Oil, Sovereign Wealth Funds and Monetary Policy

Martin Blomhoff Holm

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Department of Economics
University of Oslo

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Preface

Over the past year, I have been fortunate to work for Norges Bank. This has given me the opportunity to write a thesis on a subject I have found challenging and interesting: monetary policy and foreign price fluctuations.

I have had two supervisors for the thesis: Nina Larsson Midthjell (University of Oslo/Norges Bank) and Leif Brubakk (Norges Bank). I thank Leif Brubakk for invaluable help in formulating the research question, and building, solving and interpreting the results of the model. I thank Nina Larsson Midthjell for providing detailed, constructive and enthusiastic feedback, in particular during the writing process.

From the staff of Norges Bank, I thank Kenneth Sæterhagen Paulsen and Ørjan Robstad for help with various problems in Matlab and Dynare, and the participants on the ReFIT project for giving me valuable feedback. Furthermore, I thank Marianne Sturød of the Policy and Analysis unit of the Monetary Policy department for allowing me to work with the master thesis as part of my work at Norges Bank.

Last, but not least, I thank my wife Maia for providing feedback and for being who you are.

Although I was working for Norges Bank during the process of writing this thesis, the views in this thesis are mine, and do not necessarily reflect those of Norges Bank. Any remaining errors are my own.
Summary

This thesis studies the effects of terms of trade changes on the Norwegian economy by applying a dynamic stochastic general equilibrium framework with a sovereign wealth fund, a housing sector and finitely-lived households. The main innovation of this thesis is to explicitly model the Norwegian fiscal rule as a transmission channel for oil price shocks. Following higher oil prices, the sovereign wealth fund increases, subsequently increasing the real return and future transfers from the fund. The fiscal rule therefore effectively acts to transform temporary oil price changes into near permanent shocks to the Norwegian economy. The model is able to replicate key features of the Norwegian economy during the last decade. In particular, following a strengthening in the terms of trade, there is an increase in consumption, real wages, housing prices and household debt levels, and the real exchange rate appreciates. There is further a divergence in the relative prices of intermediate goods, as import prices decrease while domestic prices increase.

Another important result is that simulations of the model reveal that monetary policy is unable to stabilise the economy following oil price shocks within a short time-horizon. This is due to its inability to affect the main transmission mechanism, the sovereign wealth fund. Rather, to limit the effects of oil price shocks, I suggest to transform the Norwegian fiscal rule. I propose a new rule for the sovereign wealth fund, where nominal transfers from the fund grow by the rate of productivity and prices in the economy, ensuring that transfers are independent of the oil price and thereby eliminating the transmission channel of oil price shocks.

For the model solving and calculations, I used Dynare 4.2.0 and Matlab R2012a. Dynare is an open source program developed to handle a wide range of economic models and runs in Matlab. For the construction of the correlation matrix, table 1, I used EViews 8.
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1 Introduction

In many small open economies, primary resources constitute an essential part of export revenues. In these economies, the high volatility of commodity prices impacts a wide range of macroeconomic variables, creating difficulties for the conduct of optimal monetary and fiscal policy.

In particular, real exchange rates in small commodity-exporting economies are often highly volatile and negatively correlated with commodity prices, see e.g. Chen and Rogoff (2003) and Cashin, Césedes and Sahay (2004). The literature generally explains the strong negative correlation with the Balassa-Samuelson effect, i.e. an increase in prices implies higher productivity in the commodity-exporting sector and tends to drive up wages in this sector, but also in the less productive non-traded goods sector due to wage bargaining. This results in higher prices in both sectors and subsequently a real exchange rate appreciation. In countries with inflation targeting monetary policy like Norway, the common recommendation is to let the nominal exchange rate float while keeping price inflation low and stable. Exchange rate fluctuations, however, affect the stability of prices and output, henceforth affecting monetary policy decisions. For example, following a commodity price increase, economic activity increases and the exchange rate appreciates, lowering import prices and subsequently the consumer prices. In order to stabilise the domestic economy, the central bank can increase the interest rate. The result is a lowering of domestic economic activity, whereas the exchange rate appreciates further. The commodity-currency effect therefore poses a problem for optimal monetary policy as it makes it difficult to simultaneously achieve both stable nominal prices and stability in the real economy.

Fiscal policy is also affected by commodity price volatility. Resource-rich economies often apply fiscal rules to shield the economy against price uncertainty, to smooth consumption over time and to avoid resource-dependence. For example, many oil exporting economies, such as Venezuela, Russia and Saudi
Arabia, apply different fiscal rules and have acquired sovereign wealth funds with variable success. In Norway, a fiscal rule ensures that the income from oil exports is saved in foreign assets in the Government Pension Fund - Global (GPF), and only the expected real net return is available for government expenditure.\(^1\) This limits the short-run effects of oil price movements by removing the impact of temporary fluctuations on available government expenditure, but allows for more permanent effects. When the oil price increases temporarily, the size of the GPF increases, making future expected real returns larger. As a result, the future expected transfers from the sovereign wealth fund increase. In the case of the Norwegian economy, this effect has been present over the last decade, as the oil price has been continuously increasing. Hence, although succeeding in limiting the temporary effects of oil price changes, the Norwegian fiscal rule acts to transform temporary oil price changes into nearly permanent government income shocks.

An increase in the sovereign wealth fund also has important macroeconomic consequences. The government can cover its expenditures either by using the real return from the fund or by raising taxes. A larger sovereign wealth fund therefore decreases the amount of taxes needed to sustain a given level of government expenditure, and implicitly raises the lifetime disposable income of households by lowering the future tax burden. As a result, following a commodity price increase and a subsequent larger sovereign wealth fund, household’s lifetime income is higher and they will subsequently increase their demand for consumption goods and lower their supply of labour, in turn affecting consumer prices and output, and the conduct of monetary policy. In addition, the increase in lifetime income of households raises housing prices and the household’s ability to obtain credit. If the central banks are concerned with financial stability, these movements in asset prices and credit levels will increase trade-offs in monetary policy further.

\(^1\)The Norwegian budgetary rule states that 4 percent (an approximation to the expected real net return) of the Government Pension Fund should be allocated to the yearly government budget.
For small open economies, the terms of trade, i.e. the price of exports relative to the price of imports, is an important source of shocks to the domestic economy. The terms of trade is strengthened if export prices increase more than import prices, or if import prices fall more than export prices. Since the late 1990s, there has been an increase in the inflow of cheap imports from low-cost emerging economies like China, to the Western world. This fall in imported inflation has lowered consumer prices and strengthened the terms of trade. In the case of the Norwegian economy, an increase in the oil price since 2000 has further led to a significant strengthening of the terms of trade. This strengthening, however, is fragile. In the case of a diminish in the inflow of cheap imports or a fall in oil prices, the effects on the Norwegian economy could be severe. Such a reversal in the terms of trade may require a costly restructuring of the Norwegian economy, lowering consumption and output, and lead to a fall in the price and wage level. Due to these consequences, policy makers need to be concerned with the possible effects of such a reversal in the terms of trade, and seek to conduct robust fiscal and monetary policy.

This thesis studies the effects of terms of trade changes on a small open resource-rich economy with a fiscal rule similar to the Norwegian budgetary rule. I apply a DSGE framework with a sovereign wealth fund, a housing sector and finitely-lived households to allow temporary wealth and income changes to have effects on the optimal allocation of households. The main innovation of this thesis is to model the sovereign wealth fund as a transmission channel for oil price shocks. Following higher oil prices, the sovereign wealth fund increases, subsequently increasing the future transfers to households from the sovereign wealth fund. The first key result is that the model is able to replicate key features of the Norwegian economy during the last decade. In particular, following a strengthening in the terms of trade, consumption, real wages, housing prices and the level of household debt increase, the real exchange rate appreciates, and there is a divergence in the prices of intermediate goods - import prices fall
while domestic prices increase.

The results also reveal that monetary policy is unable to fully stabilise the impact of oil price changes on the Norwegian economy. Household foreign debt, for example, continues to grow for almost 10 years following an oil price shock, regardless of the interest rate response. In the benchmark calibration with a flexible inflation targeting regime, the interest rate responses of the central bank are limited due to the small effects on consumer prices and output. When applying other monetary policy regimes on the model, the central bank responds more strongly. These results show, however, that since the central bank is unable to address the main transmission channel of oil price shock, the sovereign wealth fund, it is unable to stabilise the economy within a short time-horizon. Rather, to dampen the effects of oil price shocks on the Norwegian economy, I suggest to transform the fiscal rule. I propose a new rule for the sovereign wealth fund, ensuring that nominal transfers from the fund grow by the rate of price inflation and productivity and thereby neutralising the impact of oil price shocks on the economy.

The present thesis consists of eight sections. Section 2 provides a literature review of the DSGE literature on small open resource-rich economies. Section 3 gives a series of stylised facts of the Norwegian economy during the last decade. The model is presented in section 4 with a benchmark calibration in section 5. In section 6, I present the results from numerical simulations of the model. In particular, two shocks which have been important for Norwegian terms of trade during the last decade, a positive oil price shock and a negative foreign marginal cost shock, are discussed. Section 7 is a discussion of the results and policy implications. Section 8 concludes.
2 Review of the DSGE Literature on Small Open Economies

Although commodity price fluctuations are an important source of volatility in small open economies, it has been virtually ignored by the DSGE literature on small open economies. The work of Obstfeld and Rogoff (1996), however, has inspired a growing interest in studying monetary and fiscal policy in open economies with more general price frictions. They build a bridge between the intertemporal approach of, for example, Obstfeld (1982) and the classic contributions of Fleming (1962), Mundell (1963), and Dornbusch (1976), and develop a tractable two-country model with monopolistic power and sticky prices to address global macroeconomic dynamics. Their model, however, does not capture the effects of foreign price impulses to the domestic economy. Instead, most of the open economy literature following Obstfeld and Rogoff (1996) has applied various versions of two-country models with strategic interactions in order to address the effects of international welfare spillovers of monetary and fiscal policy, in particular the fixed vs. floating debate on exchange rates, see e.g. Obstfeld and Rogoff (2002).

The dynamic stochastic general equilibrium (DSGE) framework has been applied to open economy macroeconomics since the seminal work of Galí and Monacelli (2005). They build a tractable two-country small open economy model of global interactions with Calvo sticky prices and analyse the conduct of Taylor rules for monetary policy, showing in particular that monetary policy rules differ in the relative amount of exchange rate volatility they entail.

A small branch of the literature has explicitly included commodity prices in this DSGE framework. The work of Backus and Crucini (2000) are one of the precursors to this literature. They build a dynamic general equilibrium model and show how oil price fluctuations accounts for a large share of terms of trade volatility since the Bretton Woods system ended in 1971, and subsequently that a large share of business cycle volatility can be explained by movements in the
oil price. Bodenstein, Erceg and Guerrieri (2011) build a DSGE model in which the price of oil is endogenous. They demonstrate that higher oil prices generate a reallocation of wealth between oil exporting and oil importing economies, an effect which may account for a consumption decline and an exchange rate depreciation of oil importing economies. Bodenstein and Guerrieri (2011) also show that monetary policy play a limited role in shaping the response of economic activity following oil price fluctuations in the world economy. As a consequence of their work, a growing literature focus on optimal monetary policy responses to oil price fluctuations for oil-importing economies and on the effects on the economy of oil as an input factor in production, see e.g. Bodenstein, Guerrieri and Kilian (2012) and Van Robays (2012).

The model literature on the effects of commodity price changes for small open resource-rich economies is very limited. Charnavoki (2009) and Charnavoki (2010) address the question of optimal monetary policy in a DSGE model, focusing on the classical question of fixed vs. flexible exchange rates with special emphasis on international risk sharing. He argues that flexible nominal exchange rates are not necessarily optimal in a model with complete international risk sharing. Hevia and Nicolini (2013) build a DSGE model with the purpose of analysing the effects of commodity price changes in small open resource-rich economies. They focus on the effects of price movements on the relative productivity of the commodity sector, affecting the economy through wage formation. They are able to reproduce the strong co-movements between nominal exchange rates and the price of exports. Optimal monetary policy responses to commodity price fluctuations for oil exporting economies have, to my knowledge, so far not been addressed in the literature.

Although the model literature has provided a limited focus on small open resource-rich economies, the empirical literature has addressed the importance of oil price movements on business cycle fluctuations in oil exporting economies. First of all, many empirical investigations show that oil price hikes entail a lowering of global economic activity, see e.g. Jiménez-Rodríguez and Sanchez
(2005) and Bjørnland (2009). Jiménez-Rodríguez and Sanchez (2005) analyse the effect of oil price increases on GDP in OECD countries. Two of the countries in the sample were oil exporters (Norway and United Kingdom). Interestingly, the two economies experience opposite effects from an oil price rise. While Norwegian GDP grows, UK output falls. The authors argue that the relative size of the exchange rate appreciation might be an explanatory factor as the sterling exchange rate appreciated considerably more than the Norwegian krone. This suggests that the exchange rate response might be a crucial component in understanding the propagation of oil price shocks on the economy.

Some papers indicate that the response of the Norwegian exchange rate to oil price fluctuations is limited (e.g. Bjørnland (2009)), several papers, on the other hand, have presented evidence for a contradictory interpretation, see e.g. Haldane (1997), Solheim (2008) and Akram (2004). In particular, Akram (2004) analyses the Norwegian exchange rate in a fundamental equilibrium exchange rate (FEER) framework. He focuses on the effect the sizeable position of Norwegian net foreign assets could have on the exchange rate when high net foreign wealth is associated with a strong real exchange rate. The importance of net foreign wealth suggests that the Norwegian sovereign wealth fund could be an important transmission channel from oil price movements to the exchange rate in Norway.

Furthermore, Bjørnland (2009) uses a VAR approach to analyse the effects of oil price shocks on the Norwegian economy. She finds that asset prices, inflation and the interest rate increase following an oil price shock. Moreover, Bjørnland and Jacobsen (2010) show how housing price movements are an important part of the transmission mechanism in the Norwegian economy. These papers indicate that the asset and housing markets could be important components in understanding how oil affects the economy.

The empirical literature on Norway emphasises the importance of exchange rates, net foreign wealth and asset prices for the transmission of oil price shocks to the domestic economy. This thesis addresses these issues by constructing a
tractable model of the Norwegian economy with an oil sector, a sovereign wealth fund and a housing sector modelled explicitly.

3 Stylised Facts of the Norwegian Economy During the last Decade

This section presents a series of stylised facts for the effects of terms of trade changes on the Norwegian economy. Oil is the major export commodity in Norway, making up about 1/5 of Norwegian GDP and 1/2 of exports.\(^2\) Government revenue from the oil sector in Norway is saved in the Government Pension Fund - Global (GPF) and invested in foreign assets. Since 2004, the GPF has grown almost five-fold and is at present worth more than 5 000 billion NOK (about 800 billion USD and 5/3 of GDP).

Because oil is such a large share of the Norwegian GDP, oil price fluctuations evidently have important consequences for the Norwegian economy and the Norwegian terms of trade. The oil price has increased significantly during the last decade, from around 30 dollars in 2004 to more than 100 dollars in 2013, see figure 1. Additionally, since the late 1990s, there has been an inflow of low-cost products from emerging economies into the Norwegian economy. For example, the liberalisation of trade in clothing following the establishment of WTO in 1994 has been succeeded by increases in clothing imports from cheaper countries (see Moe (2002)). The increased inflow of goods from China, in particular, which joined the WTO in 2001, has had a marked impact on import prices. This has lowered prices on imports significantly compared to domestically produced goods, see figure 2. The combination of high oil prices and low import prices has led to a strengthening in the Norwegian terms of trade during the last decade, see figure 3. It is particularly noteworthy that movements in the terms of trade are dominated by oil price movements. Hence, in the Norwegian empirical

literture, the oil price is often used as a proxy for the terms of trade.

Although the Norwegian budgetary rule seeks to limit the impact of terms of trade changes, the strong terms of trade has affected the Norwegian economy in a wide variety of dimensions. A stronger terms of trade makes imports cheap relative to exports. As a result, households can purchase more imports for the same amount of exports. Furthermore, increased export earnings and lower import prices generally lead to higher real wages. During the last decade, real wages, consumption and imports in Norway have increased strongly while non-oil exports have stagnated, accompanying the strengthening of the terms of trade.

There has also been a stark increase in real housing prices and households credit, which can be related to the strengthening of the terms of trade. A stronger terms of trade increases household lifetime income and wealth through the effects on consumer prices and real wages. As a result, households have easier access to credit, allowing them to buy more housing and assets. Although
Figure 2: The price level of domestically produced and imported goods and services. Index. 1990 Q1 = 100. 1980 Q1 - 2013 Q2
Source: Statistics Norway

Figure 3: The Norwegian terms of trade. Calculated as log of export prices divided by import prices and normalised to 2000 Q1 = 0. 2000 Q1 - 2013 Q3
Source: Statistics Norway
other factors, such as the banking system and global access to credit, may be other possible explanatory factors, it seems plausible that oil prices affect both housing prices and credit levels. Figure 4 suggests the existence of a positive correlation between the oil price, housing prices and the credit to GDP ratio.

Table 1 presents a summary of the correlations between the oil price and a series of macroeconomic variables using Norwegian quarterly data between 2000 Q1 and 2013 Q3. Contemporaneously, the oil price is positively correlated with output, consumption, credit and housing prices, while it is negatively correlated with the nominal exchange rate. The correlation table also seems to indicate that the oil price is leading GDP and credit, a result of the stimulating impulse a higher oil price has on the Norwegian economy. The oil price is, however, lagging consumption and housing prices. The lag structure between housing and the oil price might be explained with the 2008 financial crisis where housing prices fell before the actual outbreak of financial distress. Using a sample excluding the
Figure 5: The oil price and the exchange rate. Four-quarter change. Percent. 2004 Q1 - 2013 Q3.
Sources: Norges Bank and Thomson Reuters

Table 1: Correlations between macroeconomic variables and the oil price. The correlations are calculated using HP-filtered quarterly data with \( \lambda = 1600 \) from 2000Q1 to 2013Q3.
Sources: Norges Bank and Statistics Norway

<table>
<thead>
<tr>
<th></th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil Price</td>
<td>-0.09</td>
<td>0.18</td>
<td>0.66</td>
<td>1.00</td>
<td>0.66</td>
<td>0.18</td>
<td>-0.09</td>
</tr>
<tr>
<td>GDP</td>
<td>0.18</td>
<td>0.25</td>
<td>0.29</td>
<td>0.36</td>
<td>0.41</td>
<td>0.37</td>
<td>0.22</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.22</td>
<td>0.49</td>
<td>0.67</td>
<td>0.60</td>
<td>0.37</td>
<td>0.11</td>
<td>-0.07</td>
</tr>
<tr>
<td>Nominal Exchange rate</td>
<td>-0.03</td>
<td>-0.24</td>
<td>-0.52</td>
<td>-0.62</td>
<td>-0.28</td>
<td>0.19</td>
<td>0.40</td>
</tr>
<tr>
<td>Credit</td>
<td>-0.03</td>
<td>0.08</td>
<td>0.18</td>
<td>0.24</td>
<td>0.29</td>
<td>0.26</td>
<td>0.22</td>
</tr>
<tr>
<td>Housing prices</td>
<td>0.36</td>
<td>0.42</td>
<td>0.51</td>
<td>0.47</td>
<td>0.20</td>
<td>-0.01</td>
<td>-0.01</td>
</tr>
</tbody>
</table>
recent financial crisis indicates that the oil price is leading housing prices. The statistical properties of the estimates, however, are weak. Hence, only broad qualitative implications might be drawn from the correlation table.

4 The Model

Based on the stylised facts presented in the previous section, it seems evident that changes in the terms of trade have effects on the Norwegian economy. The model literature, however, has not yet addressed this aspect fully; there exists very few models where foreign price impulses matter. The direct impact from foreign prices on domestic prices and GDP matters for the conduct of monetary policy. In addition, the oil price correlates with credit and housing prices - factors that matter for the stability of the banking system. To the extent that central banks care about financial fragility in their monetary policy decisions, the effects of terms of trade movements on the financial sector are important for optimal monetary policy. In this section, I develop a DSGE model where foreign price impulses are modelled explicitly, focusing on oil price shocks and import price shocks. The model provides a tractable framework to analyse the effects of foreign price changes on small open resource-rich economies.

The model is built on a DSGE framework which is extended to incorporate overlapping generations, a housing sector, a government sector with a sovereign wealth fund and income from oil exports, and an exchange rate market. The economy consists of six types of agents: (i) finitely-lived households, (ii) perfectly competitive life-insurance companies, (iii) perfectly competitive final goods firms, (iv) a continuum of firms producing differentiated intermediate goods and setting nominal prices in a staggered fashion, (v) the monetary authority, and (vi) a government sector, which owns the natural resource and distributes the real return to households. In addition, there are foreign intermediate goods firms and foreign final goods firms trading with the domestic economy.
4.1 Households

The individual households in the economy are described by a stochastic discrete-time version of Yaari-Blanchard (Blanchard, 1984; Yaari, 1965) households, i.e. finitely-lived households with no intergenerational bequest motive, and are extended to include endogenous labour supply, a housing sector and the holding of foreign bonds.

The economy is populated by forward-looking households with identical preferences who face a constant probability of death in each time period. The population is assumed to be constant over time and normalised to one, implicitly assuming that birth and death rates are equal in each period. Since the probability of death is constant, the expected lifetime of any household is $\frac{1}{v}$.\(^3\) Note that if $v \to 0$, the expected lifetime is $\frac{1}{v} \to \infty$, i.e. the limit case is an infinite lifetime household.

A perfectly competitive life insurance market is assumed to operate, similar to Blanchard (1984) and Yaari (1965). The insurance companies collect financial wealth from the deceased and pay a fair premium to survivors. The zero profit condition in the insurance market requires that all collected wealth is redistributed to the survivors in equal proportions.

4.1.1 Individual Optimisation

Individual households face a stochastic sequence of prices, interest rates and profit shares, and decide on the future time path of consumption, housing, labour supply and wealth accumulation. Labour yields direct dis-utility and housing yields direct utility. Furthermore, housing is an asset in the economy and it allows agents to save between periods. The housing stock depreciates each year by $\delta_h$. There are complete markets in the economy and there exist two risk-free bonds: (i) domestic bonds in zero net supply and (ii) foreign bonds. Foreign bonds are assumed to be in infinite supply since the economy is assumed to be

\(^3\)The expected lifetime is the sum of the series $\sum_{t=1}^{\infty} t(1-v)^{t-1}v = v + 2(1-v)v + 3(1-v)^2v + ... = \frac{1}{v}$.
small relative to the world market. Total non-human wealth therefore consists of housing, domestic bonds and foreign bonds.

The objective of the individual household $j$ belonging to the generation born at time $s \leq 0$ is to maximise the expected lifetime utility function:

$$E_0 \sum_{t=0}^{\infty} \beta(1-\nu) U(C_{s,t}(j), H_{s,t}(j), N_{s,t}(j))$$

(1)

where $\beta \in (0,1]$ is the discount factor, $C_{s,t}(j)$ is consumption of the final good, $H_{s,t}(j)$ is consumption of housing and $N_{s,t}(j)$ denotes the amount of labour provided by household $j$.

The flow budget constraint at time $t$ for the individual household $j$ born at time $s$ and owning a life insurance is:

$$C_{s,t}(j) + B_{s,t}(j) + S_t B^*_{s,t}(j) + P_{h,t} H_{s,t}(j) \leq \frac{P_{h,t} H_{s,t-1}(j)(1-\delta_h)}{1-\nu} + \frac{R_{t-1} B_{s,t-1}(j)}{\Pi_t(1-\nu)} + \frac{S_t R^*_{t-1} B^*_{s,t-1}(j)}{\Pi_t^*(1-\nu)} + W_t N_{s,t}(j) + Div_{s,t}(j) + T_{s,t}(j)$$

(2)

The individual income side of equation (2) is composed of non-human wealth from period $t-1$ and income in period $t$. Individual non-human wealth is the sum of the present value of housing assets, $P_{h,t} H_{s,t-1}$, depreciated by $\delta_h$, risk-free domestic bonds, $\frac{R_{t-1} B_{s,t-1}(j)}{\Pi_t(1-\nu)}$, and risk-free foreign bonds, $\frac{S_t R^*_{t-1} B^*_{s,t-1}(j)}{\Pi_t^*(1-\nu)}$. $S_t$ is the real exchange rate, $R_{t-1}$ and $R^*_{t-1}$ are the gross nominal interest rate on domestic bonds and foreign bonds, respectively, and $\Pi_t$ and $\Pi_t^*$ denote the gross domestic and gross foreign inflation rate of consumption goods, respectively. The insurance market acts to discount assets by the factor $(1-\nu)$, i.e. if the household survives to period $t$ it also receives its fair share of the capital of deceased households through the insurance market. For the rest of this thesis, $A_{s,t}$ is used to denote the sum of non-human wealth at time $t$. Income in each period is the sum of wage income, $W_{s,t} N_{s,t}(j)$, dividends from the ownership

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4See appendix A for the derivation of the budget constraint for Yaari-Blanchard households.
of domestic intermediate firms, $\text{Div}_{s,t}(j)$, and transfers from the government, $T_{s,t}(j)$.

The expenditure side of the flow budget constraint of individual households is the sum of consumption, $C_{s,t}(j)$, savings in domestic and foreign bonds, $B_{s,t}(j)$ and $B^*_s(t)$, and housing, $H_{s,t}(j)$. All prices are real and the common numeraire is the price of consumption goods in the domestic market.

An optimal consumption plan for the individual household $j$ of generation $s$ is described by the initial set \( \{C_{s,0}(j), B_{s,0}(j), B^*_s(j), H_{s,0}(j), N_{s,0}(j)\} \) and the sequence of contingency plans \( \{C_{s,t}(j), B_{s,t}(j), B^*_s(t), H_{s,t}(j), N_{s,t}(j)\}_{t=1}^{\infty} \), given initial wealth $A_{s,0}(j)$ and the stochastic sequences \( \{\text{Div}_{s,t}(j), T_{s,t}(j), R_t, R^*_t, W_t\}_{t=0}^{\infty} \) whose exogenously given probability distributions are known by all households.\(^{5}\) For the sake of simplicity, newly born agents hold no financial assets and profit shares are age-independent.

The rest of this thesis focuses on the instantaneous separable utility function:

\[
U(C_{s,t}(j), H_{s,t}(j), N_{s,t}(j)) = \log(C_{s,t}(j)) + \kappa \log(H_{s,t}(j)) - \frac{N_{s,t}(j)^{1+\xi}}{1+\xi} \quad (3)
\]

The given utility function is chosen for two reasons. First, it satisfies the assumptions of concavity in all arguments and non-satiation. Second, the log formulation in consumption is important in order to derive a tractable analytical solution to the model.\(^{6}\) Furthermore, exponential smoothing and time separable utility implies time consistent behaviour. The log formulation on housing and constant relative risk aversion (CRRA) formulation on labour are used for simplicity.

The household maximises the expected lifetime utility presented in equation (1) subject to the flow budget constraint given in equation (2) and a transver-

\(^{5}\)The probability distributions are assumed to be known by all agents in the economy to avoid market failures arising from information asymmetries. Furthermore, it allows agents to optimise under rational expectations.

\(^{6}\)In particular, an analytical solution to the dynamic consumption equation hinges on the assumption of log utility in consumption.
sality condition which precludes Ponzi games:

\[
\lim_{T \to \infty} E_t\{[(1 - v)(T-t)^{-1} R_{t,T} A_s T(j)] = 0 \tag{4}
\]

where \( R_{t,T} = \prod_{k=t}^{T} R_k \) is the product of all interest rates from period \( t \) to period \( T \) and \( A_s T(j) \) is the level of non-human wealth in period \( T \).\(^7\)

The solution to the household maximisation problem yields the following necessary first order conditions:\(^8\)

\[
1 = \beta R_t E_t\left[ \frac{C_{s,t}(j)}{C_{s,t+1}(j) \Pi_{t+1}} \right] \tag{5}
\]

\[
W_t = C_{s,t}(j) N_{s,t} \tag{6}
\]

\[
\kappa C_{s,t}(j) = P_{h,t} H_t(j) - E_t\left\{ \frac{1}{R_t} P_{h,t+1} H_t(j)(1 - \delta_h) \Pi_{t+1} \right\} \tag{7}
\]

where equation (5) is the stochastic euler equation and equation (6) is the intratemporal optimality condition for the optimal choice of consumption and labour. Equation (7) is the dynamic housing equation, illustrating the choice between consumption and housing in period \( t \). Note that housing yields both direct utility today and wealth income which allows consumption tomorrow. Hence, the dynamic housing equation contains both an intratemporal and an intertemporal choice.

The stochastic discount factor of household \( j \) of generation \( s \) is defined as individual valuation of future consumption relative to consumption today and is given by:

\[
D_{t,t+1}(s,j) = \beta E_t\left\{ \frac{C_{s,t}(j)}{C_{s,t+1}(j) \Pi_{t+1}} \right\} \tag{8}
\]

\(^7\)Equation (4) has to be satisfied in order to obtain a interior optimum to the optimisation problem.

\(^8\)The problem has been solved as a Kuhn-Tucker optimisation problem with time-varying coefficients.
A combination of equation (5) and equation (8) gives:

\[ D_{t,t+1}(s, j) = E_t\left(\frac{1}{R_t}\right) \tag{9} \]

Equation (9) shows how the individual valuation of future consumption relative to consumption today corresponds to the expected interest rate in the economy, i.e. individual discounting is the same as discounting by the expected risk-free interest rate.

A key component of the model is to allow for temporary wealth and income changes to have effects on the optimal allocation of households. The limited lifetime of households allows for such effects because households are reluctant to pass away with positive wealth and are therefore more concerned about the near-future relative to the far-off future. By iterating the budget constraint forward, using equation (5) and equation (7), and imposing the transversality condition, the individual dynamic consumption is given as:

\[ C_t = (1 - \beta(1 - \nu)\left[ \frac{S_t R^*_{t-1} B_{s,t-1}(j)}{\Pi_t^*} + I_{s,t}(j) + H_{s,t}^{HH}(j) \right] \] (10)

where \( I_{s,t}(j) \) is human wealth, defined as the expected present discounted value of future labour incomes, profit shares and transfers from the government for household \( j \):

\[ I_{s,t}(j) = E_t \sum_{T=t}^{\infty} (1 - \nu)^{T-t} \frac{\Pi_t^*}{R^*_{t,T}} \left[ W_t N_{s,T}(j) + Div_{s,T}(j) + T_{s,T}(j) \right] \tag{11} \]

where the second equality follows from uncovered interest parity. \( H_{s,t}^{HH}(j) \) is expected future housing income for individual households:

\(^9\)See appendix B for a detailed description of the steps leading to equation (10).
There are now four equations describing the optimal behaviour of individual households, (i) the budget constraint given by equation (2), (ii) the intratemporal optimality condition for labour given by equation (6), (iii) the dynamic housing equation given by equation (7), and (iv) the individual dynamic consumption equation given by equation (10). These equations provide the description of the demand side of the economy at the individual level.

4.1.2 Aggregation

In the previous section, I derived the individual optimality conditions. However, to allow for macro-economic modelling, the aggregate versions of the equations are needed. The aggregate value $Q_t$ at time $t$ of a general economic variable $Q_{s;t}(j)$ is obtained as a sum across the living households in each cohort:

\[ Q_t = \sum_{s=1}^{t} (1 - \nu)^{t-s} Q_{s;t}(j) \]  

Aggregation of all generations alive at time $t$ yields the following expressions for the flow budget constraint, the first order conditions and the dynamic consumption equations, respectively:

\[ H_{s,t}^H(j) = E_t \sum_{T=t}^{\infty} \frac{(1 - \nu)^{T-t}}{\Pi_{t,T}} \left[ \frac{P_{H,T}H_{s,T-1}(j)(1 - \delta_h)}{1 - \nu} - P_{h,T}H_{s,T}(j) \right] \]

\[ E_t \sum_{T=t}^{\infty} \frac{(1 - \nu)^{T-t}}{\Pi_{t,T}} \left[ \frac{P_{H,T}H_{s,T-1}(j)(1 - \delta_h)}{1 - \nu} - P_{h,T}H_{s,T}(j) \right] \]

(12)

10 Equation (13) is composed of three parts: $Q_{s,t}(j)$ is the value of the given variable of each agent in cohort $s$, $(1 - \nu)^{t-s}$ is the number of remaining households from cohort $s$, while $\sum_{t=1}^{\infty}$ is sum over all cohorts which have existed up to time $t$. Hence, equation (13) is the sum of the variable across all living households.

11 The budget constraint now holds with equality due to the non-satiation property of the household utility function.
\[ C_t = P_{h,t} H_t + B_{HH,t} + S_t B^*_t \]
\[ W_t = C_t N_t^{i} \]
\[ \kappa C_t = P_{h,t} H_t - E_t\{ \frac{D_{t,t+1} P_{h,t+1} H_t (1 - \delta_t)}{\Pi_{t+1}} \} \]
\[ C_t = [1 - \beta (1 - \nu)] \left[ \frac{S_t R_{t-1} B^*_{t+1} H_{t-1}}{\Pi_t} + I_t \right] \]

where \( I_t \) denotes aggregate human wealth and is defined by \( I_t = E_t \sum_{T=t}^\infty [(1 - \nu)^T - \nu^T (W_T N_T + Div_T + T_T)] \) and \( B^*_{HH,t} \) is the ownership of foreign bonds by households in the economy at time \( t \). Using equations (14), (16) and (17) above, I obtain the following dynamic equation for aggregate consumption:

\[ C_t = \frac{1 - \nu}{\nu + \beta (1 - \nu)^2} E_t\{ D_{t,t+1} \Pi_{t+1} C_{t+1} \} + \frac{1 - \beta (1 - \nu)}{\nu + \beta (1 - \nu)^2} \frac{S_t R_{t-1} B^*_{t+1} H_{t-1}}{\Pi_t} \]

This dynamic consumption equation is the key component in the model for including wealth and income effects in the economy. In the case of finite lifetime (\( \nu \geq 0 \)), the time path of consumption depends on the aggregate level of non-human wealth. Note that in the case of infinite lifetime (\( \nu = 0 \)), equation (18) is reduced to the standard intertemporal Euler equation. Furthermore, note that housing has no effect on the time path of aggregate dynamic consumption. This follows from the assumptions of constant population and equal preferences implying that the aggregate household sector in the economy has to buy and sell all houses in each period. Therefore, an increase in housing prices will in

\[ ^{12}\text{See Appendix C for a detailed description of the steps leading to equation (18).} \]
the aggregate lead to more non-human wealth from the previous period, but also to more expensive housing to be bought today and has no effect on total non-human wealth. This corresponds to Buiter (2008)'s notion that 'housing wealth isn’t wealth.'

4.2 Firms

The supply side of the economy is based on the DSGE model used by Norges Bank and documented in Brubakk et al. (2006). There are two types of firms, (i) final goods firms producing in competitive markets with flexible prices, and (ii) intermediate goods firms selling goods under monopolistic competition with sticky prices. The intermediate goods sector can either export its goods or sell to the domestic final goods firms, whereas the final goods firms can either import intermediate goods from abroad or purchase from the domestic intermediate goods firm.

4.2.1 Final Goods Firms

Final goods firms are identical and produce under competitive markets without any frictions. As a result, the sector can be described by a representative firm. The representative final goods firm is assumed to produce by a CES production function:

\[ Y_t = \left( \nu^{\frac{1}{\mu}} Q_t^{1-\frac{1}{\mu}} + (1 - \nu)^{\frac{1}{\mu}} M_t^{1-\frac{1}{\mu}} \right)^{\frac{\mu}{1-\mu}} \]  

where \( Y_t \) is the amount of final goods, \( Q_t \) is the amount of domestic intermediate goods used in production, \( M_t \) is the amount of imported intermediate goods used in production, \( \nu \) measures the steady-state share of domestic intermediates if relative prices are equal to 1 and \( \mu \) is the elasticity of substitution between foreign and domestic intermediate goods in production of final goods. The firm operates under competitive markets and therefore acts as a price-taker in the economy. All prices in the model are real prices, using the price of the
consumption good (final good) as the numeraire. Profit maximisation given the real prices of the two inputs yields two demand equations:

\[ Q_t = \nu P^{-\mu}_{Q,t} Y_t \]  
(20)

and

\[ M_t = (1 - \nu) P^{-\mu}_{M,t} Y_t \]  
(21)

where \( P_{Q,t} \) is the real price of domestic intermediate goods and \( P_{M,t} \) is the real price of imported goods in domestic prices.

In addition, following competitive markets, we have the zero profit condition which implies that the real value of produced goods must be equal to the real value of input:

\[ Y_t = P_{Q,t} Q_t + P_{M,t} M_t \]  
(22)

### 4.2.2 Domestic Intermediate Goods Firms

Each domestic intermediate goods firm, \( i \), is producing a differentiated good \( Y'_t(i) \) using the linear production function:

\[ Y'_t(i) = Z_t N_t(i) \]  
(23)

where \( Z_t \) is labour productivity and \( N_t(i) \) is the amount of labour used in production. Labour productivity is assumed to be equal across firms and follow a predetermined growth path. Hence, the variation of intermediate goods production around a trend only depends on the amount of labour used in production. Given that labour is the only input in production and the real wage rate of the economy is \( W_t \), the real marginal cost is:

\[ MC_t(i) = W_t(i) \]  
(24)
Firms sell their goods under monopolistic competition to both domestic final good producers and final good producers abroad. A quadratic cost of changing prices is assumed, implying that larger changes in prices cost more on the margin (Rotemberg, 1982). Hence, firms are reluctant to adjust its prices and this cost increases with the change in price in a quadratic fashion. The cost enters the firm’s maximisation problem and is described by:

\[ \zeta_{t}^{PQ}(i) = \frac{\phi_{PQ}}{2} \left[ \frac{P_{Q,t}(i)/P_{Q,t-1}}{P_{Q,t-1}/P_{Q,t-2}} - 1 \right]^2 \]  

(25)

for domestic prices of goods and

\[ \zeta_{t}^{PM}(i) = \frac{\phi_{PM}}{2} \left[ \frac{P_{M,t}(i)/P_{M,t-1}}{P_{M,t-1}/P_{M,t-2}} - 1 \right]^2 \]  

(26)

for exported goods, where \( \phi_{PQ} \) and \( \phi_{PM} \) are the main parameters of price stickiness, determining the cost of changing prices for domestic and exported goods, respectively. A higher \( \phi \) will increase the cost of adjusting prices, thus making firms more reluctant to alter its prices, resulting in more sticky prices in the economy. The costs of changing prices is assumed to enter the firm maximisation problem as an intangible dis-utility.\(^{13}\)

The dividends are given as the sum of revenue from domestic sales and exports minus wage costs. Each period, the dividend is being paid to households and is given by:

\[ Div_{t}(i) = P_{Q,t}(i)Q_{t}(i) + S_{t}P_{M^{*},t}(i)M^{*}_{t}(i) - W_{t}N_{t}(i) \]  

(27)

The firms operate under monopolistic competition with sticky prices. As a result, the intermediate goods firms are price setters in the economy. They choose labour inputs,\(^ {14}\) by profit maximisation in order to maximise the present discounted value of profits. The aggregate first order conditions for

\(^{13}\)i.e. the cost of changing prices is an intangible cost and does not enter the cash-flows of the economy.

\(^{14}\)by profit maximisation.
optimal price setting are: \(^{15}\)

\[
P_{Q,t} = \theta \frac{MC_t}{Z_t} \left[ \frac{\Pi_{Q,t}}{\Pi_{Q,t-1}} \left( \frac{\Pi_{Q,t}}{\Pi_{Q,t-1}} - 1 \right) \right]^{-1}
- \phi^{PQ} \Phi_{Q,t} \frac{Q_{t+1}}{Q_t} \left( \frac{\Pi_{Q,t+1}}{\Pi_{Q,t}} - 1 \right)^{-1}
\]

(28)

and

\[
P_{M^*,t} = \theta \frac{MC_t}{Z_t} \left[ \frac{\Pi_{M^*,t}}{\Pi_{M^*,t-1}} \left( \frac{\Pi_{M^*,t}}{\Pi_{M^*,t-1}} - 1 \right) \right]^{-1}
- \phi^{PM^*} \Phi_{M^*,t} \left( \frac{\Pi_{M^*,t+1}}{\Pi_{M^*,t}} - 1 \right)^{-1}
\]

(29)

Here, \(\theta\) is the elasticity of substitution between domestic and foreign sales of intermediate goods. The price mark-up under monopolistic competition is given by \(\frac{\theta}{M^*+1}\). The price setting equations for period \(t\) both consist of three parts, (i) price setting under flexible prices and monopolistic competition, (ii) the cost of changing the inflation rate from period \(t-1\) to period \(t\), and (iii) the expected discounted cost of changing the inflation rate from \(t\) to \(t+1\). The first order conditions reflect the optimal way of weighting these costs together.

The foreign economy is assumed to have similar technologies as the domestic economy. Hence, the foreign final goods firm has the same type of demand equations as the domestic final goods firm, and the foreign intermediate goods firms set their prices in the same manner as the domestic intermediate goods firms.

### 4.3 The Government Sector

A key contribution of this thesis is to add a government sector which administers the natural resource in the economy, assumed to be oil to resemble the Norwegian economy. The oil is assumed to be purely an endowment with no

\(^{15}\)See appendix D for the full derivation of the first order conditions for the intermediate goods firms.
cost of extraction. In each period, the government sells all it can extract, $X_t$, in the international market at the real foreign market price, $P_{X,t}$. The rate of extraction is assumed to be exogenous and increase with labour productivity in the economy. The real profits from the resource extraction is then given by:

$$Div_{G,t} = S_t P_{X,t} X_t$$

(30)

All profit from the sale of the natural resource is placed in a sovereign wealth fund that invests in foreign risk-free bonds. The government is assumed to follow a fiscal rule similar to the Norwegian budgetary rule, hence only the real return of the fund is available to cover government expenditures each period. Since the government in this economy has no expenditures to be paid, the real return is distributed to the households as a lump sum transfer each period. Note that if the government had expenditures to cover, the transfers from the fund would still be a transfer to households by lowering their tax burden. The transfer to households in period $t$ is then given by:

$$T_t = S_t \left( \frac{R^*_t - 1}{\Pi^*_t} \right) B^*_{G,t-1}$$

(31)

where $B^*_{G,t-1}$ is the amount of foreign bonds owned by the sovereign wealth fund at time $t-1$. By combining equation (30) and equation (31), the law of motion for the assets in the sovereign wealth fund is given by:

$$S_t B^*_{G,t} = S_t \frac{R^*_{t-1} B^*_{G,t-1}}{\Pi^*_t} + Div_{G,t} - T_t = S_t \frac{B^*_{G,t-1}}{\Pi^*_t} + S_t P_{X,t} X_t$$

(32)

Hence, the sovereign wealth fund grows with the income from the sale of the natural resource in each period.\textsuperscript{16} Notice that this implies that the fund will grow to infinity when $t \to \infty$. In a DSGE-model, this will simply result in all variables growing to infinity, arguably an impossible and uninteresting

\textsuperscript{16}Assuming that foreign consumer price inflation is exogenous and equal to 1.
situation. In order to make the sovereign wealth fund stationary, the rate of extraction is assumed to grow by the growth rate of labour productivity in the economy. This allows a steady-state solution to exist where the real profits from selling oil is equal to the labour productivity discount on the sovereign wealth fund. The stationary version of equation (32) is presented in section 4.7.

### 4.4 The Foreign Exchange Market and the Risk Premium

Households have the opportunity of investing in foreign and domestic bonds. The optimal allocation between the two bonds defines the interest rates and the exchange rate for which there is no arbitrage in the bond market.

\[
S_t = \frac{S_{t+1} R^*_t \Pi_	ext{t+1}}{R_t \Pi_{t+1}} u_t
\]

(33)

where \( u_t \) is an endogenous risk premium. In open economy infinite horizon DSGE-models, there is a need for closing the model by introducing an extra equation. Following Schmitt-Grohé and Uribe (2003), an endogenous risk premium is one of several ways of closing open economy macro-models. The risk premium is assumed to depend on the amount of foreign bonds owned by the agents of the economy as a fraction of the steady-state level:

\[
u_t = e^{-\gamma \left( \frac{B_t}{B^*_t} - 1 \right)}
\]

(34)

Hence, if the agents in the domestic economy own more foreign bonds than the steady-state level, the risk premium will be lower and agents can borrow cheaper abroad than in the steady-state solution. \( \gamma (\geq 0) \) is a parameter denoting the degree of impact from bond holdings on the risk premium and \( B^*_t \) is the steady-state level of foreign bonds owned by all agents in the economy, \( B^*_t \).

The latter is given by the sum of foreign assets held by the government and the households:

\[
B^*_t = B^*_{G,t} + B^*_{HH,t}
\]

(35)
4.5 Market Clearing

All factor and goods markets must clear in equilibrium. For the final good, this implies that all produced final goods must be consumed at time $t$:

$$Y_t = C_t$$  \hspace{1cm} (36)

For the intermediate good, market clearing implies that all goods produced are sold, either domestically or in the foreign market:

$$Y'_t = Q_t + M_t^*$$  \hspace{1cm} (37)

For domestic bonds, there is zero net supply, i.e.

$$B_t = 0$$  \hspace{1cm} (38)

while in the foreign bond market, the economy is assumed to be small compared to the foreign market. Hence, the supply of foreign risk-free bonds is in infinite supply. The law of motion for foreign bonds can be derived by applying the budget constraint (equation (14)), replacing for the transfers from the sovereign wealth fund from equation (32) and using the profit equations from the intermediate and final goods sector (equations (27) and (22)). The law of motion for the ownership of foreign bonds by the domestic economy is then given by:

$$B_t^* = \frac{B_{t-1}^*}{\Pi_t} + P_{X,t}X_t + P_{M^*,t}M_t^* - \frac{P_{M,t}}{S_t} M_t$$  \hspace{1cm} (39)

The trade balance is defined as $P_{X,t}X_t + P_{M^*,t}M_t^* - \frac{P_{M,t}}{S_t} M_t$. Hence, total ownership of foreign assets in the economy evolves with the trade balance. A surplus on the trade balance implies an increase in net foreign assets.
4.6 Monetary Policy

To close the model, a monetary policy framework is needed. The monetary policy makers are assumed to conduct optimal monetary policy according to a flexible inflation targeting regime. A flexible inflation targeting regime implies that the central bank is concerned with both consumer price inflation and the output in the economy. To solve for optimal monetary policy, a loss function is applied, reflecting an approximation to the welfare function of an economy with price stickiness.

I apply a loss function similar to the one used by Norges Bank. The loss function is specified in gaps, which are defined as percentage deviations from steady-state values. The period for period loss function is then given by:

$$L(t) = \frac{1}{2}(\hat{\pi}_t^2 + \lambda \hat{y}_t^2 + \gamma_r(R_t - R_{t-1})^2)$$ (40)

A positive inflation gap, $\hat{\pi}_t$, implies inflation above the steady-state level. The output gap, $\hat{y}_t$, is defined as deviation from steady-state output for mainland Norway.\(^{17}\) In addition to inflation and output, there is a smoothing term in the loss function for the nominal interest rate, $(R_t - R_{t-1})$, adding a cost to large changes in the interest rate and resulting in smooth interest rate paths. $\lambda$ and $\gamma_r$ are parameters describing the relative weights in the loss function and are set to 0.5 and 0.05, respectively, approximately the same as the loss function of Norges Bank.\(^{18}\) All terms in the loss function are squared to ensure that both positive and negative deviations from steady-state are costly for the central bank.

Subsequently, the central bank chooses a path for the key policy rate by minimising the loss function. Monetary policy is assumed to be conducted under discretion, implying that the central bank re-optimise its monetary policy each

\(^{17}\)The difference between mainland output and output is rather trivial since oil production is fixed in the model economy. Hence, the difference only materialises in the relative size of the effects.

quarter.\footnote{The monetary policy literature often assume commitment, arguably an unrealistic assumption for most countries. I assume discretion here to more realistically represent monetary policy in practice.}

4.7 The Stationary Solution

To allow for a stationary solution to the model, the law of motion for the sovereign wealth fund has to be stationary. All aggregate equations are therefore de-trended by labour productivity, $Z_t$. The adjusted variables are denoted by small letters. The growth rate of labour productivity is assumed to be constant and equal to $g$. The rest of this section presents the stationary equations of the model:

The intratemporal optimal labour-consumption choice:

$$w_t = c_t N_t^\xi$$  \hspace{1cm} (41)

The dynamic optimal housing equation:

$$\kappa c_t = P_{h,t} - E_t \{ D_{t,t+1} P_{h,t+1} \Pi_{t+1} (1 - \delta_h)(1 + g) \}$$  \hspace{1cm} (42)

The dynamic consumption equation:

$$c_t = \frac{1 - \nu}{\nu + \beta (1 - \nu)^2} E_t \{ D_{t,t+1} c_{t+1} \Pi_{t+1} (1 + g) \}$$
$$+ \frac{\nu (1 - \beta (1 - \nu)) S_t R_{t-1}^* b_{RH,t-1}^*}{\nu + \beta (1 - \nu)^2 \Pi_t^* (1 + g)}$$  \hspace{1cm} (43)

The final goods firm production function:

$$c_t = (\nu \frac{\nu}{\nu} q_t^{1 - \frac{1}{\nu}} + (1 - \nu) \frac{1}{\nu} m_t^{1 - \frac{1}{\nu}} \frac{\nu}{\nu})$$  \hspace{1cm} (44)
The demand for domestic intermediate goods from the final goods firm:

\[ q_t = \nu (P_{Q,t})^{-\sigma} c_t \]  \hspace{1cm} (45)

The demand for imported intermediate goods from the final goods firm:

\[ m_t = (1 - \nu) (P_{M,t})^{-\sigma} c_t \]  \hspace{1cm} (46)

The domestic intermediate goods firms production function:

\[ y_t = N_t = q_t + m_t^* \]  \hspace{1cm} (47)

The marginal costs of domestic intermediate goods firms:

\[ mc_t = w_t \]  \hspace{1cm} (48)

The domestic intermediate goods price setting equation in the domestic market:

\[ P_{Q,t} = \theta mc_t \left[ (\theta - 1) + \phi_{P,Q} \Pi_{Q,t} \frac{\Pi_{Q,t-1}}{\Pi_{Q,t}} \right]^{-1} - \phi_{P,Q} E_t \left\{ D_{t+1,t} \frac{q_{t+1}(1 + g)}{q_t} \Pi_{Q,t+1} \frac{\Pi_{Q,t+1}}{\Pi_{Q,t}} \right\}^{-1} \]  \hspace{1cm} (49)

The domestic intermediate goods price setting equation in the export market:

\[ P_{M^*,t} = \theta mc_t \left[ (\theta - 1) + \phi_{P,M^*} \Pi_{M^*,t} \frac{\Pi_{M^*,t-1}}{\Pi_{M^*,t}} \right]^{-1} - \phi_{P,M^*} E_t \left\{ S_{t+1} m_{t+1} \frac{S_{t} m_{t}}{S_{t} m_{t}^*} \Pi_{M^*,t+1} \frac{\Pi_{M^*,t+1}}{\Pi_{M^*,t}} \right\}^{-1} \]  \hspace{1cm} (50)

The price setting equation of foreign intermediate goods firm in the domestic
import market:

\[ P_{Mt} = \theta^* mc^* S_t [(\theta^* - 1) + \phi_{PM} \frac{\Pi_{Mt}}{\Pi_{Mt-1}} (\frac{\Pi_{Mt}}{\Pi_{Mt-1}} - 1) - \phi_{PM} E_t \{D_{t+1} \Pi_{Mt+1} \Pi_{Mt+1} \Pi_{Mt+1} (1 + g^*) (\frac{\Pi_{Mt+1}}{\Pi_{Mt}} - 1) \}]^{-1} \]  

The demand for exported intermediate goods:

\[ m_t^* = \nu^*(P_{Mt})^{-\nu^*} y_t^* \]  

The law of motion of the sovereign wealth fund:

\[ b_{G,t}^* = b_{G,t-1}^* + P_{X,t+1} x_t \]  

The exchange rate market:

\[ S_t = S_{t+1}^* \frac{R_t^* \Pi_{t+1}}{R_e^* \Pi_{t+1}^*} e^{-\gamma \frac{g^*}{1+g}} \]  

The current account equation:

\[ b_t^* = \frac{R_{t-1}^* b_{t-1}^*}{R_e^* (1 + g)} + P_{X,t+1} x_t + P_{Mt+1} m_t^* - \frac{P_{Mt}^* m_t}{S_t} \]  

In order to use the complete stationary model for policy analysis, the numerical values of the free parameters have to be set.

### 5 Parametrisation

In this section, I document the calibration of the steady-state and dynamic parameters in the model. The parametrisation in this thesis is based on a combination of estimated parameters from the empirical literature, common values in the model literature and calibration to match dynamic features of the data.

There are two kinds of parameters in the model, steady-state parameters
and dynamic parameters. The steady-state parameters are calibrated so as to match empirically estimated parameters and long-run ratios in the Norwegian economy. The dynamic parameters matter for the impulse responses in the model and are calibrated to match dynamic results from empirical estimates of the Norwegian economy. The goal has been to produce the true parametrisation of the Norwegian economy, and the result is arguably a reasonable parametrisation which allows a discussion of the qualitative results from the model.

5.1 Steady-state Parameters

The steady-state parameters in the model are: $\Pi$, $\Pi^*$, $g$, $R$, $R^*$, $\mu$, $\mu^*$, $\nu$, $\nu^*$, $\nu$, $\xi$, $\kappa$, $\delta$, $\theta$ and $\theta^*$. The gross inflation rate for both domestic ($\Pi$) and foreign ($\Pi^*$) inflation are set to 1 to allow for tractability of the model for a diverse range of countries. The constant growth rate of labour productivity, $g$, is set to 0.004 per quarter to match the average rate of labour productivity growth for the Norwegian economy between 1990 and 2012 (Hagelund (2009) and OECD\textsuperscript{20}). Both the domestic and foreign gross nominal interest rate are set to match a long-run real interest rate of 4 percent.

The elasticity of substitution between domestic and imported intermediate goods in the production of final consumption goods, $\mu$, typically ranges between 0.5 and 2 in the business cycle literature, see e.g. Chetty et al. (2011). For the Norwegian economy, a value of the elasticity in the low range is probably appropriate due to the relative specialisation of the Norwegian production structure. Naug (2002) uses Norwegian data and finds the elasticity of substitution between domestically-produced and imported goods in the production of final goods to be 1.5. However, his study is based on the industrial sector and an economy-wide elasticity is expected to be somewhat lower. Varying the parameter, however, does not change the steady-state values of the economy much. The elasticity is set to be 0.8, and the corresponding elasticity for trading partners, $\mu^*$, is also set to be 0.8 for simplicity.

\textsuperscript{20}http://stats.oecd.org/Index.aspx?DataSetCode=LEVEL
The home bias parameter in the final good production function, \( \nu \), is chosen in order to reproduce an import share similar to the Norwegian economy, and is set to 0.7. Together, the two parameters, \( \mu \) and \( \nu \), result in a weight on imported goods in the consumer price index close to 0.3, similar to the weight of imported goods and services in the consumer price index for Norway. The home bias parameter for the foreign producers, \( \nu^* \), is also set to 0.7 for simplicity.

The probability of passing away, \( \upsilon \), is chosen to match the expected remaining lifetime of a randomly selected person in Norway. The expected lifetime is about 80 years and, assuming a symmetrical age distribution, the expected remaining lifetime of a randomly chosen person is 40 years. As a result, the probability of passing away is set to \( \frac{1}{160} \) per quarter.\(^{21}\)

Estimates for the labour dis-utility parameter, \( \xi \), vary in the literature. In the model in this thesis, \( \frac{1}{\xi} \) corresponds to the Frisch-elasticity of labour. Most microeconomic studies have estimates of the Frisch-elasticity close to zero, see e.g. Killingsworth (1984) and Killingsworth and Heckman (1986). The real business cycle literature, on the other hand, use estimates based on macro data and often work with Frisch-elasticities of much higher magnitude (often in excess of 5, see e.g. Christiano and Eichenbaum (1992)). Given the wide range of values, I follow Christiano, Eichenbaum and Evans (1997) and set \( \xi \) to 1 for simplicity.

The housing utility parameter, \( \kappa \), is set to 1. In the steady-state, \( \kappa \) only matters for the steady-state house prices and it has no effect on the dynamic solution. As a result, \( \kappa \) is set to 1 for simplicity. The housing capital depreciation rate, \( \delta_h \) is set equal to the average ratio of housing maintenance costs to housing capital in the Norwegian data, which results in a quarterly ratio of 0.012.\(^{22}\)

The elasticity of substitution between differentiated intermediate goods, \( \theta \), represents a parameter describing the degree of competition in the intermediate

\(^{21}\)Recall that the expected remaining lifetime of a household with constant probability of death, \( \frac{1}{\upsilon} \), is \( \sum_{t=1}^{\infty} (1 - \upsilon)^{t-1} \upsilon = \upsilon + 2(1 - \upsilon)\upsilon + 3(1 - \upsilon)^2\upsilon + \ldots = \frac{1}{\upsilon} \), implying that the expected remaining lifetime when \( \upsilon = \frac{1}{160} \) is 160 quarters, i.e. 40 years.

goods market since $\frac{\theta}{\sigma-1}$ can be interpreted as the price mark-up. Estimates in the literature typically range between 3 and 11, see e.g. Domowitz, Hubbard and Petersen (1988). In this model, the value for $\theta$ is chosen to be 6, equal to the parameter value used in the Norwegian Economic Model (NEMO), see Brubakk et al. (2006).\textsuperscript{23} The degree of competition in the foreign market, $\theta^*$, is also set to 6 for simplicity.

5.2 Dynamic Parameters

There are four parameters that matter for the dynamic responses of the model: the three stickiness parameters, $\phi^{PQ}$, $\phi^{PM}$ and $\phi^{PM^*}$, and the endogenous risk premium parameter, $\gamma$. The stickiness parameters are set equal to the respective values in NEMO, where these are estimated by a Bayesian approach. Hence, $\phi^{PQ}$ is set equal to 1500 and $\phi^{PM^*}$ is set equal to 2300, while the cost of changing prices for foreign intermediate goods exporters to Norway is set equal to the domestic intermediate goods import cost of changing prices, hence $\phi^{PM} = \phi^{PM^*} = 2300$.

The parameter on the endogenous risk premium determines the dynamic response of the exchange rate to shocks in the model. When $\gamma$ is set to zero, the exchange rate moves in the same qualitative direction, but the response is smaller and more permanent.\textsuperscript{24} In this thesis, the value of $\gamma$ is set to 0.01, which results in an exchange rate impulse response persistence approximately similar to Norwegian data on exchange rate movements to oil price shocks (Aastveit, Bjørnland and Thorsrud (2011)).

6 Results

This section presents the results from numerical simulations of the model from section 4. The results are presented as impulse response functions, illustrating

\textsuperscript{23}$\theta$ equal to 6 is also used by Christiano, Eichenbaum and Evans (1997).

\textsuperscript{24}Setting $\gamma$ to zero is only possible in the finite lifetime version of the model. To allow for comparison with an infinite lifetime model, $\gamma$ is therefore set different from zero.
the evolution of macroeconomic variables following a shock to stochastic state variables. All variables in the impulse response functions, except the interest rate, are presented as percentage deviations from steady state.

In a DSGE-framework, the economy is driven by exogenous shocks to stochastic state variables. Absent these shocks, the economy will converge back to a stable steady-state growth path. I consider three exogenous shocks, (i) a positive monetary policy shock, (ii) a positive oil price shock and (iii) a negative foreign marginal cost shock. The following stochastic processes are assumed for the three stochastic state variables:

\[
\begin{align*}
\Lambda_t &= \rho_{\Lambda} \Lambda_{t-1} + \epsilon_{\Lambda}^t \\
\hat{p}_{x,t} &= \rho_{\hat{p}_{x}} \hat{p}_{x,t-1} + \epsilon_{\hat{p}_{x}}^t \\
\hat{mc}^* &= \rho_{\hat{mc}} \hat{mc}^*_t - \epsilon_{\hat{mc}}^t 
\end{align*}
\]

(56)

where \( \Lambda_t \) is a monetary policy shock, \( \hat{p}_{x,t} \) is the oil price gap, \( \hat{mc}^*_t \) is the foreign marginal cost gap, \( \epsilon_{j}^t \) is assumed to be normally distributed with standard deviation \( \sigma_{j}^t \) for \( j \in \{ R, x, mc^* \} \), and \( \rho_{j} \) are persistence parameters for the shock processes.

This section is divided in three parts. First, I present the results from a positive monetary policy shock in order to clarify how monetary policy operates in the economy. Second, in order to estimate the effects of an oil price increase on the Norwegian economy, I discuss the results from a positive oil price shock under optimal monetary policy. Third, I present the results from a negative foreign cost shock to illustrate how the increased inflow of cheap imports have affected the Norwegian economy during the last decade.

6.1 A Positive Monetary Policy Shock

This subsection presents the numerical results of a positive monetary policy shock corresponding to an increase in the interest rate of 100 basis points per
year. The parameters are calibrated using the values from section 5. The persistence parameter of the monetary policy shock, $\rho$, is set to 0.5.

Figure 6 shows the impulse responses of an increase in the interest rate where each period corresponds to one quarter. Price inflation figures and interest rates are annualised. There are two transmission mechanisms of monetary policy in the model: (i) a demand channel, and (ii) an exchange rate channel. Through the demand channel, a higher interest rate induces households to save more thereby leading to lower demand. As a result, consumption and housing prices fall. The final goods sector experiences lower demand and buys less of both inputs, thus both the volume and the prices of imported and domestically produced intermediate goods fall. The domestic intermediate goods sector compensates for lower domestic demand by lowering export prices and thereby increasing their exports somewhat. However, the domestic intermediate goods firms are unable to compensate fully and therefore the total production of domestic intermediate goods is lower. As a result, they demand less labour, which subsequently leads to both lower employment and lower real wages.

The interest rate also affects the exchange rate. Investors seek to maximise profits and choose between operating in the domestic and the foreign market. The law of one price implies that the return in both markets should be equal. Hence, when the domestic interest rate increases, the exchange rate appreciates to clear the markets. A higher interest rate in Norway therefore results in a nominal exchange rate appreciation. The isolated effects from the exchange rate appreciation are lower import prices and higher export prices, resulting in higher imports and lower exports. However, from figure 6, it is evident that the demand channel dominates the import sector whereas the two effects appears to cancel out in the export sector.

In general, the effects on consumer prices and output depend on the relative change in various variables. Consumer prices are a composite of import prices and the prices on domestically sold intermediate goods and the total effect

\[ L(t) = \frac{1}{2}((\pi_t + \Lambda_t)^2 + \lambda y_t^2 + \gamma_i (R_t - R_{t-1})^2), \]

where $\Lambda_t$ is the shock process described above.

25 The monetary policy shock is modelled as a shock process to the central bank loss function,
Figure 6: Impulse responses to a positive monetary policy shock. All variables (except the interest rates) are percentage deviations from steady-state. Interest rates and inflation figures are annualised.
thus depends on the relative changes in these. Following a monetary policy shock, both import price inflation and domestically sold intermediate goods price inflation fall, resulting in lower consumer prices. For mainland output, the total effect depends on three volume components: (i) consumption, (ii) imports, and (iii) exports. Following an interest rate increase, consumption fall, implying lower output, and imports are lower, implying higher output. Exports, on the other hand, barely move. In the benchmark calibration, the effect on consumption dominates and as a result, following an interest rate increase, output falls.

The results of a monetary policy shock show clearly how interest rate changes affect in the economy. In particular, by increasing the key policy rate, the central bank is able to reduce the consumer inflation rate, output and asset prices, and to achieve a real exchange rate appreciation.

6.2 A Positive Oil Price Shock

Figure 7 shows the impulse responses of the model variables to a temporary increase in the oil price of 10 percent. All impulse responses are presented as percentage deviations from steady-state (except the interest rates) and the inflation figures and interest rates are annualised. In the simulation, $\rho_x$ is set to be 0.85 to approximately match the empirical estimates of an AR(1) process of the oil price, implying a temporary but persistent oil price shock.

The oil price shock affects the economy through two channels: (i) a demand channel and (ii) an exchange rate channel. First, through the demand channel, an increase in the oil price will increase the size of the sovereign wealth fund. Through the fiscal rule, this will increase the future transfers to households, raising the present value of future household income and subsequent demand. Second, through the exchange rate channel, a positive oil price shock results in higher net foreign assets in the economy because the size of the sovereign wealth fund increase. This affects the exchange rate by lowering the endogenous risk premium, leading to a nominal exchange rate appreciation.
Figure 7: Impulse responses following a positive oil price shock. All variables (except the interest rates) are percentage deviations from steady-state. Interest rates and inflation figures are annualised.
The impulse responses in figure 7 illustrate these mechanisms. As a result of higher lifetime income, households demand more consumption goods and housing; and they supply less labour. Subsequently, consumption and housing prices increase, and hours worked fall. Furthermore, lower willingness to work makes real wages increase through the wage bargaining process. Households want to smooth the effects of the shock across their lifetime. As a result, after the initial shock to income and the subsequent higher consumption, they will increase their lending abroad, attempting to maintain some of the high level of consumption and resulting in an increase in net foreign debt held by households.

Through the exchange rate channel, higher oil prices result in a nominal exchange rate appreciation. As a result, export prices increase and import prices decrease, and the final goods producer substitutes away from domestically produced intermediate goods toward imports. Furthermore, through the demand channel, higher oil prices result in higher demand, driving up the prices on imports and domestically sold intermediate goods. For import prices, the exchange rate effect dominates, hence following an oil price increase, import prices fall. For domestically sold intermediate goods, higher domestic demand result in higher prices, but the effect is limited because the final goods firm substitutes away from domestically produced intermediate goods. As a result, following higher oil prices, export price inflation increases, import price inflation falls while price inflation on domestically sold intermediate goods increases somewhat. The total effect on consumer prices depends on the relative changes in import prices and the prices on domestically produced intermediate goods. In the benchmark calibration, the effects from lower import prices dominate. Hence, following a positive oil price shock, consumer price inflation falls, however by a small amount.

Following higher oil prices, the volume of exports, imports and domestically sold intermediate goods change as well. Export prices increase following the nominal exchange rate appreciation, resulting in lower exports of intermediate goods. For import prices, the total effect from the oil price shock is lower import
prices which lead to higher imports. Furthermore, higher oil prices result in an increase in domestic demand, further increasing imports. The volume of imports therefore increases significantly. The increase in domestic demand also results in an increase in the volume of domestically sold intermediate goods. The total effect on mainland output is a combination of the changes in consumption and exports, minus the increase in imports. Consumption increases, implying higher output, but, on the other hand, exports fall, resulting in lower output and higher imports. Given these contrary effects, the total effect on mainland output is an increase, however by a small amount.

**Monetary policy response to a positive oil price shock**

The central bank loss function contains consumer price inflation and mainland output, both measured as percentage deviations from their steady-state levels, and an interest rate smoothing term. As explained above, following the positive oil price shock, consumer price inflation falls while output increases. The central bank therefore faces a trade-off since a higher interest rate will reduce the output gap, but at the cost of even lower inflation. The trade-off is implicitly quantified in the loss function and it is the relative effects on consumer prices and output which determine the optimal interest rate path. In the benchmark simulation, the central bank chooses to raise its interest rate somewhat, implying that the increase in output outweighs the decrease in consumer price inflation, before gradually lowering the interest rate. The interest rate response, however, is small.

The impulse responses also reveal that the effects of the oil price shock is very long-lasting. The sovereign wealth fund effectively acts to transform a temporary oil price shock into a near permanent shock to the Norwegian economy. Hence, the impulses to the macroeconomic variables are very persistent, some of them, for example the effect on household debt, lasting more than ten years before being stabilised. Furthermore, the impulse responses reveal that the central bank is unable to stabilise the economy within a short time-horizon since
it cannot, by its interest rate policy, affect the main transmission mechanism of oil price shocks, namely the increase in the sovereign wealth fund and the subsequent transfers to households. As a result, the model suggests that monetary policy has limited abilities to stabilise the economy following an oil price shock.

The numerical impulse responses following a positive oil price shock represent the first key result from this thesis. The model, with a monetary policy regime similar to that of Norges Bank, is able to replicate key features of the Norwegian economy during the last decade. The impulse responses indicate that following higher oil prices, the macroeconomic variables are affected as follows: (i) housing prices increase, (ii) household debt increases, (iii) consumption increases, (iv) output increases, (v) the real exchange rate appreciates, (vi) there is a divergence in the relative prices of the intermediate goods; prices on imports fall while prices on domestically produced goods increase, (vii) real wages increase, (viii) the central bank increases its interest rate, and (ix) the terms of trade is strengthened as a result of lower import prices, higher export prices, and higher oil prices. All these features have been prominent in Norway during the last decade.

6.3 A Negative Foreign Marginal Cost Shock

During the last decade, the Norwegian economy has experienced an increased inflow of low-cost goods from emerging economies. In this section, I study the effects of such a switch toward cheaper imports and simulate a negative shock to the foreign marginal cost. In the simulation, $p_{mc}$ is set to 0.5, implying some persistence in the shock process. The initial shock is set to a 10 percent decrease in foreign marginal costs.

Figure 8 shows the impulse responses of the economic variables from a negative foreign marginal cost shock. The shock affects the economy through import prices. Since it becomes cheaper to produce abroad, the foreign producers lower their prices to maximise profits - resulting in lower import prices in Norway.
Subsequently, the final goods firm will substitute away from domestic intermediate goods toward more imports in its production. As a result, the volume of imports increases while the sale of domestic intermediate goods falls. The initial effect on import prices also affects consumer prices, resulting in lower consumer price inflation.

Lower consumer price inflation affects the households through an income and a substitution effect. First, lower consumer price inflation increases the real income of households, resulting in higher demand for all goods. As a result, consumption is higher, housing prices increase and the supply of labour falls because households desire more leisure. Second, the substitution effect implies that consumption is cheaper relative to housing and leisure. Hence, households want to adjust toward even more consumption relative to housing and leisure. The total effect is dominated by the income effect resulting in more consumption, higher housing prices and lower supply of labour. Furthermore, following the lower labour supply, households are able to increase the real wage through a strengthened wage bargaining position.

Households use foreign assets to smooth their consumption path. As explained above, a negative foreign marginal cost shock results in a period of temporary high income. During this period, households save some of their capital in foreign assets to smooth their consumption. Hence, following a negative foreign marginal cost shock, households temporary increase their holdings of foreign assets. As the effects from the shock gradually retract, the households consume their accumulated assets, lowering their net foreign assets.

Domestic intermediate goods firms face several effects from lower foreign marginal costs. First, households lower their labour supply, resulting in higher real wages and lower domestic intermediate goods production. Higher real wages affect prices on domestic intermediate goods, resulting in higher prices on both domestically sold intermediate goods and exports. Second, there is an increase in consumer demand. As a result, the production of final goods increase, leading to more demand for domestically sold intermediate goods. Third, the lower
Figure 8: Impulse responses following a negative shock to the foreign marginal cost. All variables (except the interest rate) are percentage deviations from steady-state. Interest rates and inflation figures are annualised.
import prices following the initial shock results in a substitution away from domestically sold intermediate goods toward imports. And fourth, the central bank lowers the interest rate,\textsuperscript{26} resulting in a nominal exchange rate depreciation, and subsequently, import prices increase while export prices decrease somewhat. The total effect on the domestic intermediate goods sector is: higher price inflation on exports and lower price inflation on domestically sold intermediate goods, indicating that the substitution effect away from domestically sold intermediate goods dominates. For mainland output, consumption increase, resulting in higher output, however by less than the total effect on consumption because some of the consumption goods are imported. As a result, mainland output increase, however, by a small amount.

**Monetary policy response to a negative foreign marginal cost shock**

Following the foreign marginal cost shock, consumer price inflation is lower while output increases somewhat. Again, the central bank faces a trade-off. Lower consumer prices imply a lower interest rate while higher output implies a higher interest rate. Qualitatively, these are the same effects as the positive oil price shock, but the timing and relative size of the effects have changed. When foreign marginal costs fall, the effects on consumer prices are larger and more pronounced during the first few quarters than the effects on output. By minimising the loss function, the central bank chooses to reduce the interest rate.

The impulse responses to a negative shock to the foreign marginal cost represent the second key result of this thesis. During the last decade, the Norwegian economy has experienced lower import prices as a result of an increased inflow of goods from low-cost emerging economies. The model shows that lower foreign marginal costs affect the economy in key dimensions: (i) housing prices increase, (ii) household debt decreases somewhat, (iii) consumption increases, (iv) output increases, (v) the real exchange rate depreciates, (vi) there is a divergence in the

\textsuperscript{26}The monetary policy response will be further discussed below.
relative prices of intermediate goods, prices on imports fall more than prices on domestically sold goods, (vii) real wages increase, (viii) the central bank lowers its interest rate, and (ix) the terms of trade is strengthened as a result of lower import prices and higher export prices.

7 Result Robustness and Policy Implications

Since the early 2000s, the oil price has increased substantially and there has been an increasing inflow of imports from low-cost emerging economies. The model presented in this thesis provides impulse responses of a positive oil price shock and a negative foreign marginal cost shock that match the evolution of the Norwegian economy during the last decade well. Following these two shocks, the economy should experience an increase in consumption, production, real wages and housing prices, and a fall in the prices on imported goods. Furthermore, assuming that the oil price shock dominates, there should also be an exchange rate appreciation, an increase in the prices on domestically sold intermediate goods and an increase in household debt levels. As a result, by a combination of a positive oil price shock and a negative foreign price shock, the model is able to replicate most major movements in macroeconomic variables of the Norwegian economy during the last decade.

There are, however, some troubling aspects of these results. Note in particular that by studying the impulse responses, it is evident that the shocks to the Norwegian economy are long-lived. The sovereign wealth fund effectively acts to transform temporary changes into near permanent shocks to the Norwegian economy. The monetary policy responses of the central bank have limited effects on the evolution of the macroeconomic variables. This is a result of two features. First, both the oil price shock and the foreign marginal cost shock present the central bank with a trade-off, and as a consequence, monetary policy responses are small. Second, the central bank is unable to address one of the main transmission mechanisms of the shocks, namely the sovereign wealth
fund and the subsequent future transfers to households - perhaps suggesting that fiscal policy is better suited to respond. In this section, I investigate the robustness of the results and discuss the implications for monetary and fiscal policy, respectively.

7.1 Monetary Policy Implications

In the numerical simulations of this thesis, a flexible inflation targeting regime similar to that of Norges Bank has been applied. When responding to the oil price shock and the foreign marginal cost shock, the central bank faces a trade-off due to the fact that output and consumer price inflation move in opposite directions. Furthermore, the impulse responses of output and consumer price inflation are small. Subsequently, the interest rate responses are small and have limited effects on the evolution of macroeconomic variables. In this subsection I investigate the robustness of this result. First, the small monetary policy responses could possibly be an artifact of the calibration of the model. And second, the limited effects of monetary policy could be a result of a wrong monetary policy regime set-up.

7.1.1 Result Robustness

In the model framework in this thesis, the responses of output and consumer price inflation are sensitive to how the model is calibrated. The response of consumer price inflation depends on the relative changes in prices of imports and domestically sold intermediate goods. As a result, the consumer price inflation responses are sensitive to the numerical values given to the stickiness parameters \( \phi^{PM}, \phi^{PM^*} \) and \( \phi^{PQ} \) and the elasticity of substitution between foreign and domestically produced intermediate goods in the production of final goods, \( \mu \). Through different value combinations of these parameters, any qualitative response in the consumer price inflation is possible following the foreign price shocks. However, although a different parameter set-up is able to change the sign of the consumer price responses, the numerical movements will still be
small.

Output is also sensitive to the numerical value given to the elasticity of substitution between foreign and domestically produced intermediate goods in the production of final goods, \( \mu \). An elasticity of substitution lower than 1, for example, indicating that the demand for imports and domestically produced goods change less than one for one with price changes, generally produces a positive output response to a positive oil price shock, while setting it above 1 switches the response. In other words, it is possible to change the sign of the response, but again the numerical responses will be small.

Furthermore, the elasticity of substitution generally affects consumer price inflation and output in opposite directions. A higher \( \mu \) will for example result in lower output and higher consumer price inflation following a positive oil price shock. This is a result of the higher price sensitivity, demanding less price changes to achieve volume changes. Consequently, changes in the elasticity of substitution will generally have small effects on monetary policy as the two effects dampen each other and the trade-off in monetary policy persists. Although the sensitivity of the monetary policy responses are limited with regard to the calibration of the model, the fragile nature of the qualitative responses suggests that the researcher should strenuously seek to find the true parametrisation to allow the model to most closely represent the economy, a path sought in the work with this thesis.

Hence, changes in the calibration of the model may change the central bank responses qualitatively, but will have limited numerical effects on the economy. Whatever the calibration is, trade-offs will be prevalent under flexible inflation targeting and the interest rate responses of the central bank will be small. This leads to the result that if monetary policy is to respond more strongly to foreign price impulses, a change in the monetary policy regime is necessary.
7.1.2 Alternative Monetary Policy Regimes

Under a flexible inflation targeting regime, the central bank puts weight on consumer price inflation and the output gap when setting the interest rate policy. As shown above, trade-offs will be prevalent and the interest rate responses small following changes in the oil price or the foreign marginal cost. A monetary policy regime that explicitly puts more weight on components affected by foreign price shocks in the loss function will respond more strongly to the oil price and foreign marginal cost shocks. In particular, explicitly adding the oil price in the loss function by using the GDP deflator or nominal income as monetary policy target instead of the consumer price index, for example, will result in stronger interest rate responses to oil price shocks. Although these alternative regimes might differ from the direct welfare optimising central bank loss function, when analysing an oil price shock they might fare better in the long run by limiting the level of foreign debt and limiting financial fragility.

Figure 9 demonstrates the impulse responses from a positive oil price shock on the Norwegian economy using two monetary policy regimes, (i) the flexible inflation targeting regime presented in this thesis and (ii) an alternative regime targeting GDP deflator inflation and mainland output. For the reasons laid out above, it is clear that the central bank increases the interest rate to a larger extent when targeting the GDP deflator than the consumer price index. As a result, there are major changes in the responses of macroeconomic variables. In the alternative regime, consumption, housing prices, output, employment and wages drop significantly following the interest rate increase. Furthermore, the exchange rate appreciates almost twice as much. Although these responses seem to represent an overreaction from the central bank because some of the macroeconomic variables are strongly affected without being stabilised, the example suggests that, by putting some weight on a wider price target, the central bank will change its optimal interest rate path, and that it may be possible to achieve a more stable evolution of many macroeconomic variables in the event of an oil price shock.
Figure 9: Impulse responses following a positive oil price shock under two monetary regimes. All variables (except the interest rate) are percentage deviations from steady-state. Interest rates and inflation figures are annualised.
There are, however, two aspects of the impulse responses in figure 9 which cause unease. First, some economic variables are virtually unaltered by the interest rate policy. The inflation rate of export prices, import prices and domestically sold intermediate goods prices change very little when changing monetary policy regime. Second, the long-run path of the macroeconomic variables change very little. Hence, after the initial strong central bank response, the oil price is stabilised after about one year and the difference between consumer price targeting and GDP-deflator targeting gradually disappears. As a consequence, the two regimes start to converge to a common interest rate path. This change in regime is therefore unable to alter the long-run paths of the macroeconomic variables.

The limited long-run effects of a regime change is further a result of the inability of monetary policy to address the transmission mechanism of oil price shocks. When the oil price increases, the sovereign wealth fund accumulates assets and the future transfers to households increase. These are aspects of the economy which monetary policy has limited possibility to address because the domestic interest rate has limited effect on the future return from the sovereign wealth fund. When monetary policy tries to stabilise an economy which experiences shocks and mechanisms that monetary policy cannot address, it will either not respond, or respond by changing the variables it can affect. In other words, monetary policy will either refrain from treating its patient or be limited to marginalise the symptoms, in either case letting the illness persist. For example, when targeting GDP deflator inflation, the central bank will change the components of the GDP deflator it can affect, output and consumer prices. In order to truly stabilise the economy following an oil price shock, economic policy has to address the actual transmission mechanism, which in this case is the fiscal rule of the sovereign wealth fund.
7.2 Fiscal Policy Implications: An Alternative Fiscal Policy Rule

The Norwegian fiscal policy rule can be transformed to eliminate the effects of oil prices on future transfers to households. In order to achieve this, the future transfers from the sovereign wealth fund have to be independent of the current oil price and subsequently be independent of the current size of the fund. In other words, transfers must be determined by an exogenous rule. I suggest one such rule: the value rule.

The value rule states that the transfers from the sovereign fund should be constant, adjusted for consumer prices and productivity. I.e., in nominal terms, the transfers should grow by the consumer price inflation\(^{27}\) and the growth rate of productivity. This ensures that transfers from the sovereign wealth fund are independent of the price of oil and the size of the fund, and as a result, the transmission mechanism in the model of this thesis would be eliminated.

A major problem of this method, however, would be to set the value today and commit to this level in the infinite future. Changing political movements and circumstances could affect the commitment of the government and the value of the transfers should therefore, to the best possible extent, reflect an objectively chosen level rather than being arbitrarily chosen. One such possibility could be to use the permanent income rule, see for example Jafarov and Leigh (2007), where the transfers from the fund is set to the annuity value of net wealth, defined as the sum of net assets and the discounted value of future oil revenue. In the formulation by Jafarov and Leigh (2007), the rule is constructed such that transfers are a constant fraction of real GDP. The permanent income rule is, however, affected by oil price changes because the calculation of future oil revenue depends on the current oil price. Hence, in order to preserve the transfers from the sovereign wealth fund as independent of the oil price, the permanent income rule can only be used to calculate the initial value for the

\(^{27}\)The consumer price index which best represents the costs of the consumption without taking oil into account. In the case of Norway, the CPI-ATE (consumer prices adjusted for taxes and energy prices) represents one such measure.
Naturally, a number of assumptions must be made in order to calculate the annuity value of net wealth. For example, the future path of real returns and productivity growth are needed, in addition to the expected future price and extraction rate of oil. Furthermore, government obligations are likely to increase in the future as a result of increasing pension expenditures and the expectation of a period of restructuring when the oil resource is depleted. I will therefore argue that the estimate should be conservative to allow for some precautionary savings in the fund.

To conclude, I argue that a viable route for the Norwegian economy would be to use the value rule, calculating the initial value by a conservative permanent income rule. This would ensure that the effects of oil price changes are limited. Furthermore, to allow for some readjustments, I suggest that the value is recalculated by the permanent income rule at fixed time intervals, for example each decade, thus ensuring that the transfers from the oil fund will, in the long run, be a constant fraction of real GDP. In addition, it will enable monetary policy makers to easier achieve price and output stability, in addition to financial stability.

Although this rule succeeds in making transfers from the sovereign wealth fund independent of the current oil price, there may be other considerations which are more pressing. For example, such a change in rule could have important implications for wider aspects of fiscal policy such as taxation, or there could possibly be important intergenerational distributional and demographic aspects of such a change. These problems, however, are beyond the scope of this thesis.

8 Conclusion

The main innovation of this thesis is to model the effects of foreign price changes in a DSGE-model for a small commodity exporting economy with a sovereign

value rule.
wealth fund. In particular, I have analysed the effects of an oil price shock on the Norwegian economy through the transfers to households from the sovereign wealth fund. Following higher oil prices, the sovereign wealth fund increases, subsequently increasing the future transfers from the fund. The fund therefore effectively acts to transform temporary price changes into near permanent shocks to the Norwegian economy.

A first key result of this thesis is that the model is able to replicate key features of the Norwegian economy during the last decade. Since the early 2000s, two foreign price shocks have been prominent in Norway, (i) the oil price has almost quadrupled since 2004, and (ii) there has been a sharp increase in imports from low-cost emerging markets, resulting in lower prices on imports. Using a combination of these two shocks, the model is able to reproduce major movements in macroeconomic variables. In particular, since year 2000, there has been an increase in consumption, real wages, housing prices and household debt, a real exchange rate appreciation, and a divergence in the relative prices of intermediate goods - import prices have fallen while domestic prices have increased. All these features was replicated by the model.

As a second key result, the model shows that monetary policy under flexible inflation targeting, where the central bank’s main concern is to stabilise consumer price inflation and output, is unable to fully stabilise the economy following foreign price shocks. This is a result of two factors: there is a trade-off between the two target variables and the movements in these target variables are small. As a consequence, the interest rate response of the central bank is very limited. This result is robust to the model calibration.

Other monetary policy regimes could ensure stronger interest rate responses. For example, GDP deflator targeting implicitly targets the oil price because the oil price enters as a factor in the calculation of the GDP deflator, and effectively ensures that the central bank responds strongly to oil price movements. The comparison of different monetary policy regimes, however, reveals a further result of this thesis: namely that monetary policy is unable to stabilise the long-
run movements in macroeconomic variables following oil price shocks because it cannot address the main transmission mechanism, the sovereign wealth fund. The best monetary policy makers can do is therefore to dampen the symptoms, but not address the actual transmission mechanism. In order to truly address the effects of oil price shocks on the Norwegian economy, the fiscal rule for the sovereign wealth fund has to be transformed. A new fiscal rule, where the transfers from the sovereign wealth fund grow by the rate of productivity and price inflation, can eliminate the transmission channel from oil price shocks to the real economy and therefore effectively stabilise the economy following oil price movements.

The model in this thesis only models one explicit aspect of foreign price shocks, the transmission through the sovereign wealth fund. A possible extension is to model the effects of oil prices through the wage formation in the oil exporting sector, affecting the wage formation in the whole economy. This channel could be more short-lived and temporary. Furthermore, if wage stickiness is introduced as well, there is possibly a more appropriate and effective role for monetary policy; however beyond the scope of this thesis.
References


A Yaari-Blanchard Budget Constraint

The insurance company provides fair (competitive) insurance against passing away with positive wealth. The insurance company receives all assets of those who die and pays a fair premium to all who survive.

In a competitive environment, the profits for the insurance company is zero:

\[ Div^{ins}_t = \nu \text{Prem}_t - (1 - \nu)(R_{t-1})A_{t-1} = 0 \]

The competitive no-profit solution for the insurance premium is then \( \text{Prem}_t = \frac{1 - \nu}{\nu} R_t A_t \). Inserting this into the individual budget constraint yields:

\[
C_{s,t}(j) + B_{s,t}(j) + S_t B^*_s(j) + P_{h,t} H_{s,t}(j) \leq \\
\frac{P_{h,t} H_{s,t-1}(j)(1 - \delta_h)}{1 - \nu} + \frac{R_{t-1} B_{s,t-1}(j)}{\Pi_t(1 - \nu)} + \frac{S_t R_{t-1} B^*_s(j)}{\Pi_t^*(1 - \nu)} + \\
W_t N_{s,t}(j) + Div_{s,t}(j) + T_{s,t}(j)
\]

B Individual Dynamic Consumption

From the individual optimisation problem, we have the following budget constraint and euler-equation:

\[
C_t + B_t + S_t B^*_t + P_{h,t} H_t \leq \\
\frac{P_{h,t} H_{t-1}(1 - \delta_h)}{1 - \nu} + \frac{R_{t-1} B_{t-1}}{\Pi_t(1 - \nu)} + \frac{S_t R_{t-1} B^*_t}{\Pi_t^*(1 - \nu)} + \\
W_t N_t + Div_t + T_t
\]

and

\[
1 = \beta R_t E_t \left[ \frac{C_t}{C_{t+1}} \frac{1}{\Pi_{t+1}} \right]
\]

Simplifying the equations by noting that \( B_t = 0 \ \forall \ t \) because all households
are the same, denoting \( Inc_t = W_t N_t + Div_t + T_t \), using non-satiation to get equality in the budget constraint, and rewriting the euler-equation yields:

\[
C_t + S_t B^*_t + P_{h,t} H_t = \frac{P_{h,t} H_{t-1} (1 - \delta_h)}{1 - \upsilon} + \frac{S_t R^*_{t-1} B^*_{t-1}}{\Pi^*_t (1 - \upsilon)} + Inc_t
\]

(60)

and

\[
E_t C_T = \beta^{T-t} E_t R_t, T C_t \frac{1}{\Pi_t, T}
\]

(61)

Now, iterating the budget constraint forward yields:

\[
0 = S_t R^*_{t-1} B^*_{t-1} \frac{(1 - \upsilon)^T \Pi^*_t}{\Pi^*_t (1 - \upsilon)} C_T + \sum_{T=t}^{\infty} \frac{(1 - \upsilon)^T \Pi^*_t}{R^*_t, T S_t, T} Inc_T + \lim_{T \to \infty} \frac{(1 - \upsilon)^T \Pi^*_t}{R^*_t, T S_t, T} S_{T+1} B^*_{T+1} + \sum_{T=t}^{\infty} \frac{(1 - \upsilon)^T \Pi^*_t}{R^*_t, T S_t, T} I_t + \Pi^*_t (1 - \upsilon) P_{h,t} H_{T-1} (1 - \delta_h)
\]

(62)

Using the euler-equation to substitute for the \( C_T \), renaming the expressions

\[
I_t = \sum_{T=t}^{\infty} \frac{(1 - \upsilon)^T \Pi^*_t}{R^*_t, T S_t, T} Inc_T \quad \text{and} \quad H^*_t = \sum_{T=t}^{\infty} \frac{(1 - \upsilon)^T \Pi^*_t}{R^*_t, T S_t, T} \left( P_{h,t} H_{T-1} (1 - \delta_h) \right) - \lim_{T \to \infty} \frac{(1 - \upsilon)^T \Pi^*_t}{R^*_t, T S_t, T} S_{T+1} B^*_{T+1}
\]

and assuming \( \lim_{T \to \infty} \frac{(1 - \upsilon)^T \Pi^*_t}{R^*_t, T S_t, T} S_{T+1} B^*_{T+1} = 0 \), the result is

\[
C_t = [1 - \beta (1 - \upsilon)] \left( \frac{S_t R^*_{t-1} B^*_{t-1}}{\Pi^*_t (1 - \upsilon)} + I_t + H^*_t \right)
\]

(63)

### C Aggregate Dynamic Consumption

From the aggregate version of the household optimisation problem, we have the following budget constraint and dynamic consumption equation:

\[
C_t + P_{h,t} H_t + B^*_{H H t} + S_t B^*_{H H t} = P_{h,t} H_{t-1} + \frac{B^*_{H H t-1} R_{t-1}}{\Pi_t} + \frac{S_t R^*_{t-1} B^*_{H H t-1}}{\Pi_t} + W_t N_t + Div_t + T_t
\]

(64)
\[ C_t = [1 - \beta(1 - \nu)]\left[\frac{S_tR^*_{t-1}B_{HH,t-1}^*}{\Pi_t^*} + I_t\right] \quad (65) \]

Note that \( B_{HH,t}^* = 0 \), the net holding of domestic assets for households is zero. Now, using \( Inc_t = W^tN_t + Div_t + T_t \), assuming that the housing stock is constant, and iterating the budget constraint one step forward yields:

\[ S_tB_{HH,t}^* = \frac{S_tR^*_{t-1}B_{HH,t-1}^*}{\Pi_t^*} + Inc_t - C_t \quad (66) \]

Now, iterating the aggregate dynamic consumption equation one step forward yields:

\[ C_{t+1} = [1 - \beta(1 - \nu)]\left[\frac{S_{t+1}R^*_{t}B_{HH,t}^*}{\Pi_{t+1}^*} + I_{t+1}\right] \quad (67) \]

Using the budget constraint to substitute in for \( B_{HH,t}^* \) gives:

\[ C_{t+1} = [1 - \beta(1 - \nu)]\left[\frac{S_{t+1}R^*_{t}R^*_{t-1}B_{HH,t-1}^*}{\Pi_t^*\Pi_{t+1}^*} + \frac{S_{t+1}R^*_{t}Inc_t}{S_t\Pi_{t+1}^*} - \frac{S_{t+1}R^*_{t}C_t}{S_t\Pi_{t+1}^*} + I_{t+1}\right] \quad (68) \]

which simplifies to:

\[ C_{t+1} = \frac{S_{t+1}R^*_{t}R^*_{t-1}B_{HH,t-1}^*}{S_t\Pi_{t+1}^*} + \frac{S_{t+1}R^*_{t}Inc_t}{S_t\Pi_{t+1}^*} - \frac{S_{t+1}R^*_{t}C_t}{S_t\Pi_{t+1}^*} + I_{t+1} \quad (69) \]

where \( I_t = I_{t+1} + \frac{s_{t+1}R^*_{t}Inc_t}{S_t\Pi_{t+1}^*} \).

Now, using the aggregate dynamic consumption equation at time \( t \) and multiplying by \( \frac{s_{t+1}R^*_{t}}{S_t(1 - \nu)\Pi_{t+1}^*} \) gives:

\[ I_t' = \frac{s_{t+1}R^*_{t}}{S_t\Pi_{t+1}^*(1 - \nu)(1 - \beta(1 - \nu))} C_t - \frac{s_{t+1}R^*_{t}R^*_{t-1}}{\Pi_t^*\Pi_{t+1}^*(1 - \nu)} B_{HH,t-1}^* \quad (70) \]

Using this to replace for \( I_t' \) in equation (69) yields:

63
\[ C_{t+1} = \frac{1 - \beta(1 - v)}{v + \beta(1 - v)^2} E_t \{ D_{t+1} \Pi_{t+1} C_{t+1} \} + \frac{1 - \beta(1 - v)}{v + \beta(1 - v)^2} \frac{S_{t+1} B_{HH,t-1}^t}{S_t \Pi_{t+1}^t} \]

Solving this for \( C_t \) yields:

\[ C_t = \frac{1 - \beta(1 - v)}{v + \beta(1 - v)^2} E_t \{ D_{t+1} \Pi_{t+1} C_{t+1} \} + \frac{1 - \beta(1 - v)}{v + \beta(1 - v)^2} \frac{S_{t+1} B_{HH,t-1}^t}{S_t \Pi_{t+1}^t} \]

### D Optimal Price Setting in the Intermediate Goods Sector

In this appendix, I present the derivation of the first order condition for price setting of the domestic price, \( P_{Q,t} \). The intermediate goods firm, \( i \), produce under monopolistic competition and quadratic price adjustment costs, \( Q_{PM}^i \). The demand for the domestic intermediate goods of the individual firm is assumed to follow a CES aggregate:

\[ Q_t(i) = \frac{P_{Q,t}(i)}{P_{Q,t}} Q_t \]  

where \( Q_t(i) \) is the goods sold domestically by firm \( i \) and \( P_{Q,t}(i) \) is the price charged by firm \( i \). Furthermore, production by firm \( i \) is given by:

\[ Y_t'(i) = Z_t N_t(i) = Q_t(i) + M_t'(i) \]

i.e. the produced goods \( Y_t'(i) \) is equal to goods sold \( (Q_t(i) + M_t'(i)) \). Profits for firm \( i \) at time \( t \) is then given by:
Hence, profits are the sum of sales, $PQ_t(i)Q_t(i) + PM^*_t(i)M_t^*(i)$, minus production costs, $W_tN_t(i)$, and the price adjustment costs, $\zeta_{t}^{PQ}(i)P_{Q,t}Q_t - \zeta_{t}^{PM^*}(i)P_{M^*,t}M_t^*$. Due to the price adjustment costs, the price setting today matters for price setting tomorrow. The firm therefore has to take dynamic effects of price setting into account and set prices to maximise the discounted sum of profits:

$$
\max_{P_{Q,t}(i)} \sum_{t=0}^{\infty} \left\{ \frac{P_{Q,t}(i)}{P_{Q,t}^*} Q_t \left[ W_t - \frac{P_{Q,t}(i) - Q_t}{P_{Q,t}^*} \right] - \frac{\phi_{t}^{PQ} P_{Q,t}(i)}{2} \right\} + f(M^*)
$$

where $f(M^*)$ is some function not depending on $P_{Q,t}(i)$. The first order condition with respect to $P_{Q,t}(i)$ is then:

$$
0 = (1 - \theta) \frac{P_{Q,t}(i) - Q_t}{P_{Q,t}^*} - \theta \frac{W_t}{Z_t} \frac{P_{Q,t}(i) - Q_t}{P_{Q,t}^*} - \phi_{t}^{PQ} \frac{P_{Q,t}(i)}{P_{Q,t}^*} + \theta \frac{W_t}{Z_t} \frac{P_{Q,t}(i) - Q_t}{P_{Q,t}^*} - \phi_{t}^{PQ} \frac{P_{Q,t}(i)}{P_{Q,t}^*}
$$

Using the fact that all firms are equal and face the same constraints, $P_{Q,t}(i) = P_{Q,t}$ and $P_{Q,t+1}(i) = P_{Q,t+1}$. Furthermore, using the fact that $mc_t = W_t$ and divide through by $Q_t$ yields:

$$
\frac{\theta mc_t}{P_{Q,t}Z_t} = \theta - 1 + \phi_{t}^{PQ} \frac{\Pi_{Q,t}}{\Pi_{Q,t+1}} - \left[ \frac{\Pi_{Q,t}}{\Pi_{Q,t+1}} \right]
$$

$$
-2E_t \{ D_{t+1} \phi_{t}^{PQ} P_{Q,t+1} Q_t \Pi_{Q,t+1} \left[ \frac{\Pi_{Q,t+1}}{\Pi_{Q,t}} \right] - \left[ \frac{\Pi_{Q,t+1}}{\Pi_{Q,t}} \right] \} = \theta - 1 + \phi_{t}^{PQ} \frac{\Pi_{Q,t}}{\Pi_{Q,t+1}} - \left[ \frac{\Pi_{Q,t}}{\Pi_{Q,t+1}} \right]
$$

where $(77)$ and $(78)$.
Now, solving this for $P_{Q,t}$ yields the equation for optimal price setting:

$$P_{Q,t} = \theta \text{mc}_{t} \left[ (\theta - 1) + \phi_{PQ} \frac{\Pi_{Q,t}}{\Pi_{Q,t-1}} \left( \frac{\Pi_{Q,t}}{\Pi_{Q,t-1}} - 1 \right) \right.$$

$$\left. - \phi_{PQ} E_t \left\{ D_{t+1} \frac{Q_{t+1}}{Q_t} \frac{\Pi_{Q,t+1}}{\Pi_{Q,t}} \left( \frac{\Pi_{Q,t+1}}{\Pi_{Q,t}} - 1 \right) \right\} \right]^{-1}$$

(79)