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In this paper we suggest a new measure of the importance of a component in a coherent system and derive some of its properties. The measure is for the case of components not undergoing repair proportional to the expected reduction in the remaining system life-time due to the failure of the component. This measure seems to be a useful guide during the system development phase as to which components should receive the most urgent attention in order to increase the system's expected life-time. The properties of the measure are compared with the ones of a measure suggested by Barlow and Proschan [1].

coherent structure
component importance
structural importance
min cuts

1. Basics concerning coherent systems.

Consider a system consisting of n components. For simplicity we will first restrict to the case where the components and hence the system can not be repaired. Let $(i=1,\dots,n)$

$$X_i(t) = \begin{cases} 1 & \text{if i-th component functions at time } t \\ 0 & \text{if i-th component is failed at time } t. \end{cases}$$

Assume in the following that $X_1(t),\dots,X_n(t)$ are mutually independent for any $t \geq 0$. Introduce

$$\underline{X}(t) = (X_1(t),\dots,X_n(t))$$

and let

$$\phi(\underline{X}(t)) = \begin{cases} 1 & \text{if system functions at time } t \\ 0 & \text{if system is failed at time } t, \end{cases}$$

where ϕ is the system's structure function.

Let now the i-th component have an absolutely continuous life distribution $F_i(t)$ with density $f_i(t)$. Then the reliability of this component at time t is given by

$$P(X_i(t)=1) = 1 - F_i(t) \stackrel{\text{def}}{=} \bar{F}_i(t).$$

Introduce

$$\underline{\bar{F}}(t) = (\bar{F}_1(t),\dots,\bar{F}_n(t)).$$

Then the reliability of the system at time t is given by

$$P(\phi(\underline{X}(t))=1) = h(\underline{\bar{F}}(t)),$$

where h is the system's reliability function.

The following notation will be used

$$(\cdot_i, \underline{x}) = (x_1, \ldots, x_{i-1}, \cdot, x_{i+1}, \ldots, x_n).$$

1.1. Definition.

The i-th component is irrelevant to the structure ϕ iff

$$\phi(1_i, \underline{x}) = \phi(0_i, \underline{x}) \quad \text{for all} \quad (\cdot_i, \underline{x}).$$

Otherwise the i-th component is relevant.

We note that an irrelevant component can never directly cause the failure of the system. As an example of such a component consider a condensor being in parallel with an electrical device in a large engine. The task of the condensor is to cut off high voltages which may have destroyed the electrical device. Hence although being irrelevant the condensor can be very important in increasing the life-time of the device and hence the life-time of the whole engine.

1.2. Definition.

A system is coherent iff $\phi(\underline{x})$ is nondecreasing in each argument and each component is relevant.

In the following we restrict to coherent systems.

1.3. Definition.

A cut set is a set of components whose failure is sufficient to cause system failure. A cut set is minimal if it can not be reduced and still be a cut set.

We also need the following notation. Let $C = \{i \mid 1 \leq i \leq n\}$ be the set of components comprising the system and let $M \subseteq C$. Then

i) $\underline{x}^M$ = vector with elements $x_i, i \in M$

ii) M^c = subset of C complementary to M.

1.4. Definition.

The coherent system (M,χ) is a module of the coherent system (C,ϕ) iff

$$\phi(\underline{x}) = \psi[\chi(\underline{x}^M), \underline{x}^{M^c}],$$

where ψ is a coherent structure function and $M \subseteq C$.

Intuitively, a module is a coherent subsystem that acts as if it were just a component.

1.5. Definition.

A modular decomposition of a coherent system (C,ϕ) is a set of disjoint modules $\{(M_k,\chi_k)\}_{k=1}^{r}$ together with an organizing structure ψ; i.e.

i) $C = \bigcup_{i=1}^{r} M_i$ where $M_i \cap M_j = \emptyset \quad i \neq j$

ii) $\phi(\underline{x}) = \psi[\chi_1(\underline{x}^{M_1}), \ldots, \chi_r(\underline{x}^{M_r})]$.

2. Existing measures of importance of system components.

There seems to be two main reasons for giving a measure of importance of system components. Firstly, it permits the analyst to determine which components merit the most additional research and development to improve overall system reliability at minimum cost or effort. Secondly, it may suggest the most efficient way to diagnose system failure by generating a repair checklist for an operator to follow. Lambert [5] reviews three different measures of importance of components in a coherent system.

Birnbaum [2] defines the importance of the i-th component at time t by:

$$I_B^{(i)}(t) = P[\phi(1_i,\underline{X}(t)) - \phi(0_i,\underline{X}(t)) = 1],$$

which in fact is the probability that the system is in a state at time t in which the functioning of the i-th component is critical; i.e. the system functions if the i-th component functions and is failed otherwise.

Vesely and Fussel [3,6] suggest as a definition of the importance of the i-th component at time t:

$I_{V-F}^{(i)}(t) = P$ [A cut set containing the i-th component has failed at $t|$system has failed at t].

This definition takes into account the fact that a failure of a component can be contributing to system failure without being critical.

One objection against the mentioned definitions when applied during the system development phase, is that they both give the importance at fixed points of time leaving for the analyst to determine which points are important. This is not the case for the definition by Barlow and Proschan [1] giving the (time-independent) importance of the i-th component by:

$I_{B-P}^{(i)} = P$ (The failure of the i-th component coincides with the failure of the system).

Now obviously

$$I_{B-P}^{(i)} = \int_0^\infty I_B^{(i)}(t) f_i(t) dt$$

$$= \int_0^\infty [h(1_i, \underline{\bar{F}}(t)) - h(0_i, \underline{\bar{F}}(t))] f_i(t) dt,$$

implying that the Barlow-Proschan measure is a weighted average of the Birnbaum measure, the weight at time t being $f_i(t)$.

We close this section by listing some of the properties of the latter measure.

2.1. Theorem.

Let the i-th component be in series (parallel) with the rest of the system. Let for $j \neq i$ $F_i(t) \geq F_j(t)$ $(\bar{F}_i(t) \geq \bar{F}_j(t))$ for all $t \geq 0$. Then $I_{B-P}^{(i)} \geq I_{B-P}^{(j)}$.

2.2. Theorem.

Assume the life distributions of the components to have proportional hazards, i.e.

$$\bar{F}_i(t) = \exp(-\lambda_i R(t)) \qquad \lambda_i > 0,\ t \geq 0,\ i=1,\ldots,n.$$

Then for a series system

$$I_{B-P}^{(i)} = \lambda_i / \sum_{j=1}^{n} \lambda_j ,$$

whereas for a parallel system

$$I_{B-P}^{(i)} = \lambda_i [\lambda_i^{-1} - \sum_{j \neq i} (\lambda_i + \lambda_j)^{-1} + \sum_{\substack{j<k \\ j,k \neq i}} (\lambda_i + \lambda_j + \lambda_k)^{-1} - \ldots + (-1)^{n-1} (\lambda_1 + \ldots + \lambda_n)^{-1}].$$

2.3. Theorem.

Let the i-th component be in series (parallel) with the rest of system. Then $I_{B-P}^{(i)}$ is increasing (decreasing) in $F_i(t)$ and in $\bar{F}_j(t)$, $j \neq i$.

Barlow and Proschan [1] gives the importance of the module (M,χ) and of the minimal cut set K by respectively

$I_{B-P}^{(M)} = P$ (The failure of M coincides with the failure of the system.)

$I_{B-P}^{(K)} = P$ (The failure of K coincides with the failure of the system.)

2.4. Theorem.

$$I_{B-P}^{(M)} = \sum_{i \in M} I_{B-P}^{(i)}$$

They also extend their measure to systems of components undergoing repair.

3. A suggestion of a new measure of importance of system components.

Since one during the system development phase would like to have at hand a time-independent measure of importance that accounts for the contribution to system failure the failure of a non-critical component, one should perhaps look for measures of the type

$$I_{V-F}^{(i)} = \int_0^\infty I_{V-F}^{(i)}(t) w_i(t) dt,$$

where

$$\int_0^\infty w_i(t) dt = 1.$$

The following suggestion of a new measure of importance embodies some of the spirit of $I_{V-F}^{(i)}$.

Intuitively it seems that components that by failing strongly reduce the remaining system life-time are the most important. This seems at least true during the system development phase. However, even when setting up a repair checklist for an operator to follow, one should just not try to get the system functioning. Rather one should try to increase the time until the system breaks down next. Introduce the random variable (r.v.)

Z_i = Reduction in remaining system life-time due to the failure of the i-th component.

We then suggest the following measure of the importance of the i-th component

$$I_N^{(i)} = E(Z_i) / \sum_{j=1}^{n} E(Z_j),$$

tacitly assuming $E(Z_i) < \infty$ $i=1,\ldots,n$. Obviously

$$0 \leq I_N^{(i)} \leq 1 \quad , \quad \sum_{i=1}^{n} I_N^{(i)} = 1 .$$

These relations are of course also true for the Barlow-Proschan measure.

Under the assumptions stated in Section 1, we can prove the following theorem

<u>3.1. Theorem.</u>

Let

$$\bar{H}_{i,t}^{1}(u) = \frac{\bar{F}_i(t+u)}{\bar{F}_i(t)} \qquad \bar{H}_{i,t}^{0}(u) = 0$$

and

$$\underline{\bar{H}}_t^{\underline{x}}(u) = (\bar{H}_{1,t}^{x_1}(u),\ldots,\bar{H}_{n,t}^{x_n}(u)).$$

Then:

$$E(Z_i) = \int_0^\infty \sum_{(\cdot_i,\underline{x})} \prod_{j\neq i} F_j(t)^{1-x_j}\bar{F}_j(t)^{x_j}$$

$$\int_0^\infty [h(\underline{\bar{H}}_t^{(1_i,\underline{x})}(u)) - h(\underline{\bar{H}}_t^{(0_i,\underline{x})}(u))]du\, f_i(t)\, dt \tag{3.1}$$

Proof. First note that the vector $\underline{\bar{H}}_t^{\underline{x}}(u)$ gives the conditional reliabilities of the components at time $t+u$ given the state vector of the components, $\underline{x}$, at time t. Now introduce the r.v.

$Y_t^{\underline{x}}$ = Remaining life-time for the system given the state vector of the components, $\underline{x}$, at time t.

Then

$$P(Y_t^{\underline{x}} > u) = P[\phi(\underline{X}(t+u)) = 1 | \underline{X}(t) = \underline{x}] = h(\underline{\bar{H}}_t^{\underline{x}}(u))$$

and

$$E(Y_t^{\underline{x}}) = \int_0^\infty P(Y_t^{\underline{x}} > u)du = \int_0^\infty h(\underline{\bar{H}}_t^{\underline{x}}(u))du.$$

Hence

$$\int_0^\infty [h(\underline{\bar{H}}_t^{(1_i,\underline{x})}(u)) - h(\underline{\bar{H}}_t^{(0_i,\underline{x})}(u))]du$$

equals the conditional expected reduction in remaining system life-time given that the i-th component failed at time t and that the state vector of the components just before t was $(1_i,\underline{x})$. Now the expression for $E(Z_i)$ follows by an ordinary conditional expectation argument.

3.2. Theorem.

For a series system we have

$$I_N^{(i)} = \frac{\int_0^\infty \bar{F}_i(v)\ln(\bar{F}_i(v)) \prod_{j\neq i} \bar{F}_j(v)dv}{\sum_{i=1}^n \int_0^\infty \bar{F}_i(v)\ln(\bar{F}_i(v)) \prod_{j\neq i} \bar{F}_j(v)dv}, \tag{3.2}$$

whereas for a parallel system we get

$$I_N^{(i)} = \frac{\int_0^\infty \bar{F}_i(v)\ln(\bar{F}_i(v)) \prod_{j\neq i} F_j(v)dv}{\sum_{i=1}^n \int_0^\infty \bar{F}_i(v)\ln(\bar{F}_i(v)) \prod_{j\neq i} F_j(v)dv} \tag{3.3}$$

Proof. For a series system (3.1) immediately reduces to

$$\int_0^\infty \prod_{j\neq i} \bar{F}_j(t) \int_0^\infty \prod_{j=1}^n (\bar{F}_j(t+u)/\bar{F}_j(t))du\, f_i(t)\, dt,$$

which by changing the order of integration establishes (3.2).

For a parallel system (3.1) reduces to

$$\int_0^\infty \sum_{(\circ_i,\underline{x})} \prod_{j\neq i} F_j(t)^{1-x_j} \int_0^\infty \prod_{j\neq i} (\bar{F}_j(t)-\bar{F}_j(t+u))^{x_j} F_i(t+u)du f_i(t)/\bar{F}_i(t)dt$$

$$= \int_0^\infty \sum_{(\circ_i,\circ_k,\underline{x})} \prod_{j\neq i\neq k} F_j(t)^{1-x_j} \int_0^\infty \prod_{j\neq i\neq k} (\bar{F}_j(t)-\bar{F}_j(t+u))^{x_j}$$

$$F_k(t+u)\bar{F}_i(t+u)du\, f_i(t)\, /\bar{F}_i(t)dt$$

$$= \int_0^\infty \int_0^\infty \prod_{j\neq i} F_j(t+u)\bar{F}_i(t+u)du\, f_i(t)\, /\bar{F}_i(t)dt,$$

which by changing the order of integration establishes (3.3).

Note the similarity between the expressions (3.2) and (3.3).

3.3. Theorem.

Consider a series (parallel) system. Let for $j \neq i$ $F_i(t) \geq F_j(t)$ $(\bar{F}_i(t) \geq \bar{F}_j(t))$ for all $t \geq 0$. Then $I_N^{(i)} \geq I_N^{(j)}$.

Proof. Note that for both (3.2) and (3.3) the numerator and denominator are negative. The theorem then follows since $\ln(\bar{F}_i(v))$ is an increasing function and $\bar{F}_i(v)\ln(\bar{F}_i(v))/(1-\bar{F}_i(v))$ a decreasing function of $\bar{F}_i(v)$.

Note that the corresponding Theorem 2.1 for the Barlow-Proschan measure is stronger.

3.4. Theorem.

Assume

$$\bar{F}_i(t) = \exp(-\lambda_i R(t)) \qquad \lambda_i > 0, \quad t \geq 0, \quad i=1,\dots,n,$$

where

$$\int_0^\infty R(t) \exp((-\sum_{i=1}^{n} \lambda_i)R(t))dt < \infty. \tag{3.4}$$

Then for a series system

$$I_N^{(i)} = \lambda_i / \sum_{j=1}^{n} \lambda_j . \tag{3.5}$$

Assume furthermore that $R(t) = t^\alpha$, $t \geq 0$, $\alpha > 0$; i.e. the life lengths of the components are Weibull-distributed with the same shape parameter. Then for a parallel system

$$I_N^{(i)} =$$

$$\frac{\lambda_i[\lambda_i^{-\beta} - \sum_{j\neq i}(\lambda_i+\lambda_j)^{-\beta} + \sum_{\substack{j<k\\ j,k\neq i}}(\lambda_i+\lambda_j+\lambda_k)^{-\beta} - \ldots + (-1)^{n-1}(\lambda_1+\ldots+\lambda_n)^{-\beta}]}{\sum_{i=1}^{n}\lambda_i[\lambda_i^{-\beta} - \sum_{j\neq i}(\lambda_i+\lambda_j)^{-\beta} + \sum_{\substack{j<k\\ j,k\neq i}}(\lambda_i+\lambda_j+\lambda_k)^{-\beta} - \ldots + (-1)^{n-1}(\lambda_1+\ldots+\lambda_n)^{-\beta}]} \qquad (3.6)$$

where $\beta = (1+1/\alpha)$.

Proof. For a series system the result follows immediately from (3.2). For a parallel system the numerator of (3.3) reduces to

$$(\lambda_i/\alpha)\int_0^\infty u^{1/\alpha}\exp(-\lambda_i u)\prod_{j\neq i}(1-\exp(-\lambda_j u))du$$

$$= (\lambda_i/\alpha)[\lambda_i^{-\beta}\Gamma(\beta) - \sum_{j\neq i}(\lambda_i+\lambda_j)^{-\beta}\Gamma(\beta)$$

$$+ \sum_{\substack{j<k\\ j,k\neq i}}(\lambda_i+\lambda_j+\lambda_k)^{-\beta}\Gamma(\beta) - \ldots + (-1)^{n-1}(\lambda_1+\ldots+\lambda_n)^{-\beta}\Gamma(\beta)],$$

which gives (3.5).

Note that the result for a series system is, except for the assumption (3.4), identical to the one given for the Barlow-Proschan measure in Theorem 2.2. For a parallel system the measures are identical if $\alpha \to \infty$; i.e. no component will survive $t=0$.

3.5. Theorem.

Make the same assumptions as in the preceding theorem. Then for a series (parallel) system of n (2) components $I_N^{(i)}$ is increasing (decreasing) in $F_i(t)$ and in $\bar{F}_j(t)$, $j\neq i$. Furthermore

$$I_N^{(i)} \geq I_N^{(j)} \Leftrightarrow F_i(t) \geq F_j(t) (\bar{F}_i(t) \geq \bar{F}_j(t)) \quad \text{for all } t\geq 0. \qquad (3.7)$$

Proof. For a series system the results follow immediately from (3.5). For a parallel system of two components (3.6) reduces to

$$I_N^{(i)} = \frac{\lambda_i^{-1/\alpha}-\lambda_i(\lambda_1+\lambda_2)^{-\beta}}{\lambda_1^{-1/\alpha}+\lambda_2^{-1/\alpha}-(\lambda_1+\lambda_2)^{-1/\alpha}} \qquad i=1,2,$$

from which (3.7) follows immediately.

Now $(i \neq j)$

$$\frac{\partial I_N^{(i)}}{\partial \lambda_j} = \{\lambda_1\lambda_2(\alpha+1)[(\lambda_1^{1/\alpha}+\lambda_2^{1/\alpha})(\lambda_1+\lambda_2)^{1/\alpha}-\lambda_1^{1/\alpha}\lambda_2^{1/\alpha}]$$
$$-[(\lambda_1+\lambda_2)^{\beta}-\lambda_i^{\beta}][\lambda_j^{\beta}-(\lambda_1+\lambda_2)^{\beta}]\}/\{\alpha\lambda_i^{1/\alpha}\lambda_j^{\beta}$$
$$(\lambda_1+\lambda_2)^{2\beta}[\lambda_1^{-1/\alpha}+\lambda_2^{-1/\alpha}-(\lambda_1+\lambda_2)^{-1/\alpha}]\} ,$$

the numerator of which reduces to

$$\lambda_1\lambda_2\alpha[(\lambda_1^{1/\alpha}+\lambda_2^{1/\alpha})(\lambda_1+\lambda_2)^{1/\alpha}-\lambda_1^{1/\alpha}\lambda_2^{1/\alpha}]$$
$$+ (\lambda_1+\lambda_2)^{1/\alpha}\{2\lambda_1\lambda_2(\lambda_1+\lambda_2)^{1/\alpha}+\lambda_1^2[(\lambda_1+\lambda_2)^{1/\alpha}-\lambda_1^{1/\alpha}]$$
$$+ \lambda_2^2[(\lambda_1+\lambda_2)^{1/\alpha}-\lambda_2^{1/\alpha}]\}.$$

This expression is obviously positive. Hence $I_N^{(i)}$ is decreasing in $\bar{F}_j(t)$.

Since

$$I_N^{(i)} + I_N^{(j)} = 1,$$

$I_N^{(j)}$ is decreasing in $F_j(t)$ and the proof is completed.

Note that Theorem 2.3 which corresponds to the first part of the latter theorem for the Barlow-Proschan measure, is essentially stronger. (3.7) is obviously also valid for this measure in the case where the life distributions of the components have proportional hazards and we consider a series (parallel) system of $n(2)$

components. This suggests that we must go outside components having proportional hazards in order to find a case where the ordering of importance is different using the Barlow-Proschan measure and the measure suggested in this paper. The following conjecture seems natural.

3.6. Conjecture.

For a parallel system of 2 components with

$$\bar{F}_i(t) = \exp(-\lambda_i t^{\alpha_i}) \qquad \lambda_i>0,\ \alpha_i>0,\quad t\geq 0,\quad i=1,2$$

we can find $\lambda_1, \lambda_2, \alpha_1, \alpha_2$ such that

$$I_{B-P}^{(1)} > I_{B-P}^{(2)} \qquad I_N^{(1)} < I_N^{(2)} .$$

We close this section by defining and giving expressions for the importance of a module and of a minimal cut set.

Let the coherent system (C,ϕ) have the modular decomposition $\{(M_k,\chi_k)\}_{k=1}^r$. Introduce the r.v.

Z_{M_k} = Reduction in remaining system life-time due to the failure of the k-th module.

We then suggest the following measure of the importance of the k-th module

$$I_N^{(M_k)} = E(Z_{M_k}) / \sum_{j=1}^{r} E(Z_{M_j}) .$$

Making the same assumptions and using the same notation as in Theorem 3.1, we get

3.7. Theorem.

$$E(Z_{M_k}) = \sum_{i\in M_k}\int_0^\infty \sum_{(\cdot_i,\underline{x})} \prod_{j\neq i} F_j(t)^{1-x_j}\bar{F}_j(t)^{x_j}\int_0^\infty [h(\underline{\bar{H}}_t^{(1_i,\underline{x})}(u))$$

$$-h(\underline{\bar{H}}_t^{(0_i,\underline{x})}(u))]du[\chi_k(1_i,\underline{x}^{M_k})-\chi_k(0_i,\underline{x}^{M_k})]f_i(t)dt \ .$$

Proof. The proof is almost identical to the one of Theorem 3.1. We just have to take into account that the component whose failure coincides with the failure of the module, must be critical for the module just before failing.

Note that

$$E(Z_{M_k}) \leq \sum_{i\in M_k} E(Z_i).$$

Hence generally Theorem 2.4 is not valid for our measure. Note also that the importance of a module depends totally on the whole modular decomposition.

Let the coherent system (C,ϕ) have minimal cut sets $K_1,\ldots,K_s$. Introduce the r.v.

Z_{K_k} = Reduction in remaining system life-time due to the failure of the k-th minimal cut set.

We then suggest the following measure of the importance of the k-th minimal cut set

$$I_N^{(K_k)} = E(Z_{K_k})/\sum_{j=1}^{s} E(Z_{K_j}).$$

Making the same assumptions and using the same notation as in Theorem 3.1 we get

3.8. Theorem.

$$E(Z_{K_k}) = \sum_{i\in K_k} \int_0^\infty \sum_{(\cdot K_k,\underline{x})} \prod_{j\notin K_k} (F_j(t)^{1-x_j}\bar{F}_j(t)^{x_j}) \prod_{j\in K_k-\{i\}} F_j(t) \int_0^\infty h(\underline{\bar{H}}_t^{(1_i,\underline{0}^{K_k-\{i\}},\underline{x})}(u))du f_i(t)dt \ .$$

Proof. Again the proof is almost identical to the one of Theorem 3.1. Now we just have to note that the component whose failure coincides with the failure of the minimal cut set, must be the last one to fail within this set.

4. Final remarks on the suggested measure.

As for the Barlow-Proschan measure one obtains the structural importance of a component (module, minimal cut set) by setting

$$F_i(t) = F(t) \quad , \quad t\geq 0, \quad i=1,\ldots,n \ .$$

Now consider the case where the components undergo repair after failure, again assuming them to operate independently of one another. Specifically, while repair of one component is occuring, the remaining components continue to operate. Introduce the r.v.'s $(i=1,\ldots,n \ ; \ j=1,2,\ldots)$,

T_{ij} = Length of the j-th operating period for the i-th component,

D_{ij} = Length of the j-th repair period for the i-th component,

and assume the T_{ij}'s to be independent with distribution function $F_i(t)$ and the D_{ij}'s to be independent with distribution function $G_i(t)$. Furthermore let

V_i = Reduction in time until a functioning system fails due to the failure of the i-th component

W_i = Increase in time until a failed system functions due to the failure of the i-th component

$Z_i = V_i + W_i$.

We then suggest

$$I_N^{(i)} = E(Z_i) / \sum_{j=1}^{n} E(Z_j).$$

In order to arrive at a generalization of Theorem 3.1 one has to find the expected values of the following r.v.'s

$R_t^{\underline{x}}$ = Time until a functioning system fails given the state vector of the components, $\underline{x}$, at time t,

where $\phi(\underline{x}) = 1$ and

$S_t^{\underline{x}}$ = Time until a failed system functions given the state vector of the components, $\underline{x}$, at time t,

where $\phi(\underline{x}) = 0$. Assume

$$\bar{F}_i(t) = \exp(-\lambda_i t),\ \lambda_i > 0,\quad t \geq 0,\quad i=1,\ldots,n.$$

$$\bar{G}_i(t) = \exp(-\mu_i t),\ \mu_i > 0,\quad t \geq 0,\quad i=1,\ldots,n,$$

and remember the "lack of memory" property for the exponential distribution.

For a series system obviously

$$E(R_t^{\underline{1}}) = 1 / \sum_{i=1}^{n} \lambda_i,$$

whereas $E(S_t^{\underline{x}})$ seems hard to find.

For a parallel system

$$E(S_t^{\underline{0}}) = 1/\sum_{i=1}^{n} \mu_i,$$

whereas $E(R_t^{\underline{x}})$ is wanted. For a k-out-of-n system, which functions iff at least k out of the n components function, assuming $\lambda_i = \lambda$, $\mu_i = \mu$, $i=1,\ldots,n$, Halperin [4] gives an expression for $E(R_t^{\underline{x}})$. Using a duality argument an expression for $E(S_t^{\underline{x}})$ is straightforward. However, in this special case obviously all components must be equally important. Hence for the case where the components undergo repair a lot of research remains before our suggested measure can be of any practical value.

The same is true for the case where we allow components to be irrelevant. Our definition obviously extends tc this case. An expression for the measure seems, however, by no means easy to arrive at.

As a conclusion the measure suggested in this paper does not have as nice properties as the Barlow-Proschan measure. However, we are not sure that this is any objection. Anyway, it seems that the measure can be a useful guide, at least at the system development phase, as to which components should receive the most urgent attention in order to increase the system's expected life-time.

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