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RELIABILITY IMPORTANCE OF COMPONENTS

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ABSTRACT

The present report represents an entry RELIABILITY IMPORTANCE OF COMPONENTS which is to appear in the Encyclopedia of Statistical Sciences, Vol. 6 published by Wiley in 1985. It reviews different measures of reliability importance of components especially emphasizing recent developments.

RELIABILITY IMPORTANCE OF COMPONENTS

In reliability theory, see COHERENT STRUCTURE THEORY and MULTISTATE COHERENT SYSTEMS, one key problem is to find out how the reliability of a complex system can be determined from knowledge of the reliabilities of its components. However, trying to apply this theory on a large technological system, seems often almost impossible. This is due to a poor and often irrelevant data base, to little knowledge on human components and vague information on the dependencies coming into play. This was clearly demonstrated in the Reactor Safety Study [9] on the safety of nuclear reactors in the USA. Hence the use of risk analysis and reliability theory to back political decisions on some controversial safety issues, may at least be doubtful.

If, however, a political decision is already made, these disciplines can contribute essentially to improve the safety of a system. This seems to be the present philosophy for instance both in the existing nuclear industry and in offshore engineering. When aiming at such improvements measures of relative importance of each component to system reliability are basic tools. Firstly, it permits the analyst to determine which components merit the most additional research and development to improve overall system reliability at minimum cost or effort. Secondly, it may suggest the most efficient way to diagnose system failure by generating a repair checklist for an operator to follow.

SOME MEASURES OF IMPORTANCE OF SYSTEM COMPONENTS

Consider a system consisting of n components. As is true for most of the theory in this field, we shall here restrict to the case where the components and hence the system can not be repaired.

We shall also assume that we have a binary description of system and component states as in classical COHERENT STRUCTURE THEORY.

Let $(i=1, \dots, n)$

$$X_i(t) = \begin{cases} 1, & \text{if } i\text{th component functions at time } t, \\ 0, & \text{if } i\text{th component is failed at time } t. \end{cases}$$

For mathematical convenience the stochastic processes $\{X_i(t), t > 0\}$, $i=1, \dots, n$ are assumed to be mutually independent. Introduce

$$\underline{X}(t) = (X_1(t), \dots, X_n(t))$$

and let

$$\phi(\underline{X}(t)) = \begin{cases} 1, & \text{if system functions at time } t, \\ 0, & \text{system is failed at time } t, \end{cases}$$

where the system's structure function ϕ is assumed to be coherent.

Let now the i th component have an absolutely continuous life distribution $F_i(t)$ with density $f_i(t)$. Then the reliability of this component at time t is given by

$$P(X_i(t)=1) = 1 - F_i(t) \stackrel{\text{def}}{=} \bar{F}_i(t).$$

Introduce

$$\bar{\underline{F}}(t) = (\bar{F}_1(t), \dots, \bar{F}_n(t)).$$

Then the reliability of the system at time t is given by

$$P(\phi(\underline{X}(t))=1) = h(\bar{\underline{F}}(t)),$$

where h is the system's reliability function.

The following notation will be used

$$(\cdot, \underline{x}) = (x_1, \dots, x_{i-1}, \cdot, x_{i+1}, \dots, x_n).$$

Birnbaum [3] defines the importance of the i th component at time t by

$$I_B^{(i)}(t) = P[\phi(1_i, \underline{X}(t)) - \phi(0_i, \underline{X}(t)) = 1],$$

which in fact is the probability that the system is in a state at time t in which the functioning of the i th component is critical for system functioning. As in [1] it is not hard to see that

$$I_B^{(i)}(t) = \partial h(\bar{F}(t)) / \partial \bar{F}_i(t),$$

which is the rate at which system reliability improves as the reliability of the i th component improves. In [4] Vesely and Fussel suggest the following definition of the importance of the i th component at time t

$$I_{V-F}^{(i)}(t) = P(X_i(t)=0 | \phi(\underline{X}(t))=0).$$

Hence this definition takes into account the fact that a failure of a component can be contributing to system failure without being critical. However, also a failure of the i th component after system failure, but before time t is contributing to this measure. Another objection is that according to this measure all components in a parallel system are equally important at any time irrespective of their life distributions.

One objection against both measures above when applied during the system development phase, is that they both give the importance at fixed points of time leaving for the analyst to determine which points are important. This is not the case for the definition by Barlow and Proschan [2] giving the (time-independent) importance of the i th component by

$$I_{B-P}^{(i)} = P(\text{The failure of the } i\text{th component coincides with the failure of the system}).$$

Now obviously

$$I_{B-P}^{(i)} = \int_0^{\infty} I_B^{(i)}(t) f_i(t) dt = \int_0^{\infty} [h(1_i, \bar{F}(t)) - h(0_i, \bar{F}(t))] f_i(t) dt,$$

implying that the Barlow-Proschan measure is a weighted average of the Birnbaum measure, the weight at time t being $f_i(t)$.

Intuitively it seems that components that by failing strongly reduce the remaining system lifetime are the most important. This seems at least true during the system development phase. However, even when setting up a repair checklist for an operator to follow, one should just not try to get the system functioning. Rather one should try to increase the time until the system breaks down next. Introduce the random variable

Z_i = reduction in remaining system lifetime due to the failure of the i th component.

Natvig [5] suggests the following measure of the importance of the i th component

$$I_{N_1}^{(i)} = EZ_i / \sum_{j=1}^n EZ_j.$$

In [6] Z_i is given the following representation

$$Z_i = Y_i^1 - Y_i^0, \quad (1)$$

where

Y_i^1 = remaining system lifetime just after the failure of the i th component, which, however, immediately undergoes a minimal repair; i.e. it is repaired to have the same distribution of remaining lifetime as it had just before failing.

Y_i^0 = remaining system lifetime just after the failure of the i th component.

Also the distribution of Z_i is arrived at.

Let now T be the lifetime of a new system, and T_i the lifetime of a new system where the life distribution of the i th component is replaced by the corresponding one where exactly one

minimal repair of the component is allowed. As in [7] it follows from (1) that

$$Z_i = T_i - T,$$

which leads to

$$EZ_i = \int_0^{\infty} \bar{F}_i(t) (-\ln \bar{F}_i(t)) I_B^{(i)}(t) dt.$$

If instead a total repair of the i th component is allowed, i.e. the component is repaired to have the same distribution of remaining lifetime as originally, the expected increase in system lifetime is given by

$$EU_i = \int_0^{\infty} \int_0^t f_i(t-u) \bar{F}_i(u) du I_B^{(i)}(t) dt.$$

Finally, the expected increase in system lifetime by replacing the i th component by a perfect one, i.e. $\bar{F}_i(t)$ is replaced by 1, is given by

$$EV_i = \int_0^{\infty} F_i(t) I_B^{(i)}(t) dt.$$

Now let the components have proportional hazards, i.e.,

$$\bar{F}_i(t) = \exp(-\lambda_i R(T)) \quad \lambda_i > 0; t > 0, i=1, \dots, n,$$

where λ_i , $i=1, \dots, n$ are the proportional hazard rates.

In [6] the following measure is suggested

$$I_{N_2}^{(i)} = \frac{\partial ET}{\partial \lambda_i^{-1}} / \sum_{j=1}^n \frac{\partial ET}{\partial \lambda_j^{-1}}.$$

At least for the special case where components are exponentially distributed this measure is easily motivated since λ_i^{-1} is the expected lifetime of the i th component. As in [7] it is not hard to see that

$$\frac{\partial ET}{\partial \lambda_i^{-1}} = \lambda_i EZ_i.$$

We now define the measures

$$I_{N_3}^{(i)} = EU_i / \sum_{j=1}^n EU_j, \quad I_{N_4}^{(i)} = EV_i / \sum_{j=1}^n EV_j.$$

Hence, we see that all measures $I_{N_k}^{(i)}$, $k=1,2,3,4$ and $I_{B-P}^{(i)}$ are weighted averages of the Birnbaum measure. In [7] one is comparing the different weight functions. A preliminary conclusion seems to be that the $I_{N_1}^{(i)}$ measure is advantageous.

As a very simple example from [5] consider a series system of 2 components where

$$\bar{F}_i(t) = \exp(-\lambda_i t^{\alpha_i}) \quad \lambda_i > 0, \quad i=1,2; \quad \alpha_1=2, \quad \alpha_2=1; \quad t>0.$$

For instance for $\lambda_2/\sqrt{2\lambda_1} = 0.6$ we have

$$I_{B-P}^{(2)} = 0.494 < 0.506 = I_{B-P}^{(1)}, \text{ whereas}$$

$$I_{N_1}^{(2)} = 0.539 > 0.461 = I_{N_1}^{(1)}$$

Hence the ordering of importance is different using the two measures, illustrating the need for a theory behind the choice of measures.

Finally, the measures suggested in [2], [3] and [5] are generalized to the multistate case in [8]. As a concluding remark it should be admitted that the costs of improving the components are not entering into the measures reviewed here. Hence a continued research in this important field is needed.

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(COHERENT STRUCTURE THEORY
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