Viability and martingale measures under partial information

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Abstract

We consider a financial market with a single risky asset whose price process $S(t)$ is modeled by a jump diffusion, and where the agent only has access to a given partial information flow $\{\mathcal{E}_t\}_{t\geq 0}$. Mathematically this means that the portfolio φ is required to be $\mathcal{E}$ - predictable. We let $\mathcal{A}_{\mathcal{E}}$ denote the set of admissible portfolios. If U is a given utility function, we say that the market is $(\mathcal{E}, U)$ -viable if there exists a portfolio $\varphi^* \in \mathcal{A}_{\mathcal{E}}$ (called an optimal portfolio) such that

$$\sup_{\varphi \in \mathcal{A}_{\mathcal{E}}} E[U(X_\varphi(T))] = E[U(X_{\varphi^*}(T))]. \tag{0.1}$$

We prove that, under some conditions, the following holds:
The market is $(\mathcal{E}, U)$-viable if and only if the measure Q^* defined by

$$dQ^* = \frac{U'(X_{\varphi^*}(T))}{E[U'(X_{\varphi^*}(T))]} dP \text{ on } \mathcal{F}_T \tag{0.2}$$

is an equivalent local martingale measure (ELMM) with respect to $\mathcal{E}$ and with respect to the $\mathcal{E}_t$-conditioned price process

$$\tilde{S}(t) := E_{Q^*}[S(t) \mid \mathcal{E}_t] \; ; \; t \in [0, T]. \tag{0.3}$$

This is an extension to partial information of a classical result in mathematical finance.

We also obtain a characterization of such partial information optimal portfolios in terms of backward stochastic differential equations (BSDEs), which is a result of independent interest.

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1 Introduction

Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a filtered probability space and let $B(t) = B(t,\omega)$; $t \geq 0$, $\omega \in \Omega$ be a Brownian motion and $\tilde{N}(dt, d\zeta) := N(dt, d\zeta) - \nu(d\zeta)dt$ an independent compensated Poisson random measure, respectively, on this space.

Consider the following financial market with two investment possibilities:

(i) A risk free asset with unit price $S_0(t) = 1$; $t \geq 0$.

(ii) A risky asset, with unit price $S(t)$ given by the equation

$$\begin{aligned} dS(t) &= b(t)dt + \sigma(t)dB(t) + \int_{\mathbb{R}} \gamma(t,\zeta)\tilde{N}(dt, d\zeta) \; ; \; t \geq 0 \\ S(0) &= S_0 > 0. \end{aligned} \tag{1.1}$$

Here $b(t)$, $\sigma(t)$ and $\gamma(t,\zeta)$ are given bounded $\mathcal{F}_t$-predictable processes on [0,T], where $T > 0$ is a fixed constant. We refer to [11] for information about the stochastic calculus for Lévy processes.

Let $\mathcal{E}_t \subseteq \mathcal{F}_t$ be a given subfiltration, representing the information available to an agent at time t. For example, we could have

(i) $\mathcal{E}_t = \mathcal{F}_{(t-\delta)^+}$ (delayed information flow) or

(ii) $\mathcal{E}_t = \mathcal{F}_t^{(S)}$ (the price observation flow), where $\mathcal{F}_t^{(S)}$ is the σ-algebra generated by the price process $S(s)$; $0 \leq s \leq t$; $t \in [0, T]$.

Let $\mathcal{A}_{\mathcal{E}}$ be the family of $\mathcal{E}_t$-predictable portfolios $\varphi(t)$, representing the number of units of the risky asset held at time t, such that

$$E\left[\int_0^T \varphi^2(t)dt\right] < \infty, \tag{1.2}$$

where E denotes expectation with respect to P. We assume that φ is self-financing, in the sense that the corresponding wealth process $X_\varphi(t)$ is given by

$$\begin{cases} dX_\varphi(t) &= \varphi(t)dS(t) = \varphi(t)\left(b(t)dt + \sigma(t)dB(t) + \int_{\mathbb{R}} \gamma(t,\zeta)\tilde{N}(dt, d\zeta)\right) \; ; \; t \geq 0 \\ X_\varphi(0) &= x \in \mathbb{R}, \; t \geq 0. \end{cases} \tag{1.3}$$

Note that

$$E\left[\int_0^T X_\varphi^2(t)dt\right] < \infty. \tag{1.4}$$

Let $U : (-\infty, \infty) \to [-\infty, \infty)$ be a given utility function, assumed to be C^1 on $(0, \infty)$, concave and strictly increasing on $[0, \infty]$.We assume that

$$E\left[U'(X_\varphi(T)))^2\right] < \infty \tag{1.5}$$

for all $\varphi \in \mathcal{A}_\mathcal{E}$.

We study the following *partial information optimal portfolio problem*:

Problem 1.1 *Find $u(x)$ and $\varphi^* \in \mathcal{A}_\mathcal{E}$ such that*

$$u(x) = \sup_{\varphi \in \mathcal{A}_\mathcal{E}} E[U(X_\varphi(T))] = E[U(X_{\varphi^*}(T))]. \tag{1.6}$$

We say that the market is $(\mathcal{E}, U)$-viable if there exists an optimal $\varphi^* \in \mathcal{A}_\mathcal{E}$ satisfying (1.6).

Recently there has been much discussion in the literature concerning various concepts of arbitrage and their relation to stochastic control, viability and equivalent local martingale measures. See e.g. [5], [6] and [7]. The purpose of this paper is to prove that in our partial information financial market setting, without any no-arbitrage conditions, the following holds:

The market is $(\mathcal{E}, U)$-viable (with a growth condition added) if and only if the measure Q^* defined by

$$dQ^* = \frac{U'(X_{\varphi^*}(T))}{E[U'(X_{\varphi^*}(T))]} dP \text{ on } \mathcal{F}_T \tag{1.7}$$

is an equivalent local martingale measure (ELMM) with respect to $\mathcal{E}$ and with respect to the $\mathcal{E}_t$-conditioned process

$$\tilde{S}(t) := E_{Q^*}[S(t) \mid \mathcal{E}_t] \ ; \ t \in [0, T]. \tag{1.8}$$

See Theorem 4.1.

Remark 1.2 In the complete observation case ($\mathcal{E}_t = \mathcal{F}_t$) this result has been known for a long time in a variety of settings. One of the first results in this direction seems to be in the paper [8]. Even in a basic one-period market model a version of this result can be proved; see e.g. [10]. For a general discussion see [9] and the references therein. A recent model uncertainty version can be found in [12].

2 A BSDE characterization of optimal portfolios

In this section we give a characterization of portfolios φ^* satisfying (1.6) in terms of a *backward stochastic differential equation* (BSDE). This is obtained by applying the maximum principle for optimal control to the problem, as follows:

The Hamiltonian

$$H : [0,T] \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathbb{R} \times \mathcal{R} \times \Omega) \to \mathbb{R}$$

(where $\mathcal{R}$ is the set of functions $r(\cdot) : \mathbb{R}\backslash\{0\} \to \mathbb{R}$) is defined by

$$H(t,x,\varphi,p,q,r(\cdot),\omega) = \varphi b(t,\omega)p + \varphi\sigma(t,\omega)q + \int_{\mathbb{R}} \varphi\gamma(t,\zeta,\omega)r(\zeta)\nu(d\zeta), \tag{2.1}$$

whenever the integral converges.

Associated to each $\varphi \in \mathcal{A}_{\mathcal{E}}$ we have a BSDE in the adjoint processes $p(t), q(t), r(t,\zeta)$ given by

$$\begin{cases} dp(t) & = -\frac{\partial H}{\partial x}(t, X_\varphi(t), \varphi(t), p(t), q(t), r(t,\cdot))dt \\ & \quad +q(t)dB(t) + \int_{\mathbb{R}} r(t,\zeta)\tilde{N}(dt,d\zeta) \; ; \; 0 \le t \le T \\ p(T) & = U'(X_\varphi(T)). \end{cases} \tag{2.2}$$

The partial information necessary maximum principle (see [2]) states that if $\varphi \in \mathcal{A}_{\mathcal{E}}$ is optimal for the problem (1.6) and (2.2) has a unique solution p, q, r then

$$E\left[\frac{\partial H}{\partial \varphi}(t, X_\varphi(t), \varphi(t), p(t)q(t)r(t,\cdot)) \mid \mathcal{E}_t\right] = 0 \text{ a.s., for a.a.t.} \tag{2.3}$$

In our case (2.2) reduces to the form

$$\begin{cases} dp(t) & = q(t)dB(t) + \int_{\mathbb{R}} r(t,\zeta)\tilde{N}(dt,d\zeta) \; ; \; 0 \le t \le T \\ p(T) & = U'(X_\varphi(T)). \end{cases} \tag{2.4}$$

Note that by the Itô representation theorem this BSDE has a unique solution $p(t), q(t), r(t,\zeta)$ satisfying

$$E\left[\int_0^T \left\{p^2(t) + q^2(t) + \int_{\mathbb{R}} r^2(t,\zeta)\nu(d\zeta)\right\} dt\right] < \infty. \tag{2.5}$$

In our case equation (2.3) becomes

$$E\left[b(t)p(t) + \sigma(t)q(t) + \int_{\mathbb{R}} \gamma(t,\zeta)r(t,\zeta)\nu(d\zeta) \mid \mathcal{E}_t\right] = 0 \text{ a.s., for a.a.t.} \tag{2.6}$$

Conversely, suppose (2.4)-(2.6) hold. Then, since H is a concave function of (x,φ) we see that φ satisfies all the conditions of the partial information *sufficient* maximum principle (see e.g. [2]). Therefore we can conclude that φ is optimal.

We have proved:

Theorem 2.1 *A portfolio* φ *is optimal for the problem* (1.6) *if and only if the solution* (p,q,r) *of the BSDE* (2.4) *satisfies* (2.6).

Recall the generalized Clark-Ocone theorem (see [1] for the Brownian motion case and [3, Theorem 3.28] for the Lévy process case) which states that if $F \in L^2(P)$ is $\mathcal{F}_T$-measurable, then F can be written

$$F = E[F] + \int_0^T E[D_t F \mid \mathcal{F}_t] dB(t) + \int_0^T \int_{\mathbb{R}} E[D_{t,\zeta} F \mid \mathcal{F}_t] \tilde{N}(dt, d\zeta) \tag{2.7}$$

where $D_t F$ and $D_{t,\zeta}$ denote the generalized Malliavin derivative at t with respect to $B(\cdot)$ and at t, ζ with respect to $N(\cdot, \cdot)$, respectively.

Applying this to $F := U'(X_\varphi(T))$ we see that the solution of (2.4) is

$$p(t) = E[U'(X_\varphi(T)) \mid \mathcal{F}_t] \tag{2.8}$$

$$q(t) = E[D_t U'(X_\varphi(T)) \mid \mathcal{F}_t] \tag{2.9}$$

$$r(t, \zeta) = E[D_{t,\zeta} U'(X_\varphi(T)) \mid \mathcal{F}_t]. \tag{2.10}$$

Therefore, by Theorem 2.1 we get the following characterization of the optimal terminal wealth $X_\varphi(T)$ of the partial information portfolio problem:

Theorem 2.2 *A portfolio φ is optimal for the problem* (1.6) *if and only if the corresponding terminal wealth $X_\varphi(T)$ satisfies the following partial information Malliavin differential equation:*

$$\begin{aligned} &E[b(t)U'(X_\varphi(T)) + \sigma(t) D_t U'(X_\varphi(T)) \\ &\quad + \int_{\mathbb{R}} \gamma(t, \zeta) D_{t,\zeta} U'(X_\varphi(T)) \nu(d\zeta) \mid \mathcal{E}_t] = 0 \ \textit{a.s.};\ t \in [0, T]. \end{aligned} \tag{2.11}$$

3 Partial information equivalent local martingale measures (PIELMMs)

Let Q be a probability measure equivalent to P. Then we can write

$$dQ(\omega) = G(T, \omega) dP(\omega) \text{ on } \mathcal{F}_T, \tag{3.1}$$

where $G(T, \omega) > 0$ a.s. and $E[G(T)] = 1$. If we restrict the measures P, Q to $\mathcal{F}_t$ for $t < T$, they are still equivalent and we have

$$\frac{d(Q \mid \mathcal{F}_t)}{d(P \mid \mathcal{F}_t)} = E[G(T) \mid \mathcal{F}_t] =: G(t) > 0. \tag{3.2}$$

By the martingale representation theorem there exist predictable processes $\theta_0(t)$ and $\theta_1(t, \zeta)$ such that

$$\begin{cases} dG(t) &= G(t^-) \left[\theta_0(t) dB(t) + \int_{\mathbb{R}} \theta_1(t, \zeta) \tilde{N}(dt, d\zeta) \right] \ ; \ 0 \le t \le T \\ G(0) &= 1. \end{cases} \tag{3.3}$$

If we assume that $\theta_1(t,\zeta) > -1$ and

$$E\left[\int_0^T \left\{\theta_0^2(t) + \int_{\mathbb{R}} \theta_1^2(t,\zeta)\nu(d\zeta)\right\} dt\right] < \infty, \tag{3.4}$$

then by the Itô formula,

$$\begin{aligned} G(t) = \exp\Bigg(&\int_0^t \theta_0(s)dB(s) - \frac{1}{2}\int_0^t \theta_0^2(s)ds \\ &+ \int_0^t \int_{\mathbb{R}} \ln(1+\theta_1(s,\zeta))\tilde{N}(ds,d\zeta) \\ &+ \int_0^t \int_{\mathbb{R}} \{\ln(1+\theta_1(s,\zeta)) - \theta_1(s,\zeta)\}\,\nu(d\zeta)ds\Bigg) \ ; \ 0 \le t \le T. \end{aligned} \tag{3.5}$$

In the following we write $G(t) = G_\theta(t)$ and $Q = Q_\theta$; $\theta = (\theta_0, \theta_1)$, when $G(t)$ is represented by θ as in (3.3). We let Θ denote the family of all predictable processes $\theta = (\theta_0, \theta_1)$ such that (3.3) has a unique martingale solution $G_\theta(t)$; $t \in [0,T]$.

Definition 3.1 *We say that $S(t)$ is an $(\mathcal{E}_t, Q_\theta)$-local martingale if there exists an increasing family of $\mathcal{F}_t$-stopping times τ_k such that $\tau_k \to \infty$ when $k \to \infty$, a.s. and*

$$E_{Q_\theta}[S(t \wedge \tau_k) \mid \mathcal{E}_s] = E_{Q_\theta}[S(s \wedge \tau_k) \mid \mathcal{E}_s] \text{ a.s. for all } s < t \text{ and all } k. \tag{3.6}$$

Note that (3.6) is equivalent to requiring that the $\mathcal{E}_t$-conditional process

$$\tilde{S}(t) := E_{Q_\theta}[S(t) \mid \mathcal{E}_t] \ ; \ t \ge 0 \tag{3.7}$$

is an $(\mathcal{E}_t, Q_\theta)$ local martingale.

We now give a characterization of the measures Q_θ such that $S(t)$ is an $(\mathcal{E}_t, Q_\theta)$ local martingale:

Theorem 3.2 *The process $S(t)$ given by (1.1) is an $(\mathcal{E}_t, Q_\theta)$ local martingale if and only if*

$$\begin{aligned} E_{Q_\theta}&\left[b(t) + \sigma(t)\theta_0(t) + \int_{\mathbb{R}} \gamma(t,\zeta)\theta_1(t,\zeta)\nu(d\zeta) \mid \mathcal{E}_t\right] = 0 \\ &\text{a.s., for a.a. } t \in [0,T]. \end{aligned} \tag{3.8}$$

Proof. By the Itô formula we get (see e.g. [11][chapter1])

$$\begin{aligned} d(G_\theta(t)S(t)) &= G_\theta(t^-)dS(t) + S(t^-)dG_\theta(t) + d[G_\theta, S](t) \\ &= G_\theta(t^-)\left[b(t)dt + \sigma(t)dB(t) + \int_{\mathbb{R}} \gamma(t,\zeta)\tilde{N}(dt,d\zeta)\right] \\ &\quad + S(t^-)\left[G_\theta(t^-)\left(\theta_0(t)dB(t) + \int_{\mathbb{R}} \theta_1(t,\zeta)\tilde{N}(dt,d\zeta)\right)\right] \\ &\quad + G_\theta(t)\sigma(t)\theta_0(t)dt + \int_{\mathbb{R}} G_\theta(t^-)\gamma(t,\zeta)\theta_1(t,\zeta)N(dt,d\zeta), \end{aligned} \tag{3.9}$$

where $N(dt,d\zeta) = \tilde{N}(dt,d\zeta) + \nu(d\zeta)dt$.

Collecting the dt-terms we get

$$G_\theta(t)S(t) = S(0) + \int_0^t G_\theta(s)\left\{b(s) + \sigma(s)\theta_0(s) + \int_{\mathbb{R}} \gamma(s,\zeta)\theta_1(s,\zeta)\nu(d\zeta)\right\} dt$$
$$+ \, dB(s)\text{-integrals } + \tilde{N}(ds,d\zeta)\text{-integrals.} \tag{3.10}$$

Since the $dB(s)$-integrals and the $\tilde{N}(ds,d\zeta)$-integrals are local $\mathcal{F}_t$-martingales, they are also local $\mathcal{E}_t$-martingales. Therefore, with τ_k as above we get

$$E_{Q_\theta}[S(t\wedge\tau_k) \mid \mathcal{E}_s] - E_{Q_\theta}[S(s\wedge\tau_k) \mid \mathcal{E}_s]$$
$$= E[G_\theta(t\wedge\tau_k)S(t\wedge\tau_k) - G_\theta(s\wedge\tau_k)S(s\wedge\tau_k) \mid \mathcal{E}_s]$$
$$E\left[\int_{s\wedge\tau_k}^{t\wedge\tau_k} G_\theta(u)\left\{b(u) + \sigma(u)\theta_0(u) + \int_{\mathbb{R}} \gamma(t,\zeta)\theta_1(u,\zeta)\nu(d\zeta)\right\} du \mid \mathcal{E}_s\right].$$

This is 0 for all $s < t \le T$ and all τ_k if and only if

$$E\left[G_\theta(u)\left\{b(u) + \sigma(u)\theta_0(u) + \int_{\mathbb{R}} \gamma(u,\zeta)\theta_1(u,\zeta)\nu(d\zeta)\right\} \mid \mathcal{E}_u\right] = 0$$
$$\text{a.s. for a.a. } u, \tag{3.11}$$

which is equivalent to (3.8) □

4 Viability and ELMMs under partial information

We now combine the main results of Sections 2 and 3 to obtain a characterization of viability in terms of partial information equivalent local martingale measures. The result is the following:

Theorem 4.1 *The following are equivalent:*

(i) *The portfolio $\varphi \in \mathcal{A}_{\mathcal{E}}$ is optimal for the partial information portfolio optimization problem* (1.6)*, and the solution (p,q,r) of the BSDE* (2.4) *satisfies the growth condition*

$$E\left[\int_0^T \left\{\frac{1}{p^2(t)}[q^2(t) + \int_{\mathbb{R}} r^2(t,\zeta)\nu(d\zeta)]\right\} dt\right] < \infty \tag{4.1}$$

(ii) *The measure $\tilde{Q}$ defined by*

$$d\tilde{Q} := \frac{U'(X_\varphi(T))}{E[U'(X_\varphi(T))]} dP \text{ on } \mathcal{F}_T$$

is an equivalent local $\mathcal{E}_t$-martingale measure for $S(t)$.

Proof.
(i) ⇒ (ii): Suppose (i) holds. Then by Theorem 2.1 we know that the solution (p, q, r) of the BSDE (2.4) satisfies (2.6). Put

$$G(t) := \frac{p(t)}{p(0)} = \frac{E[U'(X_\varphi(T)) \mid \mathcal{F}_t]}{E[U'(X_\varphi(T))]} \tag{4.2}$$

and

$$\theta_0(t) = \frac{q(t)}{p(t)}, \quad \theta_1(t,\zeta) = \frac{r(t,\zeta)}{p(t^-)}. \tag{4.3}$$

Then by (2.6) and (4.2), (4.3)

$$\begin{aligned} dG(t) &= \frac{dp(t)}{p(0)} = \frac{q(t)}{p(0)} dB(t) + \int_{\mathbb{R}} \frac{r(t,\zeta)}{p(0)} \tilde{N}(dt, d\zeta) \\ &= \frac{p(t)}{p(0)} \theta_0(t) dB(t) + \frac{p(t^-)}{p(0)} \int_{\mathbb{R}} \theta_1(t,\zeta) \tilde{N}(dt, d\zeta) \\ &= G(t^-) \left[\theta_0(t) dB(t) + \int_{\mathbb{R}} \theta_1(t,\zeta) \tilde{N}(dt, d\zeta) \right]. \end{aligned}$$

Therefore $G(t) = G_\theta(t)$ satisfies (3.3), and by (4.1) we get that $G_\theta(t)$ is a martingale. So by Theorem 3.2 it suffices to verify that

$$E\left[G_\theta(t) \left\{ b(t) + \sigma(t)\theta_0(t) + \int_{\mathbb{R}} \gamma(t,\zeta)\theta_1(t,\zeta)\nu(d\zeta) \right\} \mid \mathcal{E}_t \right] = 0 \quad \text{a.s. for a.a. } t. \tag{4.4}$$

This follows by substituting (4.2)-(4.3) into the equation (2.6) for p, q and r.

(ii) ⇒ (i): Conversely, assume that (ii) holds. Define

$$G_\varphi(t) := \frac{E[U'(X_\varphi(T)) \mid \mathcal{F}_t]}{E[U'(X_\varphi(T))]} \; ; \; t \in [0, T].$$

Then by the martingale representation theorem there exists $\mathcal{F}_t$-predictable processes $\theta_0(t)$, $\theta_1(t,\zeta)$ such that

$$\begin{cases} dG_\varphi(t) &= G_\varphi(t) \left[\theta_0(t) dB(t) + \int_{\mathbb{R}} \theta_1(t,\zeta) \tilde{N}(dt, d\zeta) \right] \; ; \; 0 \le t \le T \\ G_\varphi(0) &= 1. \end{cases} \tag{4.5}$$

By Theorem 3.2 we deduce that since $S(t)$ is an $(\mathcal{E}_t, Q_\theta)$ local martingale, we have

$$E\left[G_\varphi(t) \left(b(t) + \sigma(t)\theta_0(t) + \int_{\mathbb{R}} \gamma(t,\zeta)\theta_1(t,\zeta)\nu(d\zeta) \right) \mid \mathcal{E}_t \right] = 0 \quad \text{a.s. for a.a. } t \in [0, T]. \tag{4.6}$$

Define

$$p(t) := E[U'(X_\varphi(T))]G_\varphi(t) \tag{4.7}$$

and

$$q(t) := E[U'(X_\varphi(T))]G_\varphi(t)\theta_0(t), \quad r(t,\zeta) := E[U'(X_\varphi(T))]G_\varphi(t^-)\theta_1(t,\zeta). \tag{4.8}$$

Then by substituting (4.7)-(4.8) into (4.5), we see that (p, q, r) satisfies the BSDE (2.4). We also obtain (4.1). Moreover, substituting (4.7)-(4.8) into (4.6),we see that (2.6) holds. Hence φ is optimal by Theorem 2.1. □

Remark 4.2 Theorem 4.1 does not hold if we drop the condition (4.1). *A counterexample can be found in [4]*

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