Experimental investigation of wave-induced drift in the presence of surface covers

by

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THESIS
for the degree of
Master of Science

(Master i Anvendt matematikk og mekanikk)

Faculty of Mathematics and Natural Sciences
University of Oslo

May 2015

Det matematisk- naturvitenskapelige fakultet
Universitetet i Oslo
Abstract

Wave induced drift currents and wave attenuation rates for various surface covers are investigated by means of PIV and ultrasonic wave gauges. Two sessions of experiments treating mechanically generated monochromatic waves propagating in a wave tank with and without presence of various stationary surface covers, are conducted. The results are evaluated and compared with already existing analytical solutions. A change in wave attention rates is observed for the presence of various surface covers. The wave attenuation rates increase with frequency.

The theoretical mean drift velocity results are discussed in the framework of analytical mean drift solutions developed in Christensen (2005). A pronounced backward drift is observed as a result from the experiments without presence of a surface cover. The backward drift could only be a result of longitudinal waves generated by the variations in surface tension in a thin elastic sheet. The presence of a stationary surface cover enhances mean drift velocities in the wave propagating direction compared with no surface cover.
Acknowledgements

There are many people who deserves more than just being mentioned by name. But this is all I have to offer at the moment.

First of all I want to thank my main supervisor Prof. Atle Jensen at UiO for showing me trust to work on this thesis. I would also like to thank my co-supervisors Dr. Kai Håkon Christensen and Dr. Graigory Sutherland for all your help, comments and advices. You have all taught me valuable lessons in terms of scientific work, both theoretical, experimental and out in the field. It has been a great pleasure to work with you all.

I would also like to thank Jean Rabault for helping me out with the laboratory experiments and for showing me various data processing tools. Thank you for all the valuable coffee breaks and discussions. Thank you, Olav Gundersen and Dr. Jostein Kolaas for helping me out with the experimental setup and data processing. And Bjørn Riise, for all your friendly figure encouragements.

I thank you all, dear fellow students, for giving me a great time at Blindern. I cannot imagine how this time would have been without you.

I also want to give thanks to my mum and dad and the rest of my family and my family in law, for encouraging me all the way throughout my studies and teaching me the most valuable lesson of my life; to know Jesus. A special thanks to my oldest brother Rolf, for encouraging me to study science.

Finally, I would like to thank my lovely wife, Katrine! All the best in me, is you.
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Chapter 1

Introduction

This work is a part of the Waves in Oil and Ice (WOICE) project with main attention on investigating the physical process of spreading of oil in ice free and ice covered waters. The project is relevant for increasing knowledge and reduce environmental risks associated with increasing petroleum activities in arctic environments e.g. by detection and response of oil spills. The strength of the project is the evident coupling between theory, laboratory experiments and field observations which are also reflected in this thesis.

The scope of this work is to investigate how various surface covers dampens waves and how the lost wave momentum leads to increased mean currents. Analytical transient and steady state solutions for mean horizontal drift currents are developed for waves in fluids covered by surface films in Christensen (2005). From the results, he shows that surface covers with different elastic properties yield non-similar mean current profiles. For an inextensible surface cover (e.g. ice), the mean drift is in the wave propagation direction, causing a jetlike behavior in the surface boundary layer. The jet was first observed in Weber and Saetra (1995). Surface covers with elastic properties yield a mean counterflow in the region close to the surface.

Laboratory experiments in a plunger generating wave tank are conducted in order to verify the analytical solutions.

The outline of this thesis is to first (i.e. Chapter 1) look at the historical development and the state of the art of the field of research. In Chapter 2 we consider the mathematical derivations from Christensen (2005). In Chapter 3 we look at the measurement methods and the experimental configuration. The results are presented and discussed in Chapter 4, and concluding remarks and future work are outlined in Chapter 5.

A brief summary and some preliminary results from field work conducted at Svalbard are presented in Appendix G.

1.1 Historical introduction

It is common knowledge that a surface cover on top of water waves contribute to wave attenuation. Plinius the elder stated in Secundus (77) (book 2, art.103, verse 54) that small portions of oil would smooth the small ripples at the sea. We also find some discoveries by Aristotle in his Problemata Physica. Not only did the ancient scientists
notice this effect, oil was also used by the fishermen. They poured out oil beneath the water surface, and when the oil rose to the surface the fishermen could easily see where the fish were due to improved light conditions. We also know of the miracle after St. Aiden (Scott, 1978) that English sailors in year 731 A.D. were given holy oil to use on the water surface in case of coming into a storm, and that it actually worked.

If we travel to the 18th century, Benjamin Franklin was as far as we know the first person to carry out scientific experiments. He dropped olive oil from a teaspoon on water ponds in England. A remarkable calming effect on the wind generated water waves were noticed (Franklin, 1774). In his explanation to the phenomena (in Section 453) he states that the wind does not "catch" upon the oily surface, such that the small ripples can't be made, but slides over the smooth surface and makes the oil slick slide a little bit. Franklin explains the phenomena as prevention of friction, the same effect we obtain when oiling machinery. Today we know that lack of friction is not the physical explanation to the damping, but Franklin's work contributed to the experimental method of analyzing problems, and a number of papers have been published due to his work.

From a laboratory experimental point of view, Franz Carl Achard was the first scientist to make a laboratory experiment on the topic in 1778 (Scott, 1978). He wanted to see how mechanically generated breaking waves were able to sink a ship by taking the time for the ship to be filled and sink. By using olive and fennel oil in his 4.3 x 1.2 x 1.2 meters wave tank, he discovered that oil had a calming effect on the water. However, the effect it was not remarkable. The surface was not uniformly covered by oil. Instead, the oil accumulated in small droplets, and it is likely that Achard considered the parts of the surface not covered by these droplets as clean. The effect of the wind in the breaking of waves at sea were ignored, although he refers to the article of Franklin.

The concept of surface tension was introduced by the Hungarian mathematician Johan Andreas von Segner in 1751. Although he did not give any significant contributions to the theory of capillary wave damping due to a oily surface cover, his discoveries gave rise to the theory introduced by Carlo Marangoni in 1871. Marangoni concluded that the surface had some kind of elasticity that changed the surface tension and exerted a force on the water surface. On the other hand, Marangoni gave the non-oily surface the elastic properties and not the oily one. But the important part is that the change in surface tension would be the dominating contribution to damping of water ripples caused by oil. The well known Marangoni effect is named after him, which will be described in detail later.

To the end of the 19th century people like Osborne Reynolds, Lord Rayleigh and Horace Lamb did a great breakthrough on the mathematical approach of describing the effect of waves damped by a surface slick. Reynolds is known as the first one to give a satisfactory explanation to the problem in 1880 (Scott, 1978), but he had some problems publishing his work on a complete form. Therefore, Rayleigh was in 1890 the first one to publish something clarifying the mechanics involved. Lamb was in 1895 the first publishing mathematical description of waves damped by a surface cover in his famous work "Hydrodynamics". The fundamentals in the mathematical derivations are the same as used today, which will be verified in Section 2. Although Lamb derived analytical models for the calming effect of oil on water waves, he only
solved the attenuation rate equations for extreme values of elasticity (i.e. fully elastic and inelastic). He further assumes that the attenuation rate increases monotonically from the fully elastic- to the inelastic limit.

Veniamin Grigorievich Levich was in 1941 the first to couple the wave attenuation properties with the change in the dilational elastic modulus (Hansen and Ahmad, 1971), which he defined as

\[ E = -\frac{d\sigma}{d(\ln \Gamma)}, \]

where \( \sigma, \Gamma \) is surface tension and surface concentration of surface active material respectively. The imaginary part of the dilational elastic modulus describes viscous effects from the surface and the real part describes the elastic properties of the film. Levich investigated the problem for how the change in elastic modulus affected the wave damping with pure real values. But a error in sign lead him to the result that there was no maximum damping in the same range as Lamb already had investigated.

The results from Lamb and Levich where the leading assumption until Dorrestein (1951) used the same method as Levich. But unlike previous findings, he found that due to the viscosity and compressibility of the surface film a maximum occurs in the intermediate of the elasticity, also unlike the results from Lamb. His findings gave rise to the theory that are used today. Dorrestein sort of completed the linear theory that Lamb had developed.

Some years later Lucassen-Reynders and Lucassen (1969) fulfilled the linear problem in their paper *Properties of Capillary Waves*, where they review of the works of Lamb and Dorrestein, and validate the theory with experimental results.

1.2 State of the art - literature review

In the past 40 years, there have been extensive research on the effect of elastic and inelastic covers upon the water surface. In addition to wave damping and drift velocities this review also consider experimental methods and configurations in order to measure fluid properties. Therefore, the present literature review consider a wide span of papers treating various topics.

Hühnerfuss et al. (1981) performed experiments in a wave tank equipped with a wind-wave generator. They needed results to compare with observations from the Joint North Sea Wave Project (JONSWAP) project in 1975 where they poured out several artificial oil films outside the island of Sylt and measured the wave properties with a microwave scatterometer. They showed that viscous attenuation rate from an oleyl alcohol surface cover on surface waves was confined to frequencies greater or equal to 2 Hz. At lower frequencies, the attenuation rate was explained by other effects as modified wind input and wave-wave interactions. In their wind-wave tank experiments they did the same runs with and without the oleyl alcohol and found that the clean surface did not damp as much as the covered surfaces. They compared the damping coefficient for clean surface and oil covered surface with analytical solutions for viscous fluids given by Phillips (1977)
\begin{align}
\alpha_v &= 2\nu k^2, \\
\alpha_f &= \frac{1}{2} \nu k \sqrt{\frac{\omega}{2\nu}},
\end{align}
(1.1) (1.2)

where \(\nu, k, \omega\) are kinematic viscosity, wave number and angular velocity respectively. \(\alpha_v\) is for clean water and \(\alpha_f\) is for the surface covered by an inextensible. They showed that \(\alpha_f\) could not account for higher observed values of damping.

A couple of years later, Hühnerfuss et al. (1983) compared laboratory experiments with results from the Marine Remote Sensing (MARSEN) field work in 1979 where they produced three different oil slicks, two with oleyl alcohol and one with methyl oleate and recorded the wave properties with a microwave scatterometer. The laboratory experiments were carried out in a wind-wave tank some years earlier and the wave attenuation ratio was calculated from the wind-wave spectra. The two alcohols (oleyl alcohol and methyl oleate) had identical hydrophobic compound, but different rheological properties. They found that various oil films damp waves different, and that damping on gravity waves is significant for an oleyl alcohol surface film for frequencies above 0.7 Hz. They also adjusted the “direc” damping effect from (Hühnerfuss et al., 1981) of oleyl alcohol surface films to become significant at frequencies above 1.7-2.0.

In (Hühnerfuss et al., 1984) the writers continued to address the problem of how different surface-active materials affect the wave-attenuation of surface waves. The wave experiments were performed in a wind-wave tank with plunger generated waves of 1.0 Hz, 1.3 Hz, 1.8 Hz, 2.0 Hz, 2.3 Hz and 2.5 Hz that propagated over clean water and then over a surface covered. To determine the wave-damping, they consider a wave with amplitude \(a_0\) at some initial point, and \(a_x\) after some distance \(x\) such that
\[a_x = a_0 e^{-\Delta x}\]
(1.3)

where \(\Delta\) is the damping coefficient. Among many conclusions, they state that surface active substances with straight chain saturated alkyl groups possess the most intensive wave damping potential (i.e. alkyl groups with single hydrogen atom bonds), and it is evident that various surface active materials damp waves differently. They also discuss the effect by introducing longitudinal/Marangoni waves, where the surface tension gradients may destabilize an interface which can cause waves.

Some years later, Alpers and Hühnerfuss (1989) considered the influence of the Marangoni effect due to wave attenuation. The attenuation by viscoelastic surface films is now attributed to the Marangoni effect which dampens the waves with a resonance-type interaction in the short gravity wave region, see figure 1.1. The Marangoni effect (also called Marangoni waves and longitudinal waves) occurs as a result of variations in surface tension, due to extension and contraction of the water surface. The behavior consequently generates tangential drag forces exerting waves in the longitudinal direction. In theory, the resonant behavior occurs when the dispersion relation for the Marangoni wave intersect, or approach, the curve of the dispersion relation for the gravity capillary waves. The maximum attenuation rate occurs for various frequencies and wave numbers based on the properties of the fluid and the surface cover. For different kinds of oleyl acids and alcohols, the relative maximum damping coefficient
is confined between 4 and 7 Hz. They state the fact that contrary to common belief the viscoelastic films exerts the strongest damping on the short surface waves and not the inextensible films, the same conclusion as Dorrestein.

Craik (1982) contributed with important discoveries in the sense of mean drift currents and wave damping. He discusses the importance of viscosity in producing Eulerian drift currents in the presence of small amplitude waves. Considering an incompressible/inextensible film and second order Eulerian drift currents. The main discussion is about how the mean wave induced drift currents from waves on a covered surface alters the classical Stokes drift. Energy from the spatial or temporally decaying surface are transferred to the mean drift within the fluid. At this time, the absence of precise experiments could not prove Craik’s results.

Weber and Førland (1989) continue doing a theoretical analysis as Craik, to show that an inextensible surface cover with length much larger than the wavelength has a large influence on the mean wave induced drift currents. The damping due to the presence of a surface cover will affect the drift in some way. As Craik (1982), they also compared the classical inviscid Stokes drift at the surface \( u_s = \zeta_0^2 \omega k \) (derived in Appendix B), where \( \zeta_0, \omega \) and \( k \) are amplitude, angular frequency and wavenumber respectively, with a new analytical solution \( u \) by the non-dimensional drift velocity

\[
u_s = \frac{u}{u_s}.
\]

For the small waves/capillary waves regime the horizontal drift velocity does not seem to exceed the stokes drift significantly unlike the results from Craik. For gravity waves the maximum mean velocity exceeds the stokes drift in a larger extent. For long gravity waves, as long as one could stretch the theory (i.e. 100cm) for the influence of a surface active material, the maximum mean velocity exceeds the stokes drift by a factor of 5. The authors also show that in a capillary wave field a large slick/surface cover will move slower than the surrounding water.
A few years later Weber and Saetra (1995) left the inextensible surfaces and developed theoretical analysis on how elastic insoluble surface covers affect the drift velocity of capillary-gravity waves on the basis of a Lagrangian description, excluding forcing from the atmosphere with temporally amplitude attenuation. The maximum damping occurs as a result of the resonant behavior with Marangoni waves. From theory they show that the maximum drift value is attained just below the water surface for high values of film elasticity. The damping effect modifies the drift velocity significantly, such that the mean drift profiles obtain entirely different shapes for various elasticities. The authors also state the fact that they are also lack of any experimental evidence.

In order to verify some of the theory of inextensible sheets Kang and Lee (1996) performed laboratory experiments on drift of a vinyl-sheet covering the surface for waves with various slope. In the experiment, the Stokes drift velocity was measured using a video-camera by tracing the trajectory of a cork particle of at most 1 mm diameter on the surface. The drift of the vinyl sheets was calculated the same way. They concluded that Stokes theory predicts the drift of a single particle at the surface fairly well and that the theory of Phillips’ for drift of inextensible surface viscous layers appears valid for layer length less than half of the wave-length.

Mass and Milgram (1998) studied the behavior of sea surfactants at low time scale (i.e. from 0.04s to 2s) in order to say something about the water wave behavior due to extensible surface covers. The waves were in the frequency range 4 Hz to 25 Hz. In their experiment, they used two wave tanks; One generating longitudinal waves, and one generating transverse waves. The latter wave tank was equipped with wave gauges, in order to resolve wave attenuation. The wave tanks were also equipped with devices measuring the static surface tension versus surface area. The static surface tension was measured using a Wilhelmy plate. They also measured elasticity $E_s$ from the dilational modulus $E_s = \frac{d\Pi}{d(lnA)}$, where $\Pi$ is the change in surface tension and $A$ is the surface area.

From the longitudinal wave tank experiments, they compared two methods measuring surface tension; The transverse wave laser phase meter method (TWLPM) and the Wilhelmy plate method, and obtained similar results. In contrast to earlier results, they found that the film pressure of an undisturbed oleyl alcohol film did not change with time after the surfactant was spread on the surface.

They also compared the results with theory developed in Hansen and Ahmad (1971) looking at the ratio between the amplitude of surface tension oscillations $|T'|$ (i.e. longitudinal waves) and the amplitude from transverse waves. They find excellent agreement with theory on this point.

Another paper treating experiments and theory on inextensible films is Law (1999). He developed theoretical solutions on wave-induced surface drift of inextensible sheets up to third order of wave steepness and performed some experiments to verify the analytical results. The wave flume used in this experiment has dimensions 45 x 1.6 x 3 meters long, wide and deep respectively, using a wave paddle to generate the waves. To obtain a clean surface, they used a powerful fan to collect all the impurities at the end of the tank. The fan operated for about one hour between every experiment. Law ignores accelerations which leads to no diffusion from the boundary layer. In his paper,
Law concludes that the drift velocity theory developed by Phillips (1977) is too low and that the theory developed in his paper, which estimates higher drift velocities than the results from Phillips, is better.

Gushchin and Ermakov (2004) did a laboratory study of the surfactant redistribution under action of a gravity wave field. The authors investigated stationary distributions of the Surface Tension Coefficient (STC) (i.e. the surface tension) in the film and compared it with theory.

In the experiment the authors used a wave paddle to generate water waves in a tank about 4 m long and 0.3 m wide focusing on two regions: one clean and one "polluted" with Oleic acid. 4Hz waves were generated with wave amplitude variations from 2.5 to 3.5 mm near the film boundary. The surfactant distributions in the wave field were determined from analysis of film samples taken from the working region of the tank. A vessel containing the surface film surfactant was placed on a low-frequency dynamic loudspeaker with frequency 50 Hz. From pictures, they measured the standing waves pattern produced in the vessel. The STC, \( \sigma \), was calculated from the dispersion relation for gravity-capillary waves

\[
\omega^2 = (gk + \frac{\sigma k^3}{\rho}) \tanh(kh)
\]

where \( g \), \( \rho \) and \( h \) are the acceleration of gravity, density and water depth respectively.

The results indicated that the wave amplitude decreases in distance while propagating in the polluted area. The STC also decreases in the same manner. The behavior of the STC distribution is compared with theory developed by Foss (2000). They find that the analytical solution satisfactorily describes the STC dependence of distance.

Later on, new theoretical analysis for time dependent drift currents due to inextensible surface films derived in an earlier paper by J.E. Weber was compared with experiments from Law (1999) in Christensen and Weber (2005). They conclude that the new time dependent solutions predict the results from the experiments better than the earlier steady-state solutions developed by Phillips (1977) which underestimated the drift. The lack of good experiments is still an issue in the sense of taking the spatial attenuation of waves for a freely drifting sheet to account. Since earlier experiments have considered time averaged solutions, the time development in the drift velocity was not possible to compare.

Some years later, Law and Huang (2007) presented more results from already conducted laboratory experiments on the drift of inextensible polyethylene films. The drift of the films increased rapidly and became stable after a while for deep water waves. The magnitude of the steady drift increased with the longitudinal length of the film, until the wave length was at the same size as the film.

Siddiqui and Loewen (2007) used Particle Image Velocimetry (PIV) in order to measure velocity properties of a field beneath the water surface for both contaminated and clean surfaces in presence of wind generated waves. Their results showed a viscous sublayer beneath both surfaces. The mean streamwise horizontal velocity at a depth of 4 mm beneath the surface was 25-30% larger for the contaminated surface. The mean velocity gradients are more than doubled for the contaminated surface compared to
clean water.

From the review of all these papers, this is the first time an experimental investigation of mean drift currents beneath non-drifting surface covers are carried out, as of the authors knowledge. The hypothesis is that drift velocities are higher than the classical inviscid Stokes drift for the presence of stationary surface covers, and that the attenuation rates change for various surface covers.
Chapter 2

Mathematical derivations

The linear theory from Christensen (2005) is fully derived in this chapter, and a brief explanation on how to obtain solutions up to second order are presented. We start with deriving the dynamic boundary conditions for the problem in Section 2.1. In the next Section, 2.2, we look at the fluid motion equations for a Lagrangian description. Next, first and second order theory are derived respectively by using perturbation series, in Section 2.4 and 2.5.

The main difference between the solutions for covered and non-covered viscous fluids has its origin in the dynamic boundary condition. From Appendix A the relation between the surface tension $\sigma$ and dilatation modulus $E$ is derived. The relation appears in the dynamic boundary condition yielding different mean drift solutions for various surface covers.

2.1 Dynamic boundary conditions

For a fluid film boundary (e.g. oil) that separates two fluids, see figure 2.1, we assume that the density $\rho$ in the upper fluid (e.g. air), denoted by $\wedge$, is much smaller than the density in the lower fluid (e.g. water). $\mu$ and $\nu$ are dynamic and kinematic viscosity respectively.

Cauchy’s stress tensor (Gjevik, 2002, e.g.) for the upper fluid yields

$$\hat{P} = \begin{pmatrix} -\dot{p} + \hat{\tau}_{xx} & \hat{\tau}_{xz} \\ \hat{\tau}_{xz} & -\dot{p} + \hat{\tau}_{zz} \end{pmatrix}, \quad (2.1)$$
where \( \hat{p} \) is the pressure in the upper fluid and \( \hat{\tau} \) is the shear-stress tensor is defined as

\[
\hat{\tau}_{ij} = \hat{\mu} \left( \frac{\partial \hat{u}_i}{\partial x_j} + \frac{\partial \hat{u}_j}{\partial x_i} \right). \tag{2.2}
\]

Cauchy’s stress tensor in the lower fluid yields

\[
\mathbf{P} = \begin{pmatrix} -p + \tau_{xx} & \tau_{xz} \\ \tau_{xz} & -p + \tau_{zz} \end{pmatrix}, \tag{2.3}
\]

where \( p \) is the pressure in the lower fluid and \( \tau \) is the shear-stress tensor

\[
\tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right). \tag{2.4}
\]

We consider the tension on the fluid film in figure 2.2 and 2.3 where upper and lower fluid are denoted by subscript \( u_f \) and \( l_f \) respectively, \( \tau_{f,1}, \tau_{f,2} \) is the shear-stress tensors from the system balance and \( \mathbf{n} \) and \( \mathbf{t} \) are normalized normal and tangential vectors respectively to the surface.

From balance on the upper fluid and the fluid film in figure 2.2 we obtain

\[
\tau_{f,1} = -\tau_{uf} = -\hat{\mathbf{P}} \cdot \mathbf{n} = \hat{\mathbf{P}} \cdot \mathbf{n}. \tag{2.5}
\]

The lower part on the film, from figure 2.3, yield

\[
\tau_{f,2} = -\tau_{lf} = -\mathbf{P} \cdot \mathbf{n}. \tag{2.6}
\]
2.1. Dynamic Boundary Conditions

\[ z = \zeta(x, t) \]
\[ -\sigma(x)t(x) \]
\[ \sigma(x + dx)t(x + dx) \]

Figure 2.4: Fluid interface where the surface elevation \( z = \zeta(x, t) \). \( \sigma \) is the surface tension.

Let \( z = \zeta(x, t) \) be the surface elevation, see figure 2.4. We have that \( dx = (dx, dz) = (dx, d\zeta(x, t)) \) such that the surface tension and the normalized tangential vector, \( \sigma \) and \( t \), becomes functions of \( x \) for a stationary process. The balance from figure 2.4 yields

\[ -\sigma(x)t(x) + \sigma(x + dx)t(x + dx) = 0. \quad (2.7) \]

We do a series expansion of \( \sigma \) and \( t \) for small values of \( dx \) and obtain

\[ \sigma(x + dx) = \sigma(x) + dx \frac{\partial \sigma}{\partial x} + dz \frac{\partial \sigma}{\partial z} + O(dx^2), \quad (2.8) \]
\[ t(x + dx) = t(x) + dx \frac{\partial t}{\partial x} + dz \frac{\partial t}{\partial z} + O(dx^2). \quad (2.9) \]

Inserted in the balanced equation (2.7) we obtain to first order

\[ \sigma t_x dx + \sigma_1 t_1 dx + \sigma_2 t_2 dz = 0. \quad (2.10) \]

The normalized normal and tangential vectors to the surface are known as (Gjevik, 2002, e.g.)

\[ t = \frac{1}{\sqrt{1 + \zeta_x^2}} \begin{pmatrix} 1 \\ \zeta_x \end{pmatrix}, \quad (2.11) \]
\[ n = \frac{1}{\sqrt{1 + \zeta_x^2}} \begin{pmatrix} -\zeta_x \\ 1 \end{pmatrix}. \quad (2.12) \]

We calculate the derivative of the normalized tangential vector w.r.t. \( x \)

\[ t_x = \frac{1}{\sqrt{1 + \zeta_x^2}} \begin{pmatrix} -\frac{\zeta_x \zeta_{xx}}{1 + \zeta_x^2} \\ \zeta_{xx} - \frac{\zeta_x \zeta_{xx}}{1 + \zeta_x^2} \end{pmatrix}. \quad (2.13) \]

The length of an arc element \( ds \) is obtained by

\[ ds^2 = dx^2 + dz^2 \]
\[ \frac{ds^2}{dx^2} = 1 + \frac{dz^2}{dx^2} \]
\[ ds = \sqrt{1 + \zeta_x^2} \, dx, \quad \text{at} \ z = \zeta. \quad (2.14) \]
If we consider the total balance of the stress tensor (2.5) and (2.6) and the surface tension (2.7) we get

\[(\hat{P} \cdot \mathbf{n}) ds + (P \cdot (-\mathbf{n})) ds + \sigma(x + dx) \mathbf{t}(x + dx) - \sigma(x) \mathbf{t}(x) = 0. \tag{2.15}\]

The different terms yields

\[(\hat{P} \cdot \mathbf{n}) ds = \left( \hat{p}_{xx} \zeta_x - \hat{t}_{xx} \zeta_x + \hat{t}_{xz} - \hat{t}_{xz} \zeta_x - \hat{p} + \hat{t}_{zz} \right) dx, \tag{2.16}\]

\[(P \cdot (-\mathbf{n})) ds = \left( -p_{xx} \zeta_x + \tau_{xx} \zeta_x - \tau_{xz} \zeta_x \right) dx, \tag{2.17}\]

\[\sigma t_x dx + \sigma_x t dx + \sigma_z t dz = \left( \frac{1}{\zeta_x} \right) \frac{\sigma_x dx}{\sqrt{1 + \zeta_x^2}} \tag{2.18}\]

\[+ \left( \frac{1}{\zeta_x} \right) \frac{\sigma_z dx}{\sqrt{1 + \zeta_x^2}} \tag{2.19}\]

\[+ \left( \frac{-\frac{\zeta_{xx}}{\sqrt{1 + \zeta_x^2}}}{\zeta_{xx} - \frac{\zeta_{xx}^2}{\sqrt{1 + \zeta_x^2}}} \right) \frac{\sigma dx}{\sqrt{1 + \zeta_x^2}}. \tag{2.20}\]

If we insert the different terms into (2.15), multiply with \(\frac{1}{dx}\) and do a series expansion w.r.t. small \(\zeta_x^2\) we obtain the dynamic boundary conditions truncated to \(O(\zeta_x^2)\), in horizontal and vertical direction respectively, i.e.

\[\hat{p}_{xx} \zeta_x - \hat{t}_{xx} \zeta_x + \hat{t}_{xz} - p_{xx} \zeta_x + \tau_{xx} \zeta_x - \tau_{xz} + \sigma_x + \sigma_z \zeta_x - \sigma_x \zeta_x \zeta_{xx} = 0, \tag{2.21}\]

\[-\hat{t}_{xz} \zeta_x - \hat{p} + \hat{t}_{zz} + \tau_{xz} \zeta_x + p - \tau_{zz} + \sigma_x \zeta_x + \sigma \zeta_{xx} = 0. \tag{2.22}\]

We define \(P = p - \hat{p}\) as the hydrostatic pressure in the fluid assuming constant pressure above the interface, and \(\hat{t}, \hat{\sigma}\) as external tangential and normal stresses acting on the surface. By using the stress-shear-rate (2.2) we obtain the dynamic surface conditions

\[\mu(u_z + w_x) + P \zeta_x - 2\mu u_x \zeta_x + \sigma \zeta_x \zeta_{xx} - \sigma_x - \sigma_z \zeta_x = \hat{\sigma} - \hat{\tau} \zeta_x, \tag{2.23}\]

\[-P + 2\mu w_z - \mu(u_z + w_x) \zeta_x - \sigma \zeta_{xx} - \sigma_x \zeta_x = \hat{\sigma} - \hat{\tau} \zeta_x, \tag{2.24}\]

for the horizontal and vertical direction respectively.
2.2 Lagrangian description of a fluid field

Let \( \mathbf{r}_0 = (a, c) \) be the initial position coordinates to a fluid particle at time \( t = t_0 \), as in figure 2.5. We consider \( \mathbf{r} = (x, z) \) as functions dependent on \((a, c)\) and time \( t \), such that

\[
\mathbf{r} = \mathbf{r}(\mathbf{r}_0, t - t_0); \quad \mathbf{r}(\mathbf{r}_0, 0) = \mathbf{r}_0. \tag{2.25}
\]

\( \mathbf{r} \) is regarded as the path to a particle throughout the fluid. The velocity to a fluid particle, \( \mathbf{v}_l \), is the time derivative of its position such that

\[
\mathbf{v}_l(\mathbf{r}_0, t - t_0) = \frac{\partial}{\partial t} \mathbf{r}(\mathbf{r}_0, t - t_0). \tag{2.26}
\]

From equation (2.26), we obtain the distance from \( \mathbf{r}_0 \) to \( \mathbf{r} \) by

\[
d\mathbf{r} = \int_{t_0}^{t} \mathbf{v}_l(\mathbf{r}_0, t') \, dt', \tag{2.27}
\]

as shown in figure 2.5. \( \mathbf{v}_l(\mathbf{r}_0, t) \) is in literature known as the Lagrangian velocity of a particle at time \( t \). We now let \( t_0 = 0 \).

The velocity in an Eulerian description is defined as

\[
\mathbf{v}_E = \mathbf{v}_E(\mathbf{r}, t), \tag{2.28}
\]

which describes fluid properties in a fixed point. Then, the two velocities are related as (Lamb, 1932):

\[
\mathbf{v}_E(\mathbf{r}, t) = \mathbf{v}_l(\mathbf{r}_0, t). \tag{2.29}
\]

The acceleration is defined as the time derivative of the velocity such that

\[
\frac{\partial}{\partial t} \mathbf{v}_E(\mathbf{r}, t) = \frac{\partial}{\partial t} \mathbf{v}_l(\mathbf{r}_0, t) = \frac{\partial^2}{\partial t^2} \mathbf{r}(\mathbf{r}_0, t) = \mathbf{r}_{tt}, \tag{2.30}
\]

where where the subscript denote partial differentiation, in this case with respect to time.
We want to consider the governing equations in a Lagrangian reference frame. The equation of continuity for an incompressible fluid is given by (Pierson, 1962; Lamb, 1932),

\[
\frac{\partial (x,z)}{\partial (a,c)} = 1,
\]

(2.31)

where

\[
\frac{\partial (x,z)}{\partial (a,c)} = |x_a z_c - x_c z_a| = J(x,z),
\]

(2.32)

is the Jacobian determinant referred to in Christensen (2005). The result is obtained from considering the change in volume of a parallelepiped from time \( t_0 \) to \( t \), assuming that the density, \( \rho \), is conserved for any time \( t \). We make use of the Jacobian notation further on in the thesis.

The equation of motion for an incompressible fluid, disregarding Coriolis forces, in terms of Eulerian coordinates is written as (Pierson, 1962)

\[
x_{tt} = -\frac{p_x}{\rho} + \nu \nabla^2 u,
\]

(2.33)

\[
z_{tt} = -\frac{p_z}{\rho} - g + \nu \nabla^2 w,
\]

(2.34)

where \( p_i \) is the derivative of the pressure in the respective directions for \( i = x, z \). \( u, w \) are horizontal and vertical velocities respectively and \( \nu = \frac{\mu}{\rho} \) where \( \mu \) is the dynamic viscosity. The pressure gradient in Lagrangian description is obtained from

\[
\frac{\partial p}{\partial x} = \frac{\partial}{\partial x} \frac{\partial p}{\partial a} \frac{\partial a}{\partial x} + \frac{\partial}{\partial z} \frac{\partial p}{\partial a} \frac{\partial a}{\partial z},
\]

(2.35)

and a similar equation with derivative w.r.t. \( c \). We obtain by using Cramer’s rule a relationship between the Lagrangian and Eulerian description on the form

\[
\frac{\partial p}{\partial x} = \frac{\partial (p,z)}{\partial (a,c)} = J(p,z),
\]

(2.36)

\[
\frac{\partial p}{\partial z} = \frac{\partial (x,p)}{\partial (a,c)} = J(x,p).
\]

(2.37)

The Laplacian operator \( \nabla^2 \) in (2.33) is the remaining part that must be expressed in Lagrangian coordinates. In two dimensions, the operator is expressed as
2.3. STOKES DRIFT

\[
\nabla^2 u = \frac{\partial}{\partial x} \left( \frac{\partial x_t}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{\partial x_t}{\partial z} \right)
\]

\[
= \frac{\partial}{\partial (x, c)} \left( \frac{\partial (x, z)}{\partial (x, c)} \right),
\]

(2.38)

by using the subscript notation as in (2.30). A similar expression is obtained for \( \nabla^2 w \).

Then, the equations of motion are written as

\[
x_{tt} = -\frac{1}{\rho} J(p, z) + \nu \left\{ J\left[ J(x(t, z), z] + J[x, J(x, x_t)] \right] \right\}, \quad (2.39)
\]

\[
z_{tt} = -\frac{1}{\rho} J(x, p) - g + \nu \left\{ J\left[ J(z(t, z), z] + J[x, J(x, z_t)] \right] \right\}. \quad (2.40)
\]

In order to express the Eulerian coordinates \((x, z)\) in terms of Lagrangian coordinates we do a series expansion after an ordering parameter \(\epsilon\) (Pierson, 1962):

\[
x = a + \epsilon x^{(1)} + \epsilon^2 x^{(2)} + O(\epsilon^3),\]

(2.41)

\[
z = c + \epsilon z^{(1)} + \epsilon^2 (x^{(2)}) + O(\epsilon^3),\]

(2.42)

\[
p = -\rho gc + \epsilon p^{(1)} + \epsilon^2 p^{(2)} + O(\epsilon^3).\]

(2.43)

In our case, we choose \(\epsilon = \zeta_0 \omega k\) as a dimensional variable (Christensen, 2005). If we insert the series expansions (2.41) - (2.43) into the respective equations (2.39), (2.40) and (2.31) we see that \((x, z) = (a, c)\) and \(p = -\rho gc\) are exact solutions of these equations.

2.3. Stokes drift

Wave drift is the phenomena of particles moving along with the waves in some direction. This is in its nature a Lagrangian property. In 1847 Sir Gabriel Stokes used perturbation series to derive analytical solutions for weakly non-linear wave equations. The Stokes drift as a Lagrangian quantity is derived in Appendix B which give

\[
u_s = \zeta_0^2 \omega k e^{2kz}.\]

(2.44)

The drift is uniformly decaying with the water depth, see figure 2.6. The Eulerian description is a spatial distribution of fluid motion where information of the particle movement is not available. In this description, the Stokes drift in the fluid region becomes zero, and the mass transport is confined to the area between the wave crest and troughs. The approximate formula for Eulerian mean drift is given by Broström et al. (2014) as

\[
\overline{u_E} = \zeta_0 \omega / \pi \sqrt{(1 - z^2)/\zeta_0}.\]

(2.45)
2.4 Linear motion of surface covered waves

In order to derive the linear wave motion (or the first order theory) in a Lagrangian description we use equations (2.39) and (2.40) and solve them for the series expansion in equations (2.41) to (2.43) up to $O(\epsilon)$. We get

\begin{align*}
    x_{tt}^{(1)} &= -\frac{1}{\rho} \left( p^{(1)} + \rho g z^{(1)} \right)_a + \nu \nabla^2_L x_t^{(1)}, \\
    z_{tt}^{(1)} &= -\frac{1}{\rho} \left( p^{(1)} + \rho g z^{(1)} \right)_c + \nu \nabla^2_L z_t^{(1)},
\end{align*}

(2.46, 2.47)

where the two dimensional Laplacian operator is defined as (Pierson, 1962)

\begin{equation}
    \nabla^2_L = \frac{\partial^2}{\partial a^2} + \frac{\partial^2}{\partial c^2}.
\end{equation}

The continuity equation (2.31) yield

\begin{align*}
    x_a^{(1)} + z_c^{(1)} &= 0, \quad \text{or} \\
    x_{1a}^{(1)} + z_{1c}^{(1)} &= 0.
\end{align*}

(2.49)

As in Lamb (1932) the velocities are given as functions of a potential and a stream function

\begin{align*}
    x_t^{(1)} &= -\phi_a - \psi_c, \\
    z_t^{(1)} &= -\phi_c + \psi_a,
\end{align*}

(2.50)

where the potential function is an irrotational field and the stream function a divergence free field (Lucassen-Reynders and Lucassen, 1969). From (2.49) we obtain
2.4. LINEAR MOTION OF SURFACE COVERED WAVES

\[ \nabla^2 L \phi = 0. \]  \hspace{1cm} (2.51)

Insert the velocity functions in the momentum equations (2.46) and (2.47) which yield

\[ \left( -\phi_t + \frac{1}{\rho}p^{(1)} + gz^{(1)} \right)_a + \left( -\psi_t + \nu \nabla^2 L \psi \right)_c = 0, \]  \hspace{1cm} (2.52)

\[ \left( -\phi_t + \frac{1}{\rho}p^{(1)} + gz^{(1)} \right)_c - \left( -\psi_t + \nu \nabla^2 L \psi \right)_a = 0. \]  \hspace{1cm} (2.53)

To satisfy a wave field we set

\[ \psi_t = \nu \nabla^2 L \psi, \]  \hspace{1cm} (2.54)

\[ \phi_t = \frac{1}{\rho}p^{(1)} + gz^{(1)}. \]  \hspace{1cm} (2.55)

Wave solutions of (2.51), (2.54) and (2.55) are given by

\[ \phi = z_1(c)e^{ika + nt}, \]  \hspace{1cm} (2.56)

\[ \psi = z_2(c)e^{ika + nt}, \]  \hspace{1cm} (2.57)

where \( \kappa \) and \( n \) are complex parameters measuring the periodicity in the horizontal direction and in time respectively (Lucassen-Reynders and Lucassen, 1969). We require \( \text{Re}(\kappa) > 0 \). In order to decide the two coefficients we put the solutions (2.56) and (2.57) into (2.51) and (2.54) and solve w.r.t. \( z_1, z_2 \) such that

\[ z_1(c) = Ae^{\kappa c}, \]  \hspace{1cm} (2.58)

\[ z_2(c) = Be^{mc}, \]  \hspace{1cm} (2.59)

for

\[ m^2 = \kappa^2 + \frac{n}{\nu}, \]  \hspace{1cm} (2.60)

where \( \text{Re}(m) > 0 \). Inserting the coefficient in the above equations we see that this fulfill the criteria that \( \phi \to 0 \) when \( c \to -\infty \) obtaining

\[ \phi = Ae^{\kappa c} e^{ika + nt}, \]  \hspace{1cm} (2.61)

\[ \psi = Be^{mc} e^{ika + nt}. \]  \hspace{1cm} (2.62)

We are considering spatially damped waves (Christensen, 2005) unlike time damped waves and introduce \( n = -i\omega \) and \( \kappa = k + i\hat{\alpha} \) where \( \hat{\alpha} \) is a damping coefficient or attenuation rate. All three values are real and positive.

We assume the damping coefficient \( \hat{\alpha} \) to be “small” such that \( \hat{\alpha} \ll k \). We also assume \( \nu \) to be small such that \( \frac{\omega}{\nu} \) is large.
\( m \) is a complex value. We split \( m \) it into a imaginary and a real part such that

\[
m = m_r + im_i,
\]

where subscripts denote real an imaginary part. Insert (2.63) in (2.60) gives

\[
m_r^2 - m_i^2 + i2m_r m_i = k^2 - \hat{\alpha}^2 - i\left(\frac{\omega}{v} - 2k\hat{\alpha}\right).
\]

(2.64)

Dividing (2.64) into real and imaginary parts yields

\[
m_r^2 - m_i^2 = k^2 - \hat{\alpha}^2,
\]

(2.65)

\[
2m_r m_i = 2k\hat{\alpha} - \frac{\omega}{v},
\]

(2.66)

for small \( \hat{\alpha} \). We will see a posteriori that neglecting the \( k^2 \) in (2.65) is OK such that (2.65) is approximately fulfilled for \( m_r^2 = m_i^2 \). Inserted in (2.66) gives us

\[
m_r^2 = \pm \frac{\omega}{2v}.
\]

(2.67)

We choose the positive part since we are solving for the real part. From this we get

\[
m_r = \pm \sqrt{\frac{\omega}{2v}}.
\]

(2.68)

In order to fulfill (2.61) and (2.62) we must choose the positive solution. Then, from (2.66)

\[
m_i = -\sqrt{\frac{\omega}{2v}},
\]

(2.69)

and we define the inverse boundary layer thickness as

\[
\gamma = \sqrt{\frac{\omega}{2v}},
\]

(2.70)

a result obtained from a dimensional analysis from the momentum equation. We write

\[
m = (1 - i)\gamma.
\]

(2.71)

We assumed that \( \frac{\omega}{v} \) is “large”, which indicates that \( \gamma \) is large. The assumption more precisely indicates

\[
\frac{k}{\gamma} = \sqrt{\frac{2vk^2}{\gamma}} \ll 1.
\]

(2.72)

The formulation is critical for short waves (i.e. large \( k \)) and needs to be verified later along with the assumption that \( \hat{\alpha} \ll k \). From (2.72) we see a posteriori that neglecting the \( k^2 \) term in (2.65) is OK since \( m_r \sim m_i \sim \gamma \).

We normalize equation (2.61) and (2.62) such that \( B \) is relative to \( A \) and write
\[ \phi = e^{\kappa c} e^{i \kappa a + nt}, \]  
\[ \psi = B e^{m c} e^{i \kappa a + nt}. \]

From (2.50) and (2.55) we have that

\[ x^{(1)}_t = - \phi - \psi = -i \kappa \psi - m \phi, \]  
\[ z^{(1)}_t = - \phi + \psi = -\kappa \phi + i \kappa \psi, \]  
\[ \frac{p^{(1)}}{\rho} = \phi - g z^{(1)} = n \phi + \frac{g \kappa}{n} \phi - \frac{ig \kappa}{n} \psi. \]

Integrating the two first equation w.r.t. \( t \), we obtain the solutions to first order

\[ x^{(1)} = - \frac{i \kappa}{n} \left( e^{\kappa c} - i B m e^{m c} \right) e^{i \kappa a + nt}, \]  
\[ z^{(1)} = - \frac{\kappa}{n} \left( e^{\kappa c} - i B e^{m c} \right) e^{i \kappa a + nt}, \]  
\[ \frac{p^{(1)}}{\rho} = \frac{\rho}{n} \left[ e^{\kappa c} (n^2 + g \kappa) - ig \kappa B e^{m c} \right] e^{i \kappa a + nt}, \]

which is the same result as in Christensen (2005).

The dilational modulus of a film is defined in Christensen (2005) as

\[ E = - \frac{d \sigma}{d (\ln \Gamma)} = -\Gamma \frac{d \sigma}{d (\Gamma)}, \]

where \( \sigma, \Gamma \) is the surface tension and the concentration of film material respectively. The imaginary part of \( E \) describes the viscous properties of the film and the real part describes the elastic properties of the film.

Considering an insoluble film assumes no exchange of surface material with the fluid. From conservation of surface material the surface concentration can be expressed from Weber and Saetra (1995) as

\[ \Gamma = \frac{\Gamma_0}{\sqrt{x_0^2 + z_0^2}} \]

where \( \Gamma_0 \) is the surface concentration in a undisturbed case.

If we do a series expansion in the same manner as in equations (2.41) to (2.43) for the surface tension coefficient and the concentration we obtain respectively

\[ \sigma = T + \epsilon \sigma^{(1)} + \epsilon^2 \sigma^{(2)} + \mathcal{O}(\epsilon^3), \]  
\[ \Gamma = \Gamma_0 + \epsilon \Gamma^{(1)} + \epsilon^2 \Gamma^{(2)} + \mathcal{O}(\epsilon^3). \]

In order to obtain a consistent set of equations we need to expand the external stresses from (2.1)
\[ \hat{\tau} = \epsilon \hat{\tau}^{(1)} + \epsilon^2 \hat{\tau}^{(2)} + O(\epsilon^3), \]  
\[ \hat{\sigma} = \epsilon \hat{\sigma}^{(1)} + \epsilon^2 \hat{\sigma}^{(2)} + O(\epsilon^3). \]  
(2.85)  
(2.86)

If we assume no external stress to the first order, \( \hat{\tau}^{(1)} = \hat{\sigma}^{(1)} = 0 \) we obtain the dynamic boundary conditions from (2.23) and (2.24) by taking to account the relation between the elastic dilational modulus and the surface tension from (A.10) derived in Appendix A, using the notation from the Jacobian determinant in (2.32)

\[ \mu(\chi^{(1)}_{tc} + z^{(1)}_{ta}) = E\chi_{\alpha\alpha}^{(1)}, \quad c = 0, \]  
\[ -p^{(1)} + 2\mu z^{(1)}_{tc} = T\chi_{\alpha\alpha}^{(1)}, \quad c = 0. \]  
(2.87)  
(2.88)

In order to decide \( B \) in (2.78) to (2.80) we insert (2.78) and (2.79) into the horizontal boundary condition (2.87) and define

\[ \mathcal{E} = \frac{Ek^2\gamma}{\rho \omega^2}, \]  
(2.89)

as a non-dimensional elasticity parameter, such that \( B \) is given by

\[ B = \frac{\kappa}{\gamma} \left( \frac{\kappa}{\mathcal{E}} + i \mathcal{E} \right), \]  
(2.90)

where terms of order \( \frac{\kappa^2}{\gamma^2} \) in the denominator is truncated.

To find the dispersion relation we solve the vertical boundary condition (2.88) and obtain the relation

\[ -\frac{i}{\omega} \left( \check{g}\kappa - \omega^2 - iB\kappa \right) - 2v(\kappa(\kappa - imB)) = \frac{T}{\rho} \frac{k^3}{\omega} (1 - iB). \]  
(2.91)

From the dispersion relation, we obtain by using the boundary layer thickness \( \gamma \), inserting for \( \kappa \) and write \( B \) as \( B = B_r + iB_i \) where \( B_r, B_i \) are real and imaginary part respectively

\[ \omega^2 = \omega_0^2 (1 + B_i) + O\left(\frac{k^2}{\gamma^2}\right), \quad \omega_0^2 = \check{g}k + \frac{T}{\rho} k^3. \]  
(2.92)

From the imaginary part of the dispersion relation we obtain an expression for the non-dimensional damping coefficient as a function of the non-dimensional elasticity parameter (2.89)

\[ \frac{\check{\kappa}}{\kappa} = \frac{1}{2} \frac{c_p k}{c_g} \left( \frac{\mathcal{E}^2}{F} + \frac{2k(1 - \mathcal{E})}{\gamma F} \right), \]  
(2.93)

where \( c_p = \frac{\omega}{k} \) is the phase velocity, \( c_g = \frac{d\omega}{dk} \) is the group velocity and \( F = 1 - 2\mathcal{E} + 2\mathcal{E}^2 \).
If we consider the orders of magnitude we obtain for small $\mathcal{E}$

$$\frac{\hat{\alpha}}{k} \sim \frac{k^2}{\gamma^2}.$$  

For large $\mathcal{E}$ we have that

$$\frac{\hat{\alpha}}{k} \sim \frac{k}{\gamma}.$$  

From the assumptions made earlier with $\hat{\alpha} \ll k$ and (2.72). We verify this by choosing $\nu = 0.01 \text{cm s}^{-1}$ and $\lambda = 5\text{cm}$. We use the dominating contribution $\omega_0$ from the dispersion relation (2.92). Then

$$\frac{k}{\gamma} \simeq 0.03 \ll 1,$$

and

$$\hat{\alpha} = \frac{k^2}{\gamma} \simeq 0.03 \ll k \simeq 1.3.$$  

The earlier assumptions are well fulfilled.

Christensen (2005) show that the maximum attenuation coefficient is obtained for $\mathcal{E} = 1$ for a resonant frequency between the transverse waves and the longitudinal waves, with dispersion relation

$$\Omega = \sqrt{\frac{E\gamma}{\rho}k}. \quad (2.94)$$

He also argues to consider wave motion as a superposition of an irrotational part and a rotational part such that e.g.

$$\epsilon x^{(1)} = \tilde{x} + \dot{x}, \quad (2.95)$$

where $\tilde{()}$ and $\dot{()}$ are irrotational and rotational parts respectively. Thus, the irrotational part regards the capillary-gravity wave and the rotational part regards the dilational wave.

### 2.5 Second order wave drift theory

The second order mean drift equations are derived by inserting the first order solutions in (2.78) to (2.80) into the momentum equations (2.39) and (2.40) and the continuity equation (2.31) and collect the terms from the series expansion (2.41) to (2.43) to order $O(\epsilon^2)$. In order to obtain the mean drift velocities, the second order equations are averaged over a wave period. From Christensen (2005) a steady solution of the horizontal second order wave drift is derived as
\[ u = u^{(p)} + 6 \left[ \left( \frac{c}{h} \right)^2 + \left( \frac{c}{h} \right) \right] \frac{U^{(p)}}{h} \]
\[ - \left[ 3 \left( \frac{c}{h} \right)^2 + 4 \left( \frac{c}{h} \right) + 1 \right] u^{(p)}_0 - \left[ 3 \left( \frac{c}{h} \right)^2 + 2 \left( \frac{c}{h} \right) \right] u^{(p)}_b, \tag{2.96} \]

where

\[ u^{(p)} = u^{(S)} + u^{(v)}, \]
\[ u^{(S)} = e^{2kc - 2\alpha a}, \]
\[ u^{(v)} = \frac{3}{2} e^{2\gamma c - 2\alpha a} + \frac{2E}{F} \left( [1 - 2E] \cos \gamma c - \sin \gamma c \right) e^{2\gamma c - 2\alpha a}, \]
\[ U^{(p)} = \int_{-h}^{0} u^{(p)} dc, \]
\[ u^{(p)}_0 = \frac{2 - E^2}{2F}, \]
\[ u^{(p)}_b = u^{(p)}(c = -h), \]

where \( U^{(p)} \) is the particular solution integrated in the vertical direction \( U^{(p)} = \int_{-h}^{0} u^{(p)} dc \). \( u^{(p)}_0 \) is obtained from the particular solution setting \( a = 0 \). \( u^{(p)}_b \) is the particular solution inserted at the bottom \( u^{(p)}_b = u^{(p)}(c = -h) \). (2.96) is scaled on the maximum inviscid Stokes drift \( \zeta^2_0 \omega k \), from equation (2.44).

It is convenient to write \( u \) as a solution of three parts

\[ u = u^{(S)} + u^{(v)} + u^{(h)}, \tag{2.97} \]

where \( u^{(h)} \) corresponds to the right hand side of equation (2.96) except from \( u^{(p)} \), and is described as a quasi-Eulerian mean current resulting from diffusion of vorticity from the surface. \( u^{(S)} \) represents the inviscid Stokes drift and \( u^{(v)} \) is a vorticity solution confined to the surface boundary layer. The solution (2.96) is a result of a time-dependent solution where the time dependence is neglected by assuming the surface drift velocity to decay in time. The steady state solution also takes to account that the wave tank is closed in both ends, such that the net volume flux is zero over time. The pressure term that occurs in the momentum equations, takes to account the sloping surface due to mass transport.

The solution (2.96) is plotted in figure 2.7 for various elasticities. The counterflow for \( E = 0.5 \) is largest in the boundary layer close to the surface, as seen in the figure. For the inextensible limit, \( E \to \infty \), the boundary layer flow is smaller but a jetlike behavior occur in the wave propagation direction. The remarkable result is investigated in the next sections by experiments.
Figure 2.7: Steady state solution from (2.96), \( k = 22 \text{ m}^{-1} \). Waves propagating from right to left.
Chapter 3
Experimental setup and measurement methods

In the current chapter we present the experiments conducted at the hydrodynamic laboratory at the University of Oslo (UiO). The measurement methods are explained and discussed. The outline of this chapter is to first (i.e. Section 3.1) give a general overview of the measurement situation. In Section 3.2, we look at the method for measuring surface elevation. Then, we consider the method for doing fluid field velocity measurements in Section 3.3.

We separate between two sessions of experiments: One during fall 2014, and one during spring 2015.

In every experiment it is necessary to quantify the error. Errors related to data filtering and method validation are also included in this chapter.

Errors are often divided into two groups: Systematic- and random errors. The difference between these two can be explained by rifle shooting. If you shoot ten shots with your rifle and the grouping is really good, but off the bullseye, you can have a systematic error in your aiming device. On the other hand, if the grouping is good, except for one shoot, it indicates the occurrence of a random error. Maybe the trigger was drawn to fast. In experimental data; Strong biases often indicates systematic errors and outliers often indicates random errors.

3.1 Main experimental setup

The large wave tank in the hydrodynamic laboratory were used to carry out the experiments. The wave tank dimensions are 24.6 meters long, 0.516 meters wide and 0.98 meters deep. The main experimental setup is shown in figure 3.1. The wave tank was filled with water such that the water depth was 0.6 meters. The wave paddle is a high precision pressure driven paddle, controlled by a ®LABVieW program, that generates waves with different amplitudes, frequencies and shapes. The input value for the amplitude is in voltage [V] such that it is needed to measure the wave elevation using wave gauges.

In our experiments we used a hyperbolic-tangens-sinus-function for the generated waves with various steepness.
At the end of the wave tank there is an absorbing beach. The reflection coefficient from waves generated in our experiments was measured using the WaveAr software tool from Landry et al. (2012). The resulting reflection coefficients ranged between 1 and 4% of the generated waves.

![Main experimental setup.](image)

Figure 3.1: Main experimental setup. Yellow cylinders are wave gauges attached on the wave gauge array that are moved four times during one run. The array was moved at 5, 8, 11, 14 and 17 minutes after start-up. Surface cover is attached about 6.4m in x-direction from the wave paddle. A digital camera is placed at about 10 meters from the wave paddle.

3.1.1 Measurement equipment

The wave attenuation rates and the fluid velocity field were measured by means of quantitative imaging techniques and ultrasonic wave gauges.

- Falcon 2 4M digital camera, produced by Teledyne Dalsa.
- U-gauge S18U produced by Banner Engineering.
- USS02/HFP, IP 65, M18 x 1.0 wave ULS Advanced gauges produced by Ultralab.

3.1.2 Surface covers

For the purpose of simulating an elastic membrane or ice we used different surface covers with different properties to see if we could observe the same effects we obtain from theory, see Section 2.5. We used three different covers:

- Polyethylene garden cover, $0.5 \times 17 \times 0.0003$ meters
- Thin latex cover, $0.5 \times 15 \times 0.00025$ meters
3.1. MAIN EXPERIMENTAL SETUP

(a) Polyethylene cover in wave tank.  
(b) Fixing the surface cover.

Figure 3.2: Surface cover in wave tank experiments.

- Thick latex cover, 0.5 × 15 × 0.0005 meters

The polyethylene garden surface cover is the same that is used in gardens to avoid weeds to grow, see figure 3.2a. The two other latex sheets were purchased at the same provider as in Deike et al. (2013). The sheets were attached to prevent them from drifting, see figure 3.2b.

The elasticity of the surface covers are not measured but the polyethylene cover is inelastic, the thin latex cover is the most elastic one and the thick latex cover is in between.

3.1.3 Repeatability of the waves

To calculate the repetition error of the waves in the experiment the we use the formula

\[
\zeta_{1,R} = \sqrt{\frac{\sum_{n=0}^{N-1} (\eta_1(t_n) - \eta_R(t_n))^2}{\eta_1(t_n)^2}}, \quad R = 2, 3, \tag{3.1}
\]

which calculates the relative difference in percent between run R and run 1.

From experiments carried out earlier, two irregular wave fields (i.e. a superposition of various waves) were generated with characteristic steepness

\[\epsilon_s = a_c k_p = \begin{cases} 0.02 \\ 0.1 \end{cases}\]

where \(a_c, k_p\) are characteristic amplitude and peak wavenumber respectively. For both cases the experiment were carried out three times, see figure 3.3 giving the percent difference calculation from equation (3.1)

\[\epsilon_s = 0.02:\]

\[\begin{align*}
\zeta_{1,R} &= \begin{cases} 0.67, & \text{for } R = 2 \\ 0.85, & \text{for } R = 3 \end{cases}
\end{align*}\]

\[\epsilon_s = 0.1:\]

\[\begin{align*}
\zeta_{1,R} &= \begin{cases} 0.52, & \text{for } R = 2 \\ 0.63, & \text{for } R = 3 \end{cases}
\end{align*}\]
CHAPTER 3. EXPERIMENTAL SETUP AND MEASUREMENT METHODS

Figure 3.3: Repetition error in the wave tank. Three different runs were conducted with the same voltage input in the software that controls the wave paddle in order to see if the wave field is reproducible.

The maximum repetition error of 0.85% is considered good enough to state that the repeatability of the waves is satisfactory.

3.2 Wave gauges - calibration and filtering

In the current sub-section we present the different wave gauges, raw-data filtering and calibration routines.

3.2.1 Banner U-gauges

We used ultra sonic probes (U-gauge S18U) produced by Banner Engineering. The working principal of the probes is by sending and receiving an ultrasonic signal where the signal traveling distance is calculated using the formula

\[ D = \frac{c_a t}{2}, \]

where \( D \) is the distance, \( c_a \) is the speed of sound in air, \( t \) is the transit time. The sensing range is from 30 to 300 mm with a minimum window size on 5mm. A systematic error is that the speed of sound is dependent on the air temperature, but the gauges are temperature compensated such that the sensor will maintain its limits to within 1.8% over the -20 to +60 °C range. The sampling frequency in our experiment is 256 Hz.

The response time on the probes is 30ms which gives an accuracy window within ±0.5mm.

In order to decide the window limits for the measurements there are two alternatives; the built in auto-window feature, or deciding it manually.
3.2.2 Ultralab ULS Advanced gauges

In latter experiments we used USS02/HFP, IP 65, M18 x 1.0 gauges delivered by Ultralab with technical resolution at 0.18mm. The measuring range was from 30 mm up to 250 mm. In this wave gauge system one can choose between sampling frequency at 125 Hz and 250 Hz. We used the latter.

An advantage with this wave gauge system is that one can have several gauges receiving the same transmitted signal at one place, increasing the state of redundancy. Thus, surface elevation is resolved better by interpolating the signal dropouts.

3.2.3 Calibration U-gauges

From earlier experience the U-gauges must be calibrated in order to establish that they work properly. We fixed the gauges to a rod pointing down against the water surface in the wave tank. We lowered and lifted them three cm each way from mean level. From the raw data it could be verified if the gauges worked properly. 14 wave gauges were calibrated and the 8 best of them were chosen.

3.2.4 Filtering wave elevation U-gauges

Filtering the data from the U-gauges is necessary due to noise and dropouts. Outliers were removed at a given threshold and replaced by NaNs. Missing data is replaced by using linear interpolation.

The next step is to remove the noise from the data. We used a filter made by Dr. Odin Gramstad. The filter removes sharp peaks exceeding the smallest of two neighboring points at some threshold value and replace the peak with a cubic interpolation using the neighboring points. An example of the filtered versus raw data is shown in figure 3.4.

Not all of the wave gauges worked properly during the entire run. Some of the gauges had problems detecting the signal, especially when the surface elevation was small and comparable to the resolution accuracy of the wave gauge system. For instance, in the upper right plot in figure 3.5 the filter was not able to reproduce the correct signal, and the observed surface elevation in this case is almost two times the physical surface elevation. From a visual inspection from the post-processed data about 10% of the filtered data from the polyethylene cover runs had this error. It varies a lot, and the runs with a clean surface obtained significantly better results in terms of less noise. This is the reason for the increased measurement points.

The error of the U-gauges (Section 3.2.1) is 0.5mm. Due to initial amplitudes from Table 3.4 in order of $O(1)$ mm, it is expected that the amplitudes at the end will be in the noise level due to wave damping.
Figure 3.4: Filtered vs. raw data using U-gauges. Solid line is raw data

Figure 3.5: Filtered vs. raw data using U-gauges. Solid line is raw data
3.2. WAVE GAUGES - CALIBRATION AND FILTERING

3.2.5 Amplitude extraction and cumulative mean

In order to extract the amplitudes from the data stored by the wave gauges, we use a 1D discrete Fourier transform and its inverse defined as (Brigham, 1974, e.g.)

\[ \zeta(t_n) = \sum_{n=0}^{N-1} \hat{\zeta}(\omega_n)e^{-i\omega_n t_n}, \]  
(3.3)

\[ \hat{\zeta}(\omega_n) = \frac{1}{N} \sum_{n=0}^{N-1} \zeta(t_n)e^{i\omega_n t_n}, \]  
(3.4)

where the discrete values are denoted with \( n \). The amplitudes for the various frequencies contained in the data are obtained by considering the onesided Fourier transform

\[ p_{\zeta} = 2|\hat{\zeta}(\omega_n)|. \]  
(3.5)

A standard procedure in signal processing is to use a cumulative mean to verify if a signal is converging. If \( \bar{x} \) is the arithmetic mean

\[ \bar{x} = \frac{1}{N} \sum_{n=1}^{N} x_n, \]  
(3.6)

where \( x_n \) are independent measurements, the cumulative mean is the change in mean by increasing \( n \) to a integer value \( N + K \), where \( K > 0 \), such that the difference in the mean, \( d \), is

\[ d = \text{abs} \left( \frac{1}{N} \sum_{j=1}^{N} x_j - \frac{1}{N+K} \sum_{k=1}^{N+K} x_k \right). \]  
(3.7)

This method was applied on the time series from both wave gauges, as seen in see figure 3.6. All the runs were checked and a decay in the cumulative mean was obtained, yielding a convergence in the sense of surface elevation.

3.2.6 Wave decay rates

The non-dimensional damping coefficient \( \alpha \) and initial amplitude \( a_0 \) from the exponential fit curve in later figures is a result from a least square fit method in \textcopyright{}matlab from a function on the form

\[ f_x = a_0 e^{-k x \alpha}, \]  
(3.8)

where \( a_0 \) is initial amplitude guess and \( x \) is gauge distance from the wave paddle. The reason for choosing an exponential fit has its origin from the wave equations, e.g. equation (2.78) in Section 2.4 where the damping coefficient is found in the term

\[ e^{\kappa c} = e^{(k+i\alpha)c}. \]
CHAPTER 3. EXPERIMENTAL SETUP AND MEASUREMENT METHODS

3.3 PIV

Particle Image Velocimetry (PIV) was used to extract velocities from the fluid flow. A summary of the method and important features regarding our experiments are outlined in this sub-section.

3.3.1 General overview

The article of Sveen and Cowen (2004) is used as basis in this general overview. Particle Image Velocimetry (PIV) is a quantitative imaging technique which calculates velocity fields to a flow in Eulerian reference frame by pattern recognition algorithms. PIV is a relatively new method used in fluid mechanics to describe fluid properties. Since computer- and camera-systems have developed in such a large speed, PIV results are obtained much faster than they where just a decade ago.

From PIV we can capture the entire Eulerian fluid field properties, e.g., the velocity field, $u(x,z)$, the vorticity field, $\chi(x,z)$, the Reynolds stress field, $\overline{u'w'}(x,z)$ and the turbulent dissipation field, $F_d(x,z)$. This provides knowledge about e.g. the velocities, mass-transport and the energy that occurs in the fluid.

For a regular 2D PIV, a two dimensional sheet lightens up the actual measurement area and images of the flow are captured, see figure 3.7. A regular light source or laser
3.3. PIV

Figure 3.7: PIV system in wave tank.

can be used for the two dimensional light sheet. It is desirable that the depth of the 2D sheet is so thin that particles from other focal planes are not lightning up, avoiding introduction of noise. On the other hand, it must be deep enough such that the actual particles do not jump in and out of the light sheet due to small velocities perpendicular to the flow.

In a PIV algorithm, a picture is divided into sub-windows (interrogation windows) in size depending on e.g. image resolution and fluid velocities. On each sub-window a difference algorithm is applied to calculate a correlation peak between two pictures taken with a time delay $\Delta t$. The most common difference measure is cross-correlation. It uses sub-windows $I_i, J_j$ of two images $F', F''$, where $i, j$ denotes the pixel location and $s, t$ are the two dimensional cyclic lag at which the cross-correlation is being computed, we define the cross-correlation function as

$$ R(s, t) = \frac{1}{N^2} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} F'_{i,j}(i, j) F''_{i,j}(i + s, j + t), \quad (3.10) $$

where $N$ is the width and height of picture in pixels. It can also be computed in the spectral domain, making use of $F$ and $F^{-1}$ which denote Fourier and inverse Fourier transform

$$ R(s, t) = F^{-1} \left[ F^\ast \left\{ F'_{i,j}(i, j) \right\} F \left\{ F''_{i,j}(i + s, j + t) \right\} \right], \quad (3.11) $$

where star denotes the complex conjugate. This is known as the correlation-theorem and can be found in most introduction books on the topic (Sveen and Cowen, 2004, e.g.).

From the cross correlation function we note that the maximum particle displacement which can be resolved is $\frac{N}{2}$ pixels. If the displacement is in the interval $[\frac{N}{2}, N]$ such that the images are correlated with a few particles, the correlation peak from (3.10) will alias to the location $-(N - \xi)$ where $\xi$ is the actual displacement. If the displacement
is larger than $N$, the two images will be uncorrelated and the correlation peak will result in random noise.

An example of the velocity output from PIV is shown in figure 3.8 where the velocity vectors calculated from two images are plotted with a raw image. The same velocity field is plotted in figure 3.9 where the velocity vectors and the velocity vector modulus are plotted in the same plot.
Figure 3.8: Green velocity vectors from PIV on top of raw image.

Figure 3.9: Velocity vectors and modulus plot from PIV.
3.3.2 Particle size and particle seeding

The PIV method needs tracers. The actual fluid is seeded with passive tracers i.e. particles that do not affect the fluid flow field. The issue is solved by choosing particles that efficiently scatter light and are naturally buoyant with the fluid flow. Acceleration and density differences between the fluid flow and the particles causes measurement biases. In order to see if the particle follows the flow, one can do a rough estimate of the velocity bias between tracer particles and the fluid flow as explained in Longo et al. (2004), where

\[ U_g = \frac{d_p^2 (\rho_p - \rho)}{18 \mu} g, \]  

(3.12)

is a gravitationally induced velocity, \( d_p \) is the particle diameter and \( \rho_p \) is fluid particle density. In order to calculate the velocity lag \( (U_s) \) of a particle in a continuously accelerating fluid, one can do an estimate of equation (3.12) to calculate \( U_s \),

\[ U_s = U_p - U = \frac{d_p^2 (\rho_p - \rho)}{18 \mu} a, \]  

(3.13)

where \( U_p \) and \( U \) are the particle and fluid velocities respectively and \( a \) is the acceleration of the fluid flow.

The particles used in our experiment were 50\( \mu \)m diameter Polyamid Seeding Particles (PSP) delivered from Dantec Dynamics.

An estimate of the velocity bias is obtained by calculating the velocity lag from equation (3.13). Choosing particle density \( \rho_p = 1200[kg/m^3] \), water density at 20\(^\circ\)C \( \rho = 998.2[kg/m^3] \), dynamic viscosity in water \( \mu = 1.002 \times 10^{-3}[Ns/m^2] \) and find the order of magnitude of typical accelerations in the experiment as \( \varepsilon_\omega \sqrt{g/k} \approx 2 \) (for steepness 0.1) from equation (3.15). The velocity lag \( U_s = 5.59 \times 10^{-5}[m/s] \) which is of order \( O(10^{-3}) \) of the fluid velocities, which is considered good enough in our experiments.

The particle concentration in each interrogation window is also important. It is necessary to have sufficiently many particles in order to compute the cross correlation. Too many particles will introduce noise in the picture due to particle overlap.

3.3.3 Peak locking

In PIV we separate between random errors and bias errors. Random errors can for instance be electronic noise in cameras and problems identifying the sub pixel displacement in patterns. These errors can be fatal for the accuracy of instantaneous PIV results, but can be considered negligible when PIV fields are spatially or time averaged over a suitable large ensemble.

Regarding bias errors, peak locking is the most fatal one. The error often occurs if the particle image diameter is lower than 2 pixels. The computed particle displacement will be biased to the nearest integer pixel value due to the sub pixel displacement estimation (Westerweel, 1997). An example is shown in figure 3.10 which is a histogram
of the un-calibrated displacements. The peak locking (or pixel locking) effect could be due to small particle size or placing the camera to far away from the 2D light sheet.

![Graph showing displacement vs number of vectors](image)

**Figure 3.10:** Peak locking effect. Reproduced from Westerweel (1997).

Sveen and Cowen (2004) showed that using a Gaussian peak fit always yield better result than using a center-of-mass or a parabolic fit. The authors also state that peak locking is insignificant if one is interested in a mean velocity or variance as long as the histogram spans at least two integer pixels of displacement.

### 3.3.4 Camera and imaging

For the imaging system we used the Falcon 2 4Mpx monochromatic camera with an maximum frame rate at 168 fps. This contains a Complementary Metal Oxide Semiconductor (CMOS) image sensor which can deliver 8 or 10 bits of data on 8 or 10 taps. The setup with the camera is shown in figure 3.12a. More details about the camera and imaging setup and routine can be found in Appendix C.

We need a sufficient amount of tracer particles in the field of view (FOV) such that the signal is as good as possible. The shutter speed, frame rate, aperture size, camera resolution and FOV are critical parameters. If the shutter speed (or exposure time) is too high, particle streaks will occur and the image becomes useless for PIV. In addition, the exposure time can not be too low in order to get any signal at all.

![Aperture size for camera lens](image)

**Figure 3.11:** Aperture size for camera lens. Reproduced from www.infotor.com
The camera resolution and FOV are connected in terms of particle representation. For large FOV a high resolution camera is needed to obtain represent particles over several pixels. Too large FOV introduces peak locking in PIV. See FOV for the conducted experiments in Table 3.1.

The aperture size is important due to the depth of field and light conditions, as seen in figure 3.11. If you have large aperture, more light will expose the camera chip and the signal becomes better. The drawback is that the depth of field is shallower such that contributions from other focal plans will be larger. The optimal settings would be a small aperture opening. However, lack of light could give poor results. See Table 3.1 for aperture size in the conducted experiments.

The frame rate is adjusted in order to capture the motion of the fluid flow. If the time interval $\Delta t$ between the pictures is too large we are not able to reproduce the change in pattern.
3.3. PIV

Camera parameters for experiments:

-------------------------------------------------------------
First weeks of experiment
-------------------------------------------------------------
Frame rate: $fps = 50$
Duration: $T = 7 \, [s]$
Field of View, $FUV = 14 \times 19 \, cm$
Aperture size: $ap = 1 - 2$
Garden cover:
Black level: $bl = 2000$
Shutter speed: $sh = 90 - 110$
No cover:
Black level: $bl = 2000$
Shutter speed: $sh = 110 - 130$

-------------------------------------------------------------
Last weeks of experiment
-------------------------------------------------------------
Frame rate: $fps = 160$
Duration: $T = 7 \, [s]$
Field of View, $FUV = 14 \times 18 \, cm$
Aperture size: $ap = 1 - 1.2$
Garden and thin latex cover:
Black level: $bl = 4000$
Shutter speed: $sh = 160 - 240$
No cover:
Black level: $bl = 4000$
Shutter speed: $sh = 170 - 260$

Table 3.1: Camera parameters for capturing the fluid flow.
3.3.5 Light source

The VLX2 LED Line lightning delivered by Gardasoft Vision was used as light source. The led lamp was controlled by the computer using Tera Term. The intensity range of the lamp is from 0 to 140%, where 140% is equivalent with 2.3 million Lux. The area lighted up by the LED is shown in figure 3.12b.

The main reason for not using laser in this experiment was due to work conditions. Since the laboratory is used by many people, it was not convenient to use laser goggles for protection all the time. Also, since we measured long time sequences, we did not need the light sheet to be triggered in sequences.

3.3.6 Conversion from pixel to world coordinates

In order to do a transformation from pixels into world coordinates a picture of a coordinate system in the correct focal plane, see figure 3.13, is captured. The spacing between the dots are known. It is also wise to make use of a reference point in the picture in order to relate important parameters, such as water depth and distance from the wave generator.

Using DigiFlow (©Dalziel Research Partners) as software for PIV, it is possible to make a spatial transformation structure (tform) and generate a .dfc script which can be loaded into the DigiFlow software. The processing then yields velocities in world coordinates as output. Otherwise, this could be done later in the post-processing. A script written by Dr. Jostein Kolaas was used to find the tform matrix.

If the coordinate system is not orientated vertically with the water surface, it can be corrected using the rotation matrix

$$R = \begin{pmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{pmatrix},$$

(3.14)

where $\theta$ is the rotation angle.

Figure 3.13: Picture of coordinate system in wave tank.
3.3. **PIV**

<table>
<thead>
<tr>
<th>Angle (rad)</th>
<th>0.0130</th>
<th>0.0130</th>
<th>0.0127</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>0.0103</td>
<td>0.0140</td>
<td>0.0129</td>
</tr>
</tbody>
</table>

Table 3.2: *Angles in coordinate system from figure 3.13.*

<table>
<thead>
<tr>
<th>Direction</th>
<th>px</th>
<th>cm</th>
<th>ratio (px)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>32</td>
<td>0.2532</td>
<td>0.008</td>
</tr>
<tr>
<td>y</td>
<td>16</td>
<td>0.1265</td>
<td>0.008</td>
</tr>
</tbody>
</table>

Table 3.3: *Pixels vs. world coordinates during the first weeks of experiments.*

The coordinate reference plate was somewhat skewed the first weeks of experiments, see figure 3.13. Since the camera was placed horizontal, with help from a libelle on the camera tripod, the angle was found by using Pythagoras theorem. Since the camera was angled a little bit in the vertical direction, in order to image the true interface, the distance between the points near the surface could be larger than at the bottom. Therefore, the angle was measured at the top and bottom three times each. $\phi_1, \phi_2$ are angles from the bottom and the top respectively as shown in Table 3.2.

The mean of both angles are so similar that the error can be assumed negligible.

The ratio between pixels and world coordinates is shown in Table 3.3.

### 3.3.7 Dynamic interface filter

In order to carry out PIV on the water waves, it is necessary to remove the part of the picture which is above the wave-water interface. All the images were stored as .png-files in order to preserve the entire raw signal. The images were run through a dynamic filter which detected the interface from light intensity and thickness. Outliers from the detection process were filtered out using a median filter twice. A fifth order polynomial was used to interpolate the interface from the filtered data and all values above the interface were set to zero. Since DigiFlow has a “mask zeros” alternative preventing calculations on cells with zero value, this both increased computation efficiency and accuracy. An example from the filter is shown in figure 3.14. The thickness of the interface also varies in every run due to the particle concentration. In some cases, and especially for the polyethylene cover, many of the particles clung at the interface introducing light scatter.
3.3.8 Deciding interrogation window size

As explained in Section 3.3.1, the interrogation window size is a critical parameter. Increasing the window size (width or/and height) yields longer computation and decreases spatial resolution, but increases the accuracy (see DigiFLow user manual). Decreasing the window size yields faster computation, but lack of signal could give poor result. Another feature in PIV is the spacing, which specifies the vertical and horizontal spacing between the points where the matching is undertaken. Decreasing this number yields longer computation time, and does not always give a better result depending on the interrogation window size.

A smooth velocity profile is desired in the water waves. From changing the interrogation window size and the spacing, it is possible to decide the optimal interrogation window size by a visual inspection.

To determine the interrogation size, a small amplitude run at 2.10Hz from Table 3.4 was chosen. The maximum horizontal velocity at the same point, to the extent possible, is compared for different window sizes. The interrogation window sizes are on the form $2^n \times 2^n$ or $2^n \times 2^{n-1}$ for integer values $n = 4, ..., 7$. The reason for choosing rectangular window size is to resolve the vertical components better. The spacing was set to 50% overlap in every case. The results are shown in figure 3.15.

Looking at the smoothness in the profile shown in figure 3.15, the best result is obtained with $64 \times 32$ window size. As expected, a sawtooth behavior of the profile is obtained for the small interrogation windows e.g. $32 \times 16$ and $32 \times 32$. This could be due to lack of signal inside the interrogation window which increases the uncertainty in each window where the matching is undertaken.

Due to the results, all executions are done using the rectangular window size $64 \times 32$ with 50% overlap in order to resolve the velocity profile.
3.3.9 Validation of set-up and PIV parameters used in processing

In order to validate the velocity output from PIV, a selection of vector fields were compared with an analytical solution of Stokes weakly non-linear wave solution to third order for gravity waves as explained in Grue et al. (2004). The velocity potential is written by

\[
\phi(x, z, t) = \varepsilon_s \sqrt{g/k} e^{kz} \sin \theta + \mathcal{O}(\varepsilon_s^4),
\]

where \(\varepsilon_s = \zeta_0 k\) is the steepness or wave slope and \(\theta = (kx - \omega t)\), with the dispersion relation

\[
\omega^2 = gk(1 + \varepsilon_s^2) + \mathcal{O}(\varepsilon_s^4).
\]

This is a higher order approximation of the velocity potential. The surface elevation is written as

\[
k\eta = \left(1 + \frac{1}{8} \varepsilon_s^2 \right) \varepsilon_s \cos \theta + \frac{1}{2} \varepsilon_s^2 \cos 2\theta + \frac{3}{8} \varepsilon_s^3 \cos 3\theta + \mathcal{O}(\varepsilon_s^4).
\]

From the potential we obtain the horizontal and vertical velocity components

\[
u(x, z, t) = \varepsilon_s \sqrt{g/ke^kz} \cos \theta + \mathcal{O}(\varepsilon_s^4),
\]

\[
w(x, z, t) = \varepsilon_s \sqrt{g/ke^kz} \sin \theta + \mathcal{O}(\varepsilon_s^4).
\]
In order to find the steepness one may solve the system of equation (3.16) and (3.17) truncated to the 4th order, the latter for maximum $\eta$ such that all cosine terms are 1:

\[
\omega^2 = gk(1 + \varepsilon_s^2),
\]

(3.20)

\[
k\eta_{\text{max}} = \left(1 + \frac{1}{8}\varepsilon_s^2\right) \varepsilon_s + \frac{1}{2}\varepsilon_s^2 + \frac{3}{8}\varepsilon_s^3.
\]

(3.21)

For comparison with the experiments the velocities are scaled on $\varepsilon_s \sqrt{g/k}$ and the maximum horizontal velocity is obtained when the cosine term in (3.18) is 1. The vertical velocity component is zero for maximum horizontal velocity.

Some test samples for various steepness-values were taken out of the post-processed material in order to verify the quality of the parameters chosen in processing. Crucial parameters, such as vertical coordinate $z$ and wave number $k$, were checked.

Another important parameter is the surface elevation measured by the wave gauges. We did not have a wave gauge placed exactly over the measurement area for the first experiments. The amplitude used as scaling parameter was obtained by taking a mean of surface elevation measurements within a short distance before and after the area of interest. Since the amplitudes are used as scaling parameters, there was a systematic error in the difference between the results and (3.18), as seen in figure 3.16c. From the figure we see that the largest difference appears at the bottom of the profile. As already explained, the uncertainty related to the scaling amplitude could change the occurrence of maximum difference. Therefore, it is most important to consider the profile behavior, and to find out if the error is constant.

The vertical velocity, $w(x, y, t)$ is also plotted. From theory, the vertical velocity should be zero beneath the crest. In figure 3.16b we see that this is not the case. Due to peak locking, as seen on figure 3.17, this bias effect occurs at some of the low negative velocities in the vertical direction. This could be an explanation for some of the points.

Although we have included a vertical column before and after the assumed wave crest, we cannot be entirely sure if the true vertical column for the maximum horizontal velocity is captured. From Table 3.3, the spacing between the columns is 0.2532 cm which hampers the possibility by reaching the maximum horizontal velocity. Negative vertical velocities are directed up towards the surface in world coordinates, see an example in figure 3.16b. In pixel coordinates, this corresponds to positive velocities since coordinate system reference point is changed to zero at the interface.

Another error is the lack of measurements regarding the water depth in the wave tank. The water depth was measured to 60cm at day one. The next days water depth measurements were forgotten. Although the wave tank was leaking minimal, the decrease in water depth due to evaporation (cf. information from the lab-journal two weeks earlier) were approximately 3 mm a day. This affects the zero-level in the coordinate system such that the analytical velocity profiles are underestimated.

On this basis, we consider the validation of the profiles as good enough to continue doing further PIV analysis with the same parameters.
Comparison with theoretical profiles are also carried out for frequencies 1.50, 2.10, 2.20, 2.30 and 2.40 Hz with a variety of initial amplitudes. See the figures in Appendix F.
Figure 3.16: Validation of 2.10Hz run with $a_0 = 0.387$ cm. $\bullet$ indicates vertical column beneath the crest, $\times$ the vertical column to the right, and $+$ the vertical column to the left.
3.3 PIV

3.3.10 Cumulative mean for Eulerian drift profiles

To determine if the mean profiles converge, we use the cumulative mean, explained in Section 3.2.5. This is a good exercise to verify if something unphysical happens in the post-processing.

In figure 3.18 we see the cumulative mean for the vertical and horizontal velocities in a 2.1 Hz run without cover. The result is obtained by taking the sum of all rows and columns from every single vector-field, and see if the mean changes. The wavy behavior is due to the orbital motion of the waves. Since this run lasted for 7 seconds, we count 29 peaks in figure 3.18. This corresponds to

\[ 2 \times 2.1 \times 7 = 29.4. \]

A cumulative mean close to zero indicates satisfactory results.
3.4 Improving the experimental setup

Since this was the first time this experiment was carried out in our department, it took some time to optimize and improve the experiment. The experiments carried out by Kang and Lee (1996), Mass and Milgram (1998), Gushchin and Ermakov (2004) and Siddiqui and Loewen (2007), were used as inspiration when designing the experiment.

During 2015, the Mechanics department at UiO bought a new set of wave gauges with a higher rate of accuracy compared with the old ones. Therefore, the surface elevations in the experiments conducted fall 2014 were measured using the U-gauges system produced by Banner (presented in Section 3.2.1). These are referred to as experiments using U-gauges. In the experiments conducted spring 2015 we used the ULS Advanced gauges system delivered by Ultralab (presented in Section 3.2.2). These are referred to as experiments using ULS-gauges.

The first experimental setup during fall 2014, consisted of a 5.2 meters long array consisting of eight wave gauges as shown in figure 3.1. We moved the array four times along the wave tank during every run in order to get a proper amount of data to estimate the damping coefficients. We started recording the surface elevation five minutes after the wave paddle startup, and measured for three minutes at each array location, obtaining 48 measurement points. The reason for the high number of points was prior experience with dropouts in the gauges. During each run, we took image
sequences of the flow with the camera system three times to obtain raw-data for PIV. The image sequences were carried out 8, 12 and 18 minutes after startup, to check if the flow achieved a steady state. For camera parameters, see Table 3.1 in Section 3.3.4.

In the experiments during spring 2015, the wave gauge array consisted of three dual gauges equispaced with 1.8 meters. The array was moved 2.7 meters three times in order to get the spatial damping, in the same way as in the experimental setup in figure 3.1. The total length of the surface elevation measurement area was 11.7 meters. The data used in the results were obtained by collecting data 3 minutes at every station, starting data collection 8 minutes after paddle start-up.

The imaging were conducted in a separate session running 15 minutes series with waves, recording after about 11 minutes. The camera parameters in these runs are shown in Table 3.1. Two of the new wave gauges (ULS gauges in Section 3.2.2) were placed above the field of view to record the surface elevation.

### 3.4.1 Conducted experiments

In Table 3.4 - 3.6 all the experiments carried out are listed. In the tables; \(a_0\) is the initial amplitude from the measurement array, \(\lambda\) is wavelength and \(a_0k\) is the steepness. The two latter quantities are results from using the non-linear dispersion relation from Section 3.3.9.

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Table 3.4: First week of experiments. Fall 2014, with U-banner gauges.
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Table 3.5: Wave attenuation experiments with Ultralab ULS Advanced gauges. Conducted spring 2015.
3.4. IMPROVING THE EXPERIMENTAL SETUP

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Garden cover:

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<table>
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Table 3.6: PIV experiments. The "No amplitude meas..." is lack of data from Ultralab ULS gauges. Conducted spring 2015.
Chapter 4

Results and discussion

The results from the conducted experiments are presented in the current chapter. The main topics are wave attenuation and mean drift currents. We separate between two sessions of experiments: Fall 2014, using U-gauges to measure surface elevation and spring 2015 using ULS-gauges to measure surface elevation. The water conditions were different in the two sessions which are reflected in the results.

The experiments are compared with the theory in Christensen (2005) derived in Section 2.4 and 2.5.

4.1 Wave damping - attenuation

4.1.1 Experiments using U-Gauges

After conducting the experiment described in Table 3.4, a least square fit (described in Section 3.2.6) were performed on the scattering points from the two different amplitude runs at the same frequency. The results are shown in figure 4.1.

Before the experiments were carried out, the wave tank was cleaned in order to obtain clean water. The tank was drained and a brush was used to clean the walls and collect all the contamination at the end before it was filled up. The tank was also flushed a couple of times. Despite all the cleaning, the water became "dirty" after some days with experiments. The "dirt" was a mixture of e.g. dust, algae production and pollution from human hands when seeding the water with tracer particles for the experiment.

From figure 4.1, it is evident that increasing frequency leads to higher attenuation rates, which corresponds with results from Hühnerfuss et al. (1983). From Phillips (1977), the analytical deep water wave attenuation rates for a clean water surface and a water surface covered by an inelastic sheet are derived as

\[ \alpha_v = 2 \nu k^2, \]  
\[ \alpha_{in} = \frac{1}{2} \nu k \gamma, \]

where \( \alpha_v \) is for clean water, and \( \alpha_{in} \) is for a covered surface. From both equations (4.1) and (4.2), the wave attenuation rates increases with higher wavenumber \( k \), which is
Figure 4.1: Attenuation rates from measurements using U-gauges. No cover in the left column, polyethylene cover in the right column. All results are scaled on initial amplitude and wave number in vertical and horizontal axis respectively. $\alpha, b$ indicates exponential and linear fit respectively.
4.1. WAVE DAMPING - ATTENUATION

Figure 4.2: Surface tension as function of time from two different samples of water from the wave tank. Solid and dashed line are from samples taken at 6.5 m and 16.5 m from the wave paddle, respectively. Gray field indicate surface tension for clean water in the range from 20°C to 30°C from Vargaftik et al. (1983).

related to the frequency by the dispersion relation. The same trend is described in the change attenuation rate (i.e. $\frac{\hat{\alpha}}{k}$) from Christensen (2005) in equation (2.93). For large $\mathcal{E}$ (i.e. inelastic surface covers) and deep water waves (i.e. $c_p = c_g$) we have that

$$\frac{\hat{\alpha}}{k} \simeq \frac{k}{2\gamma}.$$  \hspace{1cm} (4.3)

From equation (4.3), we observe that the attenuation rate increases with the wavenumber.

Despite the work cleaning the tank, it was apparent that the water was not entirely clean after some days with experiments. To investigate the fluid properties (i.e. surface tension and elasticity) some test samples of the water was taken at different locations in the wave tank just the minute after an experiment was carried out. The samples were evaluated at the chemistry laboratory by professor Finn Knut Hansen. He used the drop pendant method to calculate the surface tension. The results of the samples at 6.5 meters from the wave paddle (where the surface cover was attached, see figure 3.2b) and 16.5 meters from the wave paddle are shown in figure 4.2. The surface tension at 6.5 meters from the wave paddle achieves a steady state condition after about 50 seconds. The surface tension 16.5 from the wave paddle decreases over time. This indicates, according to the professor, that the water is polluted. It is reasonable to believe that after 20 minutes with continuous waves, the “dirty” water surface will drift (due to mass transport) downwards in the tank. The change in concentration of polluted material affects the boundary condition and explains why the amplitude at the end of the measurement array in figure 4.1c vs. 4.1d, 4.1e vs. 4.1f and 4.1g vs. 4.1h is about the same size - despite linear and exponential attenuation.

The high attenuation in the runs without cover (see left column in figure 4.1) could be
due to occurrence of Marangoni waves, as explained in Section 1.2. A resonant behavior between the longitudinal and transverse wave modes exert strong attenuation. The effect is well documented in oily surfaces in Alpers and Hühnerfuss (1989), Hühnerfuss et al. (1981) and Hühnerfuss et al. (1983).

If the water surface had been perfectly clean, we would expect exponential decay in the no cover runs in figure 4.1. The linear attenuation in figure 4.1c, 4.1e and 4.1g could be explained in terms of surfactant transport. Due to the enhanced surfactant transport for increasing frequencies (i.e. from the Stokes drift in equation (2.44)), the wave attenuation rate behaves as a function of space and time. The result is a non-uniform attenuation rate for a 20 minutes run. The behavior in figure 4.1g underpins the argument; At the end of the measurement array the wave attenuation is no longer linear. It looks like the wave damping could be divided in to two regimes: One before and one after $kx = 250$. From these observations it is reasonable to claim that the attenuation rate changes in space and time and that drift currents add a bias in the wave attenuation for the no cover cases in figure 4.1. The surface drift velocity is mentioned in Christensen (2005) and taken to account in the steady state solution in (2.96).

It would be preferable to decide the elastic properties of the water, in order to calculate the attenuation rate (i.e. $\alpha k$) directly from equation (2.93). But according to professor Finn Knut Hansen, the results from trying to estimate the elasticity from the drop pendant method were not trustworthy.
4.1. WAVE DAMPING - ATTENUATION

Table 4.1: Difference in wave elevation for 2.20 Hz runs in figure 4.3.

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4.1.2 Experiments using ULS gauges

A selection of the wave damping figures from the conducted measurements using the ULS gauges are shown in figure 4.3. The wave tank was cleaned carefully several times before the experiments were carried out. The difference in results from the experiments conducted fall 2014 and spring 2015 is discussed in Section 4.1.3.

By considering the attenuation rate \( \alpha = \alpha(f) \) as a function of frequency, we observe the same behavior as explained in detail in Section 4.1.1: The attenuation rate increases with frequency. The exceptions are the no cover cases in figure 4.3b and 4.3c where the amplitudes at the end of the measurement array are of the same size.

From a measurement perspective, the thick latex cover was the most difficult one to handle. Some places - water came on top of it, and sometimes, it clung against the wall. This resulted in a high RMS in some of the runs as seen in e.g. figure 4.3l.

In the literature review in Section 1.2, we outlined the theory from Dorrestein (1951), that the maximum wave attenuation takes place between the inelastic and elastic limit (i.e. in between \( E \rightarrow 0 \) and \( E \rightarrow \infty \)). Therefore, it was expected that the thin elastic cover could exert highest damping compared with the other covers. This is not observed in figure 4.3, but it could still be the case in another frequency range. Due to limitations on the wave maker, we could not generate waves above 2.4 Hz with amplitude which could be detected by the wave gauges since this is close to the noise limit.

The difference in wave attenuation between the no cover and covered cases are significant for frequencies > 1.5 Hz, see Table 4.1 and 4.2. For 2.40Hz, the wave elevation at \( kx = 100 \) (i.e. 1/3 of the measurement array) is about 2 times higher for the no cover case than the covered cases. At the end of the measurement array (i.e. \( kx = 250 \)) the wave elevation is about 5 times higher. The difference at the end of the measurement array for the 2.20Hz run (i.e. \( kx \approx 200 \) from Table 4.1) is not so pronounced, although it is more than 50% for the no cover case vs. the covered cases. For a clean water surface, most of the energy dissipation for gravity waves occurs in the irrational wave field (Craik, 1982). Contamination, or the presence of a surface cover, provides a change in the dynamic boundary conditions, see Section 2.4. A greater shear is provided in the surface boundary layer causing enhanced energy dissipation in this region. The wave attenuation increases due to this effect, which is what we observe in our results.

For additional wave damping figures see Appendix D.
Figure 4.3: Wave decay figures from measurements using ULS-gauges. From top: No cover, polyethylene cover, thin latex cover, thick latex cover. All results are scaled on initial amplitude and wave number in vertical and horizontal axis respectively. $\alpha, b$ indicates exponential and linear fit respectively.
### 4.1. WAVE DAMPING - ATTENUATION

#### 4.1.3 Comparison of attenuation rates

Regarding the no-cover cases, we observe that the attenuation rates in figure 4.4 are remarkably different. Although the spatial resolution from the experiments is not the same, the amplitude at the 2.40 Hz runs in figure 4.4b and 4.4d at $kx = 250$ is about twice the size for the experiment with the ULS-gauges. The new damping regime that occurs after $kx = 250$ in figure 4.4b is not measured with the ULS-gauges. We are therefore not able to determine if the new damping regime occurs, but by looking at the difference in surface elevation, it is reasonable to believe we would not see it. We observe the same behavior by comparing figure 4.4a and 4.4c. The steepness is the same for both these runs.

We plot the wave attenuation rates in figure 4.5 where "NoCoverOld" indicates the no cover measurements conducted fall 2014 and "NoCoverNew" are no cover measurements from spring 2015. We observe a systematic increase in wave attenuation rates for the old no cover experiments. The increase in the spring 2015 no cover experiments is about two orders of magnitude smaller than the other. It is impossible to make an experiment which is 100% reproducible. But in our case, it is quite convincing that we have a difference in water properties from the two experiment sessions. The theory outlined in Section 4.1.1 states that the presence of a thin surface cover with elastic properties (in our case due to pollution) attenuates more than a clean surface, is supported by the results. It now becomes very interesting to examine the difference in results from the polyethylene cover experiments, in order to verify the effect of an inelastic surface cover on top of fluids with various rheological properties.

From comparison of attenuation rates in figure 4.7, where "Poly old" indicates measurements of polyethylene cover with U-gauges and "Poly new" are measurements with ULS-gauges, the 95% confidence interval for $\alpha$ is higher for the latter measurements except for the thin latex cover, see figure 4.7b and 4.7d. The difference of the polyethylene cover cases is not nearly as dominant as for the no cover cases.

In figure 4.6, we consider the wave attenuation rate $\alpha = \alpha(f)$ as a function of frequency for the different surface covers. It seems to increase exponentially for all the covers. The error-bars in figure 4.6a, indicating a 95% confidence interval, is overall largest for the thick latex cover and overall smallest for the polyethylene cover case from the U-gauges experiment.

It is evident from figure 4.6b that the slope from the old polyethylene measurements is steeper than the slope from the latter measurements. Although the difference is not nearly as dominant as in figure 4.5 for the no cover case, it still underpins the

<table>
<thead>
<tr>
<th>$kx$</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
<th>250</th>
</tr>
</thead>
<tbody>
<tr>
<td>No cover</td>
<td>0.92</td>
<td>0.83</td>
<td>0.78</td>
<td>0.7</td>
<td>0.62</td>
</tr>
<tr>
<td>Polyethylene cover</td>
<td>0.59</td>
<td>0.4</td>
<td>0.24</td>
<td>0.18</td>
<td>0.1</td>
</tr>
<tr>
<td>Thin latex cover</td>
<td>0.7</td>
<td>0.5</td>
<td>0.38</td>
<td>0.24</td>
<td>0.18</td>
</tr>
<tr>
<td>Thick latex cover</td>
<td>0.5</td>
<td>0.31</td>
<td>0.2</td>
<td>0.1</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 4.2: Difference in wave elevation for 2.40 Hz runs in figure 4.3.
Figure 4.4: Replotted no cover cases from figure 4.1 and 4.3 for comparison of wave attenuation due to different water conditions. Top row are results from measurement fall 2014, bottom row are results from spring 2015.

Figure 4.5: \( b(f) \). Exponential fit to a scatter plot of the non-dimensional linear damping coefficients at different frequencies with error-bars indicating a 95% confidence interval. ◀ and ● are old and new experiments respectively.
theory of change in rheological properties in the fluid. The observations supports the statement that surface covers on top of fluids with different rheological properties dampens differently.

We do not observe a significant difference in the slopes for the covered cases in figure 4.7a and 4.7c at 2.20 and 2.40Hz. The expectation of higher wave attenuation for the thin latex cover (explained in Section 4.1.2) seems incorrect. Although the steepness in the thin latex cover case is higher than for the polyethylene cover in figure 4.7a (0.1359 vs. 0.0878, from Table 3.5), the difference in attenuation rate is within the confidence interval. Although occurrence of non-linear effects could modify the attenuation rate due to wave-wave interactions, we would expect a larger difference to state that the attenuation rate is remarkable different. From figure 4.7d we observe that the polyethylene cover obtain higher attenuation rate than the thin latex cover for 2.40Hz.

Regarding the thick latex sheet, the confidence interval in $\alpha$ is so large due to high RMS that its difficult to say if the difference is significant. As explained in Section 4.1.2, the difference in elastic properties within the sheets does not affect the attenuation significantly in the frequency domain of our experiments.
Figure 4.6: $\alpha(f)$. Upper figure is a scatter plot the exponential attenuation rates at different frequencies with error-bars indicating a 95% confidence interval. Lower figure show a exponential least square fit for the scatter points.
Figure 4.7: Comparison of attenuation rates for different covers at 2.20 Hz and 2.40 Hz. Left column are the least square curves fitted to the scatter points. Right column is the non dimensional damping coefficients for different covers within a 95% confidence interval.
4.2 Mean drift profiles

The mean drift profiles obtained from the velocity outputs are presented in the current sub-section. First we present the results obtained from the different experiments and then we compare the results with second order wave theory from Section 2.5.

4.2.1 Experiments conducted fall 2014

The horizontal mean drift profiles from the large amplitude runs (i.e. 0.075V as input value in wave maker software) during the first week of experiments fall 2014 are presented in figure 4.8 and 4.9. The horizontal mean drift profiles for the small amplitude run (i.e. 0.05V as input value in wave maker software) are found in Appendix E. Since the same overall behavior is obtained in both amplitude runs, we draw our attention to the large amplitude run.

The most striking result from figure 4.8 and 4.9 are that all the mean drift profiles from the no cover experiments have a counterflow just below the mean wave trough. The counterflow is not present using the polyethylene cover. In Christensen (2005), the counterflow is explained mathematically as a change in the homogeneous solution $u_h$ in order to balance the contribution from the particular solution $u_p$ such that $u_h = -u_p$ at the surface. $u_h$ is described as a quasi-Eulerian mean current resulting from diffusion of vorticity from the surface in equation (2.97). From theory, the counterflow is impossible to obtain without the presence of a thin elastic surface cover. In the steady state solution (2.96), the backward drift in the upper layer is obtained for all values of $\mathcal{E} < \sqrt{2}$. The film is prevented from drifting away, which requires a force from the film barrier. With elasticity, $\tilde{u}(z = 0) \neq 0$ such that work can be done by the film on the fluid causing the backward drift in the upper layer. When the non-drifting polyethylene cover is present, the boundary condition changes dramatically. The force preventing the film from drifting is transferred to the bulk fluid and momentum diffuses into deeper layers by friction (Christensen, 2005). Then, higher drift velocities are obtained in the wave propagation direction.

All the mean drift profiles in figure 4.8 and 4.9 exceeds the maximum analytic value of the Eulerian representation of the Stokes drift in figure 2.6 from equation (2.45). At the most in figure 4.8b with almost 100% for the first (□) run. For the third runs (indicated by ·) the difference with the maximum Stokes drift is overall less, except from figure 4.8c. There are also uncertainties linked to some of the velocity output close to the surface due to the image filtering process explained in Section 3.3.7. However, the uncertainties should decrease due to averaging over large data sets. Despite the difference in values, it is evident that we obtain the Stokes drift representation from our results. The characteristic horizontal parabola shape, confined to the area between the mean wave crest an trough, is observed in most of the runs in figure 4.8 and 4.9.

From the theoretical Stokes drift profile in figure 2.6, we have no horizontal mean flow below the wave trough. Although noise from the post processing (as explained in Section 3.3.9) yields small velocities, it is still evident that we obtain an entirely different solution. The largest difference from the Stokes drift is obtained in figure 4.9d. The drift profile looks unphysical compared to the other profiles. From the
4.2. MEAN DRIFT PROFILES

Figure 4.8: Eulerian measurements of horizontal drift profiles scaled by the Stokes drift at surface from (2.45). From top to bottom: 2.10Hz, 2.20Hz. Columns from left: No cover, polyethylene cover. □, △, · are mean from first, second, third sequence respectively. Dashed lines indicates trough and crest. Asymmetry for some of them are due to lack of water depth measurements. Waves propagating from right to left.
CHAPTER 4. RESULTS AND DISCUSSION

(a) 2.30Hz, no cover.

(b) 2.30Hz, polyethylene cover.

(c) 2.40Hz, no cover.

(d) 2.40Hz, polyethylene cover.

Figure 4.9: Eulerian measurements of horizontal drift profiles scaled by the Stokes drift at surface from (2.45). From top to bottom: 2.30Hz, 2.40Hz. Column from left: No cover, polyethylene cover. □, △, · are mean from first, second, third sequence respectively. Dashed lines indicates trough and crest. Asymmetry for some of them are due to lack of water depth measurements. Waves propagating from right to left.
cumulative mean, (explained in Section 3.3.10), we do not obtain any indication of something abnormal. By further investigation of the velocity outputs in figure 4.10, we see some instabilities that appears regularly just before the wave crest (i.e. snapshot 80 and 81). The remarkable observation could be a valuable result in the discussion of wave instabilities and turbulent structures. Since this is outside the scope of this thesis, the result will not considered more than a feature obtained by waves damped by a inelastic surface cover in water with other rheological properties than clean water.

The evolution in time for the different sequences in the no cover runs (in the left column in figure 4.8 and 4.9) are quite similar. The counterflow below the trough grows in time and the difference between sequence 2 and 3 ($\Delta$ and $\cdot$ in figure 4.8 and 4.9 measured 12 and 18 minutes after start-up), are relatively small, allowing us to assume that a steady-state is obtained above the lowest level considered in our measurements.

For corresponding mean vertical velocities see Appendix E. Mean vertical velocities are discussed in Section 4.2.2
Figure 4.10: Instability evolution from 2.4 Hz run with inelastic cover from figure 4.9d. Snapshots of velocity output. Left to right: Horizontal -, vertical velocities and vector modulus. From top to bottom: velocity evolution with $\Delta = 0.02s$. 
4.2. MEAN DRIFT PROFILES

4.2.2 Experiments conducted spring 2015

A selection of the horizontal and vertical mean drift profiles are shown in figure 4.11, 4.12, 4.13 and 4.14. One imaging sequence were conducted 11 minutes after startup. The experiments were conducted with higher steepness for some of the wave trains compared with previous experiments, see Table 3.6.

In figure 4.11 and 4.12, we observe the same trend for the mean drift profiles as in Section 4.2.1; The backward flow for the no cover cases are still dominant. But in this case, we also observe a counterflow in the covered cases, e.g. figure 4.12c. Although, it is much more pronounced in the no cover cases. If we compare the drift profiles from the thin latex cover runs and polyethylene cover runs in figure 4.11b, 4.11c, 4.12b and 4.12c, we observe a small difference just below the trough in the transition to the region above the wave trough. This is more pronounced in the polyethylene cover cases. Although, the differences are so small such that it is reasonable to claim that similar behavior is obtained for the two covered cases. The same statement as drawn in Section 4.1.3.

In figure 4.13 and 4.14, four of the sub-figures have vertical velocities in the same order of magnitude relative to the mean horizontal velocities at the same height in figure 4.11 and 4.12. This applies for the profiles in figure 4.13a, 4.13b, 4.13c and 4.14c.

From theory, see Appendix B, the vertical Stokes drift velocity is zero for an ideal non-viscous fluid. We have already discussed the importance of viscosity in terms of wave attenuation due to shear stresses in the fluid. In addition to the stresses due to viscosity, higher shear at the fluid interface (e.g. with surface cover and dirty water) also affect the solution. From continuity considerations, the amplitude decay would also affect the mean vertical flow. We would expect velocities against the bottom of the wave tank. In all the mean vertical velocity profiles in figure 4.13 and 4.14, this behavior is obtained. Another feature that affects the mean vertical motion is the constant counterflow in the lower vertical region of the wave tank. Due to the mass transport, a build up region is created at the end of the wave tank. The sloping surface causes a pressure gradient which generates the counterflow. An instable region occurs between the counter- and forward flow that generates vertical motions. The same shape as for Kelvin-Helmholtz instabilities are observed in the median vertical water depth of the wave tank.

From figure 4.11 and 4.12, it is still evident that the Stokes drift representation is obtained from our results. The characteristic horizontal parabola shape is observed in all of the runs in figure 4.11 and 4.12.

For additional mean profiles see Appendix E.
(a) No cover, $\epsilon_s = 0.06$.  
(b) Thin latex, $\epsilon_s = 0.09$.  
(c) Polyethylene

Figure 4.11: Eulerian measurements of mean horizontal velocity profiles scaled by the Stokes drift at surface from (2.45). Runs with lack of surface elevation measurements has dimensions. $\epsilon_s$ is steepness. All cases are 1.50Hz. Waves propagate from right to left. • is total mean from velocity output.
4.2. MEAN DRIFT PROFILES

(a) No cover, $\epsilon_s = 0.09$.

(b) Thin latex, $\epsilon_s = 0.12$.

(c) Polyethylene

Figure 4.12: Eulerian measurements of mean horizontal velocity profiles scaled by the Stokes drift at surface from (2.45). Runs with lack of amplitude measurements has dimensions. $\epsilon_s$ is steepness. All cases are 2.00Hz. Waves propagate from right to left. • is total mean from velocity output.
CHAPTER 4. RESULTS AND DISCUSSION

(a) No cover, $\epsilon_s = 0.06$.

(b) Thin latex, $\epsilon_s = 0.09$.

(c) Polyethylene

Figure 4.13: Eulerian measurements of mean vertical velocity profiles scaled on Stokes drift at surface from (2.45). Runs with lack of amplitude measurements has dimensions. $\epsilon_s$ is steepness. All cases are 1.50Hz. $\bullet$ is total mean from velocity output. Negative vertical velocity corresponds to flow towards the water surface.
4.2. MEAN DRIFT PROFILES

(a) No cover, $\epsilon_s = 0.09$.

(b) Thin latex, $\epsilon_s = 0.12$.

(c) Polyethylene

Figure 4.14: Eulerian measurements of mean vertical velocity profiles scaled on Stokes drift at surface from (2.45). Runs with lack of amplitude measurements has dimensions. $\epsilon_s$ is steepness. All cases are 2.00Hz. \(\bullet\) is total mean from velocity output. Negative vertical velocity corresponds to flow towards the water surface.
4.2.3 Comparison with second order theory

In order to do a direct comparison with the steady state solution in equation (2.96), the results should be converted into a Lagrangian reference frame. The conversion between the two coordinate systems is discussed in Section 4.2.4. Since this conversion is not trivial, the dominant behavior of the mean drift is discussed by comparison of analytical solutions of (2.96) and the results obtained from PIV.

The steady state solution in figure 4.15 show a backward drift for an elastic cover (i.e. \( E = 1 \) and \( E = 0.5 \)) and, drift in the wave propagation direction for an inelastic cover (i.e. \( E \to \infty \)). By comparing figure 4.15 and the mean drift results in figure 4.11, we do not obtained drift in the wave propagation direction for the covered cases, but the backward drift in the no cover case is much more pronounced. Thus, it is evident that the no cover case in figure 4.11a correspond to the same behavior as the steady state solution for an elastic cover in figure 4.15. Hence, it is reasonable to believe that the water surface is not entirely clean in the spring 2015 experiments as well, as discussed in previous sections. Thus, the counterflow is a result of some kind of surface active material which give rise to forces due to resistance of extension and contraction at the surface.

Almost the same behaviour as explained above is obtained for 2 Hz waves, by comparing the steady state solution in figure 4.16 and the mean drift results in figure 4.12. In this case, we have a mean flow in the wave propagation direction from the results in 4.14c which corresponds to the inelastic sheet.

If we consider the mean drift profiles in figure 4.8 and 4.9 from experiments fall 2014, it is evident that we obtain backward drift in the no cover cases and forward drift with the presence of polyethylene surface cover. The observations have better agreement with the steady state solution than the results from the experiments conducted spring 2015.
4.2. MEAN DRIFT PROFILES

Figure 4.15: Figure show steady state solution from eq. (2.96) scaled for 1.5Hz waves. Solid line $E = 1$, dashed line $E = 0.5$, dotted line $E \to \infty$. Waves propagating from right to left. $c$ in range $[0, -10]$ is corresponds to the FOV from the velocity output.

Figure 4.16: Figure show steady state solution from eq. (2.96) scaled for 2.0Hz waves. Solid line $E = 1$, dashed line $E = 0.5$, dotted line $E \to \infty$. Waves propagating from right to left. $c$ in range $[0, -10]$ is corresponds to the FOV from the velocity output.
**4.2.4 Mean Lagrangian velocity profiles**

In order to do a direct comparison with solutions of equation (2.96) we need the velocity output to be in a Lagrangian reference frame. From equation (B.7) in Appendix B, it is possible to take the spatial or time average over a wave length or period to obtain the relation

\[
\mathbf{v}_l = \mathbf{v}_E + \mathbf{v}_S, \tag{4.4}
\]

where \(\mathbf{v}_l, \mathbf{v}_E, \mathbf{v}_S\) are the Lagrangian-, Eulerian and Stokes mean velocity (Stokes drift). From (4.4), the conversion between Eulerian and Lagrangian coordinates should be trivial. However, this was not the case. At the surface boundary, looking at the horizontal velocities using Lagrangian vertical coordinates, we have that

\[
u_l = \nu_E + \nu_S = 0, \quad \text{at } c = 0 \tag{4.5}
\]

implying that \(\nu_E = \nu_S = 0\) at \(c = 0\). This is not the case for our results, since the vertical coordinate \(c\) and \(z\) do not correspond to the same height when we do an average. In our case, \(c = 0\) is defined as the fluid interface in a Lagrangian sense and \(z = 0\) is situated at the undisturbed surface in the Eulerian coordinate system. Then \(c = z = 0\) is only true if the fluid particle is in the middle of a wave crest and a wave trough. The difference in description is shown in figure 4.17. An average over (i.e. over a wave period or wave length) the velocity to a single particle, will be spatially located in the middle of the particle’s orbital motion, as seen in figure 4.17b. An average over the velocities in the Eulerian vertical coordinate \(z\), yields a mean velocity located at every value of \(z\), as seen in figure 4.17a.

Since the material surface is always at \(c = 0\) in a Lagrangian reference frame, a semi Lagrangian conversion is obtained by letting the coordinate at the surface be zero for all columns in a Eulerian frame. Then \(z = c = 0\) is always obtained at the fluid interface. The idea is shown in figure 4.18. The mean of the surface “particle” should be obtained by taking the mean of the first row. The problem with the method is that all particles are assumed to have the same vertical velocities, which is not the case. The result from the top layer should conform with the particle motion, but the difference will grow by moving further down in the fluid.

Two results from using this method is shown in figure 4.19. The characteristic shape of the mean drift in Eulerian description is obtained at the bottom of the profile. We observe a difference in smoothness, especially in the lower right plot. The error should increase exponentially down in the fluid due to the wave motion of a fluid particle. But the difference in flow properties is still obtained by using the method, i.e. mean counterflow in the no cover case and mean flow in the wave propagation direction in the thin latex cover case.

A more sophisticated way of doing a quasi-Lagrangian conversion is explained in Broström et al. (2008). From an Eulerian reference frame it is possible to define material surfaces within the flow in order to detect the particle motion from Eulerian velocities, as seen in figure 4.20. An attempt on applying this method has been carried out, but due to time limitations it was not completed. It is difficult to define these material surfaces due to the vertical motions outlined in Section 4.2.2.
4.2. MEAN DRIFT PROFILES

(a) Show instantaneous velocities (●) for 7 s located at various Eulerian vertical coordinates z. WSP is the window size spacing obtained from PIV (Section 3.3.8).

(b) Mean velocity following a particle within a wave (Lagrangian description). The mean velocity arrow should be located in the median vertical level of the fluid particle motion.

Figure 4.17: Different understanding of mean quantities in Lagrangian and Eulerian description.

Figure 4.18: Idea behind semi Lagrangian conversion. Horizontal velocities.
CHAPTER 4. RESULTS AND DISCUSSION

Figure 4.19: Conversion from Eulerian mean to semi Lagrangian mean. Top pair is 1.5 Hz no cover run, bottom pair is 1.5 Hz thin latex cover run. Waves propagating from right to left.

Figure 4.20: Idea behind quasi-Lagrangian description of Eulerian velocity field. $m_1$ and $m_2$ are material surfaces at different depths indicating the particle path as a function of depth.
Chapter 5

Concluding remarks and suggestions for future work

Wave attenuation rates and wave induced mean currents are investigated by means of PIV and ultrasonic wave gauges. Two sessions of experiments during fall 2014 and spring 2015 with various surface covers (i.e. polyethylene and two different latex covers) were conducted. The wave attenuation rates were established by surface elevation measurements along the wave propagation direction of the tank. The length of the measurement area was 15.4 meters and 11.7 meters in the fall 2014 and spring 2015 experiments respectively. The post processing were mainly carried out in matlab using Fourier Transform to extract the amplitudes at the various locations.

Velocity measurements from PIV were conducted 10 meters away from the wave paddle. Various imaging sequences were carried out in the separate experiments with clean surface, polyethylene cover and the thinnest of the latex covers. The post processing took place in matlab and DigiFlow.

5.1 Concluding remarks

Some general features are observed with the change in attenuation rates due to presence of various surface covers:

- The presence of a non-drifting surface cover (i.e. polyethylene or latex cover) enhance wave attenuation compared to a clean water surface. This matches earlier results from Hühnerfuss et al. (1981).

- Wave attenuation rates for surface covered water waves increases with frequency, which coincides with theory from Christensen (2005) and Phillips (1977).

- Contaminated water enhance wave attenuation compared with clean water, which coincide with theory from Christensen (2005).

- Wave attenuation rates change in time due to mass transport in the upper layer for drifting surface covers.

A notable difference in the mean Eulerian drift currents are observed from velocity measurements:
• The presence of a non-drifting surface cover enhances Eulerian mean drift currents in the wave propagation direction compared with a non-covered surface. We find good agreement with the behavior of the Lagrangian mean current derived in Christensen (2005).

• A mean Eulerian drift current against the wave propagation direction is observed in the experiments without the presence of a surface cover. The effect corresponds to the analytical solutions in Christensen (2005) for the presence of a thin elastic film at the water surface. The unwilling inclusion of a surface film shows how difficult it is to perform ideal laboratory experiments.

• The characteristic Stokes drift representation of a wavy surface is obtained by averaged velocity results from PIV.

To summarize; In this thesis we have observed that the presence of elastic and inelastic surface covers modifies the wave attenuation. We also observe enhanced wave-induced mean currents in the wave propagation direction for a water surface covered by an inelastic sheet, compared with no sheet.

5.2 Future work

In the future aspect of this thesis, there are several problems that are left to solve. It is possible to:

• Use a sophisticated Particle Tracking Velocimetry (PTV) algorithm to obtain particle velocities, in order to do a direct comparison with analytical Lagrangian mean drift solutions.

• Develop a robust quasi-Lagrangian conversion algorithm, in order to verify analytical Lagrangian mean drift solutions from PIV.

• Do further investigation of the occurrence of instabilities and bursts in water waves.

• Solve the boundary layer problem in Christensen (2005) by means of PIV or PTV.

• Conduct experiments with oily liquid surface covers.

• Improve field experiments in ice-covered areas.

• Start conducting mean drift velocity experiments in the cold-room-ice-wave-tank at the University Center in Svalbard.

• Measure elastic properties of surface covers in wave tank experiments and relate them to the mean drift profiles in Christensen (2005).

5.3 Errors

• The effect of the wall friction is not taken into account, but is considered small due to the width of the wave tank.


Secundus, C. P. (77). Historia Naturalis. Sampled by Plinius in 77 A.D. and many translations and editions are published since then.


Appendix A

Appendix - Surface tension and elastic modulus

We consider the connection between the surface tension and the dilatation modulus. From Section 2.4 we obtain due to conservation of fluid material

$$\frac{\Gamma_0}{\Gamma} = \sqrt{x_a^2 + z_a^2} \big|_{c=0}, \quad (A.1)$$

and the dilational modulus

$$E = - \frac{d\sigma}{d(\ln \Gamma)} = - \frac{d\sigma}{\Gamma} \frac{1}{d(\Gamma)}, \quad (A.2)$$

$$\Rightarrow \frac{d\sigma}{d(\Gamma)} = - \frac{E}{\Gamma}, \quad (A.3)$$

where $\Gamma_0, \Gamma$ are the undisturbed surface concentration and surface concentration respectively. $a, c$ are horizontal and vertical Lagrangian coordinates. Subscripts (i.e. $x_a$) denote partial differentiation. $\sigma$ is surface tension and $E$ is the dilational modulus. We assume that $\sigma = \sigma(\Gamma)$. Then

$$\frac{\partial \sigma}{\partial x} = \frac{\partial \sigma}{\partial \Gamma} \frac{\partial \Gamma}{\partial x} = \frac{\partial \sigma}{\partial \Gamma} \left( \frac{\partial (\Gamma, x)}{\partial (a, c)} \right) \simeq \left( \frac{\partial \sigma}{\partial \Gamma} \right)_0 \Gamma_a z_c, \quad (A.4)$$

since $\Gamma$ is independent of $c$. $()_0$ denotes the undisturbed case. If we consider a perturbation series

$$\Gamma = \Gamma_0 + \epsilon \Gamma^{(1)} + O(\epsilon^2), \quad (A.5)$$

$$\sigma = \sigma_0 + \epsilon \sigma^{(1)} + O(\epsilon^2), \quad (A.6)$$

where $\sigma_0 = T$. Then, a first order series expansion from (A.1) yields

$$\Gamma = \frac{\Gamma_0}{\sqrt{(a + \epsilon x_a^{(1)})^2}} = \frac{\Gamma_0}{1 + \epsilon x_a^{(1)}} = \Gamma_0(1 - \epsilon x_a^{(1)}) + O(\epsilon^2) \big|_{c=0}. \quad (A.7)$$

We insert (A.3) in (A.4) and obtain:
\[
\frac{\partial \sigma}{\partial x} = \left( - \frac{E}{\Gamma} \right)_0 \Gamma_0 z_c. \quad (A.8)
\]

Then inserting for the series expansion of \( \Gamma \) yield

\[
\frac{\partial \sigma}{\partial x} = \left( - \frac{E}{\Gamma} \right)_0 \Gamma_0 (1 - \epsilon \sigma_a^{(1)})_{a} z_c. \quad (A.9)
\]

Finally we insert for the series expansion of \( \sigma \) and obtain the result for \( \mathcal{O}(\epsilon) \):

\[
\sigma_x^{(1)} = E x_{a a}^{(1)}, \\
\sigma_a^{(1)} = E x_{a a}^{(1)}, \quad (A.10)
\]

which is the connection between the dilatation modulus and the surface tension.
Appendix B

Appendix - Derivation of Stokes drift

We derivate the classical inviscid Stokes drift from Lagrangian velocity. We define $v_l(r_0, t)$ as the velocity to a particle that in time $t_0$ was in position $r_0$, and $v_E(r, t)$ as a field velocity in position $r$ at time $t$. These can be known as the Lagrangian velocity to a particle and the Eulerian field velocity respectively.

![Figure B.1: Coordinate reference frame.](image)

We require that

$$v_l(r_0, t) = v_E(r, t). \quad (B.1)$$

From figure B.1 we have

$$r = r_0 + dr, \quad \text{where}$$

$$dr = \int_{t_0}^{t} v_l(r_0, t') dt'. \quad (B.2)$$

From (B.1) we obtain

$$v_l(r_0, t) = v_E(r_0 + dr, t). \quad (B.3)$$

We assume that $dr$ is small. Then we do the Taylor series

$$v_l(r_0, t) = v_E(r_0, t_0) + \frac{\partial v_E}{\partial t} dt + dr \cdot \nabla_0 v_E(r_0, t) + O((dr, dt)^2), \quad (B.4)$$

where

$$\nabla_0 = \mathbf{i} \frac{\partial}{\partial a} + \mathbf{j} \frac{\partial}{\partial b} + \mathbf{k} \frac{\partial}{\partial c}.$$
We obtain from second order from (B.2) and (B.4)

\[ \mathbf{v}_I(r_0, t) = \mathbf{v}_E(r_0, t) + \left( \int_{t_0}^{t} \mathbf{v}_I(r_0, t') \, dt' \right) \cdot \nabla_o \mathbf{v}_E(r_0, t). \]  

(B.5)

From (B.4): the difference between \( \mathbf{v}_I(r_0, t) \) and \( \mathbf{v}_E(r_0, t) \) is of amplitude squared. We replace \( \mathbf{v}_I(r_0, t) \) with \( \mathbf{v}_E(r_0, t) \) inside the integral such that

\[ \mathbf{v}_I(r_0, t) = \mathbf{v}_E(r_0, t) + \left( \int_{t_0}^{t} \mathbf{v}_E(r_0, t') \, dt' \right) \cdot \nabla_o \mathbf{v}_E(r_0, t). \]  

(B.6)

We can now express Lagrangian velocity in terms of the Eulerian velocity field. We write

\[ \mathbf{v}_I = \mathbf{v}_E + \mathbf{v}_S, \]  

(B.7)

where \( \mathbf{v}_S \) is the Stokes velocity defined in terms of the integral-term in (B.6).

If we write

\[ \mathbf{v}_E = \overline{\mathbf{v}_E} + \mathbf{v}', \]  

(B.8)

where \( \overline{\mathbf{v}_E} \) denotes the Eulerian mean in the sense of a wave period or a wave length and \( \mathbf{v}' \) are velocities from a periodical wave field. If we assume

\[ \overline{\mathbf{v}_E} << \mathbf{v}', \]

and that a wave component from the periodical wave field (where we neglect viscous forces and the earth’s rotation) is on the form

\[ \zeta' = \zeta_0 \cos(kx - \omega t), \]  

(B.9)

where \( \zeta_0, k, \omega \) is amplitude, wave vector and angular frequency respectively with potential

\[ \phi' = \frac{\omega}{k} \zeta_0 e^{kz} \cos(kx - \omega t). \]  

(B.10)

We know that the velocities is found from the gradient of the potential and write the velocities

\[ u' = \omega \zeta_0 e^{kz} \cos(kx - \omega t), \]

\[ w' = \omega \zeta_0 e^{kz} \sin(kx - \omega t). \]  

(B.11)

The mean, in the sense of a wave period or a wave length, of the Stokes velocity \( \mathbf{v}_S \), from (B.7) is

\[ \overline{\mathbf{v}_S} = \left( \int_{t_0}^{t} \mathbf{v}' \, dt' \right) \cdot \nabla_o \mathbf{v}'. \]  

(B.12)

We calculate the two terms choosing \( t_0 = 0 \), such that
\[ \int_{t_0}^{t} v' \, dt' = \zeta_0 e^{kz} (k(\cos(kx - \omega t) - \cos(kx)) - i(\sin(kx - \omega t) - \sin(kx))), \]

\[ \nabla_o v' = \omega k \zeta_0 e^{kz} \begin{pmatrix} -\sin(kx - \omega t) & 0 & \cos(kx - \omega t) \\ 0 & 0 & 0 \\ \cos(kx - \omega t) & 0 & \sin(kx - \omega t) \end{pmatrix}. \]  \hspace{1cm} (B.13)

Then

\[ \left( \int_{t_0}^{t} v' \, dt' \right) \cdot \nabla_o v' = \zeta_0^2 \omega k e^{2kz} i(1 + \cos(kx - \omega t) \cos(kx) - \sin(kx - \omega t) \sin(kx)) \]

\[ = \zeta_0^2 \omega k e^{2kz} i \]  \hspace{1cm} (B.14)

which is the Stokes drift.
Appendix C

Appendix - Imaging and PIV system

The imaging and PIV system is developed such that all imaging and processing can be carried out on one computer within the same software. Using the commercial software tool ®DigiFlow developed by Dalziel Research Partners, one can both take pictures with a wide variety of cameras and make use of embedded array of post-processing techniques e.g. PIV, PTV and Synthetic Schlieren.

A Windows computer was used to control the camera using a KBN-PCE-CL4-SP frame grabber from BitFlow to transfer the photos. To drive the Falcon 2 camera the Bitflow SDK 5.90 was used with the camera file Dalsa_4M180_10Tap_8Bit.r64 developed by Stuart Dalziel which runs the camera with aspect ratio 4:3, 8 bits and ten taps. DigiFlow was used to control the camera parameters e.g. black level, shutter speed and frame rate.

In order to use this high speed camera system, a RAM disk was needed to store the images in a buffer. The ram disk is a virtual hard drive that makes use of some of the system memory to make a temporary storing buffer.

In order for other people to use the imaging system, a usage file (shown in Table C.1) was developed.
This is a tutorial for using the Falcon 2 4M camera delivered by Teledyne Dalsa with Digiflow.

1. Connect the camera to the frame grabber and to the power supply. Do this before starting up the computer.

2. Install a 500Gb ramdisk. In the C:\Digiflow folder there is a file called Digiflow_Ramdisk.bat. Open it and format the Ramdisk. You can also do this using cmd. Then, when starting Digiflow by typing "idigiflow" in a cmd window, go to File - Live Video and click Setup. Then it will show a dialog box which say that Digiflow will use this Ramdisk to store all the images. If this does not work, read the HighBandwidthCameras.pdf that you find in the Digiflow folder.

3. To mirror the cameras field of view go to File - Live Video - Show Live, and give your input values for frame rate, shutter speed, gain etc.

4. To capture pictures go to File - Live Video - Capture, and set your input values.

LED lighting

1. Turn on the led light and the cooling system.

2. Open "Tera Term"

3. In dialog box "New connection" choose "Serial" and "USB-to-Serial Comm Port"

4. Restore setup in "Setup" - "Restore Setup" and choose the file "LED.INI". Then press enter.

5. Use the commands from the user manual:
   X0 - turn on
   X1 - turn off
   W"xxx" - xxx is numbers in range [10,140] where 10 is minimum and 140 is maximum light intensity. Ex: W50.

Table C.1: Usage file of imaging system.
Appendix D

Appendix - Additional wave attenuation results

Additional results from wave attenuation measurements are presented in this appendix.

Figure D.1: Wave attenuation figures at different frequencies.
Appendix E

Appendix - Additional mean velocity plots

Additional Eulerian mean horizontal and vertical drift profiles from velocity measurements are presented in this appendix.

Figure E.1: Horizontal drift profiles scaled by the Stokes drift at surface from (2.45). From top to bottom: 2.10Hz, 2.20Hz. From left: No cover, polyethylene cover. □, △, · are mean from first, second, third sequence respectively. Dashed lines indicates trough and crest. Asymmetry for some of them are due to lack of water depth measurements.
Figure E.2: Horizontal drift profiles scaled by the Stokes drift at surface from (2.45). From top to bottom: 2.30Hz, 2.40Hz. From left: No cover, polyethylene cover. □, △, · are mean from first, second, third sequence respectively. Dashed lines indicates trough and crest. Asymmetry for some of them are due to lack of water depth measurements.
Figure E.3: Mean vertical velocity profiles corresponding to figure 4.8 and 4.9. Dashed lines indicates the trough and crest detected from PIV output.
Figure E.4: Mean vertical velocity profiles corresponding to figure E.1 and E.2. Dashed lines indicates the trough and crest detected from PIV output.
Figure E.5: Mean horizontal velocity profiles scaled on Stokes drift at surface from equation (2.45). Rows from top to bottom: No cover, thin latex cover, thin latex cover. $\epsilon_s$ is steepness. • is total mean from the velocity output.
Figure E.6: Mean vertical velocity profiles scaled on Stokes drift at surface from equation (2.45). Rows from top to bottom: No cover, thin latex cover, thin latex cover. \( \epsilon_s \) is steepness. \( \bullet \) is total mean from the velocity output.
Appendix F

Appendix - Validation PIV, additional plots

(a) $u_{\text{max}}$

(b) $v_{\text{min}}$

Figure F.1: Validation 2.10Hz, $a_0 = 0.552\text{cm}$
Figure F.2: Validation 2.30Hz, $a_0 = 0.3829$ cm

Figure F.3: Validation 2.20Hz, $a_0 = 0.478$ cm
Figure F.4: Validation 2.40Hz, $a_0 = 0.512\text{cm}$

Figure F.5: Validation 2.40Hz, $a_0 = 0.512\text{cm}$
Appendix G

Appendix - Field work at Svalbard

By participating in the Waves in Oil and Ice (WOICE) project, a field work trip to Svalbard was offered in order to do wave measurements in land-fast-ice covered fjords. This was the first trip in connection to the project. Its purpose was to detect waves by using off the shelf components. We also brought some measurement devices from the Norwegian Meteorological Institute. This section contains a brief presentation of the work and some preliminary results.

G.1 Measurement equipment and preparation

A lot of preparation was needed to make everything ready for the field work at Svalbard. Many issues regarding power consumption, arctic sustainable equipment and preparation of measurement devices were discussed and solved on beforehand. The main goal for the field trip was to gain experience from working in the Arctic.

G.1.1 Measurement devices

- IMU - Inertial Motion Unit, ×9. VN-100 IMU’s produced by ®Vectornav contains accelerometer, gyroscope, magnetometer and a barometric pressure sensor. Used sampling frequency of 10Hz.

- ADV - Acoustic Doppler Velocimetry, ×1.

- Pressure sensor, ×1. Keller 33X pressure sensor.

- Weather station, ×1. Davis Vantage Pro2 Plus w/fan aspirated radiation shield.

- Kipp and Zonen CNR4 Net radiometer, ×1.

- IR 120 sensors, ×2. Delivered by Campbell Scientific.

G.1.2 Methodology

The IMU’s and Keller pressure sensor were primary measurement devices to resolve the waves. In order to resolve wave attenuation in land fast ice, we wanted to measure the waves over a large distance. See figure G.1 for a ideal setup of the IMU’s.
Figure G.1: Measurement methodology for field work. ▲ is IMU and ■ is pressure sensor.

Three of the IMUs were wired with a cable to the MOXA (the storage device). One internal and two others were connected with 10 m long cables. In order to gain large distances three, of the IMUs were made wireless by transmitting radio signal using YS-C10U units. A prototype of a self-logging device was also developed using an ©Arduino to run the IMU and log the signal on a SD-card. For protection, all IMUs were placed in waterproof pelicases, see figure G.2, which were buried in the snow to resolve the actual ice-motion.

The pressure sensor was also wired with a 10 m long cable, logging data directly to the MOXA.
G.2 Brief summary of field work

We spent two weeks at Svalbard. The first week we tested our set-up in Van Mijen-fjorden, see figure G.3, while living in the mining village Svea. Valuable lessons were gained from working conditions in the arctic and challenges w.r.t. battery consumption. The setup seemed to be working, but radio transmitting problems from the wireless IMUs and power supply issues regarding the self-logging device were taken care of. One of the biggest challenges was to maintain logging over a large period of time due to the power conditions. E.g. a couple of large 12V car batteries froze during this first week. In order to keep the batteries warm, we used a stove inside a tent to maintain temperatures above 0°C.

The plan for the second week was to move our equipment to Billefjorden near Pyramiden, but due to bad weather forecast, it was deployed it in Tempelfjorden (see figure G.3) north east of Longyearbyen. Preparations and experiences from the previous week made it possible to deploy and leave the measurement equipment out on the ice logging for approximately 24 hours. The placement of the IMUs and pressure sensors are shown in figure G.4.

We were lucky in terms of weather conditions that windy weather was approaching Svalbard from the west, generating waves in our direction. All the devices was in place and we got measurements almost 48 hours. Two of the wireless IMUs only worked for a small period of time, and the same with one of the wired ones. The battery device for the self-logging Arduino lasted for about 20 hours.
G.3 Preliminary Results from Svalbard data

Some preliminary results from the IMU’s are presented in the current section in order to show that the method of resolving waves actually works.

G.3.1 Power spectral density from IMU

The power spectral densities in figure G.5 and G.6 are calculated from the Fourier transform. The vertical accelerations are pointing perpendicular to the ice. The IMUs measured waves with periods between 5 and 11 seconds. Since two of the sensors were able to sample for such a long time, the state of redundancy is much higher. The lack of signal in figure G.5 between 20 and 25 hours is because a group went out to replace batteries and start up the power generator for a new measurement period.

The power spectral densities in figure G.6 show that the self-logging device worked. There were some problems with the internal clock so the time stamps did not work properly. However, the same waves as we see in figure G.5 are captured. It is convenient to use self-logging devices instead of having separate units logging at the same station. The results would be easy to compare if the self-logging devices were equipped with a GPS, giving accurate time stamps and positions. The self-loggers are also convenient to build and are relatively inexpensive. They are also easy to replace if something does not work.

The data from the other IMUs was either influenced by a lot of dropouts, often due to bad radio signal connection, or power related issues. The best signal were obtained by VNN2 and VNN7, shown in figure G.5.
Figure G.5: Power spectral density from 10 minutes series from acceleration in vertical direction for two different IMU’s.
APPENDIX G. APPENDIX - FIELD WORK AT SVALBARD

(a) Horizontal acceleration.

(b) Vertical acceleration.

Figure G.6: Power spectral density from 20 minutes series from accelerations measured with the self logging Arduino device.