A PROCEDURE FOR FINDING DOUBLE SAMPLING PLANS
FOR DISCRETE RANDOM VARIABLES

by

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A problem frequently encountered in quality control is the determination of sampling plans whose operating characteristic (OC) is at least $1 - \alpha_0$ if a parameter $\theta$ assumes the value $\theta = \theta_0$ and is no more than $\beta_1$ if $\theta = \theta_1 > \theta_0$. A procedure usable with existing tables for finding double sampling plans for the binomial, hypergeometric, and Poisson cases is given.

1. INTRODUCTION

Let $n$ be the sample size, $c$ the acceptance number and $y_0 = c + 1$ the rejection number of a single sample plan based upon a binomial, hypergeometric, or Poisson random variable. If we let $\theta$ be the parameter of interest, then it is a standard problem to find sampling plans which meet the requirements

$$\begin{align*}
OC & \geq 1 - \alpha_0 \quad \text{if} \quad \theta = \theta_0 \\
& \leq \beta_1 \quad \text{if} \quad \theta = \theta_1 > \theta_0
\end{align*}$$

or equivalently

$$\begin{align*}
\text{Power} & \leq \alpha_0 \quad \text{if} \quad \theta = \theta_0 \\
& \geq 1 - \beta_1 \quad \text{if} \quad \theta = \theta_1
\end{align*}$$

With a good table of the appropriate distribution such plans are found by observation and without difficulty (a fact which simplifies the double sample solution). For a given $c$ the first inequality determines an integer $n_U(c) = n_U$ such that the inequality is satisfied of $n \leq n_U$. Similarly, from the
second inequality we find \( n_L(c) = n_L \) such that all \( n \geq n_L \) satisfy the inequality. If \( n_L > n_U \) no solutions exist for the chosen \( c \) but if \( n_L \leq n_U \) then all \( n \) such that \( n_L \leq n \leq n_U \) are solutions. Usually one would list all solutions starting with \( c = 0 \) and increasing \( c \) a unit at a time, terminating when an optimum solution is found.

Since the inequalities (1.1) have an infinite number of solutions (finite in the hypergeometric case), we usually select a specific solution which minimizes a function \( G(n,c) \). Although the most familiar of such functions is \( G(n,c) = n \), other possibilities include the Dodge-Romig [1, Formula (2–8) p. 34] "average number of items inspected" and the Hald [3] linear cost function. For the more complicated \( G(n,c) \) minimization is in general not achieved with the smallest \( c \) which permits solutions or with the minimum \( n \) for a given \( c \).

Since our goal is merely to find plans satisfying (1.1), such minimization problems are beyond the scope of this paper.

We will consider double sampling plans which depend upon four predetermined constants \( n_1, n_2, c_1, c_2 \). The procedure begins by observing a random variable \( X_1 \) based upon a random sample of size \( n_1 \). If \( x_1 \leq c_1 \), we accept (the lot, process, or hypothesis whichever is appropriate) on the basis of the sample. On the other hand if \( x_1 > c_2 > c_1 \), we reject. However, if \( c_1 < x_1 \leq c_2 \) a second sample of size \( n_2 \) is taken and a second random variable \( X_2 \) is observed. Then we accept if \( x_1 + x_2 \leq c_2 \) and reject if \( x_1 + x_2 > c_2 \). When tables contain right hand sums, it is more convenient to work with power rather than \( OC \) and \( y_1 = c_1 + 1 \), \( y_2 = c_2 + 1 \) rather than \( c_1 \), \( c_2 \).

For the binomial case we will use the notation
\[
b(y;n,p) = \binom{n}{y} p^y (1-p)^{n-y}, \quad y = 0, 1, 2, \ldots, n
\] (1.3)
for the probability function and cumulative sums will be
denoted by
\[ E(r; n, p) = \sum_{y=r}^{n} b(y; n, p) \] (1.4)
Probably the two most useful tables are the Harvard [4] and
the Ordnance Corps [7] tables. The former gives \( E(r; n, p) \) to
five decimal places for \( p = .01(.01), .50 \) (plus a few rational
fractions) and \( n = 1(1)50(2)100(10)200(20)500(50)1000 \) while
the latter (which is out of print but is available in many
university libraries) gives the same sum to seven decimal
places for \( p = .01(.01), .50 \) and \( n = 1(1)150 \).

The hypergeometric probability function will be denoted by
\[ p(N, n, k, y) = \frac{\binom{k}{y} \binom{N-k}{n-y}}{\binom{N}{n}}, \quad a \leq y \leq b \] (1.5)
where \( a = \max[0, n-N+k] \), \( b = \min[k, n] \)
and cumulative sums by
\[ P(N, n, k, r) = \sum_{y=a}^{r} p(N, n, k, y) \] (1.6)
Apparently the best table of the hypergeometric is the one
prepared by Lieberman and Owen [5] which gives both (1.5) and
(1.6) to six decimal places for \( N = 1(1)50(10)100 \).

The Poisson probability function will be denoted by
\[ p(y; \mu) = \frac{e^{-\mu} \mu^y}{y!}, \quad y = 0, 1, 2, ... \] (1.7)
with cumulative sums
\[ E(r; \mu) = \sum_{y=r}^{\infty} p(y; \mu) \] (1.8)
Molina [6] gives (1.7) and (1.8) to at least six decimal places
for \( \mu = .001(.001)(.01)(.01).30(.1)15(1)100 \) and the General
Electric [2] table gives (1.7), (1.8), and left hand cumulative
sums to eight decimal places for considerably more values of
\( \mu \) if \( \mu < 2 \) but for less values of \( \mu \) if \( \mu > 2 \). In
Poisson type problems \( Y \) is usually the sum of \( n \) independent
random variables all having probability function (1.7) with \( \mu \) replaced by \( n\mu \).

For the double sample case we seek \( n_1, n_2, c_1, c_2 \) which satisfy (1.1) and (1.2). The well known expressions for power and OC which we will use are for the binomial case

\[
\text{Power} = K(p; n_1, n_2, y_1, y_2) = E(y_2; n_1, p) + \sum_{j=0}^{y_2-y_1-1} b(y_1+j; n_1, p) E(y_2-y_1-j; n_2, p) \tag{1.9}
\]

for the hypergeometric case

\[
\text{OC} = H(k; n_1, n_2, c_1, c_2) = P(N, n_1, k, c_1) + \sum_{j=1}^{c_2-c_1} p(N, n_1, k, c_1+j) P(N-n_1, n_2, k-c_1-j, c_2-c_1-j) \tag{1.10}
\]

for the Poisson case

\[
\text{Power} = K(\lambda; n_1, n_2, y_1, y_2) = E(y_2; n_1, \lambda) + \sum_{j=0}^{y_2-y_1-1} p(y_1+j; n_1, \lambda) E(y_2-y_1-j; n_2, \lambda) \tag{1.11}
\]

2. **THE DOUBLE SAMPLE SOLUTION**

Obtaining solutions for (1.1) in the double sample case involves some trial and error. However, if we use the information discussed in this section, those solutions are found very rapidly. Since an infinite (finite in the hypergeometric case) number of solutions are possible, we can select the specific solution which minimizes some function \( G(n_1, n_2, c_1, c_2) \). Possibilities for such a function include the Dodge-Romig [1, Formula (2-9), p. 34] "average number of items inspected," the two sample analogue of the Hald [3] linear cost function, or some other function of the average sample size, \( \text{ASN} \) [i.e., \( w_1(\text{ASN when } \theta=\theta_0) + w_2 (\text{ASN when } \theta=\theta_1) \), where \( w_1 \) and \( w_2 \) are positive fractions whose sum is 1]. As we stated for the single sample case, such minimization problems will not be considered
The first step is to list the single sample solutions (and non-solutions) since this information is essential to the double sample case. Reading from the standard tables which we have already mentioned we obtain the following type of information:

- If $c = 0$, $n \leq n_u(0)$, $n \geq n_L(0)$, $n_L(0) > n_u(0)$, no solution possible.
- If $c = 1$, $n \leq n_u(1)$, $n \geq n_L(1)$, $n_L(1) > n_u(1)$, no solution possible.
- If $c = r-1$, $n \leq n_u(r-1)$, $n \geq n_L(r-1)$, $n_L(r-1) > n_u(r-1)$, no solution possible.
- If $c = r$, $n \leq n_u(r)$, $n \geq n_L(r)$, solutions for $n_L(r) \leq n \leq n_u(r)$.
- If $c = r+1$, $n \leq n_u(r+1)$, $n \geq n_L(r+1)$, solutions for $n_L(r+1) \leq n \leq n_u(r+1)$.

Thus, if solutions exist for $c = r$, then solutions are possible for all $c \geq r$. Further, as $c$ increases so do $n_L(c)$ and $n_u(c)$.

Returning to the double sample case, we make the following observations:

1. Obviously double sample plans satisfying (1.1) exist for all $c_2 \geq r$ (where $r$ is defined above), $c_1 = 0, 1, 2, \ldots, c_2 - 1$ since one such plan is $n_1 = n_L(c_2)$, $n_2 = 0$. It can be shown that if solutions exist for given $c_1, c_2$ with $n_1 = n_{1m}$, the minimum $n_1$ to admit solutions, then solutions exist for all $n_1 \geq n_{1m}$ up to the maximum value $n_1$ can assume, a set which includes the single sample solutions. Hence, we cannot have $c_2 < r$ since for such $c_2$, $n_1 = n_L(c_2)$, $n_2 = 0$ is a solution and this implies that a single sample solution exists, contrary to assumption.

2. Bounds on $n_1$ for chosen $c_1$ and $c_2$ are immediately
attainable from the power and the OC. For example, in the hypergeometric case the condition \( H(k_1; n_1, n_2, c_1, c_2) \leq \beta_1 \) implies
\[
P(N, n_1, k_1, c_1) \leq \beta_1 \tag{2.1}
\]
or \( n_1 \geq n_{1L} \). Similarly \( H(k_0; n_1, n_2, c_1, c_2) \geq 1 - \alpha_0 \) implies
\[
P(N, n_1, k_0, c_2) \geq 1 - \alpha_0 \tag{2.2}
\]
or \( n_1 \leq n_{1U} \). The corresponding conditions for the binomial case are
\[
E(y_1; n_1, p_1) \geq 1 - \beta_1 \tag{2.3}
\]
\[
E(y_2; n_1, p_0) \leq \alpha_0 \tag{2.4}
\]
and for the Poisson case
\[
E(y_1; n_1 u_1) \geq 1 - \beta_1 \tag{2.5}
\]
\[
E(y_2; n_1 u_0) \leq \alpha_0 \tag{2.6}
\]
Solutions do not always exist for \( n_1 = n_{1L} \) but this number provides a lower bound in our search.

3. We must have \( n_1 + n_2 \geq n_L(c_2) \). To see this assume the converse is true, that is there exist \( n_1, n_2 \) such that \( n_1 + n_2 < n_L(c_2) \). Then the power at \( \theta = \theta_1 \) is made \( \geq 1 - \beta_1 \) by taking \( n_1 \) observations all of the time and \( n_2 \) observations part of the time. The power is not decreased of the second sample is taken with probability 1. But this means that a single sample plan with the given \( c_2 \) exists with \( n = n_1 + n_2 < n_L(c_2) \), contrary to the definition of \( n_L(c_2) \).

4. For given \( c_1, c_2 \) lower and upper bounds on \( n_2 \), say \( n_{2L}, n_{2U} \) can be found for each \( n_1 \). Of course, if \( n_{2U} < n_{2L} \) solutions are not possible for the chosen \( n_1 \) and \( n_1 \) has to be increased. Both \( n_{2L} \) and \( n_{2U} \) are non-increasing functions of \( n_1 \) which behave very well, a fact which considerably reduces the number of trials required. As \( n_1 \) increases \( n_{2L} \) rapidly approaches \( n_L(c_2) \) and \( n_{2L} = n_L(c_2) \) for at least
\( n_L(c_2) \leq n_1 \leq n_U(c_2) \). (This is readily explained in terms of the behavior of the power function.)

In summary, the recommended procedure is

1. List the single sample solutions and non-solutions.
2. Select any \( c_2 \) for which solution exist. Usually we would begin with the smallest possible \( c_2 \).
3. Select any \( c_1 \) such that \( 0 \leq c_1 < c_2 \). We would probably begin with \( c_1 = 0 \) and increase \( c_1 \) a unit at a time.
4. For chosen \( c_1, c_2 \) determine bounds on \( n_1 \). We would probably begin looking for solutions with the smallest \( n_1 \) satisfying the appropriate inequality (2.1), (2.3), or (2.5) and then increase \( n_1 \) a unit at a time.
5. By trial for the chosen \( c_1, c_2, n_1 \) find bounds on \( n_2 \) determined so that (1.1) is satisfied. If \( n_{2L} \leq n_{2U} \), then one set of solutions is the chosen \( c_1, c_2, n_1 \), and
\[
\begin{align*}
n_{2L} & \leq n_2 \leq n_{2U}.
\end{align*}
\]
6. Select another \( n_1, c_1, c_2 \) and repeat step 5, proceeding in some orderly fashion. In an application we would terminate calculations when we find a solution which minimizes a function \( G(n_1, n_2, c_1, c_2) \).

As we will demonstrate by examples in the next section, solutions are easily found. When the sum of products appearing in (1.9), (1.10), or (1.11) contains more than one term, use the accumulative multiplication feature which is built into most good desk calculators.

3. EXAMPLES

Example 3.1

Find some double sampling plans for the hypergeometric case with \( N = 50, k_0 = 3, k_1 = 12, \alpha_0 = .10, \beta_1 = .20 \). Solution

We first consider the single sample case. The conditions
(1.1) are \( P(50,n,3,c) \geq .90 \) and \( P(50,n,12,c) \leq .20 \). From the Lieberman and Owen [5] tables we find that the inequalities require

with \( c = 0 \) \( n \leq 1 \), \( n \geq 6 \) no solution possible

\( c = 1 \) \( n \leq 10 \), \( n \geq 11 \) no solution possible

\( c = 2 \) \( n \leq 23 \), \( n \geq 16 \) or \( 16 \leq n \leq 23 \)

\( c = 3 \) \( n \leq 50 \), \( n \geq 20 \) or \( 20 \leq n \leq 50 \)

\( c = 4 \) \( n \leq 50 \), \( n \geq 25 \) or \( 25 \leq n \leq 50 \)

\( c = 5 \) \( n \leq 50 \), \( n \geq 29 \) or \( 29 \leq n \leq 50 \)

etc.

Double sample solutions are possible with \( c_2 = 2,3,4,5 \), etc.

We will limit our search to the case \( c_2 = 2 \) (since the procedure is identical for other \( c_2 \)). Thus we can have \( c_1 = 0 \), \( c_2 = 2 \) and \( c_1 = 1, c_2 = 2 \).

First consider the case \( c_1 = 0, c_2 = 2 \). From inequalities (2.1) and (2.2) we know that \( P(50,n_1,12,0) \leq .20 \) or \( n_1 \geq 6 \) and \( P(50,n_1,3,2) \geq .90 \) or \( n_1 \leq 23 \).

With \( c_1 = 0, c_2 = 2, n_1 = 6 \) the OC at \( k = 12 \) is

\[
H(12;50,6,n_2,0,2) = P(50,6,12,0) + p(50,6,12,1) P(44,n_2,11,1)
+ p(50,6,12,2) P(44,n_2,10,0)
= .173729 + .379046 P(44,n_2,11,1)
+ .306581 P(44,n_2,10,0)
\]

Using the fact that \( n_1 + n_2 \geq 16 \) we find by trial

\[
H(12;50,6,15,0,2) = .173729 + (.379046)(.043689)
+ (.306581)(.008073) = .192764
\]

\[
H(12;50,6,14,0,2) = .173729 + (.379046)(.061968)
+ (.306581)(.012109) = .20930
\]

so that \( n_2 \geq 15 \) makes OC \( \leq .20 \). Similarly

\[
H(3;50,6,n_2,0,2) = P(50,6,3,0) + p(50,6,3,1) P(44,n_2,2,1)
+ p(50,6,3,2) P(44,n_2,1,0)
\]

By trial we get \( H(3;50,6,23,0,2) = .903930 \),

\( H(3;5,6,24,0,2) = .896123 \) and OC \( \geq .90 \)
provided \( n_2 \leq 23 \). Thus with \( n_1 = 6 \), solution are possible with \( 15 \leq n_2 \leq 23 \).

The solutions for \( n_1 = 6 \) are of assistance in finding solutions for \( n_1 = 7 \). When \( n_1 \) increases \( n_2 \) cannot so 15 and 23 are upper bounds for the new interval endpoints.

We find \( H(12;50,7,11,0,2) = .185866 \),

\( H(12;50,7,10,0,2) = .206891 \) so that \( n_2 \geq 11 \) and

\( H(3;50,7,21,0,2) = .900714, H(3;50,7,22,0,2) = .892143 \) which means \( n_2 \leq 21 \). Thus with \( n_1 = 7 \) solutions are possible with \( 11 \leq n_2 \leq 21 \).

Proceeding in a similar manner we find all the solutions for \( c_1 = 0, c_2 = 2 \). These are

\[
\begin{align*}
n_1 &= 6, \quad 15 \leq n_2 \leq 23 & n_1 &= 15, \quad 1 \leq n_2 \leq 9 \\
n_1 &= 7, \quad 11 \leq n_2 \leq 21 & n_1 &= 16, \quad 0 \leq n_2 \leq 7 \\
n_1 &= 8, \quad 9 \leq n_2 \leq 19 & n_1 &= 17, \quad 0 \leq n_2 \leq 6 \\
n_1 &= 9, \quad 7 \leq n_2 \leq 17 & n_1 &= 18, \quad 0 \leq n_2 \leq 5 \\
n_1 &= 10, \quad 6 \leq n_2 \leq 15 & n_1 &= 19, \quad 0 \leq n_2 \leq 4 \\
n_1 &= 11, \quad 5 \leq n_2 \leq 14 & n_1 &= 20, \quad 0 \leq n_2 \leq 3 \\
n_1 &= 12, \quad 4 \leq n_2 \leq 13 & n_1 &= 21, \quad 0 \leq n_2 \leq 2 \\
n_1 &= 13, \quad 3 \leq n_2 \leq 11 & n_1 &= 22, \quad 0 \leq n_2 \leq 1 \\
n_1 &= 14, \quad 2 \leq n_2 \leq 10 & n_1 &= 23, \quad n_2 = 0
\end{align*}
\]

For the case \( c_1 = 1, c_2 = 2 \) we still have \( n_1 \leq 23 \)

but now we must have \( P(50,n_1,12,1) \leq .20 \) so that \( n_1 \geq 11 \).

With \( c_1 = 1, c_2 = 2, n_1 = 11 \) the OC at \( k = 12 \) is

\[
H(12;50,11,n_2,1,2) = P(50,11,12,1) + p(50,11,12,2) P(39,n_2,10,0) = .184081 + .288024 P(39,n_2,10,0)
\]

Again \( n_1 + n_2 \geq 16 \) and we find by trial

\[
H(12;50,11,9,1,2) = .184081 + .288024(.047261) = .197963 \\
H(12;50,11,8,1,2) = .184081 + .288024(.069764) = .204175
\]
so we must have \( n_2 \geq 9 \). Similarly

\[
H(3;50,11,n_2,1,2) = \Pr(50,11,3,1) + \Pr(50,11,3,2) \Pr(39,n_2,1,0) = .882143 + .109439 \Pr(39,n_2,1,0)
\]

By trial we find \( H(3;50,11,32,1,2) = .901786 \),

\( H(3;50,11,33,1,2) = .898980 \) so that \( n_2 \leq 32 \)

Continuing we find all the solutions for \( c_1 = 1, c_2 = 2 \).

These are

\[
\begin{align*}
 n_1 &= 11, \quad 9 \leq n_2 \leq 32 & n_1 &= 18, \quad 0 \leq n_2 \leq 7 \\
 n_1 &= 12, \quad 5 \leq n_2 \leq 26 & n_1 &= 19, \quad 0 \leq n_2 \leq 5 \\
 n_1 &= 13, \quad 3 \leq n_2 \leq 22 & n_1 &= 20, \quad 0 \leq n_2 \leq 4 \\
 n_1 &= 14, \quad 2 \leq n_2 \leq 16 & n_1 &= 21, \quad 0 \leq n_2 \leq 3 \\
 n_1 &= 15, \quad 1 \leq n_2 \leq 14 & n_1 &= 22, \quad 0 \leq n_1 \leq 1 \\
 n_1 &= 16, \quad 0 \leq n_2 \leq 11 & n_1 &= 23, \quad n_2 = 0 \\
 n_1 &= 17, \quad 0 \leq n_2 \leq 9 \\
\end{align*}
\]

**Example 3.2**

Find some double sampling plans for the binomial case with \( p_0 = .05, p_1 = .20, \alpha_0 = .05, \beta_1 = .10 \).

**Solution**

For the single sample case the inequalities (1.2) are

\[
E(y_0;n,.05) \leq .05 \quad \text{and} \quad E(y_0;n,.20) \geq .90 .
\]

From the Ordnance Corps [7] table we find that the inequalities require

with \( c = 0 \) \( n \leq 1, n \geq 11 \) no solution possible

\( c = 1 \) \( n \leq 7, n \geq 18 \) no solution possible

\( c = 2 \) \( n \leq 16, n \geq 25 \) no solution possible

\( c = 3 \) \( n \leq 28, n \geq 32 \) no solution possible

\( c = 4 \) \( n \leq 40, n \geq 38 \) or \( 38 \leq n \leq 40 \)

\( c = 5 \) \( n \leq 53, n \geq 45 \) or \( 45 \leq n \leq 53 \)

etc.

Double sampling plans are possible for \( c_2 \geq 4 \). With
From inequalities (2.3) and (2.4) we know that
\[ E(4; n_1, 0.20) \geq 0.90 \text{ or } n_1 \geq 32 \]
\[ E(5; n_1, 0.05) \leq 0.05 \text{ or } n_1 \leq 40 \]

With \( n_1 = 32, y_1 = 4, y_2 = 5 \) the power at \( p = 0.20 \) is
\[ K(0.20; 32, n_2, 4, 5) = E(5; 32, 0.20) + b(4; 32, 0.20) E(1; n_2, 0.20) \]
\[ = 0.79562 + 0.11129 \]
Using the fact that \( n_1 + n_2 \geq 38 \) we find by trial
\[ K(0.20; 32, 13, 4, 5) = 0.79562 + 0.11129(0.94502) = 0.90079 \]
\[ K(0.20; 32, 12, 4, 5) = 0.79562 + 0.11129(0.93128) = 0.89926 \]
so that \( n_2 \geq 13 \) makes Power \( \geq 0.90 \). Similarly
\[ K(0.05; 32, n_2, 4, 5) = E(5; 32, 0.05) + b(4; 32, 0.05) E(1; n_2, 0.05) \]
and by trial we obtain \( K(0.05; 32, 15, 4, 5) = 0.04904 \),
\[ K(0.05; 32, 16, 4, 5) = 0.05028 \text{ so that Power } \leq 0.05 \text{ if } n_2 \leq 15 \].

As in the hypergeometric case, having found one solution, the next one requires less trial. With \( n_1 = 33 \) we find
\[ K(0.20; 33, 8, 4, 5) = 0.90239, K(0.20; 33, 7, 4, 5) = 0.89794 \text{ and} \]
\[ n_2 \geq 8. \text{ Then } K(0.05; 33, 12, 4, 5) = 0.04959, K(0.05; 33, 13, 4, 5) \]
\[ = 0.05115. \text{ and } n_2 \leq 12 \].

Continuing the calculations we soon have all the solutions
for \( c_1 = 3, c_2 = 4 \). These are
\[ n_1 = 32, \quad 13 \leq n_2 \leq 15 \]
\[ n_1 = 33, \quad 8 \leq n_2 \leq 12 \]
\[ n_1 = 34, \quad 6 \leq n_2 \leq 9 \]
\[ n_1 = 35, \quad 4 \leq n_2 \leq 7 \]
\[ n_1 = 36, \quad 3 \leq n_2 \leq 5 \]

Example 3.3
Find some double sampling plans for the Poisson case with
\[ \mu_0 = .05, \mu_1 = .20, \alpha_0 = .05, \beta_1 = .10. \]

**Solution**

For the single sample case the inequalities (1.2) are

\[ \mathbb{E}(y_0; .05n) \leq .05 \quad \text{and} \quad \mathbb{E}(y_0; .20n) \geq .90. \]

From the Molina [6] table (using linear interpolation) we find

with \( c = 0 \)

\[ .05n \leq .051, \quad .20n \geq 2.21 \]

\[ n \leq 1 \quad \text{no solution possible} \]

\[ c = 1, \quad .05n \leq .36, \quad .20n \geq 3.89 \]

\[ n \leq 7 \quad \text{no solution possible} \]

\[ c = 2, \quad .05n \leq .82, \quad .20n \geq 5.33 \]

\[ n \leq 16 \quad \text{no solution possible} \]

\[ c = 3, \quad .05n \leq 1.37, \quad .20n \geq 6.68 \]

\[ n \leq 27 \quad \text{no solution possible} \]

\[ c = 4, \quad .05n \leq 1.97, \quad .20n \geq 8.00 \]

\[ n \leq 39 \quad \text{no solution possible} \]

\[ c = 5, \quad .05n \leq 2.61, \quad .20n \geq 9.28 \]

\[ n \leq 52 \quad \text{or} \quad 47 \leq n \leq 52 \]

\[ c = 6, \quad .05n \leq 3.28, \quad .20n \geq 10.53 \]

\[ n \leq 65 \quad \text{or} \quad 53 \leq n \leq 65 \]

etc.

Double sampling plans are possible for \( c_2 \geq 5 \). With \( c_2 = 5 \) we can have \( c_1 = 0, 1, 2, 3, 4 \). Since all solutions are obtained in a similar manner, we will consider only the case \( c_1 = 4, c_2 = 5 (y_1=5, y_2=6) \). From inequalities (2.5) and (2.6) we know that \( \mathbb{E}(5; .20n_1) \geq .90 \) or \( n_1 \geq 40 \) and \( \mathbb{E}(6; .05n_1) \leq .05 \) or \( n_1 \leq 52 \).

With \( n_1 = 40, y_1 = 5, y_2 = 6 \) the power at \( \mu = .20 \) is

\[ K(.20; 40, n_2, 5, 6) = \mathbb{E}(6; 8) + p(5; 8) \mathbb{E}(1; .20n_2) \]

\[ = .80876 + .09160 \mathbb{E}(1; .20n_2) \]

Using \( .20n_2 = 5.5 \) and \( .20n_2 = 5.6 \) (5.5 and 5.6 are
successive table entries) yields

\[ K(.20;40, n_2^*, 5, 6) = .89999, .90002 \] respectively so we conclude

\[ .20n_2^* \geq 5.53, n_2^* \geq 28 \]. Similarly, the power at \( \mu = .05 \) is

\[ K(.05;40, n_2^*, 5, 6) = E(6;2) + p(5;2) E(1; .05n_2^*) \]

\[ = .01656 + .03609 E(1; .05n_2^*) \]

Using \( .05n_2 = 2.6 \) and \( .05n_2 = 2.7 \) yields

\[ K(.05;40, n_2^*, 5, 6) = .04997, .05022 \]

respectively so we conclude that \( .05n_2^* \leq 2.61, n_2^* \leq 52 \).

With \( n_1 = 41 \) the power at \( \mu = .20 \) is

\[ K(.20;41, n_2^*, 5, 6) = E(6;8.2) + p(5;8.2) E(1; .20n_2^*) \]

\[ = .82641 + .08485 E(1; .20n_2^*) \]

Using \( .20n_2 = 2.0 \) and \( .20n_2 = 2.1 \) yields

\[ K(.20;41, n_2^*, 5, 6) = .89978, .90087 \] respectively so we conclude

\[ .20n_2^* \geq 2.02, n_2^* \geq 11 \]. Similarly the power at \( \mu = .05 \) is

\[ K(.05;41, n_2^*, 5, 6) = E(6;2.05) + p(5;2.05) E(1; .05n_2^*) \]

\[ = .01850 + .03889 E(1; .05n_2^*) \]

where \( .01850 \) and \( .03889 \) were obtained by linear interpolation. Using \( .05n_2 = 1.6 \) and \( .05n_2 = 1.7 \) yields

\[ K(.05;41, n_2^*, 5, 6) = .04954, .05028 \]

respectively so we conclude \( .05n_2^* \leq 1.66, n_2^* \leq 33 \).

With \( n_1 = 43 \) using \( .05n_2 = .9 \) and \( .05n_2 = 1.0 \) led to \( .05n_2^* \leq .95 \). To determine whether \( n_2 \leq 19 \) or \( n_2 \leq 18 \)
the General Electric [2] table was used giving \( .05n_2 \leq .947, n_2 \leq 18 \).

Similarly, all solutions were found for \( c_1 = 3, c_2 = 4 \).

These are

\[
\begin{align*}
&n_1 = 40, \quad 28 \leq n_2 \leq 52 & n_1 = 47, \quad 0 \leq n_2 \leq 6 \\
&n_1 = 41, \quad 11 \leq n_2 \leq 33 & n_1 = 48, \quad 0 \leq n_2 \leq 5 \\
&n_1 = 42, \quad 7 \leq n_2 \leq 24 & n_1 = 49, \quad 0 \leq n_2 \leq 3 \\
&n_1 = 43, \quad 5 \leq n_2 \leq 18 & n_1 = 50, \quad 0 \leq n_2 \leq 2 \\
&n_1 = 44, \quad 3 \leq n_2 \leq 14 & n_1 = 51, \quad 0 \leq n_2 \leq 1 \\
&n_1 = 45, \quad 2 \leq n_2 \leq 11 & n_1 = 52, \quad n_2 = 0 \\
&n_1 = 46, \quad 1 \leq n_2 \leq 9 & & & & \\
\end{align*}
\]
REFERENCES


