Individual Choice Behavior with Social Interaction

An Empirical Analysis of Choice among Mobile Network Operators

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IV
Abstract

The purpose of this paper is to analyze how individual choice behavior might be influenced by the behavior of others (social interaction effect). To this end, we have conducted an empirical analysis on how individuals make choices among mobile network operators, and how they are influenced by the choices of others. To obtain data, we have carried out a Stated Preference survey in both China and Norway. Our models are based on the theory of discrete choice suitably extended to account for the social interaction effect. It is known from the literature that models with social interactions may yield multiple equilibria, provided that the effect is sufficiently strong. In our empirical analysis, we have found clear evidence of social interaction effects, however, the effects are not strong enough to imply multiple equilibria.
Preface

Writing this thesis has been a challenging yet rewarding journey. I would not have made it through without the help of many others.

First and foremost, I would like to express my sincere gratitude to my brilliant supervisor, John K. Dagsvik, for his excellent guidance, patience and support. His expertise in discrete choice theory and encouragement has been invaluable.

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Oslo, October 2015

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1. Introduction

Recently, there has been a growing interest in analyzing individual choice behavior in the presence of (potential) social interaction. By social interaction we mean that individual preferences may be affected by the behavior of others (Schelling, 1971, Becker, 1974, and Manski, 1993, 2000). In this paper we shall be particularly concerned with the role of social interaction on qualitative choice behavior. In particular, we shall analyze the influence of social interaction on individuals’ choice among Mobile Network Operators (MNO). More precisely, we shall assume that a particular form of social interaction takes place in this case, namely that an individual’s preferences may be affected by the fractions of people in the individual’s peer group that make the respective choices.

The reason for analyzing the influence of social interaction on individuals’ choice among MNOs can be motivated as follows: First, individuals might have incomplete knowledge about the attributes of different subscriptions or quality of the services (voice quality, signal strength, data speed and customer support, etc) that different operators offer, and it might be costly for them to find all the information themselves. Therefore, individuals might be better off using the choices of others as an indicator of the quality of the products and services, and hence simply imitate what the majority does (Conlisk, 1980). Second, there might be local network effect (Sundararajan, 2007) that directly benefits individual who makes the same choice as others in the group. Family Plan, operator-exclusive apps and free within-operator calls are examples that display local network effect.

There are several problems one faces when undertaking studies of social interaction. In our case, market data are not available. Second, even if market data were available, it is a problem that what constitutes each individual’s peer group is not known to us as researchers. Third, there are difficult identification and endogeneity problems related to statistical inference in models with social interaction. Brock and Durlauf (2001, 2007), Manski (1993, 2000) and Angrist (2013) have discussed particular problems that arise when one attempts to estimate structural models with social interaction. The problem stems from the fact that, loosely speaking, the dependent variable enters the model also as
explanatory variable (the reflection problem). This feature creates particular econometric problems. Models with social interaction may also give rise to multiple equilibria as demonstrated by Schelling (1971), Becker (1991) and Brock and Durlauf (2007). Whether or not multiple equilibria are possible depends heavily on the phenomenon under study and the empirical model specification. As a result, empirical results may not be robust with respect to functional form specifications.

To overcome some of these problems we have resorted to an empirical strategy based on data from a Stated Preference (SP) survey. In a SP survey, respondents are asked to express preferences for hypothetical products characterized by specific attributes. To this end we have constructed a questionnaire where the respondent is invited to rank order 3 hypothetical MNOs (alternatives) in 8 different “experiments”. Each alternative is characterized by qualitative attributes and prices. In addition, the respondent is given hypothetical fractions of other individuals’ choices in the respondent’s peer group. These qualitative attributes are properties of the respective MNOs, such as how many minutes of calls and how much data are included in their subscriptions and what are the additional services they offer to their subscribers. Whereas the qualitative attributes are fixed throughout all the experiments, the prices and aggregate choices in the respondent’s peer group vary across experiments. The fractions of persons in the individual’s peer group that make the respective choices are endogenous if market data are used, but this is not so with SP data. The reason is of course that whereas revealed preference data are determined by preferences, SP data on aggregate choices in the individual’s peer group are stated by the researcher. In this sense, SP data are similar to the data collected from experiments in the natural sciences. Our models are based on the theory of discrete choice suitably extended to account for the social interaction effect. We have used the data from the SP survey to estimate the models, and found clear evidence of social interaction effect. However, the effect is not strong enough to imply multiple equilibria.

This paper is organized as follows. The next section contains a review of the literature. In section 3 we present different model versions based on the theory of discrete choice, suitably extended to allow for social interaction. Section 4 discusses data and survey methods and in section 5 we present model estimates with interpretation. In section 6 we
discuss implications by analysing selected marginal effects and elasticities. Section 7 addresses functional form issues. Section 8 concludes and summarizes the paper.
2. Literature review on social interactions

The role of social interactions in economic analysis has become increasingly popular in recent years. Social interactions refer to particular forms of externalities, in which the utility or payoff to an individual by a given action depends directly on the actions of the individuals in his or her reference group (Schelling, 1971, Manski, 2000, Scheinkman, 2008). What makes the phenomenon of social interactions special is that it is not regulated by the market and the price mechanism.

The first contributions to the economic literature on social interactions are probably the analyses of Veblen (1934) and Dusenberry (1949) related to consumption. Social interactions have also been taken into account in other theoretical studies such as, Schelling’s (1971) analysis of the patterns of residential segregation, Loury’s (1977) study of racial inequality, volatility in financial markets (Brock, 1993) and labor market and welfare dependence (Lindbeck et al., 1999, and Nechyba, 2001). Social interaction as a determinant of behavior has also been studied in sociology. See for example, Lewis (1966) and Liebow (1967) who provide early analyses of how social interaction may imply that isolated poor groups exhibit different values towards work, childbearing and parenting from the population as a whole. Wilson (1987) discusses the interdependence between individual utility and community wide behavior in his study of ghetto poverty.

A particularly important contribution was Schelling’s (1971, 1972). He developed a formal analysis of racial dynamics, and what he termed “neighborhood tipping” was seminal and important for later developments in economics. He showed that when social interaction influences preferences among neighbors, all the whites will leave the neighborhood once the percentage of the minority of colored persons exceeds a “tipping point”. Schelling’s critical mass theory was further discussed in Schelling (1978) where he assumes that there is an activity which some individuals will always take, others will only take it if a high enough fraction of the population is engaged in the action, and still others may never undertake the action. He demonstrates that this type of models may give multiple equilibria. In an analysis of riots and revolutions, Granovetter (1978) proposes a very similar model which includes the consideration of dynamics that is not emphasized in Schelling’s model. He finds that a slight change in parameters might cause some
equilibria to disappear and lead to drastic changes in the equilibrium outcomes.
Following Schelling and Granovetter, versions of the critical mass model have been applied to a number of economic issues involving social interactions, see for example, the study of social norms by Akerlof (1980) and the study of savings and consumption norms by Lindbeck (1997).

An interesting theoretical analysis of how social interactions may affect market behavior is Becker (1991). He demonstrated that if preferences are allowed to depend on the aggregate behavior of others it would be possible to rationalize why two restaurants with comparable prices, services and food quality are widely different in popularity. Becker and Murphy (2000) developed a framework in which social interactions are included in the utility functions, and thereby provided a way of analyzing how aggregate choice behavior affect individual behavior. Furthermore, they showed how aggregate behavior might be affected by social interactions. Blume (1993) and Brock (1993) were among the first to mention how social interaction effects could be accounted for in specific discrete choice models. This analysis was developed further by Brock and Durlauf (2001) who, in the spirit of Schelling’s critical mass framework, characterized how individual decisions and the decisions of others interact to produce particular aggregate choice patterns. They considered identification issues in binary choice models with interaction effects and they showed that multiple equilibria might exist provided the social interaction effect is sufficiently strong. Brock and Durlauf (2002) generalized their results to multinomial choice problems.

So far empirical studies on social interactions are sparse compared to the theoretical literature. However, recently an increasing number of empirical studies have explored the relevance decisions by others have on individual choices and found evidence that is consistent with the presence of such interaction effects. For example, based on data from Sweden, Lindbeck et al. (2007) analyzed how sickness absence may be influence by aggregate sickness absence within the neighborhood and he found significant evidence on such interaction effects. Another example is the study of Rege et al. (2009) on disability pension participation in Norway. Their analysis indicates that there is a considerable influence on individual disability pension participation from aggregate disability pension participation in the individual’s neighborhood. Further empirical studies include the
analysis of Ioannides and Zabel (2003) on housing demand analysis in which they found that neighborhood interdependence contributes to a large extent to individual housing demand.

There are several econometric problems that arise when analyzing social interaction effects empirically. Manski (1993) and Angrist (2013) have discussed the problem that one cannot reveal from observed outcome data alone whether group behavior actually affects individual choice or is simply the result from aggregation of individual behavior. This is what Manski (1993) called the reflection problem. In empirical analysis, one problem is related to the fact that aggregates of the dependent variables enter the model also as explanatory variables. As mentioned above, Manski (1993) and Moffitt (1999) discuss several alternative strategies for obtaining identification, under various assumptions, in models with social interaction effects when using revealed preference data. A particular difficulty is that one might not be able to clearly identify the peer group of the respective individuals. Lacking empirical evidence from revealed data, economists have conventionally relied on assumptions and then proceeded with the analysis. To circumvent this problem, Dominitz and Manski (1997a, 1997b), Hurd and McGgary (1995) and Guiso et al. (1992) have initiated research using survey data and clarified how SP survey data can provide useful information for analysis on social interactions.
3. Probabilistic choice models accounting for social interaction

We shall now discuss models for individual choice behavior. This section draws heavily on Dagsvik and Ge (2015). The models are based on the theory of discrete choice (see for example McFadden, 1973 and Train, 2009).

3.1. Basic formulation of the choice model

Consider an individual that faces a set of discrete alternatives and makes choices from this set at different points in time. Let $U_{ijt}$ be the utility of individual $i$ of choosing alternative $j$ at time $t$. We assume that the individual has access to data on aggregate choice behavior in his peer group in the previous period. Here a peer group can be the whole population or a sub group of the population (possibly latent). Let $U_{ijt}$ be measured in monetary value, and assume that it has the structure

$$U_{ijt} = \alpha^*_i + y_i - w_{jt} + \gamma P_{jt,t-1}$$

(1)

where $y_i$ is the individual’s income which is assumed to be constant across time, $w_{jt}$ stands for the price of alternative $j$ at time $t$, $P_{jt}$ is the average probability of choosing alternative $j$ at time $t$ across the peer group. The term $\alpha^*_i$ represents the monetary value of the qualitative properties of alternative $j$, as perceived by individual $i$ at time $t$. The role of the parameter $\gamma$ is to transfer $P_{jt}$ into its corresponding monetary value. Thus, $\gamma P_{jt,t-1}$ is interpreted as the monetary value of the social interaction effect at time $t$. Thus, in the formulation in (1) it is assumed that the utility of individual $i$ is influenced by the aggregate choices in the peer group in the previous period. Furthermore, with no loss of generality, one can write

$$\alpha^*_i = \bar{\alpha}_j + X_i \beta_j + \sigma \epsilon_{ijt}$$

(2)

where $\bar{\alpha}_j$ denotes the average monetary value (average across time and individuals) of the qualitative properties of alternative $j$ and $\epsilon_{ijt}$ is a random term with known distribution function. The square of the parameter $\sigma$, is proportional to the variance of the error term.
The observable characteristics of the individual are represented by the vector $X_i$, which we assume does not vary across time, and $\tilde{\beta}_j$ is the associated vector of parameters specific to alternative $j$. The term $\tilde{\alpha}_j$ captures the average effect on utility of all non-pecuniary attributes of alternative $j$. In other words, it reflects the desirability of alternative $j$, adjusted for the individual characteristics, the effect of prices and social interaction. The term $\sigma \epsilon_{ijt}$ captures the remaining effect of unobserved heterogeneity in preferences across individuals and across time. This heterogeneity may be due to variables that are perfectly known to the individual but unobserved by the researcher. It may also result from factors that are uncertain to the individual himself (Manski, 1977). This latter effect stems from well-known psychological observations that individual preferences may vary in an unpredictable way because the individual may have shifting moods and perceptions, as well as having difficulties with assessing a definitive value of the respective alternatives once and for all. We assume here that $\epsilon_{ijt}$ follows a type I extreme value distribution which means that the distribution function is given by

$$P(\epsilon_{ijt} \leq x) = \exp(-e^{-x}).$$

(3)

Moreover, the variables $\epsilon_{ijt}$, $j = 1, 2, \ldots, M$, $t = 1, 2, \ldots, T$, are assumed to be independent, where $M$ is the number of alternatives in the maximal choice set, denoted $S$. From (1) it follows, after dividing through by $\sigma$ that

$$U_{ijt} \equiv \frac{\bar{U}_{ijt}}{\sigma} = \alpha_j + X_j \beta_j + \frac{y_{ijt} - w_{ijt}}{\sigma} + \gamma P_{j,t-1} + \epsilon_{ijt} \equiv \nu_{ijt} + \frac{y_{ijt}}{\sigma} + \epsilon_{ijt}$$

(4)

where $\alpha_j = \tilde{\alpha}_j / \sigma$, $\gamma = \tilde{\gamma} / \sigma$, $\beta_j = \tilde{\beta}_j / \sigma$ and $\nu_{ijt} = \alpha_j + X_j \beta_j - w_{ijt} / \sigma + \gamma P_{j,t-1}$. Since income is an individual specific variable that does not depend on the alternative, $y_i / \sigma$ cancels in utility comparisons. Intuitively, when the individual is making his choice, the magnitude of his income will not affect his decision when utility is linear in income, provided, of course, that the alternatives are affordable to the individual. Hence, from (3) and (4) it follows by standard calculus (see for example Greene, 2012) that

$$P_{ijt} = P(U_{ijt} = \max_{k \in B} U_{ikt}) = \frac{\exp(\nu_{ijt})}{\sum_{k \in B} \exp(\nu_{ikt})}$$

(5)

where $B$ is a given choice set that is equal to, or a proper subset of $S$. 


A motivation for the distributional assumption in (3) stems from the Independence from Irrelevant Alternatives assumption (IIA) (Luce, 1959). In order to explain the intuition of IIA, consider an individual with a general representation of preferences, given by the utility function $U_{ij} = v_j + \varepsilon_{ij}, j = 1, 2, \ldots, M$. Let $J(B)$ be the most preferred alternative in a given choice set $B, B \subseteq S$. In other words, $J(B)$ is the alternative that maximizes the individual’s utility among alternatives in $B$. Then IIA is equivalent to the statement that for sets $A \subset B \subset S$,

$$P(J(B) = j \mid J(B) \in A) = P(J(A) = j).$$

(6)

for $j \in A$. Equation (6) states that the probability that $j$ is the most preferred alternative in $B$, given that the most preferred alternative belongs to the subset $A$ is equal to the probability that alternative $j$ is the most preferred alternative in $A$. This is clearly an intuitive rationality assumption. Luce (1977) thus calls IIA the assumption of probabilistic rationality. Since it is a probabilistic statement, it only requires that rationality holds on average, thus allowing choice behavior of some individuals to deviate from rationality. Luce (1959) has proved that the multinomial logit choice model in (5) is equivalent to the IIA assumption. Moreover, provided the random error terms are independent and identically distributed, Yellott (1977) and others have proved that the error terms $\varepsilon_{ij}, j = 1, 2, \ldots, M$, must be extreme value distributed as specified in (3).

Another implication from IIA is that the rank ordering of alternatives within a subset $A \subset B, \ (B \subset S)$.

3.2. Long term choice probabilities and multiple equilibria, the binary case

In the context of social interaction, Schelling (1971) and Becker (1991) among others, have shown that multiple equilibria might occur. That is, for given prices and product attributes, there may be several values of long term choice probabilities. By long term choice probability we mean the limiting choice probabilities (if they exist) when time $t$ becomes very large, given that prices and other attributes remain constant over time. In a theoretical note, Becker (1991) showed how a model with social interaction, similar to the one we shall describe below, was able to explain why different restaurants with comparable prices and services are unequally popular. For the sake of clarifying this issue, we shall go through Becker’s analysis within our framework in the special case.
with binary choice, that is, in the case where \( B = \{1, 2\} \). First, we shall give a derivation of the binary choice logit model. For simplicity, we only consider the case with observationally identical individuals. In other words, there are no individual specific observable variables in the model. Although the binary logit model follows as a special case from (5) it may still be instructive to provide an explicit derivation, since the derivation in the general case with more than two alternatives is not immediate. Note first that if \( \varepsilon_{it1} \) and \( \varepsilon_{it2} \) are independent random variables with c.d.f. as in (3) it is known in the statistical literature (see Train, 2009) that \( \varepsilon_{it} = \varepsilon_{it1} - \varepsilon_{it2} \) has a Logistic c.d.f. given by

\[
P(\varepsilon_{it} \leq x) = \frac{1}{1 + e^{-x}}
\]

(7)

for all \( x \). In the following it will be convenient to consider the utility difference

\[
U_{it1} - U_{it2} = \alpha_1 - \alpha_2 - \frac{1}{\sigma}(w_{it1} - w_{it2}) + \gamma(P_{1i,t-1} - P_{2i,t-1}) + \varepsilon_{it1} - \varepsilon_{it2}
\]

(8)

\[
= \alpha_1 - \alpha_2 - \gamma - \frac{1}{\sigma} w_i + 2\gamma P_{1i,t-1} - (\varepsilon_{it1} - \varepsilon_{it2}) = \alpha - \frac{1}{\sigma} w_i + 2\gamma P_{1i,t-1} - \varepsilon_{it}
\]

where \( \alpha = \alpha_1 - \alpha_2 - \gamma, P_{2i} = 1 - P_{1i} \) and \( w_i = w_{it1} - w_{it2} \). From (7) and (8) it follows that

\[
P_{it} = P(U_{it1} - U_{it2} > 0) = P\left(\alpha - \frac{1}{\sigma} w_i + 2\gamma P_{1i,t-1} - \varepsilon_{it} > 0\right) = P\left(\varepsilon_{it} < \alpha - \frac{1}{\sigma} w_i + 2\gamma P_{1i,t-1}\right)
\]

implying that

\[
P_{it} = \frac{1}{1 + \exp\left(-\left(\alpha - \frac{1}{\sigma} w_i + 2\gamma P_{1i,t-1}\right)\right)}.
\]

(9)

Notice here that the choice probability on the left hand side does not depend on \( i \), because there are no observed individual characteristics in the model. In other words, the left hand side choice probability coincides with the aggregated choice probability in the peer group.

Let us now consider the possibility of multiple equilibria. Suppose that the price difference is constant over time, i.e., \( w_i = w \). Then, under specific conditions \( P_{it} \) will converge towards a unique equilibrium value \( P_i \) (say) as \( t \) increases. We call \( P_i \) the long term choice probability, as mentioned above. The corresponding equilibrium equation is given by
\[ p_i = \frac{1}{1 + \exp \left( -\left( \alpha - \frac{1}{\sigma} w + 2\gamma p_i \right) \right)} . \]  

(10)

From (10) it follows that

\[ w = \sigma \alpha + 2\gamma \sigma p_i - \sigma \log \left( \frac{p_i}{1-p_i} \right) . \]  

(11)

The expression in (11) can be interpreted as an inverse demand function. It yields the price difference \( w \) that corresponds to a given demand \( p_i \). From (11) it follows that

\[ \frac{\partial w}{\partial p_i} = 2\sigma \gamma - \frac{\sigma}{p_i(1-p_i)} . \]  

(12)

The largest value \( p_i(1-p_i) \) can have is 1/4, which means that the largest value \( \partial w/\partial p_i \) can have is \( \sigma(2\gamma - 4) \). Hence, if \( \gamma < 2 \), \( \partial w/\partial p_i \) will always be negative and consequently only one equilibrium will occur. If, however, \( \gamma > 2 \), several equilibria are possible. Specifically, when \( p_i \) is close to zero it follows from (12) that \( \partial w/\partial p_i \) is negative. However, as \( p_i \) increases, \( \partial w/\partial p_i \) will subsequently become positive. As \( p_i \) increases towards 1, \( \partial w/\partial p_i \) will eventually become negative again. In other words, the inverse demand function will in this case decrease until a local minimum \( r^L \) (say) is attained, and subsequently increase until a local maximum \( r^H \) (say), is attained and then decrease again. As a consequence, there may be up to 3 levels of demands (shown for example as the three red dots A, B and C in Figure 1) that give the same level of the inverse demand function (price difference).
Let $w^H$ denote the price level that corresponds to the local maxima occurring at $r^H$, and $w^L$ denote the price level that corresponds to the local maxima occurring at $r^L$. If the price difference is set to be $\bar{w}$ ($w^H > \bar{w} > w^L$), then there are three corresponding equilibrium demand levels, namely equilibrium A, B and C. Following Becker (1991) equilibrium B is unstable and equilibria A and C are stable. In other words, if the price difference is set equal to $\bar{w}$, then eventually the demand level will converge to either A or C (depends on the level of demand when the time $\bar{w}$ is set). It will never converge to B, unless (very unlikely) the starting level of the demand is exactly the same as B, in which case, the demand will remain stuck at B. If the price difference is set above $w^H$ (or below $w^L$), then, evidently, the demand will jump to a low (or high) level equilibrium, depending on the properties of the inverse demand function. Hence, the values $w^H$ and $w^L$ correspond to the “tipping” points (Schelling, 1971). Thus, we realize that social interaction may indeed lead to rather complicated behavioral patterns.

In the more general case with other distributions of the error terms and the presence of population heterogeneity Brock and Durlauf (2007) have investigated the identification problem for the binary choice model. In the general multinomial case with $J$ alternatives
in the choice set Brock and Durlauf (2002) have proved that multiple equilibria may occur when $\gamma > m$, where $m$ is the number of alternatives in the choice set $B$.

### 3.3. The case with correlated error terms in the utility function

The multinomial logit model described above is based on the IIA assumption. In some applications this assumption has been found to be violated. As discussed above, IIA relies crucially on the random error terms in the utility function being independent and identically distributed according to the distribution function given in (3). When key alternative-specific attributes are unobserved and correlated across alternatives the independence assumption may be violated. To see why, consider the case where the true utility function has the form

$$U_{ijt} = v_{ijt} + u_{ijt} \theta + \varepsilon_{ijt} = v_{ijt} + \eta_{ijt}$$

for $j = 1, 2, 3$, where $u_{ijt}$ is an unobserved attribute of alternative $j$ that is individual specific, $\theta$ the corresponding coefficient and $\eta_{ijt} = \varepsilon_{ijt} + u_{ijt} \theta$. For example, one type of specifications is given by $u_{ijt} = x_i u_j$ where $x_i$ is an unobserved individual characteristic and $u_j$ is a pure alternative-specific attribute. Accordingly, the utility function that corresponds to the actual observable setting has systematic term $v_{ijt}$ and error term $\eta_{ijt}$. Suppose that $u_{i1t}$ is uncorrelated with $u_{i2t}$ and $u_{i3t}$, but that $u_{i2t}$ and $u_{i3t}$ are correlated. Then we get that

$$\text{Corr}(\eta_{i2t}, \eta_{i3t}) \neq 0 \quad \text{and} \quad \text{Corr}(\eta_{i1t}, \eta_{ijt}) = 0$$

for $j = 2, 3$. Thus, in this case the corresponding multinomial logit model will not follow. However, McFadden (1978) has extended the multinomial model to the Generalized Extreme Value model (GEV) which allows for this type of correlation structure. In fact, the correlation structure of the error terms above is consistent with the so-called Nested logit model. A distributional assumption that yields the nested logit model for 3 alternatives is given by

$$P(\eta_{i1t} \leq x_1, \eta_{i2t} \leq x_2, \eta_{i3t} \leq x_3) = \exp(-e^{-x_1} + (e^{-x_2/\kappa} + e^{-x_3/\kappa})^\kappa)$$

(13)

where $0 < \kappa \leq 1$ is a constant that has the interpretation as

$$\text{Corr}(\eta_{i2t}, \eta_{i3t}) = 1 - \kappa^2.$$
From (13) it can be demonstrated (McFadden, 1978) that the corresponding choice probabilities are given by

$$P_{it} = \frac{\exp(v_{it})}{\exp(v_{it}) + \left(\sum_{k=2}^{3} \exp(v_{ikt} / \kappa)\right)^\kappa}$$

(14)

and

$$P_{ij} = \frac{\left(\sum_{k=2}^{3} \exp(v_{ikt} / \kappa)\right)^{\kappa-1} \exp(v_{ij} / \kappa)}{\exp(v_{i1r}) + \left(\sum_{k=2}^{3} \exp(v_{ikt} / \kappa)\right)^\kappa}$$

(15)

for $j = 2, 3$. In the case with social interaction of the type considered in this paper it is not known if the corresponding long term aggregate choice probabilities $\{P_j\}$ are uniquely determined.

3.4. The model for rank orderings

In the previous two sections, we discussed models for the most preferred alternative of an individual, from a given choice set. In this section we shall discuss the corresponding ranking model in which the individuals are asked to make a complete rank ordering of the alternatives according to their preferences. Consider, for example, the case where the individual is presented with three alternatives and asked to decide which of them is the most preferred, and which of them is the second preferred. For example, the outcome may be that alternative 2 is the first choice followed by alternative 1 as the second choice and alternative 3 as the third (last) choice. Using the same utility specification and assumptions about the error terms of the utility function as in the derivation of the multinomial logit model, one can show that the probability of this ranking can be expressed as the product of the following two terms: the trinomial logit probability of choosing alternative 2 from the choice set $\{1, 2, 3\}$, and the binary logit probability of choosing alternative 1 from the choice set $\{1, 3\}$ (Beggs et al., 1981). This is the same as

$$P(U_{2t} > U_{1t} > U_{3t}) = \frac{\exp(v_{21})}{\sum_{k=1,2,3} \exp(v_{ikt})} \cdot \frac{\exp(v_{1t})}{\sum_{k=1,3} \exp(v_{ikt})}.$$  

(16)

When some of the error terms are correlated, the formula in (16) does not hold. In this case one can either use a multinomial Probit type of framework (that is, assume that the
random error terms in the utility function are multivariate normally distributed) or logit type framework with random effects, see for example, Layton (2000) and Calfee et al. (2001). Both approaches yield models that cannot be expressed on closed form so that simulation techniques are necessary for calculating choice probabilities. When the choice set contains only 3 alternatives, Dagsvik and Liu (2009) have demonstrated that one can express the ranking probabilities on closed form in the case where the error terms of the utility function are distributes as in (13). In this case the ranking probabilities are given by

\[ P(U_{ij} > U_{ij'} > U_{ik}) = \frac{\exp(v_{ikr})}{\sum_{s=1,k} \exp(v_{ist})} \left( \sum_{x=2}^{3} \frac{\exp(v_{ist} / \kappa)}{\left( \sum_{s=2}^{3} \exp(v_{ist} / \kappa) \right)^{\kappa-1}} \right) \left( \frac{\exp(v_{ist} / \kappa)}{\sum_{s=2}^{3} \exp(v_{ist} / \kappa)} \right)^{\kappa}, \]

(17)

\[ P(U_{ij} > U_{ik} > U_{ik'}) = \frac{\exp(v_{ikr})}{\sum_{s=1,k} \exp(v_{ist})} - \frac{\exp(v_{ikr})}{\sum_{s=2}^{3} \exp(v_{ist} / \kappa)^{\kappa}}, \]

(18)

and

\[ P(U_{ik} > U_{ij} > U_{ik'}) = \frac{\exp(v_{ikr})}{\sum_{s=1,k} \exp(v_{ist})} \left( \sum_{s=2}^{3} \frac{\exp(v_{ist} / \kappa)}{\left( \sum_{s=2}^{3} \exp(v_{ist} / \kappa) \right)^{\kappa-1}} \right) \left( \frac{\exp(v_{ikr} / \kappa)}{\sum_{s=2}^{3} \exp(v_{ist} / \kappa)} \right)^{\kappa} \]

(19)

for \( j, k = 2, 3 \).
4. Data and survey method

4.1. Stated Preference

As mentioned in the introduction, there are several empirical approaches that can be employed to analyze individuals’ choice behavior for choice among MNOs. One way is to use (revealed preference) data obtained from individual purchases of subscriptions that exist in the market. There are several problems with this approach. Firstly, we do not have access to this type of market data. Secondly, even if a cross-section of micro-data were available it would not be sufficient to assess the effect of prices on demand because a single cross-section would not contain information on demand at different prices for a given alternatives. An attractive alternative way of obtaining data is to use the SP approach. In SP surveys the respondents are presented with purely hypothetical choice alternatives, characterized by specified attributes, and asked to state their most preferred alternative. Alternatively, one may ask the respondents to carry out a complete rank ordering of the alternatives presented. Although the latter strategy gives more information than the former one it may not necessarily be the preferred strategy because it may be more difficult to answer. The SP methodology has the advantage that it is possible to expose each respondent to several hypothetical choice experiments. SP data are based on what people say rather than what they do, which make some researchers sceptic towards this type of data. On the other hand SP surveys allow the researcher to specify alternatives that are not (yet) available in the market and accordingly study the potential demand for such products.

While SP techniques can, in principle, be used to value any products, tasks, phenomena and policies, there may however be limitations to stating preferences in practice. For example, people may not fully understand very small changes in risk, or highly complex products or phenomena. Sometimes one can become aware of such limitations by using focus groups. Another advantage of SP data is that it is possible to vary the (hypothetical) product attributes extensively, and thus obtain more precise estimates on the effects of attribute combinations, in contrast to revealed preference data, where the researcher may not necessarily be able to determine fully what attributes lie behind a given valuation or
the precise impact of each attribute. With surveys, the motives for preferences can also be discerned.

In the literature there has been some discussion about the external validity of SP experiments. The argument is that individuals may not treat hypothetical choice settings with the same care as they would in “real” choice contexts. Levin et al. (1983) and Pearmain et al. (1991) give a summary of the work on external validity and they conclude that in some cases there seems to be considerable evidence of external validity. Some researchers, for example Pearmain et al. (1991) claim that it appears difficult for individuals to relate to more than four attribute components. Other studies (see for example Beggs et al., 1981) have applied more complex designs with more than four attributes. In addition to each choice set a description of the choice context was provided.

4.2. Concerns related to our survey and questionnaire

A few things were under concern when we designed the survey.

(i) Online questionnaire or face-to-face interview?
It would have been best to conduct face-to-face interviews, because through face-to-face communication, respondents might be more motivated in answering questions and more clear about what they are asked to do. However, face-to-face interview is much more time- and money-consuming, and respondents might not be willing to reveal their true rankings when asked in front of an interviewer. In contrast, an online survey is free and anonymous, so the volunteering respondents might be those who are interested in the topic and are willing to express their true rankings. Therefore, we have chosen to conduct an online survey in Norway using Google Form and then post it on social media, such as, Facebook and discussion.no forum. However, in China we handed out the printed questionnaires in a university classroom.

(ii) How many choice experiments?
Different authors have used different numbers of experiments. For example, Dagsvik et al. (2002), and Dagsvik and Liu (2009) used 12 to 15 experiments. In order for the respondents to be able to finish within 10 minutes we decided to use only 11. This is
because a too long survey might scare away some respondents. The purpose of dividing the survey into two parts is first to train the respondents to answer what to some respondents may seem like a complicated survey. Adding information gradually with suitable introductory explanations might reduce the respondents’ confusion and then increase the precision of the responses. A second goal is to collect data that can be used to analyze misspecified models. By misspecified models in this context we mean models where social interaction effects (in case social interaction effects matter) are not accounted for.

(iii) How should the questions be asked?
In this survey the respondents were requested to rank order three alternatives instead of their best choice. An obvious advantage of this setup is that we then will be able to obtain two observations, instead of just one, from each experiment. There are at least two ways of asking ranking questions. One way is simply to ask the respondent to rank the alternatives from best to worse. Another possibility is to proceed in two stages as follows. In the first stage, the respondent is asked to choose his most preferred MNO among the 3 presented to him. Then in the second stage, his best choice from stage one is discarded and he is asked which of the remaining two alternatives he would choose. These two ways of asking questions should in principle reveal the same preference structure. However, this may not necessarily be the case. In fact, the probabilistic ranking model given in section 3.4 is based on the assumption that once the respondent has made his first choice, then his second choice is the most preferred choice among the two remaining alternatives. Moreover, when he selects the most preferred alternative among the two remaining ones he is assumed to have “forgotten” all about his first choice (IIA). It may therefore seem as if the second way of asking the ranking questions is closer to our choice model than the former. However, for the sake of simplicity and concision, we have chosen the former in our online survey.

(iv) How should the survey be conducted?
As mentioned above, market data of subscription rates of all different MNOs are not available to us. The survey was conducted in such a way that each respondent was presented with 11 choice experiments. The experiments consisted of two parts with 3 experiments in one and 8 experiments in the other. In all the experiments, three
hypothetical mobile network operators (MNO) were described with lists of qualitative attributes of their subscriptions and services (see Appendix A). In each experiment in part one, only the qualitative attributes and the prices of the 3 MNO were presented to the respondents, whereas in part two, information about the fractions of persons in each individual’s peer group were also present, in addition to prices and qualitative attributes. The respondents were then asked, in each experiment, to rank the three hypothetical MNOs from best to worst as first choice, second choice and last choice.

4.3. Summary statistics

4.3.1 Chinese data

The Chinese data was based on the survey of 183 undergraduates from Jiangsu University in China. Each of them was given a questionnaire attached in Appendix A. Table 1 summarizes the characteristics of the respondents. Table 2 shows the statistics of answers from each of the 11 ranking questions. For example, in Question 4, given the price of the subscription and services offered by MNO A (133 Chinese Yuan) and the subscription rate of respondent’s peer group (14%), there are 22 students chose A as “First Choice”, 94 “Second Choice” and 67 “Third Choice”.

Table 1. Summary of Respondents (Chinese Data)

<table>
<thead>
<tr>
<th></th>
<th>Male</th>
<th>Female</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>131 (71.58%)</td>
<td>52 (28.42%)</td>
<td>183 (100%)</td>
</tr>
</tbody>
</table>
Table 2. Summary of Choices (Chinese Data)

Number of respondents: 183

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First</td>
<td>Second</td>
<td>Third</td>
</tr>
<tr>
<td>1</td>
<td>(78)</td>
<td>37</td>
<td>106</td>
</tr>
<tr>
<td>2</td>
<td>(101)</td>
<td>49</td>
<td>48</td>
</tr>
<tr>
<td>3</td>
<td>(97 )</td>
<td>73</td>
<td>52</td>
</tr>
<tr>
<td>4</td>
<td>(14%, 133)</td>
<td>22</td>
<td>94</td>
</tr>
<tr>
<td>5</td>
<td>(82%, 116)</td>
<td>36</td>
<td>53</td>
</tr>
<tr>
<td>6</td>
<td>(5%, 73)</td>
<td>59</td>
<td>92</td>
</tr>
<tr>
<td>7</td>
<td>(42%, 80)</td>
<td>82</td>
<td>61</td>
</tr>
<tr>
<td>8</td>
<td>(9%, 94)</td>
<td>47</td>
<td>57</td>
</tr>
<tr>
<td>9</td>
<td>(84%, 116)</td>
<td>68</td>
<td>69</td>
</tr>
<tr>
<td>10</td>
<td>(27%, 107)</td>
<td>19</td>
<td>59</td>
</tr>
<tr>
<td>11</td>
<td>(23%, 100)</td>
<td>19</td>
<td>109</td>
</tr>
</tbody>
</table>

Note: The prices and subscription rates of respective alternatives (given only in experiments 4 to 11) are stated in parentheses and in bold. The currency used is Chinese Yuan (RMB).

4.3.2. Norwegian data

The Norwegian data was based on the online questionnaire, attached as Appendix A. Since the questionnaire was posted on different social media websites, such as, Facebook and discussion.no forum, the respondents are more diverse. Among 51 respondents (see Table 3), 35 (68.63%) are male and 16 (31.37%) are female. Their ages range from 16 to older than 51 with the majority (82.35%) of them being younger than 30. Not surprisingly, students account for 62.75% of the total 51 respondents and only 37.25% are non-students.
Table 3. Summary of Respondents (Norwegian Data)

<table>
<thead>
<tr>
<th>What is your gender?</th>
<th>Male</th>
<th>35</th>
<th>68.63%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Female</td>
<td>16</td>
<td>31.37%</td>
</tr>
<tr>
<td>Are you a student?</td>
<td>Student</td>
<td>32</td>
<td>62.75%</td>
</tr>
<tr>
<td></td>
<td>Non-student</td>
<td>19</td>
<td>37.25%</td>
</tr>
<tr>
<td>What is your age?</td>
<td>15 or younger</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td></td>
<td>16 - 20</td>
<td>2</td>
<td>3.92%</td>
</tr>
<tr>
<td></td>
<td>21 - 25</td>
<td>21</td>
<td>41.18%</td>
</tr>
<tr>
<td></td>
<td>26 - 30</td>
<td>19</td>
<td>37.25%</td>
</tr>
<tr>
<td></td>
<td>31 - 40</td>
<td>4</td>
<td>7.84%</td>
</tr>
<tr>
<td></td>
<td>41 - 50</td>
<td>3</td>
<td>5.88%</td>
</tr>
<tr>
<td></td>
<td>51 or older</td>
<td>2</td>
<td>3.92%</td>
</tr>
<tr>
<td>What is your highest</td>
<td>Primary school</td>
<td>1</td>
<td>2%</td>
</tr>
<tr>
<td>educational level?</td>
<td>High school</td>
<td>7</td>
<td>13.7%</td>
</tr>
<tr>
<td></td>
<td>Vocational education (or equivalent)</td>
<td>3</td>
<td>5.9%</td>
</tr>
<tr>
<td></td>
<td>Bachelor’s degree (or equivalent)</td>
<td>21</td>
<td>41.2%</td>
</tr>
<tr>
<td></td>
<td>Master’s degree (or equivalent)</td>
<td>17</td>
<td>33.3%</td>
</tr>
<tr>
<td></td>
<td>Ph.D (or equivalent)</td>
<td>2</td>
<td>3.9%</td>
</tr>
</tbody>
</table>

Table 4 shows the statistics of answers from each of the 11 ranking questions. Notice that there are some ties in the data. In other words, it might be the case that respondents rank alternatives A, B and C as “Second Choice”, “First Choice” and “Second Choice”. It happens because online respondents have made mistakes and clicked the wrong button when they rank ordered the alternatives by clicking “First choice”, “Second Choice” and “Third Choice”. Fortunately, there are only 8 pairs of ties among 561 observations of full ranking data. In cases mentioned above, Stata automatically ensures that data are consistent so that ties are no longer present in the data used in the estimation.
Table 4. Summary of Choices (Norwegian Data)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First</td>
<td>Second</td>
<td>Third</td>
</tr>
<tr>
<td>1</td>
<td>14</td>
<td>21</td>
<td>16</td>
</tr>
<tr>
<td>2</td>
<td>(233)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>17</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>(187)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>15</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>(202)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>22</td>
<td>22</td>
</tr>
<tr>
<td>5</td>
<td>(14%, 242)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>22</td>
<td>21</td>
<td>8</td>
</tr>
<tr>
<td>6</td>
<td>(82%, 213)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>22</td>
<td>22</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>(5%, 157)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>25</td>
<td>9</td>
</tr>
<tr>
<td>8</td>
<td>(42%, 272)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>17</td>
<td>9</td>
</tr>
<tr>
<td>9</td>
<td>(9%, 189)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>11</td>
<td>23</td>
</tr>
<tr>
<td>10</td>
<td>(84%, 206)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>24</td>
<td>3</td>
</tr>
<tr>
<td>11</td>
<td>(27%, 169)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>22</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>(23%, 174)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The prices and subscription rates of respective alternatives (given only in experiments 4 to 11) are stated in parentheses and in bold. The currency used is Norwegian Kroner (NOK).
5. Estimation results

In this section we shall discuss the estimation results based on both the Chinese and the Norwegian data sets. We have estimated different models by using the maximum likelihood estimation method. First-choice and Second-choice data sets are used separately to estimate logit models for the choice when all 3 alternatives are available and subsequently to estimate the most preferred choice among the remaining alternatives when the first choice alternatives have been removed from the choice set. The full data set is used in estimating rank-ordered logit model. See Appendix B for a detailed list of variables.

5.1. Estimation results for Chinese data

As shown in Table 5, all three data sets, namely first-choice data, second-choice data and ranking data, reveal that price has a significantly negative effect on preferences. On average, alternative B is most preferred and alternative C is second preferred when compared to alternative A, when keeping prices, social interaction and individual characteristics constant across alternatives. Sex has no significant effect on preferences. Social interaction has a significantly positive effect on preferences. In other words, the more people in an individual’s peer group that are subscribing to a certain MNO, the more likely the individual will be influenced and choose the same MNO.

One thing to notice is that, there is a significant difference between coefficient-estimates of social interaction effect obtained from the first-choice data compared to those obtained from the second-choice data. This follows because there is no overlap between the respective 95% confidence intervals for the social interaction parameter estimates, which are (1.233, 1.610) for the first choice data and (0.048, 0.521) for the second choice data. This inconsistency between the first-choice and second-choice estimates is also apparent when looking at the 95% confidence interval for the social interaction coefficient estimate using rank-ordered data, i.e. using both the first-choice and second-choice data.

---

1 All analyses in this Section are performed in Stata13.
observations. It equals (0.891, 1.178) and does not overlap the other two confidence intervals.

**Table 5.** Estimation Results for Chinese Data (Q4-11)

<table>
<thead>
<tr>
<th></th>
<th>Alternative-specific conditional logit (McFadden’s choice) model</th>
<th>Rank-ordered logistic regression</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First-choice data (1)</td>
<td>Second-choice data (2)</td>
</tr>
<tr>
<td>Price</td>
<td>-0.031*** (0.002)</td>
<td>-0.020*** (0.002)</td>
</tr>
<tr>
<td>Social interactions</td>
<td>1.421*** (0.096)</td>
<td>0.284** (0.121)</td>
</tr>
<tr>
<td>Alternative A (Base alternative)</td>
<td>Male</td>
<td>-0.086 (0.163)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.720*** (0.075)</td>
<td>0.290*** (0.086)</td>
</tr>
<tr>
<td>Alternative B</td>
<td>Male</td>
<td>-0.081 (0.164)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.441*** (0.074)</td>
<td>-0.105 (0.076)</td>
</tr>
<tr>
<td>Alternative C</td>
<td>Male</td>
<td>-0.081 (0.164)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.441*** (0.074)</td>
<td>-0.105 (0.076)</td>
</tr>
<tr>
<td>N</td>
<td>4392</td>
<td>2928</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-1298.418</td>
<td>-957.451</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
* p < 0.1, ** p < 0.05, *** p < 0.01

Why are there significant differences between the estimates from the first-choice and the second-choice data? One possible reason is that we have neglected the likely possibility that there might be serial correlation (across the panel) in the error terms, as mentioned above in Section 3. Dagsvik and Liu (2009) found that when allowing for serial correlation in a similar study based on panel data the estimates changed considerably. When they did not allow for serial correlation the IIA assumption was violated whereas when serial correlation was accounted for IIA was not rejected by the data. Another reason could be that the variance of the error terms in the utility function when the
individual makes his second choice might be larger than when he makes his first choice. The rationale is that individuals are generally more careful when making their first choice than when ranking the remaining alternatives. Hausman and Ruud (1987) have discussed this problem and developed formal tests of consistency between the parameters of the utility function when the individual makes his first choice and the corresponding parameters when choosing lower ranked alternatives. They also carried out an empirical application and they found that this the parameters associated with the first choice were different from the parameters associated with lower ranked alternatives.

Table 6. Estimation Results for Chinese Data (Q4-11 vs. Q1-3)

<table>
<thead>
<tr>
<th></th>
<th>Alternative-specific conditional logit (McFadden’s choice) model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First-choice data</td>
</tr>
<tr>
<td></td>
<td>Question 4 to 11  Question 1 to 3</td>
</tr>
<tr>
<td>Price</td>
<td>-0.031*** (0.002) -0.063*** (0.006)</td>
</tr>
<tr>
<td>Social interactions</td>
<td>1.421*** (0.096)</td>
</tr>
<tr>
<td>Alternative A (Base alternative)</td>
<td>0.720*** (0.075)  1.605*** (0.154)</td>
</tr>
<tr>
<td>Alternative B</td>
<td>0.441*** (0.074)  0.483*** (0.116)</td>
</tr>
<tr>
<td>N</td>
<td>4392  1647</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-1298.418 -506.770</td>
</tr>
</tbody>
</table>

Standard errors in parentheses
* p < 0.1, ** p < 0.05, *** p < 0.01

Table 6 shows the estimates for the misspecified model when the social interaction effect is ruled out a priori. By comparing the estimates based on the first-choice data (first column) in Table 5 and 6, we see that the price coefficient of the misspecified model displayed in Table 5 is overestimated and its absolute value is twice as large as the one in
Table 5. The misspecification also results in a biased estimate of the attractiveness parameter of alternative B, which is twice the magnitude of the one in Table 5.

5.2. Estimation results for Norwegian data

The estimates based on Norwegian data are given in Table 7. We note that price has a significantly negative effect on preferences, which is the same as what we found in the case with Chinese data.

Similarly to the Chinese case we see that there are large differences between the estimates based on the first choice data- compared to the estimates based on the second-choice data sets. As we have discussed above, serial correlation in the error terms and different variances of the error terms between first- and second-choice settings might be two possible reasons for the differences. Again, since the realized choices are the first choices, the estimates from the first-choice data are the more relevant ones. In this case, the social interaction effects play a significantly positive role in individuals’ preferences. However, the effect is not as strong as in the case with Chinese data.

The sample of Norwegian individuals include both students and non-students as well as different age groups, in contrast to the Chinese data which only include students in the beginning of their twenties. In order to investigate if preferences are dependent on age and level of schooling we extended the model by allowing the parameters that represent the effect of qualitative attributes of the alternatives to depend on age, sex and level of schooling. The estimation results show, keeping prices and social interactions fixed, that students have a higher preference for alternative B than for alternative A while being indifferent between C and A, and male respondents prefer alternative C to A while being indifferent between B and A. In addition, the younger the respondents are, the more they seem to prefer alternative C compared to alternative A. This is not contradicting the result that students are indifferent between B and A, because young non-students might have strong preference for alternative C. Last but not the least, educational level seem to have no significant impact on the individuals’ preferences.
<table>
<thead>
<tr>
<th></th>
<th>First-choice data</th>
<th>Second-choice data</th>
<th>Full data set</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative-specific conditional logit model (McFadden’s choice)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Price</td>
<td>-0.016***</td>
<td>-0.017***</td>
<td>-0.019***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
<td>(0.003)</td>
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<td>0.517***</td>
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<td></td>
<td>(0.172)</td>
<td>(0.176)</td>
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<tr>
<td>Constant</td>
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<td>-2.062</td>
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<td>1221</td>
<td>809</td>
</tr>
<tr>
<td>Log likelihood</td>
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<td>-402.824</td>
<td>-656.807</td>
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</table>

Note: 1. Ties handled via the “exactm” method in (5). 1 case (3 obs.) dropped due to no positive outcome per case in (1) and (2). 3 cases (6 obs.) dropped due to no positive outcome per case in (3) and (4). 2. Standard errors in parentheses. 3. * p < 0.1, ** p < 0.05, *** p < 0.01.
We also used the likelihood ratio test to test the null hypothesis that specification (1) and (2) are equal and we found that the null hypothesis is strongly rejected. At 0.05 confidence level, the test statistic \( D \), which is equal to twice the difference in the respective log-likelihood values, is much larger (about 30) than the critical value (about 15) of the chi-squared distribution with 8 degrees of freedom (the difference of the number of free parameters of specification (1) and (2))\(^2\). Hence, we can conclude that specification (2) fits the data better than (1) does.

Table 8. Estimation Results for Norwegian Data (Q4-11 vs. Q1-3)

<table>
<thead>
<tr>
<th>Alternative-specific conditional logit (McFadden’s choice) model</th>
<th>First-choice data</th>
<th></th>
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<tr>
<td></td>
<td>Question 4 to 11 (1)</td>
<td>Question 1 to 3 (2)</td>
</tr>
<tr>
<td>Price</td>
<td>-0.016***</td>
<td>-0.027***</td>
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<tr>
<td></td>
<td>(0.002)</td>
<td>(0.007)</td>
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<td>Social interactions</td>
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<td>(0.172)</td>
<td></td>
</tr>
<tr>
<td>Alternative A (Base alternative)</td>
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</tr>
<tr>
<td>Alternative B</td>
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<td>(0.239)</td>
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<td>Alternative C</td>
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<tr>
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<td>456</td>
</tr>
<tr>
<td>Log likelihood</td>
<td>-417.023</td>
<td>-149.329</td>
</tr>
</tbody>
</table>

Standard errors in parentheses

\( * p < 0.1 \), \( ** p < 0.05 \), \( *** p < 0.01 \)

Table 8 shows the estimation results based on the misspecified model and the data from question 1 to 3 where the information about social interaction is suppressed included. In

\[ 2 \times (-402.824 + 417.023) > \chi^2_{(0.05,8)} \]

28
this case, the coefficient associated with price is again overestimated which yields a false impression of the attractiveness of alternative B.
6. Analysis of marginal effects and elasticities

Recall that the choice probabilities are functions of qualitative attributes, prices and social interaction effects, represented by the fractions that have chosen the respective alternatives in the previous period. For both researchers and operator companies, it is of great interest to know how individuals’ responses are affected when some of these attributes change. For instance, a MNO that supplies alternative A might want to know how much the probability of choosing A will change if the price of its subscription changes, and similarly its competing MNO that supplies alternative B might be interested in knowing how much the probability of choosing B will change if the price of A changes. One way to address these questions will be to calculate the respective marginal effects or corresponding elasticities.

6.1. Short term marginal effects and elasticities

Here we shall derive what we call short term marginal effects. This means that we condition on the choice fractions in the previous period and treat them as fixed exogenous variables. We have that the own marginal effects are given by the formula

$$\frac{\partial P_{ij_t}}{\partial v_{jt}} = \frac{\partial}{\partial v_{jt}} \left[ \exp\left(v_{ij_t}\right) \right] / \sum_{k=1}^{J} \exp\left(v_{ik_t}\right) = \frac{\partial v_{ij_t}}{\partial v_{jt}} \left( P_{ij_t} - P_{ij_t}^2 \right) = -\frac{1}{\sigma} P_{ij_t} \left( 1 - P_{ij_t} \right)$$

(20)

Here it is understood that the qualitative attributes of the alternatives do not change.

Similarly, cross marginal effects are readily found to be

$$\frac{\partial P_{ij_t}}{\partial v_{kt}} = \frac{\partial}{\partial v_{kt}} \left[ \exp\left(v_{ij_t}\right) / \sum_{m=1}^{J} \exp\left(v_{im_t}\right) \right] = \frac{1}{\sigma} P_{ij_t} P_{ik_t}$$

(21)

where $k \neq j$. Similarly, we get that

$$\frac{\partial P_{ij_t}}{\partial P_{ij_{t-1}}} = \gamma (1 - P_{ij_t}) P_{ij_t} \quad \text{and} \quad \frac{\partial P_{ij_t}}{\partial P_{nj_{t-1}}} = -\gamma P_{nj_t} P_{ij_t}.$$
Consider next the corresponding elasticities. From (20) and (21) it follows immediately that the own and cross price elasticities are given by

$$ \frac{\partial \log P_{ij}}{\partial \log w_{jt}} = -(1-P_{ij}) \cdot \frac{w_{jt}}{\sigma} \quad \text{and} \quad \frac{\partial \log P_{ij}}{\partial \log w_{rt}} = P_{ir} \cdot \frac{w_{rt}}{\sigma}. $$

(23)

Furthermore, we get that

$$ \frac{\partial \log P_{ij}}{\partial \log P_{j,t-1}} = \gamma (1-P_{ij}) P_{j,t-1} \quad \text{and} \quad \frac{\partial \log P_{ij}}{\partial \log P_{r,t-1}} = -\gamma P_{ir} P_{r,t-1}. $$

(24)

6.2. Long term (equilibrium) marginal effects and elasticities

Another type of elasticities are what we call long term-or equilibrium-elasticities. For simplicity, we shall consider only the case without observable individual specific characteristics. This type of elasticities are calculated for the long term equilibrium choice probabilities. For notational simplicity let $\theta_{ij} = \partial \log P_j / \partial w_i$. In equilibrium it follows from (5) that

$$ \log P_j = v_j - \log \left( \sum_{k \in B} \exp(v_k) \right) $$

(25)

where

$$ v_j = \alpha_j - w_j / \sigma + \gamma P_j. $$

(26)

Using (25) and (26) it follows that

$$ \theta_{ij} = -1 / \sigma + \gamma \cdot \frac{\partial P_j}{\partial w_j} + \frac{\exp(v_j)}{\sum_{k \in B} \exp(v_k)} \cdot \frac{1}{\sigma} - \gamma \sum_{k \in B} \left( \frac{\exp(v_k)}{\sum_{s \in B} \exp(v_s)} \cdot \frac{\partial P_k}{\partial w_j} \right) $$

$$ = -(1-P_j) / \sigma + \gamma P_j \theta_{ij} - \gamma \sum_{k \in B} P_k^2 \theta_{kj}. $$

(27)

Similarly, we get that

$$ \theta_{jr} = \gamma \cdot \frac{\partial P_j}{\partial w_r} + \frac{\exp(v_r)}{\sum_{k \in B} \exp(v_k)} \cdot \frac{1}{\sigma} - \gamma \sum_{k \in B} \left( \frac{\exp(v_k)}{\sum_{s \in B} \exp(v_s)} \cdot \frac{\partial P_k}{\partial w_r} \right) $$
\[ = P_r / \sigma + P_j \theta_{jr} - \gamma \sum_{k \in B} P_k^2 \theta_{kr} \]  

(28)

when \( r \neq j \). Equations (26) and (28) can be written in a more compact way as

\[ \theta_{jr} = - (\delta_{jr} - P_r) / \sigma + \gamma P_j \theta_{jr} - \gamma \sum_{k \in B} P_k^2 \theta_{kr} \]  

(29)

for all \( j \) and \( r \) where \( \delta_{jr} = 1 \) if \( j = r \), and zero otherwise. To solve (29) for \( \{ \theta_{jr} \} \) let

\[ A_r = \sum_{k \in B} P_k^2 \theta_{kr}. \]

With this notation we can express \( \theta_{jr} \) as

\[ \theta_{jr} = \frac{(P_r - \delta_{jr}) / \sigma - \gamma A_r}{1 - \gamma P_j}. \]  

(30)

If we multiply both sides of (29) by \( P_j^2 \) and take the sum over all alternatives in \( B \) we obtain that

\[ A_r = \sum_{k \in B} P_k^2 \frac{(P_r - \delta_{kr}) / \sigma}{1 - \gamma P_k} - \gamma A_r \sum_{k \in B} \frac{P_k^2}{1 - \gamma P_k}. \]

The equation above can easily be solved for \( A_r \) and the solution is given by

\[ A_r = \frac{1}{\sigma} \sum_{k \in B} \frac{P_k^2 (P_r - \delta_{kr})}{1 - \gamma P_k} \]  

(31)

Consequently, when inserting (31) into (29) we get that

\[ \theta_{jr} = - \frac{1}{\sigma} \frac{1}{1 - \gamma P_j} \left( \delta_{jr} - P_r + \frac{\gamma \sum_{k \in B} P_k^2 (P_r - \delta_{kr})}{1 + \gamma \sum_{k \in B} P_k^2} \right). \]  

(32)

From (32) it follows immediately that the long term price elasticities are given by

\[ \frac{\partial \log P_j}{\partial \log w_r} = \frac{w_r \partial P_j}{P_j \partial w_r} = \frac{-1}{\sigma} \frac{w_r}{1 - \gamma P_j} \left( \delta_{jr} - P_r + \frac{\gamma \sum_{k \in B} P_k^2 (P_r - \delta_{kr})}{1 + \gamma \sum_{k \in B} P_k^2} \right). \]  

(33)

for all \( j, r \) that belong to \( B \).
The formulas for the corresponding marginal effects follows readily because

\[
\frac{\partial P_j}{\partial \log w_r} = \frac{P_j \partial \log P_j}{w_r \partial \log w_r}.
\]  

(34)

Thus, given the formulas for the elasticities given in (33) one can obtain the corresponding marginal effects readily.

6.3. Simulation experiments for the binary case

In this section we report results from some simulation experiments. That is, we simulate the fluctuations of the choice probabilities over time conditional on a particular time series of prices. We will limit our simulation to the binary case without individual characteristics and we use the estimates from the Chinese data. Recall that the utility function is independent of the choice set, and also the parameters of the utility function are independent of prices and previous choices. As a result, we can use the model in hypothetical situations where the choice set and the prices are different from the ones in the empirical setting.

6.3.1. Short term choice probabilities, marginal effects and elasticities in the binary case

The short term own and cross elasticities are given by (20) and (21). In the binary case, the corresponding short term elasticities are given by inserting (9) into (20) and (21).

Therefore we get

\[
\frac{\partial \log P_{1t}}{\partial \log w_{1t}} = \frac{-\exp\left(-\alpha + w_t / \sigma - 2\gamma P_{1,t-1}\right) w_{1t}}{1 + \exp\left(-\alpha + w_t / \sigma - 2\gamma P_{1,t-1}\right)} \frac{w_{1t}}{\sigma},
\]

\[
\frac{\partial \log P_{2t}}{\partial \log w_{2t}} = \frac{\exp\left(-\alpha + w_t / \sigma - 2\gamma P_{1,t-1}\right) w_{2t}}{1 + \exp\left(-\alpha + w_t / \sigma - 2\gamma P_{1,t-1}\right)} \frac{w_{2t}}{\sigma}.
\]

To see how the social interaction effect affects the short term choice probabilities in the binary case, we simulated a time series of 30 periods from period 1 to period 30. In each period, we randomly generated prices of the two MNO, within a reasonable price range,

\[3\] All the simulation analyses are performed in R.
and calculated the corresponding price difference $w$. We assumed that the initial choice probabilities were equal to 0.5. Using (9), we calculated the choice probability of alternative one in any period, given the prices at the same period and the corresponding choice probability in the previous period. Recall that these probabilities are what we called short term choice probabilities. Having calculated the short term choice probabilities, we then used (34) and (35) to compute corresponding short term own and cross price elasticities for alternative one. See Appendix C.1 for the table which shows the simulated prices of both alternatives, the price difference, the short term choice probability of alternative one with social interaction effect, the short term choice probability of alternative one without social interaction effect, the short term own price elasticity for alternative one with social interaction effect, the short term own price elasticity for alternative one without social interaction effect, short term cross price elasticity for alternative one with social interaction effect, and short term cross price elasticity for alternative one without social interaction effect.

Figure 2 compares the short term choice probabilities of alternative one, $P_t^1$, with and without social interaction effect. The green curve represents the price difference $w = w_2 - w_1$, the red one represents $P_t^1$ with social interaction effect, and the blue one represents $P_t^1$ without social interaction effect. We can see that the blue curve follows pretty much the same trend as the green one. In other words, without social interaction effect, the short term choice probability reflects the price difference immediately. However, with social interaction effect, the current $P_t^1$ does not only depend on the price difference at the same period, but also on the value of $P_t^1$ in the previous period. As a result, the short term $P_t^1$ does not always follow the trend of price difference. For example, from period 5 to 6, as the price difference change direction and goes up, the blue curve immediately follows, but the red one continues going down because of the social interaction effect. Another example is found when looking at periods 27 to 28. Here as the price difference and the blue curve go down, the red one keeps going up. Some other examples are found at periods 16-17, 21-22. Another thing to notice is that when $P_t^1$ in the previous period is larger (smaller) than 0.5, then social interaction will play a positive
(negative) role on $P_i$ in the current period, thus the red curve will lie above (below) the blue one for the current period. This is reasonable because if alternative one is popular in the previous period, then in the current period, people will on average respond positively and strengthen the popularity of the alternative in the presence of social interaction relative to the case without social interaction.

**Figure 2.** Short-Term Choice Probabilities
(Green: Price Difference, Red: with Social Interaction Effect, Blue: without Social Interaction Effect.)

Figure 3 compares the short term own elasticities of alternative one, with and without social interaction. The red curve indicates the one with social interaction effect and the blue one without. Looking at this figure together with Figure 2, we notice that when $P_i$ in the previous period is larger (smaller) than 0.5, then social interaction has a positive (negative) effect on the own elasticity, since the red curve lies above (below) the blue one for the current period. This is intuitive since social interaction implies that popularity of alternative one in the previous period will, in the coming period, strengthen the increase in the change of its demand caused by the decrease of the relative price (own price elasticity). Figure 4 shows the similar effect of social interaction, namely that popularity of alternative one in the previous period will, in the coming period, weaken the decrease
in the change of its demand caused by the decrease of its opponent’s relative price (cross price elasticity).

**Figure 3.** Short-Term Own Elasticities
(Red: with Social Interaction Effect, Blue: without Social Interaction Effect.)

**Figure 4.** Short-Term Cross Elasticities
(Red: with Social Interaction Effect, Blue: without Social Interaction Effect.)
6.3.2. Long term choice probabilities, marginal effects and elasticities in the binary case

In the long run, given specific values of the prices, one can solve the nonlinear equation (10) numerically for the corresponding equilibrium $P_i$. It follows readily from the previous results that the long term marginal effects with respect to prices and the corresponding price elasticities in the binary case are given by

$$\frac{\partial P_i}{\partial w_i} = \frac{-(1 - P_i) P_i}{1 - 2\gamma P_i (1 - P_i)} \cdot \frac{1}{\sigma}, \quad \frac{\partial P_i}{\partial P_i} = \frac{(1 - P_i) P_i}{1 - 2\gamma P_i (1 - P_i)} \cdot \frac{1}{\sigma}.$$  

$$\frac{\partial \log P_i}{\partial \log w_i} = \frac{-(1 - P_i)}{1 - 2\gamma P_i (1 - P_i)} \cdot \frac{w_i}{\sigma} \quad \text{and} \quad \frac{\partial \log P_i}{\partial \log w_2} = \frac{(1 - P_i)}{1 - 2\gamma P_i (1 - P_i)} \cdot \frac{w_2}{\sigma}.$$  

We know that in our case, $\gamma < 2$, so that for any given level of prices there exists one and only equilibrium $P_i$ (long term). However, the long term case is more complex than the short term one since it is not possible to calculate marginal effects and elasticities on closed form as a function of prices. This makes it difficult to use the formulas above to see how changes in prices will affect the choice probability. Nevertheless, from the simulated figures, we are able to see the role that social interaction plays in long term analysis.

The table of figures in Appendix C.2 shows the long term choice probabilities, the corresponding own and cross price marginal effects and elasticities as a function of $w_i$ (rearranged in ascending order) and for a fixed value of $w_2$. Similar to the figures above, the red curve indicates the case with social interaction and the blue one without. We now look at the first column of figures (Appendix C.2), which shows the long term choice probabilities of alternative one at different levels of prices. In both cases with and without social interaction, the own price has a negative effect on demand. However, the red curve shows an additional interesting picture. Let us look at the figure when $w_2 = 130$ for an example. When $w_1$ is relatively low (below some threshold), social interaction effect results in a higher long term $P_i$ at each level of $w_1$, while when $w_1$ is relatively high (above
(some threshold), the effect results in a lower long term $P_i$ at each level of $w_1$. Furthermore, we see from comparing the first column of figures (Appendix C.2) vertically, that the threshold increases as the level of $w_2$ goes up.

**Figure 5.** Long-Term Choice Probabilities when $w_2 = 130$
(RED: with Social Interaction Effect, BLUE: without Social Interaction Effect.)

**Figure 6.** Long-Term Own Marginal Effects when $w_2 = 130$
(RED: with Social Interaction Effect, BLUE: without Social Interaction Effect.)
Now we look at the corresponding figure of the long term own marginal effect when $w_2 = 130$ (Figure 6). We note that both the blue and red curves show a negative marginal effect, which is in line with the “negative price effect” feature we talked about above. The marginal effect without social interaction, appears as the blue curve, is a parabola rather than a horizontal line which is typical in many studies where linear probability models are used. The discrete choice model allows the marginal effect to change in a more realistic way in that the effect decreases in absolute value as the choice probability approaches zero or one. Now compare the blue and red curves in Figure 6. When the price is sufficiently low (yielding sufficiently high long term choice probability), social interaction weakens the negative marginal price effect. This weakening effect goes on until the price exceeds some threshold, from where the social interaction starts to strengthen the negative marginal price effect. If the price increases further beyond a sufficiently high level, the social interaction will again weaken the marginal effect.

Both Figure 5 and 6 illustrate the effect of social interaction. Because of the effect from social interaction, popular operator will continue benefitting from its popularity even when his price goes up, while the operator who faces a low demand will continue to face a low demand even if he reduces his price.
7. Functional form issues

So far we have mentioned only briefly how the functional form of the systematic part of the utility function should be specified. In the analysis above we have applied a specification where the utility function is a linear function in prices and the probabilities (peer group fractions) of choosing the respective alternatives. For simplicity, we only consider the case with observationally identical individuals. We shall now discuss the consequences of replacing the utility function in (4) by

\[ U_{ijt} = \alpha_j - \frac{w_{jt}}{\sigma} + \gamma \log P_{j,t-1} + \epsilon_{ijt} \]  

(35)

The only difference between the specification in (4) and the specification in (35) is that \( P_{j,t-1} \) is replaced by \( \log P_{j,t-1} \). From the specification in (35) and (5) it follows readily that, in a trinomial case, the choice probabilities are given by

\[ P_{jt} = \frac{\exp(\alpha_j - \frac{w_{jt}}{\sigma} + \gamma \log P_{j,t-1})}{\sum_{k=1}^{3} \exp(\alpha_k - \frac{w_{kt}}{\sigma} + \gamma \log P_{k,t-1})}, \quad j = 1, 2, 3. \]

From the expression above, it follows that the long term choice probabilities, if they exist, must satisfy

\[ P_j = \frac{\exp(\alpha_j - \frac{w_{j}}{\sigma} + \gamma \log P_j)}{\sum_{k=1}^{3} \exp(\alpha_k - \frac{w_{k}}{\sigma} + \gamma \log P_k)}, \quad j = 1, 2, 3. \]

In contrast to the previous case, it is now possible to obtain a closed form solution for the long term choice probabilities. To demonstrate this, note that by dividing \( P_j \) by \( P_3 \) and taking logarithm, we get the following

\[ \log \left( \frac{P_j}{P_3} \right) = \alpha_j - \alpha_3 + \frac{w_3 - w_j}{\sigma} + \gamma \log P_j - \gamma \log P_3 = \alpha_j - \alpha_3 + \frac{w_3 - w_j}{\sigma} + \gamma \log \left( \frac{P_j}{P_3} \right). \]

Rearrange both sides, we get

\[ \log \left( \frac{P_j}{P_3} \right) = \frac{\alpha_j - \alpha_3 + (w_3 - w_j)/\sigma}{1 - \gamma}, \]

which leads to
\[ P_j = P_3 \cdot \exp \left( \frac{\alpha_j - \alpha_s + \left( w_3 - w_j \right)/\sigma}{1 - \gamma} \right). \]  

(36)

We then sum up \( P_j \) over \( j \), and get

\[ 1 = \sum_{j=1}^{3} P_j = P_3 \cdot \sum_{j=1}^{3} \exp \left( \frac{\alpha_j - \alpha_s + \left( w_3 - w_j \right)/\sigma}{1 - \gamma} \right), \]

so that

\[ P_3 = \frac{1}{\sum_{j=1}^{3} \exp \left( \frac{\alpha_j - \alpha_s + \left( w_3 - w_j \right)/\sigma}{1 - \gamma} \right)}. \]

Plugging it into (36) yields

\[ P_j = \frac{\exp \left( \frac{\alpha_j - w_j / \sigma}{1 - \gamma} \right)}{\sum_{k=1}^{3} \exp \left( \frac{\alpha_k - w_k / \sigma}{1 - \gamma} \right)}. \]  

(37)

From (37) it follows that the long term price elasticities are given by

\[ \frac{\partial \log P_j}{\partial \log w_j} = \frac{(1-P_j)w_j}{\sigma(1-\gamma)} \quad \text{and} \quad \frac{\partial \log P_j}{\partial \log w_r} = \frac{P_r w_r}{\sigma(1-\gamma)}. \]  

(38)

Note that in order for long term equilibrium to exist we must have that \( \gamma < 1 \). This is easily seen from (38). There are two crucial conclusions that can be drawn from this analysis. First, the utility specification above implies that there always exists one and only one equilibrium regardless of the value of \( \gamma \). In the usual case, whether to include \( \log P_j \) or \( P_j \) in the utility function is not a big issue, but in our case, \( \log P_j \) specification a priori rule out the possibility of multiple equilibria.

Now the crucial question is which model is better? One way of examining this is to estimate both models and check which one fits the data best by comparing likelihoods. It is also possible to use statistical methods for testing non-nested hypotheses, see for example Ben-Akiva and Lerman (1985). Unfortunately, the empirical results are
inconclusive since the likelihood values are approximately the same for both models. The reason might be that all the respondents received the same questionnaire, accordingly there is no variation in the alternative specific variables across individuals in the sample.

Table 9. Estimation Results for Different Functional Form ($P$ vs. $logP$)

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P$</td>
<td>$logP$</td>
</tr>
<tr>
<td><strong>Price</strong></td>
<td>-0.031***</td>
<td>-0.030***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td><strong>Social interactions</strong></td>
<td>1.421***</td>
<td>0.424***</td>
</tr>
<tr>
<td></td>
<td>(0.096)</td>
<td>(0.029)</td>
</tr>
<tr>
<td><strong>Alternative A</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Base alternative)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Alternative B</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.720***</td>
<td>0.653***</td>
</tr>
<tr>
<td></td>
<td>(0.075)</td>
<td>(0.074)</td>
</tr>
<tr>
<td><strong>Alternative C</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.441***</td>
<td>0.575***</td>
</tr>
<tr>
<td></td>
<td>(0.074)</td>
<td>(0.075)</td>
</tr>
<tr>
<td><strong>N</strong></td>
<td>4392</td>
<td>4392</td>
</tr>
<tr>
<td><strong>Log-lik</strong></td>
<td>-1298.4183</td>
<td>-1297.5476</td>
</tr>
</tbody>
</table>

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Finally, let us consider the implication using a misspecified model. To this end, suppose for the sake of the argument that the model with utility function that is linear in the aggregate fractions of choices is the correct model. Does it really matter? From the formulas (33) and (38) of the elasticities of the respective models we note that they are fundamentally different. For simplicity, consider the special case of binary choice. From (10) and (37) we have that the long term linear $(P_i)$ and loglinear $(P_i^*)$ utility specifications yield marginal effects

$$\frac{\partial P_i}{\partial w_i} = \frac{-(1-P_i)P_i}{1-2\gamma P_i(1-P_i)} \cdot \frac{1}{\sigma}$$

and

$$\frac{\partial P_i^*}{\partial w_i} = \frac{(1-P_i^*)P_i^*}{\sigma^*(1-\gamma^*)}$$

which we note have rather different structure. For example, when $P_i = P_i^*$ we have that
\[
\frac{\partial P_1}{\partial w_i} \cdot \frac{\partial P_i^*}{\partial w_i} = \frac{(1 - \gamma^*) \sigma^* / \sigma}{1 - 2\gamma P_i (1 - P_i)}
\]

which means that the ratio between the respective marginal effects at a common level of demand is in general different from 1 and is not even constant.
8. Summary and conclusion

In this study we have analyzed how individual choice behavior might be influenced by choices of others. We have discussed how the presence of social interaction effects may yield multiple equilibria and thus explain “tipping point” aggregate behavior, in line with Schelling (1971) and Becker (1991). Our focus has been on an empirical analysis of individual choice among mobile network operators. To this end, we have conducted a Stated Preference survey in which the questionnaire was designed for this purpose. We have obtained data from a sample of Norwegian individuals who responded to the questionnaire that was posted online and a sample of Chinese undergraduate students who answered to a Chinese version of the questionnaire in class. To analyze the survey data we have applied a discrete choice model, suitably extended to allow for social interaction effects. The estimation results clearly show that there are substantial social interaction effects, in particular among Chinese students. However, the social interaction effect is not sufficiently strong to yield multiple equilibria outcomes. We have used the choice model to simulate how models with and without social interaction respond to changes in prices. The simulation experiments show that the effect of price changes on the long term demand may be strengthened or weakened, according to the level of the demand. We have also shown how functional form of the empirical specification may be critical and may lead to misleading results, in the sense that some specifications, a priori, rule out tipping point behavior.

The results of our study cannot be of immediate use of mobile network operators since our empirical analysis is based on a purely hypothetical setting. However, we believe that our results might still be of substantial interest because they show that social interaction effects may be important. Accordingly, realistic behavioral models for choice among mobile network operators should be extended to account for social interaction.
References


Appendix A

1. The Chinese version questionnaire handed out among undergraduates in class

移动通信运营商，你会选择谁？

问卷分两部分，共 12 个排序题。每一题都使用如下同样的表格，表格里罗列着由三个不同的手机运营商提供的手机套餐和服务。除表格外，每一题，我们会给你相关不同的其他信息。你将要根据提供的所有信息，按照从“好”到“坏”排列你对三个运营商的选择。谢谢！

<table>
<thead>
<tr>
<th>运营商 A</th>
<th>运营商 B</th>
<th>运营商 C</th>
</tr>
</thead>
<tbody>
<tr>
<td>套餐包含：</td>
<td></td>
<td></td>
</tr>
<tr>
<td>本地市话（分钟）</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>本地国内长途（分钟）</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>国内通话（分钟）</td>
<td>300</td>
<td>0</td>
</tr>
<tr>
<td>国内短信/彩信（条）</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>国内数据流量</td>
<td>200 M</td>
<td>2 G</td>
</tr>
<tr>
<td>其他服务和产品：</td>
<td></td>
<td></td>
</tr>
<tr>
<td>套餐外本地市话</td>
<td>0.15 元/分钟</td>
<td>0.15 元/分钟</td>
</tr>
<tr>
<td>套餐外国内长途</td>
<td>0.20 元/分钟</td>
<td>0.30 元/分钟</td>
</tr>
<tr>
<td>套餐外流量</td>
<td>29 元/100M</td>
<td>10 元/100M</td>
</tr>
<tr>
<td>其他：</td>
<td></td>
<td></td>
</tr>
<tr>
<td>免费来电显示</td>
<td></td>
<td></td>
</tr>
<tr>
<td>免费移动“支付宝”</td>
<td></td>
<td></td>
</tr>
<tr>
<td>免费短信呼叫</td>
<td></td>
<td></td>
</tr>
<tr>
<td>可选本地家庭网*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>可选本地家庭网*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>可选本地家庭网*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>可选本地同事网**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>可选本地同事网**</td>
<td></td>
<td></td>
</tr>
<tr>
<td>可选本地同事网**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*本地家庭网：当选择相同运营商套餐时，每个成员（最多 5 人）每月可享成员间 500 分钟免费通话。月功能费：3 元/成员。
**本地同事网：当选择相同运营商套餐时，若公司已开通本地同事网，加入成为成员每月可享成员间 500 分钟免费通话。月功能费：3 元/成员。

个人背景资料了解，请打勾。

你的性别是：○男 ○女

你的年龄是：○16 - 20 ○21 – 25 ○26 - 30

第一部分

移动通信运营商，比如：中国移动，中国联通，向消费者提供不同的手机套餐和服务。假设你现在要为你的手机从 A, B, C 中选择运营商（如上图）。综合考虑套餐的内容，运营商提供的其他服务和套餐的价格，请将运营商 A, B, C 从“好”到“坏”依次排序为“1”, “2”, “3”。选项 A (78 元) 表示运营商 A 提供的套餐价格为 78 元。B, C 依次类推。
参照相同图表，仅改变套餐的价格以后，请将运营商 A, B, C 重新从“好”到“坏”依次排序为“I”, “2”, “3”。

_____A (101 元) _____B (119 元) _____C (109 元)

再次仅改变套餐的价格，请将运营商 A, B, C 重新从“好”到“坏”依次排序为“I”, “2”, “3”。

_____A (97 元) _____B (118 元) _____C (123 元)

第二部分

除了套餐和服务的内容和价格以外，他人对运营商的选择也许会影响到你的。在自己选择运营商时，考虑别人的选择有时是合理的。比如，第一，你可能不确定不同运营商提供的服务质量好坏，如声音质量，信号强度，网络速度和售后服务等等。所以，你可能会把别人的选择作为参考，用来衡量服务的质量。第二，当你选择和你的家人朋友同事相同的运营商时，你可能会从比如家庭网，公司网等服务中获得直接优惠。参考相同图表和选项中的其他信息，请将运营商 A, B, C 从“好”到“坏”依次排序为“I”, “2”, “3”。

选项 A (14%, 133 元) 表示在你的家人朋友同事中，14%的人正在使用运营商 A 提供的套餐和服务，运营商 A 提供的套餐价格为 133 元。B,C 依次类推。

_____A (14%, 133 元) _____B (79%, 137 元) _____C (7%, 113 元)

表格内容不变，改变的只是套餐价格和你的家人朋友同事对于相应运营商套餐和服务的订购率，请将运营商 A, B, C 从“好”到“坏”依次排序为“I”, “2”, “3”。

_____A (82%, 116 元) _____B (16%, 71 元) _____C (2%, 90 元)

继续类似操作 6 次。

1. _____A (5%, 73 元) _____B (90%, 112 元) _____C (5%, 129 元)
2. _____A (42%, 80 元) _____B (13%, 135 元) _____C (45%, 106 元)
3. _____A (9%, 94 元) _____B (17%, 103 元) _____C (74%, 126 元)
4. _____A (84%, 116 元) _____B (12%, 132 元) _____C (4%, 88 元)
5. _____A (27%, 107 元) _____B (70%, 72 元) _____C (3%, 96 元)
6. _____A (23%, 100 元) _____B (12%, 127 元) _____C (65%, 72 元)

问卷结束！谢谢！
2. The questionnaire posted online for Norwegian data

Which Mobile Network Operator Will You Choose?
We are conducting research on individuals’ choices on Mobile Network Operators. We'd love to invite you to participate in this survey. The survey consists of two parts. In both parts, you will be presented with subscriptions (mobilabonnement) and mobile services offered by three different operators. You will be asked to rank them from the best to worst given additional information about the subscriptions. The survey should only take 10 minutes, and your responses are completely anonymous. We appreciate your input!

Your Background
Before we start, we'd love to know about your background.

- Are you male or female?
  - Female
  - Male

- Are you a student?
  - Yes
  - No

- What is your age?
  - 15 or younger
  - 16 - 20
  - 21 - 25
  - 26 - 30
  - 31 - 40
  - 41 - 50
  - 51 or older

- What is the highest level of education that you have completed or are currently pursuing?
  - Primary school
  - High school

---

4 The questionnaire stated here is slightly modified, compared with the one online which is more interactive.
There are several Mobile Network Operators in Norway, for example, Telenor, Tele2, Chess, NetCom. They offer different subscriptions (mobilabonnement) and services. Now imagine you are going to choose the operator for your mobile phone, and you’ve already narrowed it down to three operators A, B, C with their subscriptions and services (see figure above). Given the price and other features of the subscription, and the services that you will get once you subscribe to one of the operators, please rank these three operators from the best to worst as First Choice, Second Choice and Last Choice. Do the same for all 3 situations in Part 1. Choice A (233 kr.) means the subscription from operator A costs 233 kr.

1. A (233 kr.) B (210 kr.) C (184 kr.)
2. A (187 kr.) B (219 kr.) C (197 kr.)
3. A (202 kr.) B (177 kr.) C (166 kr.)

**Part 2**

In addition to the price, information about others’ choices might be relevant for yours. It may be reasonable for you to take into account the choices of others. For example, first, you might not sure about the quality of the services that different operators offer, such as, voice quality, signal strength, data speed and customer support, etc. Therefore, you might use others’ choices as an indicator of the quality of the services. Second, you might benefit from Family Plan, operator-exclusive apps and free within-operator calls with your friends or family if you subscribe to the same operator.

Please rank these three operators (see figure above) from the best to worst as First Choice, Second Choice and Last Choice. Do the same for all 8 situations in Part 2.

Choice A (14%, 242 kr.) means 14% of your Peer Group (consisting of your family, friends, colleagues etc.) are subscribers of operator A, and the subscription from operator A costs 242 kr.

1. A (14%, 242 kr.) B (79%, 181 kr.) C (7%, 222 kr.)
2. A (82%, 213 kr.) B (16%, 190 kr.) C (2%, 232 kr.)
3. A (5%, 157 kr.) B (90%, 170 kr.) C (5%, 205 kr.)
4. A (42%, 227 kr.) B (13%, 234 kr.) C (45%, 155 kr.)
5. A (9%, 189 kr.) B (17%, 194 kr.) C (74%, 235 kr.)
6. A (84%, 206 kr.) B (12%, 152 kr.) C (4%, 166 kr.)
7. A (27%, 169 kr.) B (70%, 242 kr.) C (3%, 175 kr.)
8. A (23%, 174 kr.) B (12%, 157 kr.) C (65%, 198 kr.)
## Appendix B

### List of variables

**Table B.1. Alternative-Specific Variables**

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price ((w))</td>
<td>Continuous</td>
<td>The prices used in the Chinese questionnaire are integers ranging from 58 to 138, and the currency used is Yuan/RMB. The prices used in the Norwegian questionnaire are integers ranging from 149 to 249, and the currency used is Norwegian Krone/NOK. The ranges of the prices are chosen based on the local markets.</td>
</tr>
<tr>
<td>Choice Probability ((P))</td>
<td>Continuous</td>
<td>Choice probabilities represent the subscription rates of all available MNOs in individual’s peer group, ranging from 0% to 100% and summing up to 100% across MNOs.</td>
</tr>
</tbody>
</table>

**Table B.2. Individual-Specific Variables**

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Description/Transformation from original data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>Categorical/Dummy</td>
<td>(= 1), if the respondent is male, (= 0), if female.</td>
</tr>
<tr>
<td>Student</td>
<td>Categorical/Dummy</td>
<td>(= 1), if the respondent is a student, (= 0), if not.</td>
</tr>
<tr>
<td>Age</td>
<td>Continuous</td>
<td>(= 10), if the respondent is 15 or younger, (= 18), if 16 - 20, (= 23), if 21 - 25, (= 28), if 26 - 30, (= 35), if 31 - 40, (= 45), if 41 - 50, (= 65), if 51 or older.</td>
</tr>
<tr>
<td>Educational Level</td>
<td>Continuous</td>
<td>(= 10), if the respondent completes primary school, (= 13), if high school, (= 14), if vocational education (or equivalent), (= 16), if bachelor’s degree (or equivalent), (= 18), if master’s degree (or equivalent), (= 21), if Ph.D (or equivalent).</td>
</tr>
</tbody>
</table>
Notes:
1. All the individual specific variables are used in estimating based on Norwegian data, while only variable “Male” is used in the Chinese case. This is because the sample of Norwegian individuals include both students and non-students as well as different age groups, in contrast to the Chinese data which only include students in the beginning of their twenties.
2. “Age” is transformed from categorical data to continuous data by assigning the median of each category as numerical value of that category. “15 or younger” is transformed to 10, assuming individuals younger than 5 years old are not able to make their own decision on which MNO to subscribe. “51 or older” is transformed to 65, assuming the life expectancy is 80 years old in Norway.
3. “Educational Level” is transformed from categorical data to continuous data by assigning the corresponding years of schooling\(^5\).

Appendix C

Simulation results

1. Short term simulation

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>Short-term $P_I$</th>
<th>Short-term $P_I$ (without social interaction)</th>
<th>Short-term own elasticity</th>
<th>Short-term own elasticity (without social interaction)</th>
<th>Short-term cross elasticity</th>
<th>Short-term cross elasticity (without social interaction)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68</td>
<td>51</td>
<td>17</td>
<td>0.22380998</td>
<td>0.22380998</td>
<td>-1.6256524</td>
<td>-1.6256524</td>
<td>1.2192393</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
<td>66</td>
<td>54</td>
<td>0.04038274</td>
<td>0.04038274</td>
<td>-3.5467454</td>
<td>-3.3838255</td>
<td>1.9507100</td>
</tr>
<tr>
<td>3</td>
<td>107</td>
<td>131</td>
<td>-24</td>
<td>0.21635556</td>
<td>0.50479985</td>
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</tr>
<tr>
<td>4</td>
<td>66</td>
<td>137</td>
<td>-71</td>
<td>0.65941368</td>
<td>0.81257051</td>
<td>-0.6923439</td>
<td>-0.3810067</td>
<td>1.4371308</td>
</tr>
<tr>
<td>5</td>
<td>145</td>
<td>101</td>
<td>44</td>
<td>0.16490908</td>
<td>0.11153072</td>
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<td>-3.9679038</td>
<td>2.5978088</td>
</tr>
<tr>
<td>6</td>
<td>145</td>
<td>113</td>
<td>32</td>
<td>0.06550293</td>
<td>0.15373529</td>
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<td>-3.7794182</td>
<td>3.2524268</td>
</tr>
<tr>
<td>7</td>
<td>63</td>
<td>135</td>
<td>-72</td>
<td>0.56531384</td>
<td>0.81721625</td>
<td>-0.8434650</td>
<td>-0.3546736</td>
<td>1.8874250</td>
</tr>
<tr>
<td>8</td>
<td>134</td>
<td>78</td>
<td>56</td>
<td>0.09456072</td>
<td>0.07989165</td>
<td>-3.7369290</td>
<td>-3.7977684</td>
<td>2.1752273</td>
</tr>
<tr>
<td>9</td>
<td>97</td>
<td>117</td>
<td>-20</td>
<td>0.22161865</td>
<td>0.47402341</td>
<td>-2.3254921</td>
<td>-1.5714877</td>
<td>2.8049750</td>
</tr>
<tr>
<td>10</td>
<td>185</td>
<td>65</td>
<td>40</td>
<td>0.06047412</td>
<td>0.12433544</td>
<td>-3.0384267</td>
<td>-2.8318992</td>
<td>1.8890388</td>
</tr>
<tr>
<td>11</td>
<td>105</td>
<td>149</td>
<td>-44</td>
<td>0.35116266</td>
<td>0.65366761</td>
<td>-2.0934000</td>
<td>-1.1200390</td>
<td>2.9776443</td>
</tr>
<tr>
<td>12</td>
<td>74</td>
<td>79</td>
<td>-5</td>
<td>0.27111048</td>
<td>0.36216032</td>
<td>-1.6612850</td>
<td>-1.4537642</td>
<td>1.7735340</td>
</tr>
<tr>
<td>13</td>
<td>126</td>
<td>61</td>
<td>65</td>
<td>0.03316592</td>
<td>0.06168760</td>
<td>-3.7520897</td>
<td>-3.6414082</td>
<td>1.8164879</td>
</tr>
<tr>
<td>14</td>
<td>68</td>
<td>66</td>
<td>2</td>
<td>0.18828862</td>
<td>0.31397515</td>
<td>-1.8670003</td>
<td>-1.4361804</td>
<td>1.8126789</td>
</tr>
<tr>
<td>15</td>
<td>90</td>
<td>145</td>
<td>-55</td>
<td>0.46524520</td>
<td>0.72591606</td>
<td>-1.4823403</td>
<td>-0.7597607</td>
<td>2.3882149</td>
</tr>
<tr>
<td>16</td>
<td>136</td>
<td>130</td>
<td>6</td>
<td>0.26823948</td>
<td>0.28806510</td>
<td>-3.0619158</td>
<td>-2.9821529</td>
<td>2.9299691</td>
</tr>
<tr>
<td>17</td>
<td>148</td>
<td>148</td>
<td>0</td>
<td>0.20123735</td>
<td>0.32739298</td>
<td>-3.6411417</td>
<td>-3.0660118</td>
<td>3.6411417</td>
</tr>
<tr>
<td>18</td>
<td>72</td>
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2. Long term simulation figures

\( w_2 = 70 \)

\( w_2 = 90 \)

\( w_2 = 110 \)

\( w_2 = 130 \)

\( w_2 = 150 \)


$w_2 = 70$

$w_2 = 90$

$w_2 = 110$

$w_2 = 130$

$w_2 = 150$