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Spatial relations in natural language

A constraint-based approach

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Chapter 1

Introduction

Spatial configurations and how they are described is a matter relevant to a wide range of fields of research, from mathematical geometry through studies within geographic information systems to automatic wayfinding and human-robot interaction. This thesis is concerned with how such configurations are described in natural, human language. In order to deal with this subject matter we will make some introductory assumptions, and then present some approaches with the aid of these assumptions.

- **Assumption 1:** There are sets of objects and relations between them that are conceived as spatial configurations.
- **Assumption 2:** There are different ways of describing the configurations mentioned in assumption 1.
- **Assumption 3:** There are different ways of processing the information expressed about the configurations mentioned in assumption 1.

For studies in how natural language deals with assumptions 1 and 2, we turn to spatial semantics. This area, concerning the connection between language and space, is an active area of research, and we find significant contributions in the works of, amongst others, Talmy, Langacker and Levinson[8, 9, 11, 20, 21]. This kind of linguistic research focuses on revealing certain properties of actual use of language, and in doing this tells us something about what sorts of spatial systems that are allowed and described by natural language. Such research reveals certain categories of spatial expressions, how different linguistic expressions are used to describe fundamentally different spatial properties.

Having a firm understanding of spatial semantics is useful when we move on to other areas of research that also deal with spatial configurations, as it will let us know whether or not the use of natural language is involved. Some systems are highly specialized, their linguistic components (that is, the terms they use to describe configurations, as mentioned in assumption 2) being far removed from similar use in natural language, and any similarity will be rather arbitrary. Mathematical geometry of different kinds would make up such examples. Other systems rely on how spatial expressions are used in natural language to a greater degree, even though this connection isn't necessarily explicated. If such a system claims to solve problems involving or expressed in natural language, then an investigation into the semantic foundation of the spatial expressions involved might tell us whether or not the system in question may hope to do so, regardless of how it formally operates.

Formal systems dealing with spatial configurations where descriptions in natural language are essential have been developed for specific, applied cases such as human-robot interaction [15], path descriptions [12], and navigation and way-finding [17, 22], while other, more fundamental research has been done on things like qualitative spatial representation [4, 5] and qualitative reasoning [1, 7, 13, 14, 16, 18, 19], systems also dependent on descriptions in natural language. These would be the systems mentioned in assumption 3 above. Some of these systems are based on simple formalizations of language that upon inspection shows a lacking foundation in spatial semantics, while others specifically limit themselves to certain aspects of spatial expressions.

The aim of the current thesis is to develop a formal system of inference regarding spatial relations, based on a linguistically sound and thorough interpretation of the same spatial relations. This is accomplished by first of all, in chapter 2, presenting a seminal work on spatial semantics done by Stephen C. Levinson[11]. In it, he develops a way of formalizing different kinds of spatial expressions common across languages from all over the world. We shall see how his system has visual interpretation and formalization as an inherent component, an important reason why it lends itself easily to further systematic processing. Following this, chapter 3 reviews some selected systems of qualitative reasoning. These systems are based on linguistic systems different from (and, it could be argued, less thorough) than the one presented by Levinson. They are still important to our cause, as they show a kind of reasoning that our own systems will be modelled after. The original work in this thesis, and its most important component, is presented in chapters 4 through 6: A qualitative system of reasoning based on Levinson's system. Finally, chapter 7 provides some thoughts on what has been accomplished, followed by some ideas for further development of the system introduced in chapters 4 through 6.

We mentioned that research in spatial semantics reveal different categories of spatial expressions, and our system will deal with and be limited to such a specific category. This will be the types of spatial expressions that Levinson focuses on, and the distinctions regarding what is included an not will be made clear in the chapter presenting Levinson's system.

There are projects, notably the work by Bateman et al.[2, 3], that attempt to construct a formal ontology of space based on extensive knowledge of spatial semantics, one that allows the representation of all concepts that are described across works on spatial semantics. This is very interesting work, and as it aims to cover such a wide array of spatial cases it should have a broad range of application. The mentioned works do not, however, mention how actual inference should be carried out. The aim of the current thesis is limited to a certain category of spatial expressions, the ones presented in the next chapter, and as such the broad system presented by Bateman et al. will not be the concern here. It could be that the system of inference presented in the current thesis could be applied to the relevant parts of Bateman's ontology, but this is left as a possible area of further research.

Part I

Works on spatial relations

Chapter 2

Spatial relations in natural language - Levinson's frames of reference

We would like to base our inquiry on a linguistically sound foundation, and thus we turn to Stephen C. Levinson's *Space in Language and Cognition*[11]. Levinson is a linguist at the Max Planck Institute for Psycholinguistics, and he has studied a wide range of languages across cultures from all over the world. The book mentioned is a study in how spatial concepts are expressed in different languages, and what effect this has on, and to what extent it is influenced by, the cognition of the people who use the languages and the cultures in which the languages are used. The current thesis will not go into the specifics and the differences between various languages in any great degree, but rather concentrate on the general typology that Levinson presents, a system that is able to represent the core concepts of spatial expressions independent of specific languages.

To find a common way of expressing spatial expressions in different languages, Levinson has to look into which, if any, universal spatial concepts there are in human cognition as expressed in language. He looks into how different languages from all around the world treats spatial concepts, what's shared and what's specific. He draws upon a tradition of research in this field, but notes that there has been, in general, a tendency to focus on the concepts commonly employed in western languages, failing to acknowledge different systems utilized in a broad range of languages.

Levinson goes on to introduce several formal systems able to properly represent how spatial expressions are utilized. The systems that Levinson is concerned with are restricted to describing static, projective, two-dimensional relations. To illustrate the difference between static and dynamic situations, we can compare the sentences "He moved the leftmost box" and "He moved the box towards the left". In the first one, *leftmost* is used to select a certain object at a single point in time, while the latter positions the same object differently at different points in time.

Projective relations are used to express the directon from one object to another, examples are "We walked *north*", "The monument is *in front* *of* the church" and "There's a chest *underneath* my bed". These can be compared with topological expressions, which do express a spatial relation, but not about specific directions: "The cat is *outside* the house" and "Put the salad *in* the bowl".

Restrictiong ourselves to relations in two-dimensional domains is mostly a matter of simplicity, and in some cases it would be easy to add the vertical axis. There are a lot of possible variations in dividing the horizontal, but because of the nearly ubiquitous gravitational pull, most languages' expressions regarding the vertical axis tend to converge.

The diversity of systems across languages demands different things, but Levinson recognizes a range of roles and reference points that in some cases must be filled, while in others are optional. Before we look into the systems he develops, we should look into the specifics of the building blocks they're made out of.

2.1 A system of labelled angles

In a two-dimensional domain, distinctions in projective relation based on a single point could utilize the whole 360° circle to an infinite degree of refinement. Infinite degrees of refinement are, however, not something utilized in natural language. Different languages have a lot of different ways of dividing and labelling the circle, and we find several ready examples in English: We can talk of things *in front of* and *behind* us, on a map we typically describe the relations between points using the cardinal directions(*north, east, south* and *west*), and aboard a ship we'd speak of things *port* and *starboard*, *bow* and *stern*.

The important point is that the circle around a point is divided into labelled arcs, and that these arcs are defined by certain angles. It's possible for a single system to have arcs of different angle width, and there's no requirement that the named arcs in a system comprise the whole circle.

2.1.1 Coordinates

Levinson emphasizes that one of the most important things in describing projective systems is to fix systems of coordinates on specific points. Briefly, we could say that the coordinate system in a configuration is the system of labelled angles anchored on a specific point in a specific orientation.

The coordinates in a given system are polar, that is to say that they are specified by rotation from a fixed x-axis. Some systems require more than one coordinate system, but there's always one *primary coordinate system* C_1, centered on origin X_1. It's possible to go from X_1 to a secondary origin X_2 by the following transformations (or combinations of them):

- Translation
- Rotation
- Reflection

This yields a *secondary coordinate system* C_2.

2.1.2 Roles

A spatial configuration consists of distinct entities, and these are used in deciding and defining the system used to describe them. The roles that follow aren't all mandatory in all systems, but they will all be presented here, and they will be exemplified when we go on to describing the different systems in detail. Quite often, single entities will fill several roles, but it's necessary to get a grasp of each role on its own terms. While most entities appearing in projective relations have spatial extension, we'll mostly be concerned with zero-dimensional points. In some cases, we'll see how this can be expanded upon.

Levinson uses the label *points* for all of the following, but seeing as several of them are, and some necessarily, spatially extended beyond a single point, we've opted for the *role* label.

- F = Figure, sometimes called *referent*, with centre point at volumetric centre F_c. This is the entity whose position we're interested in. In a projective analysis of the sentence "There's a lion behind you", the entity referred to by "lion" would be the figure.

- G = Ground, sometimes called *relatum*, with volumetric centre G_c and a surrounding region G_r. This is the entity in relation to which the figure F is described. In the sentence we used to explain the figure, the ground would be the entity referred to by "you".

- V = Viewpoint of observer. This role is usually filled by a person, but it could also be filled by e.g. a camera. The important thing is that it's able to direct its gaze in a specific direction.

- A = Anchor point. This is used to fix the system of labelled angles to a coordinate system. When looking into the different types of system, we'll see how this can be done.

- L = Designated landmark[1]. In some systems this is used to fix the whole coordinate system, while others use it to fix single angles.

Levinson mentions that some of these concepts, like figure and ground, are familiar from earlier studies, while many of the notions have been left implicit in the literature. He goes on to show how these primitives are combined to form three *frames of reference*, which together are able to account for all orientational spatial expressions grammaticalized or lexicalized in language. In the following we will present these three frames, how they assign the primitives and explain what sort of expressions that utilise them.

[1]It's important to note that this type of landmark is different from the one appearing in systems utilizing "trajector" and "landmark" roles, e.g. Langacker[9]. Levinson's landmarks can be points in the terrain or conceptualized ones, while Langacker's landmark would correspond more to Levinson's ground.

2.2 The intrinsic frame of reference

The intrinsic frame of reference is used in expressions such as “The ball is to my right” and “There’s a lion behind you”. One of the most important differences between languages (in our current area of interest) is how they divide and label the angles in a projective system, and this is perhaps especially apparent in the intrinsic frame of reference. In this frame, the system of angles and their anchoring is based on what’s often called ‘intrinsic’ features of objects. An example in English is how a person would describe entitities’ positions in relation to themselves. We have the four arcs *front, right, back/behind* and *left*, and the *front* arc is anchored to the persons front side, the side where the person normally would aim his gaze. When looking across languages it’s noticeable that there’s no universal way of naming these ‘intrinsic’ features, as Levinson mentions. Some languages assign these based on shape or size of objects, and some utilize features in a specific type of object and then metaphorically project these features onto other objects. Levinson mentions systems basing the arcs on animal body-parts and plant-parts, using labels like *heads, feet, horns, roots* etc. He notes that the angles can be assigned according to some algorithm, or on a case-by-case basis, or more often a combination of these. In English, the procedure is largely functional, e.g. the *front* of a car lies in the direction of typical motion. It should be noted that we aren’t necessarily able to fix an intrinsic frame of reference to all objects, it’s only possible with objects where we recognize the mentioned intrinsic features. Thus, a round object without any markings would typically not be assigned an intrinsic frame of reference.

Regarding the primitives: An intrinsic spatial relation R names a certain projective relation, which is typically the name of a labelled angle. The name of the angle is typically used to name a part of G. It takes two arguments, G and F. There’s one coordinate system C, with it’s origin X on the volumetric centre G_c. The anchor point A is assigned to a part of G according to the procedure of the language, and the labelled angles are distributed accordingly. In English A is often the *front* of G, and the remaining angles are distributed to 90° arcs in a clockwise manner.

The relation R(G,F) asserts that F lies in a search domain specified by R extending from G_c, outwards for a certain distance. It can be difficult to determine just how far the search domain extends, but it is assumed that there is a limit to it. R can be an internal projective relation, in which case it is used to name F as a part of, or the whole of, the volumetric mass of G that falls within the search space of R. We have an example of this in the sentence “The front of the car was all rusted”. When R is an external projective relation, F is not part of G. An example of this is the sentence “There’s a lion behind you”.

We often use this frame of reference in such a way that G is ego. This in turn makes it useful when we describe projective relations to other people: We explain from their point of view, making it immediately accessible. In these cases we would say that V = G, and it seems that in the case of people the system of angles is usually anchored by a neutral direction of gaze. We

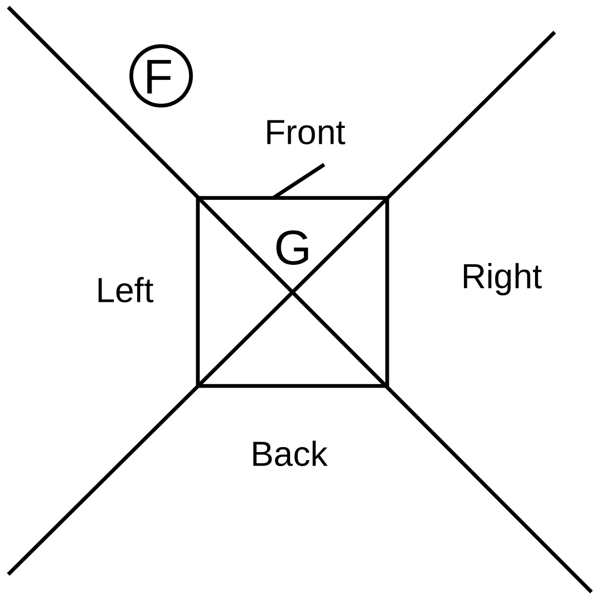


Figure 2.1: An example of the intrinsic frame of reference

may not wish to ascribe viewpoints to inanimate objects such as cars and computer screens, even though these are objects that we typically assign the same system of angles as people. Therefore, V is optional in this system.

It's worth noting that it well may be that there aren't labelled search areas all around G. Given an anchor point *entrance* of a church, we can derive an axis from the centre of the church G_c to the entrance, and designate a search domain as *at the entrance of the church*, but this doesn't necessarily imply that there are intrinsic parts covering the rest of the circle around G_c.

2.2.1 An example

Figure 2.1 shows an abstract example of an application of the intrinsic frame of reference, viewed from above. We'll say that the ground G in the figure is a house, and that on one side of the house there's a main entrance. This is usually called the *front door*, which also gives us the *front side* of the house. This becomes the anchor point A of the *front* arc, and the other arcs are distributed accordingly. This particular house happens to have a square shape, giving us an even distribution of 90° arcs, but a differently shaped house would give different distributions.

Now, we'll say that the figure F is a ball, and in English we could describe its position with the sentence "The ball is in front of the house", regardless of the position of the speaker. In the formal system we've introduced, the correct instantiation of R(G,F) would be front(house, ball).

2.3 The relative frame of reference

The intrinsic frame of reference allowed us to described the direction from an object to another, and now we'll see how the relative frame of reference allows us to describe the position of an object in relation to another, seen from a third, separate one. We find examples of this in the sentences "There's a girl standing in front of the tree" and "John kicked the ball to the left of the goal".

It's very common for languages that utilize a relative system to have an intrinsic system sharing at least some of the terms, and this more or less guarantees the ambiguities of some projective terms. Consider an example where one is told the following sentence: "The table's to the left of the chair". Most people would agree that this could both mean (in the relative interpretation) that the table is to the left of the chair from where the speaker is standing and (in the intrinsic interpretation) that the table is on the left hand when seated in the chair, and that these two need not be the same. The two interpretations can be syntactically disambiguated, the intrinsic one being specified by "The table's on the chair's left", and after developing the relative frame of reference we'll see how the distinction can be presented in a formalized manner.

To begin with, we have the viewpoint V given by the position of an observer. The observer must, as mentioned, be capable of directing his gaze in a specific direction. Furthermore, V must be distinct from G. The primary coordinate system C_1 always has its origin X_1 centered on V, with the important angle being the direction of view. G is used as an anchor point to lock, in English, the *view* or *front* angle, running in a straight line from V to G. Then, just as in the case of the intrinsic frame of reference, we distribute the *right, back* and *left* angles in the same clockwise fashion.

Next, we place a secondary coordinate system C_2, its origin X_2 being the volumetric centre G_c. The anchor point for locking the angles of C_2 is V. Now, we mentioned earlier that there are specific ways of getting from C_1 to C_2, and in English this is done by a *reflection* over an axis perpendicular to the line between X_1 and X_2. This means that the *front* arc of C_2 is the one going from G to V, and therefore meeting the *front* arc of C_1, while the *back* arc of C_2 is its opposite. The *left* arc of C_2 will be on the same side of the line between X_1 and X_2 as the *left* arc of C_1, and likewise for the *right* arc of C_2 and the *right* arc of C_1.

Other languages may use other transformations, some languages assign the *front* arc of C_2 to the same direction from X_2 as the *front* arc of C_1 from X_1. This would be a *translation*.

Finally, we have F. The relative relation R names a projective relation R, which is typically the name of a specified arc of C_2, and the ternary relation R(V,G,F) asserts that F is located in the search space R of C_2, as specified by viewpoint V. We can still express both internal and external relations, e.g. "The windows on the right side of the wall" vs. "The windows to the right of the wall", and the distinction is made in the same way as for intrinsic relations.

Now we can specify the reason of the previously mentioned ambiguity.

In the intrinsic frame, the anchoring of the coordinate system centered on G_c depended on intrinsic features of G, while in the relative frame, the intrinsic features of G are irrelevant. This allows us to describe projective relations from objects that have no intrinsic projective features, giving us unambiguous sentences (in English) like “The girl’s standing in front of the tree” and “Andrew chose the box to the right of the ball”, but we get ambiguities when we’re used to ascribing intrinsic features to G, such as in the case of the chair. To avoid the ambiguity completely in conversation the viewpoint would have to be included in the utterance, making it clear that we’re dealing with a ternary relation. In context, this is more often made clear by visual and contextual clues, and it seems people are able to disambiguate, but this is a clear problem if we’re trying to create a formalized parse.

2.3.1 An example

Figure 2.2 shows an abstracted example of an application of the relative frame of reference. We base the frame on viewer V, which we’ll say is a person named John. The ground G is the same house that we used in figure 1. John directs his gaze towards G_c, and this line becomes the anchor point of the *front* arc of coordinate system C_1, centered on V. Its other arcs are distributed as they would be in an intrinsic frame of reference.

Now we anchor the coordinate system C_2, centered on G_c. The anchor point of the *front* arc is, as mentioned above, V, and we distribute the other arcs by *reflecting* the arcs of C_1. We’ll say that the figure F is a ball, and we see how John could describe its position in relation to the house with the sentence “The ball is to the right of the house”. In the formal system we’ve introduced, we could instantiate R(V,G,F) in this case with right(John, house, ball). It’s important to note that the coordinate system C_2 would stay the same if we rotated G around G_c. The features of G are irrelevant.

We can compare figure 2.1 and figure 2.2 to make the possibilities of ambiguity clearer. Since the ground we’re dealing with has intrinsic properties, John could, from his position in figure 2.2, describe the position of the ball in relation to the house both with “The ball is in front of the house” and “The ball is to the right of the house”. The problem is that we utilize the same set of labels for the arcs in both frames. He could specify that he’s utilizing a relative frame of reference by expanding his previous sentence to “The ball is to the right of the house, from my point of view” or something of the like. This would emphasize that the relation is ternary, but this inclusion of the viewer is typically left implicit in conversation.

2.4 The absolute frame of reference

Absolute relators are familiar to us in the form of cardinal directions(*north, east, south, west*), and appear in many languages in different ways. Some languages use designated landmarks, Levinson mentions that many

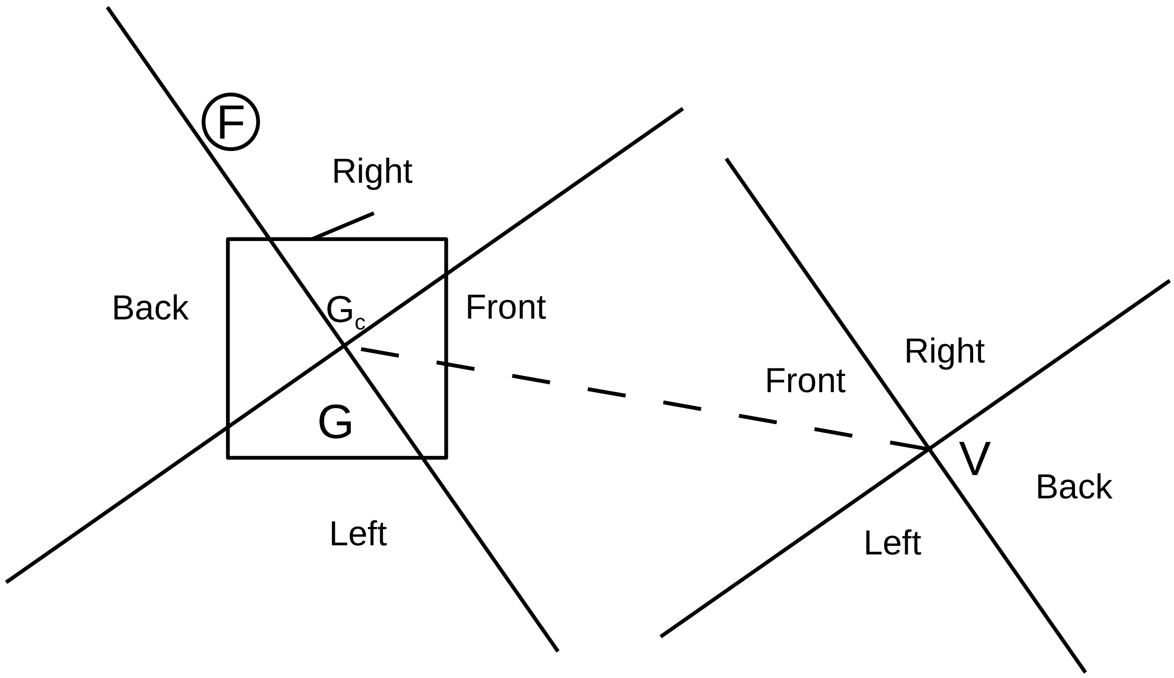


Figure 2.2: An example of the relative frame of reference

Austronesian systems have directions towards and away from the central mountain on islands, combined with a fixed bearing determined by monsoons. Some languages make almost exclusive use of such a frame of reference on the horizontal plane, and might describe an array of, say, a spoon in front of a cup, as 'spoon to north of cup' without any reference to the viewer/speaker's location.

In an absolute frame of reference, point G_c of every possible ground G has a fixed bearing anchored by a landmark L. In the case of the cardinal directions we have a landmark designated as absolute north, and we distribute the other directions accordingly. We mention north, but can of course mark other directions as a starting point, e.g. using moss growth on trees to mark south. In the case of the monsoon and central mountain, the monsoon-axis would be parallell all over the island, while the mountain-axis would change while circumventing the island.

The geometry of the labelled coordinate system is linguistically/culturally variable, the quadrants of 90° from the familiar compass rose is not universal. There are reports of abstract systems based on star-setting points and winds, which tend to have uneven distribution around the horizon.

The absolute relation R(G,F) asserts that F lies within a given angle R in the coordinate system centered on G_c. It's important to note that in a system utilizing abstract or conceptual landmarks (it could be argued that *north* and *south* in most cases are conceptual), there won't be a limit to the extent of the search domains, while in landmark systems the domains are, naturally, limited by the landmark(peak of the central mountain, the magnetic north pole).

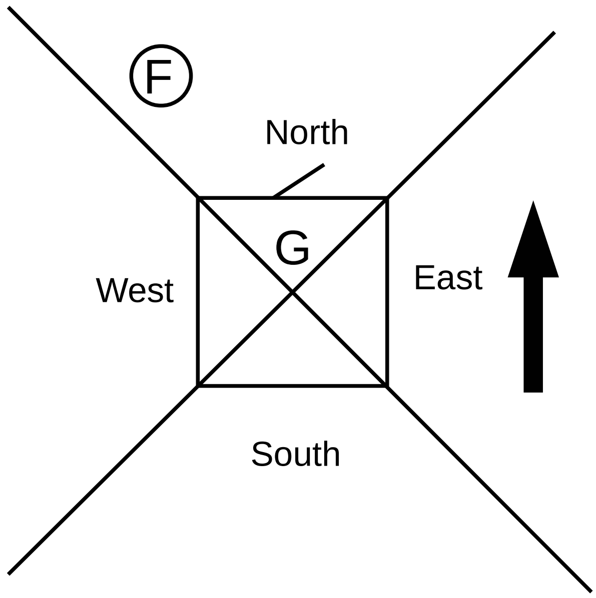


Figure 2.3: An example of the absolute frame of reference

2.4.1 An example

Figure 2.3 shows the house familiar from figure 2.1 and figure 2.2, this time in an absolute frame of reference. The bearing *north* is given by, say, a magnetic compass (depicted by the arrow on the right), and the other arcs are distributed accordingly. In this case, we could describe the position of the figure F (still a ball) with the sentence "The ball is to the north of the house".

2.5 Logical properties of the frames of reference

Now we look into what logical properties, if any, the three frames of reference have. It seems that the intrinsic frame of reference doesn't offer much in terms of spatial inference possibilities. There's no guarantee of transitivity, a counterexample is easily found in a configuration where person A is facing north, and person B is standing on the right side of A facing south. B is to the right of A and A is to the right of B, but it's not the case that A is to the right of A. If the whole configuration rotates, the intrinsic relations stay the same. This is also the case if V is separate and rotates around the configuration. If G rotates, the relations change.

Absolute relations are binary, asymmetric and transitive. If person B is to the north of A, and person C is to the north of B, then it must be the case that person C is to the north of A. It's possible to find the converse of a relation given equal distribution of angles, for the cardinal directions we have the opposite pairs (*north, south*) and (*east, west*). Absolute relations are constant under the separate rotation of figure and ground, but not under rotation of the whole configuration. The relation is independent of the viewers position.

Relative relations have some interesting logical properties. The whole ternary configuration can be rotated while maintaining the same relations, this also goes for the separate rotations of figure and ground. The relations depend on a straight line from the front of V to ground, therefore the relations are not constant under rotation of V. There are other properties as well, but these will be discussed as we discover them during the development of a greater system of inference.

Levinson doesn't enter into it, but the relation expressed by the word "between" merits an interesting analysis in the relative frame of reference, and it seems it can designate two different things. First, in a sentence like "The box between us", it means that the figure, in this case the box, is in a search space between V and G. The shape of this search space isn't given, but it could be the intersection of the *front* search space of V and the *front* search space of G.

If we have a sentence like "The box between the tree and the house", then we're dealing with a quaternary relation. If it's the case that, in this configuration, the tree is to the left of the house, then the search space for *between* could be the intersection of the *left* search space of the house and the *right* search space of the tree. A requirement in this case is that the coordinate systems centered on the tree and the house have been anchored by the same viewpoint V.

It could be that we could find interesting possibilities by introducing more complex relations and schematic deductions in the intrinsic frame of reference. Furthermore, knowledge of several intrinsic relations in a configuration could imply certain relative relations in a frame based on the same configuration. Levinson is mostly concerned with the linguistic aspects of these matters, and therefore we shouldn't be surprised if there are logical aspects in his system left unexplored. Such possibilities will not be developed in the current thesis.

It should be noted that visual interpretations are a basic part of Levinson's system, drawings of objects and their coordinate systems that visualize linguistic expressions. Not all work on spatial semantics take this approach, but the fact that Levinson does makes his system appropriate for use in further reasoning.

Chapter 3

Approaches to qualitative spatial calculi

We've seen the framework that Levinson develops to represent spatial expressions. This chapter will present some systems that are designed to carry out inferences in a manner similar to the one we'll develop for Levinson's framework, systems based on different linguistic foundations and utilizing different formalizations.

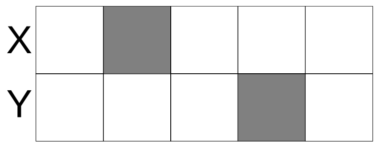
3.1 Allen's interval algebra

James F. Allen developed a calculus for temporal reasoning in 1983 [1]. It may seem strange to present a system dealing with temporal matters when the ones we're interested in are spatial, but there are several reasons why this is useful: First of all, many languages use the same, or at least similar, expressions to describe both temporal and spatial situations. Secondly, we would expect a calculus used to describe one-dimensional situations to be useful when moving on to descriptions of two-dimensional ones. And finally, this system has been very influential on qualitative spatial reasoning, and as such we should familiarize ourselves with it.

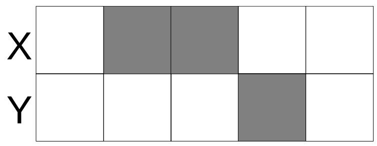
This calculus describes the possible relations between two extended intervals along a one-dimensional line. The primitives in the system are temporal intervals rather than points, avoiding some problems that atomic points of time cause in temporal logic. Another reason for using these primitives is that the system is meant to deal with stories in natural language, in which temporal expressions often express precisely such intervals. There is no absolute way of ordering these intervals other than the direction of the time line, i.e. the time line has no beginning and no end and no absolutely marked points, meaning that the intervals are only ordered among each other.

The ordering of any two intervals on the one-dimensional time line is a matter of placing four points: The start and end of each interval. These four points can be ordered in a large number of ways, but we will provide some constraints. We will say that the time line runs from left to right, and that any point of the line is taken to be a point chronologically earlier than any point further to the right of this point. Furthermore, the line is assumed to continue infinitely in each direction. We've mentioned that the entities involved are extended entities, in the time line depiction this means that the beginning and the end of the same interval cannot be placed at the same point. We also have a final axiom saying that the beginning of an interval must happen earlier than the end of the same interval, ruling out a number of possible orderings.

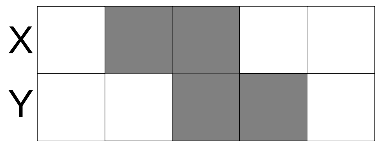
This gives us the basic relations in the calculus, shown in figure 3.1. The timeline in each subfigure runs from left to right, and is assumed to continue infinitely in each direction. The two intervals X and Y are marked on separate lines, but these lines are taken to occur on the same time line. We see that all relations have a defined inverted relation, typically marked by adding i to the operator. The "is equal to"-relation is commutative while the others aren't, giving a total of 13 relations.



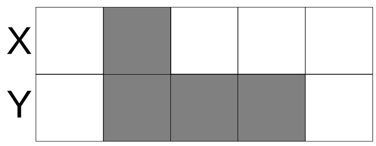
(a) $X < Y, Y > X$
X happens before Y



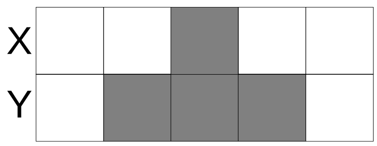
(b) $XmY, YmiX$
X meets Y



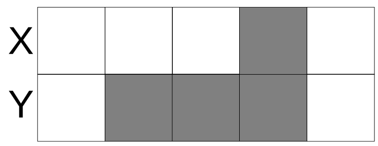
(c) $XoY, YoiX$
X overlaps with Y



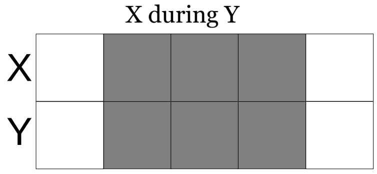
(d) $XsY, YsiX$
X starts Y



(e) $XdY, YdiX$
X during Y



(f) $XfY, YfiX$
X finishes Y



(g) $X = Y, Y = X$
X is equal to Y

Figure 3.1: The basic relations in Allen's interval algebra

Allen shows how these relations can be used to formalize expressions involving temporal intervals, given that they're (i) distinct, meaning that no pair of definite intervals can be related by more than one of the relations, (ii) exhaustive, meaning that any pair of definite intervals are related by one of the relations, and (iii) qualitative, because no numeric time spans are considered.

He presents a table of what he calls *transitivity relations*, meaning the possible relations from X to Z given known relations from X to Y and from Y to Z. Such a table is often called a *composition table* in other systems. This table will not be reproduced here, but in short it can be said that its content is found by systematically combining pairs of relations and seeing which possibilities this leaves for the third relation involved. As an example, if we know that XmY and $YsiZ$, then we can see from the depiction of the relations that it must be the case that XmZ. The information this gives us can be applied iteratively to infer possible values of other, unknown relations, and in this way the constraints given by known relations are propagated to the possible values of unknown ones.

We are now able to distinguish between *global* and *local* constraints in this system. The axioms introduced earlier that lead to the 13 basic relations in the system are global constraints, they limit the relations that might possibly occur between two entities(intervals) in the system. These constraints are not dependent on any actual occurences of any entity, they are rather consequences of our definitions of how the entities may occur in the system. The transitivity table is also a presentation of such global constraints, as it lists all possible outcomes of all possible combinations. These constraints can be contrasted to those that are provided by actual knowledge of a relation in any given configuration: If we have definite knowledge about a given relation, then we are able to constrain the domain of possible values of some other relation. A known value does not give us general knowledge about the possibilities in the system, it is rather a local constraint that tells us something about some specific other relation. Another important constraint is the conversion of a given relation. We saw in figure 3.1 that all relations have exactly one conversion, meaning that when we know the relation from X to Y then we have constrained the possible values of the relation from Y to X to a single relation.

The constraints we've introduced are ways of ensuring local consistency among sets of relations. A set of relations can be said to be consistent if they may be realized as intervals on the same time line, satisfying the demands we've presented earlier. Any given relation from X to Y has, as we've seen, exactly one possible conversion, which is to say that, e.g. XmY and $YmiX$ is a consistent set of relations while XmY and YsX aren't. As long as X and Y denote the same intervals in both expressions, they cannot both be realized on the same time line. If we look at a transitivity that has several possible outcomes, such as $XdiY$ and YsZ, then the table will tell us that $XdiZ, XfiZ$ and XoZ are all possible values of the relation from X to Z, which again is to say that the three sets $\{XdiY, YsZ, XdiZ\}, \{XdiY, YsZ, XfiZ\}$ and $\{XdiY, YsZ, XoZ\}$ are all consistent sets of relations.

3.2 Freksa's double cross calculus

Christian Freksa, cognitive/computer scientist at the University of Bremen, wrote an article in 1992 where he develops a system for representing projective spatial relations, and a way of calculating over this system[7]. The article is a bit dated, but it's still mentionened in articles discussing qualitative spatial reasoning(QSR), and newer systems often expand upon or present alternatives to Freksa's system rather than replace it.

Before writing this article, Freksa developed a similar system for describing and calculating over temporal relations[6]. This explains many facets of his approach to the spatial domain, in that he's trying to apply some of the same techniques and explanations as he does in the temporal. What he does is to take the logical constraints considered by Allen, and adds cognitive aspects and considerations to the system. This means that the entities and relations involved should be ones that are used and acknowledged in sciences of human cognition. As an example, this means that Allen's use of intervals rather than points transfers very well, as the sort of events that are to be modelled must have a certain extent to be perceivable. More importantly, Freksa specifies that while Allen deals with situations where we have certain knowledge about the relations involved, not certain in a quantitative way but certain in that a relation is deemed to be one of the thirteen ones that Allen lists, that he wants to model situation where there is incomplete knowledge about events, in which we may infer partial answers to a query. This is possible to do in Allen's algebra by presenting a series of disjunctions, which Freksa finds to be cognitively inadequate. He talks instead of *conceptual neighbourhoods*, stating that two relations are conceptual neighbours if a description of a situation changes from one to the other when altering a single thing in the situation, showing conceptual similarity between the two relations. If a relation is limited to being in such a conceptual neighbourhood, rather than known to have a specific value, then Freksa calls this information *coarse knowledge*. He goes on to show how this leads to neighbourhood-based temporal reasoning, but we will rather look at how these things are applied to spatial matters.

Freksa notes how important and fundamental physical space and its properties are in all sorts of actions and decisions. As a consequence, the ability to reason in and about physical space is crucial if we wish to create systems able to perform such actions and make such decisions. The system he decides to explore is one based on an actor positioned in physical space, its spatial knowledge based on its perception of the environment. This means that a system based on, say, Cartesian coordinates won't make sense, but rather one based on relative spatial orientation as it is presented to a perceiving entity. Note that while the information involved is based on a subjective experience, the representation will be one 'from above', the perceiver's knowledge being mapped to a two-dimensional plane.

3.2.1 Dimensionality of space

Our goal is then to investigate a system representing orientation in two-dimensional space. To begin with, we should look into how a similar thing could be done in one-dimensional space. In this domain, we're able to relate extended lines and zero-dimensional points. If we have an extended interval $[a, b]$ and a point x, then the relation space consists of nine disjoint classes: $x < a, x < b; x = a, x < b; x > a, x < b; x > a, x = b; x > a, x > b; x = a, x > b; x < a, x > b; x < a, x = b; x = a = b$. This is useful in models of temporal events, but in that case we'd not permit $b < a$ (time is unidirectional) or $b = a$ (we only model extended intervals), as we mentioned when presenting Allen's algebra. In the current description a single point is allowed, however. So in the temporal domain, the relation space reduces to five relations.

These observationse are useful when we move on to two-dimensional space. Freksa presents the following definition: *Directional orientation in 2-dimensional space is a 1-dimensional feature which is determined by an oriented line; an oriented line, in turn, is specified by an ordered set of two points.* Orientations are denoted by a line *ab* through points *a* and *b*. What we'll be interested in in the following are *relative orientations*, which is to say the orientation of line *bc* relative to the orientation of line *ab*, corresponding to describing the point location of *c* with respect to reference location *b* and reference orientation *ab*.

3.2.2 Dividing the plane

Since we're dealing with qualitative reasoning, we won't describe the relative orientation of *c* by giving exact degrees. Rather, we follow Freksa as he gradually enriches the possible relation space by dividing the plane into an increasing number of areas. First, we extend the line *ab* infinitely in both directions. This gives us four possible projections from *b*, in addition to the non-projective case where $c = b$. The two cases where *c* is somewhere along the infinite line will be denoted as *front* and *back*, where *back* refers to the case where *c* is on the same side of *b* as *a*, and *front* its opposite. As of yet we're only dealing with orientation and not distance, so these two relations are absolute and each have a possible variation of 0°.

This leaves us with the two relations *left* and *right*. They name a semi-plane each, distributed in the same way as they would've been in an intrinsic frame of reference utilizing the *front* relation already established. These two relations allow variation, as opposed to the other we've seen, but we're still not concerned with distance. An example of this system is seen in figure 3.2.

We note some interesting things in our current system. The *front* relation is transitive: If *c* is in front of *ab*, and *d* is in front of *bc*, then *d* is in front of *ab*. The *back* relation doesn't have the same property, given that so far we've only dealt with projective relations without specifying their possible extension. If *c* is to the back of *ab*, we don't know where *c* is positioned in relation to *a*. The relation does, however, have a certain periodic quality when combined with the *left* and *right* relations: If *c* is to the *back* of *ab*, and *d* is to the *left* of *bc*, then *d* is to the *right* of *ab*. If *c* is to the *back* of *ab*, *d* is to the *back* of *bc*, and *e* is to the *left* of *cd*, then *e* is to the *left* of ab. These two cases apply for all applications of odd and even numbers, respectively, of the *back* relation.

3.2.3 Augmenting qualitative orientation relations: The Double Cross Calculus

We saw that the ability to express the position of *c* along line *ab* in relation to *a* was lacking in our previous system, suggesting that this should be included. Further, a system designed to express orientational information

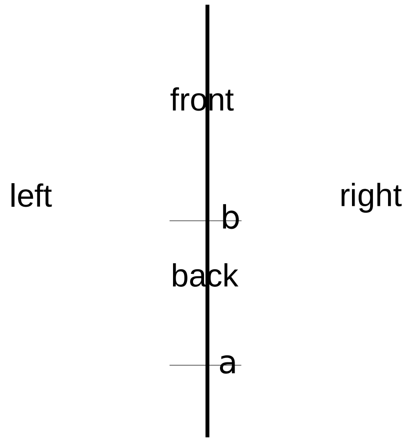


Figure 3.2: Freksa's intitial division of the plane

with two relations covering a semi-plane each isn't very expressive. What Freksa does is to introduce two more infinite lines: Both are perpendicular to the infinite line trough a and b, one intersects point a and one point b. These three lines form two crosses, and the system is known in the literature as the Double Cross Calculus(DCC). This new system expands the relation space of the position of c in relation to line ab into 15 relations. Figure 3.3 shows the double cross and its regions.

There are two absolute relations, the cases where $c = b$ and $c = a$. In the first of these cases there's no projection, while the last case denotes a projection of a specific distance. There are seven relations along the three lines, dividing them into a line segment and six rays. The *front* relation is the same as it was in our previous, less expressive system. We'll designate the cases where c is on the line segment between a and b as *middle*, and the cases where c lies past a on the ray from b through a as *back*. Along the line intersecting b we have *b-left* along the ray extending into the semi-plane previously known as *left*, and the corresponding case for *b-right*. The two relations *a-left* and *a-right* are in the same semi-plane as their b-counterparts, but lying along the line intersecting a.

The remaining relations allow c to be placed in six distinct, two-dimensional areas. The area bordering on *b-left* and *front* will be designated as *front-left*, with *front-right* being its reflection over *front*. *Between-left* is the area bordering on *b-left, between* and *a-left*, with *between-right* being its reflection over *between*, and finally we get *back-left* bordering on *a-left* and *back*, its reflection *back-right* over *back*.

While the system shown in figure 3.2 only was able to express projective relations, the DCC allows richer expressions of the position of a third point relative to two others. Given that the total search space is infinite, only three of the relations have finite search spaces, namely $c = a, c = b$ and *middle*.

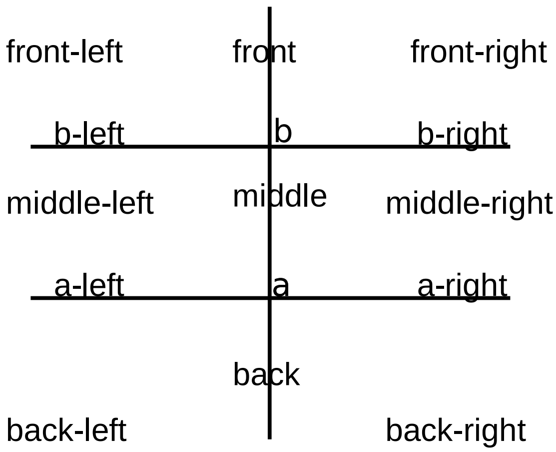


Figure 3.3: Freksa's double cross and its regions

The set of relations are jointly exhaustive and pairwise disjoint wrt. the complete search space.

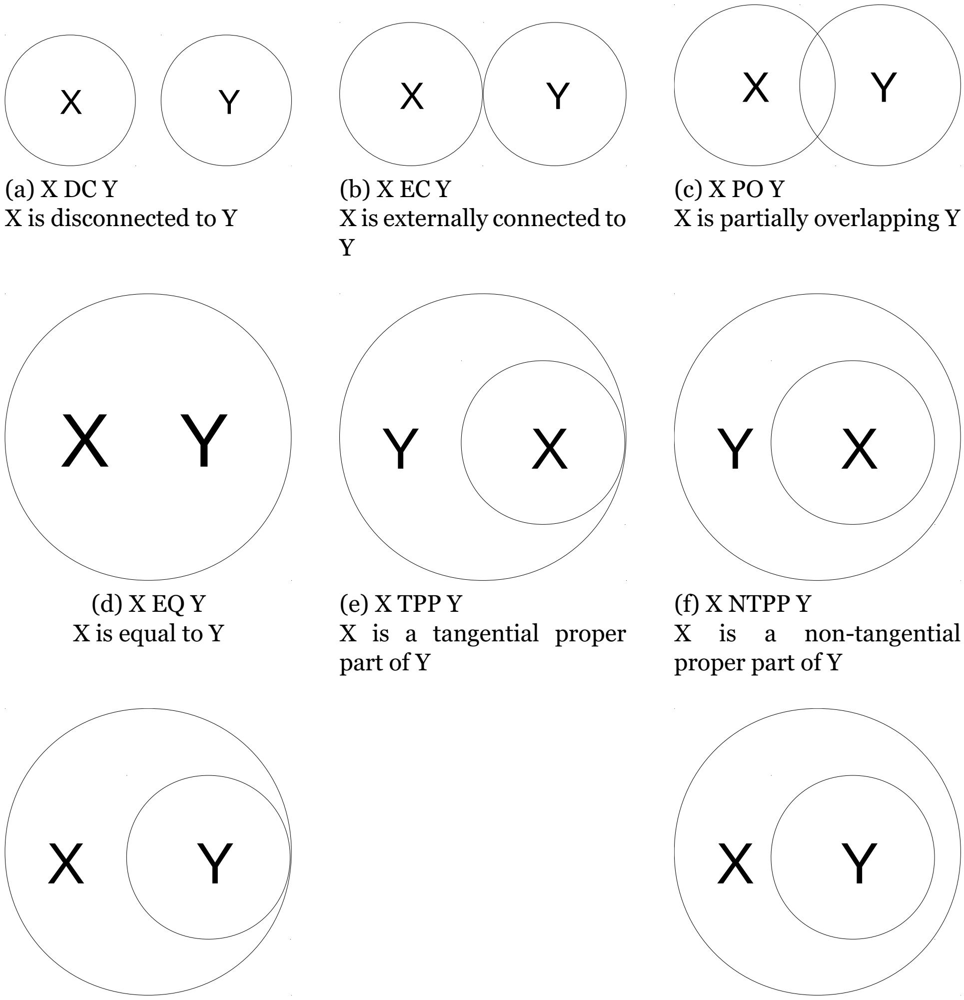
Freksa presents a similar system of conceptual neighbourhoods in this case as he did in the temporal one, where the relations in the DCC are seen as conceptual neighbours if their search spaces are directly connecting.

3.2.4 Applying the DCC

Freksa goes on to show one application of his system: how we can infer, from knowledge of the projection of point *c* relative to line segment *ab* and knowledge of the projection of point *d* relative to line segment *bc*, to knowledge of the projection of point *d* relative to line segment *ab*. This is done by presenting a composition table, where possible values of the unknown relation is given by the known ones. Here he also shows how the composition table can be altered based on the resolution of the known relation, meaning a variation in which and how many relations (which will be conceptual neighbours) that are included in the entries of the known relations. In cases that allow a set of several possible relations, the inferences can be refined if we add multiple evidence regarding some of the same points.

3.3 The regional connection calculus

A formal system that has been very important within spatial reasoning is the regional connection calculus(RCC), presented by Randell et al. in 1992[16]. While the DCC was concerned with spatial relations, the RCC is a system used for describing topological situations, or more specifically the connection between regions.



(a) X DC Y
X is disconnected to Y

(b) X EC Y
X is externally connected to Y

(c) X PO Y
X is partially overlapping Y

(d) X EQ Y
X is equal to Y

(e) X TPP Y
X is a tangential proper part of Y

(f) X NTPP Y
X is a non-tangential proper part of Y

(g) X TPPi Y
X is a tangential proper part of Y inverted

(h) X NTPPi Y
X is a non-tangential proper part of Y inverted

Figure 3.4: The relations in RCC8

The basic part of the formalism in the system assumes one primitive relation: X C Y, read as "X connects with Y". This relation is reflexive and symmetric, and using this a basic set of relations are defined. There are different variants of the system using different sets of relations, one that is widely applied is RCC8, containing the 8 relations seen in figure 3.4. They are built up as axioms, we have e.g. that X DC Y when it's not the case that X C Y. From these relations it's possible to build a transitivity table like the one Allen presents, giving the possible relations between X and Z given known relations between X and Y and Y and Z. These are local constraints similar to those we saw in Allen's algebra, allowing us to carry out reasoning, finding possible values of unknown values based on known ones.

3.4 Comparing the systems

There are some apparent similarities between Allen's algebra and RCC8. This is not surprising, Randell et al. explicitly state that their system is meant to be a version of Allen's system applicable to spatial reasoning. If we removed the directional aspect of Allen's algebra, then all inversions would be superfluous, it wouldn't matter whether one interval finishes or starts another(XsY and XfY become the same relation), and we would in fact have the same relations that we have in RCC8. It is not, however, supposed to deal with projective relations, which is exactly the point of DCC, which again is based on Freksa's temporal development of Allen's algebra. The two systems, DCC and RCC8, can thus be seen as two separate successors of Allen's system, dealing with projective and topological relations, respectively.

The mentioned systems are also very interesting in how they deal with inference and reasoning: Knowledge about how different values may be combined, as found in the transitivity and composition tables of the systems, is applied together with known values to constrain the possible domain of unknown relations. This is exactly the approach that will be used when developing calculi for Levinson's systems. A few differences should be noted, though: Allen's calculus and RCC8 talk about relations between distinct entities, all of which have a clearly defined inverse or converse relation. With the DCC this isn't exactly the same matter, as the relations in this system are based on where an object is placed in a coordinate system based on the position of two other objects.

Speaking of Levinson, it should be mentioned that it's clear that he is not a logician, and equally clear that Allen, Freksa and Randell aren't linguists. None of the systems we've just seen discuss the linguistic background of the relations they describe at any length, neither do they claim to do so: The intent is to create a logical system capable of reasoning over a set of relations. Freksa uses the same relation names as Levinson does, but it could well be said that their distribution isn't necessarily according to usage in natural languages. At any rate, his primary concern is sets of relations that are in accordance with cognitive theory, and specifically the relation between these relations in his conceptual neighbourhoods. The relations in RCC8 are based on properties from mathematical topology, and as such can be said to be somwehat removed from how such things are expressed in natural languages: Compare "X is inside Y", as Levinson mentions, to "X is a non-tangential proper part of Y". The relations in Allen's algebra can be said to be closer to usage in natural language, and one of his goals was to allow inference based on stories in natural language. Levinson's frames of reference get a lot of credence from his extensive research into various languages, and how they're able to adapt to different systems. This makes them an interesting starting point for systems of constraint-based reasoning, of which we've just seen several examples. When developing such systems we will choose single interpretations for each frame of reference, meaning a determined distribution of relations and way of assigning the coordinate systems using these, and we'll see how a constraint-based approach may deal with such situations.

Part II

Qualitative calculi for Levinson’s frames of reference

Chapter 4

Calculi for frames of reference

Chapter 2 presented three different interpretations of spatial expressions in natural language, presented in three different frames of reference. Levinson mentions some of the logical properties of the different frames, mainly the properties of configurations consisting of one referent object and one relatum object. Our goal here will be to expand upon this, and look at situations where we have several objects, and where we place all objects relative to all other objects. This means combining the coordinate systems anchored to each object, and we will see what sort of relations this gives rise to and what information we may be able to infer about such configurations.

For such inference to make sense at all, the objects involved must have something in common that decides their frames of reference. In the case of absolute frames of reference this will be the case by definition, as the frames of reference involved are assumed to have the same orientation for all possible objects appearing in the same space. When dealing with relative frames of reference, this condition will be met by having a single observer anchoring the frames of reference for all objects in any single configuration. This commonality makes inference possible. When it comes to intrinsic frames of reference we find that there is no such common anchoring point, as all application of the internal frame of reference is based on features contained within a single object. One might argue that separate objects appearing within a larger object that has an intrinsic frame of reference, such as pieces of furniture within a house, could share the intrinsic frame of reference of the larger object, but this would in reality mean using the intrinsic frame of reference of the larger object as an absolute frame of reference for all objects contained within it, reducing the problem to the absolute case mentioned above.

For any configuration of n elements, there will be a total of $n * (n - 1)$ relations involved, and information about a configuration is said to be complete if all of these are known. The following calculi will deal with configurations involving three separate objects. The spatial relations involved depend upon the frame of reference, but in all of these cases there will be a total of six relations. This means that we could have from none to six known relations, and leaving out the cases of none and six relations(the first giving none and the second giving perfect information) we get a range of possible combinations of which relations that are known and which we may choose to investigate.

Both calculi will deal with three distinct objects p, q and r. In these systems we will be concerned with the arrangement of points, and thus the objects are taken to be without spatial extension beyond single points. We will write pq for "The relation from p to q", X for "The relation X", pXq for "p is in the relation X to q", Xp for "The area in relation X to p", which is the same to say "All points that are in relation X to p", and finally pX for "The area to which p is in relation X". This last expression is a bit convoluted, but it is used to designate an area in which placement of another object q would make pXq true. We will seldom make use of the names of the roles in each frame of reference that we introduced with Levinson in chapter 2, and rather refer to specific points. Because of this, the notation in this part may differ slightly from what we saw then, but this will all be explained and made clear.

When we talk about the two relations between two specific objects we will talk about the *composition* of these objects. This use of the word differs a bit from how Freksa uses it when presenting his DCC, but it should simply be thought of as the way that two objects are placed relative to each other.

We will approach the problem of finding possible values of unknown relations in a constraint-based manner, defining the possible domain of such relations through global constraints, and then going through possible combinations of values and seeing what possibilities such constraints leave us with. When introducing Allen's algebra we saw several sets of constraints. We will use one of these, *conversion*, telling us which relations from object q to object p that are possible given a known relation from p to q. The second constraint we will develop is a bit different. When we know where two objects are placed in a configuration, then we know the two relations between them. The coordinate systems of these two objects will now partition the field into a number of areas, and each of these areas can be characterized by the relations a third object will have to each of the two already placed objects, if the object is placed within. Thus, this constraint will tell us the possible pairs of relations from third object r to objects p and q, given that we know the relations between p and q. The required knowledge here is about the composition of two objects, as described above. The result of such a constraint is knowledge about where a third object is placed relative to both of these object. When talking about these relations, an objects relation to two other objects, we'll talk about this object's *placement*.

After introducing these constraints, we will go through all possible combinations of one or more known relations and one unknown one, and see how we can apply the constraints to arrive at possible values of the unknown one. This process is the *inference* that we're able to carry out in each system.

The following will show that the calculus for the relative frame of reference is far more complex than the one for the absolute frame of reference. The absolute one can be seen as a kind of proof-of-concept, introducing the core concepts that the relative one requires us to expand upon. None the less, the absolute one is a complete system in its own regard.

Chapter 5

A calculus for absolute frames of reference

In this chapter we will develop a calculus for the absolute frame of reference, and we'll be using the familiar one that utilizes the cardinal directions. The relations in this system are shown in figure 5.1. There's no necessity in using these exact relations, what's important here is that the frame of reference gives each object in the physical plane a similar coordinate system, and that these systems have the same orientation. The distribution of relations are equal, meaning that each of the four main relations get a 90° part of the circle surrounding an object. The relations are jointly exhaustive and pairwise disjoint, meaning that any object p will be in one and only one of these eight relations to q, given that p and q are separate objects. The names of the relations are taken to be abbreviations of the familiar cardinal directions, N meaning "north", NW meaning "north-west", and so on. The areas designated by N, E, S and W are two-dimensional, while the ones designated by the remaining four are one-dimensional. Two two-dimensional areas (and corresponding relations) separated by a single one-dimensional one are said to be neighbouring areas(or neighbouring relations), such as N and E, but not E and W.

The global constraints in this calculus is simply that a relation has to have one of the values shown in figure 5.1, and that it never has more than one of these values.

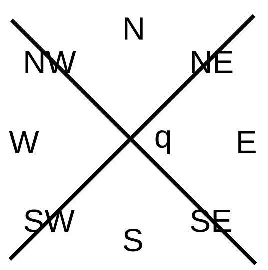


Figure 5.1: The relations in 3PCAFOR

ab	ba
N	S
NE	SW
E	W
SE	NW
S	N
SW	NE
W	E
NW	SE

Table 5.1: Conversion table for 3PCAFOR

5.1 The constraints in the system

5.1.1 Conversion

To find the values of a relation qp that are consistent with a known relation pq, that is to say that the two may cooccur, we systematically move p through all its possible relations to q, and for each of these take note of which relations from q to p this gives us. As the orientation of the coordinate systems of both objects stay the same at all times, we find that each relation has exactly one possible conversion. The possible conversion for each relation is given in table 5.1. We see that the function is a binary relation over the set of relations in the system, and that it is both one-to-one, left-total and surjective, making it a bijective function. Furthermore, we see that it's irreflexive, as no relation is its own conversion, and it's symmetric, because if qp is the conversion of pq then pq is the conversion of qp. This is similar to the relations we saw in Allen's algebra and in RCC8: Each relation is consistent with exactly one conversion.

The bijectivity of conversion reduces the number of possible unknown relations in a configuration of three objects from six to three, greatly simplifying the number of possible combinations and thus the entire calculus. When considering the different cases of known relations we will speak of "two known relations" when we know e.g. pq and pr, even though this in reality implies four known relations.

5.1.2 Composition and placement

A composition is a configuration of two objects, implying two specified relations between the two. When drawing out the coordinate systems of both objects, any composition will divide the plane into a certain number of distinct areas. All of these areas can be referred to by the conjunction of their relation to the two objects in the composition, which is the same as naming the relations that all objects within the area will have to the two objects. The composition in figure 5.2, where pNq, gives the following two-dimensional areas: $Np\&Nq, Ep\&Nq, Sp\&Nq, Wp\&Nq, Ep\&Eq, Sp\&Eq, Sp\&Sq, Sp\&Wq$ and $Wp\&Wq$. In addition to these nine areas we get four line segments,

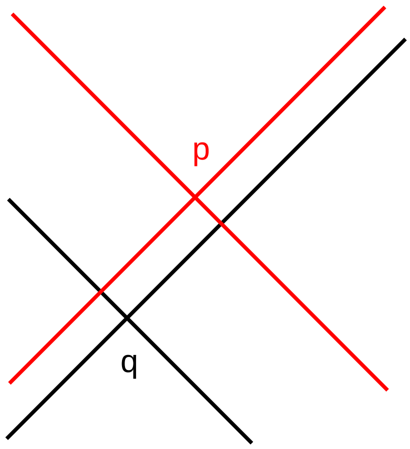


Figure 5.2: A sample composition

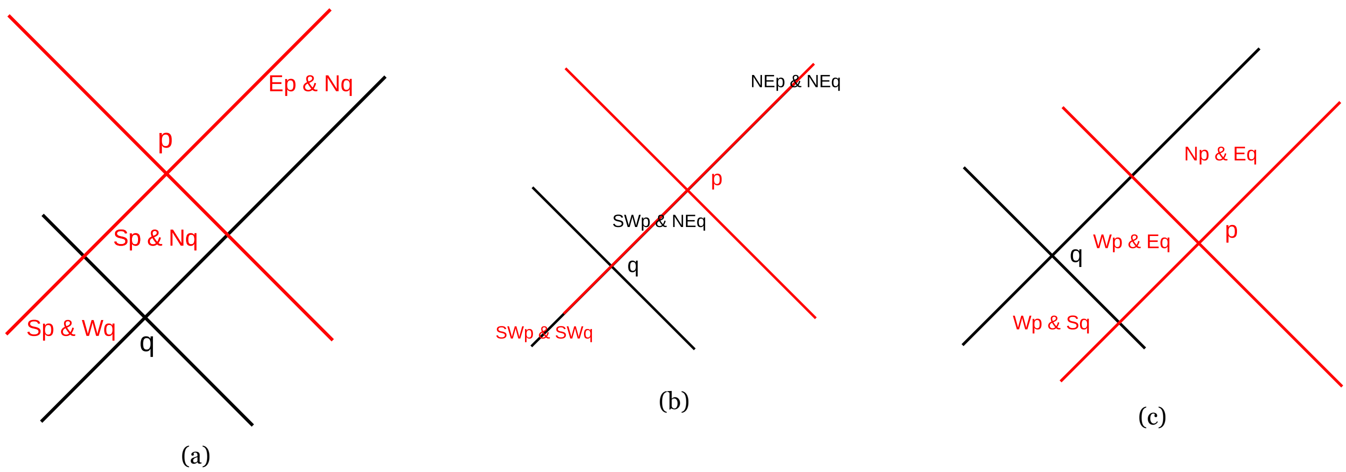


Figure 5.3: Neighbouring configurations

eight rays and two points, all of which can be calculated from the overlapping extended areas.

Because each relation in this system has exactly one conversion, it is sufficient to know the relation from one object to another to know the complete composition of these objects. We find that all possible instances of each single composition allows the same set of placements, which is to say that the coordinate systems of the two objects involved divide the field into the same areas.

If the relation between the two objects is one of the one-dimensional ones (NE, SE, SW, NW), the area collapses into six distinct extended areas. Figure 5.3 shows the transition from pNq to neighbouring pEq through $pNEq$. Figure 5.3b shows how the number of areas collapse, and the named areas in figures 5.3a and 5.3c are the ones that appear/disappear in the transition.

The complete list of areas for all compositions are given in table 5.2. The table should be read as follows: Each line represents a possible configuration of two objects, specified by the relation ab. The header of

	ca	N	N	N	N	E	E	E	E	S	S	S	S	W	W	W	W
ab	*cb*	N	E	S	W	N	E	S	W	N	E	S	W	N	E	S	W
N		1	0	0	0	1	1	0	0	1	1	1	1	1	0	0	1
NE		1	0	0	0	0	1	0	0	0	1	1	0	1	0	0	1
E		1	1	0	0	0	1	0	0	0	1	1	0	1	1	1	1
SE		1	1	0	0	0	1	0	0	0	0	1	0	0	0	1	1
S		1	1	1	1	0	1	1	0	0	0	1	0	0	0	1	1
SW		1	0	0	1	0	1	1	0	0	0	1	0	0	0	0	1
W		1	0	0	1	1	1	1	1	0	0	1	1	0	0	0	1
NW		1	0	0	0	1	1	0	0	0	0	1	1	0	0	0	1

Table 5.2: Composition table for 3PCAFOR

the columns list all possible combinations of relations a third object may have to the two objects. When reading the entry for a given configuration pq, p is mapped to a and q to b in the table, and the corresponding line is found. A "1" indicates that a third object r, mapped to c in the table, may have the relations specified in the header to p and q in combination. A "0" indicates that this isn't possible, again telling us that the configuration doesn't yield such an area. As knowledge about a relation gives us absolute knowledge about its conversion, the composition table also tells us the relations that are possible from the objects involved in the composition to the third, placed one.

We can take note of a few things from the table. First of all, areas that have the same relations to both objects, such as $Sp\&Sq$, exist in all configurations. Secondly, areas that have opposing relations to the two objects, such as $Np\&Sq$, appear in one configuration each. Finally, we see that the remaining areas appear in three configurations each.

The table lists 16 possible combinations of relations, using the two-dimensional relations from our basic set. This is mainly done to avoid having to list the 64 possible combinations that occur when we're dealing with all eight relations in the system. To find the fields not mentioned in the table, such as $NWp\&Nq$, we look for neighbouring relations to one object cooccurring with a single relation to the other object. Example: The entry for pNq contains $Np\&Nq$ and $Wp\&Nq$, and because N and W are neighbouring relations that both occur together with Np we can infer that there is an area $NWp\&Nq$ when pNq.

5.2 Inference in the system

In this section we will go through all possible cases and explain how the calculations are carried out.

5.2.1 One known relation

This is the basic case that we considered when presenting the conversion table. Knowing pq allows us to infer qp from the conversion table. This

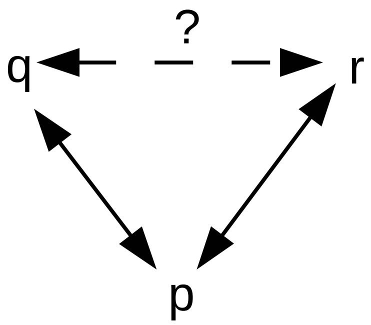


Figure 5.4

	rp	E	E	E	E
pq	rq	N	E	S	W
S		0	1	1	0

Table 5.3: Extract of composition table

doesn't give us any certain information about the other relations in the configuration, but it limits the possible placement of r, meaning the distinct combinations of rp and rq, to the ones listed in the entry for pq in the composition table.

Example: We know pNq. We know that rp and rq individually could have any value, but the conjunction of the two are constrained by the table: it couldn't be the case that rNp and rWq, because this conjunction isn't found in the table, which again is a way of stating that these are inconsistent with the premises. This can be verified visually from figure 5.2.

5.2.2 Two known relations

In the case of two known relations, the bijectivity of conversion leaves only one actual case, the one where we know pq and pr and wish to make inference regarding rq. Figure 5.4 illustrates the situation, where continuous lines represent known relations and the dashed line the unknown. The algorithm proceeds as follows: For a given combination of pq and pr, take rp. In the composition table, find the line for pq. Any area in this line satisfying rp gives a possible value for rq.

Example: We know pSq and pWr. From table 5.1 we know that rEp. The part of the composition table that we're interested in is the entry for S. The fields we're looking for satisfy rEp, and the extract of the composition table satisfying these two constraints are shown in table 5.3. When inspecting the entry for S we find that only two of these fields actually exist in the configuration. The relation to q specified in the header gives a possible value of rq, and we get the possibilities rEq and rSq. These are neighbouring relations, leaving a third possibility $rSEq$.

Table 5.4 shows the full set of inferences that can be made from pq and pr to rq in this system. The symbol $\forall$ signifies that all eight relations are

					pr			
		N	NE	E	SE	S	SW	W	NW
	N	∀	N,NW,W	N,NW,W	N	N	N	N,NE,E	N,NE,E
	NE	S,SE,E	SW,NE	N,NW,W	N	N	NE	E	E
	E	S,SE,E	S,SE,E	∀	N,NE,E	N,NE,E	E	E	E
	SE	S	S	S,SW,W	SE,NW	N,NE,E	E	E	SE
pq	S	S	S	S,SW,W	S,SW,W	∀	S,SE,E	S,SE,E	S
	SW	S	SW	W	W	N,NW,W	SW,NE	S,SE,E	S
	W	S,SW,W	W	W	W	N,NW,W	N,NW,W	∀	S,SW,W
	NW	S,SW,W	W	W	NW	N	N	N,NE,E	SE,NW

Table 5.4: Inference table the absolute calculus

possible in the given case. This table is similar to the transitivity table that Allen uses, and the problem at hand does in fact have many similarities to his. It would actually be possible to create a table like our composition table for Allen's algebra, showing the possible pair of relations from interval r to both intervals q and p, given a defined relation between the latter two. It would've been possible to infer the transitivity table from such a composition table, but the cases in Allen's system are easy enough to inspect iteratively.

Chapter 6

A calculus for relative frames of reference

In this chapter we will proceed to a more demanding task, that of developing a calculus for relative frames of reference. Whenever we speak in general terms about objects and their properties we'll denominate them by variables a, b and c. When any of these objects are instantiated we will refer to them as p, q and r. The basic configuration, as we saw when we introduced relative frames of reference, is one involving a single object p and an observer o, giving us the set of relations shown in figure 6.1. The relations are jointly exhaustive and pairwise disjoint, as in the case of the absolute frame of reference. The names of the relations are taken to be abbreviations of the familiar relations in English shown when introducing the relative frame of reference, B for "behind", BR for "behind-right of", and so on. B, R, F and L have the same properties as N, E, S and W had in the absolute calculus, such as two-dimensionality and pairwise neighbourhood, likewise one-dimensionality for the remaining four.

In the presentation of the frames of reference we were concerned with the placement of a figure relative to a ground, here we will se how separate objects are placed relative to each other. This means seeing where they are placed in each other's frames of reference. As we've mentioned, there must be something binding these systems together for any system of inference to make sense, and this common point is a single observer anchoring the coordinate system of each object. This again means that for any configuration involving n objects and an observer o, we will have n coordinate systems anchored to the objects, all of them a transformation of the coordinate system anchored to o across a straight line between o and the object. As the orientation of the coordinate system is determined by the direction from o to the object, we should expect the coordinate systems of the different objects to have different orientations. We will be concerned with configurations of three objects and an observer.

We will limit our investigation to configurations where the angle formed by the line from an observer o to an object p and the line from o to another object q is less than $90°$, except for some specific cases that we will present later.

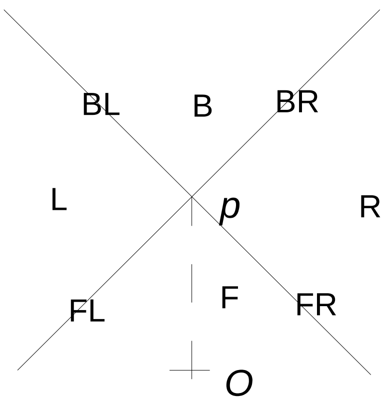


Figure 6.1: The relations in 3PCRFOR

As we've mentioned earlier, any configuration of three objects contain six relations in total. As each of these can have any of the eight relations shown in figure 6.1, our starting point is that there could be a total of 8^6 (262144) possible combinations of relations describing equally many possible configurations. Many of these possibilities are simply not possible given our two-dimensional space, and the main goal of this calculus is to show how this domain is constrained, both in general and given certain premises about the relations involved. It's also a fact that many of these possible combinations are symmetric. This is because we're not concerned with any inherent difference between the objects in any configuration, we're rather interested in the relations between them. Thus, finding the possible values of relations from p to r based on known relations from p to q and q to r is symmetric to finding the possible values of relations from q to r based on known relations from q to p and p to r.

6.1 The constraints in the system

6.1.1 Conversion

A relation pq may have any of the eight values. Without any form of constraint, its opposite qp may have eight values as well, giving $8 * 8$ possibilities for pq and qp. If we place these objects in the physical space we're operating with, together with their coordinate systems, then we see that a lot of these combinations simply aren't possible. It's not as simple as it was in the absolute system, where every relation had exactly one conversion, as we see in figure 6.3: both configurations shown are situations where qRp, but we have pLq in figure 6.2a and pFq in figure 6.2b. Neither do we have a system completely without constraints, as we would if we had two objects using an intrinsic frame of reference: Here, rotation of p would allow all values of qp given any single value of pq. The fact that each coordinate system is anchored to o provides a constraint for each specified relation, and if we go through all of them we find that there's a set of values of qp compatible with each value of pq. This operation,

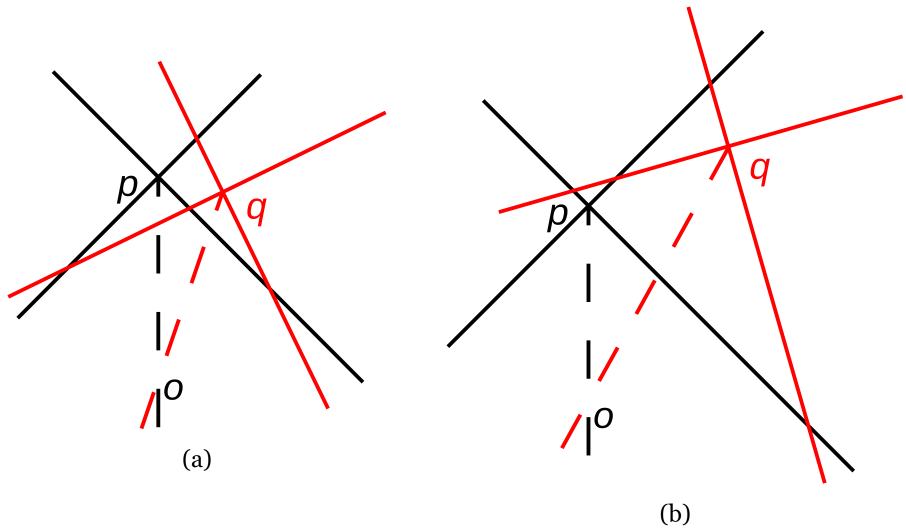


Figure 6.3

		ba							
		B	BR	R	FR	F	FL	L	BL
ab	B	0	0	0	0	1	0	0	0
	BR	0	0	0	0	1	0	0	0
	R	0	0	0	0	1	1	1	0
	FR	0	0	0	0	0	1	1	0
	F	1	1	1	0	0	0	1	1
	FL	0	0	1	1	0	0	0	0
	L	0	0	1	1	1	0	0	0
	BL	0	0	0	0	1	0	0	0

Table 6.1: Conversion table for 3PCRFOR

giving the set of compatible values, will be referred to as *conversion*, and all such compatibilites are shown explicitly in table 6.1.

The table should be read as follows: If we have a known relation pq, we map this relation to ab in the table, and its opposite qp to ba. The value of pq specifies which row in the table we look up, and a "1" in any column tells us that the value shown in the header row is a possible value of qp, while a "0" tells us that such a value is impossible. The table is a syntactic way of telling which pairs of relations between two objects that could be modeled as a physical configuration. We see that some relations clearly specify what the value of its conversion has to be, while others allow a range of possibilities. No relation is its own conversion, and all of the relations have at least one possible conversion.

We should dwell on the properties of conversion. Intuitively, one is likely to assume that one direction is clearly coupled with a single opposite direction, as was the case in absolute frames of reference: The opposite of "right" is always "left", the opposite of "behind" always "in front of". However, as this calculus is based on relative frames of reference,

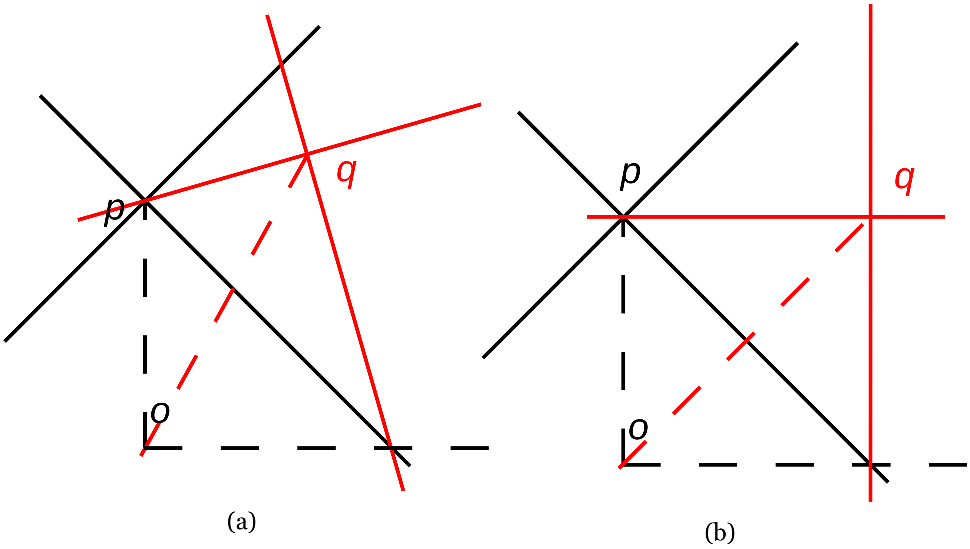


Figure 6.5: Possible conversions of qRp

the distribution of relations relative to different objects depend on the perspective from a single observer to all of these objects, and this creates a diversity invalidating this intuition.

Figure 6.5 shows two configurations where the converse value of qRp is $pFLq$. If we moved q somewhere further along the line from o through q in any of them, then we would have pFq, and if we moved q closer to o we would have pLq. The specific points where q is placed marks the singular border between these two cases given the specific orientation of the line from o to q. If we rotate this line then we would expect to find such a border point for each orientation, points where $pFLq$ and qRp are the case. What are the general propertise that characterizes these configurations and these points?

A geometrical proof

In order to better explain these properties, we will make a detour where we present a geometrical proof. It will be based on the principle of inscribed angles[23], the basic properties of which can be traced back to Euclid's *Elements*. We will tailor the proof to our specific needs, and prove the following claims:

- **Claim 1:** Whenever we have two points A and B that both are placed on the edge of a circle with central point O, then there will exist an angle $\alpha < 180°$ such that for any point C placed on the edge of the circle one of the following will apply:
  - If C is on the same side of AB as O, then $\angle ACB = \alpha$
  - If C is on the opposite side of AB as O, then $\angle ACB = 180 - \alpha$
- **Claim 2:** Given two points A and B and an angle $\alpha < 180°$, then any

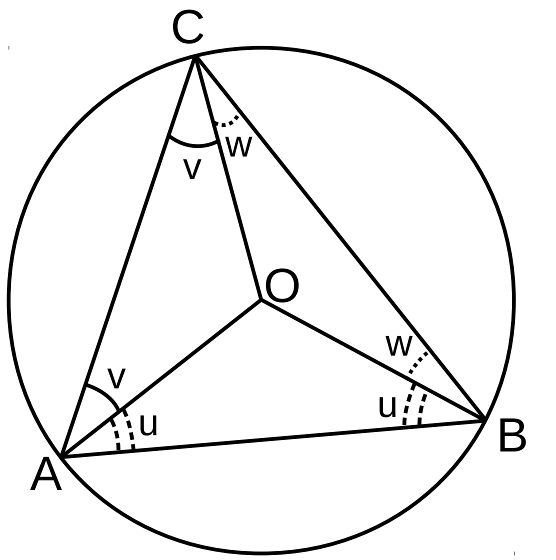


Figure 6.6

point C placed such that $\angle ACB = \alpha$ will be placed on the edge of a circle with central point O where the following applies:

- A and B are also on the edge of the circle
- The exact value of α can be found based on $\angle AOB$

For the first claim, consider figure 6.6. As O is the centre point of the circle, we know that OA and OB are equal, and as these two lines make up two of the sides of triangle ABO, we know that $\angle OAB$ and $\angle OBA$ are equal. We will call this angle u. It should be noted that the placement of C has no effect on u. For any placement of C, we have the same case of equal pairs of sides of triangles ACO and BCO, and see that $\angle OAC$ and $\angle OCA$ are equal, here known as v, and likewise $\angle OBC$ and $\angle OCB$, here known as w.

We know that the angles in the triangle ABC are 180° in total, now we can say that 180° = (v + u) + (v + w) + (w + u), which again tells us that v + w + u = 90°. The angle in which we are interested in in this claim, $\angle ACB$, is equal to, as we see in the figure, v + w. Thus, we now know that $\angle ACB$ is equal to 90° - u, and as u was independent of C so is the value of $\angle ACB$.

We should also consider the case whown in figure 6.7. Here, the triangle ABC is made up by (v + u) + (v - w) + (u - w), and we find that u + v - w = 90^{circ}. We also see that $\angle ACB$ is equal to v - w, and thus that $\angle ACB$ is equal to 90° - u, the same as in figure 6.6. We get a symmetrical case if C is closer to B, on the opposite side of the circle.

Figure 6.8 shows the case where C is on the opposite side of AB than O. The angles marked in the figure are made up the same way as the ones in figures 6.6 and 6.7, and we find that the triangle ABC is made up by (v - u) + (v + w) + (w - u). This gives us v + w - u = 90°, and as $\angle ACB$ is equal to v + w we find that it's also equal to 90° + u, and thus independent of the placement of C. This concludes the proof of the first claim.

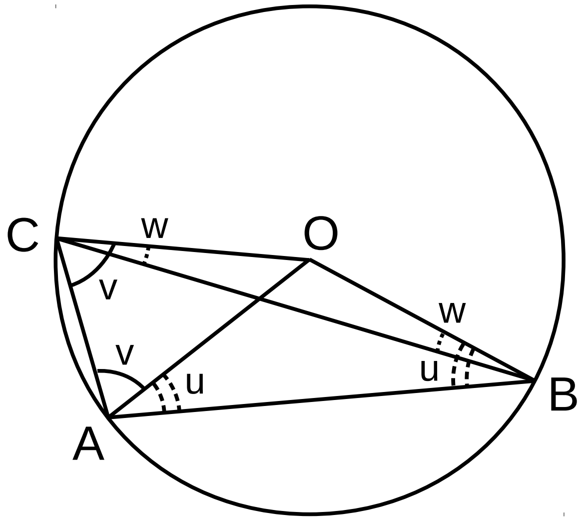


Figure 6.7

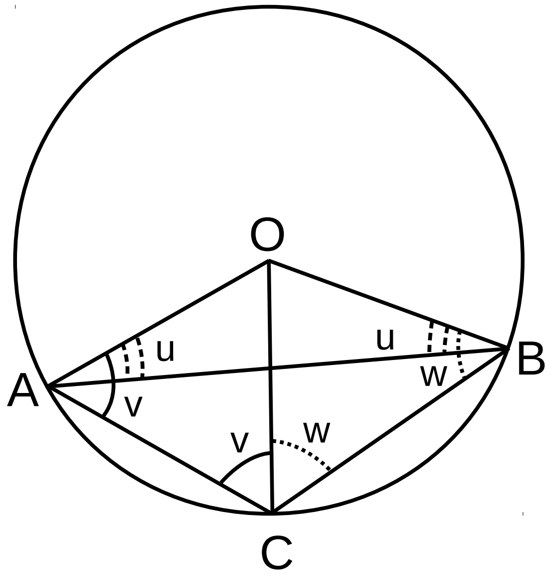


Figure 6.8

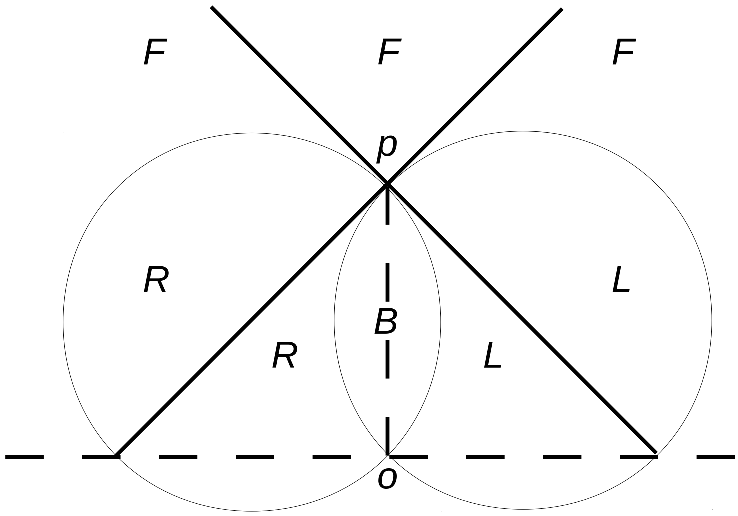


Figure 6.9: A visualization of which relation q would have to any object placed around it

To prove the second claim, we will first look at the relationship between α and $\angle$AOB. In figure 6.6, we saw that $\angle$ACB was equal to 90° - u. We know that the sum of the angles in triangle AOB is 180°, and thus we know that $\angle$AOB is equal to 180° - 2u. So in this case, $\angle$AOB is twice as large as α. In figure 6.8 we'll find that the relation is a bit different, with α being equal to 90 + u, but we see that the value of $\angle$AOB can still be used to find this.

If we move C outside the circle, then $\angle$ACB will be narrower, and the relation to $\angle$AOB will no longer apply. Conversely, if we move C inside the circle then the angle grows wider. Thus, the mentioned properties will guarantee that the three points are placed on a circle. By mirroring O across AB and applying the same α we get a second circle of the same size.

Conversion, continued

We are now better able to characterize the situations shown in figure 6.5. In these situations, we had $pFLq$. We know that $\angle oqp$ is 45° in all these situations, and by mapping o, p and q to A, B and C in the proof, and using the mentioned angle as α, we can draw the two circles. The results of this can be seen in figure 6.9.

Here we have a single object p, and the straight, continuous lines divide the plane into the basic relations seen in figure 6.1. Those relations are the ones where p is the ground, meaning that they denote where a second object q is relative to p. They are not the ones denoted by the capital letters in the figure, however: These letters show what relation p would have to this second object q when q is placed anywhere in the plane, the relations where p is the figure placed relative to the ground q. Thus, if q is placed anywhere on the half-circle to the right of p, then we would know that $pFLq$. From our proof we know that $\angle oqp$ is 45° at all these points, which we know to mean $pFLq$. We get the symmetric case in the area to the left of p, the half-circle showing where $pFRq$ is the case. For the part of the circles between o and p

we know from our proof that any $\angle oqp$ is 135°. If we look at the definition of the current frame of reference we find this corresponds to BRq and BLq. The dashed line drawn orthogonically to the line from o to p through o shows the limit in either direction to which fields we'll acknowledge and include. We see that the circles cross this line in the same point as FLp and FRp.

Figure 6.9 shows two layers of information: Information about both relations the object p may be in with another object. One comes from the coordinate system we're familiar with, the other comes from the circles just added. These circles, partioning the field around an object into the possible conversions, will be referred to as the object's *converted coordinate system*. As the size of the circles in the converted coordinate system is determined by the distance from the observer to the object, it will scale with this distance. This information is also presented in the conversion table, but in this figure we see the actual parts of the areas relative to p that are covered by each possible conversion. If q was placed anywhere behind p, then p would have to be in front of q. We already knew this from the table, but here it's visualized. In fact, if we placed q behind p and drew the converted coordinate system of q, similar to the one for p in the figure but scaled up, then p would have to be placed within the area marked B in this converted coordinate system.

The figure also shows how, when two relations share the same conversion, such as pLq being a possible conversion of both qRp and qFp, how these converted areas form a continuous area. This is not unexpected, but the figure shows it clearly.

The information we will be dealing with in any situation will consist of explicit relations between any two objects, as these are the qualitative qualifications we're dealing with in the calculus. Figure 6.9 shows several things, first how a single pair of relations allows a wide range of geometrical instantiations, all of which expressing or being a result of the same qualitative information, and secondly which part of one relation that allows which parts of its converse. The double coordinate systems shows something about the semantic possibilities of model, information for which we don't have a direct syntactic counterpart.

6.1.2 Composition and placement

When we place a third object r anywhere in the plane where we already have objects p and q, this object will participate in the four relations pr, qr, rp and rq. Conversion provided some constraints on possible values of qp given pq, and now we'll see how this can be used to provide further constraints on the relations between a third object r and these two objects in a configuration.

When talking about the two relations between two objects we talk about the objects' *composition*. The possible pairings of compositions are given by the constraints on conversion, and by listing all possible pairs of pq and qp allowed by conversion we get all possible compositions of two objects. Figure 6.10 shows an example of a composition, where we have

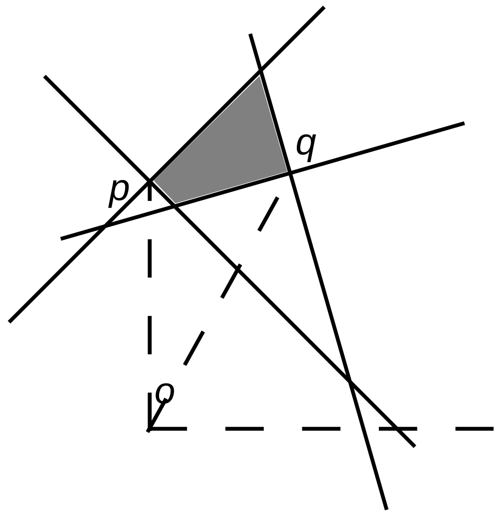


Figure 6.10

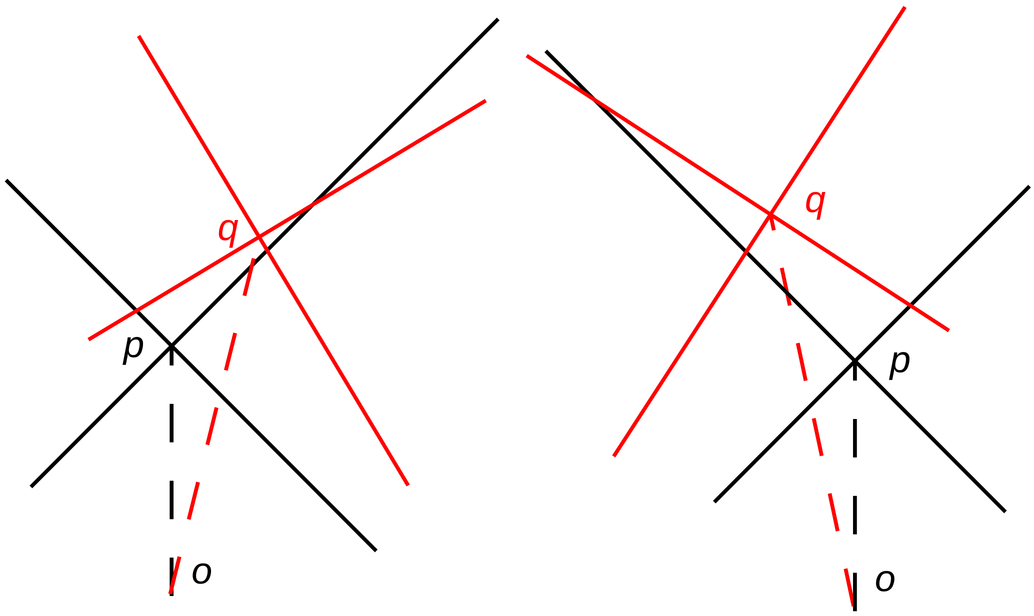
pLq and qRp. We see that the coordinate systems of the two objects in this composition divides the plane into a certain number of areas. Placing a third object r within any of these areas will satisfy a specific pair of values of relations rp and rq, eg. placing r within the gray area would make rRp and rLq true. We can refer to this area as $Rp\&Lq$.

Such areas are defined by which of the eight relations they have to each of the two objects. As such, there are a total of $8 * 8$ possible ways to qualify such an area. Figure 6.10 shows, however, that this specific composition does not yield 64 different areas. Rather, we can see how the composition divide the plane into 11 two-dimensional areas, 8 line segments, 8 rays and 4 points of intersection, giving 31 possible pairs of relations.

In most cases we find that all possible instances of a given composition produces the same set of areas, no matter where specifically we place the objects. This means that for all possible compositions we can be sure which areas are and which aren't possible, making each composition a constraint on a pair of relations. We will go on to present this information, but first we will take note of some important exceptions.

We limited the possible conversions of all relations to those where the resulting angle would be less than 90°, and as the compositions we're dealing with are produced from the conversion table this requirement is also met by the possible compositions we're dealing with. If this weren't the case we could have a situation where two objects p and q could be on opposite sides of o, and we would have both pFq and qFp. There is in itself nothing wrong with this, but it would require different methods than the ones we present in this system, and thus these situations are outside the scope of the current system. This constraint is important in all parts of the system, and we'll see how we apply different methods and rules to make sure that this requirement is satisfied in the results of our methods of inference.

We'll take note of the configurations in figure 6.11. Both of these



(a) (b)

Figure 6.11: Variatons of $aFb\&bBa$

configurations are cases where pFq and qBp, but they differ in which areas they produce when we extend the lines of their coordinate systems: In figure 6.11a we have $Rp\&Bq$ and $Fp\&Rq$, while in figure 6.11b we have $Lp\&Bq$ and $Fp\&Lq$. It should be noted that $Fp\&Rq$ and $Fp\&Lq$ in these cases will appear somewhere behind o, which is to say that if a line is drawn through o that runs orthogonal to the line from o to p, then they would appear on the side of this that's opposite to the objects. This in turn means that there will be an angle greater than 90° between a line from the observer to an object r in any of these areas and a line from the observer to p, which we'll take as an incentive and an argument not to include either of these two fields in the entry for pFq and qBp. This will also be the case for some fields in other compositions, which we'll get back to. The important point is that we have a single composition that may be associated with different sets of areas. We will include both $Rp\&Bq$ and $Lp\&Bq$ in our overview of fields produced by this composition, and later see whether we need to differentiate between the two.

We get a problematic category of properties when the angle between the line from o to an object p and the line from o to another object q is either 0° or 90°. These are exemplified in figure 6.12. They all produce nine two-dimensional areas rather than the 11 that appear in general cases such as in figures 6.2a and 6.2b. This is not in itself problematic, the problem is that we aren't able to specify this peculiarity with our set of relations, we can't compositionally differentiate between the case in figure 6.12a and one where the dashed lines form an angle of less than 90°. This is why we have the angle requirement. In the following we will consider the case shown in figure 6.12b simply as $pFq\&qBp$, and not include the cases shown in figures

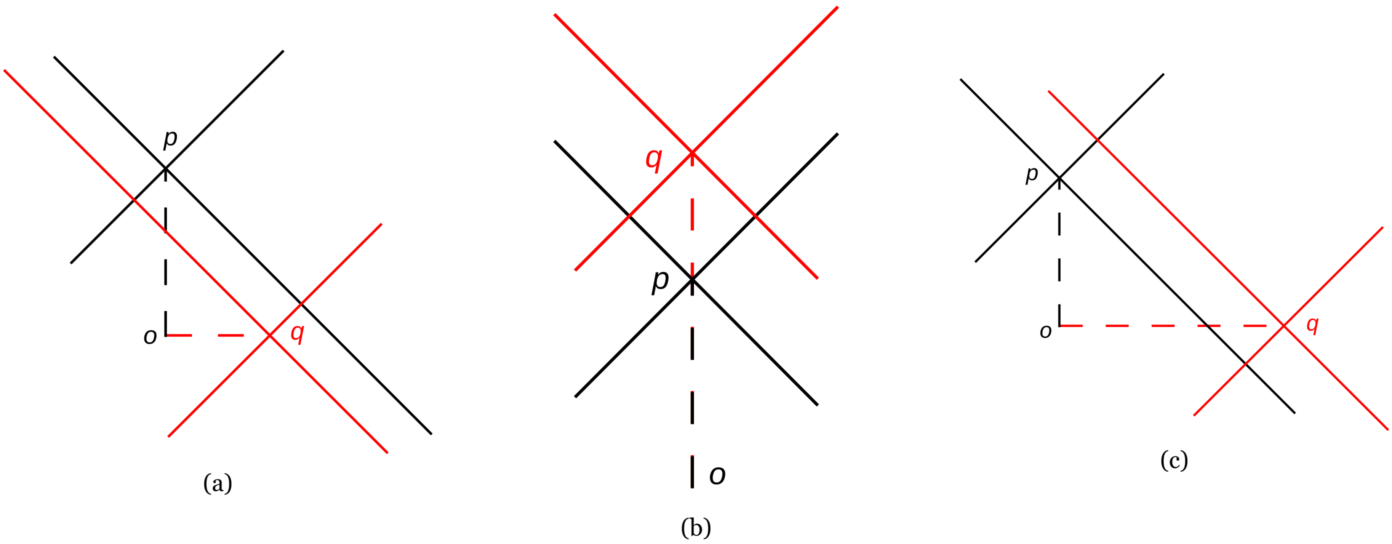


Figure 6.12: Problematic configurations

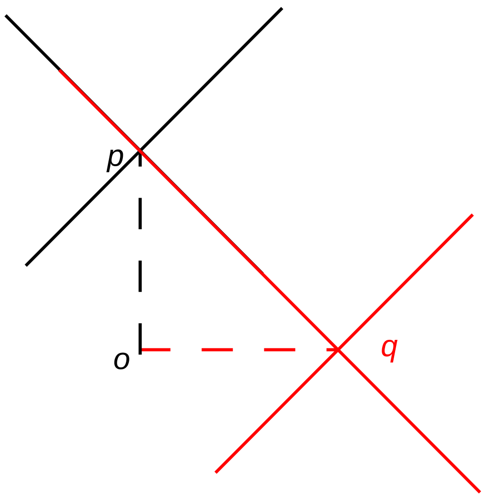


Figure 6.13: Instantiation of $aFLb\&bFRa$

6.12a and 6.12c. Figure 6.13 shows an exception to this: We will be able to deal with configurations where $pFLq$ and $qFRp$, where the angle must be exactly 90°. Here, the number of areas collapse into six.

Table 6.2 shows the complete composition table for 3PCRFOR. It should be read similarly to the one for 3PCAFOR: The two leftmost columns list the possible combinations of relations between two reference objects, while the two lines in the header list the possible relations a third object may have to the two reference objects. When we have a composition of two objects p and q, we map these objects to a and b in the table. This lets us find the entry for the given composition, where a "1" indicates that a third object r, mapped to c in the table, may have the relations specified in the header to p and q in combination. A "0" indicates that this isn't possible, telling us that the configuration doesn't yield such an area. The pair of relations

		ca	B	B	B	B	R	R	R	R	F	F	F	F	L	L	L	L
ab	ba	cb	B	R	F	L	B	R	F	L	B	R	F	L	B	R	F	L
B	F		1	1	0	1	1	1	0	0	1	1	1	1	1	0	0	1
BR	F		1	1	0	0	0	1	0	0	1	1	1	1	1	0	0	1
R	F		1	1	0	0	0	1	0	0	1	1	1	1	1	1	0	1
R	FL		1	1	0	0	0	1	0	0	0	1	1	1	1	1	0	1
R	L		1	1	0	0	0	1	1	0	0	1	1	1	1	1	1	1
FR	FL		0	1	0	0	0	0	0	0	0	0	1	0	1	1	0	0
FR	L		1	1	0	0	0	1	1	0	0	0	1	0	1	1	1	1
F	L		1	1	1	0	0	1	1	0	0	0	1	0	1	1	1	1
F	BL		1	1	1	0	0	1	1	0	0	0	1	0	1	0	1	1
F	B		1	1	1	1	1	1	1	0	0	0	1	0	1	0	1	1
F	BR		1	0	1	1	1	1	1	0	0	0	1	0	0	0	1	1
F	R		1	0	1	1	1	1	1	1	0	0	1	0	0	0	1	1
FL	R		1	0	0	1	1	1	1	1	0	0	1	0	0	0	1	1
FL	FR		0	0	0	1	1	0	0	1	0	0	1	0	0	0	0	0
L	R		1	0	0	1	1	1	1	1	0	1	1	1	0	0	1	1
L	FR		1	0	0	1	1	1	0	1	0	1	1	1	0	0	0	1
L	F		1	0	0	1	1	1	0	1	1	1	1	1	0	0	0	1
BL	F		1	0	0	1	1	1	0	0	1	1	1	1	0	0	0	1

Table 6.2: Composition table for 3PCRFOR

named in the leftmost columns, ab and ba, will be called *compositional relations*, as they name the relations between two objects. The ones named in the header, ca and cb, will be called *placemental relations*, as they place a third object in certain relations to the two other objects.

It should be noted that this table can be read in two ways: We may know a pair of compositional relations, as we've just seen, but we may also use it if we know a pair of placemental relations. If we know such relations we find the appropriate column satisfying these, and a "1" indicates that the compositional relations in the leftmost columns are possible given the placemental relations we have.

When describing the situations in figure 6.11 we mentioned that we wouldn't include relations in the composition table that require an angle greater than 90° from the placed object to any of the two compositional objects. This applies to other possible compositions as well, and table 6.3 shows a list of compositions that produce fields that always have this property, together with a list of thus excluded fields. The reason for excluding these are that they would allow situations where a third object would have relations with one or both of the other two objects which would not be possible based on the conversion table, which again would not have a corresponding entry in the composition table. Figure 6.14 shows an example of this: In this situation we have $pFRq$, qLp, and an r such that rFp and rLq. We also have pFr, which is the only possible value of this relation in this situation. Now we have two values, pFr and rFp, which are not allowed conversions and for which we don't have a compositional entry.

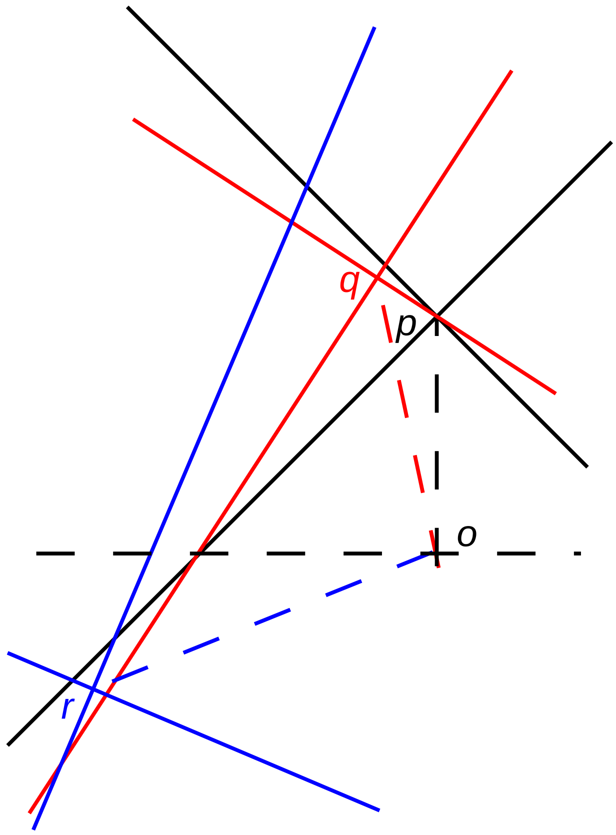


Figure 6.14: Example of excluded area

This is a breach of a basic premise in our calculus, and excluding such areas is necessary to ensure its completeness.

The fields that involve one-dimensional relations, such as $BLa\&Lb$, are not expressed in the table. This is done to somewhat confine the table. They can be found in the same way we did in 3PCAFOR, by looking for neighbouring relations to one object cooccuring with the same relation to other objects. As an example, pRq and qLp allow both $rRp\&rRq$ and $rRp\&rFq$, which means that it also allows $rRp\&rFRq$.

6.2 Consistency in the system

The two tables presented are created as a way of describing certain physical properties: combinations of relations that may occur in the physical plane. We've seen several figures showing possible physical or spatial instantiations of certain sets of relations, these can be seen as semantic models of such sets. The conversion table describes which values of qp may cooccur with a given value of pq, and as it is an exhaustive table it rules out which values of qp that are impossible given the same pq. This is simply a syntactic way of telling which pairs of relations that are and which aren't situations that the spatial properties of the calculus allows, situations for which we're able to construct a spatial model. The same goes for the

ab	ba	Areas excluded
B	F	$Ra\&Fb$, $La\&Fb$
BR	F	$Ra\&Fb$
R	F	$Ra\&Fb$
R	FL	$Ra\&Fb$
FR	L	$Fa\&Lb$
FR	FL	$Ra\&Fb$, $Fa\&Lb$
F	L	$Fa\&Lb$
F	BL	$Fa\&Lb$
F	B	$Fa\&Lb$, $Fa\&Rb$
F	BR	$Fa\&Rb$
F	R	$Fa\&Rb$
FL	R	$Fa\&Rb$
FL	FR	$La\&Fb$, $Fa\&Rb$
L	FR	$La\&Fb$
L	F	$La\&Fb$
BL	F	$La\&Fb$

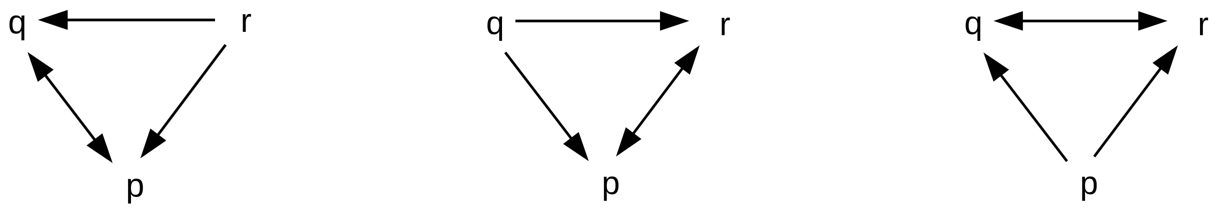
Table 6.3: Areas excluded from the composition table

composition table: It describes which set of the four relations involved that may cooccur as a physical situation.

Any syntactic description of a configuration, partial or complete, is a set of valued relations. We've mentioned, both when discussing Allen's algebra and when presenting the calculus for the absolute frame of reference, that the local constraints ensure consistency of sets of relations. This concept is no less important in the current system, and we will restate its definition as we'll use it here: *A set of valued relations are consistent if they may cooccur in a physical situation*. The two tables we've created are methods that can be used for checking that values of relations involved are consistent. The requirement that the angle between the lines from o to one object and o to another object be less than 90° is a limit on the physical situations that we'll allow, and limiting the conversion table and excluding certain areas from the composition table is a syntactic way of ensuring consistent results regarding this.

6.3 Inference in the system: The general approach

Inference always goes from one or more known relation to possible values of unknown ones. We will now introduce two core concepts used in carrying out inference in the system: (i) Different mappings from a configuration to the composition table and (ii) constraining said table.



(a) (b) (c)

Figure 6.15: The three possible mappings for any given configuration

		rp	L
pq	qp	rq	F
R	L		1

		qp	L
pr	rp	qr	B
R	L		1

		pr	R
rq	qr	pq	R
F	B		1

Table 6.5: Mappings to the compositional table for figure 6.16

6.3.1 Possible mappings to the composition table

Any configuration of three objects will contain a total of six relations. Any entry in the composition table expresses the relation from one object to two others given the relation between these two, or conversely the relation between two objects given a third objects relation to both of these. This means that the composition table expresses knowledge about four relations in the configuration, which again means that the composition table never expresses complete knowledge about a configuration.

To bridge the gap between these two facts we need to note that there are three possible mappings from a configuration of three objects to the composition table: any of the three pairs pq and qp, rq and qr or pr and rp may be assigned the role of compositional relations in the table. This will not change the distribution of existing fields in the table, but they will be able to express different information about the configuration. It should also be noted that the ordering of the pairs doesn't matter, as the composition table is symmetric in this regard.

Figure 6.15 visualizes all the mappings given a configuration. The double-headed arrows are the compositional relations, the single-headed the placemental ones. Any two of these mappings together would express information about all six relations, or in other words: complete information about a configuration. We also see that specifying either the compositional or the placemental relations (the objects participating in them) is enough to characterize a mapping.

Example: Figure 6.16 is a syntactic presentation of a configuration where we have complete knowledge about the relations involved. This knowledge gives us the three mappings shown in table 6.5. Any two of these tables together would be enough to provide complete knowledge about the configuration, but they are all included to show that they are based on different pairs of compositional and placemental relations.

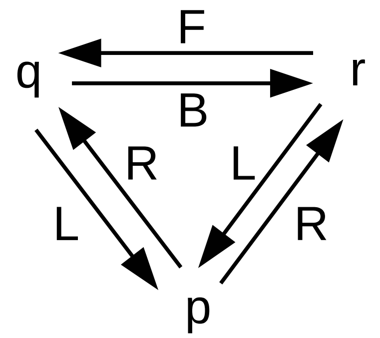


Figure 6.16: Complete knowledge about a configuration

6.3.2 Propagating constraints in the system

When nothing is known about the relations in a configuration, all mappings to the composition table gives us all possible combinations of values that the relations involved may have. Our task when carrying out inference in this system is to limit this domain of possible values. There is some constraint built into the composition table and the conversion table, as they specify which combinations that are possible, but the main part of the inference is carried out by applying the knowledge we have about any of the relations in a given configuration. This knowledge provides constraints on the domain of possible values, and propagating these constraints allow us to select a subset of the entire composition table. In some cases this will leave us with a single entry and thus exact knowledge about the relation in question, but in most cases we will have a set of possible values. In this section we will look at how this is carried out in various situations.

How we go about selecting the table subset depends on which relations that are known and which we're asking about. In the first situations we'll look at, shown in figure 6.17, we know three of the relations involved in the composition table when mapping pq and qp as compositional relations and are asking about a fourth. In figure 6.17a we know both compositional relations, and can thus limit the table to the corresponding line. This will limit the possible placemental relations, as no matter which value the composition has, there's no line in the composition table that allows all of their pairings. Furthermore we can rule out the placemental pairings that do not include rq. This leaves a number of possible table entries, and thus a number of possible values of rp. In figure 6.17b we know both placemental relations, limiting the composition table to a single column. Not all lines in the composition table will allow the values of rp and rq that we have, and those who do must include pq. This leaves a number of possible table entries, and thus a number of possible values of qp.

We could have dealt with both of these cases in both ways: For figure 6.17a we could've selected the columns satisfying rq and then found the one line satisfying the compositional relations, selecting existing fields in the resulting table as possible values of rp. For 6.17b we could've selected the lines satisfying pq, found the one column satisfying the placemental

(a)
(b)

Figure 6.17

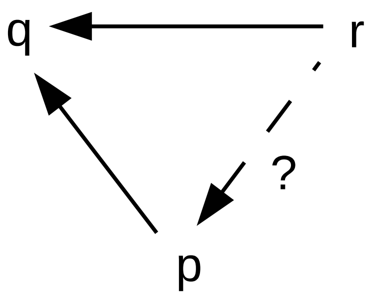


Figure 6.18

relations and then selected existing fields in the resulting table as possible values of qp. All constraints are applied in both cases, the ordering is not relevant to the results.

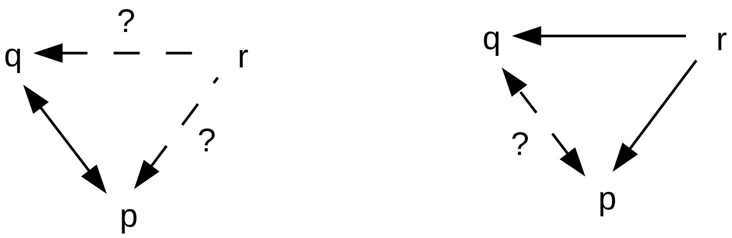
In figure 6.18 we only know two of the relations when mapping pq and qp as compositional relations. Our first step could be to limit the table to the lines satisfying pq. This will include all values of qp that are converse possibilities of pq, possibly giving us more lines to consider than in figure 6.17a. For all of these lines we find the fields satisfying the placemental relation rq, and get the possible values of rp from these.

Figure 6.19 shows us situations where we need to apply conversion when mapping pq and qp as compositional relations. In figure 6.19a we find the possible values of rq by applying one placemental and both compositional constraints, like we did for figure 6.17a, and we get the set of possible values of qr by taking the conversions of these. In figure 6.19b we first select the line satisfying the compositional relations pq and qp. Now we don't have a direct constraint on any placemental relation, but we apply what we get from the known pr by taking its conversion. This leaves one or more possible values of rp, and by selecting these fields we get a set of possible values of rq. Finally, we apply conversion to these to get the possible values of qr. In both of these cases we find that the relation in question is not a direct participant in the composition table, but we're still able to find possible values through first constraining the composition table and then taking conversions.

Figure 6.20 shows two configurations where, when mapping pq and qp as compositional relations, taking the converse of any single known relation won't bring us any closer to the unknown relations. In the case shown in

(a) (b)

Figure 6.19



(a) (b)

Figure 6.20

figure 6.20a we know the compositional relations in the table, but this won't give us information about either rp or rq individually. By inspecting the composition table we can see that all compositions allow all placemental relations *individually*, but the table will tell us which pairings of the two placemental relations that are possible and which aren't. Constraining the table in this way, not knowing either of the placemental relations, will be called *placemental completion*. Figure 6.20b shows the opposite case, where we know both placemental relations. This allows us to exclude some compositional relations entirely, and thus provides more constraints than knowing only the compositional ones. Their pairings will also be constrained. Constraining the table in this way will be called *compositional completion*.

We've looked at some examples of how we have to utilize conversions or possible pairings to find possible values of the relations in a mapping. There are a number of ways to combine these procedures when applying our constraints, and a very important thing should be noted: All such applications potentially weaken the available constraints. A conversion typically allows a disjunction of possibilities, and when combining several of these operations we may end up with a vast array of possibilities. A way of characterizing how well we're able to utilize the constraints we have is to talk of the *cost* when propagating them to a mapping, where conversion and compositional completion are seen as costly operations, because they weaken the constraints. To rectify this, we look back at figure 6.15 and remember that there are three possible mappings to the composition table for every configuration. Many situations contain a set of known relations that do not participate in the same mapping, but every possible known relation is directly part of exactly two of these. Thus, propagating our constraints in one mapping may include applications that produce possibilities that are directly prohibited by propagating the same constraints in another mapping. To fully utilize the constraints from a given configuration we must propagate them to all mappings in which any of the known relations participate directly. This will give us several sets of possibilities, and the final set of possibilities will be the intersection of these, the values that are possible in all mappings.

At this point we should note something about relations denoting one-dimensional areas. We mentioned that we would calculate this for placemental relations based on neighbouring areas co-occurring with the same area. This is because these are left out of the listing of placemental relations in the composition table, and as such this is the only time we should do this. There are two other ways of finding possible relations: Either as a compositional relation or as possible conversions. In both of these cases the possible one-dimensional relations are listed, and as such we shouldn't carry out any extra calculations to find them. If we come across cases where the results of our calculations don't comply with our expectations in this regard, such as B and L being in the final result set and not BL, then we've either made a mistake or we've come across a peculiar situation. In any case, such outcomes will be further investigated.

The reason that we're utilizing different mappings is that each composition table expresses easily understandable information, requiring moderately complex calculations. If we were to include all possible and consistent values of all six relations, this would require a much larger set of assumptions, making the calculation a lot more costly in terms of complexity. Intersecting constraint propagations to different mappings is a way of utilizing available knowledge without requiring too much of the calculation.

6.3.3 Inference in the system: A detailed example

We will now present a detailed example of inference in the system, to properly illustrate the methods presented. The case we'll consider is the one shown in figure 6.21, where we know pRq and qRr, and wish to find rp. We will go through each possible mapping, explain which operations we need to perform for each of them, and present the results.

The first mapping we consider is the one where pq and qp are compositional relations, as shown in figure 6.15a. The relation we're after takes direct part in this table, as does the known pq, but we need the converse possibilities of pq to complete the compositional pairings and the conversion of qr to get a placemental constraint. Thus we need to make two conversions, making this the cost of constraint propagation in this mapping.

First we see that we need to consider the cases where we have pRq and either $qFp, qFLp$ or qLp, these being the possible compositional pairings. Furthermore we use conversion and find that the possible values of rq are $rFq, rFLq$ and rLq. These constraints leave us with the part of the

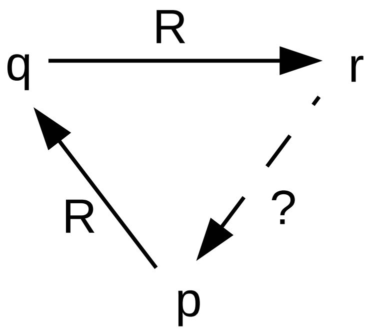


Figure 6.21

		rp	B	B	R	R	F	F	L	L
pq	qp	rq	F	L	F	L	F	L	F	L
R	F		0	0	0	0	1	1	0	1
R	FL		0	0	0	0	1	1	0	1
R	L		0	0	1	0	1	1	1	1

Table 6.6: Extract of the mapping of pq and qp as compositional relations

composition table shown in table 6.6. In this mapping, the constraints allow rRp, rFp and rLp, and as these are placemental relations we have to include $rFRp$ and $rFLp$ as well.

Next we consider the mapping of pr and rp as compositional relations, as shown in figure 6.15b. Here we have one placemental relation, qr, and we get the other by conversion of pq. We have none of the compositional relations, and get their possibilities by compositional completion based on our assumptions of placemental relations. The possibilities of rp are included here. Cost of constraint propagation in this mapping: One conversion and one compositional completion.

Converting pRq leaves us the possibilities $qFp, qFLp$ and qLp, and sets of placemental constraints when paired with qRr. This leaves the part of the composition table shown in table 6.7. We see that the constraints allow $rRp, rFRp, rFp, rFLp$ and rLp in this mapping.

The final mapping we consider is the one where rq and qr are compositional relations, as shown in figure 6.15c. Both of our known relations participate directly in this table, and we select all possible conversions of rq to get our compositional relations. This will leave us with a range of possible values of pr. The last step is converting these possible values to get the ones we're after. Cost of constraint propagation in this mapping: Two conversions.

Converting qRr gives us the possible compositional relations $rFq, rFLq$ and rLq. Together with pRq this leaves the part of the composition table shown in table 6.8. We get the possibilities $pRr, pFRr$ and pFr, and by converting these the possibilities $rFp, rFLp, rLp, rBLp, rBp, rBRp$ and rRp. Note that $rFRp$ isn't included, as it isn't included in the conversions.

We now have three sets of possible values of rp due to constraining the

		qp	F	L
pr	rp	qr	R	R
B	F		1	0
BR	F		1	0
R	F		1	1
R	FL		1	1
R	L		1	1
FR	FL		0	1
FR	L		0	1
F	L		0	1
F	BL		0	0
F	B		0	0
F	BR		0	0
F	R		0	0
FL	R		0	0
FL	FR		0	0
L	R		1	0
L	FR		1	0
L	F		1	0
BL	F		1	0

Table 6.7: Extract of the mapping of pr and rp as compositional relations

		pq	R	R	R	R
qr	rq	pr	B	R	F	L
R	F		0	1	0	0
R	FL		0	1	0	0
R	L		0	1	1	0

Table 6.8: Extract of the mapping of qr and rq as compositional relations

pq	qp(A)	qr	rq(A)	rp(A)
R	L	R	F	R

(a) Compositional: pq and qp

pr(A)	rp(A)	qr	qp(A)	pq
L	R	R	F	R

(b) Compositional: pr and rp

rq(A)	qr	pq	pr(A)	rp(A)
L	R	R	F	R

(c) Compositional: rq and qr

Table 6.9: Solution sets leading to rRp

composition table, and finally their intersection: $rRp, rFp, rFLp$ and rLp.

6.4 Consistency, continued

The results of the example that we just saw quickly raises a question: How are rRp and rFp possible solutions, when $rFRp$ is left out? The result seems anomalous. To better understand the result, we will expand upon the concept of consistency.

Every possible solution we arrive at using the methods of inference described above will involve a set of known and assumed relations. The known relations here are those that we take as given in a case, and the assumed ones are those whose values we arrive at using the methods of inference described earlier. Our goal in each case is to find the value of a specific relation, which naturally always will be an assumed relation, but the inference may often include other assumptions as well. There may be different ways of arriving at the same possible answer, if we look back at table 6.7 we see that both qFp and qLp produce the possible value rFp. If we consider the values of each relation involved as part of a solution, this example would give two different solutions, as qp differs. Such a specification of which values of which relations that leads us to an answer, including the known relations, will be called a *solution set*. For each possible problem we can create a list of such sets.

This leads us to an important demand on our methods of inference: Each solution set must be consistent for it to be considered an actual solution. This seems obvious, the solutions we're interested in must be the kind of solutions that may actually occur physically, but how can we make sure that the results we get from our inference satisfies this demand? The two tables we have ensure this to a great degree already, as the composition table ensures the consistency of the four relations involved. However, as we've mentioned several times, there are six relations involved in any configuration of three objects. Are there situations where our current methods are insufficient?

The results we got in our example could imply this. We will look at rRp in particular, as the intermediary relation $rFRp$ is not given as a possible solution, making rRp the odd one in the collection. To better understand how we got this result we will show all solution sets that lead to it. Table 6.9 shows the complete list, where a relation marked with (A) is an assumption in the solution. We should note that the only assumption that appears in all three tables is rp, and that qp appears in 6.9a and 6.9b, rq in 6.9a and 6.9c and pr in 6.9b and 6.9c.

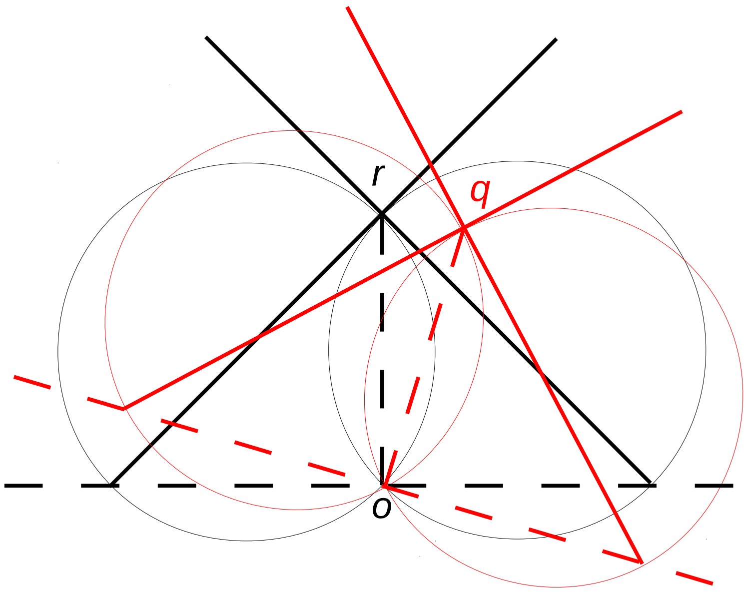


Figure 6.22

As pq and qr are known relations, they appear in all solution sets. If we look back at figure 6.9 we see the physical areas a second object p would have to be placed in relative to first object q for q to be in the relations specified to p. We will utilize the information we get from this figure to investigate our current situation. We'll begin with the single solution set shown in 6.9c, where we know qRr and assume rLq. Figure 6.22 illustrates this situation, and the converted coordinate systems of each object is added to the diagram. To figure out whether our inference went wrong we will carry out a visual inference. We know that pRq, and find the corresponding area in the figure. We assumed placemental relation pFr, as the composition table shows that there is an area in front of r and to the right of q. We can see how this is a sub-area of the one to the right of q. However, the final assumption, rRp, is clearly not covered by this area. No matter where we move q within the area right of r, the area to the right of q and in front of r simply won't reach that far to the left within the area in front of r. It seems we've found a situation that can't exist as a physical situation in our system, and thus an inconsistent set. If we had investigated the other possibilities of rq given the known qr we would have found that an area in front of r and to the right of q simply wouldn't exist, as these are excluded from these compositions. Thus, the assumption of rq is inconsequential in this matter.

How did we end up with this? The answer lies in the fact that the relation in question, rp, was found by converting possible values of placemental relation pr. As we found pFr to be a possibility, conversion gives a range of possible results, including rRp. We used the composition table to find possible values of pr, but none of the existing constraints limits the conversion of these results. Assuming pFr would also give us rBp as a possibility in this mapping, but as the other mappings don't produce this possibility it's left out. This inconsistency must be remedied, but first we'll see how the other mappings led to the same mistake.

In the case in table 6.9b we can use the same illustration, as we have qRr. We know that pRq is still the case, and we find this area in the diagram. It's limited further by the assumption qFp, to the area to the right of q outside of the half-circle marking the areas where we would have qLp. What's most important are the assumed compositional relations pLr and rRp. This area can be seen as the intersection of the area left of r and the circle that r is to the right of. This area does not in any way overlap with the area to the right of q, which we know must be the case. The set is inconsistent.

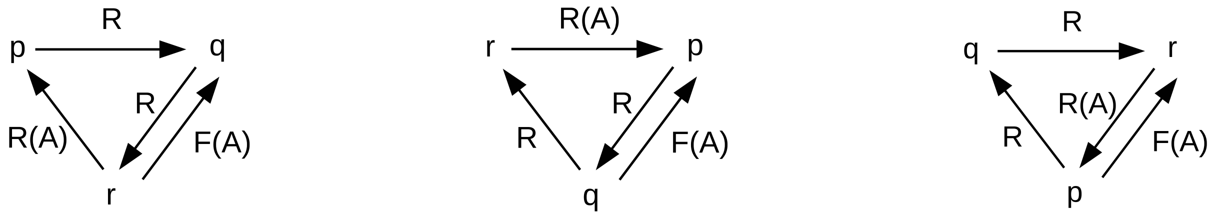
In this mapping, q is placed relative to the composition of p and r. One of them, qRr, is given, and the other is inferred by conversion. The composition pLr and rRp does in fact create an area that is to the right of r and in front of p, but pRq is not true in this area. The problem is that the known relation pRq isn't included when constraining the composition table, other than offering all its conversions as possible values of qp. We could've gotten qFp from pLq as well, which would lead to the compositional assumptions pRr and rLp, and the set would've been consistent.

Finally we get to the cases in table 6.9a. This is actually the same case that we had for table 6.9c, with different relations in the same roles. If we read qp and pq as compositional relations, in that order, we see that they match the ones in 6.9c. The placemental relation towards the first member of the first compositional relation, rq here and pr in 6.9c, both have the value F, and their conversions qr and rp are valued R. The second placemental relation, rp and pq, nhave the value R.

However, the resulting relation rp is here a direct participant in the composition table. The area leading to our dismay is the one in front of q and right of p, similar to the one in front of r and to the right of q in 6.9c. We get the possibility of rFq from converting qRr, but this does not specify that r has to be placed in the area in front of q where qRr is the case.

We have three ways of reaching the same erronous result. It will be sufficient to show what characterizes the inconsistent set in one of the mappings and remove the result to remove it completely from the final set of solutions, as this is an intersection of the three sets of results. In table 6.9c, we got an erronous result because we allowed all conversions of F. As we saw, in all situations where we have qRr, pRq and pFr, then the only possible value of rp will be L. This defines a necessary property of how we're able to construct configurations, and it can be generalized as a rule. It should be noted that the value of rq isn't necessary to involve in our rule. The reason for this is that the relations pRq and pFr only are possible when we have rLq paired with qRr, due to exclusion of areas. We also get the mirrored case: If we know qLr, pLq and pFr, then it must be the case that rRp.

We investigated possible models to derive a syntactic rule. When looking into each of the solution sets we specified which assumptions that



(a) (b) (c)

Figure 6.23

lead to which erronous result, and we explained in the model how this was possible. If we draw the solution sets as syntactic situations, as shown in figure 6.23, then we see that they all express the same thing, and that the newly developed rule is equally applicable to all of them. By solving one of the problems in our constraint propagation we've solved all of them. All of these solution sets include the conversion of a placemental relation, and our previous methods of consistency checking had no ways of ensuring the consistency of such a set. The added rule takes care of this specific case.

We can go further into the syntactic reasons that this case was accepted. We based our investigations on specific values of the relations qr and pq. These values allowed the three different mappings to assign the same values to the relations involved. That is, not the same relations in the actual configuration we're considering, but the same roles the relations play in the composition table: In figure 6.23, we see that each mapping has one placemental relation valued R, one placemental relation valued F, the converse of this last one valued R, and the compositional relation from the object of the unconverted placemental relation to the object of the converted one valued R. If we go through every single solution set for every possible combination of known relations in this specific situation, then we would find that the only ones where this happens, where all mappings can assign the same values to their roles, are these two: where we get rRp from pRq and qRr, and where we get rLp from pLq and qLr. In fact, if we extend the solution sets so that they each contain all six relations in the configuration, requiring us to check a great number of assumptions and adding to the complexity of the calculation, then these two sets of assumptions still allows such a situation to happen, where all mappings may use the same valuations. We will later show which assumptions that are involved in each possible combination of known and unknown relations, but if we go through all the solution sets for all the possible situations, the one just shown is actually the only one where this occurs. We've seen that this property cooccurred with the need to apply our new rule. If the rule is applicable only when the property just described occurs, then the rule has a rather narrow use. None the less, it has to be applied if our calculations are to give consistent results.

What lead us into this consideration of full solution sets applicable to all mappings was a way of telling which part of the front area of an object another object would have to be in, and thereby constraining the possible

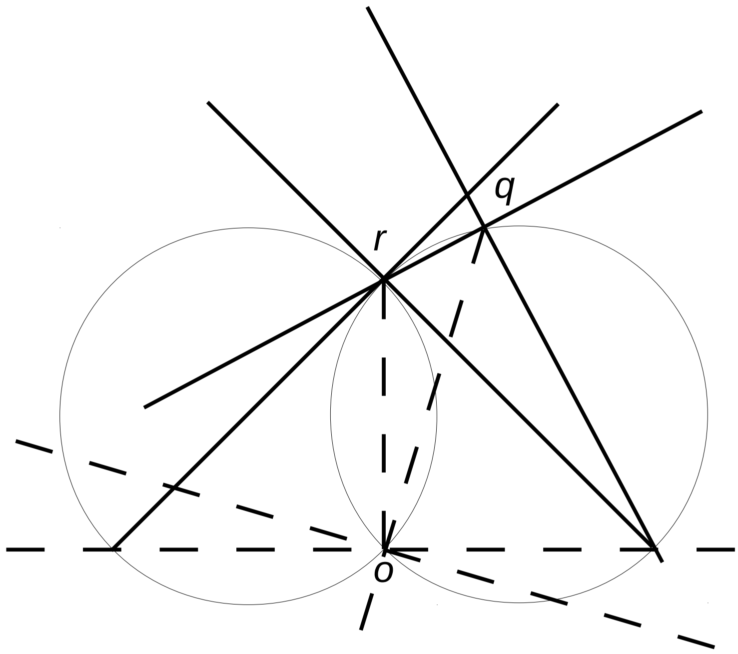


Figure 6.24

values of conversion. The areas to the left and right of an object also have multiple possible conversions, and there are compositions that leave areas within these that only allow one conversion. Do we need similar rules in these cases?

Consider the situation shown in figure 6.24. Here we have $rFLq$, and we can see that placing p in the part of the area right of r that's also behind q does not allow rLp as a conversion. This is similar to our previous rule, and we could create a rule covering this case. The solution set we would wish to rule out would be $rFLq, pBq, pRr$ and rLp. If we map rq and qr as compositional relations in the composition table, we get an area satisfying Rr and Bq. If we map rp and pr as compositional relations we only have the converted values of the placemental relations, but pBq only allows qFp, and we can take qRr from $rFLq$. This is also possible in the composition table. The final mapping, where pq and qp are compositional tables, is more problematic. When we have pBq, there is no area to the left of p and to the front-left of q, and the possibility of rLp has to be left out. We see that multiple mappings to the composition table took care of our problem, and we don't need to apply any extra rules. This is the case for other sub-areas of R or L in other compositions as well, none of the potentially problematic solution sets pass through all mappings.

In cases where we have $rFLq$ and $qFRr$, as seen in figure 6.25, we get a special concern regarding consistency. We've said that we would only allow configurations where the angle between lines from the observer to any of the objects involved is at most 90°, and this should limit where we allow a third object p to be placed in the configuration shown. The area to the left of p is excluded according to the exclusion table, so both pLr and $rFLp$

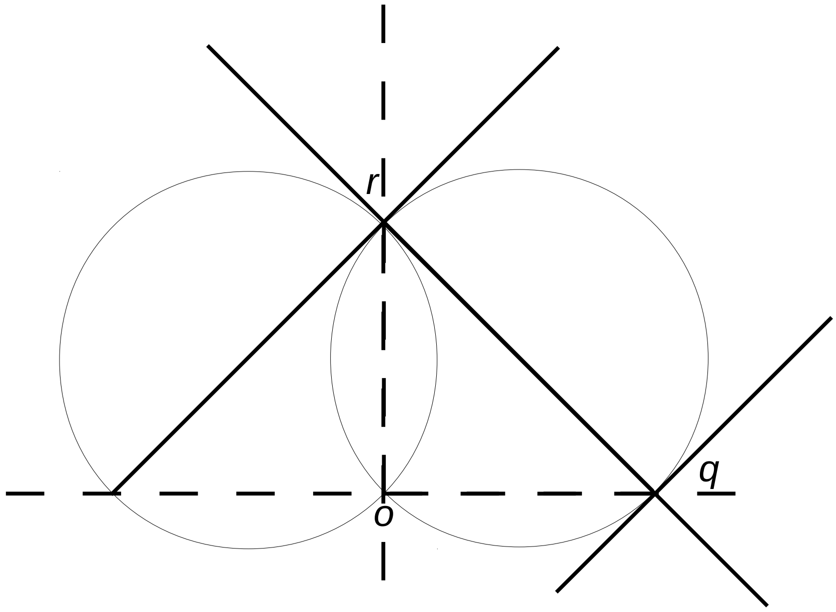


Figure 6.25

are none of our concern. However, the right half of the area in front of r should be included, while the left half should be left out. Is this taken care of in our current system? If we look up the entry for $rFLq$ and $qFRr$ in the composition table, we see that there is an area in front of both, as should be expected. The problem here is that arriving at pFr in this way allows both $rBRp$ and rRp, both of which are areas that we wish to exclude. Even though we utilize all mappings, both of these possibilities would pass through. This is not too surprising, as we're able to express this problematic solution set without including anything about the relations between p and q, thus providing no constraints on these. We will have to provide some constraints of our own, and say that whenever two objects r and q are in the relations $rFLq$ and $qFRr$, then there can't be any third object p so that $rBRp$ or rRp. We also get the mirrored case: There can be no placement of p so that $qBLp$ or qLp. This rule will be referred to as the FR/FL-rule.

There is a final case that we need to take into account, also applicable when we have $rFLq$ and $qFRr$, like in figure 6.25. If we know these two things, and we also know rBp, then we know that p must be placed in the elliptical field between o and r. Normally, knowing $qFRr$ and rBp would allow qBp. This can be verified in the composition table, in the entry where rBp and pFr are compositional relations and $qFRr$ and qBp are placemental ones. This simply means that the compositional relations produce a field behind p and to the front-right of r. However, this specific case requires q to lie at the maximum distance from r along FRr, as we have $rFLq$. This is not covered by the area behind p when we place p within said elliptical field. This also applies when we have $rBLp$. There is currently no way of excluding this possibility in the system, and thus we need to add a third rule: If we have $rFLq, qFRr$ and either rBp or $rBLp$, then it must be the case that qRp. We also get the mirrored case: If we have $rFRq, qFLr$

Conditions	Result
aRb, cRa and cFb	bLc
aLb, cLa and cFb	bRc
$aFLb$ and $bFRa$	not $aBRc$, not aRc, not $bBLc$, not bLc
$aFLb, bFRa$ and (aBc or $aBLc$)	bRc
$aFRb, bFLa$ and (aBc or $aBRc$)	bLc

Table 6.10: Conditional rules

and either rBp org $rBRp$, then it must be the case that qLp.

Table 6.10 summarizes these rules. We apply them for each solution set in each mapping: If any of the sets break with the rules, they must be inconsistent and can thus be removed from the list of actual solutions. The reason that they must be applied for each mapping rather than the final intersection of answers is that the full solution sets, with all their assumptions, aren't included here.

6.5 Inference in the system: The possible cases

Now that we've explained the general approach to inference in the system we can proceed to deal with the possible cases, varying in which relations that are known and which we're asking about. All configurations involve the three objects p, q and r. There is no ordering between these objects other than the known relations, so symmetrical cases will not be considered.

6.5.1 One known relation

When there's only one known relation pq in a configuration, the only single other relation we can infer anything about is its converse, qp, given by the conversion table. This gives us a set of possible compositional relations, which leads us to the situation shown in figure 6.20a. The possible inferences from this point are, as mentioned, the possible pairings of placemental relations.

We will not gain any information by considering the known relation as a placemental relation, as the composition table will show that all values of placemental relations could co-occur with all values of both composition relations.

Example: We know that pBq. From the conversion table we can infer that qFp. We can't know anything about rp or rq individually, but we know that they together have to form a pair that exists in the entry for pBq and qFp. This means that it could be the case that $rBp\&rRq$, but not that $rBp\&rFq$. If pq had more than one conversion, we would have to look up more than one entry to find the possible pairings.

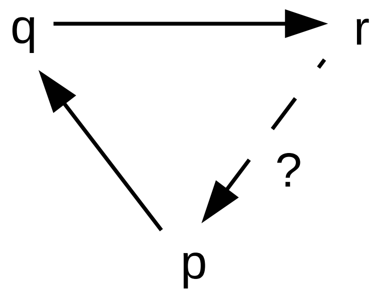


Figure 6.26

6.5.2 Two known relations

There are a number of different situations where two of the relations in a configuration are known, and we inquire about a third, unknown one. We looked at one in figure 6.20a, and saw that the only thing we could infer about the unknown relations was their pairing. This information is fully expressed in the composition table. If we take the conversions of these possible pairs we would get the possible values of pairings of pr and qr.

The first case we'll consider in depth is shown in figure 6.26, where we know pq and qr and wish to figure out rp. This is the one we showed in detail earlier. We'll go through each of the three possible mappings and show how they let us calculate possible values, and specify which assumptions we need to make.

- *Mapping pq and qp as compositional relations* - The relation we're after takes direct part in this table, as does the known pq, but we need the converse possibilities of pq to complete the compositional pairings and the conversion of qr to get a placemental constraint. Cost of constraint propagation in this mapping: Two conversions. Assumptions beside answer: qp and rq.

- *Mapping pr and rp as compositional relations* - Here we have one placemental relation, qr, and we get the other by conversion of pq. We have none of the compositional relations, and get their possibilities by compositional completion based on our assumptions of placemental relations. The possibilities of rp are included here. Cost of constraint propagation in this mapping: One conversion and one compositional completion. Assumptions beside answer: qp and pr.

- *Mapping rq and qr as compositional relations* - Both of our known relations participate directly in this table, and we select all possible conversions of rq to get our compositional relations. This will leave us with a range of possible values of pr. Finally we add the step of converting this relation to find the ones we're after. Cost of constraint propagation in this mapping: Two conversions. Assumptions beside answer: rq and pr.

Table 6.11 shows the resulting possibilities for rp given all possible combinations of pq and qr. The symmetric properties of this table should be noted: If we know the two relations in the situation it does not, in fact, matter which of them has which value, both combinations will lead to the same set of possibilities.

	qr								
		B	BR	R	FR	F	FL	L	BL
	B	F	F	F,FL,L	F,FL,L	∀	R,FR,F	R,FR,F	F
	BR	F	F	F,FL,L	F,FL,L	∀	R,FR,F	R,FR,F	F
	R	F,FL,L	F,FL,L	F,FL,L	F,FL,L	∀	∀	∀	F,FL,L
	FR	F,FL,L	F,FL,L	F,FL,L	L	L,BL,B,BR,R	L,BL,B,BR,R	∀	F,FL,L
pq	F	∀	∀	∀	L,BL,B,BR,R	L,BL,B,BR,R	L,BL,B,BR,R	∀	∀
	FL	R,FR,F	R,FR,F	∀	L,BL,B,BR,R	L,BL,B,BR,R	R	R,FR,F	R,FR,F
	L	R,FR,F	R,FR,F	∀	∀	∀	R,FR,F	R,FR,F	R,FR,F
	BL	F	F	F,FL,L	F,FL,L	∀	R,FR,F	R,FR,F	F

Table 6.11: Table showing possible values of rp when pq and qr are known, as depicted in figure 6.26

The next case we'll consider is the one shown in figure 6.27, where we know pq and qr, and wish to find pr.

- *Mapping pq and qp as compositional relations* - pq is given, and we use its converse to find the lines to consider. We choose the columns where rq is a possible value of the conversion of qr, and take the conversions of the resulting possible values of rp. Cost of constraint propagation in this mapping: Three conversions. Assumptions beside answer: qp, rq and rp.

- *Mapping pr and rp as compositional relations* - Here we'll consider all columns that satisfy qr and any converse of pq as placemental relations. We have none of the compositional relations, so compositional completion gives us the possible calues of pr. Cost of constraint propagation in this mapping: One conversion and one compositional completion. Assumptions beside answer: qp and rp.

- *Mapping rq and qr as compositional relations* - We have one compositional relation, qr, and together with its converse limit the lines to consider. The columns satisfying pq will give all possible values of pr. Cost of constraint propagation in this mapping: One conversion. Assumptions beside answer: rq.

The complete table of inferences for this situation is shown in table 6.12 Here we see the same symmetric properties that we saw in table 6.11.

Next we get the case in figure 6.28, where we know rq and rp and wish to figure out qp.

- *Mapping pq and qp as compositional relations* - We see that we have a complete constraint of the placemental relations. This lets us select

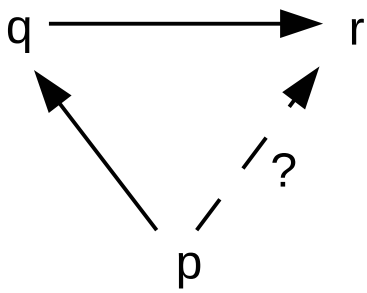


Figure 6.27

		qr							
		B	BR	R	FR	F	FL	L	BL
	B	L,BL,B,BR,R	B,BR,R	B,BR,R	B,BR,R	∀	L,BL,B	L,BL,B	L,BL,B
	BR	B,BR,R	R	R	R	∀	L,BL,B	L,BL,B	B
	R	B,BR,R	R	R,FR,F	R,FR,F	∀	∀	∀	B,BR,R
	FR	B,BR,R	R	R,FR,F	F	F	FR,F,FL	∀	B,BR,R
pq	F	∀	∀	∀	F	F	F	∀	∀
	FL	L,BL,B	L,BL,B	∀	FR,F,FL	F	F	F,FL,L	L
	L	L,BL,B	L,BL,B	∀	∀	∀	F,FL,L	F,FL,L	L
	BL	L,BL,B	B	B,BR,R	B,BR,R	∀	L	L	L

Table 6.12: Table showing possible values of pr when pq and qr are known, as depicted in figure 6.27

a single column, and all resulting values of pq are possibilities. Cost of constraint propagation in this mapping: Compositional completion. Assumptions beside answer: pq.

- *Mapping pr and rp as compositional relations* - We choose the lines satisfying rp and all its converse values. The columns satisfying the converse of rq gives us possible values of qp. Cost of constraint propagation in this mapping: Two conversions. Assumptions beside answer: pr and qr.

- *Mapping rq and qr as compositional relations* - We have one compositional relation, rq, and use its converse values to select lines for inquiry. The converse of rp gives a placemental constraint, and we select the corresponding values of pq. Finally, we take the converse values of these. Cost of constraint propagation in this mapping: Three conversions. Assumptions beside answer: qr, pr and pq.

Table 6.13 shows the complete list of inferences for this situation. It should be noted that this table doesn't have the same symmetric property that we found in the two previous situations. The table shows some peculiar results that should be noted:

- $rBRq$ and $rBRp$ - Here we have B and F as possible values of qp. To understand this, we can look back at figure 6.9. If r is the object

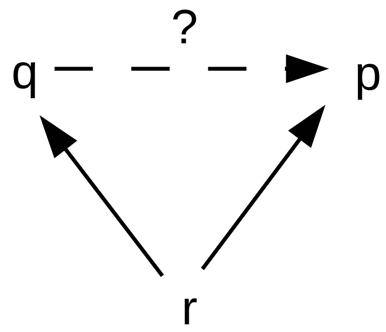


Figure 6.28

depicted, we know that both p and q have to be placed on the part of the circle in front of r that lies in the left half. This area has no width, and thus we get the two possible values of qp. We get the same result for $rBLq$ and $rBLp$.

- $rFRq$ and $rFRp$ - This is similar to the case above, but here p and q have to be placed on the half-circle to the left of r. We get the added possibility $qFLp$, made possible when we have $qFLr$, the extreme case of 90°. We get a similar case when we have $rFLq$ and $rFLp$.
- $rFRq$ and $rFLp$ - Looking back at the same figure as before, here we'll find that q has to be placed along the half-circle to the left of r, and p along the half-circle to the right. We get the possibility $qFLp$ when we have $pFRq$ and $qFLp$, the extreme case of 90°. We get a similar case when we have $rFLq$ and $rFRq$.

		rp							
		B	BR	R	FR	F	FL	L	BL
rq	B	∀	B,BR,R,FR,F	B,BR,R,FR,F	F	F	F	F,FL,L,BL,B	F,FL,L,BL,B
	BR	F,FL,L,BL,B	B,F	B,BR,R,FR,F	F	F	F	F,FL,L	F,FL,L
	R	F,FL,L,BL,B	F,FL,L,BL,B	∀	R,FR,F,FL,L	R,FR,F,FL,L	F,FL,L	F,FL,L	F,FL,L
	FR	L,BL,B	L,BL,B	L,BL,B,BR,R	R,FL,L*	R,FR,F,FL,L	FL,L	L	L
	F	L,BL,B,BR,R	L,BL,B,BR,R	L,BL,B,BR,R	L,BL,B,BR,R	∀	L,BL,B,BR,R	L,BL,B,BR,R	L,BL,B,BR,R
	FL	B,BR,R	R	R	R,FR	R,FR,F,FL,L	R,FR,L	L,BL,B,BR,R	B,BR,R
	L	B,BR,R,FR,F	R,FR,F	R,FR,F	R,FR,F	R,FR,F,FL,L	R,FR,F,FL,L	∀	B,BR,R,FR,F
	BL	B,BR,R,FR,F	R,FR,F	R,FR,F	F	F	F	F,FL,L,BL,B	B,F

Table 6.13: Table showing possible values of qp when rq and rp are known, as depicted in figure 6.28

Now we move on to the situation shown in figure 6.29, where we know pq and rq, and wish to find pr.

- *Mapping pq and qp as compositional relations* - We have one compositional constraint provided and one of the placemental ones. Using the converse of the compositional one we get a selection of possible values of rp, the converse of these gives us the possible

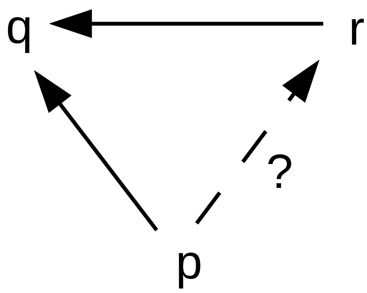


Figure 6.29

answers. Cost of constraint propagation in this mapping: Two conversions. Assumptions beside the answer: qp and rp.

- *Mapping pr and rp as compositional relations* - We have to consider all conversions of both rq and pq to get possible placemental constraints, and then select the corresponding values of pr through compositional completion. Cost of constraint propagation in this mapping: Two conversions and one compositional completion. Assumptions beside the answer: rp, qp and qr.

- *Mapping rq and qr as compositional relations* - We have one compositional relation, rq, and one placemental one, pq, and we select the resulting possibilities of pr. Cost of constraint propagation in this mapping: One conversion. Assumptions beside the answer: qr.

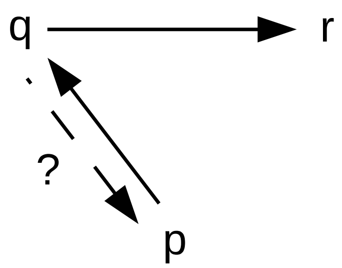
pq \ rq	B	BR	R	FR	F	FL	L	BL
B	∀	F,FL,L,BL,B	F,FL,L,BL,B	L,BL,B	L,BL,B,BR,R	B,BR,R	B,BR,R,FR,F	B,BR,R,FR,F
BR	B,BR,R,FR,F	B,F	F,FL,L,BL,B	L,BL,B	L,BL,B,BR,R	R	R,FR,F	R,FR,F
R	B,BR,R,FR,F	B,BR,R,FR,F	∀	L,BL,B,BR,R	L,BL,B,BR,R	R	R,FR,F	R,FR,F
FR	F	F	R,FR,F,FL,L	R,L,FL	L,BL,B,BR,R	R,FR	R,FR,F	F
F	F	F	R,FR,F,FL,L	R,FR,F,FL,L	∀	R,FR,F,FL,L	R,FR,F,FL,L	F
FL	F	F	F,FL,L	FL,L	L,BL,B,BR,R	R,L,FR	R,FR,F,FL,L	F
L	F,FL,L,BL,B	F,FL,L	F,FL,L	L	L,BL,B,BR,R	L,BL,B,BR,R	∀	F,FL,L,BL,B
BL	F,FL,L,BL,B	F,FL,L	F,FL,L	L	L,BL,B,BR,R	B,BR,R	B,BR,R,FR,F	B,F

Table 6.14: Table showing possible values of pr when pq and rq are known, as depicted in figure 6.29

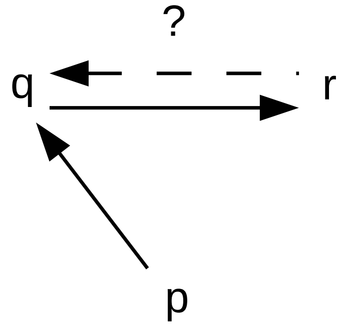
The resulting inferences possible in this situation is shown in table 6.14. We'll find some of the same peculiar result sets as we did in the previous case, such as for $pBRq$ and $rBRq$ resulting in pBr or pFr, and the reasons are similar to the ones we saw.

These four cases cover all situations where the relation in question is not the converse of any of the known ones. What about the situations where this is the case? Figure 6.30 shows all possible situations where we have two known relations and the one in question is the converse of one of these.

When carrying out constraint propagation in these cases we find that the results are very much like the values we find in the conversion table. There are some exceptions, though, and rather than giving the complete table for each case these exception will be explained.



(a)



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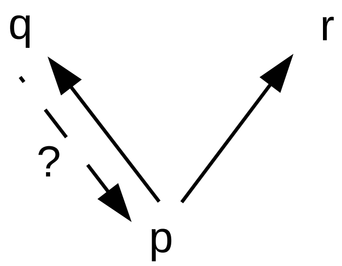

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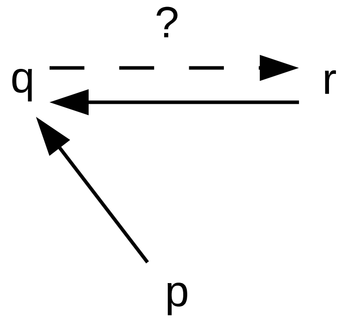

(d)

Figure 6.30: Possible situations with two known relations where the relation in question is the converse of a known one

- Figure 6.30a: When we have $pFRq$, any of $qBRr, qRr$ or $qFRr$ only allows qLp and not $qFLp$. We also have the symmetric case: Given $pFLq$, any of $qFLr, qLr$ or $qBLr$ only allows qRp and not $qFRp$.
- Figure 6.30b: When we have $qFRr$, then pRq or $pFRq$ only allows rLq and not $rFLq$. Oppositely: Given $qFLr$, either $pFLq$ or pLq only allows rRq and not $rFRq$.
- Figure 6.30c: Given $pFRq$, any of $pFLr, pLr$ or $pBLr$ only allows qLp and not $qFLp$. Oppositely: Given $pFLq$, any of $pBRr, pRr$ or $pFRr$ only allows qRp and not $qFRp$.
- Figure 6.30d: Given $rFRq$, any of $pFLq, pLq$ or $pBLq$ only allows qLr and not $qFLr$. Oppositely: Given $rFLq$, any of $pBRq, pRq$ or $pFRq$ only allows qRr and not $qFRr$.

We see that all of these cases are directly connected to the the FL/FR-rules, and the exceptions are simply the cases that have to be left out if the results are to be expressable in the calculus.

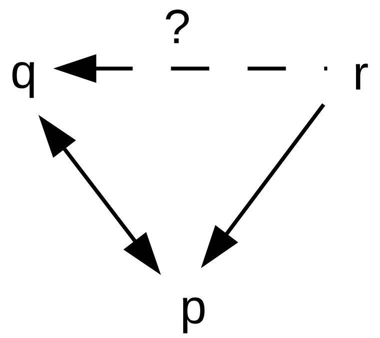

Figure 6.31

6.5.3 Three known relations

Now we move on to situations where three of the relations are known. This will in general mean that we can carry out constraint propagation at a lower cost in most mappings, and we'll see what effect this has on the resulting inference tables. In the first situations we'll look at one set of compositional relations will be known. These situations are the only ones where the relation in question is not the converse of any of the known ones, an thus possibly those of greatest interest. They will be followed by all situations where the relation in question is a converse of one of the known ones, and we'll see whether this is different from the corresponding situations where two relations are known.

Our first situation is the one seen in figure 6.31, where we know pq, qp and rp, and wish to find rq.

- *Mapping pq and qp as compositional relations* - Here we have both compositional relations and one placemental one, mirroring the case we saw when first presenting constraint propagation. Cost of constraint propagation: None. Assumptions beside answer: None.

- *Mapping pr and rp as compositional relations* - Here we need the conversion of rp to complete the compositional relations, and we use qp as a placemental one to get qr. Finally, we take the converse of these to get the possible values of rq. Cost of constraint propagation: Two conversions. Assumptions beside answer: pr and qr.

- *Mapping rq and qr as compositional relations* - We take the converse of rp to get a set of placemental relations. Through compositional completion we find the possible values of rq. Cost of constraint propagation: One conversion and one compositional completion. Assumptions beside answer: pr and qr.

This gives us the results shown in table 6.15. The entries marked as "-" means that the combinations of relations are impossible.

Next we get the situation shown in figure 6.32, where we know pq, qp and rp, and wish to find qr.

		rp							
pq	qp	B	BR	R	FR	F	FL	L	BL
B	F	L,BL,B,BR,R	B,BR,R	B,BR,R	B,BR,R	∀	L,BL,B	L,BL,B	L,BL,B
BR	F	B,BR,R	R	R	R	∀	L,BL,B	L,BL,B	B
R	F	B,BR,R	R	R	R	∀	L,BL,B,BR,R	L,BL,B,BR,R	B,BR,R
R	FL	B,BR,R	R	R	R,FR	R,FR,F,FL,L	R,L	L,BL,B,BR,R	B,BR,R
R	L	B,BR,R	R	R,FR,F	R,FR,F	R,FR,F,FL,L	R,FR,F,FL,L	∀	B,BR,R
FR	FL	R	-	-	-	F	FR	B,BR,R	R
FR	L	B,BR,R	R	R,FR,F	F	F	F,FL	∀	B,BR,R
F	L	B,BR,R,FR,F	R,FR,F	R,FR,F	F	F	F	∀	B,BR,R,FR,F
F	BL	B,BR,R,FR,F	R,FR,F	R,FR,F	F	F	F	F,FL,L,BL,B	B,F
F	B	∀	B,BR,R,FR,F	B,BR,R,FR,F	F	F	F	F,FL,L,BL,B	F,FL,L,BL,B
F	BR	F,FL,L,BL,B	B,F	B,BR,R,FR,F	F	F	F	F,FL,L	F,FL,L
F	R	F,FL,L,BL,B	F,FL,L,BL,B	∀	F	F	F	F,FL,L	F,FL,L
FL	R	L,BL,B	L,BL,B	∀	FR,F	F	F	F,FL,L	L
FL	FR	L	L	L,BL,B	FL	F	-	-	-
L	R	L,BL,B	L,BL,B	∀	R,FR,F,FL,L	R,FR,F,FL,L	F,FL,L	F,FL,L	L
L	FR	L,BL,B	L,BL,B	L,BL,B,BR,R	R,L	R,FR,F,FL,L	FL,L	L	L
L	F	L,BL,B	L,BL,B	L,BL,B,BR,R	L,BL,B,BR,R	∀	L	L	L
BL	F	L,BL,B	B	B,BR,R	B,BR,R	∀	L	L	L

Table 6.15: Table showing possible values of rq when pq, qp and rp are known, as shown in figure 6.31

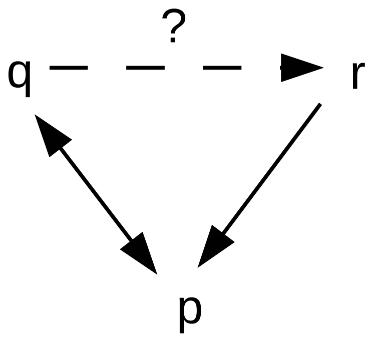

Figure 6.32

- *Mapping pq and qp as compositional relations* - Here we have both compositional relations and one placemental one, and we get possible values of rq. Taking the converse values of these gives us the possible values of qr. Cost of constraint propagation: One conversion. Assumptions beside answer: rq.

- *Mapping pr and rp as compositional relations* - We have one compositional relation, and need its converse. qp provides one placemental constraint, and togethere these give us the possible values of qr. Cost of constraint propagation: One conversion. Assumptions beside answer: pr.

- *Mapping rq and qr as compositional relations* - Here we use the converse of known rp to get placemental constraints together with known pq, and compositional completion to find possible values

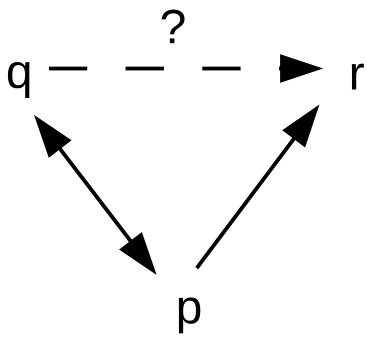

Figure 6.33

of qr. Cost of constraint propagation: One conversion and one compositional completion. Assumptions beside answer: pr and rq.

This gives us the results shown in table 6.16.

		rp							
pq	qp	B	BR	R	FR	F	FL	L	BL
B	F	F	F	F,FL,L	F,FL,L	∀	R,FR,F	R,FR,F	F
BR	F	F	F	F,FL,L	F,FL,L	∀	R,FR,F	R,FR,F	F
R	F	F	F	F,FL,L	F,FL,L	∀	R,FR,F,FL,L	R,FR,F,FL,L	F
R	FL	F	F	F,FL,L	FL,L	L,LB,B,BR,R	R,FR,L	R,FR,F,FL,L	F
R	L	F,FL,L	F,FL,L	F,FL,L	L	L,BL,B,BR,R	L,BL,B,BR,R	∀	F,FL,L
FR	FL	F	-	-	-	L,BL,B	L	F,FL,L	F
FR	L	F,FL,L	F,FL,L	F,FL,L	L	L,BL,B,BR,R	L,BL,B,BR,R	∀	F,FL,L
F	L	F,FL,L,BL,B	F,FL,L	F,FL,L	L	L,BL,B,BR,R	L,BL,B,BR,R	∀	F,FL,L,BL,B
F	BL	F,FL,L,BL,B	F,FL,L	F,FL,L	L	L,BL,B,BR,R	B,BR,R	B,BR,R,FR,F	B,F
F	B	∀	F,FL,L,BL,B	F,FL,L,BL,B	L,BL,B	L,BL,B,BR,R	B,BR,R	B,BR,R,FR,F	B,BR,R,FR,F
F	BR	B,BR,R,FR,F	B,F	F,FL,L,BL,B	L,BL,B	L,BL,B,BR,R	R	R,FR,F	R,FR,F
F	R	B,BR,R,FR,F	B,BR,R,FR,F	∀	L,BL,B,BR,R	L,BL,B,BR,R	R	R,FR,F	R,FR,F
FL	R	R,FR,F	R,FR,F	∀	L,BL,B,BR,R	L,BL,B,BR,R	R	R,FR,F	R,FR,F
FL	FR	F	F	R,FR,F	R	B,BR,R	-	-	-
L	R	R,FR,F	R,FR,F	∀	L,BL,B,BR,R	L,BL,B,BR,R	R	R,FR,F	R,FR,F
L	FR	F	F	R,FR,F,FL,L	R,FL,L	L,BL,B,BR,R	R,FR	R,FR,F	F
L	F	F	F	R,FR,F,FL,L	R,FR,F,FL,L	∀	R,FR,F	R,FR,F	F
BL	F	F	F	F,FL,L	F,FL,L	∀	R,FR,F	R,FR,F	F

Table 6.16: Table showing possible values of qr when pq, qp and rp are known, as shown in figure 6.32

Next up is the situation shown in figure 6.33, where we know pq, qp and pr, and wish to find qr.

- *Mapping pq and qp as compositional relations* - We have both compositional relations. Taking the converse of known pr gives us placemental constraints that lets us find possible values of rq, and taking the converse of these again lest us find possible values of qr. Cost of constraint propagation: Two conversions. Assumptions beside answer: rp and rq.

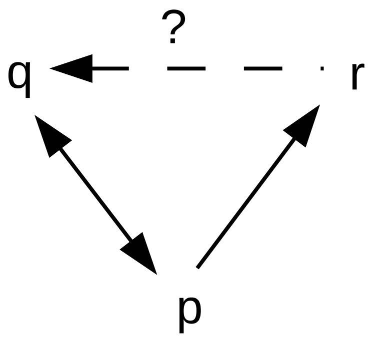

Figure 6.34

- *Mapping pr and rp as compositional relations* - We have one compositional relation, and use its converse for the compositional constraint. We get a placemental constraint from qp, giving us possible values of qr. Cost of constraint propagation: One conversion. Assumptions beside answer: rp.
- *Mapping rq and qr as compositional relations* - We have both placemental constraints, and use compositional completion to find possible values of qr. Cost of constraint propagation: Compositional completion. Assumptions beside answer: rq.

This gives us the possible inferences shown in table 6.17.

		pr							
pq	*qp*	B	BR	R	FR	F	FL	L	BL
B	F	∀	B,BR,R,FR,F	B,BR,R,FR,F	F	F	F	F,FL,L,BL,B	F,FL,L,BL,B
BR	F	F,FL,L,BL,B	B,F	B,BR,R,FR,F	F	F	F	F,FL,L	F,FL,L
R	F	F,FL,L,BL,B	F,FL,L,BL,B	∀	F	F	F	F,FL,L	F,FL,L
R	FL	L,BL,B	L,BL,B	∀	FR,F,FL	F	F	F,FL,L	L
R	L	L,BL,B	L,BL,B	∀	R,FR,F,FL,L	R,FR,F,FL,L	F,FL,L	F,FL,L	L
FR	FL	L,BL,B	L,BL,B	L,BL,B	FL	F	-	-	-
FR	L	L,BL,B	L,BL,B	L,BL,B,BR,R	R,FL,L	R,FR,F,FL,L	FL,L	L	L
F	L	L,BL,B	L,BL,B	L,BL,B,BR,R	L,BL,B,BR,R	∀	L	L	L
F	BL	L,BL,B	B	B,BR,R	B,BR,R	∀	L	L	L
F	B	L,BL,B,BR,R	B,BR,R	B,BR,R	B,BR,R	∀	L,BL,B	L,BL,B	L,BL,B
F	BR	B,BR,R	R	R	R	∀	L,BL,B	L,BL,B	B
F	R	B,BR,R	R	R	R	∀	L,BL,B,BR,R	L,BL,B,BR,R	B,BR,R
FL	R	B,BR,R	R	R	R,FR	R,FR,F,FL,L	R,FR,L	L,BL,B,BR,R	B,BR,R
FL	FR	B,BR,R	-	-	-	F	FR	B,BR,R	B,BR,R
L	R	B,BR,R	R	R,FR,F	R,FR,F	R,FR,F,FL,L	R,FR,F,FL,L	∀	B,BR,R
L	FR	B,BR,R	R	R,FR,F	F	F	FR,F,FL	∀	B,BR,R
L	F	B,BR,R,FR,F	R,FR,F	R,FR,F	F	F	F	∀	B,BR,R,FR,F
BL	F	B,BR,R,FR,F	R,FR,F	R,FR,F	F	F	F	F,FL,L,BL,B	B,F

Table 6.17: Table showing possible values of qr when pq, qp and pr are known, as shown in figure 6.33

The final situation is the one shown in figure 6.34, where we know pq, qp and pr, and wish to find rq.

- *Mapping pq and qp as compositional relations* - Both compositional relations are known. We take the converse of pr to get placemental constraints, and use these to find possible values of rq. Cost of constraint propagation: One conversion. Assumptions beside answer: rp.

- *Mapping pr and rp as compositional relations* - One compositional relation is known, we take its converse to get these constraints. Placemental constraint qp gives us possible values of qr, which again we take the converse of to get possible values of rq. Cost of constraint propagation: Two conversions. Assumptions beside answer: rp and qr.

- *Mapping rq and qr as compositional relations* - We have both placemental relations, and use compositional completion to find possible values of rq. Cost of constraint propagation: Compositional completion. Assumptions beside answer: qr.

The resulting possibilities are shown in table 6.18.

					pr				
pq	qp	B	BR	R	FR	F	FL	L	BL
B	F	∀	F,FL,L,BL,B	F,FL,L,BL,B	L,BL,B	L,BL,B,BR,R	B,BR,R	B,BR,R,FR,F	B,BR,R,FR,F
BR	F	B,BR,R,FR,F	B,F	F,FL,L,BL,B	L,BL,B	L,BL,B,BR,R	R	R,FR,F	R,FR,F
R	F	B,BR,R,FR,F	B,BR,R,FR,F	∀	L,BL,B,BR,R	L,BL,B,BR,R	R	R,FR,F	R,FR,F
R	FL	R,FR,F	R,FR,F	∀	L,BL,B,BR,R	L,BL,B,BR,R	R	R,FR,F	R,FR,F
R	L	R,FR,F	R,FR,F	∀	L,BL,B,BR,R	L,BL,B,BR,R	R	R,FR,F	R,FR,F
FR	FL	F	F	R,FR,F	R	B,BR,R	-	-	-
FR	L	F	F	R,FR,F,FL,L	R,FL,L	L,BL,B,BR,R	R,FR	R,FR,F	F
F	L	F	F	R,FR,F,FL,L	R,FR,F,FL,L	∀	R,FR,F	R,FR,F	F
F	BL	F	F	F,FL,L	F,FL,L	∀	R,FR,F	R,FR,F	F
F	B	F	F	F,FL,L	F,FL,L	∀	R,FR,F	R,FR,F	F
F	BR	F	F	F,FL,L	F,FL,L	∀	R,FR,F	R,FR,F	F
F	R	F	F	F,FL,L	F,FL,L	∀	R,FR,F,FL,L	R,FR,F,FL,L	F
FL	R	F	F	F,FL,L	FL,L	L,BL,B,BR,R	R,FR,L	R,FR,F,FL,L	F
FL	FR	F	-	-	-	L,BL,B	L	F,FL,L	F
L	R	F,FL,L	F,FL,L	F,FL,L	L	L,BL,B,BR,R	L,BL,R,BR,R	∀	F,FL,L
L	FR	F,FL,L	F,FL,L	F,FL,L	L	L,BL,B,BR,R	L,BL,B,BR,R	∀	F,FL,L
L	F	F,FL,L,BL,B	F,FL,L	F,FL,L	L	L,BL,B,BR,R	L,BL,B,BR,R	∀	F,FL,L,BL,B
BL	F	F,FL,L,BL,B	F,FL,L	F,FL,L	L	L,BL,B,BR,R	B,BR,R	B,BR,R,FR,F	B,F

Table 6.18: Table showing possible values of rq when pq, qp and pr are known, as shown in figure 6.34

What, then, of situations where the relation in question is a converse of one of the known ones? When dealing with three known relations, this premise opens up for more cases than we saw in the section dealing with two known relations. Figure 6.35 shows all these possibilities. In the situations shown in figure 6.35a and 6.35b we only need to consider the combinations of pq and qp that we find in the composition table, as we did in our previous cases of three known relations. In the situation shown in figure 6.35c we only need to check the combinations of known relations where pq and qr allow rp, according to table 6.11, while the three last situations require us to check the combinations of known relations where pq and qr allow pr according to table 6.12.

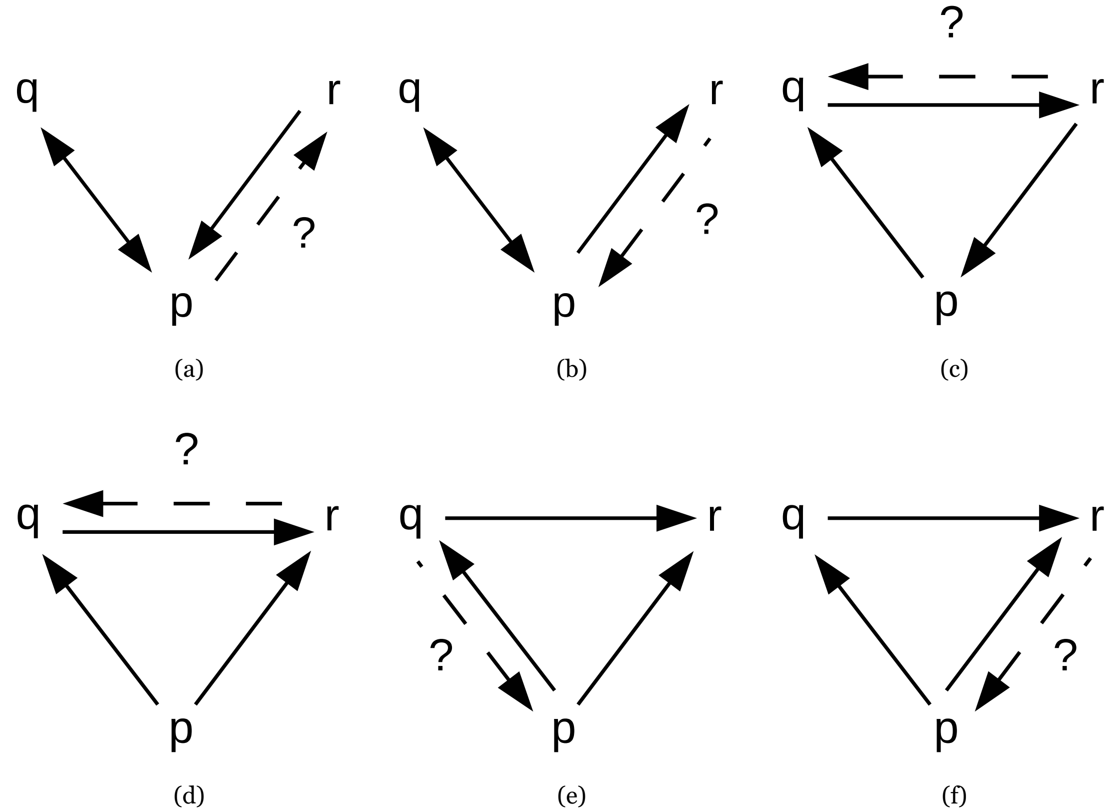

Figure 6.35: All possible situations where three relations are known and the one in question is the converse of one of these

If we go through the results of all these cases, we find that the results are the same as we find in the conversion table, with the expected exceptions similar to those we found for the cases we saw in figure 6.30.

6.5.4 Four known relations

When moving onto situations where four of the relations are known we get only one case where the relation in question isn't the converse of any of the known ones, the one shown in figure 6.36, where we know pq, qp, rq and qr and wish to find pr.

- *Mapping pq and qp as compositional relations* - We have both compositional relations, and the placemental relation rq. We use this to get possible values of rp, and take the conversion of these the get possible values of pr. Cost of constraint propagation: One conversion. Assumptions beside answer: rp.
- *Mapping pr and rp as compositional relations* - Here we have both placemental relations, and use compositional completion to find

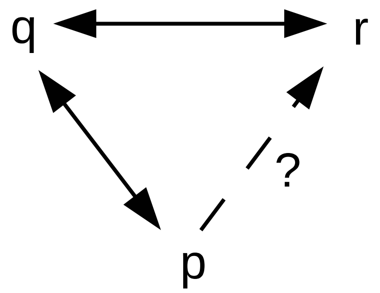

Figure 6.36

possible values of pr. Cost of constraint propagation: Compositional completion. Assumptions beside answer: rp.

- *Mapping rq and qr as compositional relations* - We have both compositional relations, and placemental relation pq. Applying these to the composition table gives us possible values of placemental relation pr. Cost of constraint propagation: None. Assumptions beside answer: None.

This leaves us the possibilities shown in table 6.19. This table is, not surprisingly, rather large, and has been split into three separate ones.

Inspecting this table, we find that it has the same symmetric properties that we saw in tables 6.12 and 6.11, but here the symmetry is between compositional pairs rather than single relations.

6.5.5 Five known relations

The only cases that are left are five known relations, which will simply consist of listing all possible combinations of relations. If we do calculate these possibilities, by going through all eight possible values for all six relations in turn, and apply the constraints we've developed for all mappings in each case, then we'd find that there are a total of 1290 combinations that are left. This is quite a drop from our initial 8^6 possibilities. Such a calculation is costlier than the ones we've shown here, and the resulting complete table will not be reproduced here.

Table 6.19: (Next page) Table showing possible values of pr when pq, qp, qr and rq are known, as shown in figure 6.36

		qr	B	BR	R			FR	
pq	qp	rq	F	F	F	FL	L	FL	L
B	F		L,BL,B,BR,R	B,BR,R	B,BR,R	B,BR,R	B,BR,R	R	B,BR,R
BR	F		B,BR,R	R	R	R	R	-	R
R	F		B,BR,R	R	R	R	R	-	R
R	FL		B,BR,R	R	R	R	R	-	R,FR
R	L		B,BR,R	R	R	R	R,FR,F	-	R,FR,F
FR	FL		R	-	-	-	-	-	-
FR	L		B,BR,R	R	R	R,FR	R,FR,F	-	F
F	L		B,BR,R,FR,F	R,FR,F	R,FR,F	R,FR,F	R,FR,F	-	F
F	BL		B,BR,R,FR,F	R,FR,F	R,FR,F	R,FR,F	R,FR,F	-	F
F	B		∀	B,BR,R,FR,F	B,BR,R,FR,F	R,FR,F	R,FR,F	F	F
F	BR		F,FL,L,BL,B	B,F	B,BR,R,FR,F	R,FR,F	R,FR,F	F	F
F	R		F,FL,L,BL,B	F,FL,L,BL,B	∀	R,FR,F,FL,L	R,FR,F,FL,L	F	F
FL	R		L,BL,B	L,BL,B	L,BL,B,BR,R	R,L	R,FR,F,FL,L	FR	F,FL
FL	FR		L	L	L,BL,B	L	L	-	FL
L	R		L,BL,B	L,BL,B	L,BL,B,BR,R	L,BL,B,BR,R	∀	R	R,FR,F,FL,L
L	FR		L,BL,B	L,BL,B	L,BL,B,BR,R	L,BL,B,BR,R	L,BL,B,BR,R	R	R,FL,L
L	F		L,BL,B	L,BL,B	L,BL,B,BR,R	L,BL,B,BR,R	L,BL,B,BR,R	B,BR,R	L,BL,B,BR,R
BL	F		L,BL,B	B	B,BR,R	B,BR,R	B,BR,R	R	B,BR,R

		qr			F			FL	
pq	qp	rq	L	BL	B	BR	R	R	FR
B	F		B,BR,R,FR,F	B,BR,R,FR,F	∀	F,FL,L,BL,B	F,FL,L,BL,B	L,BL,B	L
BR	F		R,FR,F	R,FR,F	B,BR,R,FR,F	B,F	F,FL,L,BL,B	L,BL,B	L
R	F		R,FR,F	R,FR,F	B,BR,R,FR,F	B,BR,R,FR,F	∀	L,BL,B,BR,R	L,BL,B
R	FL		R,FR,F	R,FR,F	R,FR,F	R,FR,F	R,FR,F,FL,L	R,FR,L	L
R	L		R,FR,F	R,FR,F	R,FR,F	R,FR,F	R,FR,F,FL,L	R,FR,F,FL,L	L
FR	FL		-	-	F	F	F	FR	-
FR	L		F	F	F	F	F	FR,F	FL
F	L		F	F	F	F	F	F	F
F	BL		F	F	F	F	F	F	F
F	B		F	F	F	F	F	F	F
F	BR		F	F	F	F	F	F	-
F	R		F	F	F	F	F	F	-
FL	R		F	F	F	F	F	F	-
FL	FR		F	F	F	-	-	-	-
L	R		R,FR,F,FL,L	F,FL,L	F,FL,L	F,FL,L	F,FL,L	F,FL,L	-
L	FR		R,FR,F,FL,L	F,FL,L	F,FL,L	F,FL,L	F,FL,L	FL,L	-
L	F		∀	F,FL,L,BL,B	F,FL,L,BL,B	F,FL,L	F,FL,L	L	-
BL	F		B,BR,R,FR,F	B,F	F,FL,L,BL,B	F,FL,L	F,FL,L	L	-

		qr		L		BL
pq	qp	rq	R	FR	F	F
B	F		L,BL,B	L,BL,B	L,BL,B	L,BL,B
BR	F		L,BL,B	L,BL,B	L,BL,B	B
R	F		L,BL,B,BR,R	L,BL,B,BR,R	L,BL,B,BR,R	B,BR,R
R	FL		L,BL,B,BR,R	L,BL,B,BR,R	L,BL,B,BR,R	B,BR,R
R	L		∀	L,BL,B,BR,R	L,BL,B,BR,R	B,BR,R
FR	FL		R	R	B,BR,R	R
FR	L		R,FR,F,FL,L	R,L	L,BL,B,BR,R	B,BR,R
F	L		R,FR,F,FL,L	R,FR,F,FL,L	∀	B,BR,R,FR,F
F	BL		F,FL,L	F,FL,L	F,FL,L,BL,B	B,F
F	B		F,FL,L	F,FL,L	F,FL,L,BL,B	F,FL,L,BL,B
F	BR		F,FL,L	F,FL,L	F,FL,L	F,FL,L
F	R		F,FL,L	F,FL,L	F,FL,L	F,FL,L
FL	R		F,FL,L	FL,L	L	L
FL	FR		-	-	-	-
L	R		F,FL,L	L	L	L
L	FR		L	L	L	L
L	F		L	L	L	L
BL	F		L	L	L	L

Chapter 7

Conclusion

We've seen how we were able to develop a system for qualitative reasoning using Levinson's frames of reference. The problem turned out to be similar to, but more complex than the system for Allen's algebra. This is not unexpected, as it involves a move from a one-dimensional time line to a two-dimensional field, and we should comment on what the transition involves.

In the system using the relative frame of reference, the relations do not have unique converted values. This is the main reason for the increase in complexity, as the relations involved in any configuration doubles, giving us six relations when there are three objects involved. This was not the case in Allen' algebra, nor in RCC8 or in the calculus for the absolute frame of reference that we developed. This required a composition table different from the one Allen and Freksa presents, one in which four of the six relations are represented. By applying three different mappings to this composition table for each problem we were still able to limit the possible values of unknown relations.

In chapter 6 we mentioned that the tables presented in part 6.5 do not contain information about the assumed values of relations not expressed in the tables. This means that the tables presented are not ideal for iterative application, but rather that the general approach described earlier in the chapter should be used, as this approach aims to conserve consistency of all relations involved in a solution set. As such, part 6.5 should be seen as an exhaustive display of the results that our general approach gives us rather than tools for further appliance.

Levinson states that the relative frame of reference supports transitivity if the observer is kept constant. He quotes Levelt[10] on this, in which we find the claim that "left" and "right" are necessarily transitive in these cases. We saw in table 6.11 that this isn't the case in our system. Our conclusion on this matter is a result of strictly applying a certain coordinate system to all objects in the configuration, a system of which Levelt has no counterpart.

It can be argued that some of the cases we've looked at differ from how natural language is used, something that was an initial aim. This is mostly shown if the combination of relations in a configuration of more than two objects are taken into consideration, any single relation between two objects has its firm backing in Levinson's system. The result on transitivity mentioned above is an example of this, where the single relations in our system have a direct correspondence with the underlying spatial semantics, while the results we get differs from linguistic conclusions. This will always be a challenge in a system of reasoning where the basic entities and relations are drawn from natural language: Expressions are translated into formal symbols, and some form of calculation is carried out on these symbols. The results of such calculation will be of the same kind as the initial input, and as such they can be translated back to natural language, but we may not agree that the results make sense. This happens because the formal processing typically utilizes a set of methods and tools for which there are no counterparts in natural language, and which may differ from how we would carry out reasoning ourselves.

7.1 Further research

It would be interesting to see whether the system developed for the relative frame of reference could be expanded to deal with configurations involving more than three objects. As all such configurations could be interpreted as a set of configurations of three objects it should be possible to apply the constraints we have, but it could also be that including more objects create new possibilities of constraining the possible domain of unknown values. At any rate, such an expansion should be based on the general approach described in chapter 6 rather than the tables of specific results in 6.5, as mentioned above.

Another possible area of further research is other kinds of frames of reference. There are many possibilities here, for one thing the distribution of angles could be altered from the even 90° portions we've seen here. One simplistic frame is one where we partition the plane into two halves for each object, the orientation of the dividing line being the same for all such coordinate systems. This would make it impossible to differentiate between objects if they we're placed in such a manner that their partitionings overlapped completely, even if they we're placed at different positions in the plane. If we allow one of these halves to include the line on which the object is placed, then we could introduce a relation like "north of or equal to", and we could investigate what properties such a relation would lead to.

Such a system could be expanded to one where two coordinate systems, each partitioning the plane in half and one running orthogonal to the other, are superimposed on the same object. This would create sets of relations. In an absolute case we would get a familiar coordinate system, and it could be investigated what qualitative properties such a system would produce. The relative case is shown in figure 7.1, where the horizontal line marks the division between "behind *q*" and "in front of *q*", while the horizontal one, anchored to *o*, marks the division between "left of *q*" and "right of *q*". This would lead to other kinds of compositions than the ones we found in our system, figure 7.2 shows a configuration where we have both "*p* is in front of *q*" and "*q* is in front of *p*".

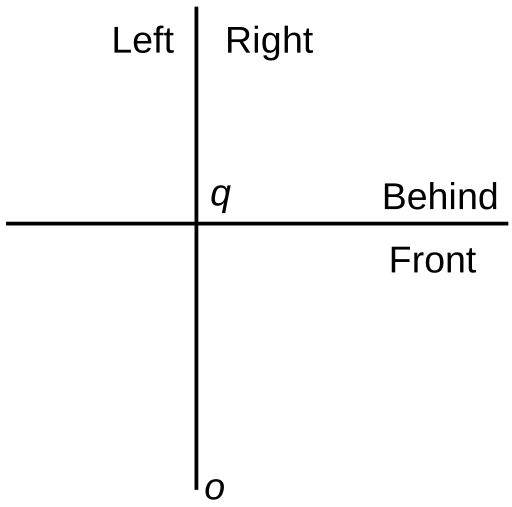

Figure 7.1

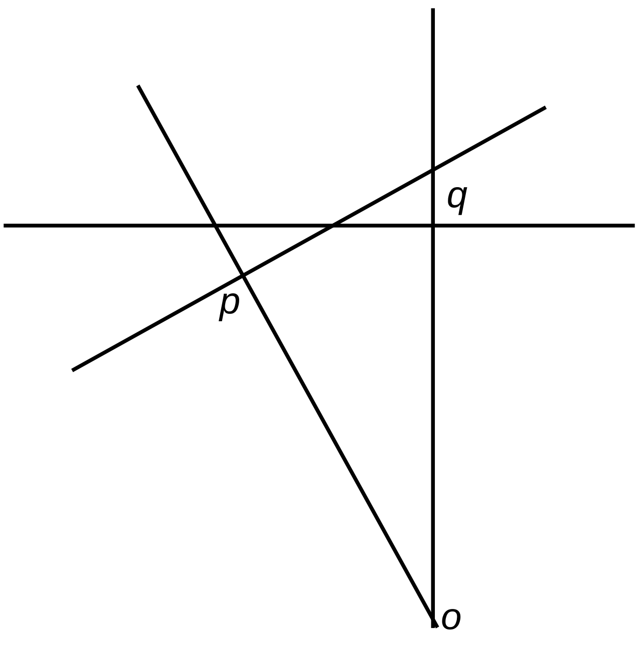

Figure 7.2

It would also be interesting to see whether a temporal system like Allen's algebra and the current systems could be combined, enabling calculation over expressions involving dynamic spatial expressions. This could simply mean a number of static expressions arranged chronologically, but an investigation into temporal semantics would give a clearer impressions of which features that would have to be represented.

We mentioned above that the general method is a better tool for further work than the result tables. This matter could also be investigated further. If the table being used is the complete one showing all 1290 possible combinations of the six relations involved, then consistency would of course be ensured, but so would the application of the general method.

As we mentioned in the introduction, it could be that the calculi developed in this thesis could serve as a computational component in the part of Bateman's ontology[3] that is concerned with Levinson's frames of reference.

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