text{aut}}$. 
Proof. See Appendix 3.A.2. The assumption of quadratic utility and cost functions is sufficient, but not necessary, for the proof.

The equilibrium price (see Lemma 3.2) is the average across all countries’ marginal damages, times a mark-up factor. When countries differ in their marginal damages, it may thus be the case that some countries have a marginal damage that exceeds the equilibrium permit price under international trade. Consumers and producers in such countries will then face a lower price under trade than they do under autarky. This means that emissions from these countries would increase under trade. At the same time, other countries’ consumers and producers will face a price increase which will lead to reduced emissions. However, by restricting the analysis to the case where supply and demand are identical and linear so that the reaction to a given price change is the same for all consumers and investors, the effect of introducing international permit trade is still clearcut: aggregate emissions decrease, and aggregate welfare increases. This is because the strategic complementarity in emission levels results in a price increase in the average country.

If countries differ both in their marginal damages and their cost and utility functions at the same time, the effect on aggregate emissions of introducing international permit trade is ambiguous. The net effect of introducing international permit trade could be positive or negative: if there are consumers and producers facing a price decrease who have sufficiently strong reactions to the price change compared to the reactions in the other countries, total emissions may increase when international trade is introduced. Due to this fact, it is also difficult to discuss the implications of countries being of different sizes, as this involves particular correlations between the different parameters. For a study on this interaction between marginal damage and demand and supply responses in a static setting, see Holtsmark and Sommervoll (2012). However, if the strategic incentive to withhold permits that we have identified is sufficiently strong, no country will face a price decrease when international trade is introduced. In this case, aggregate welfare will increase.

We have seen that aggregate welfare increases when international permit trade is introduced, both when countries are identical, and when they differ in their marginal damages or in their cost and utility functions. In the following, we discuss the determinants of the size of these welfare gains. We first consider the effect of the total number of countries, \( N \), and then move to a discussion of the effect of depreciation and discounting.

Proposition 3.5. As the number of countries, \( N \), increases, the gain to the average country from participation in the international permit market also
increases:
\[
\frac{\partial}{\partial N} \left( \frac{1}{N} \sum_{i=1}^{N} V_{i}^{\text{trade}} - \frac{1}{N} \sum_{i=1}^{N} V_{i}^{\text{aut}} \right) > 0,
\]
provided that the characteristics \((R_i, D_i, u_i(\cdot) \text{ and } c_i(\cdot))\) of the average country do not change.

**Proof.** From Lemma 3.2, it follows that \(\partial p^{\text{trade}}/\partial N > 0\), while from Equation (3.14) it follows that \(\partial p^{\text{aut}}/\partial N = 0\). Average welfare increases with the permit price and the result follows.

Proposition 3.5 implies that the average welfare gains from introducing international permit trade are larger if the number of participating countries is larger. The reason is that when the number of countries in the market is large, the scope for each country to reduce the externalities inflicted on themselves by other countries is also large. In other words, the more countries that participate in the market, the more countries will be affected when country \(i\) withholds permits. One permit withheld has a smaller impact on the international permit price when \(N\) is large. At the same time, the effect of a given price increase on the aggregate foreign stock of renewables is larger when \(N\) is larger. The latter effect dominates, resulting in a higher future welfare gain to country \(i\) from a decrease in \(\omega_i\).

For a given number of participating countries, the strength of the incentives countries have to withhold permits depends strongly on the discount factor and the depreciation rate of the renewables stocks. Rewrite the discount factor as \(\beta = e^{-\rho \Delta}\), where \(\Delta\) is the length of a period, and \(\rho\) is the continuous time-discount rate. Similarly, rewrite the survival rate as \(\delta = e^{-\eta \Delta}\). The extent to which countries limit their permit issuance depends on the countries’ patience, the stock durability, and the length of the time periods, as highlighted in the following proposition.

**Proposition 3.6.** The aggregate gain from linking permit markets \(\sum_i V_{i}^{\text{trade}} - \sum_i V_{i}^{\text{aut}}\) is higher, if

- the length of each time period, \(\Delta\), is shorter, or
- either the depreciation rate of the renewables stocks, \(\eta\), or the time discount rate, \(\rho\), is smaller.

Furthermore, when \(\beta \delta\) is close to 1, aggregate welfare under international permit trade is close to the first-best welfare level: for every \(\varepsilon > 0\), there exists \(\mu > 0\) such that whenever \(1 - \beta \delta < \mu\), \(W^{\text{FB}} - \sum_i V_{i}^{\text{trade}} < \varepsilon\).
3.3. INTERNATIONAL PERMIT TRADE

Proof. Recall that $\beta \delta = e^{-\sigma \Delta}$, where $\sigma = \eta + \rho$. Both $\frac{\partial \beta \delta}{\partial \sigma} < 0$, and $\frac{\partial \beta \delta}{\Delta} < 0$. Furthermore, $p^{\text{trade}}$ increases in $\beta \delta$, while $p^{\text{aut}}$ is a constant. Thus, the gains to introducing linkages increases in $\beta \delta$. Note that $\lim_{\beta \delta \to 1} \Omega = \infty$, which gives $p^{\text{trade}} \to p^{FB}$. Thus, if $\beta \delta$ close to 1 satisfies Equations (3.10) and (3.11), the equilibrium price will be close to the first-best price. Furthermore, we can always choose cost and utility functions $c(\cdot)$ and $u(\cdot)$ such that Equations (3.10) and (3.11) are satisfied, as long as $\beta \delta < 1$. Finally, given the expression for $V^{\text{trade}}_i$ in Equation (3.A.9), $\lim_{p^{\text{trade}} \to p^{FB}} \sum_i V^{\text{trade}}_i = W^{FB}$.

The proposition states that when $\beta \delta$, the product of the discount factor and the survival rate, is higher, the equilibrium permit price will also be higher. Additionally this product is high if either countries are very patient and the stocks are very durable, or if the time periods are short. Recall that neither the first-best price nor the autarky prices depend on the discount factor or the survival rate of the renewables stocks. However, in the presence of international permit trade, these two parameters become relevant, because they determine the strength of the countries’ incentives to withhold permits. A higher survival rate means that withholding permits today will affect future permit issuance more, because increased investments in renewables today will give more long-lasting effects on the stocks. A higher discount factor means that each country values the effect they can get from withholding permits more in terms of welfare. Hence, both long-lasting renewables stocks and patience strengthen the incentives to incur current costs by withholding permits. Shorter time periods will have exactly the same effect, since the future gains become both bigger and more important if the future is closer in time. However, it should be noted that the mechanism behind all our results hinges on the assumption that the renewables producers have sufficient time to react to a change in the equilibrium price before the cap is reset. Very short time periods will of course inhibit such reactions. Furthermore, the assumption of an interior solution also places restrictions on how high the product $\beta \delta$ can be for given parameters in the rest of the model, since renewables investments in each time period will increase monotonically in $\beta \delta$.

From Lemma 3.2, it is also clear that $\delta = 0$ gives $\Omega = 0$ and $p^{\text{trade}} = \mathcal{D}$. This means that if the renewables stocks are not durable, the equilibrium price under international permit trade is equal to the average of all countries’ autarky prices. With no durability, the countries cannot affect their trading partners’ permit issuance in the future even if there is international trade, because increased investments following a price increase will only lead to more renewables available within the same time period. In this case, our model only constitutes a static stage game repeated infinitely many times. This stage
game is studied by Helm (2003), who indeed shows that the permit price in equilibrium will be equal to the average marginal damage across countries.

Finally, we show that if the cap on emissions is set once and for all, meaning that countries issue permits only once, there is no strategic incentive to withhold permits. In this case, the strategic complementarity in issuance vanishes because there is no future permit issuance that can be affected by withholding permits today. As long as the renewables producers have incentives to react to current price changes, there is a clear benefit to resetting the cap often. This conclusion stands in contrast to the conclusions of several papers in the literature. Harstad and Eskeland (2010) find that permits should be long-lasting to avoid costly signaling by firms with private abatement costs. Harstad (2015) finds that climate agreements should be long-lasting to avoid that the costly hold-up problem appears “too often”. Battaglini and Harstad (2015) find the same, and demonstrate that the endogenous duration of the climate treaty can be leveraged to support equilibria with large coalitions. Our conclusions are in line with Battaglini and Harstad (2015) as we show that endogenous and non-contractible technology investments may lead to higher emission reductions.

To summarize, in this sections we show that introducing international permit trade may substantially increase welfare. We identify a mechanism that creates strategic complementarity in permit issuance among countries, which is only in place when there is a common permit price. Furthermore, we show that the size of the welfare gains which can be reaped by introducing such trade depends strongly on the durability of the renewable energy production capacity stocks, and on the level of patience of the governments in countries participating in the market.

3.3.4 Implications for different countries

In the previous section, we argue that some countries may benefit more than others when international permit trade is introduced. In this section, we study which country characteristics that determine this heterogeneity in outcomes. Whether or not a particular country gains depends on the extent to which this country benefits from reduced emissions and to what extent the country benefits from buying and selling permits in the international market. Countries with higher marginal damage gain more from the reduced emissions following the introduction of international permit trade, but, as the next proposition demonstrates, will also to a larger extent import permits, which is costly.

**Proposition 3.7.** Consider two countries, $i$ and $j$. Country $i$ will import more permits than country $j$ if either
1. country $i$ has a higher marginal damage, $D_i > D_j$, all else equal, or

2. country $i$ has less price-responsive renewables producers, $r'_i(\cdot) < r'_j(\cdot)$, all else equal.

**Proof.** Insert the expression for the continuation values from Lemma 3.1.2 into the first-order condition (Equation (3.16)). Then insert from the definition of $\Omega_i = -\frac{\beta \delta}{1 - \delta} r'_i(\hat{p})p'(s) > 0$. A country’s net sales of permits in the international permit market is given by its trade balance ($TB_i = \omega_i + R_i + r_i - e_i$). Solve the first-order condition for $TB$, to get:

$$TB_i = \frac{D_i (1 + \Omega)}{p'(s) \left( \frac{1 + \Omega}{1 + \Omega} - \frac{D_i}{D} \right)} > 0.$$  \hspace{1cm} (3.18)

If countries only differ in marginal damage and we sort them along a line by this parameter, countries with higher-than-average marginal damage will be importers of permits, while countries with lower-than-average marginal damage will be permit exporters. Similarly, with the price-responsivity of their renewables producers, countries with the least price-responsive renewables producers will be permit importers. These countries face stronger incentives to withhold permits as their trade partners are more price-responsive and will reduce their future permit issuance the most in response to a permit being withheld today.

Countries with higher-than-average marginal damage gain from reduced emissions but they must buy permits from the low-damage countries in order to reduce emissions. A priori it is not, therefore, obvious whether high- or low-damage countries gain the most from introducing international permit trade. As the next proposition demonstrates, this depends on the parameters of the model.

**Proposition 3.8.** Assume that countries have identical, quadratic utility and cost functions but their marginal damages differ ($D_i \neq D_j$, if $i \neq j$). Then:

1. In the static model ($\beta = \delta = 0$), low-damage countries gain more from introducing permit market linkages than do high-damage countries: $V^{\text{trade}}_i - V^{\text{aut}}_i > V^{\text{trade}}_j - V^{\text{aut}}_j$, if $D_i < D_j$.

2. There exists a threshold $\beta \delta \in (0,1)$ such that if $\beta \delta > \beta \delta$, high-damage countries gain more from introducing permit market linkages than do low-damage countries: $V^{\text{trade}}_i - V^{\text{aut}}_i > V^{\text{trade}}_j - V^{\text{aut}}_j$, if $D_i > D_j$. 

\hfill \Box
Proposition 3.8.1 is a corollary to Helm (2003)'s Proposition 1, which states that low-damage countries are permit sellers in the static model. Under constant marginal damages, the static permit market delivers no emission reductions “on average,” and the permit market is merely a transfer scheme from high- to low-damage countries. Thus, the low-damage countries benefit and the high-damage countries lose when international permit trade is introduced.

Proposition 3.8.2 states that in the dynamic model, as the countries become patient enough and the renewables stocks become durable enough, this ranking is reversed. In this case, high-damage countries gain more from introducing international permit trade than do low-damage countries. Although according to Proposition 3.7, high-damage countries are still permit importers, when $\beta \delta$ is high enough, the permit market delivers sufficient emission reductions for the high-damage countries to gain more than low-damage countries.

In this section, we demonstrate how high-damage countries tend to be permit importers, while low-damage countries are exporters. When the world resembles the static model ($\beta \delta$ low), this means that low-damage countries benefit more from international trade. As the dynamic dimension becomes more pronounced ($\beta \delta$ high enough), high-damage countries will gain more than low-damage countries. The reason is that the incentives to withhold permits increase strongly in the discount factor and the survival rate of the renewables stocks, as demonstrated in Proposition 3.6.

### 3.4 Trade in technology

Under neither autarky nor international permit trade do we allow for international trade in renewables. This is done in order to focus on the effect of opening up the permit market. However, the mechanism leading to welfare gains from introducing trade in permits is driven by the common price on emission permits—and thus renewables—among countries. It is this common price that makes it possible for each country to affect renewables investments in other countries, and by extension, future permit issuance in other countries. The common price and the possibility of affecting issuance decisions in other countries can, however, also be achieved by simply establishing trade in renewables, even absent international permit trade. Given an assumption that guarantees an interior solution to the problem of each country, a stricter emission cap in one country would—through the common price on renewables, and thus permits—affect investments in exactly the same way as in our basic
model with international permit trade. Our next proposition underlines that trade in renewables is sufficient to generate welfare gains.

**Proposition 3.9.** International trade in renewable energy alone is sufficient for countries to face strategic incentives to limit domestic permit issuance and for the welfare effects established in earlier results to accrue. Specifically, Propositions 3.1, 3.2, 3.3, 3.4, 3.5, and 3.6 carry over to a setting with international trade only in renewables provided that \( e_i(p^{\text{trade}}) > \omega_i > 0 \) for every country \( i \).

**Proof.** Consider international trade only in renewables. Market clearing requires

\[
f_{jt} = \omega_{jt}, \quad \forall j, t,
\]

\[
\sum_j z_{jt} = \sum_j (R_{jt} + r_{jt}), \quad \forall t
\]

\[
= \sum_j (f_{jt} + z_{jt}) = \sum_j (e_{jt}) = \sum_j (R_{jt} + r_{jt} + \omega_{jt}) \quad \forall t,
\]

which is the same aggregate condition as for the case with international permit trade only. As long as \( e_i(p^{\text{trade}}) > \omega_i > 0 \), the equilibrium remains unchanged. This condition is trivially satisfied if countries are identical.

The model presented in this paper has been applied to climate change. Though every common good problem is different, some of our findings might also be useful for other international common good problems. In these cases, Proposition 3.9 can potentially be of importance. Permit trade, including trade between consumers in different countries, has been established in several places. One example is the EU Emissions Trading System. Another example is the link between the permit markets of California and Quebec, implemented as of January 2014. Introducing international permit trade therefore seems feasible in the case of climate change. Furthermore, because international trade in renewables could potentially involve large transaction costs, there is reason to believe that trade in emission permits is the simplest way to reap the gains from a common price. However, it is likely that there are other international common good problems where trade in substitute goods or technologies can more easily be implemented than trade in allowances to exploit the good. Furthermore, in various international common good problems, there may be political constraints that make permit markets difficult to establish. Proposition 3.9 shows that the positive welfare effects can still be reaped if there are durable substitutes to exploitation.
3.5 Discussion

In this section, we discuss some of the assumptions we make in Section 3.3.

3.5.1 Endogenous fossil energy

Throughout Section 3.3, we simplify the way fossil energy supply enters our model by allowing fossil energy to be available at zero price to all consumers. We claim that this does not drive our results. Though a full analysis of the inclusion of fossil energy in our model is out of scope, here we provide the basis for that claim. Our results rely on the fact that international permit trade turns permit issuance into intertemporal strategic complements between countries. This complementarity arises because fewer permits in the market today increases investments in all countries, leading to reduced permit issuance by every country in the future.

We argue that endogenous fossil energy supply could provide a separate channel of intertemporal strategic complementarity. Within a time period, the equilibrium producer price of fossil energy increases with the number of permits available in the market, all else equal. Hence, by withholding permits, a country will decrease the price of fossil energy. Furthermore, we assume that investments in fossil energy production capacity, such as exploration of new fields, are increasing functions of the current producer price. If so, any permit withheld will lead to lower fossil production capacity in countries where fossil energy production is possible. Finally, the higher the production capacity for fossil energy in a country, the more costly a tight cap on emissions will be for this country. Thus, lower fossil investments in one time period will lead the country to issue fewer permits in future periods because its marginal cost of lowering the price of fossil energy is lower. This means that any country can, by withholding permits today, induce other countries to reduce their future permit issuance not only by increasing investments in renewables, but also by reducing investments in fossil energy. By this logic, endogenous fossil energy supply provides a separate channel for strategic complementarity in permit issuance. Though a model taking all features of fossil energy supply into account is out of scope for this paper, we provide a simple two-period model in Appendix 3.C.1, illustrating the mechanism outlined here.

3.5.2 Politically determined investments

So far, we have assumed that investments in renewables are made by price-taking private investors and that the governments employ no policy instrument
other than the traded emission permits. There are results in the literature indicating that if countries are allowed to set their own domestic policies in addition to participating in a permit market, the benefits of the permit market may be dissipated. Godal and Holtsmark (2011) show that, when allowed to, every country will implement policies that maximize its welfare ex post, and the permit market will only act as a transfer mechanism from low- to high-damage countries.

It is also the case that investments in renewable energy are highly politicized in many countries. Therefore, we have briefly investigated how robust our results are to allowing the government in each country to regulate its own renewables producers. In Appendix 3.C.2, we solve a two-period model where the governments politically determine investments in renewables under the same timing as in the basic model. However, we do not assume that the governments act as price takers when they decide on the optimal investments. Instead, they take the price decrease following higher investments into account. We show that withholding permits today also affects renewables investments in the case where the governments determine these investments. The equilibrium permit price under international permit trade will therefore be higher than the average price under autarky, even in a situation in which the governments determine renewables investments. Thus, the main result from our basic model also prevails in this setting.

Although somewhat weakened, the strategic mechanism created by international permit trade is still in place. This is because each government will let investments react to price changes, meaning that in this case, as in the basic model, a higher permit price results in higher investment in all countries. As in our basic model, there is still, therefore, a benefit to withholding permits that goes beyond the direct effect on emissions.

### 3.5.3 The shape of the damage function

In Lemma 3.1.1 we see that the linear damage function results in countries reducing their permit issuance one for one when their stocks of renewable energy capacity increases. If instead we assume the damage function to be convex in the atmospheric stock, countries would not respond by reducing issuance one for one. Instead, they would reduce issuance by \(1 - \varepsilon\). The basic reason is that a convex damage function introduces a strategic substitutability, as explained for instance by Hoel (1991) or Fershtman and Nitzan (1991). To see this, note that if country \(i\) responded by reducing its issuance one for one when its stock of renewables increased, the marginal damage faced by other countries would be reduced, while their marginal revenue would remain unchanged. All other
countries would thus want to increase their issuance, which in turn decreases the permit price. The equilibrium response with convex damages for country \(i\) would then be to decrease its issuance by \(1 - \varepsilon\), while the other countries marginally increase their issuance. Although a convex damage function would complicate the analytical solution to our model, this argument shows that the main mechanism we identify in this paper does not depend on our simplification of the damage function. Even with a convex damage function, countries could still induce their trading partners to emit less in the future, by emitting less themselves today. The intuition from Lemma 3.1.1 thus carries over to a setting with convex damages. Furthermore, the incentive to react slightly less to a higher renewables stock induced by the convexity would be exactly the same under autarky as under international permit trade.

3.5.4 Excess renewables supply

Throughout the paper, we have assumed that Equations (3.10) and (3.11) hold in order to ensure that both permits and renewable energy are consumed in all countries. This ensures that the international permit price will equalize the price of renewable energy across countries. In Appendix 3.C.3 we present a two-period model in order to shed some light on the case when these conditions fail to hold for some subset of countries. We consider a situation where, at the current international permit price, domestic energy demand in some country \(i\) is completely saturated by domestic renewable energy. In this case, the consumers in country \(i\) demand no permits, but their government can still issue and sell permits on the international market. In a one-shot game, or if the decision-makers were completely myopic, this would only have a distributional impact on the equilibrium. The efficiency-properties—here understood in terms of the implemented permit price—would be independent of the fact that one country no longer demands permits. The marginal effects of issuing another emission permit—increased domestic damage from an increase in global emissions and a depressed permit price—does not depend on whether domestic consumers consume fossil energy. The positive marginal utility stemming from increased consumption of the cheaper fossil energy is canceled against the price paid for the cheaper permits. Thus the marginal tradeoff a myopic government faces does not change.

In the dynamic model, however, the fact that international permit demand is absent from a subset of countries changes the equilibrium more substantially. If domestic demand in country \(i\) is saturated by renewables, the renewables producers of country \(i\) do not react to a change in the international permit price. Only the domestic energy price in country \(i\) will be relevant and this
price is decoupled from the international permit price. Neither will the permit issuance decision of government \( i \) be affected by a change in its domestic stock of renewable energy capacity, as its marginal revenue from issuing another permit only depends on the aggregate number of permits issued. So far, this is parallel to the static case.

Imagine a situation where all countries start out with a renewables stock, \( R_{i0} \), which is below its steady-state value. For countries in which the steady-state value is such that Equations (3.10) and (3.11) do not hold, this means that the renewables stock will eventually reach a point where the country is saturated, meaning that at the international permit price, there is no demand for permits from this country. Since the energy price in a saturated country is decoupled from the international permit price, renewables producers in these countries do not react to increases in this price. Furthermore, since the permit price does not affect its consumers and producers, the government in such a country does not adjust its future permit issuance in response to changes in the current price. Therefore, the more countries whose domestic demands are saturated, the weaker the intertemporal strategic complementarity. The future permit issuance of saturated countries cannot be affected by the current issuance and permit price. As more countries become saturated, there remains fewer countries that can be affected by withholding permits. As a result, the equilibrium permit price decreases. This suggests the following intuition: at first, every country has some residual demand for permits, and the international permit price is given by the stationary expression in Lemma 3.2. Then, as the renewable energy stocks increase, some countries may gradually be saturated by renewables and Equations (3.10) and (3.11) no longer hold for these countries. They thus become unresponsive to changes in the permit price. This weakens the incentive other countries face to withhold permits and the permit price gradually declines as more and more countries become saturated. The permit price stabilizes at some level between the price given in Lemma 3.2 and the myopic price \( D \), where potentially only a subset of the countries still consume fossil fuels.

3.6 Conclusion

The global climate is an international common good and suffers from the tragedy of the commons. Since there is no super-national decision maker who can implement efficient emission levels, it is important to identify institutions and mechanisms that can change the incentives countries face. Linkages between national emission permit markets could potentially constitute such an
institution and the number of existing domestic and regional emission permit markets is high and increasing. The effect of linkages between such markets is therefore important to understand.

In this paper, we consider a situation in which there are investments in renewable energy production capacities such as hydro power plants or wind farms. We show that, even if countries do not cooperate on the emission caps they set, a simple linkage between their emission permit markets leads to reduced emissions and higher welfare. This is the case even if we allow for countries to be identical so that no trade takes place in equilibrium.

The findings in this paper highlight the importance of including dynamics when studying international permit trade. Without investments in durable renewable energy capacity, there are no links between current and future permit issuance in the market. Countries are then unable to influence each other, and have no incentives to reduce their permit issuance. The existing literature on non-cooperative permit trade typically concludes that there is no a priori reason to expect permit trade to reduce emissions. We show that allowing for dynamics changes this conclusion.

In the wider literature on provision of public goods, the typical finding is that the outcome is worse when dynamics are taken into account. Our conclusions challenge this finding, as including dynamics in our model leads to increased welfare from linking permit markets. Furthermore, our conclusions also differ from those of the existing literature along other dimensions. One important difference is that while we find that welfare increases when permits are short-lived, the typical finding is that permits should be long-lasting.

According to our results, there can be substantial gains from linking permit markets. An important issue for future research is to identify which links are most beneficial to undertake. This will depend on properties of the links that are considered, such as country characteristics, linking protocols or the timing of linkages. Finally, the insights from this paper are also applicable to other international common good problems. Further research should seek to shed light on the dynamic effects of introducing international trade in either exploitation allowances or substitute technologies for other international common good problems.
3.A Appendix - the dynamic game

3.A.1 Equilibrium under autarky

The first-order condition solving country $i$’s maximization problem becomes:

$$0 = u'_i(e_{it})c'_i(p_{it}) - c'_i(r_{it})r'_i(\hat{p}_{it}) \frac{d\hat{p}_{it}}{dp_{it}} - D_i \left( c'_i(p_{it}) - r'_i(\hat{p}_{it}) \frac{d\hat{p}_{it}}{dp_{it}} \right) + \beta \delta r'_i(\hat{p}_{it}) \frac{d\hat{p}_{it}}{dp_{it}} \frac{\partial V_{it+1}^{out}}{\partial R_{it+1}}$$

(3.A.1)

To find the continuation values, we differentiate through (3.13) with respect to $R_{it}$, using the envelope theorem:

$$\frac{\partial V_{it}^{out}}{\partial R_{it}} = D_i + \beta \delta \frac{\partial V_{it+1}^{out}}{\partial R_{it+1}}$$

$$= D_i + \beta \delta D_i + (\beta \delta)^2 D_i + \cdots$$

$$= \frac{D_i}{1 - \beta \delta} = \frac{\partial V_{it+1}^{out}}{\partial R_{it+1}}$$

Inserting for $c'_i(r_{it})$ from (3.6), for $u'_i(e_{it})$ from (3.3) and for $\frac{\partial V_{it+1}^{out}}{\partial R_{it+1}}$, the first order condition can now be rewritten as follows:

$$0 = c'_i(p_{it})(p_{it} - D_i) - r'_i(\hat{p}_{it}) \frac{d\hat{p}_{it}}{dp_{it}} \left( \hat{p}_{it} - \frac{D_i}{1 - \beta \delta} \right),$$

and we see that $p_{it}^{out} = D_i$, giving $\hat{p}_{it} = \frac{D_i}{1 - \beta \delta}$, solves the problem in every period. Future prices are then independent of the price set today, hence $d\hat{p}_{it}/dp_{it} = 1$.

The value function is linear, and given by:

$$V_{it}^{out}(R_1, \ldots, R_N) = \frac{1}{1 - \beta} \left[ u_i(e_i(D_i)) - c_i\left( r_i\left( \frac{D_i}{1 - \beta \delta} \right) \right) - D_i \sum_j c_j(D_j) \right.

+ \frac{D_i}{1 - \beta \delta} \sum_j r_j\left( \frac{D_j}{1 - \beta \delta} \right) \left. + \frac{D_i}{1 - \beta \delta} \sum_j R_j \right]$$

(3.A.2)

where time indices are dropped for simplicity.

3.A.2 Markov perfect equilibrium under permit trade

Throughout this appendix, we omit the superscript trade on the value functions.
Proof of Lemma 3.1

The government solves

\[ V_{i,t}(R_{1t}, \ldots R_{Nt}) = \max_{\omega_{it}} \left\{ u_i(e_i(p_t(s_t))) - c_i(r_i(\hat{p}_t)) - D_i \sum_j \omega_{jt} 
\right. \]

\[ + p_t(s_t) \cdot (\omega_{it} + R_{it} + r_i(\hat{p}_t) - e_i(p_t(s_t))) 
\]

\[ + \beta V_{i,t+1}\left(\delta(R_{1t} + r_1(\hat{p}_t)), \ldots, \delta(R_{Nt} + r_N(\hat{p}_t))\right) \right\} \]

subject to Equations (3.10) and (3.11), and where \( e_i(p) \) is given by the solution to the representative consumer’s problem, and \( r_i(\hat{p}) \) by the representative renewables producer’s solution. Furthermore, \( p_t(s_t) \) is implicitly given by the market clearing condition:

\[ \sum_j e_{jt}(p_t) = \sum_j (\omega_{jt} + R_{jt} + r_{jt}(\hat{p}_t)) \quad \text{with} \quad \sum_j (\omega_{jt} + R_{jt}) \equiv s_t. \]

(3.A.4)

\( \hat{p}_t \) can also potentially depend on \( s_t \) through the effect current supply will have on future stocks, and not only in the current period.

We get the following \( N \) first-order conditions:

\[ u_i'e_i'p_t' + p_t' \cdot (\omega_{it} + R_{it} + r_{it} - e_{it}) + p_t \cdot (1 + r_{it}' \frac{dp_t}{d\omega_{it}} - e_{it}'p_t') 
\]

\[ - c_i'r_{it}' \frac{dp_t}{d\omega_{it}} - D_i + \beta \delta \frac{dp_t}{d\omega_{it}} \sum_j r_{jt}' \frac{\partial V_{i,t+1}}{\partial R_{j,t+1}} = 0 \quad \forall i, t. \]

(3.A.5)

Using the consumers’ first-order condition, we can eliminate \( e_{it}'p_t'(u_i' - p_t) = 0. \) In order to find the continuation values, differentiate through (3.3) wrt. \( R_{jt} \):

\[ \frac{\partial V_{i,t}}{\partial R_{jt}} = u_i'e_i'p_t' \frac{ds_t}{dR_{jt}} + p_t' \cdot (\omega_{it} + R_{it} + r_{it} - e_{it}) + p_t \cdot (1 + r_{it}' \frac{dp_t}{dR_{jt}} - e_{it}'p_t') 
\]

\[ + p_t' \frac{ds_t}{dR_{jt}} \cdot (\omega_{it} + R_{it} + r_{it} - e_{it}) - c_i'r_{it}' \frac{dp_t}{dR_{jt}} - D_i \sum_k \frac{\partial \omega_{kt}}{\partial R_{jt}} 
\]

\[ + \beta \frac{\partial V_{i,t+1}}{\partial R_{j,t+1}} + \beta \delta \frac{dp_t}{dR_{jt}} \sum_k r_{kt}' \frac{\partial V_{i,t+1}}{\partial R_{k,t+1}}, \]

which of course depends on the policy functions \( \omega_{it}(\{R_{jt}\}_{j=1}^N) \). To find these, differentiate through the first-order conditions (Equation (3.5)) with respect to \( R_{jt} \). Guess that because of the linear damage function, the value functions will be linear in each technology stock. This guess will later be verified.
\[ \frac{\partial \omega_{it}}{\partial R_{jt}} = -1, \quad \frac{\partial \omega_{jt}}{\partial R_{jt}} = 0, \quad j \neq i, \quad \Rightarrow \sum_k \frac{\partial \omega_{kt}}{\partial R_{jt}} = -1, \quad \frac{ds_t}{dR_{jt}} = 0, \quad \forall j, t, \] (3.A.7)

which is stated in Lemma 3.1.1.

To see this, note that given these reactions, an increase in the renewables stock of country \( j, R_{jt} \), will not change the equilibrium price, since \( s_t \) is independent of \( R_{jt} \). Neither will it change future prices, since these can only be affected through the future supply \( s_t \), which is independent of the renewables stocks:

\[
\frac{ds_{t+\tau}}{dR_{jt}} = \sum_k \frac{ds_{t+\tau}}{dR_{kt+\tau}} \frac{dR_{kt+\tau}}{dR_{jt}} = 0, \quad \forall k, t.
\]

\[
\Rightarrow \frac{d\hat{p}_t}{dR_{jt}} = \sum_{\tau=t}^{\infty} (\beta \delta)^{\tau-t} p_{\tau} (s_{\tau}) \frac{ds_{\tau}}{dR_{jt}} = 0 \quad \forall k, t.
\]

Furthermore, changes in the renewables stocks will not affect the price change following from an increase in the permit supply, since it follows that:

\[
\frac{d}{dR_{jt}} \left( \frac{d\hat{p}_t}{d\omega_{it}} \right) = 0, \quad \forall k, t.
\]
It then follows that future prices are independent of changes in the current supply, $s$

$$\frac{d\hat{p}_t}{ds_t} = \sum_{\tau=t}^{\infty} (\beta \delta)^{\tau-t} p'_\tau(s_\tau) \frac{ds_\tau}{ds_t} = p'_t(s_t) + \sum_{\tau=t+1}^{\infty} (\beta \delta)^{\tau-t} p'_\tau(s_\tau) \frac{ds_\tau}{dR_j\tau} \frac{dR_j\tau}{ds_t} = p'_t(s_t),$$

By the market clearing condition (Equation (3.A.4)), we must have:

$$\sum_j e'_jt p'_t(s_t) = 1 + \sum_j r'_jt \frac{d\hat{p}_t}{ds_t} = 1 + \sum_j r'_jt p'_t(s_t) \Rightarrow p'_t(s_t) = \frac{1}{\sum_j (e'_jt - r'_jt)} < 0.$$  

Note here, that given the market clearing condition, the price in period $t$ is determined only by $s_t$. This means that $p_t$ will be independent of state and time.

Returning to the continuation values, we now have

$$\frac{\partial V_i,t}{\partial R_j,t} = -D_i + \beta \delta \frac{\partial V_i,t+1}{\partial R_j,t+1},$$

which clearly is independent of both $j$ and $t$. It follows that:

$$\frac{\partial V_i}{\partial R_j} = \frac{D_i}{1 - \beta \delta}, \ \forall \ i, j, t,$$

as stated in Lemma 3.1.2.

The proof of Lemma 3.1 is concluded by verifying our assumption of a linear value function:

**Verifying linear value function**

By assuming the value function of each country to be linear in the state variables, we solved the dynamic game, and found the reaction functions stated in Lemma 3.1.1;

$$\frac{\partial \omega^{eq}_i}{\partial R_j} = \begin{cases} -1 & \text{if } j = i \\ 0 & \text{if } j \neq i. \end{cases}$$
In this section, we show explicitly that given these policy responses to changes in the state variables, the value functions will indeed be linear, and we calculate the value functions.

Given the assumption that:

\[ V_{it}(R_1, \ldots, R_N) = A_{it} + \sum_j B_{ijt}R_{jt}, \]

we must of course have that:

\[ A_{it} + \sum_j B_{ijt}R_{jt} = \max_{\omega_{it}} \left\{ u_i(\epsilon_i(p_t(s_t))) + p_t(s_t) \cdot (\omega_{it} + R_{it} + r_i(p_t) - e_i(p_t(s_t))) \right\} + c_i(r_i(p_t)) - D_i \sum_j \omega_{jt} + \beta A_{it} + \beta \delta \sum_j B_{ijt}(R_{jt} + r_{jt}(\hat{p}_t)) \}

(3.A.8)

Differentiating through (3.A.8) with respect to \( R_j \) gives:

\[
B_{ijt} = u_t'(\epsilon_t'(p_t') \frac{ds_t}{dR_{jt}} + p_t' \frac{ds_t}{dR_{jt}} (\omega_{it} + R_{it} + r_{it} - e_{it}) + p_t \left( \frac{\partial \omega_{it}}{\partial R_{jt}} + \frac{dR_{it}}{dR_{jt}} + r_{it}' \frac{d\hat{p}_t}{ds_t} \frac{ds_t}{dR_{jt}} - e_{it}' \frac{d\hat{p}_t}{ds_t} \frac{ds_t}{dR_{jt}} \right) - c_i'(r_i(p_t)) - D_i \sum_k \frac{\partial \omega_{kt}}{\partial R_{jt}} + \beta \delta B_{ijt+1} + \beta \delta \sum_k B_{ikt+1} r_{kt}' \frac{d\hat{p}_t}{ds_t} \frac{ds_t}{dR_{jt}},
\]

and if we insert for the reaction functions, it follows that:

\[ B_{ijt} = D_i + \beta \delta B_{ijt+1} = D_i \beta \delta D_i (\beta \delta)^2 D_i + \cdots \]

\[ \Rightarrow B_{ijt} = \frac{D_i}{1 - \beta \delta} \quad \forall j, t, \]

verifying the slope of the value function found earlier.

Secondly, inserting this in Equation (3.A.8) gives:

\[ A_{it} + \frac{D_i}{1 - \beta \delta} \sum_j R_{jt} = u_i(\epsilon_i(p_t(s_{t^{trade}}))) - c_i(r_i(p_t^{trade})) - D_i \sum_j \omega_{jt}^{trade} + p_t(s_{t^{trade}}) \cdot (\omega_{it}^{trade} + R_{it} + r_i(p_t^{trade}) - e_i(p_t(s_{t^{trade}}))) + \beta A_{it} + \beta \delta \frac{D_i}{1 - \beta \delta} \sum_j (R_{jt} + r_{jt}(\hat{p}_t^{trade})), \]

where \textit{trade} denotes Markov perfect equilibrium values under permit trade.
Using the fact that market clearing implies $\sum_j \omega_{jt} = \sum_j (e_{jt} - R_{jt} - r_{jt})$ in all time periods, we can solve for $A_{it}$:

$$A_{it} = \frac{1}{1 - \beta} \left[ u_i(e_i(p_{t}^{\text{trade}})) - c_i(r_i(p_{t}^{\text{trade}})) + p_{t}^{\text{trade}} \cdot TB_i + D_i \sum_j r_j(p_{t}^{\text{trade}}) - D_i \sum_j e_j(p_{t}^{\text{trade}}) \right] \forall t,$$

which is independent of $R_{jt}, \forall j, t$, since $TB_i \equiv \omega_{it}^{\text{trade}} + R_{it} - r_i(p_{t}^{\text{trade}}) - e_i(p_{t}^{\text{trade}})$ is independent of $R_{it}$.

Since $A_{it}$ and $B_{jt}$ solves Equation (3.A.8), the value function is indeed linear, and given by:

$$v_i^{\text{trade}}(R_1, \ldots, R_N) = \frac{1}{1 - \beta} \left[ u_i(e_i(p_{t}^{\text{trade}})) - c_i(r_i(p_{t}^{\text{trade}})) + p_{t}^{\text{trade}} \cdot \left( \omega_{it}^{\text{trade}} + r_i(p_{t}^{\text{trade}}) + R_i - e_i(p_{t}^{\text{trade}}) \right) - D_i \sum_j e_j(p_{t}^{\text{trade}}) + \frac{D_i}{1 - \beta \delta} \sum_j r_j(p_{t}^{\text{trade}}) \right] + \frac{D_i}{1 - \beta \delta} \sum_j R_j,$$

where time-subscripts are dropped for simplicity.

**Proof of Lemma 3.2**

Given Lemma 3.1, the first-order conditions (Equation (3.A.5)) can now be simplified to:

$$0 = p_t + p_t' \cdot (\omega_{it} + R_{it} + r_{it} - e_{it}) - p_t' r_{it}'(c'_{it} - p_t) - D_t + \beta p_t' \frac{D_t}{1 - \beta \delta} \sum_j r_{jt}'$$

when we note that $dp_t/d\omega_{it} = p_t'(s_t)$. We also know that $c'_{it} - p_t = \sum_{\tau=t+1}^{\infty} (\beta \delta)^{\tau-t} p_{\tau}$, and we define $r_{it}' = \sum_j r_{jt}'$. We can insert this into the first-order condition, sum over all $i$ and divide over by $N$ to get (in three steps)

$$0 = p_t + p_t' \cdot (\omega_{it} + R_{it} + r_{it} - e_{it}) - p_t' r_{it}' \left( \sum_{\tau=t+1}^{\infty} (\beta \delta)^{\tau-t} p_{\tau} \right) - D_t + \beta p_t' \frac{D_t}{1 - \beta \delta},$$

$$0 = Np_t + p_t' \left( \sum_i \{\omega_{it} + R_{it} + r_{it} - e_{it}\} \right) - p_t' r_{it}' \sum_{\tau=t+1}^{\infty} (\beta \delta)^{\tau-t} p_{\tau} - N\delta + p_t' r_{it}' \frac{N\delta}{1 - \beta \delta},$$

$$p_t = \frac{p_t' r_{it}'}{N} \sum_{\tau=t+1}^{\infty} (\beta \delta)^{\tau-t} p_{\tau} + \delta - p_t' r_{it}' \frac{\beta \delta}{1 - \beta \delta}.$$
3.A. APPENDIX - THE DYNAMIC GAME

Given that the supply in the market, $s_t$, is independent of the renewables stocks by Lemma 3.1, the price is independent of state and time, and solving for a constant $p$ gives:

$$p^{\text{trade}} = D \frac{1 - \frac{\beta \delta}{1 - \beta \delta} p' r'}{1 - \frac{\beta \delta}{1 - \beta \delta} p'^r N} > D, \forall t, \text{ for } N > 1.$$ 

This is what is stated in Lemma 3.2.

The MPE price will depend on the parameters, $\beta$, $\delta$, $D$, $N$, $e'$ and $r'$, which will also be constant over time, given the same price in all periods.  

**Proof of Proposition 3.4**

We want to prove that when countries share the same quadratic utility and cost functions, yet differ in their marginal damage, emissions decrease and aggregate welfare increases when international trade in permits is introduced.

The value functions for country $i$ under autarky and permit trade, respectively are given by Equations (3.A.2) and (3.A.9), respectively. For notational simplicity, we will in the following let $p$ denote the equilibrium price under permit trade, $p^{\text{trade}}$. For each country $i$, define the welfare gain from introducing permit trade as

$$\Delta_i = (1 - \beta) \left( V_i - V_i^{\text{Aut}} \right)$$

$$= \left[ u_i(e_i(p)) - c_i(r_i(\frac{p}{1 - \beta \delta})) + pTB_i - D_i \sum_j e_j(p) + \frac{D_i}{1 - \beta \delta} \sum_j r_j(\frac{p}{1 - \beta \delta}) \right]$$

$$- \left[ u_i(e_i(D_i)) - c_i(r_i(\frac{D_i}{1 - \beta \delta})) - D_i \sum_j e_j(D) + \frac{D_i}{1 - \beta \delta} \sum_j r_j(\frac{D_j}{1 - \beta \delta}) \right]$$

$$= u_i(e_i(p)) - u_i(e_i(D_i)) + c_i(r_i(\frac{D_i}{1 - \beta \delta})) - c_i(r_i(\frac{p}{1 - \beta \delta})) + pTB_i$$

$$+ D_i \sum_j e_j(D_j) - D_i \sum_j e_j(p) + \frac{D_i}{1 - \beta \delta} \sum_j r_j(\frac{p}{1 - \beta \delta}) - \frac{D_i}{1 - \beta \delta} \sum_j r_j(\frac{D_j}{1 - \beta \delta}).$$

Throughout this section, we will assume that all countries share the same utility and cost functions, and that both these functions are quadratic. These assumptions are sufficient, but not necessary to obtain the results stated in Proposition 3.4.

The common utility from consumption and cost of investment, where indices are suppressed, are given by:
where all parameters are non-negative. Given the behavior of consumers and producers derived in Section 3.2.1, and these functional forms, we have that:

\[ u_i(e_i(p)) - u_i(e_i(D_i)) = \frac{1}{2} e'_i p^2 - \frac{1}{2} e'_i D_i^2, \]

\[ c_i(r_i(D_i)) - c_i(r_i(p)) = \frac{1}{2} r'_i D_i^2 (1 - \beta \delta)^2 - \frac{1}{2} r'_i p^2 (1 - \beta \delta)^2. \]

\[ D_i \sum_j e_i(D_j) - D_i \sum_j c_i(p) = D_i e'_i \sum_j D_j - D_i e'_i Np. \]

We can insert this back into the expression for \( \Delta_i \) to get

\[
\Delta_i = \frac{1}{2} e'_i p^2 - \frac{1}{2} e'_i D_i^2 + \frac{1}{2} r'_i D_i^2 (1 - \beta \delta)^2 - \frac{1}{2} r'_i p^2 (1 - \beta \delta)^2 + D_i e'_i \sum_j D_j - D_i e'_i Np
\]

\[ + \frac{D_i}{(1 - \beta \delta)^2} r'_i Np \]

\[ = \left( \frac{r'_i}{(1 - \beta \delta)^2} - e'_i \right) \left( \frac{1}{2} D_i^2 - \frac{1}{2} p^2 + D_i Np - D_i \sum_j D_j \right) + pTB_i. \] (3.A.12)

Note here that \( \Delta_i \) can be written as follows when \( D_i = \overline{D} \), and when we recall that the equilibrium permit price \( p = B\overline{D}, B \in [1, N) \):

\[ \Delta_i = K\overline{D}^2 (B - 1)(N - \frac{1}{2}(B + 1)) \]

This expression equals 0 when \( B = 1 \) and is strictly increasing in \( B \) for \( B \in [1, N) \). Hence, we have that \( \Delta_i > 0 \ \forall i \) for \( B > 1 \), when \( D_i = \overline{D} \), which is exactly what is stated in Proposition 3.2.
In order to show that aggregate welfare increases when permit trade is introduced and \( D_i \neq \overline{D} \), we sum over all \( \Delta_i \):

\[
\sum_i \Delta_i = K \cdot \left( \frac{1}{2} \sum_i (D_i^2) - \frac{1}{2} p^2 + \sum_i D_i N p - \sum_i D_i \sum_j D_j \right) + 0.
\]

Then we insert for \( p \) and join terms:

\[
\sum_i \Delta_i = K \cdot \left( \frac{1}{2} \sum_i (D_i^2) + \left( \sum_i D_i \right)^2 \left( B - 1 - \frac{1}{2} \frac{B^2}{N} \right) - \frac{1}{2} \frac{B^2 N^2}{\alpha} \right).
\]  
(3.A.13)

The parenthesis labeled \( \alpha \) is non-decreasing in \( B \) in the relevant region. Hence, if we can prove that \( \sum_i \Delta_i \) is positive for \( B = 1 \), we have proved it for every relevant \( B \). Insert for \( B = 1 \) to get

\[
\sum_i \Delta_i = K \frac{1}{2} \left( \sum_i (D_i^2) - \frac{1}{N} \left( \sum_i D_i \right)^2 \right),
\]

where the last term is the non-negative sample variance of \( D_i \). Hence, we have proved that aggregate welfare increases when international permit trade is introduced, when countries share identical quadratic utility and cost functions. Only if countries are identical in every respect \( (D_i = \overline{D}, \forall i) \) and we are in the static model \( (\beta \delta = 0 \Rightarrow B = 1) \), are there no positive aggregate gains from introducing trade in this case. This concludes the proof of Proposition 3.4.

**Proof of Proposition 3.8**

Also in this proof, we assume quadratic utility and cost functions (see Equations (3.A.10) and (3.A.11)), to simplify the calculations. We start out by inserting in Equation (3.A.12) for the expression for the trade balance (Equation (3.18)). We can then separate the gain to country \( i \) from introducing trade into a term that depends on \( D_i \) and a term that is independent of \( D_i \):

\[
\Delta_i = A + f(D_i), \text{ where } A = e'_i \frac{p^2}{2} - r'_i \frac{1}{(1 - \beta \delta)^2} \frac{p^2}{2} + p \overline{D} N \left( \frac{1}{1 - \beta \delta} r'_i - e'_i \right), \text{ and } f(D_i) = D_i \left( \frac{D_i}{2} - N \overline{D} \right) \left( \frac{r'_i}{(1 - \beta \delta)^2} - e'_i \right) + D_i N p r'_i \frac{\beta \delta}{(1 - \beta \delta)^2}.
\]
We are interested in whether \( f(D_i) \) is increasing in \( D_i \) or not. We start with the proof for Proposition 3.8.2, and take it step by step.

1. For simplicity, assume that there is a continuum of different \( D_i \)'s, such that we can differentiate \( f \). We thus want to know the sign of \( f'(D_i) \).

2. We have that
\[
\begin{align*}
f'(D_i) &= (D_i - ND) \left( \frac{r'_{i}}{(1 - \delta)q} - e'_{i} \right) + Npr'_{i} \frac{\beta \delta}{(1 - \beta \delta)^2}, \\
f''(D_i) &= \left( \frac{r'_{i}}{(1 - \beta \delta)^2} - e'_{i} \right) > 0.
\end{align*}
\]

\( f(D_i) \) is thus convex, so if \( f'(0) > 0 \), then \( f' > 0 \) for all relevant \( D_i \), and we have that high-damage countries gain more from introducing trade than low-damage countries.

3. We have that \( f'(0) < 0 \) for \( \beta \delta = 0 \), while \( \lim_{\beta \delta \rightarrow 1} f'(0) = \infty \), thus by the intermediate value theorem, there exists some \( \beta \delta \) such that \( f'(0) > 0 \) for \( \beta \delta > \beta \delta \). This \( \beta \delta \) is the highest \( \beta \delta \) for which \( f'(0) = 0 \), where we need to take into account that as \( \beta \delta \in [0,1) \), \( p \in (D, ND) \).

4. We now restate Equation (3.10) for quadratic utility and cost functions:
\[
\frac{1}{1 - \delta} r'_i \frac{p}{1 - \beta \delta} < e_i(0) + e'_i \cdot p. \tag{3.10 LQ}
\]

As \( p \in (D, ND) \), we can, for any \( \beta \delta < 1 \), find some \( e_i(0) \) such that there exist \( \beta \) and \( \delta \) (i.e. a pair \( (\beta, \delta) \)) such that Equation (3.10 LQ) is satisfied, given the other parameters, yet \( \beta \delta > \beta \delta \).

5. For such a pair \( (\beta, \delta) \), we have that \( f'(0) > 0 \), and as \( f(D_i) \) is convex, we must have that \( f'(D_i) \) is positive for all relevant \( D_i \). For such a pair, it is therefore the case that high-damage countries gain more from introducing international permit trade than low-damage countries do. This concludes the proof of Proposition 3.8.2.

To prove Proposition 3.8.1, note that \( f'(0) < 0 \), while \( f'(ND) = 0 \), for \( \beta \delta = 0 \), and \( f \) is still convex. Thus \( f'(D_i) \leq 0 \) for all relevant \( D_i \), and the result follows immediately.

Thus, as we move from the static case \( (\beta = \delta = 0) \) to the limit of the dynamic case \( (\beta, \delta \rightarrow 1) \), we move from a case where the low-damage countries gain more to a case where the high-damage countries gain more from introducing permit trade. This coincides with the permit market delivering lower and lower emissions, and a higher and higher equilibrium permit price.
3.B Appendix - finite-horizon convergence

We here find the unique subgame perfect equilibrium (SPE) in the finite horizon game, and let the number of periods, $T$, run to infinity. We verify that the infinite horizon-equilibrium with a constant price is the limit of the unique finite-horizon SPE. The way we do this is to start in the last period and solve backwards, until we can guess some pattern for the price $t$ periods from the end. We then take this guessed pattern and prove it is true by induction. Given this price function $p_t$, we can see what happens to the price as the length of the horizon approaches infinity. In order to get an analytical solution to this problem, we assume in the following that the utility function $u_i(\cdot)$ and the investment cost function $c_i(\cdot)$ are both quadratic.

In every period, the firms consuming the energy solve a static problem, and we will have that

$$u'_i(e_{it}) = p_t,$$

for every period $t$, defining demand as a linear function of the price. For convenience, we will count time backwards. In the last period, 0, the renewables producers also solve a static problem, leading us to

$$c'_i(r_{i0}) = p_0,$$

giving the renewables supply as a linear function of the price in period 0.

Define the supply of energy before the period-$t$ investments by $s_t \equiv \sum_j R_{jt} + \sum_j \omega_{jt}$. The above first-order conditions imply that $p_0$ is a function of $s_0$, and that $p'_0$ is a constant, denoted $p'$ and given by:

$$p' = \frac{1}{\sum_j e'_j - \sum_j r'_j}.$$

In earlier time periods, the price $p_t$ may depend on changes in supply also through changes in future prices, through the effect these will have on the renewables investments. However, the effect of increased supply in period $t$, $s_t$, on the price, $p_t$, conditional on the future prices, will always be given by $p'$.

In the following we will simplify notation by denoting the sum over all countries of the respective variables as $e_t, r_t, \omega_t$ and $R_t$.

The government in period 0 solves

$$V_{i0}(\{R_{j0}\}_{j=1}^N) = \max_{\omega_{i0}} \left\{ u_i(e_i(p_0)) + p_0 \cdot (\omega_{i0} + R_{i0} + r_i(p_0) - e_i(p_0)) - c_i(r_i(p_0)) - D_i \omega_0 \right\},$$
CHAPTER 3. THE DYNAMICS OF LINKING PERMIT MARKETS

with first-order condition

\[ u_i' e_i' p + p_0 \cdot (1 + r_i' p' - c_i' p') + p' \cdot (\omega_i R_i + r_i - e_i) - c_i' r_i' p' - D_i = 0 \]
\[ e_i' p' \cdot (u_i' - p_0) - p' r_i' \cdot (c_i' - p_0) + p_0 + p' \cdot (\omega_i R_i + r_i - e_i) = D_i \]
\[ p_0 + p' \cdot (\omega_i R_i + r_i - e_i) = D_i. \] (3.B.1)

If we sum over all \( i \) and divide by \( N \), we get
\[ p_0 = D. \] (3.B.2)

The equilibrium price is therefore independent of the current stock of renewables in the final period. If, say, country \( i \) experiences an increase in its stock, equilibrium reactions must ensure that the price remains constant. If country \( i \) is to maintain Equation (3.B.1), the only possible solution is that it issues fewer permits, one for one with the increase in \( R_i \). The reason is that there is only one net position in the energy market country \( i \) is willing to take for the constant price \( p_0 \), when the marginal damage is constant. So higher supply of renewables implies lower supply of permits. When country \( i \) keeps its net position fixed, and the price remains constant, no other country has any incentive to react. As in the basic model, this is due to the constant marginal damage in other countries. In total, we have that there is one unique equilibrium, and it satisfies:

\[ \frac{d\omega^o}{dR_i} = -1, \quad \frac{d\omega^o}{dR_j} = 0 \quad \forall j \neq i, \quad \frac{d\omega}{dR_j} = 0, \quad \frac{dV}{dR_j} = D_i. \]

In any period \( t > 0 \), the renewables producers solve a dynamic problem, with the solution

\[ c_i'(r_{it}) = \sum_{s=0}^{t} p_s(\beta \delta)^{t-s} \equiv \hat{p}_t, \]

and the renewables investments are linear in \( \hat{p}_t \).

Equation (3.B.2) implies that \( d\hat{p}_1/d\omega_1 = dp_1/d\omega_1 = p' \), as the equilibrium price in period 0 is independent of the history.

In period 1, the government then solves the following problem

\[ V_{11}(\{R_{1j}\}_{j=1}^N) = \max_{\omega_1} \left\{ u_i(e_i(p_1)) + p_1 \cdot (\omega_i + R_i + r_i(p_1 + \beta \delta p_0) - e_i(p_1)) \right. \\
\left. - c_i(r_i(p_1 + \beta \delta p_0)) - D_i \omega_1 + \beta V_0(\{\delta(R_{1j} + r_{1j}(p_1 + \beta \delta p_0))\}_{j=1}^N) \right\}. \]
with first-order condition
\[
0 = u'_i e'_i p' + p_1 \cdot (1 + r'_i p' - e'_i p') + p' \cdot (\omega_{ii} + R_{ii} + r_{ii} - e_{ii}) \\
- c'_i r'_i p' - D_i + \beta \delta r' p' \nu_{i0}
\]
\[
0 = e'_i p' \cdot (u'_i - p_1) - p' r'_i \cdot (c'_i - p_1) + p_1 + p' \cdot (\omega_{ii} + R_{ii} + r_{ii} - e_{ii}) \\
- D_i + \beta \delta r' p' D_i
\]
\[
0 = p_1 - r'_i p' \beta \delta p_0 + p' \cdot (\omega_{ii} + R_{ii} + r_{ii} - e_{ii}) - D_i + \beta \delta r' p' D_i. \tag{3.B.3}
\]

Summing over this, we get
\[
p_1 = \frac{r' p'}{N} \beta \delta p_0 + D - \beta \delta r' p' D,
\]
which is again independent of the state variables.

Taking the derivative of the first-order condition, (3.B.3), with respect to 
\( R_{j1} \) now gives:
\[
p' \left(1 + \frac{d\omega_{i1}}{dR_{j1}}\right) \left(1 + r'_i p' - e'_i p'\right) + p' \left(\frac{d\omega_{i1}}{dR_{j1}} + \frac{dR_{i1}}{dR_{j1}}\right), \tag{3.B.4}
\]
and summing over \( i \) we get:
\[
p' \left(1 + \frac{d\omega_{i1}}{dR_{j1}}\right) \cdot N = 0.
\]

From the last two equations, we see that we must have:
\[
\frac{d\omega_{i1}}{dR_{i1}} = -1, \quad \frac{d\omega_{i1}}{dR_{j1}} = 0 \forall j \neq i.
\]

Given these reaction functions, we must have:
\[
\frac{dV_{i1}}{dR_{j1}} = D_i + \beta \delta D_i.
\]

This again implies that \( dp_2/d\omega_2 = dp_2/d\omega_2 = p' \).

In period 2, the government solves
\[
V_{i2}(\{R_{j2}\}_{j=1}^N) = \max_{\omega_{i2}} \left\{ u_i(e_i(p_2)) + p_2 \cdot (\omega_{i2} + R_{i2} + r_i(p_2) - e_i(p_2)) \\
- c_i(r_i(p_2)) - D_i \omega_2 + \beta V_{i1}(\{\delta(R_{j2} + r_{j2})\}_{j=1}^N) \right\},
\]
whose first-order condition reduces to
\[
0 = p_2 - r'_ip' (\beta \delta p_1 + (\delta \beta)^2 p_0) + p' \cdot (\omega_{i2} + R_{i2} + r_{i2} - c_{i2}) - D_i + \beta \delta r' p' D_i (1 + \delta)\).
Also in period 2, we can use the first-order condition to show that we must have:

\[
\frac{d\omega_{i2}}{dR_{i2}} = -1, \quad \frac{d\omega_{j2}}{dR_{j2}} = 0 \forall j \neq i, \quad \frac{dV_{i2}}{dR_{j2}} = D_i(1 + \beta \delta + (\beta \delta)^2). \]

Next, we sum over all \(i\) to get

\[
p_2 = \frac{r'p'}{N} (\delta p_1 + (\beta \delta)^2 p_0) + D - \beta \delta r'p'\bar{D}(1 + \beta \delta),
\]

which, if we insert for \(p_1\), simplifies to

\[
p_2 = \left(\frac{r'p'}{N} \beta \delta \right) (\frac{r'p'}{N} \beta \delta + \beta \delta) p_0 + \bar{D} (1 + \frac{r'p'}{N} \beta \delta) - \left(\frac{r'p'}{N} \beta \delta \right) r'p' \beta \delta \bar{D} - \frac{r'p'}{N} \beta \delta (1 + \beta \delta) \bar{D}.
\]

We hypothesize

\[
\begin{align*}
p_t &= ap_0 d^{t-1} + b(1 + a \sum_{s=0}^{t-2} d^s) - c_t = a \sum_{s=1}^{t-1} c_s d^{t-1-s}, \forall t \geq 2 \quad \text{and} \quad (3.5) \\
\sum_{s=0}^{t} p_s (\beta \delta)^{t-s} &\equiv \hat{p}_t = p_0 d^t + b \sum_{s=0}^{t-1} d^s - \sum_{s=1}^{t} c_s d^{t-s}, \forall t \geq 2 \quad \text{where} \quad (3.6) \\
&= a \frac{r'p'}{N} \beta \delta, \quad b = \bar{D}, \quad d = a + \beta \delta, \quad c_t = r'p'\bar{D} \sum_{\tau=1}^{t} (\beta \delta)^\tau,
\end{align*}
\]

implying that the price is independent of the state variables in all time periods.

This reduces \(p_2\) to \(adp_0 + b(1 + a) - ac_1 - c_2\). We will now prove by induction that the equilibrium defined by Equation (3.5) solves the problem in all time periods. We show that given that (3.5) and (3.6) hold in period \(t\), (3.5) and (3.6) will also characterize the unique subgame perfect equilibrium in period \(t+1\), for any \(p_0\) independent of stocks. Given that we know that (3.5) and (3.6) hold in period 2, this would be sufficient in order to prove that the two equations characterize the equilibrium price in this game, for any finite horizon.

Assume (3.5) and (3.6) hold in period \(t\). Then in period \(t+1\), we have that \(dp_{t+1}/d\omega_{t+1} = dp_{t+1}/d\omega_{t+1} = p'\), and the government solves

\[
\begin{align*}
V_{it+1}(\{R_{jt+1}\}_{j=1}^{N}) &= \max_{\omega_{i,t+1}} \left\{ u_i(e_i(p_{t+1})) + p_{t+1} \cdot (\omega_{i,t+1} + R_{i,t+1} + r_i(\hat{p}_{t+1}) - e_i(p_{t+1})) \\
&- c_i(r_i(\hat{p}_{t+1})) - D_i\omega_{t+1} + \beta V_t([\delta(R_{jt+1} + r_{jt+1}(\hat{p}_{t+1}))]_{j=1}^{N}) \right\},
\end{align*}
\]

whose first-order condition reduces to

\[
0 = p_{t+1} - r'p' (\delta p_i + (\beta \delta)^2 p_{t-1} + \cdots + (\beta \delta)^t p_0) \\
+ p' \cdot (\omega_{i,t+1} + R_{i,t+1} + r_{i,t+1} - e_i,t+1) - D_i + \beta \delta \sum_j \frac{\partial V_t}{\partial R_{jt}} r'p'.
\]
Given that the price in period $t$ is independent of the period $t$-stocks, we know that the value function must be linear in these stocks. Using this, we can take the derivative of the first-order condition with respect to $R_{jt+1}$, and show that we must have:

\[
\frac{d\omega_{it+1}}{dR_{it+1}} = -1, \quad \frac{d\omega_{it+1}}{dR_{jt+1}} = 0 \quad \forall j \neq i,
\]
as before. Finally, we can then again find the derivative of the value function:

\[
\frac{dV_{it+1}}{dR_{jt+1}} = D_i + \beta \delta \frac{dV_{it}}{dR_{jt}} \\
= D_i(1 + \beta \delta + (\beta \delta)^2 + \ldots + (\beta \delta)^{t+1}).
\]
The first-order condition then reduces to:

\[
0 = p_{t+1} - r_i'p'\left(\beta \delta p_t + (\beta \delta)^2 p_{t-1} + \ldots + (\beta \delta)^{t+1} p_0\right) \\
+ p' \cdot (\omega_{it+1} + R_{it+1} + r_{i,t+1} - e_{i,t+1}) - D_i + r'p'D_i \sum_{s=1}^{t} (\beta \delta)^s.
\]

We can sum over all $i$ to get

\[
p_{t+1} = \frac{r'p'}{N} \left(\beta \delta p_t + (\beta \delta)^2 p_{t-1} + \ldots + (\beta \delta)^{t+1} p_0\right) + \bar{D} - r'p'D \sum_{s=1}^{t} (\beta \delta)^s \\
= \beta \delta \frac{r'p'}{N} \sum_{s=0}^{t} p_s (\beta \delta)^{t-s} + \bar{D} - r'p'D \sum_{s=1}^{t+1} (\beta \delta)^s \\
= a \sum_{s=0}^{t} p_s (\beta \delta)^{t-s} + b - c_{t+1} \\
= a \left(p_0d^t + b \sum_{s=0}^{t-1} d^s - \sum_{s=1}^{t} c_s d^{t-s}\right) + b - c_{t+1} \\
= ap_0d^t + b(1 + a \sum_{s=0}^{t-1} d^s) - c_{t+1} - a \sum_{s=1}^{t} c_s d^{t-s},
\]
which fits the hypothesized form (3.B.5).
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For the sum, we have

\[
\sum_{s=0}^{t+1} p_s (\beta \delta)^{t+1-s} = p_{t+1} + \beta \delta \sum_{s=0}^{t} p_s (\beta \delta)^{t-s}
\]

\[
= p_{t+1} + \beta \delta p_0 d^t + \beta \delta \sum_{s=0}^{t-1} d^s - \beta \delta \sum_{s=1}^{t} c_s d^{t-s}
\]

\[
= ap_0 d^t + b(1 + a \sum_{s} d^s) - c_{t+1} - a \sum_{s} c_s d^{t-s}
\]

\[
+ \beta \delta p_0 d^t + \beta \delta \sum_{s=0}^{t-1} d^s - \beta \delta \sum_{s=1}^{t} c_s d^{t-s}
\]

\[
= (a + \beta \delta) p_0 d^t + b(1 + (a + \beta \delta) \sum_{s=0}^{t-1} d^s) - c_{t+1} - (a + \beta \delta) \sum_{s=1}^{t} c_s d^{t-s}
\]

\[
= p_0 d^{t+1} + b + b \sum_{s=1}^{t} d^s - c_{t+1} - \sum_{s=1}^{t} c_s d^{t+1-s}
\]

\[
= p_0 d^{t+1} + b \sum_{s=0}^{t} d^s - \sum_{s=1}^{t+1} c_s d^{t+1-s},
\]

exactly the hypothesized form (3.B.6).

So now we have proved that the price in the finite horizon-game follows the form (3.B.5). What remains is to demonstrate that this price converges to the infinite horizon price as the length of the horizon, \( T \), runs to infinity. First, rewrite the last term in (3.B.5). We have

\[
\sum_{s=0}^{t-1} c_s d^{t-1-s} = r' p'D \sum_{s=0}^{t-1} d^{t-1-s} \sum_{u=1}^{s} (\beta \delta)^u = r' p'D \sum_{s=0}^{t-1} ((\beta \delta)^s \sum_{u=0}^{t-1-s} d^u)
\]

which is better seen by example. For \( t = 4 \), we have:

\[
\sum_{s=1}^{3} c_s d^{3-s} = d^2 c_1 + d c_2 + c_3
\]

\[
= r' p'D \left( d^2 \beta \delta + d (\beta \delta + (\beta \delta)^2) + (\beta \delta + (\beta \delta)^2 + (\beta \delta)^3) \right)
\]

\[
= r' p'D \left( \beta \delta (1 + d + d^2) + (\beta \delta)^2 (1 + d) + (\beta \delta)^3 \right)
\]

\[
= r' p'D \sum_{s=1}^{3} \left( (\beta \delta)^s \sum_{u=0}^{3-s} d^u \right).
\]
Since $p' = 1/(r' - e')$, we have $d \in (0, 1)$, so in total, as $t \to \infty$, the sum converges to:

$$r'p'D \frac{\beta \delta}{1 - \beta \delta} \frac{1}{1 - d}.$$  

Substituting this, we can restate (3.B.5):

$$p_t = apd^{t-1} + b(1 + a \sum_{s=0}^{t-2} d^s) - c_t - ar'p'D \sum_{s=1}^{t} ((\beta \delta)^s \sum_{u=0}^{t-s} d^u).$$

Letting $t$ run to infinity, we have

$$\lim_{t \to \infty} p_t = 0 + b(1 + \frac{a}{1 - d}) - \lim_{t \to \infty} c_t - ar'p'D \frac{\beta \delta}{1 - \beta \delta} \frac{1}{1 - d}$$

$$= b(1 + \frac{a}{1 - d}) - r'p'D \frac{\beta \delta}{1 - \beta \delta} \frac{1}{1 - d} - r'p'D \frac{\beta \delta}{1 - \beta \delta} \frac{a}{1 - d}$$

$$= (b - r'p'D \frac{\beta \delta}{1 - \beta \delta})(1 + \frac{a}{1 - d})$$

$$= \frac{b - r'p'D \frac{\beta \delta}{1 - \beta \delta}}{1 - \frac{a}{1 - \beta \delta}}$$

$$= \frac{D}{1 - \frac{r'p'D \frac{\beta \delta}{1 - \beta \delta}}{N}} = \frac{1 + \Omega}{1 + \frac{\Omega}{N}}.$$  

Thus we have proved that the infinite-horizon equilibrium with a constant price is the limit of the unique SPE of the finite-horizon game.

3.C Appendix - extensions

3.C.1 Endogenous fossil energy

Here we endogenize fossil energy supply in the same way as renewables are endogenized in the main body of the paper. We thus abstract away from concerns of exhaustibility. We consider only a two-period game.

Firstly, we introduce a positive price $\phi_t$ on fossil energy. The permit price is now $\tau_t$, and the price to renewables producers is $p_t$. We count time backwards, thus period 0 is the last, while period 1 is the second-to-last. The investment cost of a fossil producer in country $i$ of increasing the capacity in period $t$ with $g_{it}$, is given by the increasing and convex cost function $h_i(g_{it})$. As for the renewables stock, we assume that the stock of fossil energy production capacity develops according to $G_{it+1} = \delta(G_{it} + g_{it})$, with $\delta$ as the survival rate (equal to
the survival rate of the renewables stock). The fossil energy producers thus solve a problem equivalent to that of the renewables producers, given by (3.6). The consumers and the renewables producers solve the same problems as in the basic model. Perfect substitutability for consumers implies that in equilibrium we have \( p_t = \phi_t + \tau_t \).

The solutions to the producers' problems now gives investments in each time period given by the following:

\[
\begin{align*}
    c_i'(r_i) &= p_0 \equiv \hat{p}_0 \Rightarrow r_i(p_0), \\
    c_i'(r_i) &= p_1 + \beta p_0 \equiv \check{p}_1, \Rightarrow r_i(\check{p}_1), \\
    h_i'(g_i) &= \phi_0 \equiv \hat{\phi}_0 \Rightarrow g_i(\phi_0), \\
    h_i'(g_i) &= \phi_1 + \beta \phi_0 \equiv \check{\phi}_1, \Rightarrow g_i(\check{\phi}_1).
\end{align*}
\]

(3.C.1) (3.C.2)

Market clearing requires

\[
\begin{align*}
    \sum_j \omega_{jt} &= \sum_j G_{jt} + \sum_j g_{jt}(\check{\phi}_t) \\
    \sum_j \epsilon_{jt} &= \sum_j \omega_{jt} + \sum_j R_{jt} + \sum_j r_{jt}(\hat{p}_t).
\end{align*}
\]

Defining \( \omega_t, \epsilon_t, r_t, g_t, G_t \) and \( R_t \) as the sum over all countries of the respective variables, the market clearing conditions define the equilibrium prices \( \phi_0(\omega_0 - G_0), \phi_1(\omega_1 - G_1|\phi_0), p_0(\omega_0 + R_0) \) and \( p_1(\omega_1 + R_1|p_0) \). Together, these define the equilibrium permit prices \( \tau_0(\omega_0, R_0, G_0) \), and \( \tau_1(\omega_1, R_1, G_1|p_0, \phi_0) \). Differentiation gives us

\[
\begin{align*}
    \phi'_0 &= \frac{1}{g_0'}, \\
    \phi'_1 &= \frac{1}{g_1'}, \\
    p'_0 &= \frac{-1}{r_0' - c_0'}, \\
    p'_1 &= \frac{-1}{r_1' - c_1'}, \\
    \frac{\partial \tau_0}{\partial \omega_0} &= p_0' - \phi'_0, \\
    \frac{\partial \tau_0}{\partial G_0} &= \phi'_0, \\
    \frac{\partial \tau_0}{\partial R_0} &= \phi'_0, \\
    \frac{\partial \tau_1}{\partial \omega_1} &= p_0' - \phi'_0, \\
    \frac{\partial \tau_1}{\partial G_1} &= \phi'_0, \\
    \frac{\partial \tau_1}{\partial R_1} &= \phi'_0.
\end{align*}
\]

(3.C.3)

The problem facing country \( i \) in period 0 is now

\[
\begin{align*}
    V_{i0}(R_{10}, \ldots, R_{N0}, G_{10}, \ldots, G_{N0})
    &= \max_{\omega_{i0}} \left\{ u_i(e_i(p_0)) - c_i(r_i(p_0)) - h_i(g_i(\phi_0)) - D_i \sum_j \omega_{j0} + \tau_0 \omega_{i0} \\
    &\quad + \phi_0(G_{i0} + g_i(\phi_0)) + p_0(R_{i0} + r_i(p_0) - e_i(p_0)) \right\},
\end{align*}
\]

(3.C.4)

with first-order condition

\[
\begin{align*}
    0 &= u_i' e_i p_0' - c_i' r_{i0} p_0' - h_i' g_{i0} \phi_0' - D_i + \tau_0 + (p_0' - \phi_0') \omega_{i0} + \phi_0'(G_{i0} + g_{i0}) \\
    &\quad + \phi_0 g_{i0} \phi_0' + p_0'(R_{i0} + r_{i0} - e_{i0}) + p_0'(r_{i0} p_0' - e_{i0} p_0').
\end{align*}
\]
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Given that \( u'_{i0} = p_0, e'_{i0} = p_0 \) and \( h'_{i0} = \phi_0 \), this simplifies to:

\[
D_i = \phi'_0(G_{i0} + g_{i0} - \omega_{i0}) + p'_0(R_{i0} + r_{i0} - e_{i0} + \omega_{i0}) + \tau_0. \tag{3.C.5}
\]

Given the market clearing conditions, summing (3.C.5) over \( i \) gives us the equilibrium permit price:

\[
\tau_0 = \overline{D}, \tag{3.C.6}
\]

which is independent of any stock.

Turning to period 1, the problem facing country \( i \) is now

\[
V_{11}(R_{11}, \ldots, R_{N1}, G_{11}, \ldots, G_{N1}) = \max_{\omega_{i1}} \left\{ u_i(e_i(p_1)) - c_i(r_i(\hat{p}_1)) - h_i(g_i(\hat{\phi}_1)) - D_i \sum_j \omega_{j1} + \tau_1 \omega_{i1} - \phi_1(G_{i1} + g_{i1}) + p_1(R_{i1} + r_{i1} - e_{i1}) + \phi'_1(G_{i1} + g_{i1}) + \beta V_{i0}(R_{i0}, \ldots, R_{N0}, G_{i0}, \ldots, G_{N0}) \right\}.
\]

Given the constant second-period price, we have that \( \frac{\partial \hat{\phi}_1}{\partial \omega_{i1}} = p'_1 \) and \( \frac{\partial \phi'_1}{\partial \omega_{i1}} = \phi'_1 \), and we get the first order condition:

\[
0 = u'_i e'_i p'_1 - c'_i r'_i p'_1 - h'_i g'_i \phi'_1 - D_i - \tau_1 + (p'_1 - \phi'_1)\omega_{i1} + \phi'_1(G_{i1} + g_{i1}) + p'_1(R_{i1} + r_{i1} - e_{i1}) + p_1 r'_1 p'_1 - p_1 e'_1 p'_1 + \beta \sum_j \frac{\partial V_{i0}}{\partial R_{j0}} r'_{j1} p'_1 + \beta \sum_j \frac{\partial V_{i0}}{\partial G_{j0}} g'_{j1} \phi'_1,
\]

which after canceling terms becomes

\[
D_i = \tau_1 - r'_{i1} p'_1 \beta \delta p_0 - g'_{i1} \phi'_1 \beta \delta \phi_0 + p'_1(\omega_{i1} + R_{i1} + r_{i1} - e_{i1}) + \phi'_1(G_{i1} + g_{i1} - \omega_{i1}) + \beta \sum_j \frac{\partial V_{i0}}{\partial R_{j0}} r'_{j1} p'_1 + \beta \sum_j \frac{\partial V_{i0}}{\partial G_{j0}} g'_{j1} \phi'_1.
\]

To find the price, we sum over all \( i \) and divide by \( N \) to find:

\[
\tau_1 = \overline{\tau} - \frac{\beta \delta}{N} \left[ p'_1 \sum_j r'_{j1} \cdot \left( \sum_i \frac{\partial V_{i0}}{\partial R_{j0}} - p_0 \right) + \phi'_1 \sum_j g'_{j1} \cdot \left( \sum_i \frac{\partial V_{i0}}{\partial G_{j0}} - \phi_0 \right) \right].
\]
We see that we need to find $\sum_i \partial V_i / \partial R_j$ and $\sum_i \partial V_i / \partial G_j$ in order to derive the equilibrium period 1 permit price. The reason is that countries will, when issuing permits in the first period, take into account its effect on investments in both renewables and fossil energy, and the impact on the number of permits that will be issued in the last period. From market clearing, we have that $p'_i$ is negative, while $\phi'_i$ is positive. It means that the permit price in period 1 will be higher to the extent that the social value of a higher future stock of renewables (fossil energy) is higher (lower) than its private value $p_0$ ($\phi_0$).

To derive these values, we differentiate through (3.C.4), and use the investors’ and consumers’ first-order conditions to find

$$\sum_i \frac{\partial V_i}{\partial G_j} = \phi_0 - (N\bar{D} - \tau_0) \frac{\partial \omega^q_0}{\partial G_j},$$

and

$$\sum_i \frac{\partial V_i}{\partial R_j} = p_0 - (N\bar{D} - \tau_0) \frac{\partial \omega^q_0}{\partial R_j}.$$

Finally, from (3.C.6) we deduce that $\partial \omega^q_0 / \partial R_0$ is positive while $\partial \omega^q_0 / \partial G_0$ is negative. The equilibrium permit price in period 0 reduces to

$$\tau_1 = \bar{D} \left( 1 + \frac{N-1}{N} \beta \delta \left[ p'_1 r'_1 \frac{\partial \omega^q_0}{\partial R_j} + \phi'_1 g'_1 \frac{\partial \omega^q_0}{\partial G_j} \right] \right) > \bar{D}. \quad (3.C.7)$$

It is clear that the permit price implementing the first best in this economy - as in the basic model - would be $p^{FB} = \sum_j D_j$, and that the autarky price in country $i$ would be $D_i$. Hence, the results derived here are qualitatively identical to the results stated in Section 3.3, even though the supply of fossil energy is endogenously determined. We have that the fossil energy channel and the renewables channel both contribute towards a higher first-period permit price, and that the permit price exceeds the average marginal damage. Endogenous fossil energy alone would be sufficient for our mechanism to arise. Thus, our mechanism is still at work, and it is strengthened, not weakened by the presence of endogenous fossil energy.

### 3.C.2 Politically determined investments

We here let the government in each country regulate the producers, by letting the government decide the size of the renewables investment in each time period. We only consider a two-period model. Furthermore, we assume that the representative consumers in every country share the same, quadratic, utility function, and that investment costs are quadratic and identical across countries (see Equations (3.A.10) and (3.A.11)). The timing within each time period is
as in the basic model, and we now disregard renewables investments in the last time period as these are not affected by strategic incentives. The last period is denoted by 0, while 1 denotes the first period.

Consumers behave as before, and the market clearing conditions are now given by:

\[ \sum_i e_i(p_1) = \sum_i \omega_i + \sum_i R_i + \sum_i r_i, \]
\[ \sum_i e_i(p_0) = \sum_i \omega_i + \sum_i R_i, \]

determining the prices, as functions of total supply:

\[ p_1 \left( \sum_i \omega_i + \sum_i R_i + \sum_i r_i \right), \quad p_1' = \frac{1}{\sum_i e_i'}, \]
\[ p_0 \left( \sum_i \omega_i + \sum_i R_i \right), \quad p_0' = \frac{1}{\sum_i e_i'}. \]

In the last period, the governments solve the following problem

\[ W_{i0}(R_{10}, \ldots, R_{N0}) = \max_{\omega_{i0}} \left\{ u_i(e_i(p_0)) - D_i \sum_j \omega_j + p_0(R_{i0} + \omega_{i0} - e_i(p_0)) \right\}, \]

which produces the following equilibrium

\[ p_0 = D, \quad \partial \omega_{i0} / \partial R_{j0} = \begin{cases} -1 & i = j \\ 0 & i \neq j \end{cases}, \quad \partial W_{i0} / \partial R_{j0} = D_i, \ \forall i, j. \quad (3.C.8) \]

In period 1, the governments make decisions in two stages. Let

\[ W_{i1}(R_{11}, \ldots, R_{N1}) = \max_{\omega_{i1}} \left\{ -D_i \sum_j \omega_j + V_{i1}(R_{11}, \ldots, R_{N1}, \omega_{i1}, \ldots, \omega_{N1}) \right\} \quad (3.C.9) \]

be the government’s value function at the permit decision stage, where \( V_{i1} \) is the value function at the investment stage. Then we have

\[ V_{i1}(R_{11}, \ldots, R_{N1}, \omega_{i1}, \ldots, \omega_{N1}) = \max_{r_{11}} \left\{ u_i(e_i(p_1)) - c_i(r_{11}) + p_1(\omega_{i1} + R_{i1} + r_{i1} - e_i(p_1)) \\
+ \beta W_{i0}(\delta(R_{11} + r_{11}), \ldots, \delta(R_{N1} + r_{N1})) \right\}. \quad (3.C.10) \]

The first-order condition for this problem is given by:

\[ 0 = p_1 - c_i'(r_{11}) + p_1' (\omega_{i1} + R_{i1} + r_{i1} - e_i(p_1)) + \beta \delta D_i, \quad (3.C.11) \]
CHAPTER 3. THE DYNAMICS OF LINKING PERMIT MARKETS

determining renewables investments as functions of permit issuance and renewables stocks:

\[ r_{i1}(R_{11}, \ldots, R_{N1}, \omega_{11}, \ldots, \omega_{N1}). \]

We now turn to the permit issuing stage, (3.C.9). When the governments issue permits in the first stage, they will take into account how their issuance affects investments, now chosen by governments. Their first-order condition is given by:

\[
D_i = \frac{\partial V_i}{\partial \omega_i} = p'_1 \left( 1 + \sum_j \frac{\partial r_{j1}}{\partial \omega_i} \right) TB_i + p_1 \left( 1 + \frac{\partial r_{i1}}{\partial \omega_i} \right) - c'_i \frac{\partial r_{i1}}{\partial \omega_i} + \beta \delta D_i \sum_j \frac{\partial r_{j1}}{\partial \omega_i}.
\]

From (3.C.11) we have that \( p_1 - c'_i = -(p'_1 TB_i + \beta \delta D_i) \). So in deciding on permits, the government can ignore the effect on their own investments, since these are set optimally from the government’s perspective (the envelope theorem). This is different from the case with price-taking investors in the main body of the paper. Use this to get:

\[
D_i = p_1 + p'_1 TB_i + \sum_{j \neq i} \frac{\partial r_{j1}}{\partial \omega_i} (p'_1 TB_i + \beta \delta D_i) + \frac{\partial r_{i1}}{\partial \omega_i} (p_1 - c'_i).
\]

From (3.C.11) we have that \( p_1 - c'_i = -(p'_1 TB_i + \beta \delta D_i) \). So in deciding on permits, the government can ignore the effect on their own investments, since these are set optimally from the government’s perspective (the envelope theorem). This is different from the case with price-taking investors in the main body of the paper. Use this to get:

\[
D_i = p_1 + p'_1 TB_i + \sum_{j \neq i} \frac{\partial r_{j1}}{\partial \omega_i} (p'_1 TB_i + \beta \delta D_i).
\]  

(3.C.12)

where \( \sum_{j \neq i} \frac{\partial r_{j1}}{\partial \omega_i} \in [-1, 0] \). Sum over (3.C.12) to find

\[
p_1 = \overline{D} \left( 1 - \beta \delta \sum_{j \neq i} \frac{\partial r_{j1}}{\partial \omega_i} \right) > \overline{D}.
\]

(3.C.13)

Hence, the strategic incentive to withhold permits in order to reduce future issuance in other countries through increasing their stocks, is still present.

### 3.C.3 Excess renewables supply

Here we solve a two-period model for the case when Equations (3.10) and (3.11) do not hold for all countries.

Time is counted backwards, and the time periods are \( t = 1 \) and \( t = 0 \) (last period). Consumers behave as before, leading to a static demand \( e_i(p_{it}) \) as in previous sections.
The domestic renewables producers solve the following problem:

\[
\max_{r_{it}} \{ \hat{p}_{it} r_{it} - c_i(r_{it}) \} \Rightarrow r_i(\hat{p}_{it}).
\]

where, as before, \(\hat{p}_{it}\) denotes the sum of (discounted) current and future domestic prices, hence \(\hat{p}_{i0} = p_{i0}\) and \(\hat{p}_{i1} = p_{i1} + \beta \delta p_{i0}\).

The domestic energy (and permit) price may or may not be equal to the international permit price, \(p_t\). In period \(t\), domestic market clearing requires that \(R_{it} + r_i(\hat{p}_{it}) = e_i(p_{it})\). If this domestic market clears at a price \(p_{it} < p_{eq}\), domestic consumers demand no emission permits, and we say that country \(i\) is saturated: \(i \in S_t\). Since the domestic energy price in such a country is lower than the international permit price, the supply of renewables from producers in this country is independent of the permit price. If \(i \notin S_t\), we say \(i \in NS_t\).

In the international permit market, market clearing requires:

\[
\sum_i \omega_{it} = \sum_i f_{it} = \sum_i e_i(p_{it}) - R_{it} - \sum_i r_i(\hat{p}_{it}) = \sum_{i \in NS_t} e_i(p_{it}) - R_{it} - \sum_{i \in NS_t} r_i(\hat{p}_{it}) = 0
\]

This market clearing condition defines the price in each time period as a function of supply, \(s_t\), in that period: \(\tilde{p}(s_t)\), with slope given by:

\[
\tilde{p}'(s_t) = \frac{1}{\sum_{i \in NS_t} e_i'(p_{it}) - \sum_{i \in NS_t} r_i'(\hat{p}_{it})}.
\]

The government in country \(i \in S_0\) solves the following problem in period 0:

\[
V_{0,i,S_0} = u_i(e_i(p_{i0})) - c_i(r_i(\hat{p}_{i0})) + \max_{\omega_{i0}} \left\{ \tilde{p}(s_0) \omega_{i0} - D_i \sum_j \omega_{j0} \right\},
\]

giving the first-order condition

\[
\tilde{p}(s_0) + \tilde{p}'(s_0) \cdot \omega_{i0} = D_i.
\]
The non-saturated countries solve the same problem as in our the basic model:

\[ V_{0,i \in NS_0} = \max_{\omega_{i0}} \left\{ u_i (e_i (\tilde{p}(s_0))) - c_i (r_i (\hat{p}_0)) + \tilde{p}(s_0) (\omega_{i0} + R_{i0} + r_i (\hat{p}_0) - e_i (\tilde{p}(s_0))) - D_i \sum_j \omega_{j0} \right\}, \]

and the first-order condition becomes (after applying the envelope theorem)

\[ \tilde{p}(s_0) + \tilde{p}'(s_0) \cdot (\omega_{i0} + R_{i0} + r_i (\hat{p}_0) - e_i (\tilde{p}(s_0))) = D_i. \]

Summing over the \( N \) first-order conditions, and invoking market clearing, we get the equilibrium price in the last time period:

\[ p_0 = \bar{D}_i, \]

which is independent of the number of saturated countries. Furthermore,

\[ \frac{\partial \omega_{i0}^{eq}}{\partial R_{j0}} = \begin{cases} -1, & i = j, i \in NS_0 \\ 0, & \text{else.} \end{cases} \]

When \( i \in NS_0 \) this happens for the same reason as in the basic model. For \( i \in S_0 \), it follows directly from the first-order condition given by Equation (3.C.14). The permit issuance of saturated countries is independent of their domestic renewables stocks, while that of the non-saturated countries is not. This means that

\[ \frac{\partial V_{0,i}}{\partial R_{j0}} = \begin{cases} D_i, & j \in NS_0 \\ p_{0,i} & i = j, j \in S_0 \\ 0, & i \neq j, j \in S_0 \end{cases} \]

A given country benefits only from more renewable energy in other countries when these are not saturated.

In period 1, a saturated country solves

\[ V_{1,i \in S_1} = u_i (e_i (p_{11})) - c_i (r_i (\hat{p}_{11})) + \max_{\omega_{i1}} \left\{ \tilde{p}(s_1) \cdot \omega_{i1} - D_i \sum_j \omega_{j1} + \beta V_{0,i} \right\}, \]

with first-order condition

\[ \tilde{p}(s_1) + \tilde{p}'(s_1) \cdot \omega_{i1} = D_i - \beta \delta \sum_{j \in NS_1} \frac{\partial V_{0,i}}{\partial R_{j0}} \tilde{p}'(s_1) r'_j (\hat{p}_1). \]
Whereas a non-saturated country solves

\[ V_{1,i \in NS_1} = \max_{\omega_{i1}} \left\{ u_i(e_i(\tilde{p}(s_1))) - c_i(r_i(\hat{p}_1)) 
+ \tilde{p}(s_1) \cdot (\omega_{i1} + R_{i1} + r_i(\hat{p}_1) - e_i(\tilde{p}(s_1))) - D_i \sum_j \omega_{j1} + \beta V_{0,i} \right\}, \]

with first-order condition

\[ \tilde{p}(s_1) + \tilde{p}'(s_1)(\omega_{i1} + R_{i1} + r_i(\hat{p}_1) - e_i(\tilde{p}(s_1))) \]
\[ = D_i + \beta \delta \tilde{p}'(s_1) r_i'(\hat{p}_1) D - \beta \delta \tilde{p}'(s_1) \sum_{j \in NS_1} \frac{\partial V_{0,i}}{\partial R_{j0}} r_j'(\hat{p}_1). \]

Employing the information above and using the market clearing condition, the \(N\) first-order conditions sum to give us

\[ p_1 = D \left[ 1 - \beta \delta \tilde{p}'(s_1) \left( \sum_{j \in NS_1} r_j'(\hat{p}_1) - \frac{1}{N} \sum_{i \in NS_1} r_i'(\hat{p}_1) \right) \right] \geq D. \]

Recall that the autarky price is given by \(D\). The price above is greater than the autarky price whenever \(|NS| > 0\), and converges on \(D\) when \(|NS| \rightarrow 0\). We thus show that the mechanism identified in the main part of this paper is in play also in the case where there is excess supply of renewable energy in some countries, resulting in zero permit demand from consumers in these countries.

References


REFERENCES


REFERENCES


Chapter 4

Markov-perfect cooperation in continuous-time prisoners’ dilemmas

Kristoffer Midttømme

Abstract

I provide a twist on the iterated prisoners’ dilemma: I recast the game in continuous time and derive Markov-perfect equilibria in which two players are able to cooperate. As opposed to a discrete-time game, there is a payoff-relevant difference between defecting when your opponent is currently cooperating and when he is already defecting. Thus, cooperation comes about even if players are unable to credibly promise future cooperation and threaten to punish defectors. Although many symmetric equilibria potentially exist in the game, they belong to the same family and all involve randomization to some extent. Recent economic lab experiments find more cooperation in prisoners’ dilemmas played in continuous time, than in the same game played in discrete time. If players are playing Markov-perfect strategies – as suggested by another lab experiment – my findings could help explain this finding, as players will not cooperate in discrete-time Markov-perfect equilibria.

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Introduction

The question of how players can cooperate despite their incentives not to, is old and well-studied in the economics literature. The prisoners’ dilemma offers the archetypical dilemma between the social good and what is privately optimal. If both players cooperate, they achieve the best social outcome. From that socially desirable situation, however, both players have an incentive to defect. When both players defect, the social good is lost and they end up with the worst possible outcome. Such mutual defection is, sadly, the unique Nash equilibrium of this game. A well-studied problem is how the players can resolve this tension between the privately and the socially optimal. If the game is played between the same players over and over, a possible solution is to rely on history-dependent strategies. By conditioning their current behavior on what happened previously, the players can device punishment schemes that motivate them to take the socially desirable actions. Such strategies can become quite complicated, see for instance Osborne and Rubinstein (1994, pp. 143-148).

If the players are restricted to employ strategies that only depend on the payoff-relevant parts of the history (i.e. Markov-perfect strategies, in the sense of Maskin and Tirole (2001)), the punishment schemes discussed above are ruled out. When two players forever repeat a prisoners’ dilemma in discrete time, no parts of history are payoff relevant: the future payoff matrix is independent of the history, and the history does not restrict the actions that can possibly be taken today or in the future. Markov perfection then requires that the strategies be stationary, i.e. history-independent. The only Markov-perfect equilibrium (MPE) in the discrete-time repeated prisoners’ dilemma is therefore the infinite repetition of the static Nash equilibrium.

In this paper I offer a slightly different take. Instead of studying a repeated prisoners’ dilemma in discrete time, I rather study the game played over and over in continuous time. Payoffs form a flow of benefits to the players, and the players are able, but not required, to switch between cooperation and defection at every instant. First, I argue that the currently observed pair of behaviors (the state) is payoff-relevant in this continuous-time game. Second, I demonstrate how this means that there exist MPEs in which the players do cooperate.

Markov-perfect equilibria are interesting for several reasons. First, as is well known from the literature on folk theorems, there is a huge range of subgame-perfect equilibria (SPEs) in discrete-time games. As MPEs only comprise a subset of the SPEs, there are generally fewer of them. Often, as in the case above, there is a unique Markov-perfect equilibrium, and this provides clearly testable predictions. An interesting question is then whether the same is true
for a corresponding continuous-time game. Second, they are simpler, since they only depend on the payoff-relevant parts of history, not all of history. Third, at least one experiment suggests that lab participants coordinate on the unique MPE in settings with multiple SPEs (Battaglini et al., 2012), so perhaps MPEs are more salient in real-world situations.

In order to understand the difference between the discrete and the continuous models, consider the following transformation. Instead of thinking that each player picks an action over and over at discrete points in time, consider rather that the players initially pick a behavior, and then between each discrete point in time they decide whether they want to switch or remain with their current behavior. In the discrete-time game this changes nothing. There is still a unique MPE in which both players always defect. But when the periods become shorter and shorter, and the players choose to switch behavior only occasionally, the probability that two players switch behavior within the same time period becomes negligible. As a result, for instance, the continuous-time game cannot transit from both players cooperating to both players defecting, without passing through a phase in which only one player is cooperating, i.e. on player has to defect first. With perfect monitoring, the other player immediately observes this and can choose whether to react. This means that the current pair of behaviors now has become payoff relevant, as it determines the set of possible successor states. A consequence of this, as I demonstrate, is that there will exist MPEs in which the players cooperate. I briefly discuss the difficulties in comparing games played in discrete and continuous time in Section 4.3. Note that these difficulties do not arise when only considering MPEs.

This story suggests in which situations the continuous-time model is the more appropriate modeling choice. When a decision actually has to be made over and over again, and a status quo cannot be said to exist in any meaningful sense, then the discrete-time game is the more appropriate. But if a status quo exists, and any decision is actually about whether to move away from the status quo or not, then the continuous-time model is perhaps more appropriate. Consider a case where the players are countries, for instance. When the legislative system is involved in deciding on the actions, that introduces a status quo: current policies such as laws, tax rates, regulations, tolls, and tariffs are sticky. Unless an active decision to change them is made, the status quo replicates itself. Furthermore, no two countries pass bills in parliament at the exact same instant – each country could respond to a move by the other country. A cross-boundary pollution game played between two nations would therefore fit the continuous-time framework very nicely. On the other hand, consider a situation in which commuters every day have to choose between
commuting by car or by train, and everyone commutes to work at the same
time. Under suitable assumptions on the payoffs, this is a repeated prisoners’
dilemma, yet it does not fit the continuous-time framework very well: choosing
a mode of commuting does not have a clear status quo, and a new decision
has to be made every day, revealed and relevant at discrete points in time.

I present different versions of the model and show that – broadly speaking –
there is a unique family of symmetric equilibria of the following form. Players
do not necessarily switch behavior immediately, but switches do occur – players
are thus randomizing in equilibrium. The equilibrium divides the game into
three phases, potentially repeated over and over. There is an attrition phase
characterized by mutual defection, and both players are waiting for the other
player to take the lead. Once one takes the lead, the game moves into the
leadership phase where the follower is free riding and enjoys the highest possible
payoff. From the leadership phase the leader can either take the players back
to the attrition phase, or the follower can take them to the cooperative phase
in which they both cooperate. The follower will not be willing to follow suit
unless it is sufficiently likely that the leader terminates his leadership, while
the leader is not willing to uphold his leadership unless it is sufficiently likely
that the follower follows suit. If they reach the cooperative phase, they remain
there for some time while each player fears that the other will defect first,
a so-called preemption game. Eventually one of them might defect and the
game moves back to the leadership phase. In equilibrium, the players will
jump back and forth on the state space. The main result of the paper is that I
demonstrate that there are equilibria in which cooperation is achieved swiftly,
and is long-lasting. Moreover, a higher discount rate does not diminish the
players’ changes at achieving mutual cooperation, contrary to the intuition
from the literature on the folk theorem.

The game I present is contained in a class of games known in the literature
on differential games as piecewise deterministic games (see e.g. Dockner et al.
(2000)). Examples of Markov-perfect strategies and piecewise deterministic
games are sparse in the more applied literature. The most common application
are within the domain of R&D competition, in which companies invest in R&D
efforts and a higher level of efforts increases the rate at which a successful
discovery is made (see for instance Harris and Vickers (1995)).

---

2What I call states are usually referred to as modes or regimes within this literature.
The term state is usually reserved for the parts of the state space that feature continuous
transitions, not discrete jumps. Herein I stick to the term state, since it is the more common
term outside of this more technical literature. This class of games has also been called
Markov jump games.
Continuous-time prisoners' dilemmas have received increased attention lately. Recent economic experiments report significantly higher cooperation rates in continuous-time prisoners' dilemmas compared to the same game played in discrete time (Friedman and Oprea, 2012; Bigoni et al., 2014). Some experiments are played with fixed, finite horizons, while others are played with stochastic horizons. Friedman and Oprea (2012) report that players cooperate around 90% of the time in continuous-time prisoners' dilemmas played for 60 seconds. Cooperation usually comes about early on, with one player taking the lead and the other player rapidly following. Towards the end of the game, cooperation unravels, but usually only in the last few seconds. Their results are closely replicated by Bigoni et al. (2014), who also report other results: unraveling does not occur towards the end when the horizon is stochastic; and they report that as the players gain experience, unraveling in the finite-horizon games occur later and later in the game. To explain why cooperation is easier to sustain, they invoke the insight of Simon and Stinchcombe (1989) and Bergin and MacLeod (1993) that players in continuous time can react almost immediately if they are preempted, thus the incentive to preemptively defect is radically diminished. The model I present features continuous time, but an infinite horizon. If – as is alluded to above – subjects are indeed playing Markov-perfect strategies, my model could contribute to explaining these experimental findings, as Markov-perfect cooperation only comes about in continuous time.

An infinite-horizon approach is of course not the only way to explain these findings. While several authors have rationalized cooperation in finitely repeated discrete-time prisoners' dilemmas by resorting either to incomplete information or imperfect rationality (see the literature review in Park (2014, p. 17)), less work has been done on continuous-time dilemmas. Park (2014) studies continuous-time dilemmas in a finite-horizon setting in which players have heterogeneous reaction lags, but these lags are private information. Park demonstrates that the swift, but non-immediate establishment of cooperation and the late unraveling observed in experiments, are consistent with the unique Perfect Bayesian equilibrium of his game.

I first present a continuous-time game with only a finite number of stages in Section 4.1, in order to introduce the concepts and provide the intuition. Then I present the full infinite-horizon model in Section 4.2. In Section 4.3, I compare the model to a discrete-time counterpart in order to discuss when the continuous-time model could be the more appropriate modeling choice. Section 4.4 concludes.
4.1 A finite number of stages

Consider the following game depicted in Figure 4.1. Time is continuous and the horizon is infinite. Two players have the possibility to either cooperate on some task or project or not, and cooperation is binary. The game starts out in an attrition phase where no player cooperates. If one player starts to cooperate, the game moves to the leadership phase. This phase lasts until either the leader gives up and ends his cooperative behavior, in which case the game ends in mutual defection, or until the free rider decides to follow suit, in which case the game ends in mutual cooperation. In either case it is assumed that the game ends and no further decisions can be made. This game is modeled as a prisoners’ dilemma and flow payoffs per unit of time are given in Table 4.1. That the horizon is infinite means that if the players never reach one of the two final nodes, the game never ends. Only if one of these nodes are reached, will the game end.

As mentioned, I only consider symmetric Markov-perfect equilibria. Markov-perfection requires that the strategies depend only on the payoff-relevant parts of the history, which in this game amounts to the current pair
of behaviors. Since the horizon is infinite, this holds, even though there are only a finite number of stages in the game. Markov-perfection therefore implies stationarity. In a discrete-time counterpart to this model, the actions the players could have chosen from would have been the probabilities with which they switch from their current behavior. In continuous time, such a probability translates to a (Poisson) \textit{rate}. The permissible actions are therefore the non-negative rates at which a player switches his current behavior (\textit{i.e.} from defect to cooperate). Stationarity requires these rates to be constant within each phase, which means that the players either switch their behavior immediately, never, or at some constant, positive rate. The time spent in each state will then be exponentially distributed. Finally, a strategy for each player is to choose such a rate for each possible state (attrition, leader in the leadership phase or follower in the leadership phase).

Since being the only one who cooperates is costly (\textit{i.e.} leadership is privately costly), absent any commitment devices we would expect a player to initiate cooperation from the attrition phase only if it is a best response for the other player to follow suit. Since the underlying game is a prisoners’ dilemma, being the free rider is the best situation possible in this game. The prospects for cooperation seem bleak.

As the logic involved in resolving this conundrum is important throughout the entire paper, I state it clearly here before solving the model formally. Consider two players currently in the leadership phase. The only reason for the follower to willingly abandon free-ridership for mutual cooperation would be if not doing so meant that the leader would instead cease his cooperative efforts. That is, if the leader is about to end his cooperative behavior, the follower would rather have the game ending in mutual cooperation than in mutual defection. Thus, the follower would then choose to follow suit and start cooperating. The problem is that without the possibility to commit, the

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<td>(0, 0)</td>
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\textbf{Table 4.1:} Flow payoffs per unit of time.
leader cannot credibly tell the follower that he will cease cooperation unless
the follower follows. As the leader also prefers mutual cooperation to mutual
defection, such a threat carries with it no credibility. The follower faces a
similar problem; he currently enjoys the best possible payoff and would like
the status quo to remain. Also he would like to make a promise of following
suit if the leader would just uphold his cooperative efforts for a little longer.
Clearly there are no equilibria in pure strategies. If the follower immediately
follows suit, the leader would never want to terminate his leadership. But
if the leader is willing to sustain his leadership, the follower would not want
to follow suit. Then again, if the follower does not follow suit, the leader is
not willing to sustain his leadership. And if the leader is about to cease his
cooperative efforts, the follower would rather follow suit immediately. Clearly
the best responses result in a cycle. The only possible equilibrium is there-
fore one in mixed strategies. At every point in time, the leader has to cease
cooperation with a probability that is so high that the follower is indifferent
between following suit and continuing his free-ridership; while the follower has
to follow suit with a probability that is high enough for the leader to be will-
ing to uphold his cooperative efforts for just a little longer. This only possible
equilibrium can thus be seen to involve delay: in order to get the follower to
follow suit, the leader has to endure this costly leadership phase for some time.

In order to solve the game, I employ backward induction on the state space.
When either mutual cooperation or mutual defection is reached, the game is
over. No more decisions are to be made. From Figure 4.1 we see that the final
node in which decisions are made is then the leadership phase. I therefore first
solve for the MPE in the leadership phase, then I move on to solve for the
MPE in the attrition phase.

4.1.1 The leadership phase

As I only consider symmetric equilibria, it does not matter whether we consider
state \((C, D)\) or state \((D, C)\). I will stick to \((C, D)\) throughout. The tool of
analysis will be continuous-time value functions and the so-called Hamilton-
Jacobi-Bellman equations (see Dockner et al. (2000) for the theory of and
examples on differential games and their solution methods).

Value functions are denoted by \(V(\cdot)\) and the continuous-time discount rate
is \(r\). I introduce some jump costs or -benefits with the following convention:
let \(l\) denote the jump cost or benefit incurred by the leader when the players
leave the leadership phase, and let \(f\) denote the corresponding costs/benefits
for the follower; an upper bar indicates the costs incurred by the player who
causes them to leave the state, while an underscore indicates the costs incurred
4.1. A FINITE NUMBER OF STAGES

by the player who did not cause the transition. Recall the flow payoffs from Table 4.1. For the leader, the value of being in state \((C, D)\) is then given by

\[
rV(C, D) = -c + \alpha (V(C, C) + l/r - V(C, D)) + \max_{\beta} \{\beta (V(D, D) + l/r - V(C, D))\}.
\]

(4.1)

The left-hand side denotes the “return” to being the leader. The first term on the right-hand side is the flow cost of providing leadership. Then at a rate \(\alpha\), the follower decides to follow suit. In that case the game jumps to state \((C, C)\) and the current value is replaced by \(V(C, C)\). In addition, the leader experiences a jump benefit \(l/r\), which may be negative. Finally, the leader can himself decide on the rate \(\beta\) at which he terminates his leadership and takes the players to mutual defection. If he does, the current value will be replaced by \(V(D, D)\) and he experiences the (possibly negative) jump benefit \(l/r\). As the states \((C, C)\) and \((D, D)\) are absorbing, the values \(V(C, C)\) and \(V(D, D)\) are exogenously given, and equal to the discounted present value of the flow payoffs in the respective states. This means that \(V(C, C) = a/r\) and \(V(D, D) = 0\), respectively. The value to the follower in the leadership phase is leading is given by

\[
rV(D, C) = b + \beta (V(D, D) + l/r - V(D, C)) + \max_{\alpha} \{\alpha (V(C, C) + l/r - V(D, C))\},
\]

(4.2)

with a similar interpretation as the one for the leader.

The jump costs and benefits could have several interpretations. If the players are countries, they could represent the costs of implementing new laws and regulations in order to change current policy. Or they could represent political and public relation costs in response to a change in the opponent’s behavior. If the players are firms, they could encompass capital costs incurred when installing new, or fine tuning existing, machinery to new production strategies. The jump costs and benefits are not always necessary, but do play a role in some of the results below.

To get at the equilibrium, imagine first that the leader never terminates his leadership, and is thus willing to forever supply this benefit to the follower. The follower could enjoy the flow benefit \(b\) forever, and he would only be willing to follow suit and give up \(b\) in exchange for the benefit \(a\) if the flow-equivalent jump benefit from doing that exceeds the loss in flow benefits, \(l > b - a\). This would be a trivial case in which cooperation is individually rational and, as such, is not very interesting. If instead \(l < b - a\), the follower would not want to follow suit unless the leader terminates his cooperation at a sufficiently high
rate. In general, the follower prefers to continue free riding (i.e. play $\alpha = 0$) whenever

$$V(D, C) \geq V(C, C) + \frac{f}{r}$$

$$\Rightarrow \beta \leq \frac{r(b - a - \overline{f})}{a + \overline{f} - \overline{l}} \equiv \hat{\beta}. \quad (4.3)$$

So if the leader terminates his leadership at a rate $\beta < \hat{\beta}$, the follower would prefer to continue free riding at the risk of ending up in mutual defection.

If the follower never follows suit, the leader would only be willing to uphold his leadership if it is sufficiently costly for him to stop. That will only be the case if he has to pay a jump cost to stop, and this jump cost exceeds the cost of leadership: $-\overline{I} > c$. This is also a rather trivial case, and one in which the game would never leave the leadership phase. If, however, $-\overline{I} < c$, the leader is only willing to continue cooperation as long as the follower is sufficiently likely to follow. In general, the leader is only willing to continue cooperating (i.e. play $\beta < \infty$) if

$$V(C, D) \geq V(D, D) + \frac{l}{r}$$

$$\Rightarrow \alpha \geq \frac{r(\overline{l} + c)}{a + \overline{l} - \overline{l}} \equiv \hat{\alpha}. \quad (4.4)$$

In order to rule out the two trivial cases where either the follower finds it individually rational always to follow suit or where the leader finds it individually rational to uphold his leadership no matter what, I will make the following assumption.

**Assumption 4.1.** Throughout it will be assumed that $a + \overline{f} - f > 0$ and $a + \overline{l} - \overline{l} > 0$, as well as that $\overline{f} < b - a$ and $-\overline{l} < c$.

Assumption 4.1 guarantees that $\hat{\alpha}$ and $\hat{\beta}$ are non-negative. Note that the assumption is satisfied for instance if all the jump benefits are equal to $\varepsilon$, where $-c < \varepsilon < b - a$. Zero jump costs and benefits will for instance satisfy Assumption 4.1. This leads to the following equilibrium.

**Lemma 4.1.** Under Assumption 4.1 there is a unique MPE in the leadership phase given by

$$\alpha = \hat{\alpha}, \quad \beta = \hat{\beta}, \quad V(C, D) = \frac{l}{r}, \quad V(D, C) = \frac{(a + \overline{f})}{r},$$

where $\hat{\alpha}$ and $\hat{\beta}$ are given by (4.4) and (4.3).

**Proof.** If $\beta < \hat{\beta}$, then the follower’s best response is $\alpha = 0$. But the leader’s best response to $\alpha < \hat{\alpha}$ is $\beta > \hat{\beta}$. The follower’s best response to such $\beta$ is $\alpha > \hat{\alpha}$, which implies $\beta = 0$, a cycle. The best responses only intersect once, at $\hat{\alpha}$ and $\hat{\beta}$. This intersection is then the unique MPE in the leadership phase. \qed
4.1. A FINITE NUMBER OF STAGES

The unique equilibrium involves some positive, but finite delay. When both players leave the leadership phase at some constant positive rate, the state transitions are Poisson distributed. The expected time spent in the leadership phase (the holding time) is accordingly exponentially distributed with a parameter equal to the sum of the rates at which the players leave. The expected delay in the leadership phase is then shorter, whenever the rates at which the players leave the state are higher, as the following result demonstrates.

Proposition 4.1. Subject to Assumption 4.1, the expected time spent in the leadership phase is positive and

- increases in $a$, $f$, and $l$, while it decreases in $c$, $b$, $r$, $f$, and $l$.

Proof. The holding time is exponentially distributed with parameter $(\alpha + \beta)$, thus the mean holding time in equilibrium is $1/(\hat{\alpha} + \hat{\beta})$, where $\hat{\alpha}$ and $\hat{\beta}$ are given by Equations (4.4) and (4.3). The results follow by differentiation. \hfill \Box

If the cost to the leader, $c$, is higher, then the follower has to follow suit at a higher rate in order for the leader to be willing to sustain cooperation. The expected delay is accordingly shorter. Similarly, if the free-rider payoff $b$ is higher, the leader has to cease his efforts at a higher rate in order for the follower to still be willing to follow suit. If mutual cooperation becomes more attractive ($a$ increases), then the leader will be willing to sustain leadership even if the follower follows suit at a lower rate, but the the follower will also be willing to follow even if the rate at which the leader ceases cooperation is lower. The expected delay is accordingly longer. Finally, if the discount rate $r$ increases, both players put more emphasis on the present relative to the future. The effect is similar to the effect of increasing $c$ and $b$.

So far I have only described the expected delay. Since both $\alpha$ and $\beta$ are non-negative in equilibrium, it should be clear that the players could reach mutual cooperation, but it is not inevitable. It depends on the equilibrium rates $\hat{\alpha}$ and $\hat{\beta}$.

Proposition 4.2. Subject to Assumption 4.1, the probability that the players reach mutual cooperation, conditional on being in the leadership phase

- increases in $c$, $l$, and $f$, while it decreases in $b$, $l$, and $f$,
- is independent of the discount rate $r$. 
Proof. There are only two possibilities for escape from the leadership phase. The probability that the game escapes to mutual cooperation is $\frac{\hat{\alpha}}{\hat{\alpha} + \hat{\beta}}$. The result follows from differentiation.

As the follower’s rate increases in $c$ while the leader’s rate increases in $b$, the two parameters affect the probability that the players reach mutual cooperation in opposite directions. When leadership becomes more costly, the follower has to increase the rate at which he follows suit, thus making cooperation more likely. On the contrary, when free riding becomes more lucrative, the leader has to increase the rate at which he terminates his leadership, thus making mutual cooperation less likely. This has some interesting real-world applications. Consider for instance the current state of global climate-change cooperation. If we abstract away the stock effects of carbon in the atmosphere (or consider a damage function that is linear in the stock), the game presented here nicely fits the climate problem. The E.U. currently has some unilateral carbon abatement policies in place, and can be considered a leader, whereas China and the U.S., on the other hand, currently have little or no policies in place and can be considered followers. Based on the proposition above, one could argue that the E.U. can make it more likely that China and the U.S. will follow suit if they could only make it less attractive to free ride. One way to reduce the attractiveness of free-ridership is to implement so-called border carbon adjustment policies (BCA) in which goods imported to the E.U. would be subject to a carbon tariff depending on the carbon content of the imports. In a computational general equilibrium model, such border carbon adjustment has been found to “reduce greatly the abatement cost to the coalition states, shifting a large part of the cost to the non-coalition regions through international trade” (Babiker and Rutherford, 2005). The E.U. has indeed tried to enact such BCA on international flights originating or terminating on E.U. airports\(^3\).

Note that it is not the case that cooperation is more likely to take place when the players discount the future at a lower rate. The effect of an increased $\alpha$ is ambiguous and depends on the model parameters. To highlight it’s impact, the next result concerns a case where all the jump benefits are of the same sign and magnitude.

Proposition 4.3. Consider a situation where all the jump benefits are equal, and equal to $\varepsilon$. The probability that mutual cooperation is reached is given by

$$\frac{c + \varepsilon}{b + c - a},$$

and mutual cooperation is more likely when it is more attractive ($a$ is higher).

To get back to the climate example, a policy of clean technology development is likely to make it cheaper to follow suit (*i.e.* increase $a$). This should then also be a candidate E.U. policy if they want the U.S. and China to follow suit.

### 4.1.2 The attrition phase

In the attrition phase there are no flow payoffs. Each player is waiting, hoping the other player will take the lead first. If the opponent makes the first move, you will be able to free ride on his cooperative efforts for some time. The value functions are given by (where a tilde denotes the action of the opponent)

$$rV(Attr) = \tilde{\gamma}(V(D,C) + \frac{\xi}{r} - V(Attr))$$

$$+ \max_{\gamma} \{ \gamma(V(C,D) + \frac{\pi}{r} - V(Attr)) \}.$$  \(4.5\)

Again, the left-hand side is the return to being in the attrition phase. There are no flow payoffs, the value stems in its entirety from the future possibilities. At a rate $\tilde{\gamma}$ the other player takes the lead, and you can free ride on his efforts. You decide yourself the rate $\gamma$ at which you take the lead. In both cases the current value $V(Attr)$ is lost. $e$ denotes the jump benefits each player experiences when leaving the attrition phase. As above, an upper bar indicates the benefit when you yourself are causing the jump, while an underscore indicates the benefit you get when the other player causes the jump. The higher the rate at which your opponent starts to cooperate, the more likely it is that you will become the free rider. It is thus less attractive to assume leadership when your opponent starts to lead at a high rate, and the rates are therefore strategic substitutes. There is potentially a family of asymmetric equilibria in which one player moves immediately and the other moves at a very low rate. Within the class of symmetric equilibria, however, the equilibrium is unique.

**Lemma 4.2.** There is a unique symmetric MPE in the attrition phase. Given Assumption 4.1, the equilibrium is given by

$$\gamma = \max \{0, \tilde{\gamma} \}, \quad \text{where} \quad \tilde{\gamma} = \frac{r(1 + \pi)}{a + f - l + e - \pi}.$$  

**Proof.** Enter the values for $V(C,D)$ and $V(D,C)$. If the opponent plays $\tilde{\gamma} = \hat{\gamma}$, then the player is indifferent between playing $\gamma = 0$ and $\gamma = \infty$. If $\tilde{\gamma} > \hat{\gamma}$ the best response is 0, whereas the best response is $\infty$ if $\tilde{\gamma} > \hat{\gamma}$. The only symmetric equilibrium is therefore $\gamma = \tilde{\gamma} = \hat{\gamma}$.  \(\square\)
Whenever $l + r > 0$ the players eventually leave the attrition phase almost surely, but there could be a sizeable delay.

**Proposition 4.4.** When Assumption 4.1 holds and $l + r > 0$, the expected holding time in the attrition phase

- increases in $a$ and $e$, while it
- decreases in $r$, $l$, $e$ and $f$.

**Proof.** The expected holding time is $1/(2\gamma) = 1/(2\hat{\gamma})$. The results follow from differentiation. \(\square\)

Again we see that the expected delay increases in the gain from mutual cooperation, $a$. As mutual cooperation becomes more attractive, the value of being the free rider increases (as the free rider’s outside option is mutual cooperation). And when free-ridership becomes more attractive, the equilibrium involves the players taking the lead at a lower rate, since free-riding is now more attractive. Both the attrition phase and the leadership phase will last longer when cooperation in more attractive, so the model unequivocally predicts that it will take more time before the game is resolved when the cooperative outcome is more attractive.

Furthermore note that neither $b$ nor $c$ enter the attrition-phase equilibrium, as they per Lemma 4.1 play no role in the values the players experience in the leadership phase – only the outside options $l$ and $a + f$ matter once the leadership phase is reached. Thus the cost of providing leadership, $c$, does not affect the rate at which either player wants to take the lead.

Finally, note that if a potential leader can never expect any jump benefits ($l + r \leq 0$), no player would ever want to take the lead in a symmetric equilibrium, as argued in the following proposition.

**Proposition 4.5.** Whenever Assumption 4.1 holds and $l + r \leq 0$, there is a unique symmetric MPE in which the game never leaves the attrition phase ($\gamma = 0$).

**Proof.** Follows directly from Lemma 4.2: whenever $l + r \leq 0$, then $\hat{\gamma} \leq 0$ and $\gamma = 0$. \(\square\)

In the absence of positive jump benefits, the leader’s outside option is non-positive, and so the value of being the leader is also non-positive. The value of being in the attrition phase, however, will be positive if your opponent is currently starting to cooperate at a positive rate, since there then is a positive probability that he becomes the leader and that you can free ride on his efforts. This possibility is an option value that will be lost the moment you yourself
4.2. AN INFINITE NUMBER OF STAGES

take the lead, thus the players cannot benefit from taking the lead when there are no jump benefits and only a finite number of stages. Consequently there are no symmetric MPEs in which the game ever leaves the attrition phase.

This result will not resurface in Section 4.2, where I discuss the model with an infinite number of stages. The reason is that the option value alluded to above does not vanish when the number of stages is unrestricted – a discontent leader could always defect to restart the attrition phase. To sum up this finite-stages game, there is a unique symmetric MPE, in which cooperation only will happen if the net jump benefits to the leader are positive. Before any player has taken the lead, the game is a war of attrition in which both players wait for their opponent to assume leadership. Once a player has taken the lead, there is a an intermediate phase in which the leader tries to induce the follower to follow suit, while the follower tries to motivate the leader to uphold his leadership a little longer.

To sum up the effect of the discount rate $r$, both the attrition phase and the leadership phase last shorter when the discount rate is higher. This means that the players reach a resolution faster when there is more heavy discounting. The probability that they end up in mutual cooperation does not go down, it remains unchanged. This means that “more” cooperation will be observed when the discount rate is higher than when it is lower. This is counter to the insights from the folk theorem literature, where cooperation is more likely to be sustained when the future is less heavily discounted.

4.2 An infinite number of stages

I now present the model where the number of stages is not limited. If the leadership phase ends, the game still moves to either mutual cooperation or mutual defection. The difference from the model in Section 4.1 is that any player may restart the leadership phase from mutual defection, and any player can defect even if they both are currently cooperating. Mutual cooperation and mutual defection are thus no longer absorbing states. The state transitions are then no longer accurately represented by Figure 4.1. When there is an infinite number of stages, the state space is no longer a tree with unidirectional state transitions, but rather as depicted in Figure 4.2.

With only a finite number of stages the values $V(C,C)$ and $V(D,D)$ where exogenously given, since the respective states where absorbing. With an infinite number of stages, this is no longer the case. In terms of analysis, this means that the value of reaching mutual cooperation or returning to mutual defection is endogenously determined in equilibrium, and depends on the rates
CHAPTER 4. CONTINUOUS-TIME PRISONERS’ DILEMMAS

Attrition
\((D, D)\)

Leadership
\((C, D)/(D, C)\)

Mutual cooperation
\((C, C)\)

Figure 4.2: The states of the game with an infinite number of stages. The edges indicate the possible transitions. Only the leader can take the players from Leadership to Attrition, while only the follower can take the players from Leadership to Cooperation.

at which the players again return to the leadership phase. In order to solve the model, I start out by stating the values in the most general case with different jump benefits in each state and for each player. The expressions for the value functions below are identical to the ones with a finite number of stages: Equations (4.1), (4.2) and (4.3) correspond to Equations (4.5), (4.1) and (4.2), respectively. Equation (4.4) is new. Here, \( \delta \) represents the rate at which each player is defecting when both players are currently cooperating, while \( d \) represents the jump benefit experienced by either player if such a
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defection occurs.

$$\begin{align}
r V(D, D) &= \gamma \left( V(D, C) + \frac{c}{r} - V(D, D) \right) \\
&\quad + \max \gamma \left\{ \gamma \left( V(C, D) + \frac{c}{r} - V(D, D) \right) \right\}, \quad (4.1)
\end{align}$$

$$\begin{align}
r V(C, D) &= -c + \alpha \left( V(C, C) + \frac{1}{r} - V(C, D) \right) \\
&\quad + \max \beta \left\{ \beta \left( V(D, D) + \frac{1}{r} - V(C, D) \right) \right\}, \quad (4.2)
\end{align}$$

$$\begin{align}
r V(D, C) &= b + \beta \left( V(D, D) + \frac{f}{r} - V(D, C) \right) \\
&\quad + \max \alpha \left\{ \alpha \left( V(C, C) + \frac{f}{r} - V(D, C) \right) \right\}, \quad (4.3)
\end{align}$$

and

$$\begin{align}
r V(C, C) &= a + \delta \left( V(C, D) + \frac{d}{r} - V(C, C) \right) \\
&\quad + \max \delta \left\{ \delta \left( V(D, C) + \frac{d}{r} - V(C, C) \right) \right\}. \quad (4.4)
\end{align}$$

As in the previous section, the game is divided into three phases, the attrition phase, the leadership phase, and the cooperation phase. As the analysis will demonstrate, also in this section will there be a unique equilibrium in the leadership phase. The following thresholds prove useful in establishing that result.

**Lemma 4.3.** In the leadership phase, the leader is only willing to uphold his leadership (play $$\beta = 0$$) if

$$\alpha \geq \hat{\alpha} \equiv \frac{r V(D, D) + I + c}{V(C, C) - V(D, D) + \frac{I - r}{r}}. \quad (4.5)$$

while the follower is only willing to follow suit (play $$\alpha > 0$$) if

$$\beta \geq \hat{\beta} \equiv \frac{b - \frac{f}{r} - r V(C, C)}{V(C, C) - V(D, D) + \frac{f}{r}}. \quad (4.6)$$

**Proof.** From (4.2), $$\beta = 0$$ is only optimal when $$V(C, D)|_{\beta=0} \geq V(D, D) + \frac{I}{r}$$. It is straightforward to solve for $$V(C, D)|_{\beta=0} = (-c + \alpha(V(C, C) + \frac{I}{r}))/\alpha$$. Solve this for $$\alpha$$ to arrive at (4.5). Furthermore, from (4.3), $$\alpha > 0$$ is only optimal if $$V(C, C) + \frac{f}{r} \geq V(D, C)|_{\alpha=0}$$. Solve $$V(D, C)|_{\alpha=0} = (b + \beta(V(D, D) + \frac{f}{r}))/\beta$$ for $$\beta$$ in order to arrive at (4.6). 

□
Again, as was the case with only a finite number of stages, the leader is only willing to sustain leadership if the follower is sufficiently likely to follow, while the follower is only willing to follow if the leader is sufficiently likely to terminate his leadership.

**Lemma 4.4.** Consider the case where \( \hat{\alpha}, \hat{\beta} > 0 \). Then there is a unique MPE in the leadership face, given by \( \alpha = \hat{\alpha}, \beta = \hat{\beta}, V(C,D) = V(D,D) + l/r, \) and \( V(D,C) = V(C,C) + f/r \).

**Proof.** If \( \alpha < \hat{\alpha} \), the leader’s best response is \( \beta > \hat{\beta} \), to which the follower’s best response is \( \alpha > \hat{\alpha} \). The leader’s best response to this is \( \beta = 0 \), to which the follower responds by \( \alpha = 0 \); a cycle. The pair of best responses in Lemma 4.3 intersect only once. This intersection determines the unique equilibrium in states \((C,D), (D,C)\), which – as above – is characterized by each player getting a value equal to his outside option. Note that \( \hat{\alpha} \) and \( \hat{\beta} \) still depend on \( \gamma, \delta \).

Following the analysis in Section 4.1, one might rightfully expect that for different parameter values, there could be different kinds of equilibria. Below, I therefore focus on two cases: the case without any jump costs; and the case where each player incurs a jump cost of \( \varepsilon \) whenever he himself switches his behavior, but not when his opponent switches.

### 4.2.1 Costly switches

I here consider the case where both players have to pay a positive jump cost whenever they themselves switch behavior, but not when their opponent switches (\( e = l = f = d = -\varepsilon \), while \( e = l = f = d = 0 \)). This interpretation fits nicely if for instance the behaviors are something physical, so that it actually requires energy or effort to switch from your current behavior. This specification pins down a unique symmetric equilibrium. By Lemma 4.4, the players will be made indifferent between remaining in the leadership phase and leaving. Since they have to pay a jump cost in order to leave, the value they get before they leave will be pushed down below the value they can obtain once they have left and paid the jump cost – they will still be indifferent to leaving. This further means that it will be very unattractive to enter the leadership phase in the first place.

**Lemma 4.5.** The symmetric equilibrium when switching behavior is costly \((e = l = f = d = -\varepsilon \) and \( e = l = f = d = 0 \)), is unique and given by

\[
\gamma = 0, \quad \delta = 0, \quad \alpha = \frac{c - \varepsilon}{a + \varepsilon}, \quad \beta = \frac{b - a + \varepsilon}{a - \varepsilon},
\]

and

\[
V(D, D) = 0, \quad V(C, C) = \frac{a}{r}, \quad V(D, C) = \frac{a - \varepsilon}{r}, \quad V(C, D) = \frac{-\varepsilon}{r}.
\]
Proof. From Lemma 4.4 it immediately follows that \( V(C, D) = V(D, D) - \varepsilon/r \), and \( V(D, C) = V(C, C) - \varepsilon/r \). When plugging these into Equations (4.1) and (4.4), the optimal strategies in states \((C, C)\) and \((D, D)\) are clearly \( \delta = 0 \) and \( \gamma = 0 \), so these states are absorbing. This means that \( V(D, D) = 0 \) and \( V(C, C) = a/r \). Plugging these into Lemma 4.3, one arrives that the stated expressions for \( \alpha \) and \( \beta \).

If a player ever takes the lead, he will regret. He cannot escape without paying the jump cost, thus his value will be pushed down below his outside option, which is just the value he had before he took the lead. This in turn means that neither player will want to take the lead. Similarly, from a state of mutual cooperation a defector would regret it if he defected and started to freeride. Thus both mutual cooperation and mutual defection are absorbing – once the players enter either of those states, they will remain there forever.

**Proposition 4.6.** In the equilibrium when switching is costly, if the players ever reach mutual cooperation, they will cooperate forever.

Proof. Follows immediately from Lemma 4.5 as \( \delta = 0 \). This means that the players never leave state \((C, C)\).

Whether the players will be able to cooperate in the first place is an open question. Mutual defection is also an absorbing state, so as long as the game does not start out in mutual defection, it is possible that mutual cooperation is reached. The following proposition demonstrates how the jump cost, \( \varepsilon \), affects the equilibrium.

**Proposition 4.7.** When the jump cost \( \varepsilon \) increases, the expected holding time in the leadership phase goes down, and it becomes more likely that the players end up in mutual defection.

Proof. From Lemma 4.5 we have that \( \alpha \) is decreasing and \( \beta \) is increasing in \( \varepsilon \), while \( \alpha + \beta \) increases in \( \varepsilon \) over the relevant range \([0, c]\). The expected holding time in the leadership phase is \( 1/(\alpha + \beta) \), which is decreasing in \( \varepsilon \). The probability that the players end up in mutual defection is given by \( \beta/(\alpha + \beta) = 1/(1 + \alpha/\beta) \) which is increasing in \( \varepsilon \).

When the jump cost increases, it affects the rates \( \alpha \) and \( \beta \) in opposite directions. The increased jump cost decreases the outside option of both players (as they have to pay the now-higher jump cost in order to get there). In order to understand why \( \alpha \) and \( \beta \) move in opposite directions, we need to recall the no-arbitrage conditions given in Lemma 4.4. As \( \varepsilon \) increases, both \( V(C, D) \) and \( V(D, C) \) must go down. This means that the leader must make it more likely
that the follower ends up in mutual defection (so $\beta$ must go up), while the follower must make it less likely that the leader ends up in mutual cooperation (so $\alpha$ must go down).

Note that in the limit when the jump cost vanishes, the equilibrium rates are $\alpha = cr/a$ and $\beta = (b - a)r/a$, which is the exact same equilibrium as the one with a finite number of stages presented in Lemma 4.1 (as long as one takes into account zero jump costs and benefits). As is demonstrated also in the next section, the model with a finite number of stages is just a special case of a more general model, but one in which mutual cooperation and defection are assumed absorbing up front.

The following proposition demonstrates how the payoff parameters affect the equilibrium.

**Proposition 4.8.**

- When the benefit to mutual cooperation, $a$, increases, it becomes more likely that mutual cooperation eventually will occur, but the expected holding time in the leadership phase increases.

- When the cost to the leader, $c$, increases, it becomes more likely that mutual cooperation eventually will occur, and the expected holding time in the leadership phase decreases.

- When the benefit to the free rider, $b$, increases, it becomes less likely that mutual cooperation eventually will occur, and the expected holding time in the leadership phase decreases.

**Proof.** The expected holding time is $1/(\alpha + \beta)$, while the probability that they reach mutual cooperation is $\alpha/(\alpha + \beta)$. The results follow from differentiation. \qed

When the benefit to mutual cooperation, $a$ increases, both the leader’s defection rate $\beta$ and the follower’s following rate $\alpha$ decrease. The follower can now follow at a lower rate, since the carrot he is presenting to the leader is now more attractive. Similarly, when mutual cooperation becomes more and more attractive the follower needs less of a motivation to be willing to abandon his free-ridership. The delay in the leadership phase becomes accordingly longer. That mutual cooperation becomes more likely, stems from the fact that the leader’s defection rate decreases relatively more than the follower’s follow rate. When the benefit to the free rider, $b$, increases, the leader has to increase the rate at which he defects. This decreases the delay in the leadership phase, and makes mutual cooperation less likely. In terms of observed outcomes, $a$ and $b$ act in opposite directions, and an increased $a$ would be observationally
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indistinguishable from a decreased $b$. The effect of $c$ is distinct, however: when
the cost to the leader, $c$, increases, the follower has to increase the rate at which
he follows, in order for the leader to still be willing to uphold his leadership.
The delay in the leadership phase thus goes down, and mutual cooperation
becomes more likely.

When the discount rate, $r$, increases, both the leader and the follower in-
crease their rates in the leadership phase. As a result, the expected holding
time in the leadership phase goes down. Moreover, as demonstrated in the fol-
lowing proposition, the probability that mutual cooperation is achieved does
not go down.

**Proposition 4.9.** When the discount rate, $r$, increases, the expected delay in
the leadership phase goes down, but the probability that mutual cooperation is
reached remains unchanged.

**Proof.** The expected holding time is $1/(\alpha + \beta)$ while the probability that mu-
tual cooperation is reached is given by $\alpha/(\alpha + \beta)$. The results follow from
differentiation of Lemma 4.5. \hfill \Box

It is therefore not the case that increased discounting means that less time
will be spent in mutual cooperation.

I now present a case without any jump costs and benefits.

4.2.2 No jump costs or benefits

Herein I consider the knife’s edge case that all jump costs and benefits are
zero. As per Lemma 4.4, the equilibrium in the leadership phase will be given
by $\alpha = \hat{\alpha}$ and $\beta = \hat{\beta}$. Furthermore, the values will equal the outside options,
which absent jump costs and benefits means that

$$V(C, D) = V(D, D), \quad (4.7)$$
$$V(D, C) = V(C, C). \quad (4.8)$$

The values in the leadership phase are now identical to the respective out-
side options. The value of being the leader is identical to the value you can
get if you terminate your leadership. Similarly, the value the follower gets is
identical to the value he can get if he follows suit.

By the value functions (Equations (4.1) and (4.4)), we can see that the
gain to a player who considers defecting from mutual cooperation, or from
taking the lead from the attrition phase is zero. Consider mutual cooperation.
Whatever gain one might hope to achieve by defecting, will be completely
offset by ones opponent responding with a defection rate, $\beta$, which is precisely
so high that \(V(D,C) = V(C,C)\). As the net benefit of leaving either of the states \((C,C)\) and \((D,D)\) are zero, any pair of rates can be an equilibrium in those states.

**Lemma 4.6.** In the game without jump costs there are infinitely many symmetric MPEs which all belong to the same family. In particular, any non-negative tuple \((\alpha, \beta, \gamma, \delta)\) constitute a symmetric MPE as long as

\[
\alpha = \gamma \frac{a + c}{a} + \frac{c}{a} (r + \delta),
\]

\[
\beta = \frac{b}{a} \delta + \frac{b - a}{a} (r + \gamma).
\]

**Proof.** By the equilibrium in the leadership phase (Lemma 4.4), we have that \(V(C,D) = V(D,D)\) and \(V(D,C) = V(C,C)\). By Equations (4.1) and (4.4), any non-negative \(\gamma\) is a best response in state \((D,D)\), and any non-negative \(\delta\) is a best response in state \((C,C)\). Furthermore, in a symmetric equilibrium, \(\tilde{\gamma} = \gamma\) and \(\tilde{\delta} = \delta\). We can then solve for the values in states \((C,C)\) and \((D,D)\):

\[
V(C,C) = \frac{a}{r + \delta} + \frac{\delta}{r + \delta} V(C,D),
\]

and \(V(D,D) = \frac{\gamma}{r + \gamma} V(D,C)\).

Insert these into the values \(V(C,D)\) and \(V(D,C)\) in (4.2) and (4.3), and solve for the rates \(\alpha\) and \(\beta\) such that the no-arbitrage conditions (4.7) and (4.8) hold.

There thus exists an infinite number of MPEs, but they all belong to the relatively straight-forward family presented in Lemma 4.6. We see that what matters is the rates played in the leadership phase relative to the rates played under mutual cooperation or defection. Consider for instance an increase in \(\delta\). Then the value of being in mutual cooperation goes down, as the cooperation phase becomes shorter. In the leadership phase, the leader then has to terminate his leadership at a higher rate in order for the follower to be willing to give up his freeridership. Similarly, when mutual cooperation becomes shorter-lasting, and hence less attractive, the freerider has to follow suit at a higher rate in order for the leader to still be willing to uphold his leadership.

**Proposition 4.10.** When the rate at which the players defect from mutual cooperation, or the rate at which they initiate cooperation from a state of mutual defection, increases, both the rate at which the leader terminates his leadership and the rate at which the follower follows suit will increase:

\[
\frac{\partial \alpha}{\partial \gamma} > 0, \quad \frac{\partial \alpha}{\partial \delta} > 0, \quad \frac{\partial \beta}{\partial \gamma} > 0, \quad \frac{\partial \beta}{\partial \delta} > 0.
\]
Proof. Follows directly from Lemma 4.6.

Recall that in equilibrium, the value of mutual cooperation must equal the value of freeridership, $V(C,C) = V(D,C)$. When $\delta$ is reduced, $V(C,C)$ increases. Accordingly it has to become more lucrative to free ride in order for the equilibrium to remain. This can be achieved only by the leader lowering the rate at which he abandons his leadership, thus $\alpha$ must go down. The leader is only willing to reduce $\alpha$, however, if the follower reduces $\beta$. Mutual cooperation can thus be longer-lasting in equilibrium if both $\alpha$ and $\beta$ go down when $\delta$ goes down – thus also the leadership phase is longer-lasting in equilibrium. Equilibrium cooperation is therefore longer-lasting only when the player who eventually defects can expect to be able to free ride for a longer period. Note how this is opposite to the intuition in Simon and Stinchcombe (1989) and Bergin and MacLeod (1993) in which continuous-time preemption is delayed in equilibrium because the players expect their opponent to immediately defect if they themselves defect.

The next result just establishes that there are equilibria in which you can decrease $\delta$ all the way to zero – within this family of infinite-stages equilibria there do exist equilibria in which mutual cooperation can be sustained almost indefinitely.

Proposition 4.11. There are Markov-perfect equilibria in which cooperation will be sustained almost indefinitely.

Proof. As any tuple satisfying (4.9) and (4.10) can constitute an MPE, consider the MPE where $\delta \to 0$. In the limit, mutual cooperation is an absorbing state.

Contrast this to the discrete-time game in which the unique MPE implies eternal mutual defection.

Note that if we insert $\gamma = \delta = 0$ in (4.9) and (4.10), we arrive at $\alpha = cr/a$ and $\beta = (b-a)r/a$, which is the exact same equilibrium as the one with a finite number of stages presented in Lemma 4.1 (as long as one takes into account zero jump costs and benefits). As is also demonstrated in the previous section, an infinite number of stages is therefore just a generalization of the previous model in which mutual cooperation and defection are no longer necessarily absorbing. Recall, however, that when there are only a finite number of stages, no player would want to take the lead if there are no jump benefits in the model. This is not the case with an infinite number of stages.

Proposition 4.12. In contrast to the game with only a finite number of stages, there exist no-jump-benefit MPEs in which a player takes the lead from a state of mutual defection.
Proof. Follows directly from Lemma 4.6.

Recall that the reason no player wanted to take the lead in a symmetric MPE when there was only a finite number of stages, was that the option value of becoming the freerider was lost the moment you took the lead. The outside option the leader was left with was eternal mutual defection. With an infinite number of stages, the option value of becoming the freerider will not be lost when one player takes the lead. When a player takes the lead from a state of mutual defection, he always has the option to defect and be back in the exact same state with the option value intact. This possibility to reverse makes it more attractive to be the leader, and accordingly makes it possible that a player will want to take the lead in equilibrium. The fact that this possibility is only available at a cost is the intuition for why the unique equilibrium has $\gamma = 0$ with costly behavior switching (Lemma 4.5).

The final result I present is akin to Proposition 4.8 in Section 4.2.1, in which I investigate how the different payoff parameters affect the probability that the players reach mutual cooperation. With that model specification, both mutual cooperation and mutual defection are endogenously absorbing, hence it makes sense to discuss the probability of reaching a state. In the present model specification it is more illuminating to investigate the fraction of time that will be spent in cooperation relative to the other phases. A little more machinery is needed to arrive at that result. First, note that the equilibrium analyzed induces a continuous-time discrete state Markov chain on the state space. Define the transition rate matrix by

$$
Q = \begin{pmatrix}
-2\gamma & \gamma & \gamma & 0 \\
\beta & -\alpha - \beta & 0 & \alpha \\
\beta & 0 & -\alpha - \beta & \alpha \\
0 & \delta & \delta & -2\delta 
\end{pmatrix}.
$$

(4.12)

The transition rate matrix resembles a discrete time Markov transition matrix. There are four states, $(D,D)$, $(C,D)$, $(D,C)$ and $(C,C)$, respectively, and entry $(i,j)$ denotes the rate at which the game jumps from state $i$ to state $j$. The negative elements on the main diagonal are there for accounting purposes, such that the sum along each row is zero. The zero entries along the antidiagonal tell us that the game cannot jump from the attrition phase to mutual cooperation without passing through the leadership phase, and that the an immediate switch of leader is not possible.

A stationary distribution over the state space tells us the fraction of time that will be spent in each state. Such a distribution, $\pi$, solves $\pi Q = 0$ under the restriction that $\sum_i \pi_i = 1$. 
Lemma 4.7. The stationary distribution is \( \pi = \frac{1}{\pi} \cdot (\pi_1, \pi_2, \pi_3, \pi_4) \) where
\[
\pi_1 = \frac{\delta((r + \gamma)(b - a) + b\delta)}{\pi}, \\
\pi_2 = \pi_3 = \frac{a\gamma\delta}{\pi}, \\
\pi_4 = \frac{\gamma(c + a) + c(r + \delta)}{\pi},
\]
and \( \pi = \gamma(rc + \gamma(c + a)) + \gamma\delta(a + b + c) + \delta(r(b - a) + b\delta) \).

Proof. \( \pi Q = 0 \) solves to \( \pi = \frac{1}{\pi}(\beta\delta, \gamma\delta, \gamma\delta, \gamma\alpha) \), where \( \pi \) ensures that the four parameters sum to 1. To arrive at the actual \( \pi \), insert from Lemma 4.6.

Given this lemma, I can now investigate how the different payoff parameters affect the fraction of time that will be spent in mutual cooperation.

Proposition 4.13. Conditional on the rate \( \delta \) in state \((C,C)\) and the rate \( \gamma \) in state \((D,D)\), more time will be spent in mutual cooperation when:

- the benefit of mutual cooperation, \( a \), is higher;
- the private gain to defecting, \( b \), is lower; and when
- the cost to unilateral cooperation, \( c \), is higher.

Proof. Follows by differentiating through Lemma 4.7.

With some modifications, these correspond to the results in Proposition 4.8. In Proposition 4.8 the result was that for a higher \( c \), less time would be spent in the leadership phase, and mutual cooperation became more likely. That fits this result. For an increased \( a \) and a decreased \( b \), the result was that the time spent in the leadership phase would go up, but that mutual cooperation became the more likely outcome. In the present context, this translates to more time spent in mutual cooperation. Even though a partial effect of a reduced \( b \) is that the delay in the leadership phase increases, the full equilibrium result here is that more time will be spent in mutual cooperation. The effects of the discount rate \( r \) unfortunately depends on the values of \( \gamma \) and \( \delta \) in the equilibrium studied.

This concludes the formal analysis. In the next section I discuss the similarities between, and the problems in comparing, discrete-time and continuous-time games – both within this model and more generally.
4.3 Discussion

In Appendix 4.A I demonstrate – step by step – how a standard, repeated discrete-time prisoners’ dilemma converges to the model presented in Section 4.2 as the length of each time period shrinks. In this section I only present a sketch of the analysis.

Step 1 is to allow for mixed strategies, so that in each period, each player is allowed to randomize between defecting and cooperating the following period. Step 2 is more about semantics. We can change the interpretation of each player’s moves. Instead of thinking about each player making a fresh choice over and over, imagine instead that each player currently has made a choice and then ponders whether to stick with his choice or to switch to the other available behavior. So far, this changes nothing in terms of equilibrium outcomes – the unique Markov-perfect equilibrium is still mutual defection with certainty. Step 3 is to let the time periods shrink in length. As the players no longer have to act every period, i.e. they can choose to remain with their current behavior, and the length of each period shrinks, the probability that both players act in the same period becomes negligible. One can think of this as two independent draws from distributions with overlapping continuous supports – clearly two identical draws are extremely unlikely to occur. This rules out certain state transitions, namely back and forth between mutual cooperation to mutual defection, and from one player being the leader to the other player being the leader. These transitions require that both players switch behavior at the same time—something that was just ruled out.

In practice this means that the currently observed pair of behaviors becomes payoff relevant in the sense of Maskin and Tirole (2001). The current pair of behaviors limits the set of states that can be reached in the immediate future. One cannot move from mutual cooperation to mutual defection without passing through a leadership phase. A rational player who is currently cooperating but considers whether to defect, should therefore take the current behavior of his opponent into account, as there is a difference between defecting to mutual defection and defecting to a leadership phase. A Markov-perfect equilibrium then allows for strategies that depend on the currently observed behaviors – something that was ruled out in the discrete-time game. In the discrete-time game, the currently observed behaviors do not affect the set of possible state transitions, and do not in any way affect the future payoff parameters – the currently observed behaviors are therefore payoff irrelevant. In the discrete-time game, Markov-perfect strategies are then not allowed to depend on the current behaviors, and both players are restricted to defecting with the
same probabilities, irrespective of what the opponent is currently playing. The unique Markov-perfect equilibrium is then mutual defection with certainty.

As mentioned in the introduction, this suggests in which situations the continuous-time model is the more appropriate modeling choice. When a decision actually has to be made over and over again, and a status quo cannot be said to exist in any meaningful sense, then the discrete-time game is the more appropriate model. But if a status quo exists, and any decision is actually about whether to move away from the status quo or not, then the continuous-time model is perhaps more appropriate. It is – unfortunately – an open question whether there are other discrete-time formulations that converge to the same continuous-time limit. If other such discrete-time games could be found, even more light could be shed on when the continuous-time model is the more appropriate modeling choice.

A problem in comparing continuous-time games to their discrete-time counterparts is that the notion of a subgame is not well defined in continuous time. The reason is that the rational numbers are not well ordered, there is no natural first moment after time $t$, for instance (see Fudenberg and Tirole (1985), Simon and Stinchcombe (1989) and the discussion in the introduction to Hörner et al. (2014)). This problem is circumvented when studying Markov-perfect equilibria (MPEs) as compared to subgame-perfect equilibria, since MPEs only rely on the current state of the game. The notion of a subgame then poses no problems. My results coincide with the observation in Hörner et al. (2014, p. 2) that (especially within the literature on experimentation with exponential “bandits”) “there are well-known examples in which the continuous-time definition of Markov equilibrium yields a set of payoffs that does not coincide with the limit of the set of Markov equilibrium payoffs for the discrete-time approximation.” Figuring out which model is the appropriate one in each situation is therefore important, as there is a qualitative difference between their predictions.

As a final note on the literature, note that cooperation can actually be achieved in Markov-perfect equilibrium in discrete-time games under suitable parameter restrictions, if the players are exogenously restricted to alternating moves (as in Maskin and Tirole (1988)). With reference to my Table 4.1, there exists an MPE in which the best response to a cooperative move by your opponent is to cooperate, if $a > b - c$ and the players move sequentially. The best response if your opponent defects is in mixed strategies. As Bhaskar et al. (2013) note (p. 932), cooperation in a Markov-perfect equilibrium is thus only achieved by randomization off the equilibrium path. If $a = b - c$, the equilibrium involves randomization also in response to a cooperative move by your opponent, but when $a = b - c$ the payoffs are additively separable in
the two players' actions, and the opponent's move is no longer payoff relevant. Note that no parameter restrictions are needed in my model.

An interesting venue for future research is the inclusion of imperfect information through detection lags, as the ability to immediately detect your opponent's move seems an important factor in the analysis in this paper.

4.4 Conclusion

In this paper I offer a slightly different take on the familiar repeated prisoners' dilemma. By rephrasing the game in continuous time, I am able to construct Markov-perfect equilibria in which cooperation does take place in equilibrium. In the continuous-time game, the two players cannot act at the same moment in time. This makes the current pair of behaviors payoff relevant, since the set of successor states is limited by the current state. The players cannot switch from mutual cooperation to mutual defection without passing through a leadership phase in which one player cooperates and the other free rides on the cooperative efforts of the leader.

The equilibrium requires the players to randomize in the leadership phase. The free rider is only willing to follow suit if the leader is sufficiently likely to terminate his leadership, while the leader is only willing to uphold his leadership if the follower is sufficiently likely to follow suit. This leads to interesting comparative statics, where the cooperative phase is longer-lasting only when the expected length of the ensuing free-ridership is also longer-lasting, and where a higher discount rate does not diminish the players' ability to achieve cooperation.

4.A Appendix - a discrete-time game

In this section I present the standard repeated prisoners' dilemma and describe the sense in which it corresponds to the continuous-time game presented in Section 4.2. I will show that when we allow for mixed strategies in the standard repeated game, the continuous-time game is the limit when the time periods shrink to zero.

In the model setup time is discrete, each period lasts $dt$ units of time, the horizon is infinite and each player has two possible actions, Cooperate or Defect. To allow for mixed strategies, the players control the probability with which they choose $C$ or $D$ as their next action. To ease the transition to continuous time, one can think of the players as deciding on whether they will play $C$ or
4.A. APPENDIX - A DISCRETE-TIME GAME

D sometime in between the two periods. When the next period arrives, both players have decided. For sake of comparison I will denote the probabilities by different variables for different states, even though – as will be argued below – Markov perfection of course requires the players to defect with the same probability in every state. Tildes indicate the strategy of the opponent, which is exogenous to the player. The equilibrium solves the following program:

\[ V(D, D) = \max_{\gamma} \left\{ \rho \left[ \gamma dt \tilde{\gamma} dt V(C, C) + \gamma dt (1 - \tilde{\gamma} dt)V(C, D) \right] 
+ (1 - \gamma dt) \right\}, \tag{4.A.1} \]

\[ V(C, D) = \max_{\beta} \left\{ - c dt + \rho \left[ \beta dt \alpha dt V(D, C) + \beta dt (1 - \alpha dt)V(D, D) \right] 
+ (1 - \beta dt) \right\}, \tag{4.A.2} \]

\[ V(D, C) = \max_{\alpha} \left\{ b dt + \rho \left[ \alpha dt \beta dt V(C, D) + \alpha dt (1 - \beta dt)V(C, C) \right] 
+ (1 - \alpha dt) \beta dtV(D, D) + (1 - \alpha dt) (1 - \beta dt)V(D, C) \right\}, \tag{4.A.3} \]

and

\[ V(C, C) = \max_{\delta} \left\{ a dt + \rho \left[ \delta dt \tilde{\delta} dt V(D, D) + \delta dt (1 - \tilde{\delta} dt)V(D, C) \right] 
+ (1 - \delta dt) \tilde{\delta} dtV(\tilde{C}, D) + (1 - \delta dt) (1 - \tilde{\delta} dt)V(C, C) \right\}. \tag{4.A.4} \]

Consider for instance state \((C, C)\) and Equation (4.A.4). The players control the probability \(\delta dt\) with which they defect before the next period. With probability \(\delta dt \cdot \tilde{\delta} dt\), both players defect and in the next period they are in state \((D, D)\), and so on. As mentioned above, this discrete-time game constitutes the familiar iterated prisoners’ dilemma. As such, Markov perfection requires that players defect next period with the same probability in every state today, as the current pair of behaviors is payoff-irrelevant (in the sense of Maskin and Tirole (2001)). That is, Markov perfection implies stationarity. It is therefore well-known that the only MPE in this game is that both players always defect with probability 1.

To understand the stark difference between this result and the results in Section 4.2, I will here derive the continuous-time limit of this game. To that effect, rewrite \(\rho = e^{-rdt} \equiv \sum_{n=0}^{\infty} (-rdt)^n / n! = 1 - rdt + o(dt)\), where \(o(dt)\) captures all terms that are of second or higher order in \(dt\). Thus for \(dt\) small, \(\rho = e^{-rdt} = 1 - rdt\) is a good approximation, and one which holds exactly at the
limit. Below I clearly spell out the continuous-time transformation of Equation (4.A.3) into Equation (4.3). The other equations follow similarly. First insert for the approximation of $\rho$ to get

$$V(D,C) = \max_\alpha \left \{ bdt + (1 - rdt) \left [ \alpha dt \beta dt V(C,D) + \alpha dt (1 - \beta dt) V(C,C) \right. \right.$$ 

$$+ (1 - \alpha dt) \beta dt V(D,D) + (1 - \alpha dt) (1 - \beta dt) V(D,C) \left. \right] + o(dt) \right \}.$$

Then expand the terms inside the brackets

$$V(D,C) = \max_\alpha \left \{ bdt + (1 - rdt) \left [ \alpha dt \beta dt V(C,D) + \alpha dt V(C,C) - \alpha dt \beta dt V(C,C) \right. \right.$$ 

$$+ \beta dt V(D,D) - \alpha dt \beta dt V(D,D) \right.$$ 

$$+ V(D,C) - \alpha dt V(D,C) - \beta dt V(D,C) + \alpha dt \beta dt V(D,C) \left. \right] + o(dt) \right \},$$

and again gather all terms of order 2 or higher in $dt$

$$V(D,C) = \max_\alpha \left \{ bdt + (1 - rdt) \left [ \alpha dt V(C,C) + \beta dt V(D,D) + V(D,C) \right. \right.$$ 

$$- \alpha dt V(D,C) - \beta dt V(D,C) + o(dt) \right.$$ 

$$+ o(dt) \right \}.$$

Note that all transitions from state $(D,C)$ to state $(C,D)$ are now gathered in $o(dt)$, because in order for such a transition to take place, both players have to switch within the same time period. The probability with which that occurs goes to zero at a quadratic rate when we take the continuous-time limit. The same would apply to the transitions between $(C,C)$ and $(D,D)$. In the discrete-time game, each player has to act once every period, thus these double transitions can take place. But as we transition to continuous time and, and both players no longer have to act every period, the probability that two jumps occur simultaneously goes to zero very fast. To derive the final expression, multiply in $(1 - rdt)$ and once more gather all higher-order $dt$ terms in $o(dt)$:

$$V(D,C) = \max_\alpha \left \{ bdt + \alpha dt V(C,C) + \beta dt V(D,D) \right.$$ 

$$+ V(D,C) - rdt V(D,C) - \alpha dt V(D,C) - \beta dt V(D,C) + o(dt) \right \}.$$

The final step before we take the limit is to cancel $V(D,C)$ from both the left- and the right-hand-side, and move $rdt V(D,C)$ to the left-hand side:

$$rdt V(D,C) = \max_\alpha \left \{ bdt + \alpha dt (V(C,C) - V(D,C)) + \beta dt (V(D,D) - V(D,C)) + o(dt) \right \}.$$
Finally, divide by $dt$ and take the limit $dt \to 0$. As all terms in $o(dt)$ are of order higher than one in $dt$, $\lim_{dt \to 0} o(dt)/dt = 0$, and we are left with Equation (4.3) (absent the jump benefits):

$$rV(D, C) = b + \beta(V(D, D) - V(D, C)) + \max_{\alpha} \{\alpha(V(C, C) - V(D, C))\}.$$ 

These operations clearly demonstrate how the continuous-time model distinguishes itself from the discrete-time model. In discrete time, both players act every period, while this is not the case in the continuous-time model. In continuous time, each player acts only at discrete points in time—although perhaps very often—and simultaneous transitions occur with zero probability. This means that the current pair of behaviors (i.e. the state) becomes payoff-relevant, as it determines the set of possible future states. Continuous time therefore introduces some inertia into the system, in that the current pair of behaviors in some sense becomes sticky. There is an option for each player not to take any action, and that would replicate the status quo. In the discrete-time model, both players are forced to act every period.

References


REFERENCES


Bibliography


