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A PROCEDURE FOR FINDING DOUBLE SAMPLING PLANS FOR DISCRETE RANDOM VARIABLES

by

Visiting professor
from University of Wyoming
William C. Guenther

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WILLIAM C. GUENTHER
UNIVERSITY OF WYOMING

A problem frequently encountered in quality control is the determination of sampling plans whose operating characteristic (OC) is at least $1 - \alpha_0$ if a parameter θ assumes the value $\theta = \theta_0$ and is no more than β_1 if $\theta = \theta_1 > \theta_0$. A procedure usable with existing tables for finding double sampling plans for the binomial, hypergeometric, and Poisson cases is given.

1. INTRODUCTION

Let n be the sample size, c the acceptance number and $y_0 = c + 1$ the rejection number of a single sample plan based upon a binomial, hypergeometric, or Poisson random variable. If we let θ be the parameter of interest, then it is a standard problem to find sampling plans which meet the requirements

$$\begin{aligned} OC &\geqq 1-\alpha_0 \quad \text{if} \quad \theta = \theta_0 \\ &\leqq \beta_1 \quad \text{if} \quad \theta = \theta_1 > \theta_0 \end{aligned} \qquad (1.1)$$

or equivalently

$$\begin{aligned} \text{Power} &\leqq \alpha_0 \quad \text{if} \quad \theta = \theta_0 \\ &\geqq 1 - \beta_1 \quad \text{if} \quad \theta = \theta_1 \end{aligned} \qquad (1.2)$$

With a good table of the appropriate distribution such plans are found by observation and without difficulty (a fact which simplifies the double sample solution). For a given c the first inequality determines an integer $n_U(c) = n_U$ such that the inequality is satisfied of $n \leqq n_U$. Similarly, from the second inequality we find $n_L(c) = n_L$ such that all $n \geq n_L$ satisfy the inequality. If $n_L > n_U$ no solutions exist for the chosen c but if $n_L \leq n_U$ then all n such that $n_L \leq n \leq n_U$ are solutions. Usually one would list all solutions starting with $c = 0$ and increasing c a unit at a time, terminating when an optimum solution is found.

Since the inequalities (1.1) have an infinite number of solutions (finite in the hypergeometric case), we usually select a specific solution which minimizes a function $G(n,c)$. Although the most familar of such functions is $G(n,c) = n$, other possibilities include the Dodge-Romig [1, Formula (2-8) p. 34] "average number of items inspected" and the Hald [3] linear cost function. For the more complicated $G(n,c)$ minimization is in general not achieved with the smallest c which permits solutions or with the minimum n for a given c. Since our goal is merely to find plans satisfying (1.1), such minimization problems are beyond the scope of this paper.

We will consider double sampling plans which depend upon four predetermined constants n_1, n_2, c_1, c_2. The procedure begins by observing a random variable X_1 based upon a random sample of size n_1. If $x_1 \leq c_1$, we accept (the lot, process, or hypothesis whichever is appropriate) on the basis of the sample. On the other hand if $x_1 > c_2 > c_1$, we reject. However, if $c_1 < x_1 \leq c_2$ a second sample of size n_2 is taken and a second random variable X_2 is observed. Then we accept if $x_1 + x_2 \leq c_2$ and reject if $x_1 + x_2 > c_2$. When tables contain right hand sums, it is more convenient to work with power rather than OC and $y_1 = c_1 + 1$, $y_2 = c_2 + 1$ rather than c_1, c_2.

For the binomial case we will use the notation

$$b(y;n,p) = \binom{n}{y} p^y(1-p)^{n-y} , \quad y = 0, 1, 2,\ldots,n \qquad (1.3)$$

for the probability function and cumulative sums will be denoted by

$$E(r;n,p) = \sum_{y=r}^{n} b(y;n,p) \tag{1.4}$$

Probably the two most useful tables are the Harvard [4] and the Ordnance Corps [7] tables. The former gives $E(r;n,p)$ to five decimal places for $p = .01(.01).50$ (plus a few rational fractions) and $n = 1(1)50(2)100(10)200(20)500(50)1000$ while the latter (which is out of print but is available in many university libraries) gives the same sum to seven decimal places for $p = .01(.01).50$ and $n = 1(1)150$.

The hypergeometric probability function will be denoted by

$$p(N,n,k,y) = \frac{\binom{k}{y}\binom{N-k}{n-y}}{\binom{N}{n}}, \quad a \leq y \leq b \tag{1.5}$$

where $a = \max[0,n-N+k]$, $b = \min[k,n]$

and cumulative sums by

$$P(N,n,k,r) = \sum_{y=a}^{r} p(N,n,k,y) \tag{1.6}$$

Apparently the best table of the hypergeometric is the one prepared by Lieberman and Owen [5] which gives both (1.5) and (1.6) to six decimal places for $N = 1(1)50(10)100$.

The Poisson probability function will be denoted by

$$p(y;\mu) = \frac{e^{-\mu}\mu^{y}}{y!}, \quad y = 0, 1, 2, \ldots \tag{1.7}$$

with cumulative sums

$$E(r;\mu) = \sum_{y=r}^{\infty} p(y;\mu) \tag{1.8}$$

Molina [6] gives (1.7) and (1.8) to at least six decimal places for $\mu = .001(.001)(.01)(.01).30(.1)15(1)100$ and the General Electric [2] table gives (1.7), (1.8), and left hand cumulative sums to eight decimal places for considerably more values of μ if $\mu < 2$ but for less values of u if $\mu > 2$. In Poisson type problems Y is usually the sum of n independent random variables all having probability function (1.7) with μ replaced by $n\mu$.

For the double sample case we seek n_1, n_2, c_1, c_2 which satisfy (1.1) and (1.2). The well known expressions for power and OC which we will use are for the binomial case

$$\text{Power} = K(p;n_1,n_2,y_1,y_2)$$
$$= E(y_2;n_1,p) + \sum_{j=0}^{y_2-y_1-1} b(y_1+j;n_1,p)\ E(y_2-y_1-j;n_2,p) \quad (1.9)$$

for the hypergeometric case

$$OC = H(k;n_1,n_2,c_1,c_2)$$
$$= P(N,n_1,k,c_1) + \sum_{j=1}^{c_2-c_1} p(N,n_1,k,c_1+j)\ P(N-n_1,n_2,k-c_1-j,c_2-c_1-j) \quad (1.10)$$

for the Poisson case

$$\text{Power} = K(\mu;n_1,n_2,y_1,y_2)$$
$$= E(y_2;n_1\mu) + \sum_{j=0}^{y_2-y_1-1} p(y_1+j;n_1\mu)\ E(y_2-y_1-j;n_2\mu) \quad (1.11)$$

2. THE DOUBLE SAMPLE SOLUTION

Obtaining solutions for (1.1) in the double sample case involves some trial and error. However, if we use the information discussed in this section, those solutions are found very rapidly. Since an infinite (finite in the hypergeometric case) number of solutions are possible, we can select the specific solution which minimizes some function $G(n_1,n_2,c_1,c_2)$. Possibilities for such a function include the Dodge-Romig [1 , Formula (2-9), p. 34] "average number of items inspected," the two sample analogue of the Hald [3] linear cost function, or some other function of the average sample size, ASN [i.e., w_1(ASN when $\theta=\theta_0$) + w_2 (ASN when $\theta=\theta_1$), where w_1 and w_2 are positive fractions whose sum is 1]. As we stated for the single sample case, such minimization problems will not be considered here.

The first step is to list the single sample solutions (and non-solutions) since this information is essential to the double sample case. Reading from the standard tables which we have already mentioned we obtain the following type of information:

if $c = 0$ $\quad n \leqq n_U(0)$, $n \geqq n_L(0)$, $n_L(0) > n_U(0)$, no solution possible

$c = 1$ $\quad n \leqq n_U(1)$, $n \geqq n_L(1)$, $n_L(1) > n_U(1)$, no solution possible

$c = r-1$ $\quad n \leqq n_U(r-1)$, $n \geqq n_U(r-1)$, $n_L(r-1) > n_U(r-1)$, no solution possible

. . . .

$c = r$ $\quad n \leqq n_U(r)$, $n \geqq n_L(r)$, solutions for $n_L(r) \leqq n \leqq n_U(r)$

$c = r+1$ $\quad n \leqq n_U(r+1)$, $n \geqq n_L(r+1)$, solutions for $n_L(r+1) \leqq n \leqq n_U(r+1)$

etc.

Thus, if solutions exist for $c = r$, then solutions are possible for all $c \geqq r$. Further, as c increases so do $n_L(c)$ and $n_U(c)$.

Returning to the double sample case, we make the following observations:

1. Obviously double sample plans satisfying (1.1) exist for all $c_2 \geqq r$ (where r is defined above), $c_1 = 0, 1, 2, \ldots, c_2-1$ since one such plan is $n_1 = n_L(c_2)$, $n_2 = 0$. It can be shown that if solutions exist for given c_1, c_2 with $n_1 = n_{1m}$, the minimum n_1 to admit solutions, then solutions exist for all $n_1 \geqq n_{1m}$ up to the maximum value n_1 can assume, a set which includes the single sample solutions. Hence, we cannot have $c_2 < r$ since for such c_2 $n_1 = n_L(c_2)$, $n_2 = 0$ is a solution and this implies that a single sample solution exists, contrary to assumption.

2. Bounds on n_1 for chosen c_1 and c_2 are immediately

attainable from the power and the OC . For example, in the hypergeometric case the condition $H(k_1;n_1,n_2,c_1,c_2) \leqq \beta_1$ implies

$$P(N,n_1,k_1,c_1) \leqq \beta_1 \tag{2.1}$$

or $n_1 \geqq n_{1L}$. Similarly $H(k_0;n_1,n_2,c_1,c_2) \geqq 1 - \alpha_0$ implies

$$P(N,n_1,k_0,c_2) \geqq 1 - \alpha_0 \tag{2.2}$$

or $n_1 \leqq n_{1U}$. The corresponding conditions for the binomial case are

$$E(y_1;n_1,p_1) \geqq 1 - \beta_1 \tag{2.3}$$

$$E(y_2;n_1,p_0) \leqq \alpha_0 \tag{2.4}$$

and for the Poisson case

$$E(y_1;n_1\mu_1) \geqq 1 - \beta_1 \tag{2.5}$$

$$E(y_2;n_1\mu_0) \leqq \alpha_0 \tag{2.6}$$

Solutions do not always exist for $n_1 = n_{1L}$ but this number provides a lower bound in our search.

3. We must have $n_1 + n_2 \geqq n_L(c_2)$. To see this assume the converse is true, that is there exist n_1, n_2 such that $n_1 + n_2 < n_L(c_2)$. Then the power at $\theta = \theta_1$ is made $\geqq 1-\beta_1$ by taking n_1 observations all of the time and n_2 observations part of the time. The power is not decreased of the second sample is taken with probability 1. But this means that a single sample plan with the given c_2 exists with $n = n_1 + n_2 < n_L(c_2)$, contrary to the definition of $n_L(c_2)$.

4. For given c_1, c_2 lower and upper bounds on n_2 , say n_{2L} , n_{2U} can be found for each n_1 . Of course, if $n_{2U} < n_{2L}$ solutions are not possible for the chosen n_1 and n_1 has to be increased. Both n_{2L} and n_{2U} are non-increasing functions of n_1 which behave very well, a fact which considerably reduces the number of trials required. As n_1 increases n_{2L} rapidly approaches $n_L(c_2)$ and $n_{2L} = n_L(c_2)$ for at least

$n_L(c_2) \leq n_1 \leq n_U(c_2)$. (This is readily explained in terms of the behavior of the power function.)

In summary, the recommended procedure is

1. List the single sample solutions and non-solutions.
2. Select any c_2 for which solution exist. Usually we would begin with the smallest possible c_2 .
3. Select any c_1 such that $0 \leq c_1 < c_2$. We would probably begin with $c_1 = 0$ and increase c_1 a unit at a time.
4. For chosen c_1, c_2 determine bounds on n_1 . We would probably begin looking for solutions with the smallest n_1 satisfying the appropriate inequality (2.1), (2.3), or (2.5) and then increase n_1 a unit at a time.
5. By trial for the chosen c_1, c_2, n_1 find bounds on n_2 determined so that (1.1) is satisfied. If $n_{2L} \leq n_{2U}$, then one set of solutions is the chosen c_1, c_2, n_1 , and $n_{2L} \leq n_2 \leq n_{2U}$.
6. Select another n_1, c_1, c_2 and repeat step 5, proceeding in some orderly fashion. In an application we would terminate calculations when we find a solution which minimizes a function $G(n_1, n_2, c_1, c_2)$.

As we will demonstrate by examples in the next section, solutions are easily found. When the sum of products appearing in (1.9), (1.10), or (1.11) contains more than one term, use the accumulative multiplication feature which is built into most good desk calculators.

3. EXAMPLES

Example 3.1

Find some double sampling plans for the hypergeometric case with $N = 50$, $k_0 = 3$, $k_1 = 12$, $\alpha_0 = .10$, $\beta_1 = .20$.

Solution

We first consider the single sample case. The conditions

(1.1) are $P(50,n,3,c) \geq .90$ and $P(50,n,12,c) \leq .20$. From the Lieberman and Owen [5] tables we find that the inequalities require

with $c = 0$ $n \leq 1$, $n \geq 6$ no solution possible

$c = 1$ $n \leq 10$, $n \geq 11$ no solution possible

$c = 2$ $n \leq 23$, $n \geq 16$ or $16 \leq n \leq 23$

$c = 3$ $n \leq 50$, $n \geq 20$ or $20 \leq n \leq 50$

$c = 4$ $n \leq 50$, $n \geq 25$ or $25 \leq n \leq 50$

$c = 5$ $n \leq 50$, $n \geq 29$ or $29 \leq n \leq 50$

etc.

Double sample solutions are possible with $c_2 = 2,3,4,5$,etc. We will limit our search to the case $c_2 = 2$ (since the procedure is identical for other c_2). Thus we can have $c_1 = 0$, $c_2 = 2$ and $c_1 = 1$, $c_2 = 2$.

First consider the case $c_1 = 0$, $c_2 = 2$. From inequalities (2.1) and (2.2) we know that $P(50,n_1,12,0) \leq .20$ or $n_1 \geq 6$ and $P(50,n_1,3,2) \geq .90$ or $n_1 \leq 23$.

With $c_1 = 0$, $c_2 = 2$, $n_1 = 6$ the OC at $k = 12$ is

$$H(12;50,6,n_2,0,2) = P(50,6,12,0) + p(50,6,12,1)\,P(44,n_2,11,1) + p(50,6,12,2)\,P(44,n_2,10,0)$$

$$= .173729 + .379046\,P(44,n_2,11,1) + .306581\,P(44,n_2,10,0)$$

Using the fact that $n_1 + n_2 \geq 16$ we find by trial

$$H(12;50,6,15,0,2) = .173729 + (.379046)(.043689) + (.306581)(.008073) = .192764$$

$$H(12;50,6,14,0,2) = .173729 + (.379046)(.061968) + (.306581)(.012109) = .20930$$

so that $n_2 \geq 15$ makes OC $\leq .20$. Similarly

$$H(3;50,6,n_2,0,2) = P(50,6,3,0) + p(50,6,3,1)\,P(44,n_2,2,1) + p(50,6,3,2)\,P(44,n_2,1,0)$$

By trial we get $H(3;50,6,23,0,2) = .903930$,

$H(3;,6,24,0,2) = .896123$ and OC $\geq .90$

provided $n_2 \leqq 23$. Thus with $n_1 = 6$, solution are possible with $15 \leqq n_2 \leqq 23$.

The solutions for $n_1 = 6$ are of assistance in finding solutions for $n_1 = 7$. When n_1 increases n_2 cannot so 15 and 23 are upper bounds for the new interval endpoints.
We find $H(12;50,7,11,0,2) = .185866$,
$H(12;50,7,10,0,2) = .206891$ so that $n_2 \geqq 11$ and
$H(3;50,7,21,0,2) = .900714$, $H(3;50,7,22,0,2) = .892143$ which means $n_2 \leqq 21$. Thus with $n_1 = 7$ solutions are possible with $11 \leqq n_2 \leqq 21$.

Proceeding in a similar manner we find all the solutions for $c_1 = 0$, $c_2 = 2$. These are

$n_1 = 6$,	$15 \leqq n_2 \leqq 23$	$n_1 = 15$,	$1 \leqq n_2 \leqq 9$
$n_1 = 7$,	$11 \leqq n_2 \leqq 21$	$n_1 = 16$,	$0 \leqq n_2 \leqq 7$
$n_1 = 8$,	$9 \leqq n_2 \leqq 19$	$n_1 = 17$,	$0 \leqq n_2 \leqq 6$
$n_1 = 9$,	$7 \leqq n_2 \leqq 17$	$n_1 = 18$,	$0 \leqq n_2 \leqq 5$
$n_1 = 10$,	$6 \leqq n_2 \leqq 15$	$n_1 = 19$,	$0 \leqq n_2 \leqq 4$
$n_1 = 11$,	$5 \leqq n_2 \leqq 14$	$n_1 = 20$,	$0 \leqq n_2 \leqq 3$
$n_1 = 12$,	$4 \leqq n_2 \leqq 13$	$n_1 = 21$,	$0 \leqq n_2 \leqq 2$
$n_1 = 13$,	$3 \leqq n_2 \leqq 11$	$n_1 = 22$,	$0 \leqq n_2 \leqq 1$
$n_1 = 14$,	$2 \leqq n_2 \leqq 10$	$n_1 = 23$,	$n_2 = 0$

For the case $c_1 = 1$, $c_2 = 2$ we still have $n_1 \leqq 23$ but now we must have $P(50,n_1,12,1) \leqq .20$ so that $n_1 \geqq 11$.

With $c_1 = 1$, $c_2 = 2$, $n_1 = 11$ the OC at $k = 12$ is

$$H(12;50,11,n_2,1,2) = P(50,11,12,1) + p(50,11,12,2)\ P(39,n_2,10,0)$$
$$= .184081 + .288024\ P(39,n_2,10,0)$$

Again $n_1 + n_2 \geqq 16$ and we find by trial
$H(12;50,11,9,1,2) = .184081 + .288024(.047261) = .197963$
$H(12,50,11,8,1,2) = .184081 + .288024(.069764) = .204175$

so we must have $n_2 \geqq 9$. Similarly

$$H(3;50,11,n_2,1,2) = P(50,11,3,1) + p(50,11,3,2)\, P(39,n_2,1,0)$$
$$= .882143 + .109439\, P(39,n_2,1,0)$$

By trial we find $H(3;50,11,32,1,2) = .901786$,

$H(3;50,11,33,1,2) = .898980$ so that $n_2 \leqq 32$

Continuing we find all the solutions for $c_1 = 1,\ c_2 = 2$. These are

$n_1 = 11,$	$9 \leqq n_2 \leqq 32$	$n_1 = 18,$	$0 \leqq n_2 \leqq 7$
$n_1 = 12,$	$5 \leqq n_2 \leqq 26$	$n_1 = 19,$	$0 \leqq n_2 \leqq 5$
$n_1 = 13,$	$3 \leqq n_2 \leqq 22$	$n_1 = 20,$	$0 \leqq n_2 \leqq 4$
$n_1 = 14,$	$2 \leqq n_2 \leqq 16$	$n_1 = 21,$	$0 \leqq n_2 \leqq 3$
$n_1 = 15$	$1 \leqq n_2 \leqq 14$	$n_1 = 22,$	$0 \leqq n_1 \leqq 1$
$n_1 = 16$	$0 \leqq n_2 \leqq 11$	$n_1 = 23$	$n_2 = 0$
$n_1 = 17$	$0 \leqq n_2 \leqq 9$		

Example 3.2

Find some double sampling plans for the binomial case with $p_0 = .05,\ p_1 = .20,\ \alpha_0 = .05,\ \beta_1 = .10$.

Solution

For the single sample case the inequalities (1.2) are $E(y_0;n,.05) \leqq .05$ and $E(y_0;n,.20) \geqq .90$.

From the Ordnance Corps [7] table we find that the inequalities require

with	$c = 0$	$n \leqq 1,\ n \geqq 11$	no solution possible
	$c = 1$	$n \leqq 7,\ n \geqq 18$	no solution possible
	$c = 2$	$n \leqq 16,\ n \geqq 25$	no solution possible
	$c = 3$	$n \leqq 28,\ n \geqq 32$	no solution possible
	$c = 4$	$n \leqq 40,\ n \geqq 38$	or $38 \leqq n \leqq 40$
	$c = 5$	$n \leqq 53,\ n \geqq 45$	or $45 \leqq n \leqq 53$

etc.

Double sampling plans are possible for $c_2 \geqq 4$. With

$c_2 = 4$ we can have $c_1 = 0, 1, 2, 3$. Since all solutions are obtained in a similar manner we will consider only the case $c_1 = 3$, $c_2 = 4$ ($y_1=4$, $y_2=5$) . From inequalities (2.3) and (2.4) we know that

$E(4;n_1,.20) \geqq .90$ or $n_1 \geqq 32$ and

$E(5;n_1,.05) \leqq .05$ or $n_1 \leqq 40$

With $n_1 = 32$, $y_1 = 4$, $y_2 = 5$ the power at $p = .20$ is

$$K(.20;32,n_2,4,5) = E(5;32,.20) + b(4;32,.20)\, E(1;n_2,.20)$$
$$= .79562 + .11129\, E(1;n_2,.05)$$

Using the fact that $n_1 + n_2 \geqq 38$ we find by trial

$K(.20;32,13,4,5) = .79562 + .11129(.94502) = .90079$

$K(.20;32,12,4,5) = .79562 + .11129(.93128) = .89926$

so that $n_2 \geqq 13$ makes Power $\geqq .90$. Similarly

$K(.05;32,n_2,4,5) = E(5;32,.05) + b(4;32,.05)\, E(1;n_2,.05)$

and by trial we obtain $K(.05;32,15,4,5) = .04904$,

$K(.05;32,16,4,5) = .05028$ so that Power $\leqq .05$ if $n_2 \leqq 15$.

As in the hypergeometric case, having found one solution, the next one requires less trial. With $n_1 = 33$ we find $K(.20;33,8,4,5) = .90239$, $K(.20;33,7,4,5) = .89794$ and $n_2 \geqq 8$. Then $K(.05;33,12,4,5) = .04959$, $K(.05;33,13,4,5) = .05115$. and $n_2 \leqq 12$.

Continuing the calculations we soon have all the solutions for $c_1 = 3$, $c_2 = 4$. These are

$n_1 = 32$,	$13 \leqq n_2 \leqq 15$	$n_1 = 37$,	$1 \leqq n_2 \leqq 3$
$n_1 = 33$,	$8 \leqq n_2 \leqq 12$	$n_1 = 38$,	$0 \leqq n_2 \leqq 2$
$n_1 = 34$,	$6 \leqq n_2 \leqq 9$	$n_1 = 39$,	$0 \leqq n_2 \leqq 1$
$n_1 = 35$,	$4 \leqq n_2 \leqq 7$	$n_1 = 40$,	$n_2 = 0$
$n_1 = 36$,	$3 \leqq n_2 \leqq 5$		

Example 3.3

Find some double sampling plans for the Poisson case with

$\mu_0 = .05$, $\mu_1 = .20$, $\alpha_0 = .05$, $\beta_1 = .10$.

Solution

For the single sample case the inequalities (1.2) are $E(y_0;.05n) \leqq .05$ and $E(y_0;.20n) \geqq .90$. From the Molina [6] table (using linear interpolation) we find

with	$c = 0$	$.05n \leqq .051$	$.20n \geqq 2.21$	
		$n \leqq 1$	$n \geqq 12$	no solution possible
	$c = 1$	$.05n \leqq .36$	$.20n \geqq 3.89$	
		$n \leqq 7$	$n \geqq 20$	no solution possible
	$c = 2$	$.05n \leqq .82$,	$.20n \geqq 5.33$	
		$n \leqq 16$	$n \geqq 27$	no solution possible
	$c = 3$	$.05n \leqq 1.37$	$.20n \geqq 6.68$	
		$n \leqq 27$	$n \geqq 34$	no solution possible
	$c = 4$	$.05n \leqq 1.97$,	$.20n \geqq 8.00^-$	
		$n \leqq 39$	$n \geqq 40$	no solution possible
	$c = 5$	$.05n \leqq 2.61$,	$.20n \geqq 9.28$	
		$n \leqq 52$	$n \geqq 47$	or $47 \leqq n \leqq 52$
	$c = 6$	$.05n \leqq 3.28$	$.20n \geqq 10.53$	
		$n \leqq 65$	$n \geqq 53$	or $53 \leqq n \leqq 65$

etc.

Double sampling plans are possible for $c_2 \geqq 5$. With $c_2 = 5$ we can have $c_1 = 0, 1, 2, 3, 4$. Since all solutions are obtained in a similar manner, we will consider only the case $c_1 = 4$, $c_2 = 5$ $(y_1=5, y_2=6)$. From inequalities (2.5) and (2.6) we know that $E(5;.20n_1) \geqq .90$ or $n_1 \geqq 40$ and $E(6;.05n_1) \leqq .05$ or $n_1 \leqq 52$.

With $n_1 = 40$, $y_1 = 5$, $y_2 = 6$ the power at $\mu = .20$ is

$$K(.20;40,n_2,5,6) = E(6;8) + p(5;8)\, E(1;.20n_2)$$
$$= .80876 + .09160\, E(1;.20n_2)$$

Using $.20n_2 = 5.5$ and $.20n_2 = 5.6$ (5.5 and 5.6 are successive table entries) yields

$K(.20;40,n_2,5,6) = .89999, .90002$ respectively so we conclude $.20n_2 \geqq 5.53$, $n_2 \geqq 28$. Similarly, the power at $\mu = .05$ is

$$K(.05;40,n_2,5,6) = E(6;2) + p(5;2)\,E(1;.05n_2)$$
$$= .01656 + .03609\,E(1;.05n_2)$$

Using $.05n_2 = 2.6$ and $.05n_2 = 2.7$ yields

$K(.05;40,n_2,5,6) = .04997, .05022$

respectively so we conclude that $.05n_2 \leqq 2.61$, $n_2 \leqq 52$.

With $n_1 = 41$ the power at $\mu = .20$ is

$$K(.20;41,n_2,5,6) = E(6;8.2) + p(5;8.2)\,E(1;.20n_2)$$
$$= .82641 + .08485\,E(1;.20n_2)$$

Using $.20n_2 = 2.0$ and $.20n_2 = 2.1$ yields

$K(.20;41,n_2,5,6) = .89978, .90087$ respectively so we conclude $.20n_2 \geqq 2.02$, $n_2 \geqq 11$. Similarly the power at $\mu = .05$ is

$$K(.05;41,n_2,5,6) = E(6;2.05) + p(5;2.05)\,E(1;.05n_2)$$
$$= .01850 + .03889\,E(1;.05n_2)$$

where .01850 and .03889 were obtained by linear interpolation. Using $.05n_2 = 1.6$ and $.05n_2 = 1.7$ yields

$K(.05;41,n_2,5,6) = .04954, .05028$

respectively so we conclude $.05n_2 \leqq 1.66$, $n_2 \leqq 33$.

With $n_1 = 43$ using $.05n_2 = .9$ and $.05n_2 = 1.0$ led to $.05n_2 \leqq .95$. To determine whether $n_2 \leqq 19$ or $n_2 \leqq 18$ the General Electric [2] table was used giving $.05n_2 \leqq .947$, $n_2 \leqq 18$.

Similarly, all solutions were found for $c_1 = 3$, $c_2 = 4$. These are

$n_1 = 40$,	$28 \leqq n_2 \leqq 52$	$n_1 = 47$,	$0 \leqq n_2 \leqq 6$
$n_1 = 41$,	$11 \leqq n_2 \leqq 33$	$n_1 = 48$,	$0 \leqq n_2 \leqq 5$
$n_1 = 42$,	$7 \leqq n_2 \leqq 24$	$n_1 = 49$,	$0 \leqq n_2 \leqq 3$
$n_1 = 43$,	$5 \leqq n_2 \leqq 18$	$n_1 = 50$,	$0 \leqq n_2 \leqq 2$
$n_1 = 44$,	$3 \leqq n_2 \leqq 14$	$n_1 = 51$,	$0 \leqq n_2 \leqq 1$
$n_1 = 45$,	$2 \leqq n_2 \leqq 11$	$n_1 = 52$,	$n_2 = 0$
$n_1 = 46$,	$1 \leqq n_2 \leqq 9$		

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