ISBN 82-553-0612-9 No 3
Applied Mathematics 1986

THE FORCES ON AN OSCILLATING FOIL MOVING NEAR A FREE SURFACE IN A WAVE FIELD

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PREPRINT SERIES - Matematisk institutt, Universitetet
i Oslo

ABSTRACT

The forces upon a foil moving below and close to a free surface are examined. The foil moves with a forward speed U and is subjected to heaving and pitching motions in calm water, head waves or following waves. The model is two-dimensional and all equations are linearized. The fluid is assumed to be inviscid and the motion irrotational, except for the vortex wake. The fluid layer is infinitely deep.

The problem is solved by applying a vortex distribution along the center line of the foil and the wake. The local vortex strength is found by solving a singular Fredholm equation of first kind, which appropriately is transformed to a non-singular Fredholm equation of second kind. The vortex wake, the forward thrust upon the foil and the power supplied to maintain the motion of the foil are investigated. The scattered free surface waves are computed. For moderate value of $U\sigma/g$ (U = forward speed of the foil, σ = frequency of oscillation, g = acceleration due to gravity) it is found that the free surface strongly influences the vortex wake and the forces upon the foil. When the foil is moving in incoming waves it is found that a relatively large amount of the wave energy may be extracted for propulsion. As application of the theory the propulsion of ships by a foil propeller is examined.

1. INTRODUCTION

The purpose of this paper is to study the forces, in particular the horizontal force, acting upon an oscillating foil moving close to a free surface. Our interest in the problem was raised by a series of experiments performed by Jakobsen (1981). He studied a model ship moving in a wave field, with a foil fixed to it. The waves caused a heaving motion of the ship which resulted in a heaving of the foil. By an arrangement of springs the foil also was able to perform a pitching motion. The result was that the foil obtained a thrust of considerable magnitude which gave the ship a relatively high forward speed. The system was working in head waves as well as in following waves.

Jakobsen also applied a set of horizontal foils fixed to a vertical axis in a fan-like manner. Placed in a wave field, the fan would start rotating about its axis. According to Jakobsen, this can be an effective way of extracting energy out of the waves for other purposes than propulsing a ship.

The idea that an oscillating foil may create a forward thrust, is not new. The theory for an oscillating foil in an unbounded fluid was discussed thoroughly already in 1934 (von Karman and Burgers, 1934). It also turns out that experiments somewhat related to those performed by Jakobsen - but far more primitive - was carried out as early as the end of the last century by H.F.L. Linden. He applied, instead of a foil, a flexible, horizontal plate which was placed in the rear of the ship close to the free surface.

More recently Wu (1961, 1971a, 1971b) in a series of papers on the hydromechanics of swimming propulsion, has studied the optimum oscillating motion of a two-dimensional flat, flexible plate. Since around a thin flat plate. The fluid layer is supposed to be of infinite depth.

As appropriate mathematical tool we shall apply the theory of integral equations. It will be shown that the governing integral equation is a singular Fredholm equation of first kind. By a simple transformation this is brought into a form of an ordinary Fredholm equation of second kind, which is solved numerically by a collocation method.

It is found that the free surface may have a pronounced effect on the magnitude of thrust, efficiency and supplied power. The effect is in particular large for small values of $\tau = U\sigma/g$ (U = speed of the foil, σ = frequency of encounter and g = acceleration due to gravity). Thus, oscillating the foil close to the free surface with U and σ small, we find when there is no incoming waves, that the forward thrust may be doubled without increasing the wasted energy. In this case the free surface acts as a rigid wall, enlarging the lift and the forward thrust. For larger values of U and σ the surface waves become important. The momentum flux in the waves may be responsible for a considerable part of the horizontal force acting upon the foil. For $\tau < 1/4$ the effect of the momentum flux is to reduce the thrust whereas for $\tau > 1/4$ the effect is usually to increase the thrust. Also the energy waste due to the waves may be of great importance, being in some cases the main part of the total energy waste.

When the foil is moving in waves it may extract energy from the waves for propulsion. For example, when the amplitudes of the incoming waves and the vertical motion of the foil are of the same

2. THE BOUNDARY VALUE PROBLEM

We shall assume that the hydrofoil has a small camber and angle of attack. The foil is also assumed to be thin, though sufficiently rounded at the leading edge to keep the flow from being separated there. For the oscillatory part of the flow the effects of camber and thickness are then only secondary, and the foil may mathematically be replaced by a flat plate. Furthermore the amplitudes of the oscillations of the foil and the amplitudes of the incoming waves are small. Hence, the boundary conditions at the free surface and at the foil may be linearized, even if the foil is placed relatively close to the free surface.

Let coordinates be taken with the origin in the mean free surface of the fluid. The x-axis is horizontal and the y-axis positive upwards, see figure 1. The fluid is assumed incompressible and the motion irrotational. Considering the problem from the frame of reference fixed to the mean position of the foil, the water flows with a horizontal speed U along the negative x-axis. The fluid velocity may then be written

$$\vec{v} = \nabla\phi - U\vec{e}_x \tag{2.1}$$

where ϕ is a velocity potential and $\vec{e}_x$ is the unit vector along the x-axis. ϕ satisfies the two-dimensional Laplacian

$$\nabla^2\phi = 0 \tag{2.2}$$

We shall consider a fluid of infinite depth. The boundary condition at $y=-\infty$ is then

$$\nabla\phi \to 0 \quad y \to -\infty \tag{2.3}$$

$$\phi_0 = \mathrm{Re}_j \mathrm{Re}_i f_0(z) \exp(j\sigma t)$$
$$\phi_1 = \mathrm{Re}_j \mathrm{Re}_i f_1(z) \exp(j\sigma t) \tag{2.10}$$

with

$$f(z) = f_0(z) + f_1(z) \tag{2.11}$$

Finally, at the trailing edge the Kutta condition is applied, ensuring that the velocity is finite at this point. Also, at $x=\pm\infty$ the radiation conditions must be satisfied.

3. THE INTEGRAL EQUATION

To derive an integral equation for the motion we express $f_1(z)$ as a continuous distribution of vortices. Since the fluid is oscillating in time, due to the motion of the foil or an incoming wave, a vortex wake will be formed behind the foil, extending from the trailing edge to $x=-\infty$. With sufficient accuracy the wake may be considered to be located along the line $y=-d$. $f_1(z)$ is therefore expressed as an integral from $x=-\infty$ to $x=1$. Let $G(z,z_0)$ denote the complex potential for a vortex of strength unity located at $z=z_0$. $G(z,z_0)$ fulfils the boundary condition at the free surface, the radiation conditions at $x=\pm\infty$ and (2.3) at $y=-\infty$. $f_1(z)$ may then be written

$$f_1(z) = \int_{-\infty}^{1} \gamma(\xi) G(z, \xi - id) d\xi \tag{3.1}$$

Here γ is real with respect to i. This is to secure that the boundary condition at $y=0$ and the radiation conditions are satisfied. γ is, however, complex in j being of the form $\gamma=\gamma_1+j\gamma_2$ where γ_1 and γ_2 are real. $G(z,z_0)$ is derived in appendix 1, and is shown to be given by

Let now $z \to x-id$ $(x<1)$ from below and above. (3.9) then gives

$$u-iv = \frac{df_0}{dz} + \int_{-\infty}^{1} \gamma(\xi)\frac{\partial G}{\partial z}(z,\xi-id)d\xi \pm \frac{1}{2}\gamma(x) \qquad z=x-id \tag{3.10}$$

Here the + sign and - sign correspond to $z \to x-id$ from below and above, respectively, and the bar through the integral sign indicates the principal value. To identify γ we subtract the two equations in (3.10) from each other. Thereby

$$\gamma = \Delta u \tag{3.11}$$

where Δ denotes the difference between the lower and upper value along the cut $-\infty<x<1$, $y=-d$. To obtain the governing integral equation we take the imaginary part with respect to i of (3.10). For $|x|<1$ this yields

$$-v = \mathrm{Im}_i \frac{df_0}{dz} + \int_{-\infty}^{1} \gamma(\xi)\mathrm{Im}_i \frac{\partial G}{\partial z}(z,\xi-id)d\xi \qquad z=x-id \tag{3.12}$$

where v is given by the kinematic boundary condition at the body (2.5),

$$\mathrm{Re}_j v \exp(j\sigma t) = \left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)\eta \tag{3.13}$$

To find γ for $-1<x<-\infty$ we apply the fact that the vorticity in the wake is conserved. Hence

$$\left(\frac{\partial}{\partial t} - U\frac{\partial}{\partial x}\right)\Delta\frac{\partial\phi}{\partial x} = 0 \tag{3.14}$$

The solution of this equation is

$$\Delta\frac{\partial\phi}{\partial x}(x,t) = \Delta\frac{\partial\phi}{\partial x}(x+Ut) \tag{3.15}$$

From (3.8) and (3.11)

$$\gamma = \gamma_0 \exp(jkx) \qquad -\infty<x<-1 \tag{3.16}$$

problem in an unbounded fluid. Considering for the moment the r.h.s. of (3.20) as known, the solution of the equation is (see Newman 1977, p. 182)

$$\gamma(x) = \frac{1}{\pi^2}(1^2-x^2)^{\frac{1}{2}}\left[\ \displaystyle⨍_{-1}^{1} \frac{(1^2-x^2)^{\frac{1}{2}}}{x-\eta}(H(\eta)+F(\eta))d\eta + \pi\Gamma\right] \qquad (3.24)$$

From residue calculation we obtain

$$⨍_{-1}^{1} \frac{(1^2-\eta^2)^{\frac{1}{2}}}{(x-\eta)(\eta-\xi)}\, d\eta = \pi - \frac{\pi}{x-\xi}(\xi^2-1^2)^{\frac{1}{2}} \qquad \xi<-1,\ -1<x<1 \qquad (3.25)$$

By using (3.25) and (3.18) we may after some algebra write (3.24) in the form

$$\kappa(x) + \int_{-1}^{1}\kappa(\xi)\frac{\tilde{K}(x,\xi)}{(1^2-\xi^2)^{\frac{1}{2}}}\, d\xi = \tilde{H}(x) + \gamma_0 P(x) \qquad (3.26)$$

Here the tilde defines the transform

$$\tilde{f}(x) = \frac{1}{\pi^2} ⨍_{-1}^{1} \frac{(1^2-\eta^2)^{\frac{1}{2}}}{x-\eta}\, f(\eta)d\eta \qquad (3.27)$$

Furthermore,

$$\kappa(x) = (1^2-x^2)^{\frac{1}{2}}\gamma(x) \qquad (3.28)$$

and

$$P(x) = -\int_{-\infty}^{-1}\exp(jk\xi)\left[\frac{j}{k\pi}\frac{d}{d\xi}\left(\frac{(\xi^2-1^2)^{\frac{1}{2}}}{x-\xi}\right) + \tilde{K}(x,\xi)\right]d\xi \qquad (3.29)$$

For numerical purpose $P(x)$ may be considerably simplified, see appendix 2.

To close the problem we apply the Kutta condition requiring $\gamma(-1)$ to be finite. Hence, from (3.26) and (3.28)

$$\gamma_0 = \left[\int_{-1}^{1}\kappa(\xi)\frac{\tilde{K}(-1,\xi)}{(1^2-\xi^2)^{\frac{1}{2}}}\, d\xi - \tilde{H}(-1)\right]\frac{1}{P(-1)} \qquad (3.30)$$

Combining (3.30) and (3.26) we finally obtain presented N=25 is applied, giving an accuracy of 1% or better. As a check of the computations the wasted energy is calculated by (5.6) and by far-field analysis (5.12).

5. EFFECT OF THE FREE SURFACE. NO INCOMING WAVES.

To solve the governing equation (3.31) the incoming wave and the motion of the foil must be given. In this section there is no incoming waves. We shall assume that the foil is a rigid body performing oscillations in heave and pitch. Its forward speed U is given by the Froude number

$$Fr = U/(gl)^{\frac{1}{2}} \tag{5.1}$$

Following Lighthill (1970) we write the vertical coordinate of the plate in the form

$$\eta(x,t) = \mathrm{Re}_j[h+j\alpha(x-b)]\exp(j\sigma t) \tag{5.2}$$

where h, α and b are real. (5.2) spesifies a heaving of amplitude h and a pitching of amplitude α at a fixed phase lag of 90° before the heaving, with the axis of pitch at $x=b$.

The foil will set up surface waves which at $x=\pm\infty$ are harmonic waves with wave numbers k_1, k_2, k_3 and k_4 (see (3.6)). In the frame of reference where the current is zero, later called the relative frame of reference, both the k_1 wave and the k_2 wave have positive phase velocities larger than U. The k_1 wave has group velocity less than U whereas the k_2 wave has group velocity larger than U. The k_3 wave has positive phase velocity being smaller than U, and the k_4 wave has negative phase veloscattering of surface waves. The thrust is composed of two terms: a suction force, T_s, acting at the leading edge, and a sideforce due to the pressure difference along the foil, T_p. Hence, the mean thrust is given by

$$T = \bar{T}_p + \bar{T}_s \tag{5.7}$$

where

$$T_p = \int_{-1}^{1} \Delta p \frac{\partial \eta}{\partial x} dx \tag{5.8}$$

Introducing the lift L acting upon the plate

$$L = \int_{-1}^{1} \Delta p dx \tag{5.9}$$

(5.8) may be written

$$T_p = L \frac{\partial \eta}{\partial x}. \tag{5.10}$$

Applying Blasius formula, T_s is found to be

$$T_s = \frac{\pi\rho}{8l}[\mathrm{Re}_j(\kappa(1)\exp(j\sigma t))]^2 \tag{5.11}$$

where ρ is the density.

The mean waste of energy may be shown to be given by

$$E = E_2(c_{g2}-U) - E_1(c_{g1}-U) - E_3(c_{g3}-U) - E_4(c_{g4}-U) + \frac{1}{8k}\rho U|\gamma_0|^2[1-\exp(-2kd)] \tag{5.12}$$

Here E_n $(n=1,2,3,4)$ is the wave energy density

$$E_n = \frac{1}{2}\rho g|a_n|^2 \qquad n=1,2,3,4 \tag{5.13}$$

and c_{gn} $(n=1,2,3,4)$ denotes the group velocities

$$\begin{aligned} c_{gn} &= \frac{1}{2}c_n = \frac{1}{2}(g/k_n)^{\frac{1}{2}} \qquad n=1,2,3 \\ c_{g4} &= \frac{1}{2}c_4 = -\frac{1}{2}(g/k_4)^{\frac{1}{2}} \end{aligned} \tag{5.14}$$

For values of the reduced frequency $\sigma l/U$ less than about unity, figure 2a shows that the thrust becomes considerable larger for the foil moving close to the free surface than being deeply submerged. The figure further reveals that the efficiency is slightly larger for small values of d/l. Hence, for small Froude numbers and values of the reduced frequency less than about unity, it is favourable moving the foil close to the free surface. The interpretation of these results is simple. For these values of the Froude number and the reduced frequency, the free surface condition can be approximated by the rigid wall condition. In this approximation no waves occur. From figure 2b we see that E increases for decreasing values of d/l. Then, using (5.12), it follows that γ_0 and thereby the circulation around the foil, Γ, increases for decreasing d/l. The results obtained above are thus a result of the ground effect modified to apply for an oscillating foil.

A typical feature revealed in figures 2 and 3 is that for τ smaller than 1/4, the generated waves play an important role. Thus, for $Fr=0.4$ the energy waste due to the waves, E_w, is up to 60% of E, and for $Fr=1.0$ E_w is close to 100% of E. We also notice that the momentum flux in the waves is large. Since the k_1 and k_2 waves are the dominating ones, the flux is negative. For τ very close to 1/4, both the thrust and the energy waste become very small. The energy flux for k_1 and k_2 waves are exactly zero when $\tau \to 1/4$ and it turns out that the energy flux for k_3 and k_4 waves are negligible in this τ-region. More surprising is, however, that for τ close to 1/4, γ_0 becomes very small, implying that the wake is almost vanishing due to the free surface. For values of τ slightly larger than 1/4, γ_0 increases drastically.

6. EFFECT OF THE FREE SURFACE. INCOMING WAVES

Let the wave elevation η_0 of the incoming wave be given by

$$\eta_0 = a_0 \sin(k_0 x \pm \sigma t) \quad (6.1)$$

and the corresponding complex velocity potential by

$$f_0(z) = \delta a_0 \left(\frac{g}{k_0}\right)^{\frac{1}{2}} (1 \pm ij) \exp(-ik_0 z) \quad (6.2)$$

Here a_0 is the wave amplitude, k_0 the wave number of the incoming wave, $\delta=1$ for incoming head waves, i.e. $k_0=k_4$, $\delta=-1$ for incoming following waves, i.e. $k_0=k_1, k_2, k_3$. The + sign is applied for incoming k_3 and k_4 waves, the - sign for incoming k_1 and k_2 waves.

Denoting the mean wave energy power of the incoming wave by P_0, the energy equation (5.6) then takes the form

$$P + P_0 = TU + E \quad (6.3)$$

P_0 is given by

$$P_0 = E_0 |c_g - U| \quad (6.4)$$

where $E_0 = \frac{1}{2}\rho g a_0^2$ is the energy density of the incoming wave and c_g is the group velocity. $c_g = \frac{1}{2}\sqrt{g/k_0}$ for incoming following waves and $c_g = -\frac{1}{2}\sqrt{g/k_0}$ for incoming head waves.

The existence of an incoming wave obviously complicates the study of the problem considerably. It introduces into the problem three new parameters, viz. the wave amplitude, the phase between the wave and foil motion and the frequency of the wave. We shall only consider the case when the foil and the wave motion are oscillating with the same frequency. Hence only two new parameters are introduced by the incoming wave.

The basis for this definition is long incoming waves. We shall, however, also apply it for moderate wave lengths. This only means that instead of specifying the pitching angle α, we specify θ and find α from (6.9).

Obviously, when the heaving motion of the foil and the wave motion (the wave length assumed large) are 180 degrees out of phase, the generated fluid motion is equivalent to the motion generated by the foil oscillating with the same pitching motion, no incoming waves, but a larger amplitude in the heaving motion. Correspondingly, when the heave motion and wave motion are in phase, the equivalent heave motion with no incoming waves has a smaller amplitude. For incoming head waves, for instance, the first of these cases corresponds to $\beta=\pi/2$, giving $\hat{h}=h+\hat{a}_0$, $\hat{b}=b$ and $\theta=\alpha U/\sigma(h+\hat{a}_0)$, whereas the latter case is obtained for $\beta=-\pi/2$, giving $\hat{h}=h-\hat{a}_0$, $\hat{b}=b$ and $\theta=\alpha U/\sigma(h-\hat{a}_0)$. On the other hand, when the foil and wave motion are 90 degrees out of phase, the generated motion is equivalent to the motion generated by the foil for no incoming waves, the same heaving motion but the pitch axis moved from b to $\hat{b}$. This case, with the foil and wave motion 90° out of phase, corresponds to $\beta=0,\pi$ and the foil is at its highest or lowest position when the vertical velocity is maximum upwards. For incoming head waves and $\beta=\pi$, for instance, we have that $\hat{h}=h$, $\hat{b}=b+\hat{a}_0/\alpha$ and $\theta=\alpha U/\sigma h$. Thus the only effect of the incoming wave is to move the pitch axis to the point $x=b+\hat{a}_0/\alpha$.

We shall in this section study the thrust acting upon and the power supplied to the foil for two different kinds of motions. In the first case, denoted as motion 1, the heaving motion has opposite phase of the incoming wave. This corresponds to $\beta=\pi/2$

this case, the largest effect is obtained when the foil is moving in the short k_1 waves. For motion 2, in particular, the power supply becomes very small, while the thrust retains a significant value. The value of the thrust also remains finite for $\sigma l/U \to 0$ in this case.

We see from figures 8 and 9 that for $\tau > 1/4$ the largest thrust is obtained in head waves. The opposite was true for $\tau < 1/4$.

For motion 1 and $\theta = 0.6$ we have found that $\bar{T}_s$ is considerably smaller than T in all cases. For motion 2 $\bar{T}_s$ is close to T.

The examples studied above clearly show that a part of the energy of the incoming waves is utilized to produce a forward thrust on the foil. In the special cases where the power supplied to the foil is zero or very small, the ratio TU/P_0 gives the part of P_0 which is utilized for propulsion. Figures 6-9 show cases with P being very small. As an illustrious example we refer to the case with $d/l=1$, $b/l=-0.5$, $a_0/h=1$ and motion 2 which gives the following table

	$\sigma l/U$	Fr	τ	$k_0 l$	θ	$T/\rho g h^2$	TU/P_0
k_4 waves	0.5	0.5	0.125	0.05	0.6	0.066	0.024
k_2 waves	0.5	0.5	0.125	0.09	0.6	0.113	0.091
k_1 waves	0.125	1	0.125	0.73	0	0.141	0.676
k_3 waves	0.125	1	0.125	1.24	0	0.093	0.338
k_3 waves	0.3	1	0.3	1.54	0.6	0.093	0.130

Table 1

The table shows that for waves of moderate wave length, with moderate amount of wave energy, a considerable part can be utilized for propulsion. However, for long incoming waves, with larger wave energy, only a small fraction can be extracted.

is very close to the free surface. For $\tau<1/4$, however, when wave scattering effects are important, the approximate theory strongly overpredicts the forward thrust when the submergence of the foil is small. When the ground effect dominates, i.e. for small values of U and σ, the approximate theory underpredicts the forward thrust.

7. PROPULSION OF SHIP BY FOIL PROPELLER

In this section we shall apply our two-dimensional theory to study the propulsion of ship by foil propeller. The force is then found by applying the strip-theory approximation. We assume that the ship, with a foil propeller placed in the front, is advancing with a speed U in regular waves. Due to the incoming waves the ship will undergo heaving and pitching motions. Hence, the foil is forced by the ship motion to be moving up and down with a frequency equal to the frequency of encounter. The oscillatory vertical motion of the foil, as well as the oscillatory motion of the wave field, will produce a forward thrust on the foil and thereby a forward thrust on the ship.

The vertical motions of a ship advancing in waves have been studied in many papers. A thorough discussion of ship motions is given by Newman (1978). A main result concerning the motion of a ship in head waves is that its vertical displacements become larger than the amplitude of the incoming waves when the wave length λ satisfies $1.2<\lambda/L<5$, L being the ship length. For values of λ/L smaller than 1.2 the vertical ship motions rapidly become very small. The vertical motions of a ship in following waves are generally somewhat smaller than in head waves. Wachnik and Zarnik show that it is favourable to operate the foil close to the free surface.

In figure 10d T is displayed for $h/a_0=1$, $d/l=1$ and $Fr=0.5,1,2$. For $\tau>1/4$ and fixed value of $k_0 l$, T increases for increasing value of U, as expected. The figure also shows that for fixed values of U and k_0, and varying l, the largest thrust is obtained for smallest $U/\sqrt{gl}$, i.e. largest value of the chord length $2l$.

In figures 11a-c T is displayed for a foil moving in following waves. The figures show that the thrust is generally much smaller for following waves than for head waves. Comparing figures 10a and 11a we also see that the heaving motion in the waves leads to a considerably larger thrust in head waves than in following waves. Fig. 11b shows that in following waves, in opposition to in head waves, the thrust has not its maximum value when the foil is operated very close to the free surface. From figure 11c we see that for long incoming waves, the thrust is largest for small Froude numbers.

A general result of the various computations is that maximum thrust is obtained for $\theta=0$. However, according to Lighthill (1970), stall may occur when the leading edge suction is large compared to the total thrust. We have therefore in figures 12a-12b compared the leading edge suction and the total thrust for various values of θ, assuming that $Fr=1$, $d/l=1$ and $h/a_0=1$. We see that by increasing θ from 0 to 0.3, $\bar{T}_s$ is reduced essentially whereas T is only decreased slightly. For $\theta=0.6$ the side force gives the main contribution to the total thrust.

$U<0.3\sqrt{gL}$, a reasonable value for the drag coefficient is $C_D=3\cdot10^{-3}$ (see e.g. Newman 1977, pp. 30-31).

The added resistance D_w for a ship moving in head waves is discussed by Faltinsen et al. (1980) and may be written

$$D_w = C_w \rho g a_0^2 B^2/L \tag{7.4}$$

where C_w is a dimensionless function of $U/\sqrt{gL}$ and λ/L. Computations displayed in the paper by Faltinsen et al. show that the values of C_w depend strongly on the wave length of the incoming waves and the response of the ship. It turns out that the maximum value of C_w approximately occurs when the vertical motions of the ship are largest. The maximum value of C_w, occurring for $\lambda/L\simeq1$, is about 6 for $U/\sqrt{gL}=0.2$, and about 8 for $U/\sqrt{gL}=0.3$. For longer or shorter waves, i.e. smaller vertical motions of the ship, the value of C_w becomes rapidly smaller than 2.

For a ship moving in following waves, little is published about the actual values of C_w. We know that for U very small, C_w is negative. It is expected that for moderate values of $U/\sqrt{gL}$, C_w is small.

Balance between T and $D+D_w$ for following and head waves gives

$$\hat{T}\rho g a_0^2 B = \frac{3\pi}{4}\cdot10^{-3}\rho LBU^2 + C_w \rho g a_0^2 \frac{B^2}{L} \tag{7.5}$$

or approximately

$$U = 21(\hat{T} - C_w \frac{B}{L})^{\frac{1}{2}} a_0\sqrt{g/L} \tag{7.6}$$

Let us assume that the beam to the length of the ship is $B/L=1/8$ and that the Froude number is 1. For incoming head and following waves we obtain the following table

When no incoming waves occur, we find that for τ very small (or more precisely that both U and σ are small) the free surface acts as a rigid wall, leading to a pronounced ground effect. For higher values of τ, but τ still less than 1/4, the generated waves have rather large amplitudes. This results in a relatively large energy waste, and a momentum flux in the wave which reduces the thrust. Close to $\tau=1/4$, the thrust and necessary power are usually very small. For $\tau>1/4$ the thrust increases rapidly with τ. For these τ-values usually a k_4 wave of considerable intensity is generated when the foil is close to the free surface. This wave conveys a momentum flux which contributes to a positive thrust.

When a foil is moving in incoming waves, it is found that often a relatively large part of the wave energy may be utilized for propulsion. The foil may move such that the total energy supply for propulsion is due to the incoming waves.

It is found that a passive foil propeller may give a ship a forward speed of considerable magnitude in both head waves and following waves. The foil propeller gives largest forward speed when the ship is moving in head waves, with wave lengths close to the length of the ship.

the solution of the boundary value problem (A1.3)-(A1.8) is obtained as

$$F(z) = \frac{1}{2\pi i}\frac{1-ij}{\sqrt{1-4\tau}}\left(\int_{C_1}^{z}\frac{\exp(-ik_1(z-u))du}{u-\bar{z}_0} - \int_{C_2}^{z}\frac{\exp(-ik_2(z-u))du}{u-\bar{z}_0}\right.$$

$$\left. + \frac{1}{2\pi i}\frac{1+ij}{\sqrt{1+4\tau}}\left(\int_{\infty}^{z}\frac{\exp(-ik_3(z-u))du}{u-\bar{z}_0} - \int_{\infty}^{z}\frac{\exp(-ik_4(z-u))du}{u-\bar{z}_0}\right) \quad (A1.9)$$

For $\tau<1/2$, $C_1=\infty$, $C_2=-\infty$. For $\tau>1/2$, $C_1=i\infty/k_1$, $C_2=i\infty/k_2$, to ensure that $F(z)$ is bounded in the entire fluid.

It should be noted that the values obtained for C_1 and C_2 when $\tau>1/2$ are not identical to those obtained by Haskind in his derivation of the Green function for an oscillatory source. The values of C_1 and C_2 must be the same in the two cases. We therefore believe that Haskind's results for C_1 and C_2 for $\tau>1/2$ are incorrect and should be those given in this paper.

Introducing (A2.5-A2.8) in (A2.2), applying a partial integration and manipulating with Hankel functions we finally obtain

$$P_0(x) = \frac{2}{\pi}\exp(jkx)(l^2-x^2)^{\frac{1}{2}}[\frac{\pi}{2} - \arctan(\frac{l+x}{l-x})^{\frac{1}{2}}] + \frac{l}{2}H_1^{(2)}(kl)$$
$$- \frac{jx}{2}H_0^{(2)}(kl) - \frac{l}{2}(l^2-x^2)\exp(jkx)\int_0^k \exp(-jux)H_0^{(2)}(ul)du \quad (A2.9)$$

The last term in (A2.1), P_1, may by changing the order of integration be written

$$P_1(x) = -\frac{1}{\pi^2}\int_{-l}^{l}\frac{(l^2-\eta^2)^{\frac{1}{2}}}{x-\eta}d\eta \int_{-\infty}^{-l}\exp(jk\xi)K(\eta,\xi)d\xi \quad (A2.10)$$

where

$$K(\eta,\xi) = -\frac{1}{2}[\frac{1}{\eta-\xi-2jd} + \frac{1}{\eta-\xi+2jd}]$$
$$+ \frac{j}{(1-4\tau)^{\frac{1}{2}}}[k_1F_1(\eta-jd,\xi-jd)-k_2F_2(\eta-jd,\xi-jd)]$$
$$- \frac{j}{(1-4\tau)^{\frac{1}{2}}}[k_3\overline{F_3(\eta-jd,\xi-jd)}-k_4\overline{F_4(\eta-jd,\xi-jd)}] \quad (A2.11)$$

where

$$F_n(\eta-jd,\xi-jd) = \exp(-jk_n\eta-k_nd)\int_{C_n}^{\eta-jd}\frac{\exp(jk_nu)}{u-\xi-jd}du \quad (A2.12)$$

C_n, k_n and τ are defined in section 3 and a bar denotes complex conjugate. Changing the order of integration and applying partial integration, we obtain

$$\int_{-\infty}^{-l}\exp(jk\xi)K(\eta,\xi)d\xi =$$
$$\exp(-jkl)\{[\frac{1}{2}\exp(v)E_1(v)] + \frac{1}{2}[\overline{\exp(-v)E_1(-v)}]$$
$$+ \frac{1}{(1-4\tau)^{\frac{1}{2}}}[\frac{k_1}{k_1+k}F_1(\eta-jd,-l-jd) - \frac{k_2}{k_2+k}F_2(\eta-jd,-l-jd)]$$
$$+ \frac{1}{(1+4\tau)^{\frac{1}{2}}}[\frac{k_3}{k_3-k}\overline{F_3(\eta-jd,-l-jd)} - \frac{k_4}{k_4-k}\overline{F_4(\eta-jd,-l-jd)}]\} \quad (A2.13)$$

APPENDIX 3.

THE WAVE AMPLITUDES IN THE FAR FIELD

The wave amplitudes in the far field are derived from the complex velocity field (3.9)

$$u - iv = \frac{df_0}{dz} + \int_{-\infty}^{1} \gamma(\xi)\frac{\partial G}{\partial z}(z,\xi-id)d\xi \tag{A3.1}$$

for $x\to\pm\infty$. Here $f_0(z)$ is the complex potential for the incoming wave and $G(z,z_0)$ is the vortex potential defined in section 3. On the interval $(-1,1)$ $\partial G/\partial z$ reduces to terms of the form (3.4) which for $x\to\pm\infty$ become

$$F_n(z,z_0)_{z\to\pm\infty} = \exp(-ik_n z) \int_{C_n}^{z} \frac{\exp(ik_n u)}{u-\bar{z}_0}du \ , \quad z\to\pm\infty,\ n=1,2,3,4 \tag{A3.2}$$

Here C_n and k_n are defined in section 3. (A3.2) is easily evaluated by contour integration.

The contribution from the wake is

$$\lim_{x\to\pm\infty} \gamma_0 \int_{-\infty}^{-1} \exp(jk\xi)\frac{\partial G}{\partial z}(z,\xi-id)d\xi \tag{A3.3}$$

The terms of the form (A3.2) are evaluated by changing the order of integration, applying partial integration and contour integration. The remaining part of (A3.3), due to terms of the form $1/(z-z_0)$ and $1/(z-\bar{z}_0)$, are found by contour integration. These terms tend towards zero for $x\to\infty$. For $x\to-\infty$ they are non-zero. They give, however, no contribution to the vertical displacement of the free

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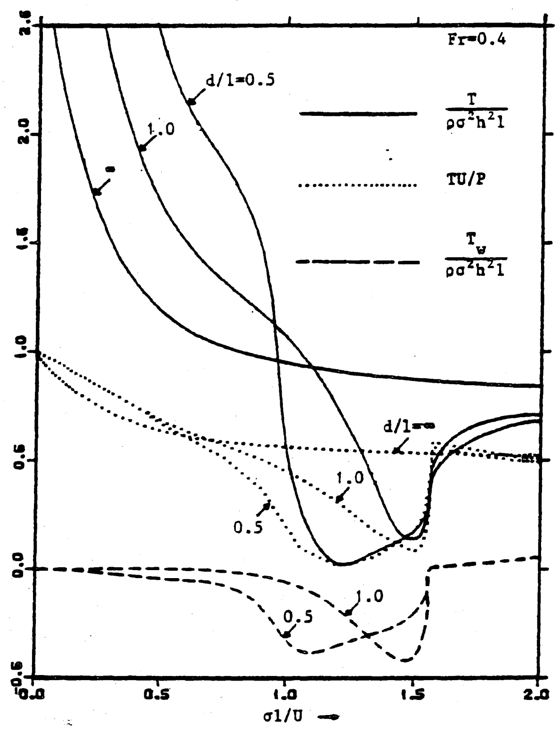

Figure 2a.

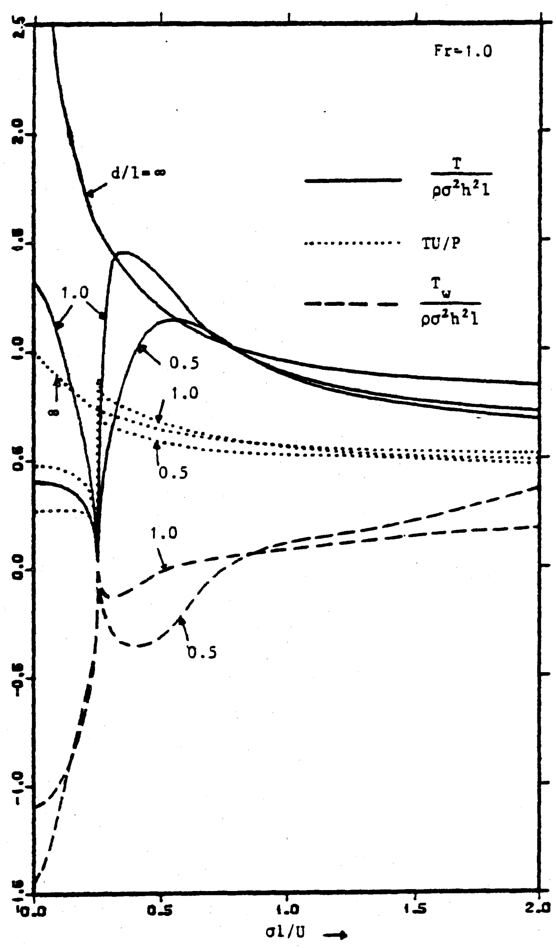

Figure 3a.

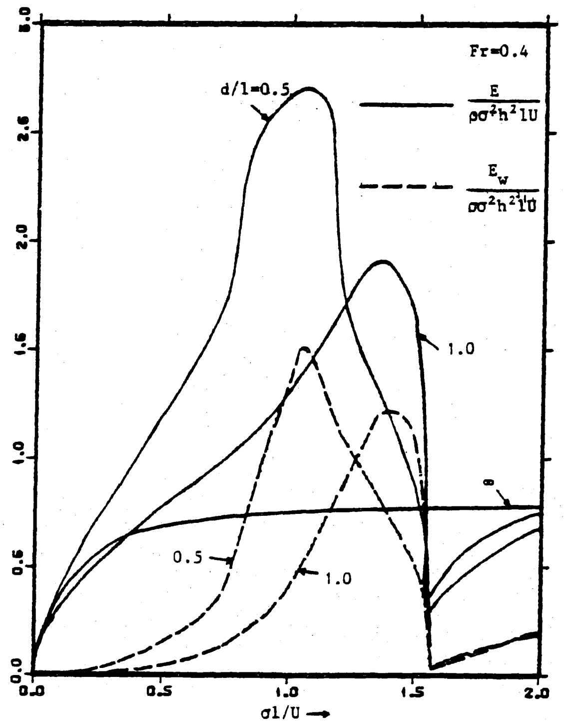

Figure 2b.

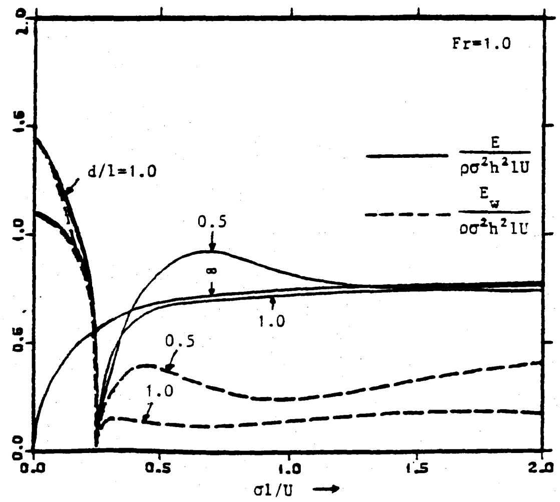

Figure 3b.

Figures 2 and 3. Thrust T, efficiency TU/P, waste of energy E, and the part of the thrust and waste of energy due to the waves, T_w and E_w, respectively. Foil oscillating in heave without pitching. $d/l=0.5, 1.0, \infty$, $Fr=0.4$ (figures 2a,b), $Fr=1.0$ (figures 3a,b).

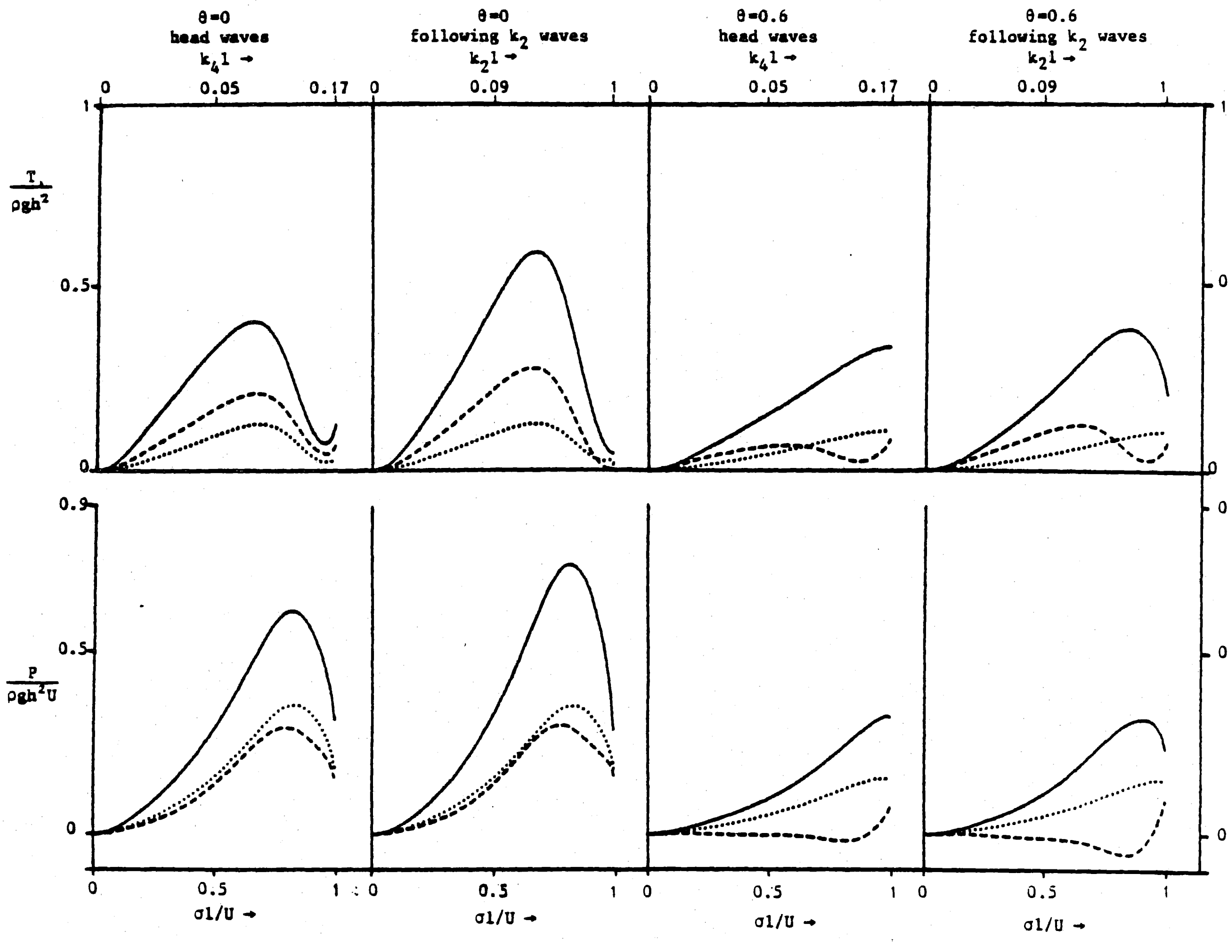

Figure 6. Thrust T and power P vs. reduced frequency $\sigma l/U$. Foil moving in incoming head waves or following waves. $d/l=1$, $b/l=-0.5$, $a_0/h=1$, $Fr=0.5$, $\theta=0,0.6$.

—— motion 1, ---- motion 2, ···· no waves.

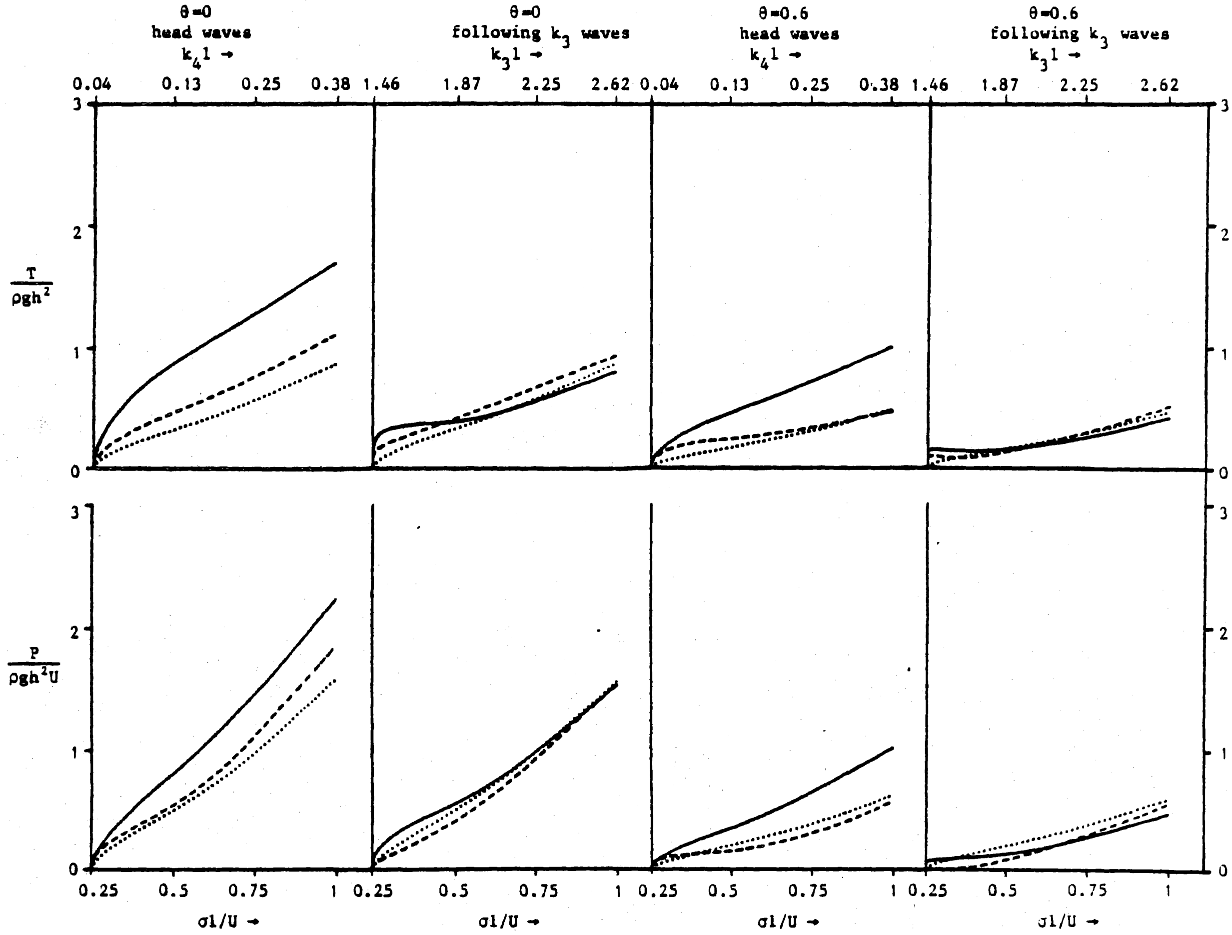

Figure 8. Thrust T and power P vs. reduced frequency σl/U. Foil moving in incoming head waves or following k_3 waves. d/l=1, b/l=-0.5, a_0/h=1, Fr=1, θ=0,0.6, τ < 1/4.

—— motion 1, ---- motion 2, ···· no waves.

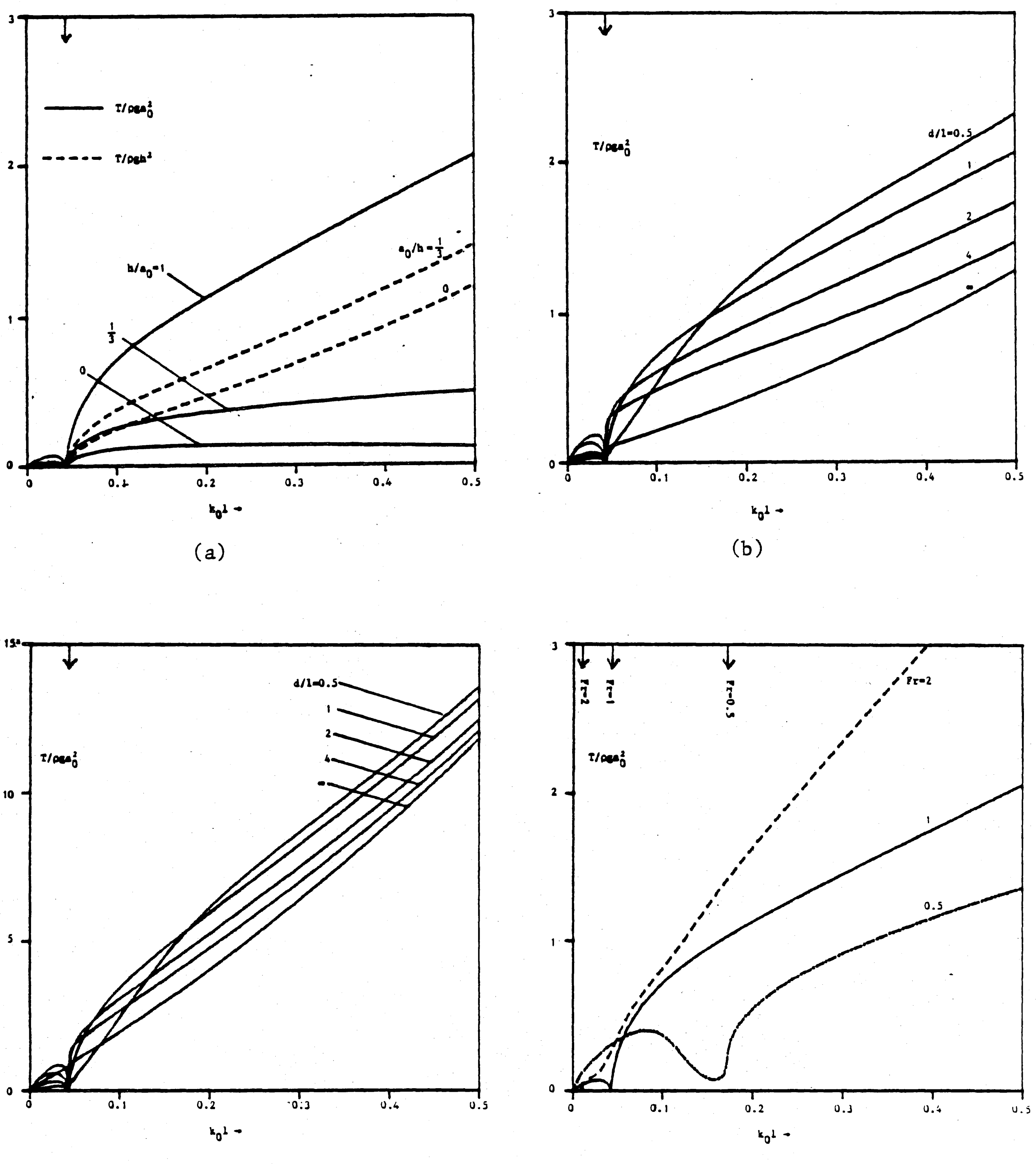

Figure 10. Maximum thrust T vs. wave number $k_0 l$ of incoming waves. Foil moving in head waves. θ=0.

a) Fr=1, d/l=1, h/a_0=1,1/3,0, a_0/h=1/3,0.

b) Fr=1, h/a_0=1, d/l=0.5,1,2,4,∞

c) Fr=1, h/a_0=3, d/l=0.5,1,2,4,∞

d) h/a_0=1, d/l=1, Fr=0.5,1,2.

The small arrows denote the occurrence of τ=1/4

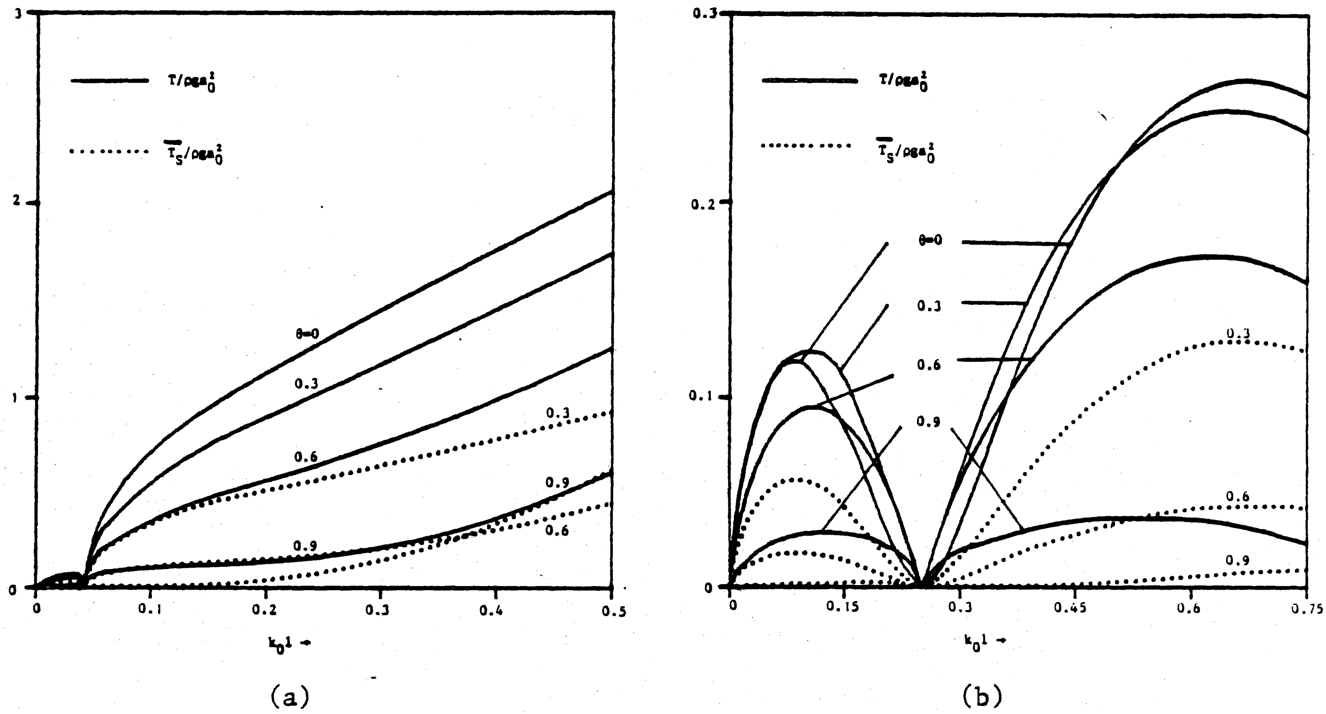

Figure 12. Maximum thrust T and corresponding suction force $\bar{T}_S$ vs. wave number $k_0 l$ of the incoming waves. $d/l=1$, $b/l=0$, $Fr=1$, $h/a_0=1$, $\theta=0,0.3,0.6,0.9$.

a) Foil moving in head waves.

b) Foil moving in following waves.