A note on the amplitude equations in Bénard convection.

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The non linear amplitude equations are derived for the motion of a fluid heated from below for the case where the motion possesses a discrete spectrum of rolls. The equations constitute a set of coupled Landau equations where the coefficients may be written as energy integrals for a linear differential operator. It is also shown that such amplitude equations are not obtained for the case of a continuous spectrum of rolls.

1. Introduction.

In a previous paper by Palm, Ellingsen and Gjevik (1967) some cellular motions in a fluid heated from below were studied. The investigation was limited to cases where the cell patterns were of either hexagonal or of purely two-dimensional form. The stability of these cell forms were studied and special attention was given to the role of the effect of a temperature dependent viscosity coefficient.

In the present note the motion is assumed to possess an arbitrary discrete spectrum of rolls, all having the same wave-length, but different orientation and (unknown) time dependent amplitudes. The amplitude equations are derived and are shown to have the form of a set of coupled Landau equations. These equations may be considered as generalizations of the equations derived by Schlüter, Lortz and Busse (1965) for the corresponding stationary amplitudes and the linear perturbations of these.

It is shown that the coefficients in the amplitude equations can be written as energy integrals for a linear differential operator, namely the four-dimensional second order operator which appears in the linearized equations of motion and of heat conduction. Since no directions in the horizontal plane are preferred there is a symmetry in the coefficients. Due to this symmetry it is possible to define a function of the amplitudes such that this function has a (local) maximum for the set of amplitudes for which the motion is steady and stable. This was pointed out and further discussed by Palm (1970).

Some discussion is also given to the assumption that the spectrum of rolls under consideration should be discrete. It is argued that when the spectrum is assumed to be continuous, no amplitude equations are obtained in the limiting case of an infinite horizontal extension of the fluid. The interpretation of this result seems to be that in such cases, some lateral wall effects must be important, and the amplitudes and the stability of the motion must strongly depend on the size and the form of the container in which the motion takes place.

2. The basic equations.

The motion under consideration is governed by the following dimensionless equations as derived by Palm et al. (1967)

$$(2.1) \qquad \frac{\partial u_i}{\partial t} + u_k \frac{\partial u_i}{\partial u_k} = - \frac{\partial p}{\partial x_i} + P\,\theta\,\delta_{i3} + P\nabla^2 u_i - \Gamma R \frac{\partial (x_3 u_{ik})}{\partial x_k} + \Gamma \frac{\partial(\theta u_{ik})}{\partial x_k} ,$$

$$(2.2) \qquad \frac{\partial \theta}{\partial t} + u_k \frac{\partial \theta}{\partial x_k} = \nabla^2 \theta + R\, u_3 ,$$

$$(2.3) \qquad \frac{\partial u_i}{\partial x_i} = 0 .$$

Here u_i are the velocity components (u_3 vertical), u_{ik} the deformation tensor,

$$(2.4) \qquad u_{ik} = \frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} ,$$

while θ and p are the deviations of temperature and pressure from those of the purely heat conducting (motionless) case. The scaling length, time, velocity, temperature and pressure are h, $h^2\kappa^{-1}$, $h^{-1}\kappa$, $R^{-1}\Delta T$ and $h^{-2}\kappa\rho_0$, respectively. Here h is the depth of the fluid layer, κ the thermal diffusivity, ΔT the temperature difference between the lower and the upper boundary and ρ_0 a standard density.

The density ρ and the kinematic viscosity are assumed to be linear functions of the temperature T, with $\frac{1}{\rho_0}\frac{d\rho}{dT} = -\alpha$ and $\frac{1}{\nu_0}\frac{d\nu}{dT} = \gamma$. The Rayleigh number R, the Prandtl number P and the parameter Γ in (2.1) are then defined by

$$R = \frac{\alpha g h^3 \Delta T}{\nu_0 \kappa}, \quad P = \frac{\nu_0}{\kappa}, \quad \Gamma = \frac{\nu_0^{\,2}\gamma}{\alpha g h^3}. \tag{2.5}$$

3. <u>Series expansions.</u>

Equations (2.1) to (2.3) are solved by the following expansions

$$\begin{aligned} u_i &= \varepsilon u_i^{(1)} + \varepsilon^2 u_i^{(2)} + \varepsilon^3 u_i^{(3)} + \cdots, \\ \theta &= \varepsilon\theta^{(1)} + \varepsilon^2\theta^{(2)} + \varepsilon^3\theta^{(3)} + \cdots, \\ p &= \varepsilon p^{(1)} + \varepsilon^2 p^{(2)} + \varepsilon^3 p^{(3)} + \cdots, \\ R &= R^{(o)} + \varepsilon R^{(1)} + \varepsilon^2 R^{(2)} + \cdots. \end{aligned} \tag{3.1}$$

With the use of some approximations, discussed by Palm et al. (1967), we arrive at the following equations for the first, the second and the third order terms.

$$(3.2) \qquad \begin{aligned} P\nabla^2 u_i^{(1)} + P\theta^{(1)}\delta_{i3} - \frac{\partial p^{(1)}}{\partial x_i} &= 0, \\ \nabla^2\theta^{(1)} + R^{(o)} u_3^{(1)} &= 0, \end{aligned}$$

$$(3.3) \qquad \begin{aligned} P\nabla^2 u_i^{(2)} + P\theta^{(2)}\delta_{i3} - \frac{\partial p^{(2)}}{\partial x_i} &= u_k^{(1)} \frac{\partial u_i^{(1)}}{\partial x_k}, \\ \nabla^2\theta^{(2)} + R^{(o)} u_3^{(2)} &= u_k^{(1)} \frac{\partial \theta^{(1)}}{\partial x_k}, \end{aligned}$$

$$(3.4) \qquad \begin{aligned} P\nabla^2 u_i^{(3)} + P\theta^{(3)}\delta_{i3} - \frac{\partial p^{(3)}}{\partial x_i} &= S_i, \\ \nabla^2\theta^{(3)} + R^{(o)} u_3^{(3)} &= S_\theta . \end{aligned}$$

S_i and S_θ are given by

$$(3.5) \qquad S_i = u_k^{(1)} \frac{\partial u_i^{(2)}}{\partial x_k} + u_k^{(2)} \frac{\partial u_i^{(1)}}{\partial x_k} - \Gamma \frac{\partial(\theta^{(1)} u_{ik}^{(1)})}{\partial x_k} + \frac{\partial u_i^{(1)}}{\partial t},$$

$$(3.6) \qquad S_\theta = u_k^{(1)} \frac{\partial \theta^{(2)}}{\partial x_k} + u_k^{(2)} \frac{\partial \theta^{(1)}}{\partial x_k} - \Delta R u_3^{(1)} + \frac{\partial \theta^{(1)}}{\partial t} .$$

Γ is assumed to be small and is omitted in all terms which would give only quantitative corrections to the solutions for $\Gamma = 0$, and is retained in the only term which causes a qualitatively new effect. Furthermore, instead of normalizing the solutions we put $\varepsilon = 1$ and let $u_i^{(1)}$, $u_i^{(2)}, \dots$ be small of successive orders. We have also used the fact that $\frac{\partial u_i^{(1)}}{\partial t}$ and $\frac{\partial \theta^{(1)}}{\partial t}$ are third order terms and that $R^{(1)} = 0$, giving $R^{(2)} = R - R^{(o)} = \Delta R$ to the order considered.

The solvability condition for (3.4) is

$$\int (R^{(o)} u_i' S_i + P\theta' S_\theta) dV = 0 \ , \tag{3.7}$$

which has to be satisfied for any solution (u_i', θ') of the linearized equation (3.2) satisfying the boundary conditions. The integration is to be taken over the fluid volume. (3.7) can be written

$$\begin{aligned} \int (R^{(o)} u_i' \frac{\partial u_i^{(1)}}{\partial t} + P\theta' \frac{\partial \theta^{(1)}}{\partial t}) dV &= \Delta R P \int \theta' u_3^{(1)} dV + \\ + \tfrac{1}{2} P \frac{\Delta \nu}{\nu_o} \int u_{ik}' \theta^{(1)} u_{ik}^{(1)} dV &+ \int (R^{(o)} u_i^{(2)} u_k^{(1)} \frac{\partial u_i'}{\partial x_k} + P\theta^{(2)} u_k^{(1)} \frac{\partial \theta'}{\partial x_k}) dV . \end{aligned} \tag{3.8}$$

Comparing with (3.3) we see that the last integral in (3.8) can be written

$$\int \left[R^{(o)} u_i^{(2)} (P\nabla^2 \tilde{u}_i^{(2)} + P\tilde{\theta}^{(2)} \delta_{i3}) + P\theta^{(2)} (\nabla^2 \tilde{\theta}^{(2)} + R^{(o)} \tilde{u}_3^{(2)}) \right] dV \tag{3.9}$$

where $\tilde{u}_i^{(2)}$ and $\tilde{\theta}^{(2)}$ are solutions of

$$\begin{aligned} P\nabla^2 \tilde{u}_i^{(2)} + P\tilde{\theta}^{(2)} \delta_{i3} - \frac{\partial \tilde{p}}{\partial x_i} &= u_k^{(1)} \frac{\partial u_i'}{\partial x_k} \ , \\ \nabla^2 \tilde{\theta}^{(2)} + R^{(o)} \tilde{u}_3^{(2)} &= u_k^{(1)} \frac{\partial \theta'}{\partial x_k} \ . \end{aligned} \tag{3.10}$$

4. The discrete spectrum of rolls.

A characteristic solution of the linearized equation (3.2) is

$$u_3 = f(z)\exp\left[ia(x\cos\phi + y\sin\phi)\right],$$

(4.1)

$$\theta = g(z)\exp\left[ia(x\cos\phi + y\sin\theta)\right],$$

with $f(z)$ and $g(z)$ satisfying

$$\left(\frac{d^2}{dz^2} - a^2\right)^2 f(z) - a^2 g(z) = 0\ ,$$

(4.2)

$$\left(\frac{d^2}{dz^2} - a^2\right)g(z) + R^{(o)}f(z) = 0\ .$$

u_1 and u_2 are then given by

$$u_1 = \frac{1}{a^2}\frac{\partial^2 u_3}{\partial x \partial z}\ ,$$

(4.3)

$$u_2 = \frac{1}{a^2}\frac{\partial^2 u_3}{\partial y \partial z}\ .$$

The first approximation $u_i^{(1)}$ and $\theta^{(1)}$ will be taken to be

$$u_i^{(1)} = \sum_\phi A(\phi,t)f_i(z,\phi)\exp\left[ia(x\cos\phi + y\sin\phi)\right],$$

(4.4)

$$\theta^{(1)} = \sum_\phi A(\phi,t)g(z,\phi)\exp\left[ia(x\cos\phi + y\sin\phi)\right],$$

where $A(\phi+\pi,t) = A(\phi,t)^*$, and

$$f_1(z,\phi) = \frac{i\,a\cos\phi}{a^2} f'(z),$$

(4.5) $$f_2(z,\phi) = \frac{i\,a\sin\phi}{a^2} f'(z),$$

$$f_3(z,\phi) = f(z).$$

The second order terms $u_i^{(2)}$ and $\theta^{(2)}$ may then be written

$$u_i^{(2)} = \sum_{\phi_1}\sum_{\phi_2} A(\phi_1,t)A(\phi_2,t)\; F_i(z,\phi_1,\phi_2) \times \exp\left[2i\,a\cos\frac{\phi_1-\phi_2}{2}\left(x\cos\frac{\phi_1+\phi_2}{2} + y\sin\frac{\phi_1+\phi_2}{2}\right)\right],$$

(4.6

$$\theta^{(2)} = \sum_{\phi_1}\sum_{\phi_2} A(\phi_1,t)A(\phi_2,t)\; G(z,\phi_1,\phi_2) \times \exp\left[2i\,a\cos\frac{\phi_1-\phi_2}{2}\left(x\cos\frac{\phi_1+\phi_2}{2} + y\sin\frac{\phi_1+\phi_2}{2}\right)\right].$$

It is not difficult to see that $F_3(z,\phi_1,\phi_2) = F(z,\lambda_{12})$ and $G(z,\phi_1,\phi_2) = G(z,\lambda_{12})$ will depend upon ϕ_1 and ϕ_2 through a parameter $\lambda_{12} = 4\cos^2\frac{\phi_1-\phi_2}{2}$. In fact, by eliminating $u_1^{(2)}$ and $u_2^{(2)}$ from (3.3), the equations for $u_3^{(2)}$ and $\theta^{(2)}$ give

$$\left(\frac{d^2}{dz^2} - \lambda_{12}a^2\right)^2 F(z,\lambda_{12}) - \lambda_{12}\, a^2 G(z,\lambda_{12}) =$$

$$= -\tfrac{1}{4} P^{-1}\lambda_{12}\left[\lambda_{12}(f'^2-a^2f^2) - 2(f'^2+ff''-2a^2f^2)\right]',$$

(4.7)

$$\left(\frac{d^2}{dz^2} - \lambda_{12}\, a^2\right)G(z,\lambda_{12}) + R^{(o)}F(z,\lambda_{12}) = (fg)' - \tfrac{1}{2}\,\lambda_{12}\, f'g.$$

$F_1(z,\phi_1,\phi_2)$ and $F_2(z,\phi_1,\phi_2)$ are then determined by

$$F_1(z,\phi_1,\phi_2) = \frac{2ia}{\lambda_{12}a^2} \cos\frac{\phi_1-\phi_2}{2} \cos\frac{\phi_1+\phi_2}{2} F'(z,\lambda_{12}) \ ,$$

(4.8)

$$F_2(z,\phi_1,\phi_2) = \frac{2ia}{\lambda_{12}a^2} \cos\frac{\phi_1-\phi_2}{2} \sin\frac{\phi_1+\phi_2}{2} F'(z,\lambda_{12}).$$

Considering now the solvability condition (3.8) we put

$$u'_i = f_i(z,\phi+\pi)\exp\Big[ia(x\cos(\phi+\pi) + y\sin(\phi+\pi))\Big] \ ,$$

(4.9)

$$\theta' = g(z) \exp\Big[ia(x \cos(\phi+\pi) + y \sin(\phi+\pi))\Big],$$

with arbitrarily chosen ϕ. Since all of the integrands in (3.8) are sums of products of two rolls, contributions to the integrals will be obtained only when the wave number vectors of the two rolls are opposite and of equal length.

Let us first discuss the last integral of (3.8) in terms of (3.9) and (3.10). $\tilde{u}_i^{(2)}$ and $\tilde{\theta}^{(2)}$ have the form (compare with (4.6))

$$\tilde{u}_i^{(2)} = \sum_{\phi_3} A(\phi_3,t)F_i(z,\phi_3,\phi+\pi) \times \exp\Big[2ia \cos\frac{\phi_3-\phi-\pi}{2}(x\cos\frac{\phi_3+\phi+\pi}{2} + y\sin\frac{\phi_3+\phi+\pi}{2})\Big] \ ,$$

(4.10)

$$\tilde{\theta}^{(2)} = \sum_{\phi_3} A(\phi_3,t)G(z,\phi_3,\phi+\pi) \times \exp\Big[2ia \cos\frac{\phi_3-\phi-\pi}{2}(x\cos\frac{\phi_3+\phi+\pi}{2} + y\sin\frac{\phi_3+\phi+\pi}{2})\Big] .$$

There will be contributions to the integral (3.9) whenever $\phi_1 = \phi_3 - \pi$, $\phi_2 = \phi$ or $\phi_2 = \phi_3 - \pi$, $\phi_1 = \phi$, and we find that (3.9) can be written in the following way

$$(4.11)\quad \begin{aligned} &\int\left[R^{(o)}u_i^{(2)}(P\nabla^2\tilde{u}_i^{(2)} + P\tilde{\theta}^{(2)}\delta_{i3}) + P\,\theta^{(2)}(\nabla^2\tilde{\theta}^{(2)} + R^{(o)}\tilde{u}_3^{(2)})\right]dV \\ &= -\,2P\sum_{\phi'} A(\phi)A(\phi')A(\phi'+\pi)B(\lambda)\ , \end{aligned}$$

where

$$(4.12)\quad \lambda = 4\cos^2\frac{\phi'-\phi}{2}\,,\qquad \text{and}$$

$$(4.13)\quad \begin{aligned} B(\lambda) = -\int&\left[R^{(o)}F_i^*\left(\left(\frac{d^2}{dz^2} - \lambda a^2\right)F_i + G\delta_{i3}\right)\right. \\ &\left. + G^*\left(\left(\frac{d^2}{dz^2} - \lambda a^2\right)G + R^{(o)}F\right)\right]dz. \end{aligned}$$

Here we have put $F_i = F_i(z,\phi',\phi)$, $F_3 = F(z,\lambda)$ and $G = G(z,\lambda)$ according to (4.7) and (4.8).

The other integrals in (3.8) are easily computed and the result is found to be

$$(4.14)\quad \begin{aligned} \frac{d}{dt}A(\phi)\int\left[R^{(o)}\left(\frac{1}{a^2}f'^2+f^2\right) + Pg^2\right]dz &= A(\phi)\Delta RP\int gf\,dz \\ &- A(\phi+\tfrac{\pi}{3})A(\phi-\tfrac{\pi}{3})\frac{1}{a^2}\,P\,\frac{\Delta\nu}{\nu_o}\int g\left[5f'^2 + \frac{1}{a^2}(f''+a^2f)^2\right]dz \\ &- A(\phi)2P\sum_{\phi'\neq\phi}|A(\phi')|^2\,B(\lambda) - A(\phi)|A(\phi)|^2P(B(4) + B(o)). \end{aligned}$$

To interpret the function $B(\lambda)$, we consider the linear four-dimensional operator

$$(4.15) \qquad \begin{Bmatrix} \delta_{ij}\nabla^2 & \delta_{i3} \\ \delta_{3j}R^{(o)} & \nabla^2 \end{Bmatrix}$$

$B(\lambda)$ is seen to be minus the integral over the fluid volume of a function

$$(4.16) \qquad \{U_i^* \quad \Theta^*\} \begin{Bmatrix} \delta_{ij}\nabla^2 & \delta_{i3} \\ \delta_{3j}R^{(o)} & \nabla^2 \end{Bmatrix} \begin{Bmatrix} U_i \\ \Theta \end{Bmatrix}$$

where (U_i, Θ) represents a roll with wave number $a\sqrt{\lambda}$ and z-dependence equal to that of the second order solution $(u_i^{(2)}, \Theta^{(2)})$. Since there are no subcritical instabilities in Bénard convection, it follows that the operator (4.15) is negative definite for $R^{(o)}$ less than $R_{crit.}$ With the restriction that (U_i, Θ) shall have the form of a second order solution, $B(\lambda)$ will certainly be positive for all λ for $R^{(o)} = R_{crit}$, and also for some positive values of $R^{(o)} - R_{crit}$.

5. A comment on Palm's maximum principle.

The amplitude equations (4.14) can be written in a simpler form

$$(5.1) \qquad \begin{aligned} \frac{d}{dt} A(\phi) &= \alpha_1 A(\phi) - \alpha_2 A(\phi + \tfrac{\pi}{3}) A(\phi - \tfrac{\pi}{3}) \\ &- \sum_{\phi' \neq \phi} \beta(\phi' - \phi) A(\phi) |A(\phi')|^2 - \tfrac{1}{2}\beta(o) A(\phi) |A(\phi)|^2, \end{aligned}$$

where $\beta(\phi'-\phi) = \beta(\phi-\phi')$.

The maximum principle discussed by Palm (1970) expresses that when the motion is represented by trajectories in an amplitude space, these trajectories will be perpendicular to the potential surfaces of a certain function V, and the direction of the motion will be towards increasing V. In the case of $\alpha_2 = 0$ (constant viscosity) there are no phase shifts in the amplitudes so that (5.1) reduces to equations for the amplitude modulus $|A(\phi)|$ alone. For a spectrum of n rolls, V will thus be a function of the n amplitudes $|A(\phi)|$. In the case $\alpha_2 \neq 0$, however, there are $2n$ variables to be considered, namely the amplitudes $|A(\phi)|$ and the phase angles θ. We shall show that the phase angles can be eliminated and that the maximum principle can be put in terms of the n variables $|A(\phi)|$ also in this case.

Writing $A(\phi) = |A(\phi)|\exp(i\theta)$ and $A(\phi \pm \frac{\pi}{3}) = |A(\phi \pm \frac{\pi}{3})|\exp(i\theta_\pm)$, (5.1) gives

$$\frac{d}{dt}|A(\phi)| = \alpha_1|A(\phi)| - \alpha_2\,|A(\phi+\tfrac{\pi}{3})||A(\phi-\tfrac{\pi}{3})|\cos(\theta-\theta_+-\theta_-) \tag{5.2}$$
$$- \sum_{\phi'\neq\phi}\beta(\phi'-\phi)|A(\phi)||A(\phi')|^2 - \tfrac{1}{2}\beta(o)|A(\phi)|^3,$$

$$\frac{d\theta}{dt} = \alpha_2\,\frac{|A(\phi+\frac{\pi}{3})||A(\phi-\frac{\pi}{3})|}{|A(\phi)|}\sin(\theta-\theta_+-\theta_-). \tag{5.3}$$

From (5.3) we deduce

$$\frac{d}{dt}(\theta-\theta_+-\theta_-) = \alpha_2 K\sin(\theta-\theta_+-\theta_-), \tag{5.4}$$

where K is positive. A stable stationary solution of (5.4) is characterized by

$$(5.5) \qquad \alpha_2 \cos(\theta - \theta_+ - \theta_-) = -|\alpha_2| .$$

From (5.2) it follows that $|A(\phi)|$ tends toward values for which V has a maximum when V is defined by

$$(5.6) \qquad \begin{aligned} V = {} & \tfrac{1}{2}\alpha_1 \sum_{\phi} |A(\phi)|^2 + \tfrac{1}{3}|\alpha_2| \sum_{\phi} |A(\phi)|\,|A(\phi+\tfrac{\pi}{3})|\,|A(\phi-\tfrac{\pi}{3})| \\ & - \tfrac{1}{4} \sum_{\phi \neq \phi'} \sum \beta(\phi'-\phi)|A(\phi)|^2 |A(\phi')|^2 - \tfrac{1}{8}\beta(o) \sum_{\phi} |A(\phi)|^4 , \end{aligned}$$

since, with the condition (5.5), (5.2) can be written

$$(5.7) \qquad \frac{d}{dt}|A(\phi)| = \frac{\partial V}{\partial |A(\phi)|} .$$

6. Discussion.

In deriving the amplitude equations (4.14), the assumption was made that the solution (u_1, θ) could be written as a sum of rolls, i.e. a discrete spectrum of rolls, as (4.4) indicates. We now want to discuss the modifications when the assumption of discreteness is removed and the sums in (4.4) are replaced by integrals. What we especially have in mind is motion with concentric circular cells which is sometimes observed in circular dishes heated from below, as reported by Koschmieder (1967). An investigation of a motion of this kind would require use of a continuous spectrum of rolls. In general, to consider some motion in a circular dish, a reasonable first order solution would be of the form

$$(6.1) \qquad w = \sum_n A_n(t)f(z)\exp(in\phi)J_n(ar),$$

in terms of cylindrical coordinates (r,ϕ,z), and this sum can again be written as an integral of plane rolls. The determination of the amplitudes $A_n(t)$ will therefore be equivalent to the determination of the amplitudes of a continuous spectrum of rolls. As we shall see, these amplitudes are not determined with the approximation used in this note, i.e. by considering the dish to be of infinite horizontal extension.

Returning now to the integrals in (3.8), we note that they are determined in the following way. The integrations are carried out over a domain of linear extension L in the horizontal plane, where $L \gg 1$, and the asymptotic values for $L \to \infty$, are considered. For a discrete spectrum it is found that all the integrals in (3.8) are of order L^2 and in the limit $L \to \infty$ (4.14) is obtained. For a continuous spectrum, however, the integral on the left side and the first integral on the right side in (3.8) are of order L while the last two integrals tend to finite values in the limit $L \to \infty$. The amplitudes of the continuous spectrum will therefore not be determined in the above approximation.

If a spectrum of rolls (discrete or continuous) is taken to represent the motion of the fluid in a given container, it must be interpreted as follows. The exact solution satisfying the boundary conditions, will consist of these rolls plus some correction terms like boundary layer terms and amplitude modulation terms, determined by the size and form of the container. And it may happen that these correction terms can be neglected as a first approximation. Segel (1969) considers cellular convective motion in a rectangular dish of finite size. And in his paper it is well demonstrated how the roll solutions for an infinite dish represent a first approximation and the amplitude modulation terms a second approximation to the solution.

Considering again the concentric circular cells in a circular dish, the situation is seen to be different. The results above indicate that the boundary layer terms and the amplitude modulation terms will be most important in the determination of the amplitudes and the criterion for the stability of the cells of this form. An attempt to take into account such wall effects will be made in another paper.

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