Model simulations of large water waves due to landslides

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Abstract

A mathematical model based on the hydrodynamic shallow water equations is developed for numerical simulation of slide generated waves in fjords. The equations are solved numerically by a finite difference technique. Special algorithms are developed to handle the effect of depth changes due to the slide. To examine the performance of the numerical model we have simulated the slide catastrophe in Tafjord, western Norway, 1934. The predicted run-up heights are in good agreement with measured run-up heights. Amplification effects are estimated in run-up zones with gentle beach slopes. The model results reveal wave energy trapping due to the fjord geometry. This causes standing wave oscillations in accordance with the observations.
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1 Introduction

There are several historic records of the occurrence of large destructive water waves associated with rock slides and avalanches into fjords and lakes. Jørstad [10] gives a review of a large number of Norwegian events. The most well-known are referred in table 1.

Table 1: Data from Norwegian catastrophes caused by slide generated water waves.

<table>
<thead>
<tr>
<th>Place</th>
<th>Year</th>
<th>Max. height of run-up (m)</th>
<th>Slide volume Rocks ($m^3$)</th>
<th>Scree ($m^3$)</th>
<th>No. of perished</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loen, Norway</td>
<td>1905</td>
<td>40</td>
<td>$5 \cdot 10^4$</td>
<td>$3 \cdot 10^6$</td>
<td>61</td>
</tr>
<tr>
<td>Tafjord, Norway</td>
<td>1934</td>
<td>62</td>
<td>$1 \cdot 1.5 \cdot 10^6$</td>
<td>$1 \cdot 1.5 \cdot 10^6$</td>
<td>41</td>
</tr>
<tr>
<td>Loen, Norway</td>
<td>1936</td>
<td>74</td>
<td>$10^6$</td>
<td></td>
<td>73</td>
</tr>
</tbody>
</table>

Devastating slide generated waves have also occurred in Alaska, USA, and Pugh and Harris [22] review data from some events. In 1958 a rock slide of estimated volume $3.1 \cdot 10^7 m^3$ slid into Lituya Bay and generated a wave that stripped forest off 1100 m inland from the high tide line. According to Pugh and Harris the wave height in the bay was about 30 m. However, it is unclear whether this was observed directly or estimated from the run-up heights.

One of the most damaging reservoir disasters of all time occurred in 1963 at Vaiont dam in Italy. A rock slide in excess of $2.4 \cdot 10^6 m^3$ slid into the reservoir and generated a 100 m wave which overtopped the dam and destroyed a downstream located town killing about 2600 people.

To evaluate the risk for such disasters there have been several attempts to model slide generated waves both with laboratory models (Eie et al [3], Norwegian Hydrotechnical Laboratory [15] and Pugh and Harris [22]) and by model simulations based on numerical models (Gjevik and Pedersen [5], Harbitz [7, 8], Koutilas [13], Raney and Butler [23] and Vila [25]). Numerical simulations have proved to lead to realistic values of wave heights in a few test cases. However, an assessment of the performance of the models and a validation of the model predictions for historic events remains to be done.

Wave formation and propagation due to land slides and avalanches is a complex phenomena which may be divided in three parts: Energy transfer from slide motion to water motion, wave propagation in open water and wave run-up along the shores. From a modelling point of view the second part may be the easiest one, since the hydrodynamic equations may be directly applied. The physical processes involved in the first part is much more complex and no common model equations are available which can describe the motion of different slide materials as rock, clay, mud, ice and snow and the energy transfer mechanism between the slide and the fluid. The run-up part has
received relatively little attention in modelling slide generated waves, but run-up of water waves was discussed in general by Carrier [1], Carrier and Greenspan [2], Gjevik and Pedersen [4, 16, 17], Hall and Watts [6], Pedersen [18], Spielvogel [24] and Wiegel [27]. Similar problems in connection with tsunami run-up have been discussed extensively and are referred in the review paper by Voigt [26].

The aim of this paper is to simulate slide generated waves with a numerical model of a well documented Norwegian event, Tafjord 1934, analyse effects that occurred in this particular case and finally compare the results of the simulations with the observed data. Estimates of run-up heights are based on the numerical methods described by Gjevik and Pedersen [4] and Pedersen [18].

2 Hydrodynamic equations

2.1 The linearized shallow water equations

Waves generated by slides can often be classified as long waves. In other words most of the energy that is transferred from the slide to water motion is distributed on waves with typical wave-length, \( \ell \), which is much larger than the characteristic water depth, \( h_0 \). From the assumption \( h_0 / \ell \ll 1 \) it may be deduced that the pressure is approximately hydrostatic and that the vertical variations of the horizontal velocity are small. We will also assume that the characteristic amplitude of the waves, \( a \), is much less than \( h_0 \). On basis of these assumptions we may derive the linearized shallow water equations (see [20, 28]).

The equations are formulated in a Cartesian coordinate system with horizontal axes, \( O\hat{x} \) and \( O\hat{y} \) in the undisturbed water level and the vertical axis, \( O\hat{z} \), pointing upwards. The fluid is confined to \(-h < z < \eta\) where \( h \) is the depth referred to the datum \( z = 0 \), \( \eta \) the water surface displacement and we denote the total water depth by \( H = h + \eta \). Since the slide introduces bathymetric changes, \( h \) will be a function of time (\( t \)). Mass conservation in a vertical fluid column leads to a continuity equation of the form:

\[
\frac{\partial H}{\partial t} \equiv \frac{\partial \eta}{\partial t} + \frac{\partial h}{\partial t} = -\nabla \cdot \bar{Q}
\] (1)

where \( \bar{Q} \) is the vertically integrated volume flux density and

\[
\nabla \equiv i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y}
\] (2)

is the horizontal component of the gradient operator. In terms of the averaged horizontal velocity, \( \bar{u} = u \hat{i} + v \hat{j} \), the volume flux density can be approximated by:

\[
\bar{Q} = h \bar{u}
\] (3)
and by substitution in eq.(1)

\[ \frac{\partial H}{\partial t} = -\nabla \cdot (h \vec{u}) \]  

(4)

We note that while eq.(1) is an exact form of the continuity equation, the use of eq.(3) introduces relative errors of order \( a/h_0 \) in eq.(4). Provided the pressure is hydrostatic and the nonlinear terms can be neglected the momentum equation becomes:

\[ \frac{\partial \vec{u}}{\partial t} = -g \nabla \eta + \frac{\vec{f}}{\rho h} \]  

(5)

where \( g \) is the acceleration of gravity, \( \rho \) is the density of the fluid and \( \vec{f} \) is the shear stress at the interface between the water and the slide body. From the discussions in sec.4 it is established that \( \vec{f} \) is of minor importance in most cases and this term will therefore be omitted from here. Eq.(5) inherits relative errors of order \( a/h_0 \) as well as order \((h_0/t)^2\).

2.2 Numerical solution of the linearized shallow water equations

The numerical approximation to a parameter \( f \) at a grid-point with coordinates \((\beta \Delta x, \gamma \Delta y, \kappa \Delta t)\) where \( \Delta x, \Delta y \) and \( \Delta t \) are the grid increments, is denoted by \( f_{\beta,\gamma} \). In order to make the difference equations more readable we introduce the symmetric difference operator, \( \delta_x \), by:

\[ \delta_x f_{\beta,\gamma} = \frac{1}{\Delta x}(f_{\beta+\frac{1}{2},\gamma} - f_{\beta-\frac{1}{2},\gamma}) \]  

(6)

and the midpoint average operator, \( \bar{x} \), by:

\[ (f)_{\beta,\gamma} = \frac{1}{2}(f_{\beta-\frac{1}{2},\gamma} + f_{\beta+\frac{1}{2},\gamma}) \]  

(7)

Difference and average operators with respect to the other coordinates \( y \) and \( t \) are defined correspondingly. We note that all combinations of these operators are commutative. To abbreviate the expressions further we also group terms of identical indices inside square brackets, leaving the super- and subscripts outside the bracket.

The equations (4) and (5) are discretized on a grid that is staggered both in time and space. Fig.1 shows the spatial distribution of the nodes which is often referred to as the Arakawa C grid [14]. Velocities and surface elevations are evaluated at different values for \( t \). The discrete quantities are denoted by:

\[ \eta_{i,j}, u_{i+\frac{1}{2},j}, v_{i,j+\frac{1}{2}}, h_{i,j} \]  

(8)
A finite difference version of (4) reads:

$$[\delta_t \eta = -\delta_t h - \delta_x (\bar{h}^x u) - \delta_y (\bar{h}^y v)]_{i,j}^{n+\frac{1}{2}}$$  \hspace{1cm} (9)$$

Equation (9) assures volume conservation.

Discretization of the components of the momentum equation, (5), (with \(\vec{r}\) omitted) gives:

$$[\delta_t u = -g \delta_x \eta]_{i+\frac{1}{2},j}^n \hspace{1cm} (10)$$

$$[\delta_t v = -g \delta_y \eta]_{i,j+\frac{1}{2}}^n \hspace{1cm} (11)$$

Equation (9) through (11) defines an explicit finite difference method of second order accuracy for which \(\vec{u}\) and \(\eta\) are evaluated at adjacent half time steps.

A rigid impermeable wall is represented by a sequence of line segments parallel to the axes. Each segment passes through nodes for the corresponding normal velocity component which is set to zero. This representation is accurate only to the first order in the grid increments and may in some cases give rise to spurious trapping effects (see [19]). Still, such boundaries are extensively used in models of the ocean and lakes because of their simplicity. A slide penetrating the water surface at a shore gives a non-zero normal flux at the boundary. Additional complications will arise from the fact that the shore line under such circumstances must be regarded as time dependent.

A grid cell is defined as the volume element circumvented by straight lines normal to the velocity directions in four velocity points around one point of surface elevation, fig.1. When the motion of the slide causes the depth in a grid cell to become negative (i.e. \(h_{i,j}^{n+1} < 0\)), the shore line is moved to the seaside of this cell. This is accomplished by setting the velocities along the line segments representing the new shore line equal to zero. The last calculated fluid volume in the cell, defined by

$$V_{i,j}^n = \Delta x \Delta y (\eta + h)_{i,j}^n \hspace{1cm} (12)$$

is spread equally over cells which have boundaries in common with the one drained, and still have a positive depth (i.e. \(h_{i,j}^{n+1} > 0\)), fig.2. Thus the total fluid volume is kept constant.

The time increment \(\Delta t\) is determined by the Courant Friedrich Levy (CFL) stability criterion

$$\Delta t \leq \left(gh_{\text{max}}(1/\Delta x^2 + 1/\Delta y^2)\right)^{-\frac{1}{2}} \hspace{1cm} (13)$$

If the slide velocity is so large that the slide moves more than one grid distance per time step, a reduced time increment is used during the slide event to avoid this. Reducing the time step this way do not introduce any significant numerical damping.
3 The slide model

Energy transfer from the slide mass to the water is clearly a very complicated process which is impossible to model in detail. Firstly the composition of the slides may vary over a wide range from large blocks to fine particles. In the former case water will flow between the blocks and each component will experience a resistance from viscous drag, form drag and added mass. The slide will also lose energy because of collisions and friction between the slide particles and because of the bottom friction. Secondly the slide characteristics may change considerably during the slide process; blocks may be crushed, mass may be released or deposited along the sea bed and water may be admitted into the total slide mass.

The process of water wave generation by slides may be controlled by the global characteristics of the slide. We shall therefore focus only on two important large scale energy transfer mechanisms; the total water displacement and the shear stress acting between the slide masses and the fluid. These effects are easily parameterized by the shallow water equations.

The total water displacement is determined by the aggregated displacement thickness of the slide. The slide will therefore be described as one body with a prescribed motion. This corresponds to a time dependent water depth

\[ h(x, y, t) = h_0(x, y) - h_s(x - x_s(t), y - y_s(t)) \]

where \( h_0(x, y) \) represents the rigid sea floor, and \( h_s \) describes the water displacement by the slide body. The coordinates \( (x_s(t), y_s(t)) \) defined by

\[
\begin{align*}
  x_s &= x_0 + (R \sin \frac{\pi t}{T}) \cos \varphi \quad (0 < t < T) \\
  y_s &= y_0 + (R \sin \frac{\pi t}{T}) \sin \varphi \\
  x_s &= x_0 + R \cos \varphi \quad (t \geq T) \\
  y_s &= y_0 + R \sin \varphi
\end{align*}
\]

specify the motion of the slide and \( \varphi \) is the angle between the propagation direction of the slide and the \( x \)-axis. \( (x_0, y_0) \) is the position of the front of the slide at \( t = 0.0 \) s. \( R \) is the total horizontal displacement during the time interval \( T \). We shall refer to \( R \) as the retardation distance and to \( T \) as the running time of the slide. The velocity of the slide at the time it starts to penetrate the water surface \( (t = 0.0 \) s) is \( U_0 \) and from eq.(15) we have

\[ T = \frac{\pi R}{2 U_0} \]

The velocity \( U_0 \) may be determined from the "Perla slide model" [21] based on estimates of slide and terrain parameters. \( R \) will depend on the maximum kinetic energy of the slide, \( E_s = \frac{1}{2} M_s U_0^2 \), where \( M_s \) is the mass of the
slide. However, this functional relationship is unknown. An estimate of $R$ is therefore found simply by assuming that the slide will stop at a well defined breaking point along the depth profile. We introduce the energy ratio

$$R_E = \frac{E_I}{E_s}$$  \hspace{1cm} (17)

where $E_I$ is the total mechanical energy in the fluid motion. $R_E$ represents the fraction of slide energy that is transferred to fluid motion.

The shape of the slide is represented by a box form of length $L$, width $B$ and maximum thickness $\Delta h$. To avoid sharp gradients in $h$, the edges of the box form is smoothed over a distance equal to $B$ along both sides and in the front by an exponential function of the form

$$h_s = \begin{cases} 
\Delta h \exp\left(-\left(\frac{2z}{B}\right)^4\right) & -(L + B) \leq x' < -B \\
\Delta h \exp\left(-\left(\frac{2y + B}{B}\right)^4 - \left(\frac{2z}{B}\right)^4\right) & -B \leq x' < 0 
\end{cases} \hspace{1cm} (18)
$$

where

$$x' = (x - x_s) \cos \varphi + (y - y_s) \sin \varphi$$
$$y' = -(x - x_s) \sin \varphi + (y - y_s) \cos \varphi$$

The $x'$-axis is directed along the direction of the slide motion, and the $y'$-axis in the transverse direction, with the origin in the front of the slide, fig.3.

The width of the slide, $B$, constitutes the width of that part of the box which is thicker than $0.37 \cdot \Delta h$. With this definition of $h_s$ the slide volume $V$ is

$$V = 0.90B\Delta h(L + 0.46B) \hspace{1cm} (19)$$

where the factors 0.90 and 0.46 arises due to the smoothening. The actual shape of the slide is sketched in fig.3. To ensure that the whole slide volume enters the water, we will choose $R = L + B$.

The slide model above may also be used for slides released under water. In this case eq.(15) is modified to

$$x_s = x_0 + \left\{ \frac{1}{2} R (1 - \cos \frac{n_l}{T}) \right\} \cos \varphi$$
$$y_s = y_0 + \left\{ \frac{1}{2} R (1 - \cos \frac{n_l}{T}) \right\} \sin \varphi$$
$$x_s = x_0 + R \cos \varphi$$
$$y_s = y_0 + R \sin \varphi$$

$$0 < t < T$$
$$t \geq T \hspace{1cm} (20)$$

to allow for zero initial velocity of the slide mass. The volume, eq.(19), will be increased due to smoothening in the rear end as well.

The shear stress acting on the water by the slide mass is assumed to be directed along the direction of propagation (given by $\varphi$) and is expressed by

$$\tau_\varphi = \frac{1}{2} C_D \rho \left| v_s - u_s \right| (v_s - u_s) \hspace{1cm} (21)$$
where \( v_\bullet = \sqrt{x'_2 + y'_2} \) is the slide velocity (the dot denotes differentiating with respect to \( t \)), \( u_\bullet \) is the component of the fluid velocity along the direction of propagation and \( C_D \) is the drag coefficient. For most slides we have \( |v_\bullet| >> |u_\bullet| \) and the fluid velocity may be neglected in eq.(21).

4 A simple, analytical solution for wave excitation by slides

An analytical solution for a two-dimensional, idealized case is presented in order to study the influence of the governing parameters. We also obtain an expression for the relative importance of the effect of volume displacement versus the effect of shear stress on the interface between the fluid and the slide masses. The slide is assumed to maintain its initial shape and to move in the positive \( x \)-direction upon a horizontal sea-bed. It is at rest for \( t \leq 0 \), moves with constant velocity, \( U \), for \( 0 < t < T \), and stops instantaneously at \( t = T \). The length of the slide is \( L \), the height from the sea bed to the undisturbed free surface is denoted by \( h_0 \) and the height of the slide, \( h_s \), is assumed small when compared to \( h_0 \). A definition sketch of the slide is depicted in fig.4. Although the present slide model is extremely simple it still contains the essential parameters and the results obtained with this model may apply to realistic slide events as well.

Under the assumptions listed above the set of equations (4), (5) is easily solved by integration along characteristics or by combining free and forced wave solutions. For \( t < T \) the solution consist of free waves, with phase speed \( \pm c_0 = \pm \sqrt{g h_0} \) and a forced wave with speed \( U \). The free and forced parts of the total solution are related by the initial conditions \( \eta(x, 0) = u(x, 0) = 0 \). At \( t = T \) the forced solution gives rise to another family of free waves which are determined by patching \( u \) and \( \eta \). As a result, the wave system that propagates in the positive \( x \)-direction for \( t > T \), can be described as the super position of two families of free waves which are phase shifted a distance \( x_f = |c_0 - U|T \). The leading family corresponds to a surface elevation, whereas the trailing one corresponds to a depression. We may recognize two contributions to the surface displacement, \( \eta^r \) and \( \eta^d \), due to the shear stress and the volume displacement respectively. We omit the details in the calculations and present only the right going solution for \( t > T \):

\[
\eta^d = \frac{F_r}{2|1 - F_r|} (h_s(x - b - c_0 t) - h_s(x - b + x_f - c_0 t)) \tag{22}
\]
\[
\eta^r = \frac{1}{2\rho c_0^2 |1 - F_r|} (\gamma(x - b - c_0 t) - \gamma(x - b + x_f - c_0 t)) \tag{23}
\]

where \( b = \max((U - c_0)T, 0) \) and the Froude number \( F_r = U/c_0 \). \( \gamma \) is a piecewise linear function of \( x \) defined as:
\[
\gamma(x) = \begin{cases} 
0 & x_s < x \\
\tau(x_s - x) & x_s - L < x < x_s \\
\tau L & x < x_s - L 
\end{cases}
\]  
(24)

where \(x_s\) is the position of the front of the slide at \(t = 0\). The solution is sketched in fig.5. For small \(x_f\) the solution (22) may be approximated by:

\[
\eta^d \approx -\frac{UT}{2} \frac{dh_s}{dx}(x - c_0 t) 
\]  
(25)

whereas \(\eta^\tau\) has a constant value \(\tau T/2\rho c_0\) for \(x_s - L < x < x_s - x_f\). When \(F_r \to 1\) we have \(x_f \to 0\) and eq.(25) is valid exactly. We note that \(\eta^\tau\) is discontinuous for this case.

Assuming \(dh_s/dx \approx -2\Delta h/L\), where \(\Delta h\) is the maximum value of \(h_s\), we obtain from eq.(25) and eq. (24):

\[
\eta^d_{\text{max}} \approx \frac{UT}{L} \Delta h 
\]  
(26)

\[
\eta^\tau_{\text{max}} \approx \frac{\tau T}{2\rho c_0} 
\]  
(27)

For the other limiting case, \(x_f > L\), we have

\[
\eta^d_{\text{max}} \approx \frac{F_r}{2|1 - F_r|} \Delta h = \frac{UT}{2x_f} \Delta h 
\]  
(28)

\[
\eta^\tau_{\text{max}} \approx \frac{\tau L}{2\rho c_0^2 |1 - F_r|} 
\]  
(29)

Substituting for \(\tau\) the expression for \(\tau_{\varphi}\), eq.(21), with \(u_s = 0.0 \text{ m/s}\) and \(\varphi = 0.0\), we may in the latter case define a number:

\[
\kappa = \frac{\eta^\tau_{\text{max}}}{\eta^d_{\text{max}}} = C_D \frac{F_r L}{2\Delta h} 
\]  
(30)

which is a measure of the relative importance of displacement and shear stress. For small \(x_f\), we find that the corresponding ratio is \(1/2\kappa\). We note that the expression for \(\kappa\) is consistent with the natural assumption that a large horizontal extension of the slide favours the shear stress effects while the displacement becomes dominant for high slides. For slide events the travelling distance \(UT = R\) can often be estimated. For a fixed \(R\) the expressions for \(\eta_{\text{max}}^d\) indicates that for nearly critical Froude numbers the largest \(\eta^d\) will occur if \(L \ll R\).
5 The slide catastrophe in Tafjord, April 7th 1934

5.1 Historical background

The slide catastrophe in Tafjord in 1934 has been chosen for a complete analysis of a real event, mainly because a relatively large number of measured run-up heights is available, fig.6.

The Tafjord is the inner part of the Norddalsfjord which is a branch of the Storfjord at Sunnmøre, western Norway. The length of the Tafjord is about 9 km from the innermost village Tafjord to the village Fjøra. The depth is about 200 – 220 m and the average width a little more than 1 km. There is no sill in the bend between Fjøra and Sylte. Depth contour lines are depicted in fig.7.

The position of the slide area is shown in fig.6. The upper part of the slide was released from about 730 m above the fjord. Before the slide, the main rock reposed on a plane with an inclination varying from 35° on top to 45° by the fjord. The rock slide, consisting of gneiss, constituted a volume of 1 – 1.5 mill. m³ with an estimated mass of 4 – 5 mill. tons. Beneath the rock a moraine scree of at least the same volume (Heggura) was released.

According to the eye witnesses the slide produced three separated huge waves, with 3-5 minutes in between. The occurrence of three huge waves has previously been associated with the way the rock fell down [12]. However, too many slide events are known in which three or more successive waves have been reported to permit such a specific explanation (confer Lyngen ca. 1830, Skafjell, Stranda 1938 [10]). The occurrence of several huge waves in the Tafjord will in this paper definitely be linked to wave reflections in the fjord, sec.7,8.

The sea was very rough for more than half an hour, and did not become calm until 7-8 hours after the slide occurred. The measured run-up heights revealed large local variations, fig.6. 41 people living in the villages Fjøra and Tafjord were killed and there were severe damages along the shore line.

The details about the slide and the waves given here are collected from the references [9, 10, 11, 12]. The information is verified against contemporary newspaper articles and field studies.

5.2 The simulated wave pattern

The coordinate system is oriented with the x-axis pointing to the east and the y-axis to the north, fig.7. Estimated values of the slide parameters, henceforth referred to as the standard model run values, are listed in table 2. The values are based on the information given in [11, 12], on maps [29] on the scale 1:50.000 and on bathymetric charts on the scale 1:20.000 for the
Table 2: Estimated slide parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U_0$</td>
<td>velocity of the slide at $t = 0.0$ s</td>
</tr>
<tr>
<td>$L$</td>
<td>length of the slide</td>
</tr>
<tr>
<td>$B$</td>
<td>width of the slide</td>
</tr>
<tr>
<td>$\Delta h$</td>
<td>height of the slide</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>direction of the slide motion relative to the $x$-axis</td>
</tr>
</tbody>
</table>

depth profile outside the slide area, fig.31.

The model domain constitutes $290 \times 158$ grid cells, which covers $14.5 \ km \times 7.9 \ km$ ($114.55 \ km^2$). The grid increments are $\Delta x = \Delta y = 50.0 \ m$. The depth in each grid point has been computed by interpolation between depth contour lines supplemented with depth sounding data.

The simulated wave pattern with the model parameters listed above is shown in fig.8-fig.14. The sea surface displacement introduced by the slide appears at $t = 10.0 \ s$ after the release of the slide as a semi circular disturbance outside the slide area and the maximum surface elevation is about 45 m in front of the slide, fig.8 and fig.11. At $t = 30.0 \ s$, fig.9 and fig.12, the primary wave has reached the opposite side of the fjord and the wave height outside the shore is about 25 m. At $t = 75.0 \ s$, fig.10 and fig.13, the disturbance has spread out and a primary wave with amplitude $2 - 3 \ m$ is propagating out the fjord and approaching the fjord bend at Fjøra. Another primary wave with amplitude of about $3 \ m$ is propagating towards the fjord head. Between these two primary waves the sea surface displacement form a complex pattern due to multiple reflections at the shore. At $t = 120.0 \ s$, fig.14, the primary wave is diffracted in the bend and the wave pattern in the inner part of the fjord will now be even more complex due to reflections also in the fjord bend, see sec.7.

5.3 Comparison with measurements, standard model run.

The modelled wave heights in grid points nearest to the $10.0 \ m$ depth contour line are taken as a measure of the wave heights at the shore. We shall refer to this wave height as $\eta_{10}$. By choosing a minimum depth of $10.0 \ m$, we have excluded grid points situated very close to the shore line where the water depth may be zero during back wash of the waves. $\eta_{10}$ will represent a good estimate of the run-up heights where the bottom slope is steep. On the other hand, if the bottom slope is gentle, nonlinear effects will be important for run-up computations. In these cases estimates of the run-up heights are obtained by multiplying $\eta_{10}$ by an amplification factor, see sec.9.

A comparison between $\eta_{10}$ and measured run-up heights are given in fig.15. The high frequency noise on the $\eta_{10}$ graph is partly due to the grid
representation of the shore line, and partly due to local physical effects. On the north side of the fjord there is good agreement between \( \eta_{10} \) and measured run-up heights in Fjøra, while \( \eta_{10} \) is smaller than the observed run-up height in Muri, Sylte and Muldal. \( \eta_{10} \) is large around the slide area as expected. The main rock of the slide was wedge-shaped with the tip pointing downwards. The peculiar shape of the rock might have caused a special way of falling. This fact has given birth to probably correct explanations of why a maximum run-up height of 62.3 m was found 200 m beside the slide area, not on the opposite side as expected. These local effects are of minor importance some distance away and a proper description of such details are of course beyond the scope of this model. Nevertheless local effects are probably responsible for the extreme run-up heights traced in the slide area.

On the south side of the fjord \( \eta_{10} \) is particularly smaller than the measured run-up heights between Oksneset and Gjeitvika and near Sødalsvik, where amplification effects due to gentle beach slopes may be important.

6 Parameter dependence

To study the relative importance of the parameters governing the volume, the shape and the movement of the slide, the values of these will be varied systematically throughout nine different model setups defined in tables 3, 4. The results are visualized by time series from 8 points along the center line of the fjord, fig.16, and by contour plots of the wave pattern outside the slide area. Comparisons are made with results of the standard model run.

6.1 Dependence on the initial velocity of the slide

To study the effect of the initial velocity of the slide, \( U_0 \) was first reduced to 30.0 m/s (case no. 1), fig.17. Compared with the standard model run (case no. 2), the time series show the same characteristics. However, the wave heights are slightly reduced, and the first wave crests are delayed 5-10 s. Correspondingly, for \( U_0 = 70.0 \text{ m/s} \) (case no. 3), the wave heights are slightly increased and the signals accelerated, fig.18. This is in accordance with the analytical results, eq.(25). It is clear from table 3 that if \( U_0 \) is reduced to 30.0 m/s while the retardation distance \( R \) is kept constant, the energy ratio \( R_E \) will be very large. Conversely will \( U_0 = 70.0 \text{ m/s} \) (case no. 3) give a very low energy ratio.

6.2 Dependence on the length, the width and the height of the slide

The dependence on length, width and height is investigated by varying the parameters in seven case studies, table 4. The results, fig.19-20, confirm that
Table 3: Energy ratio $R_E$ for varying $U_0$; $L = 400.0 \, m$, $R = 530.0 \, m$, $\Delta h = 75.0 \, m\, (t = 30.0 \, s)$.

<table>
<thead>
<tr>
<th>Case no.</th>
<th>$U_0$ ($m/s$)</th>
<th>$T$ (s)</th>
<th>$R_E$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30.0</td>
<td>27.8</td>
<td>71.8</td>
</tr>
<tr>
<td>2</td>
<td>50.0</td>
<td>16.7</td>
<td>28.4</td>
</tr>
<tr>
<td>3</td>
<td>70.0</td>
<td>11.9</td>
<td>14.6</td>
</tr>
</tbody>
</table>

Table 4: Energy ratio $R_E$ for varying $L$, $B$ and $\Delta h$; $U_0 = 50.0 \, m/s\, (t = 25.0 \, s)$.

<table>
<thead>
<tr>
<th>Case no.</th>
<th>$L$ ($m$)</th>
<th>$B$ ($m$)</th>
<th>$\Delta h$ ($m$)</th>
<th>$R = L + B$ ($m$)</th>
<th>$T$ (s)</th>
<th>$R_E$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>275</td>
<td>130</td>
<td>75</td>
<td>405</td>
<td>12.7</td>
<td>22.9</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
<td>130</td>
<td>75</td>
<td>530</td>
<td>16.7</td>
<td>28.6</td>
</tr>
<tr>
<td>5</td>
<td>400</td>
<td>130</td>
<td>52</td>
<td>530</td>
<td>16.7</td>
<td>29.4</td>
</tr>
<tr>
<td>6</td>
<td>500</td>
<td>130</td>
<td>75</td>
<td>630</td>
<td>19.8</td>
<td>31.7</td>
</tr>
<tr>
<td>7</td>
<td>500</td>
<td>130</td>
<td>60</td>
<td>630</td>
<td>19.8</td>
<td>32.2</td>
</tr>
<tr>
<td>8</td>
<td>500</td>
<td>150</td>
<td>52</td>
<td>650</td>
<td>20.4</td>
<td>33.1</td>
</tr>
<tr>
<td>9</td>
<td>600</td>
<td>130</td>
<td>75</td>
<td>730</td>
<td>22.9</td>
<td>33.2</td>
</tr>
</tbody>
</table>

the modelled wave heights depend on the volume of the slide rather than the shape. The volume is reduced by approximately 30% by reducing $L$ (case no. 4) and $\Delta h$ (case no. 5) respectively. In both cases the height of the first wave crests is reduced by 20–30%. Changing $L$, $B$ and $\Delta h$ simultaneously (case no. 8) while keeping the volume constant, does not have substantial influence on the wave heights, fig.21.

From table 4 we see that the energy ratio again provides additional information: $R_E$ increases when $L$ (and thereby $R$) increases, cases no. (4,2,6,9). This shows that the increase in the kinetic energy of the slide due to increased volume, is exceeded by the increase of wave energy in the fjord due to the elongated retardation distance. Keeping $L$ constant and reducing $\Delta h$ increases the energy ratio further, cases no. (2,5) or (6,7).

### 6.3 Dependence on the direction of the slide motion

The direction of the slide motion influences the distribution of wave energy in the fjord. As an example the slide motion is now directed to the south,
(i.e. $\varphi = 270.0^\circ$) as opposed to the southwest direction used in the standard model run. A larger part of the wave energy is now transmitted inwards in the fjord, fig.22. In points 1-5, fig.16, situated on the outside of the slide area, the wave heights are reduced, while in points 6-8 situated on the inside, the wave heights are increased. Wave heights may also be slightly reduced because a small part of the model slide does not enter the water when the direction of the slide motion is not perpendicular to the shore line.

The time of arrival of the waves is affected in the same way. Because the wave generation area is now shifted inwards in the fjord, the waves propagating outwards will be delayed, while the waves propagating inwards appear earlier.

Fig.23-24 show the wave pattern outside the slide area at $t = 5.0, 10.0, 15.0$ s for both the standard model run and the run with a southward direction of the slide motion. As expected, the wave patterns are very similar, except for minor differences due to reflections along the shore line in the wave generation area.

The time series from point 5, fig.16, might give the faulty impression that the wave heights are significantly reduced when the slide motion is directed to the south. This is because point 5 is now situated outside the propagation line of the slide, and does not measure the maximum height of the primary wave.

7 Reflection of wave energy in the fjord bend

At several locations there were traces of only one wave, meaning that the last significant wave was the largest one. This is in agreement with the results of the model simulations, which indicate that maximum wave height in the eastern part of the fjord occurred later than the time of arrival of the primary wave, fig.25. The occurrence of several huge waves with increasing height and the long duration of wave motion after the slide event, suggest that the damping is low and that the wave energy is trapped in the inner part of the fjord. In view of the geometry of the fjord, fig.6, it is reasonable to link the energy trapping to the bend near Fjøra.

The reflection and transmission in the bend is examined by computing the mechanical energy in the fjord basin inside and outside the bend (confer fig.26) at $t = 150.0$ s. At this time the outward propagating primary wave has passed the bend. A reflection coefficient of the fjord bend is defined as

$$C_r = \frac{E_o - E_w}{E_o}$$

where $E_o$ is the energy in the waves approaching the bend and $E_w$ is the energy in the western part of the fjord. $C_r$ represents the fraction of this
energy that is left in the eastern part of the fjord, i.e. inside the bend, after reflection.

In the standard model run the wave energy is almost totally reflected in the fjord bend with a reflection coefficient of $C_r = 98.6\%$. The trapping of energy is also verified by the time series from the standard model run, fig.17-22. In all cases the wave height is significantly reduced between point 2 in the bend and point 1 outside the bend.

In order to examine the surprisingly strong reflection at the fjord bend further, we have designed three experiments with different initial disturbances imposed outside the slide area, all with a maximum height of 30.0 m, fig.26a,b,c respectively. The first one corresponds to a disturbance with the wave crest extending across the fjord. This will lead to an outward propagating wave with small crosswise variation. The second one is box formed and include both crosswise and lengthwise wave components. The third one has a similar shape as the second one, but covers a smaller area and is more like the wave form produced by the standard model run. All imposed wave forms are smoothed by the same exponential function used for the box form of the slide. The extension of the initial disturbance in the crosswise and lengthwise direction of the fjord is $\frac{1}{2}B_c$ and $B_l$ respectively. $B_c$ and $B_l$ are defined in correspondence with the width of the slide, $B$, defined in sec.3. The values of $B_c$, $B_l$ and $C_r$ for the three different cases are given in table 5. The imposed wave forms are symmetric about a cross section of the fjord. We therefore assume that approximately one half of the initial wave energy is propagating towards the bend.

All three cases visualize the dependence of $C_r$ on the extension of the initial disturbance. In case no. 1 only 33.4 % of the energy approaching the bend is reflected. By decreasing $B_c$ (case no. 2), we notice how $C_r$ increases to 66.2 % when the imposed wave is released closer to the slide area. By decreasing $B_c$ and $B_l$ further (case no. 3), $C_r$ increases to 93.2 %. For $t = 300.0 \text{ s}$ we find $C_r = 92.1\%$ in case no. 3. This confirms that almost all the energy leakage in the bend is connected to the primary wave. The more the extension of the initial wave is reduced, i.e. the closer we come to a slide generated wave, the higher is the value of $C_r$. This is because only a small part of the energy in waves of initially small extension is transferred to waves propagating along the fjord. This verifies that the main part of the energy in slide generated waves will not escape. The reflection at the bend explains the observations of at least three separated huge waves, and why there was rough sea for more than half an hour.

The time series from the standard model run show that in the eastern part of the fjord, (points 7 and 8), the wave height of the first wave is not the highest one. This also indicates energy trapping and amplification which is supported by observations from the village Tafjord, saying that the third wave was the highest. For $U_0 = 30.0 \text{ m/s}$ (case no. 1, table 3) there is an exception. This is because the height of the primary wave will be unrealistically large
Table 5: Coefficients of reflection, $C_r$, for the standard model run and three initial surface displacements. $B_1$, $B_c$: Lengthwise and crosswise extension of initial surface displacements.

<table>
<thead>
<tr>
<th>Orientation of initial surface displacement</th>
<th>Case no.</th>
<th>$B_1$ (m)</th>
<th>$B_c$ (m)</th>
<th>$t$ (s)</th>
<th>$C_r$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Crosswise</td>
<td>1</td>
<td>1000.0</td>
<td>across</td>
<td>150.0</td>
<td>33.4</td>
</tr>
<tr>
<td>Lengthwise/crosswise, large</td>
<td>2</td>
<td>1000.0</td>
<td>1000.0</td>
<td>150.0</td>
<td>66.2</td>
</tr>
<tr>
<td>Lengthwise/crosswise, small</td>
<td>3</td>
<td>250.0</td>
<td>250.0</td>
<td>150.0</td>
<td>93.2</td>
</tr>
<tr>
<td>Standard model run:</td>
<td></td>
<td></td>
<td></td>
<td>150.0</td>
<td>98.7</td>
</tr>
</tbody>
</table>

when $U_0$ is reduced to 30.0 m/s while $R$ are kept constant, confer table 3. By reducing $U_0$ and $R$ simultaneously, the time series again reveal that later waves exceeds the primary one.

8 Standing waves - time series analysis

In sec. 7 it was shown that the eastern part of the fjord may be considered as an almost closed basin due to the strong effect of wave reflection in the bend. Trains of huge waves may therefore be explained as crosswise and lengthwise standing waves. In order to document this further, we have compared the periods of eigen oscillations in the fjord basin with the periods found by spectral analysis of the simulated time series.

8.1 Spectral analysis

We will first examine time series from the standard model run, given in fig. 17, for points 2-8 located as shown in fig. 16. The resulting power spectra are shown in fig. 28. Lengthwise eigen oscillations with $n$ nodal points are sought for $n = 1 - 5$. For higher values of $n$, the modes are hard to separate from crosswise eigen oscillations. Estimates of lengthwise eigen periods are calculated from Merian's formula $T_n^l = 2L/(n\sqrt{gh_m})$ where $L$ is the length of the fjord inside the bend and $h_m$ is the mean depth, see table 6.

The time series are cut after $t = 360.0$ s in order to avoid that reflections, however small, from the western boundary of the model domain affect the time series from points in the outer part of the fjord. The time series are also truncated in the beginning, by removing all the zero values until the first disturbance occurs.

Thus the time series from points 2,7,8 far from the slide area are too short to reveal the period $T_1^l$ of the basic longitudinal mode of the basin. In addition, this mode may not reach any significant height in points 4,5,6.
which are situated close to the nodal point. This may be the reason why a spectral peak corresponding to a basic lengthwise mode is found only in point 3. Moreover lengthwise modes will be damped faster than crosswise modes due to less reflection in the bend, as shown in sec. 7. (One should be aware that the detected basic period in point 3 is equal to the length of the analysed time series, and does therefore not necessarily reveal any physical period).

Spectral peaks corresponding to $T_f$ are found in points 2, 5 and 6. This eigen mode should have a nodal point between point 3 and 4, and another around point 6. The spectral peak in point 6 probably occurs because the wave is not an exact standing wave. Spectral peaks corresponding to $T_f$ are found in points 3 and 6, to $T_f$ in points 2, 3, 4 and 7 and to $T_f$ in points 2, 4, 7 and 8.

Crosswise eigen modes with $m$ nodal points are sought for $m = 1, 2$. Higher values of $m$ will correspond to waves too short to fulfill the shallow water approximation.

Crosswise eigen modes with nodal points on the mid line of the fjord will not be revealed by the time series from the eight points introduced so far. Therefore eight new points located outside the mid line, fig. 27, are considered. The power spectra for these additional points are shown in fig. 29. Crosswise eigen periods $T_c$, table 7, are estimated for seven cross sections (i-vii) of the fjord, located as shown in fig. 27.

Spectral peaks corresponding to $T_c$ are found in points 2, (3, 9, 10), (4, 11, 12), (5, 13) and (15, 16). The crosswise mode appears in points (2, 3, 4, 5) probably because the nodal points are not situated exactly here, even though these points are close to the mid line.

We also find spectral peaks corresponding to $T_c$ in all points along the seven sections.

We therefore conclude that both lengthwise and crosswise eigen modes constitute an important part of the slide induced wave motion in the fjord.

8.2 Effects of interference

Sections 7 and 8.1 have confirmed that the special geometry of the Tafjord causes energy trapping and standing waves in the eastern part of the fjord. In view of the repeated reflections we may expect the waves to interfere. In
Table 7: Period of crosswise eigen oscillations.

<table>
<thead>
<tr>
<th>Cross section</th>
<th>Mean depth (m)</th>
<th>Width (m)</th>
<th>$T_{1}^{c}$ (s)</th>
<th>$T_{2}^{c}$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i</td>
<td>200</td>
<td>600</td>
<td>27</td>
<td>14</td>
</tr>
<tr>
<td>ii</td>
<td>200</td>
<td>1200</td>
<td>54</td>
<td>27</td>
</tr>
<tr>
<td>iii</td>
<td>200</td>
<td>1000</td>
<td>45</td>
<td>23</td>
</tr>
<tr>
<td>iv</td>
<td>170</td>
<td>1000</td>
<td>49</td>
<td>24</td>
</tr>
<tr>
<td>v</td>
<td>190</td>
<td>1000</td>
<td>46</td>
<td>23</td>
</tr>
<tr>
<td>vi</td>
<td>80</td>
<td>350</td>
<td>25</td>
<td>13</td>
</tr>
<tr>
<td>vii</td>
<td>70</td>
<td>500</td>
<td>38</td>
<td>19</td>
</tr>
</tbody>
</table>

addition to varying run-up conditions along the shore line, this may explain the large observed local variations in run-up height. The rapid change of the time when maximum wave height occurred along the shore line, fig.25, proves that the traces of the highest wave were not set by the same wave. This is a further indication of interference phenomena.

8.3 Comments to the numerical computations.

Most eye witnesses described that three huge waves occurred. On the other hand the simulated time series reveal more than three huge waves. This is most likely because energy loss due to wave breaking and imperfect reflection along the shore line is neglected in the model. These effects would also smooth the time series by first damping the shorter waves. In addition the eye witness descriptions differ, and the exact number of three huge waves may be called into question.

Spectral analysis reveal significant peaks corresponding to periods varying between $T_{1} = 315\ s$ and $min[T_{2}] = 13\ s$. The shortest crosswise wave modes do not fulfil the assumption $h_0/\ell \ll 1$ given in sec.2.1. However, all wave modes will travel short distances between each reflection. Errors due to uncorrect reflection conditions may therefore be much larger than errors introduced by neglecting dispersive effects.

Wave modes with periods less than ca. 10 s are found in the spectra from almost all 16 points. These are mainly caused by numerical effects introduced by the coarse grid representation of the shore line, and may not represent waves of any physical interest.

The linearization assumption $a/h_0 \ll 1$ is fulfilled except just outside the slide area and in the run-up zones where nonlinear effects may occur, see sec.9.

Internal waves are most likely of minor interest because the stratification in the fjords is unsignificant at this time of the year. However, if internal
waves really occur, they may cause large current variations, but the surface displacement associated with these waves are very small. Stratification may also affect the energy transfer between the slide and the water.

9 Estimated run-up heights

Our model based on linear equations and no flux conditions along the steep boundaries is not valid for gentle beach slopes where the wave approaching the shore line will amplify and wash up the beach slope.

By the methods described in sec.A, the values of \( \eta_{10} \) (defined in sec.5), will be used to estimate the run-up height at seven locations (1-7) with gentle beach slopes: Sylte, Fjøra, Sødalsvik, Tafjord, Seineset, south of Korsnes and south of Oksneset, fig.30. The depth profiles along the indicated lines outside these points are given in fig.31. Between the breaking point in the bottom profile (marked with * on each depth profile) and the shore line, the actual bottom slope is replaced by a linear one as described in sec.A.4.

The maximum run-up heights \( \eta_{10}/A \) and \( R_u/A \), where \( A \) is the height of the incident wave and \( R_u \) is the run-up height along a gentle beach slope, are both functions of the inclination angle of the slope, \( \theta \), the period of the incident wave, \( T^i \), and the water depth at the lower end of the slope, \( h_0 \). \( \eta_{10}/A \) also depends on the depth \( d \) in the grid points where \( \eta_{10} \) is read. The effect of an oblique angle of incidence has proved to be negligible along the profiles where this might occur. All values are given in table 8. The values of \( T^i \) for the seven profiles are found from the spectra of the points 1,2,8,7,15,13,9 respectively, fig.28-29.

Wave modes with short periods lead to high values of \( R_u \). When more than one mode is present, \( \eta_{10} \) is the maximum wave height obtained by interference. In this case, the value of \( R_u \) will probably be between the values based on the longest and shortest periods in question. Fig.32 shows the values of \( R_u \) for the two shortest significant periods in the seven profiles, together with the measured values and the values from the basic model, see also table 8.

In the profiles 2-5 the measured run-up heights lie between the two estimated values \( R_u \). In profile 1 the run-up height is underestimated by the two values of \( R_u \). This may be explained by the grid representation of the shore line which causes an exaggerated reflection in the fjord bend. This may again cause too small values of \( \eta_{10} \) on the outside. Three dimensional focusing effects, or stem effects combined with topography (see [27]) may also be essential. In profile 7 the shortest period provides the best correction factor. Field studies have revealed that this is an area extremely exposed for run-up effects. Underestimation may also occur because the measured values in many places certainly corresponds to heights reached by the splash from the breaking waves only. This is supported by the suspiciously large local
Table 8: Parameter values used in run-up studies.

θ: Average angle of slope between defined breaking point and shore line, 
\( h_0 \): water depth above lower end of the slope, 
\( d \): water depth in the points where \( h_0 \) is read, 
\( T_1^i \): second shortest significant period of incident waves, 
\( T_2^i \): shortest significant period of incident waves, 
\( \eta_{10} \): modelled wave height in grid point nearest to the 10.0 m depth contour line (from standard model run, \( t_{\text{max}} = 360.0 \) s), 
\( R_{u1} \): estimated run-up height on gentle beach slope for wave mode with second shortest period, 
\( R_{u2} \): estimated run-up height on gentle beach slope for wave mode with shortest period, 
\( r \): measured run-up height.

<table>
<thead>
<tr>
<th>Prof.no.</th>
<th>θ (deg)</th>
<th>( h_0 ) (m)</th>
<th>( d ) (m)</th>
<th>( T_1^i ) (s)</th>
<th>( T_2^i ) (s)</th>
<th>( \eta_{10} ) (m)</th>
<th>( R_{u1} ) (m)</th>
<th>( R_{u2} ) (m)</th>
<th>( r ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17.9</td>
<td>100.0</td>
<td>24.6</td>
<td>114</td>
<td>76</td>
<td>1.2</td>
<td>1.2</td>
<td>1.3</td>
<td>≈ 4</td>
</tr>
<tr>
<td>2</td>
<td>26.1</td>
<td>216.0</td>
<td>15.7</td>
<td>31</td>
<td>15</td>
<td>6.7</td>
<td>8.0</td>
<td>16.8</td>
<td>≈ 10</td>
</tr>
<tr>
<td>3</td>
<td>15.9</td>
<td>100.0</td>
<td>14.7</td>
<td>22</td>
<td>16</td>
<td>12.7</td>
<td>26.7</td>
<td>52.1</td>
<td>≈ 33</td>
</tr>
<tr>
<td>4</td>
<td>5.9</td>
<td>103.0</td>
<td>15.0</td>
<td>89</td>
<td>15</td>
<td>3.0</td>
<td>4.5</td>
<td>30.3</td>
<td>≈ 12</td>
</tr>
<tr>
<td>5</td>
<td>20.5</td>
<td>127.0</td>
<td>19.9</td>
<td>23</td>
<td>15</td>
<td>16.6</td>
<td>28.2</td>
<td>56.4</td>
<td>≈ 32</td>
</tr>
<tr>
<td>6</td>
<td>25.7</td>
<td>183.0</td>
<td>16.6</td>
<td>21</td>
<td>18</td>
<td>27.8</td>
<td>44.5</td>
<td>52.8</td>
<td>≈ 37</td>
</tr>
<tr>
<td>7</td>
<td>17.9</td>
<td>181.0</td>
<td>10.1</td>
<td>33</td>
<td>16</td>
<td>6.2</td>
<td>8.1</td>
<td>21.1</td>
<td>≈ 24</td>
</tr>
</tbody>
</table>

In profile 6 the run-up heights are overestimated. The difference between the two shortest significant periods is here only 3 s. The two values of \( R_u \) will in this case both be close to an upper estimate of the run-up height. By including the third shortest significant period (57 s), \( R_u = 30.6 \) m becomes a lower estimate of the run-up height along profile 6.

We may conclude that the method of sec.A.4 generally provides run-up heights of the correct magnitude. In view of the simplicity of this method, as opposed to the complexity of the real event, this is as good as can reasonably be expected. Even though the amplitude of each incident wave mode were known, it would be hard to accomplish more exact calculations, because the phase of each wave mode in the gentle slope case would still be unknown.

The run-up process has been studied in detail at seven selected locations where the wave reached certain heights or did severe damage. At these locations a high number of measured values are available. Moreover such studies indicate how similar destructions today may be avoided, through location of installations and breakwaters. Unfortunately the built-up areas along the Norwegian fjords are still concentrated close to the shore line, mainly in areas with gentle beach slopes exposed for run-up effects.
10 Conclusions

We find that a numerical model for simulation of slide generated water waves is capable to predict wave run-up heights for the Tafjord event on April 7th 1934 with surprisingly good agreement in view of the complexity of the problem.

The model is based on the linearized hydrostatic shallow water equations for wave propagation in open sea regions and a slide model for describing the dynamics of the slide body including the effects of changes in bottom topography and the coastline due to the deposited slide mass. Local wave amplification effects in the wave run-up zone are estimated by comparing results from an idealized numerical run-up model using a no-flux boundary condition with an analytical model for calculation of run-up heights on a gentle beach slope.

The fjord bend near Fjøra is found to reflect the outgoing slide generated waves almost totally, hence leading to a trapping of the wave energy in the inner part of the fjord. This causes standing waves, which explains why trains of huge waves occur in the model results in accordance with observations by eye witnesses to the Tafjord event.

The assumption of linear wave propagation is fulfilled except in the vicinity of the slide area and in run-up zones with gentle beach slopes. All but the shortest crosswise wave modes are clearly within the range of long waves. Normally wave modes will travel short distances between each reflection. Errors due to incorrect reflection conditions may therefore be much larger than errors introduced by neglecting dispersive effects. The effect of viscous shear stress on the interface between the fluid and the slide masses is found to be negligible when compared with the effect of volume displacement due to the slide motion.

The experience we have gained from the Tafjord event leads us to the following conclusions which may be of general validity for slide generated waves in fjords: Large run-up heights occur at places with gentle beach slopes. The documentation of wave reflection and standing wave oscillations is convincing and we believe the same effects may be expected in other fjord systems.

Parameter variation reveals that the volume of the slide has strong influence upon the form and the amplitude of the waves, while the shape of the slide body is of minor importance. The velocity of the slide when it hits the water surface affects the wave heights only slightly. If the initial velocity of the slide is reduced while the retardation distance remains unchanged, the fraction of kinetic slide energy transferred to wave energy will be very large.

The importance of nonlinearity and dispersion in cases where these effects are more critical for the run-up height than in the Tafjord event, will be critically assessed in coming studies based on the nonlinear Boussinesq equations.
In spite of all simplifications introduced, the results from the Tafjord case study are so auspicious that we believe the model also can be used to predict slide generated waves and to estimate the risk of disastrous waves in other potential slide threatened areas.

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References


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A Theory for estimation of run-up heights

A.1 General considerations

The numerical model presented in sec. 2.2 is primarily designed to handle generation and propagation of waves in the deeper regions of the fluid domain. At the shores the model hopefully provide an adequate representation of the reflected waves, but will generally not produce correct run-up heights. The major shortcomings and difficulties concerning run-up calculations can be listed as follows:

1. The grid is too coarse for a proper resolution of the beach topography.

2. Large scale roughness in shallow regions, due to vegetation, boulders, buildings etc., are not accounted for.

3. The model does not reproduce the freely moving shore line, but invokes a no-flux condition at some limiting depth.

4. The basic theory is linear, whereas the flow field near the shores must be expected to be substantially influenced by nonlinearity.

5. It is not a trivial task to extract the characteristics of the incident waves, due to reflection and interference. Hence, a straightforward calculation of the run-up height predictions based on the incident wave parameters becomes difficult.

6. Wave breaking is not taken into consideration neither in deep water nor in the run-up zone. Breaking is generally assumed to reduce wave heights as well as run-up heights.

In view of point 5, and to some extent point 1, we have preferred to develop run-up predictions from the modelled surface elevation, $\eta_d$, at a reference depth $d$ near the shore. As long as the length of the incident wave is large, point 1 and 2 above will probably be of minor importance. In earlier studies it has been demonstrated that linear theory often provides surprisingly good run-up values (see [4]). Therefore, point 4 is not necessarily crucial either. However, point 3 really implies that our model really corresponds to an incorrect physical reality, save the parts of the shore displaying steep slopes. We will therefore discuss this point in some detail. These considerations will be based on calculations for a simple geometry defined by depth function, $h(x)$, that equals a constant, $h_0$, for $x > x_L > 0$ and becomes zero for $x = 0$. The shore line, which runs parallel to the $y$-axis, is introduced either as a vertical wall at $x = x_0 > 0$ or as a freely moving shore line around $x = x_0 = 0$. As incident wave we choose a single periodic harmonic, that allows for simple analysis and may represent the actual waves in the Tafjord as least as well as any other simple wave form (see time series in fig. 17-22).
A.2 Analytical run-up calculations

For the incident and reflected waves in the region $x > x_L$ we may write respectively:

\[
\begin{align*}
\eta_{in} &= Ae^{i(k(x-x_L)+\ell y+\omega t)} & A \in \mathbb{R} \\
\eta_{ref} &= Be^{i(-k(x-x_L)+\ell y+\omega t)} & B \in \mathbb{C}
\end{align*}
\]  

(32)

where $k, \ell$ and $\omega$ fulfills the dispersion relation and the real parts, only, have physical significance. For all $x$ we correspondingly write:

\[
\eta = \zeta(x)e^{i(\ell y+\omega t)}
\]  

(33)

Assuming $\tau = 0$, eliminating $u$ and $v$ from equations (4) and (5), and finally inserting the expression (33) we arrive at the ordinary differential equation:

\[
\frac{d}{dx}(gh\frac{d\zeta}{dx}) + (\omega^2 - \ell^2 gh)\zeta = 0
\]  

(34)

From (32) we obtain as off shore boundary condition:

\[
\frac{d\zeta}{dx} + ik\zeta = 2ikAe^{i(k(x-x_L)}
\]  

(35)

which can be applied whenever $x \geq x_L$. We define $F(x; x_0)$ as the solution of (34) that also fits the conditions $F(x_0; x_0) = 1$ and $dF(x_0; x_0)/dx = 0$ if $x_0 > 0$. The first requirement suffices to determine $F$ uniquely if $x_0 = 0$, due to the singularity of (34). In terms of $F$ the solution for $\zeta$ becomes:

\[
\zeta(x) = \frac{2ikA}{F'(x_L; x_0) + ikF(x_L; x_0)}F(x; x_0)
\]  

(36)

where $F'$ denotes the derivative of $F$. The maximum run-up height, $R_u$, becomes accordingly:

\[
\frac{R_u}{A} = \frac{2k}{\sqrt{F'(x_L; x_0)^2 + k^2F(x_L; x_0)^2}}
\]  

(37)

It is fairly easily realized that $R_u/A$ is a continuous function of $x_0$, also for $x_0 = 0$. This implies that the solution of the "rigid wall problem" becomes a close approximation to the solution of the "true run-up problem" for small $x_0$. To prove the continuity of $R_u/A$ we start by stating that the general solution of (34) is a linear combination of $F(x; 0)$ and another function, $R(x)$, that inherits a logarithmic singularity at $x = 0$, as can be shown by standard application of the Frobenius method. Thus, we may write $F(x; x_0) = \gamma(x_0)F(x; 0) + \kappa(x_0)R(x)$ where $\gamma$ and $\kappa$ are determined through the boundary conditions: $F(x_0; x_0) = 1$ and $F'(x_0; x_0) = 0$. This implies that $\kappa/\gamma$ tends to zero sufficiently fast for $F(x; x_0)$ to approach $F(x; 0)$ for all $x \geq x_0$ as $x_0 \rightarrow 0$. 


The solutions for the case \( x_0 = 0 \) and a linear bottom profile, \( h(x) = h_0 x / x_L \) for \( x < x_L \), is thoroughly discussed in [18]. For this bottom topography \( F \) may be expressed in terms of confluent hypergeometric functions, that may be derived from Bessel functions when \( \ell = 0 \). The latter case is analyzed for more general incident waves in [4].

### A.3 Discrete run-up calculations

Even though the two boundary value problems are nicely related, additional difficulties may arise during the discretization. Most considerations concerning accuracy and convergence of numerical methods of the present type, rely on Taylor series expansion. Unfortunately, the use of this expansion may be inappropriate in run-up calculations where \( \Delta x \) is larger than or comparable to \( x_0 \), which equals the radius of convergence for \( \zeta \) at \( x = x_0 \). We will thus analyze the discrete problem along the lines of the previous subsection.

We assume that the geometry is discretized with \( u \)-nodes at the boundary \( x = x_0 \). The arithmetics runs almost as for the analytical case. Again we assume (32), but this time \( k, \ell \) and \( \omega \) have to obey the numerical dispersion relation:

\[
\ddot{\omega}^2 = gh_0(\ddot{k}^2 + \ddot{\ell}^2) \tag{38}
\]

where

\[
\ddot{k} = \frac{2}{\Delta x} \sin\left(\frac{k\Delta x}{2}\right) \quad \ddot{\ell} = \frac{2}{\Delta y} \sin\left(\frac{\ell\Delta y}{2}\right) \quad \ddot{\omega} = \frac{2}{\Delta t} \sin\left(\frac{\omega\Delta t}{2}\right) \tag{39}
\]

The discrete analogue to (33) defines discrete values \( \zeta_j \) which have to be determined. Elimination of velocities from the discrete equations (9), (10) and (11) followed by separation of variables (introduction of \( \zeta \)) yields in analogy to (34):

\[
[\delta_x (g\ddot{\zeta}) + (\ddot{\omega}^2 - \ddot{\ell}^2 gh)\zeta] = 0 \big|_j \tag{40}
\]

where the position number \( j = \frac{1}{2} \) corresponds to \( x = x_0 \). The off shore boundary condition now reads:

\[
[m\delta_x \zeta + ik\zeta_x]\big|_{j+\frac{1}{2}} = 2im\ddot{k}A e^{ik(j\Delta x + x_0 - x_L)} \tag{41}
\]

where \( m = \cos\frac{1}{2}k\Delta x \). When \( x_0 > 0 \) the discrete \( F \) is defined through:

\[
[\delta_x F = 0]\big|_{\frac{1}{2}} \quad F_1 = 1 \tag{42}
\]

where \( F_0 \) is introduced as a fictitious value. The analytical case \( x_0 = 0 \) has a proper numerical counterpart only if \( [h_x]_{\frac{1}{2}} = 0 \), which implies that the fictitious value \( \zeta_0 \) does not enter the difference equation (40). A unique solution for \( F \) can then be obtained by requiring \( F_1 = 1 \) and solving the
tridiagonal system (40). As a consequence the discrete method automatically reproduce the nonsingular solution for $F$. Defining $\zeta_1$ as $R_u^*$ we then find:

$$\frac{R_u^*}{A} = \frac{2m\hat{k}}{\sqrt{(m^2(\delta_x F)^2 + \hat{k}^2(\bar{F}_{x_0})^2)_{j-\frac{1}{2}}}}$$

(43)

where $(j - \frac{1}{2})\Delta x + x_0 > x_L$. A study of calculated solutions show that the discrete results generally are in excellent agreement with the analytical results in spite of the singularity at $x=0$.

A.4 Correction factors for run-up

In the Tafjord model we have extracted the surface elevation, $\eta_d$, at a reference depth, $h = d$, near the shore. For steep sections of the beach the maximum value of $\eta_d$ approximates the run-up height closely, while an estimation of a correction factor would be desirable elsewhere. We proceed as follows:

1. An average bottom slope, $\gamma$, and an appropriate abyss depth, $h_0$, are estimated from the depth matrix. We then define a simplified geometry by a linear depth function for $x < x_L = h_0/\gamma$.

2. One (or several) characteristic period(s) and angle(s) of incidence are extracted from time-series analysis and contour diagrams for $\eta$.

3. The method of sec.A.3 is applied to the simplified geometry and wave characteristics from point 2 to yield a ratio $\zeta_d/A$ at the point $h = d$. When $\zeta_d$ is identified with the maximum value of $\eta_d$ from the Tafjord model, we find an estimate on $A$.

4. Using the value of $A$ from the previous point and the techniques described in sec.A.2 and [4] we find a run-up height $R_u$.

The resulting expression for $R_u$ should give a reasonable approximation unless there are several nodes between $h = d$ and the beach, or a node is situated close to $h = d$. 

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Figures

Figure 1: The spatial distribution of grid points.
- Locations of $\eta$-points, - : locations of $u$-points,
| : locations of $v$-points, -- : boundaries of one grid cell.

Figure 2: Distribution of fluid volume from grid cells with negative depth.
---: Line segments along the shore line at time $t = n \cdot \Delta t$,
-- : line segments along the shore line at time $t = (n + 1)\Delta t$,
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Figure 3: Sketch of slide body with characteristic parameters.

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Figure 5: The surface elevation due to volume displacement (above) and shear stress (below) for a typical slide.
Figure 6: Map of the Tafjord: Run-up heights in metres after rock slide April 7th, 1934. (From [10, 11, 12]).

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Figure 10: Simulated wave pattern at $t = 75.0 \text{ s}$. Contour line interval: 1.0 m. Dotted lines indicate negative values. Height of wave propagating out the fjord: 2 – 3 m, height of wave propagating towards the fjord head: ca. 3 m.
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Figure 12: Perspective view towards the fjord head at $t = 30.0$ s.
Figure 13: Perspective view towards the fjord head at $t = 75.0$ s.

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- - - : simulated time series from standard model run values, case no. 2.
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- - - : simulated time series from standard model run values, case no. 2.
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Figure 29: Power spectra (scaled) for time series based on standard model run values, points 9-16. ● ○: Lower and upper limit of period for crosswise wave modes between points 3 and 6, \( m = 2 \), ○ ○: lower and upper limit of period for crosswise wave modes between points 3 and 6, \( m = 1 \).
Figure 30: Location of the seven depth profiles used for run-up studies.

Figure 31: The seven depth profiles used for run-up studies, and the depth profile outside the slide area. 1: Sylte, 2: Fjøra, 3: Sødalsvik, 4: Tafjord, 5: Seineset, 6: south of Korsnes, 7: south of Oksneset, 8: outside slide area, *: defined breaking point in the bottom profile, o: stopping point for the slide.
Figure 32: *: Measured run-up heights along shore line, ---: modelled wave height in grid point nearest to the 10.0 m depth contour line (from standard model run, $t_{\text{max}} = 360.0$ s), o: values of $R_u$ for the two shortest significant periods in profile 1-7 (in pairs).