ON COLUMN SLENDERNESS LIMITS

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Content

1 Introduction

Presently, revision work on the European Standard EN 1992-1-1 /1/, also referred to as Eurocode 2 or simply EC2, is in progress. The relevant institutions in the participating countries follow up this work, and present their comments etc. as the process goes on. In Norway, this activity is the responsibility of the Norwegian Council for Building Standardization (NBR). The writer has for a number of years had an interest in design of slender compression members and slenderness limit formulations, and has undertaken to provide views and comments to NBR. The topics dealt with here covers both lower and upper slenderness limit formulations as well as actual limits. Limits are proposed based on numerical studies that considers the nonlinear behaviour of r.c. compression members.

2 Comments on slenderness limits for EC2

2.1 Existing slenderness limit proposals

In Section 5.8.2 of the first draft for EN 1992-1-1/1/, a lower slenderness limit is defined in terms of the geometrical slenderness $\lambda = l_0/i$, where l_0 is the buckling, or effective, length and i the radius of gyration. The lower slenderness limit, below which second order effects can be neglected, is given as

$$lim\lambda = \lambda_0(2 - r_1) \qquad where \qquad \lambda_0 = 7.5/\sqrt{n} \leq 12 \tag{1}$$

where, for convenience of writing and to simplify later discussion, r_1 is introduced here to define the moment ratio, i.e.

$$r_1 = M_1^0/M_2^0 \tag{2}$$

Further, $n = N/f_{cd}A_c$, and M_1^0 and M_2^0 are first order moments at column end 1 and 2, respectively. End 2 is the end with the numerically largest moment.

For non-sway members in single curvature bending (tension on the same side), the ratio should be introduced with a positive value, and for double curvature bending with a negative value.

For sway members, the moment ratio $r_1 = 1$ should be used.

The similar limit given in MC90/4/ may be written

$$lim\lambda = \lambda_0(2 - r_1) \qquad where \qquad \lambda_0 = 7.5/\sqrt{n} \geq 12 \tag{3}$$

This limit is identical to that in Eq. 1 except for the inequality (smaller and greater than 12, respectively).

A lower slenderness limit proposed by Westerberg/2/ is given by

$$lim\lambda = 5(5 - \phi_{ef})(1.5 - r_1)/\sqrt{n} \tag{4}$$

where ϕ_{ef} is the effective creep ratio. To facilitate discussion, the expression above may be rewritten, in order to more clearly reflect the individual influencing effects, as follows:

$$lim\lambda = 12.5(1 - 0.2\phi_{ef})(3 - 2r_1)/\sqrt{n} \tag{5}$$

Here the influence of creep is reflected by $k_\phi = 1 - 0.2\phi_{ef}$ and of a linearly varying first order moment along the column by $k_m = 3 - 2r_1$.

Comments

The upper limit of 12 on λ_0 in Eq. 1 seems to be very strict, and represents a significant deviation from the similar limit given in MC90. A reconsideration is probably appropriate (possibly a typing mistake has been made?).

The moment ratios above,Eq. **??**, are intended to be, it is believed, between 1st order moments that include the effect of imperfections. It is questioned whether the complications of including imperfection effects in the moment ratio is warranted. This aspect is commented on further in Section 2.5 (where ratios between end moments for simplicity are labelled r_1, as above, when imperfections are included, and r_0 when they are not included).

Although not specifically stated in the draft, it is believed that the applicability to non-sway members of the slenderness limits above, are limited to such members without tranverse loading. This should be clearly stated. The case of non-sway members with transverse loading is not mentioned, but ought to be. It is believed that this case can be covered in the same way that sway members were, i.e. by setting $r_1 = 1$.

The neglect of a reinforcement term, the effect of creep and moment gradient and how these effects are accounted for in the slenderness formulations above, are considered in the following.

2.2 Reinforcement

The effect of reinforcement is not included in the proposals above, even though the reinforcement has a strong effect. Not as much as the axial load, but still a substantial effect.

It has been argued that it is unpractical to account for reinforcement since it is not known at the time when slenderness criteria are being checked /2/. This may not be a good enough argument for leaving it out. In a good many instances in reinforced concrete design, it is necessary to make some assumption at some stage in the design process. If the assumption turns out to be unconservative it is necessary to go back and check consequences.

Also, the designer can always insert minimum reinforcement values if he has no better clue as to an approximate amount. However, fairly early in the design process the designer will know fairly well the approximate reinforcement requirements. A slenderness limit that also reflects reinforcement effects will then be a most useful tool.

It is believed that the designer will be well served with a slenderness limit that is reasonably transparent with respect to all major influencing parameters, as long as the formulation does not become too complex.

A slenderness limit that reflects the reinforcement effect has been in use in Norway the last 10 years, since the introduction of NS 3473/10/. To the writers knowledge, there has been no negative feedback with regard to complexity. Indeed, rather the opposite. The advantage of a more representative limit is considered to more than balance a minor increase in complexity. Also, such a limit has been found to be a useful tool in the preliminary design process. For instance, for estimating whether a chosen cross-section and reinforcement will require 2nd order effects to be accounted for, or neglected.

Conclusion/Recommendation

It will be useful to include the effect of reinforcement on the column stiffness, and it is recommended that this effect be included in the lower slenderness limit formulation.

Limits that reflects both axial force and reinforcement are proposed in Section 4.

2.3 Creep

The limit proposed by Westerberg, Eq. 4, includes an effect of creep. He suggests that the creep parameter may either be included with some "default value", and then provide a formulation in which the creep effect is not "visible", or retain the creep parameter in the formulation in order to avoid too conservative limits for cases with little creep /2/.

In principle, the writer prefers the formulation to be transparent with regard to the important factors affecting the problem. The question is, however, if creep is an important parameter at low slenderness values. If it is not, a formulation explicitly reflecting a creep parameter should be avoided. Thereby the designer will neither have to calculate creep factors nor moments in order to arrive at the effective creep factor.

There is various opinions on the magnitude of creep effects. The method adopted in the computations on which Westerberg based his proposal is not given. However, the effect reflected in his formulation seems quite strong. Stronger than expected. It may therefore be of interest to compare with other results.

Fig. 1 shows results obtained by K. Aas-Jakobsen /6/ using finite element analyses. The same results are also given in CEB/FIP Buckling Manual /5/. All necessary information, including the loading history, is given in the figure. The condition analysed is rather severe since the axial load that is sustained also is the axial load at failure. For a cantilever column, as in this case, the moment ratio should be taken equal to 1.0 in the lower limits above. Then, for $n = 0.4$ and $\omega_t = 0.4$ in Fig. 1c, $\lambda = 19.8$ and $l_0/h = 5.7$ are obtained from Eq. 5 with $\phi_{ef} = 0$. For this slenderness, Fig. 1 indicates completely negligible creep effects. Comparison with Fig. 1b, Eq. 5 with $\phi_{ef} = 0$ and limλ=20/0.29=69 yields a relative axial load of $n = 0.03$. At this level it is seen, Fig. 1b, that creep effects again are negligible.

It should be noted that no effects of continued hydration following loading, and therefore no concrete strength increase beyond that at loading, have been included in the results of Fig. 1. Had even a modest strength increase been included, it would have more than cancelled creep effects up to values of l_0/h and n considerably in excess of those computed in the comparisons above.

The effects of the combination of a modest strength increase of 30 percent and creep during sustained loading are shown in Fig. 2 for an unrestrained column subjected to constant and equal end eccentricities /7/. The effect of creep was accounted for by the rate of creep method. The material law included both the detrimental effects of high sustained stresses and the beneficial effects of strength increase due to continued hydration. The nonlinear creep law adopted in the time propagation analysis yielded a creep factor of about 2.5 in the linear creep-stress range, and larger values at higher stresses. At low sustained load levels, continued hydration is seen to more than cancel creep effects, even for the very slender and lightly reinforced column labelled S8 in the figure.

The vertical lines that correspond to a limit labelled $\lambda_{NL} = 10$ in the figure will be discussed in more detail later. They correspond to a level below which, it will be proposed, it is not necessary to account for creep effects. The limits given by Eq. 3 and Eq. 5 are not indicated in the figures, but fall well below (to the left of) the indicated "creep limits". In other words, for columns at these limits the creep effects are more than cancelled by the modest strength increase. However, even without such a strength increase, the effect of creep would have been negligible at the lower slenderness limits, similarly to that seen in Fig. 1.

It is emphasized that the studies referred to above were of unrestrained columns. Creep effects in such columns are generally more severe than creep effects in columns of most frame structures. Columns in frames will typically be restrained at column ends by beams or foundations. Due to sustained loading, creep will normally lead to a moment transfer from the column to the restraining elements. Both experimental and theoretical evidence /8,9 and others/ indicate that this in itself will strengthen the columns compared to the unrestrained column case. This further strengthens the argument that creep effects can be neglected for columns at the lower slenderness limits. Indeed, in a great many cases, creep effects can be neglected at levels considerably beyond these limits.

Conclusion/Recommendation

Based on the comparisons and arguments presented above, it would appear that lower slenderness limits, below which 2nd order effects can be neglected, can be computed without considering creep effects.

Also, it is proposed that a more relaxed limit be established that indicates when it is necessary to consider creep (Section 4.3).

2.4 Moment gradient

The effect of 1st order moment gradients on an end loaded, perfectly straight, elastic column has been computed and is presented in Fig. 3. Column ends are denoted A and B. The 1st order end moments for this column, without imperfections, are denoted M_{0A} and M_{0B}. The numerically largest 1st order end moment is that at end B. The ratio between these moments is for convenience denoted r_0.

Thus,

$$r_0 = M_{0A}/M_{0B} \tag{6}$$

Results are shown both for an unrestrained (pin-ended) column and a restrained column, pinned at one end and restrained by a very stiff beam at the other end. The curves labelled "5" and "10" percent correspond to the limits at which the total moment (that includes 2nd order effects) is equal to 1.05 and 1.10 times the largest 1st order end moment, repectively. Results are given in terms of the values of the unrestrained column in single, symmetrical curvature ($r_0 = 1$).

Near $r_0 = 1$ it can be seen that there is very little difference between the unrestrained and the restrained column response. However, as the moment gradient increases, the difference becomes significant. For columns with different end restraint conditions, the moment gradient effect is not as strong as it is for columns with equal end restraints. Upper bounds are obtained for the unrestrained, pin-ended column ("pinned/pinned"), and lower bounds for the column with zero restraint at one end and an approximately infinite rotational restraint at the other end. The latter restraint combination will for convenience be referred to as "pinned/fixed"-restraint.

The difference between the gradient effects at the 10 and 5 percent limits are seen to be quite significant.

The moment gradient effects reflected in the slenderness proposals above, Eq. 1, Eq. 3 and Eq. 5, are also included in the figure and denoted k_m. The MC90 assumption, $k_m = 2 - r_0$ (when $r_1 = r_0$), is a reasonably close approximation of the lower 10 percent bound. Westerberg's assumption, $k_m = 3 - 2r_0$, is, on the other hand, close to the upper 5 percent bound. It is in other words very unconservative when the moment gradient effect is seen in isolation.

The lower curve, $k_m = 1.8 - 0.8r_0$, represents the gradient effect included in the Norwegian slenderness limit/10/. It was chosen rather conservatively, in part in order to allow the moment ratio to be based on 1st order end moments without imperfection effects included/11/.

It is not sufficient, of course, to consider the moment gradient effect in isolation. It must be seen in combination with the assumed base value at $r_0 = 1$. For r.c. columns, this base value varies considerably with level of axial force and reinforcement. This will be discussed in more detail later in conjunction with Figs. 5-9.

2.5 Moment gradient–Imperfections/Notation

In principle, the moment ratio in the lower slenderness limit formulations should be calculated with 1st order end moments that include imperfections. The additional load eccentricities, to account for imperfections(e_a), must be added such as to produce the algebraically largest possible moment ratio.

However, it is also possible to account for imperfections implicitly by prescribing a slightly conservative effect of moment gradients, as illustrated in Fig. 4.

In the figure, ratios of end moments at end A and B (with the numerically larger moment) are defined by either

$$r_1 = M_{1A}/M_{1B} \qquad or \qquad r_0 = M_{0A}/M_{0B} \tag{7}$$

where the following **moment notation** has been adopted:

a. M_0 – 1st order moments **without** effects of imperfections included ,i.e. moments based on the intended structural geometry.

b. M_1 – 1st order moments **with** effects of imperfections included ,i.e. moments that include effects of unintentional deviations from the intended structural geometry (these moments are denoted M^0 in the present EC2 draft/1/).

c. (M_2 – 2nd order moments (included for the sake of completeness))

Since r_1 includes the effect of imperfections, it will be algebraically larger than r_0 and thus give smaller k_m-values than r_0. Then, in order to obtain the same moment gradient effect for the two definitions, it is seen in the figure that the curve given in terms of r_0 must be rotated (about the base value) slightly downwards compared to the curve given in terms of r_1.

It is probably quite clear that an expression in terms of r_0, not considering imperfections explicitly, would be preferable from a designers point of view as it does not require any efforts with regard to imperfections. Considering that an effect of imperfections easily can be accounted for implicitly, as described above, and that the formulation at any rate will be approximate, there is a strong argument for choosing the simplest form.

Possibly the main objection against the moment ratio definition in terms of r_0, is that cases involving very small eccentricities are not covered particularly well. One way of dealing with this may be to include a provision that requires that the end moment ratio be taken equal to 1.0 if the largest first order end moment is less than a certain value (equal to that due to a specified eccentricity). The disadvantage of such a provision is that it introduces a discontinuity.

An alternative approach, that also is quite simple and that does not introduce any discontinuity, is to adopt a definition in terms of r_1 where an additional moment is added at each end of the member such that the line of thrust remains parallell to the nominal line of thrust. This gives

$$r_1 = \frac{M_{0A} + Ne_{add}}{M_{0B} + Ne_{add}} \tag{8}$$

where Ne_{add} is to be included with the same sign as the numerically larger moment M_{0B}, and where M_{0A} is to be included with the same sign as M_{0B} when giving tension on the side of the member as M_{0B}, and with the opposite sign otherwise. The additional end eccentricity, e_{add}, needs not be the same as the specified imperfection or the minimum eccentricity. It should be chosen such as to reflect what may be considered to be reasonable uncertainties in end moment ratios at small nominal eccentricities. The actual value to be adopted needs to be considered further, but possibly a constant value in the range $h/20$ to $h/10$ may be appropriate.

Conclusion/Recommendation

Based on the arguments given above, it is recommended that the effect of a moment gradient in the lower slenderness limit (for a non-sway column without transverse loading) be allowed to be based on 1st order moments without imperfections included, i.e., to be based on r_0, *but with some restriction at small end moment values.* The imperfection effect is instead to be incorporated implicitly, in the slenderness limit value itself. *Alternatively, it is recommended that a simple* r_1*-definition be adopted, such as for instance defined by Eq. 8.*

For the sake of precision, ease of formulation and communication, it is strongly recommended that two separate symbols be used for 1st order moments, i.e. M_0 and M_1 as defined above. The moment type is in both cases identified by a subscript, as is also the case for the 2nd order moment (M_2).

In the present EC2 draft/1/, a superscript 0 is used to indicate 1st order moments (with imperfection effects included), while a subscript 2 is used for 2nd order moments. Use of either subscripts only or superscripts only would be more consistent, and preferable. With regard to subscripts, this would, in the writer's opinion, be the case even in a combination with additional subscripts that sometimes need to be added, like in M_{0Sd} or M_{1Sd}.

3 Slenderness parameters

3.1 General

Slenderness, and slenderness limit formulations, can be given in various forms. So-far, it has been most common to give slenderness limits relative to the geometrical slenderness λ or l_0/h, where l_0 is the buckling length or effective length (and often denoted l_e).

In an elastic 2nd order analysis, the sensitivity to 2nd order effects is, as is well known, a function of the parameter $l\sqrt{N/EI}$ in addition to the boundary conditions. Here l is the column length, N the axial force and EI the cross-sectional stiffness. In the trigonometric solution to the differential equation, this parameter represents the column length. In approximate analyses, where the effect of boundary conditions can be accounted for through the buckling length, the parameter can be replaced by $l_0\sqrt{N/EI}$ or $\lambda\sqrt{N/EA}$. These can very appropriately be termed **normalized slenderness** parameters.

As it is such a parameter, reflecting the combined effect of length, axial load and section stiffness, that is of importance for 2nd order effects, and not the individual parameters themselves, it would seem like a very sensible, and not least rational, choice to give slenderness limits in terms of such a "combination" parameter, suitably modified to be of use for r.c. compression members.

In a study on slenderness bounds in 1983, Menegotto/15/ discussed what might be an appropriate parameter in slenderness bounds. He suggested the parameter $\lambda\sqrt{n}$. If one wanted to include the effect of reinforcement, he proposed to base the radius of gyration (inertia), in the λ-definition, on the transformed, uncracked cross-section with a modular ratio of 20. However, in the ultimate limit state, this approach significantly underestimates the effect of reinforcement. Another study /16/ that included the effect of both load and reinforcement, included these parameters in the limit or bound and not in the slenderness parameter itself. Also in that study, the effect of reinforcement was rather underestimated.

3.2 Normalized slenderness of r.c. member

Motivated by the studies mentioned above, the writer proposed /11/ a **normalized slenderness** defined by

$$\lambda_N = \lambda\sqrt{\frac{n}{1 + k_t\omega_t}} \tag{9}$$

The effect of the reinforcement on stiffness is reflected in the term $k_t\omega_t$. The factor k_t is a new factor that is a function of a number of variables, including concrete and steel properties, cross-section and the placing of reinforcement. But simplifications are warranted, not least with respect to the concrete strength, modulus and concrete stress-strain diagram shape in general/14/.

A good representation of the factor k_t is given by the expression

$$k_t = 2.1\left(\frac{i_s}{i_c}\right)^2 \frac{0.0025}{\epsilon_{yd}} \tag{10}$$

Here, ϵ_{yd} is the strain at the design yield stress f_{yd}. (Alternatively, the influence of steel quality may be taken into account in a slightly different form presented in /11/).

The other parameters involved,

$$\lambda = \frac{l_0}{i_c}, \quad i_c = \sqrt{\frac{I_c}{A_c}}, \quad i_s = \sqrt{\frac{I_s}{\sum A_s}}, \quad n = \frac{N}{f_{cd}A_c}, \quad \omega_t = \frac{\sum f_{yd}A_s}{f_{cd}A_c} \tag{11}$$

are well known parameters that are used also in other contexts: l_0 is the buckling (or effective) length, N the (factored) design value of the axial load, i_c the elastic radius of gyration (inertia) based on the second moment of area I_c (about the centroid) and the area A_c of the concrete cross-section. Further, i_s, I_s and $\sum A_s$ are the similar parameters for the total reinforcement.

The design yield strain may be included explicitly in the expression as shown above, or a reasonably conservative value may be introduced in order to simplify the expression. A sufficiently conservative value would be $\epsilon_{yd} = 0.0025$. Then,

$$k_t = 2.1\left(\frac{i_s}{i_c}\right)^2 \tag{12}$$

It may also be convenient to define a **reference cross-section** and establish a reference k_t. Here, the reference section is taken as a rectangular section with reinforcement placed symmetrically in the two faces perpendicular to the plane of bending and at a distance $h' = 0.8h$ apart. For this case, $i_s = 0.5h' = 0.4h$ and

$i_c = 0.29h$. With these values, $k_t = 4$ is obtained. Eq. 12 can then be written in the following form ,

$$k_t = 4k_a \qquad where \qquad k_a = 0.52\left(\frac{i_s}{i_c}\right)^2 \tag{13}$$

where $k_a = 1$ is obtained for the reference case.

The factor k_a may be seen as an "adjustment" or" arrangement" factor that adjusts for reinforcement arrangements and cross-section shapes that deviates from the reference case.

For the standard cases of rectangular or circular cross sections with $h' = 0.8h$, k_a is normally 1 or 2/3. For rectangular sections with corner reinforcement, or reinforcement placed as described for the reference case, $k_a = 1$. For rectangular sections with distributed and equal amount of reinforcement in the four faces, $k_a = 2/3$. The same value is obtained for circular sections with distributed reinforcement.

Considering that an inaccuracy in the k_t - value in Eq. 9 does not affect λ_N all that much, there is room for further simplifications. In NS 3473:1998/10/, the normalized slenderness has been defined by

$$\lambda_N = \lambda\sqrt{\frac{n}{1 + 4\omega_t}} \tag{14}$$

where "the reinforcement area can be entered with its full value for rectangular sections with corner reinforcement or reinforcement distributed along the faces perpendicular to the direction of the displacement. For other cross-section shapes and with distributed reinforcement along all faces, two thirds of the reinforcement area can be entered if more accurate values are not established".

It is often a matter of personal preference what form to choose. From a simplicity of description point of view, use of Eq. 12 might be preferred, and possibly with the factor 2.1 rounded down to 2, in a code or standard format.

4 Alternative slenderness limits

4.1 Lower slenderness limit results

Computational results

Increasing slenderness or 2nd order effects reduce the external moment, i.e., the 1st order moment, that can be applied to a compression member. A precise measure of this effect is for a given member defined by the ratio between the external or **1st order design moment capacity**, here denoted $M_{1d}(= M_{1Rd})$, and the ultimate **cross-sectional moment capacity**, here denoted $M_d(= M_{Rd})$. M_{1d} will be limited by instability prior to, or at, exhaustion of the sectional capacity.

Member parameter combinations that give

$$M_{1d}/M_d = 0.95 \qquad and \qquad 0.90 \tag{15}$$

correspond to 5 and 10 percent capacity reduction, respectively. The 10 percent limit corresponds to the limit in the EC2 draft /1/, where it is stated that "Second order effects may be neglected if they are less than 10 percent of the corresponding first order effects".

Results at the 5 and 10 percent limits have been computed for a wide range of parameters, partly based on an assumed deflection shape /11,14/ and by numerical integration of curvatures along the length/13/.

The latter approach is required for non-sway members when 1st order moment variations along the length are specified. It was assumed that section, reinforcement and material properties were constant along the length. An elasto-plastic stress-strain diagram was assumed for the reinforcing steel. Results were obtained for $\epsilon_{yd} = 0.002, 0.0025, 0.003$. Those reported here are for 0.0025. For the concrete, most results, including those reported here, were obtained with the parabola-rectangle diagram ($\epsilon_{co} = 0.002, \epsilon_{cu} = 0.0035$).

The peak stress in the parabola-rectangle diagram used in the computations is the same as the strength f_{cd} that is used in the nondimensional axial load n and mechanical reinforcement ratio ω_t in the normalized slenderness, Eq. 9.

Results for different cross-section and reinforcement arrangements and locations ($h'/h = 0.7, 0.8, 0.9$) have been obtained. The results reported here are for rectangular cross-sections with either corner reinforcement or with distributed and equal amounts of reinforcement in the four faces.

Results are presented for end loaded non-sway members that are pinned at both ends and members that are pinned at one end and restrained by a very stiff beam at the other end. In the presentation of results in terms of λ or λ_N, the elastic buckling lengths are used. For these two cases they are 1.0l and 0.7l, respectively

Typical results are given in Figs. 5–9. The results are for members without any imperfections, i.e., $r_1 = r_0$. All figures include results at both the 5 and 10 percent capacity reduction limits. Results are included for both a small amount, $\omega_t = 0.2$, and a large amount, $\omega_t = 1.0$, of reinforcement. This covers the whole range from approximately minimum reinforcement to an, in practice, upper limit.

The normalized slenderness versus moment gradient results in Fig. 6 to 9 are in part for members with corner reinforcement ($k_t = 4$), and in part with distributed reinforcement ($k_a = 2/3$ and $k_t = 8/3$). Comparable results for these two cases can be seen to be very close.

Geometrical versus normalized slenderness

The geometrical slenderness at the defined limits is strongly dependent on both axial load and reinforcement, Fig. 5a. When the same results are given in terms of the normalized slenderness, they are reasonably constant at one and the same limit, Fig. 5b. They vary most for the small reinforcement ratio, $\omega_t = 0.2$. The lowest λ_N-values result at axial loads below the balance point, which in the figure is roughly at $n = 0.4$.

Although one ideally would have liked all results to lie on a horizontal line for each of the 5 and 10 percent limits, the normalized slenderness λ_N must be considered successful in achieving a reasonable approximation.

The results in Fig. 5c is for the base case of $r_0 = 1.0$, i.e., for a member with a uniform 1st order moment distribution.

Comparisons with existing proposals

The slenderness limits according to MC90, Eq. 3, and Westerberg's proposal, Eq. 5 (labelled BW in the figures), are compared to numerical results for different moment gradients in Fig. 6 and 7, where results are plotted in terms of the normalized slenderness.

The MC90 limits are consistently conservative, which was to be expected with the adopted low base value and arelatively modest moment gradient effect.

Westerberg's limits vary considerably more. It gives results that are conservative when compared to results for pinned/pinned members. However, compared to the results for the pinned/fixed members, his limits can be quite unconservative for members in double curvature. This was also to be expected, and is due to the strong moment gradient influence in his slenderness limit formulation.

Comparisons with alternative lower limits

The following alternative limits will be briefly discussed:

$$lim\lambda_N = 10(1.8 - 0.8r_0) = 18 - 8r_0 \tag{16}$$

$$lim\lambda_N = 12(1.8 - 0.8r_0) \approx 22 - 10r_0 \tag{17}$$

$$lim\lambda_N = 10(2 - r_0) = 20 - 10r_0 \tag{18}$$

$$lim\lambda_N = 10\sqrt{5 - 4r_0} \tag{19}$$

The first of these is identical to the one adopted in NS 3473/10/ in 1989. Numerical evidence /13/ obtained after its inclusion indicated that it may be somewhat conservative.

The two next limits are both compared to the numerical results in Figs. 6-9. They are, in the writer's opinion, both acceptable approximations. The second limit gives somewhat larger values than the third.

The last limit is included in Fig. 9a only. It gives somewhat larger values than the second limit in a portion of the moment gradient range, but has the same "end" values as the third limit. The square root makes it less attractive than the other two.

In the comparisons, it should be noted that the results for the pinned/fixed case are rather conservative in themselves. For more practical boundary conditions, results between those for the pinned/pinned and the pinned/fixed can be expected. However, in order to allow moment ratios to be calculated based on 1st order end moments without imperfection effects included, it is felt that the pinned/fixed results may be good target values.

Results at small moment gradients (r_0 close to 1) are probably mainly of theoretical interest. In practice, a uniform 1st order moment is not very likely. In the comparisons, therefore, it is results at about $r_0 = 0.9 - 0.8$ and less that are of most interest.

At the same time, the base value at $r_0 = 1$ in the approximate limits should also be applicable to sway members and non-sway members with transverse loading. Results for such cases are available elsewhere /13,14/. The limits considered above are considered to be acceptable in that regard.

4.2 Lower limit proposal

Of the limits above, either the second or third is recommended. Then, with the normalized slenderness and k_t in the following form,

$$\lambda_N = \lambda\sqrt{\frac{n}{1 + k_t\omega_t}} \qquad k_t = 2.1\left(\frac{i_s}{i_c}\right)^2 \tag{20}$$

or in alternative forms, the provision might go as follows:

a) For non-sway members without transverse loading, second order effects may be neglected provided

$$\lambda_N \le 20 - 10M_{0A}/M_{0B} \qquad (22 - 10M_{0A}/M_{0B}) \tag{21}$$

and provided all cross-sections are designed for the numerically largest first order end moment. If the largest first end moment is less than that due to the required minimum eccentricity, the end moment ratio should be taken equal to 1.0. (*Alternatively, an end moment ratio definition in terms r_1 given for instance by Eq. 8 may be adopted as described earlier*).

b) For non-sway members with transverse loading, and for sway members, second order effects can be neglected when the limitation above with $M_{0A}/M_{0B} = 1$ is satisfied.

Lower limit with "modified" peak stress

In the formulations in this chapter, it is assumed that f_{cd} (in n and ω_t) is the peak stress of the concrete design stress-strain diagram. If another notation for the peak stress is adopted, such as for instance αf_{cd} as in the EC2 prestandard /3/, f_{cd} must be replaced by αf_{cd} in n and ω_t, Eq. 11. Alternatively, and this may be the desired approach, one may retain the present definitions of n and ω_t, and instead include α in the k_t definition, and in the limit or bound itself (i.e., the right hand side) by multiplying it with $\sqrt{\alpha}$. Provided α is not smaller than about 0.85, it is still acceptable to retain the lower of the two limits indicated in Eq. 21. Then, with α=0.85, the following modified relations are obtained,

$$\lambda_N = \lambda\sqrt{\frac{n}{1 + k_t\omega_t}} \qquad k_t = 2.5\left(\frac{i_s}{i_c}\right)^2 \tag{22}$$

$$\lambda_N \le 20 - 10M_{0A}/M_{0B} \tag{23}$$

4.3 Lower "creep" limit

Second order effects of creep during sustained loading can be neglected if the normalized slenderness, calculated with the characteristic long term axial force

N_L (rather than the ultimate design axial load),

$$\lambda_N = \lambda_{NL} = \lambda\sqrt{\frac{n_L}{1 + k_t\omega_t}} \tag{24}$$

does not exceed the lower limits given above.

Limits corresponding to this provision are shown in Figs. 1 and 2. They appear to be rather conservative. It would be worthwhile considering a possible relaxation of these "creep" limits in a future study. *Such a study must include members with small end eccentricities.* The same creep limit formulation as that above is incorporated in NS 3473/10/. The concept of using the same type of formulation for creep effects as that in the lower slenderness limits above, but with n replaced by n_L, is due to Menegotto, who suggested it in his study on slenderness bounds/15/.

4.4 Upper slenderness limit

The normalized slenderness parameter is also suitable in conjunction with upper slenderness limits. A study of such formulations, and comparisons with other upper limits in various codes and standards is available/12/. The upper limit included in NS 3473 /10/ is based on that study, and can be stated as follows:

The normalized slenderness λ_N of a compression member should normally not exceed the larger value of

$$45 \qquad and \qquad 80\sqrt{n} \tag{25}$$

The latter of these limitations corresponds in terms of the geometrical slenderness to

$$\lambda < 80\sqrt{1 + k_t\omega_t}$$

and represents an upper bound on $\lambda_N = 45$. These limits where developed for compression members with an initial imperfection, and account for some creep effects. For further details, see /12/. If a change of peak stress notation is made, as discussed above, the first limit above should in theory be replaced by $45\sqrt{\alpha}$. The second limit will not be affected, since there will be the samme change in n on both sides of the inequality.

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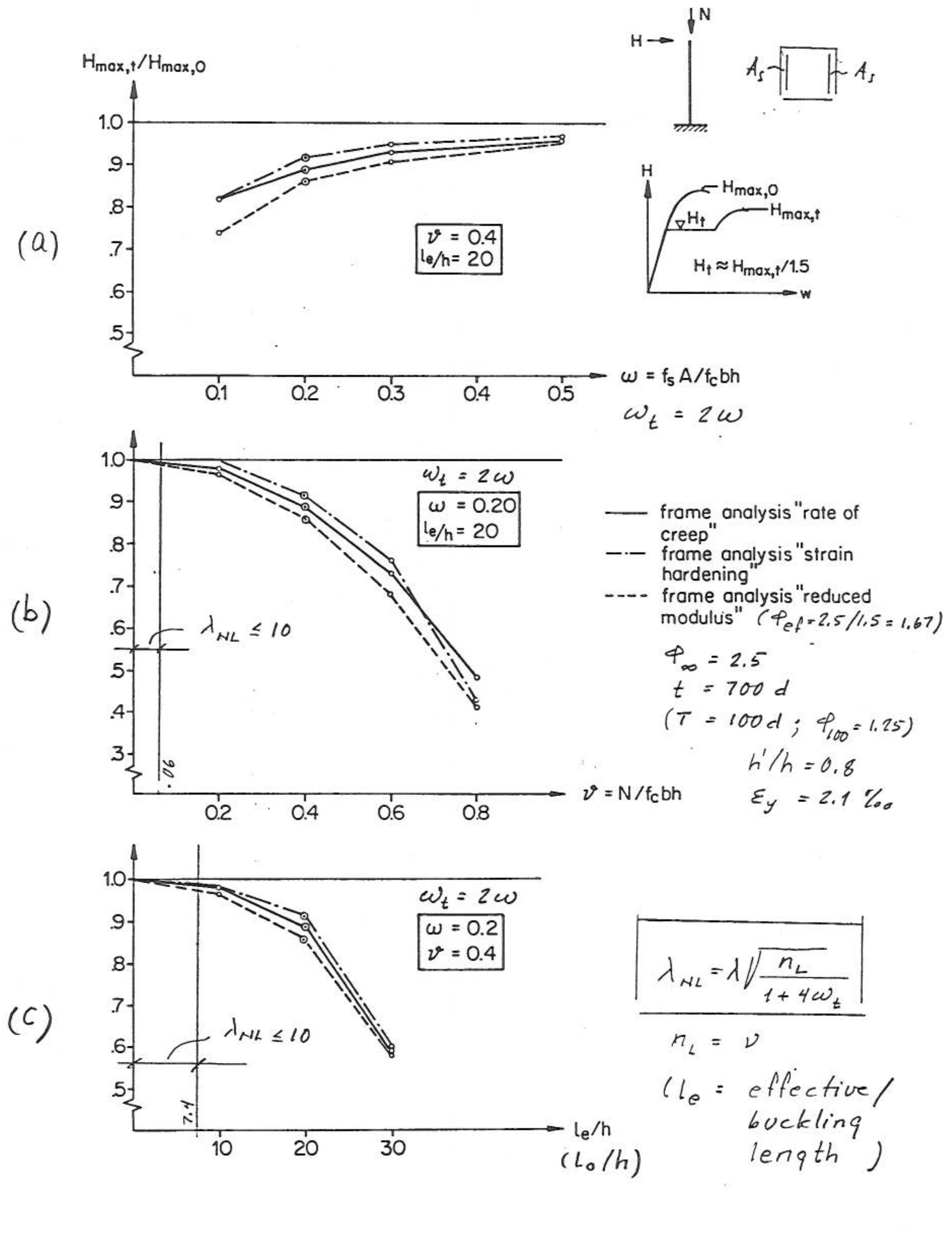
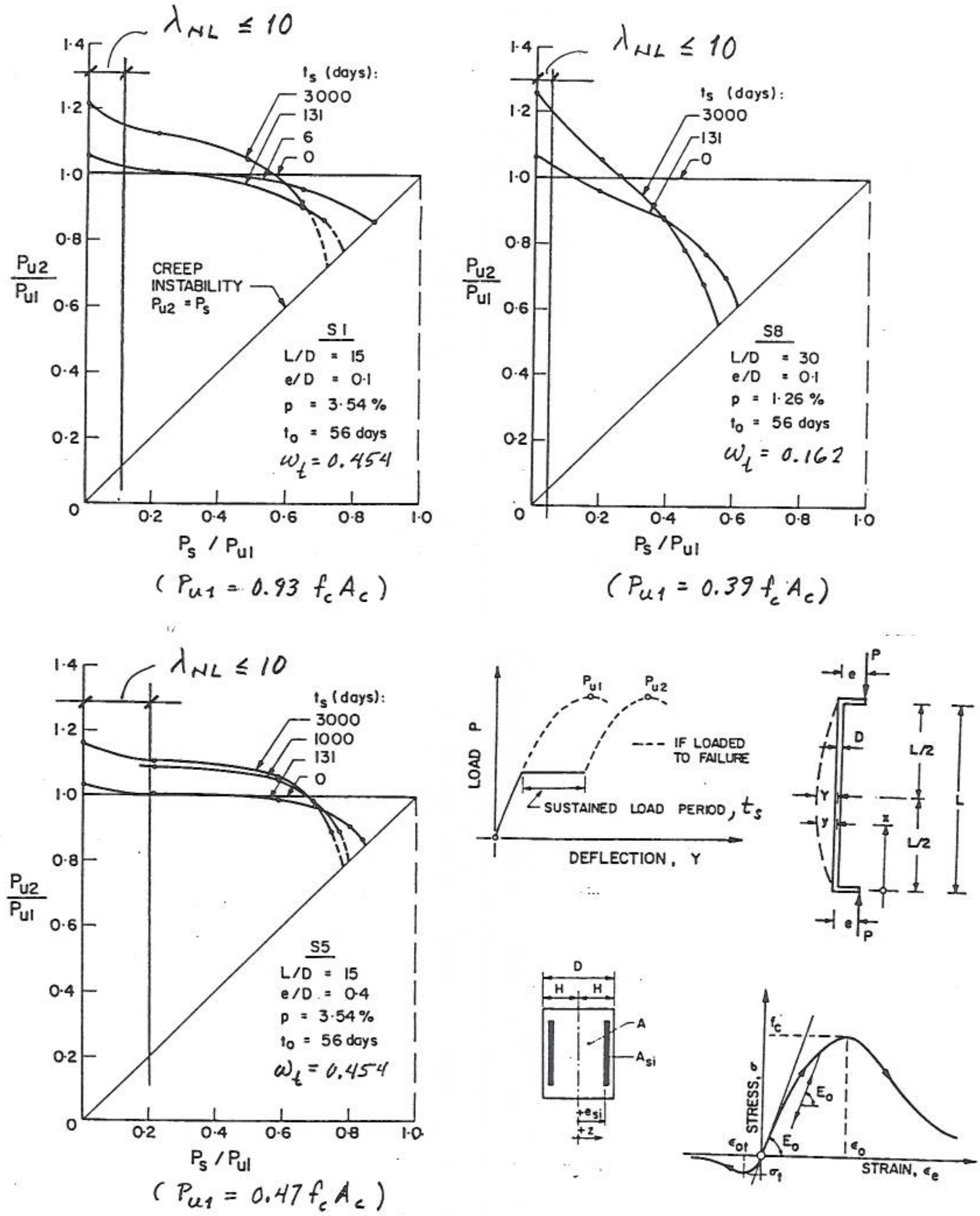


Fig. 1 Effect of creep on capacity of transversely loaded columns (from /5,6/)



$$h'/h = 2e_s/D = 0.65\ ,\quad \varepsilon_y = 1.93\ ‰\ ,\quad f_y / f_{c,56} = 12.83$$

$$f_{c,\infty} = 1.30\, f_{c,56}\ ,\quad \varphi_\infty = 2.5 \text{ (in linear range)}$$

$$\lambda_{NL} = \lambda \sqrt{\frac{n_L}{1 + k_t \omega_t}} \qquad n_L = P_s / f_{c,t_0} A_c \qquad \omega_t = \Sigma f_y A_s / f_{c,t_0} A_c$$

$$k_t = 3.4 \text{ (used above)}$$

Fig. 2 Effect of creep on capacity of columns with constant and equal end eccentricities (from /7/)

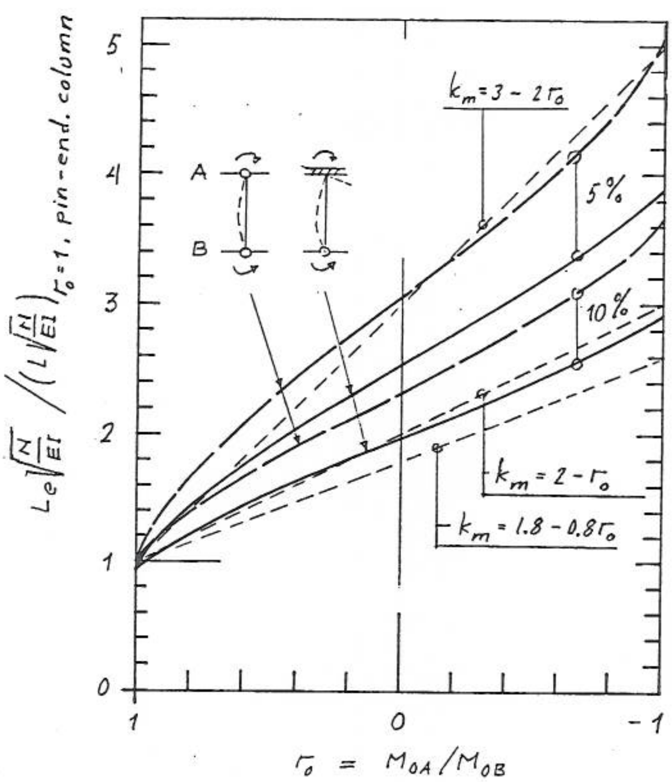


Fig. 3 Effect of 1st order moment gradient on elastic slenderness limits at which the total moment in columns without transverse loading exceeds the larger end moment by 5 or 10 percent

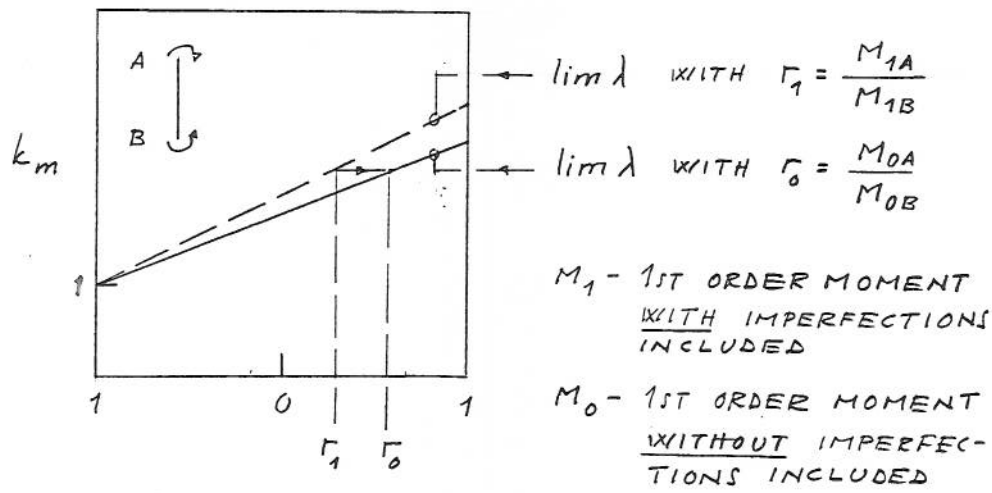


Fig. 4 Effect of moment gradient on lower slenderness limits
-in terms of 1st order moments with imperfections included, or
-in terms of 1st order moments without imperfections included

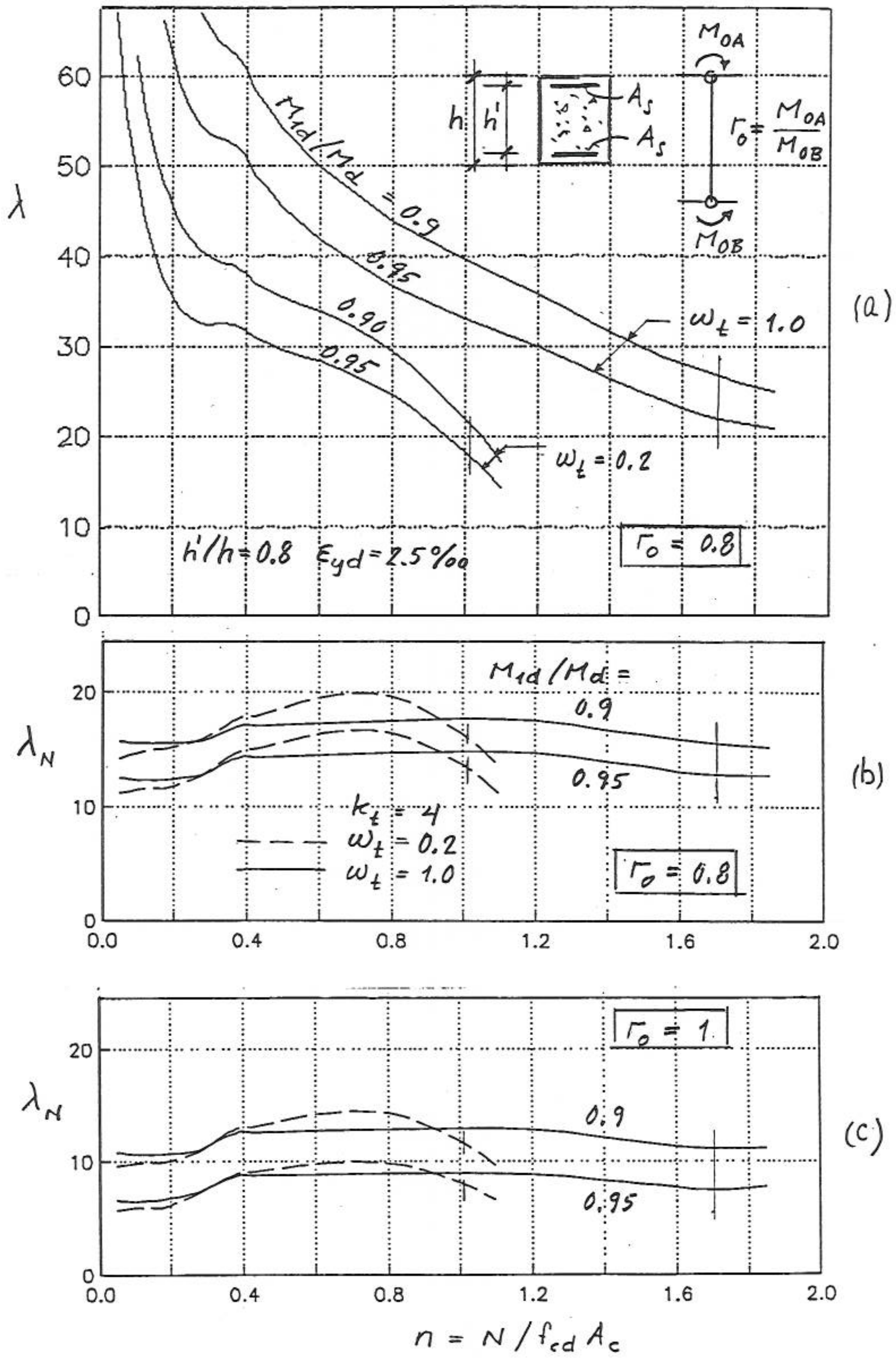
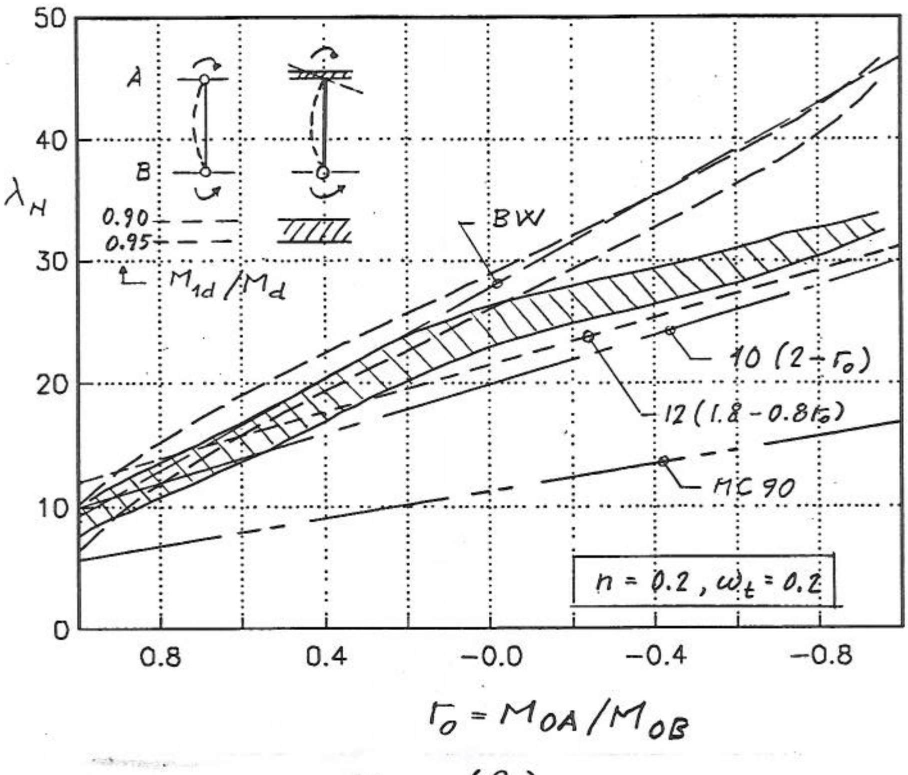
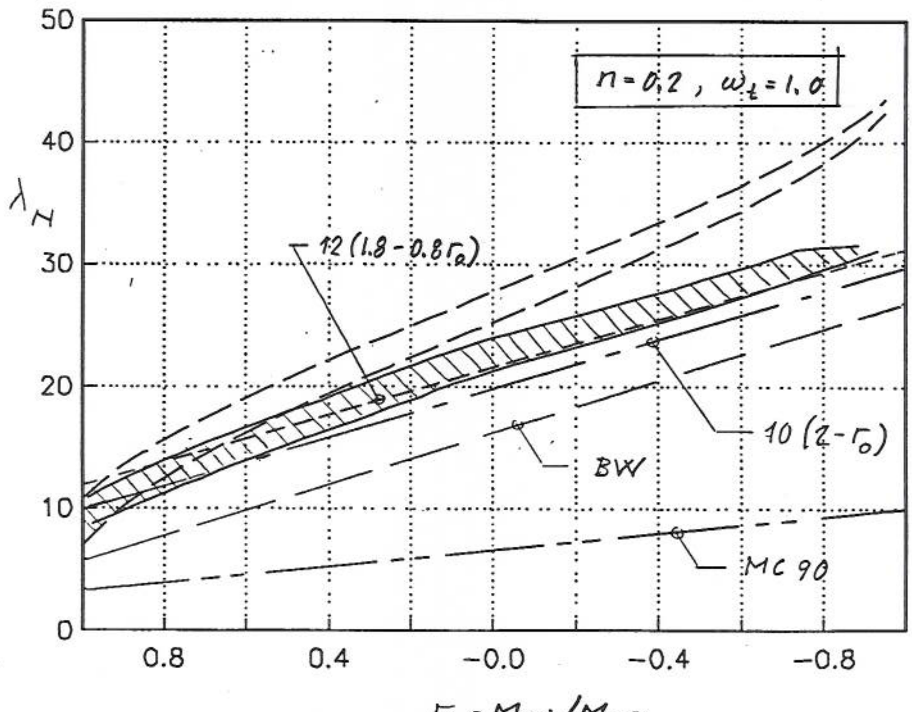


Fig. 5 Slenderness limits for unrestrained r.c. columns subjected to end moments vs. axial load for two different mechanical reinforcement ratios– (a) Geometrical slenderness for $r_0 = 0.8$ and (b) corresponding normalized slenderness, (c) normalized slenderness for $r_0 = 1.0$



(a)



(b)

Fig. 6 Slenderness limits for unrestrained and restrained r.c. columns vs. 1st order end moment ratios for $n = 0.2$ and (a) $\omega_t = 0.2$ and (b) $\omega_t = 1.0$. Rectangular corner reinforced cross-section, $h'/h = 0.8$, $\epsilon_{yd} = 0.0025$, $k_t = 4$

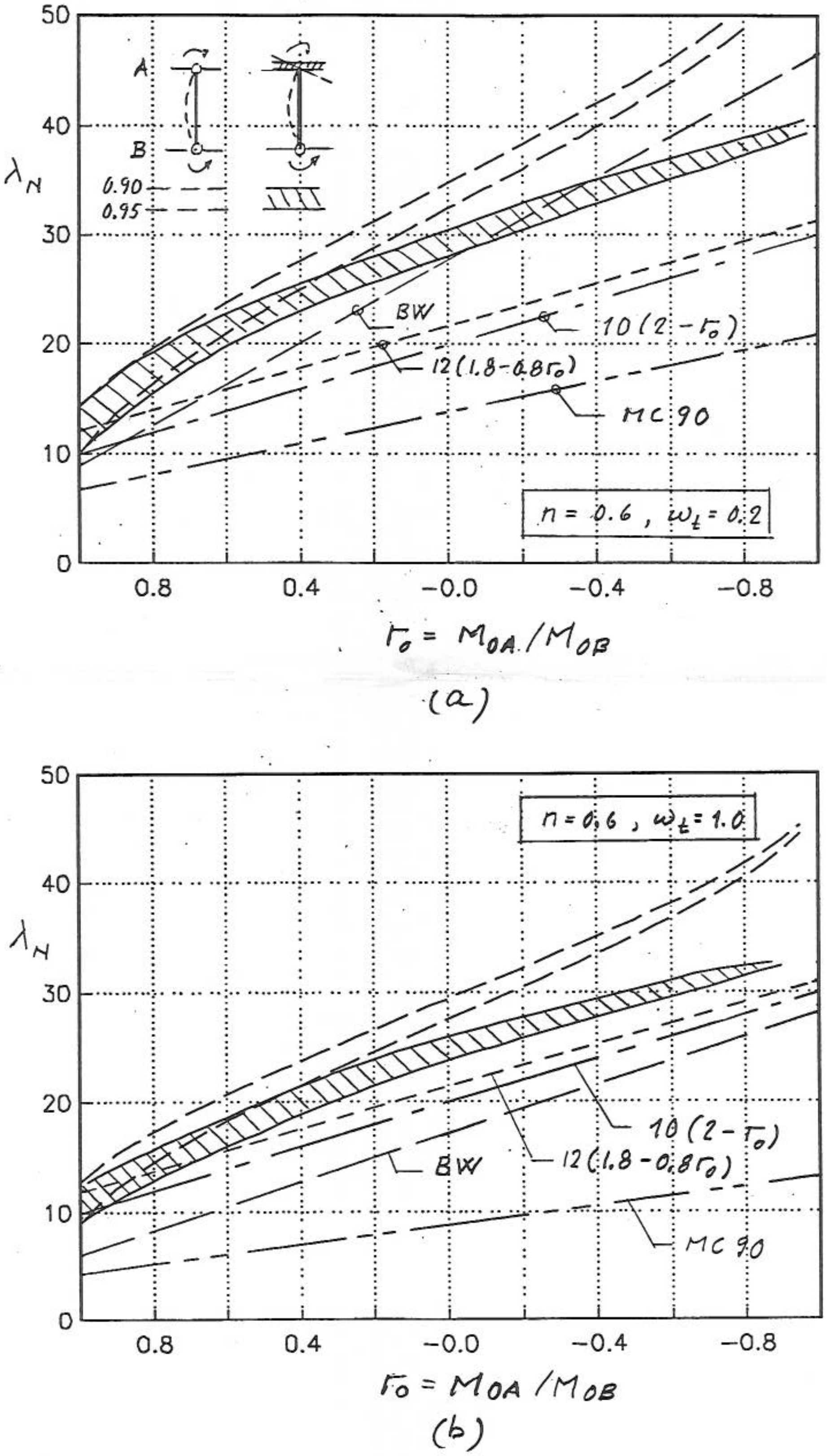
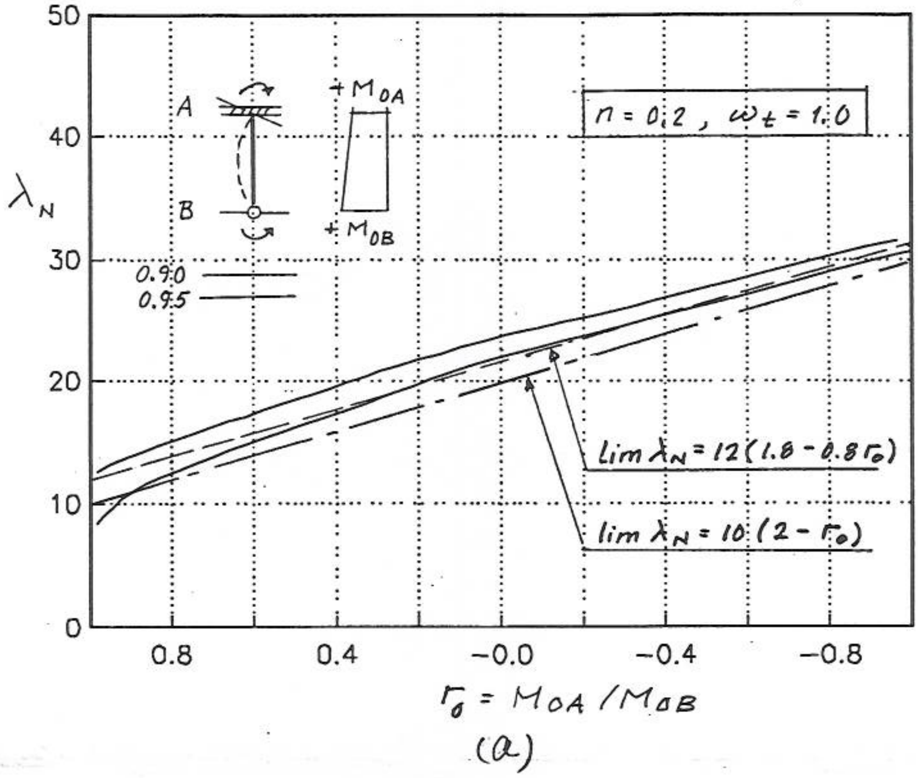


Fig. 7 Slenderness limits for unrestrained and restrained r.c. columns vs. 1st order end moment ratios for $n = 0.6$ and (a) $\omega_t = 0.2$ and (b) $\omega_t = 1.0$. Rectangular corner reinforced cross-section, $h'/h = 0.8$, $\epsilon_{yd} = 0.0025$, $k_t = 4$.



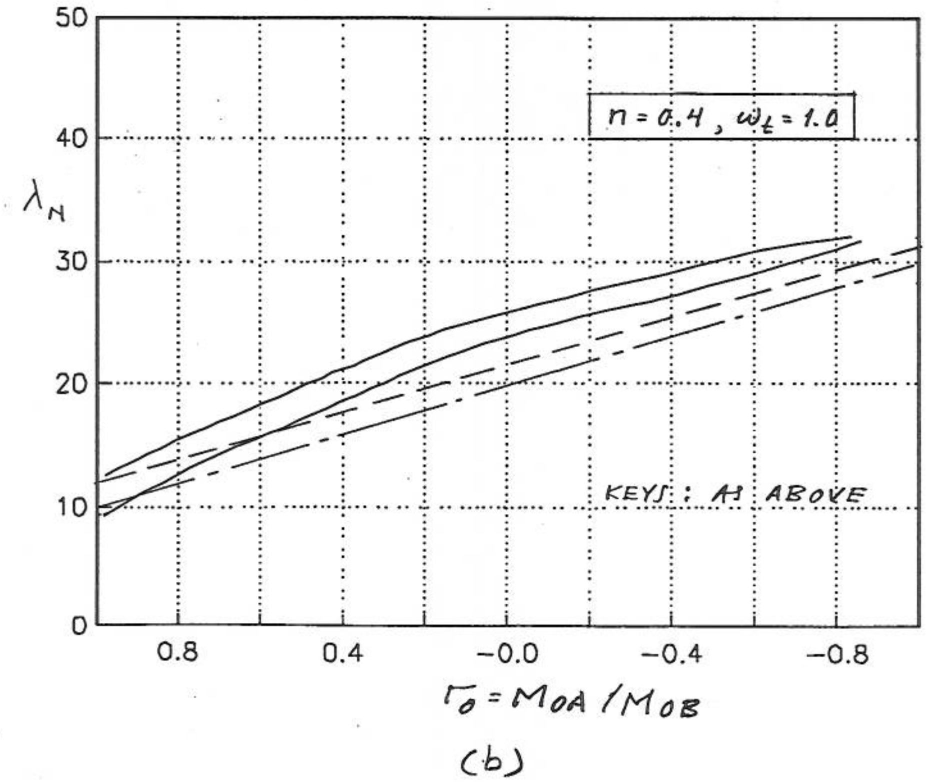


Fig. 8 Slenderness limits for restrained r.c. columns vs. 1st order end moment ratios for $\omega_t = 1.0$ and (a) $n = 0.2$ and (b) $n = 0.4$. Rectangular cross-section with equal and distributed reinforcement in the four faces, $h'/h = 0.8$, $\epsilon_{yd} = 0.0025$, $k_t = 8/3$.

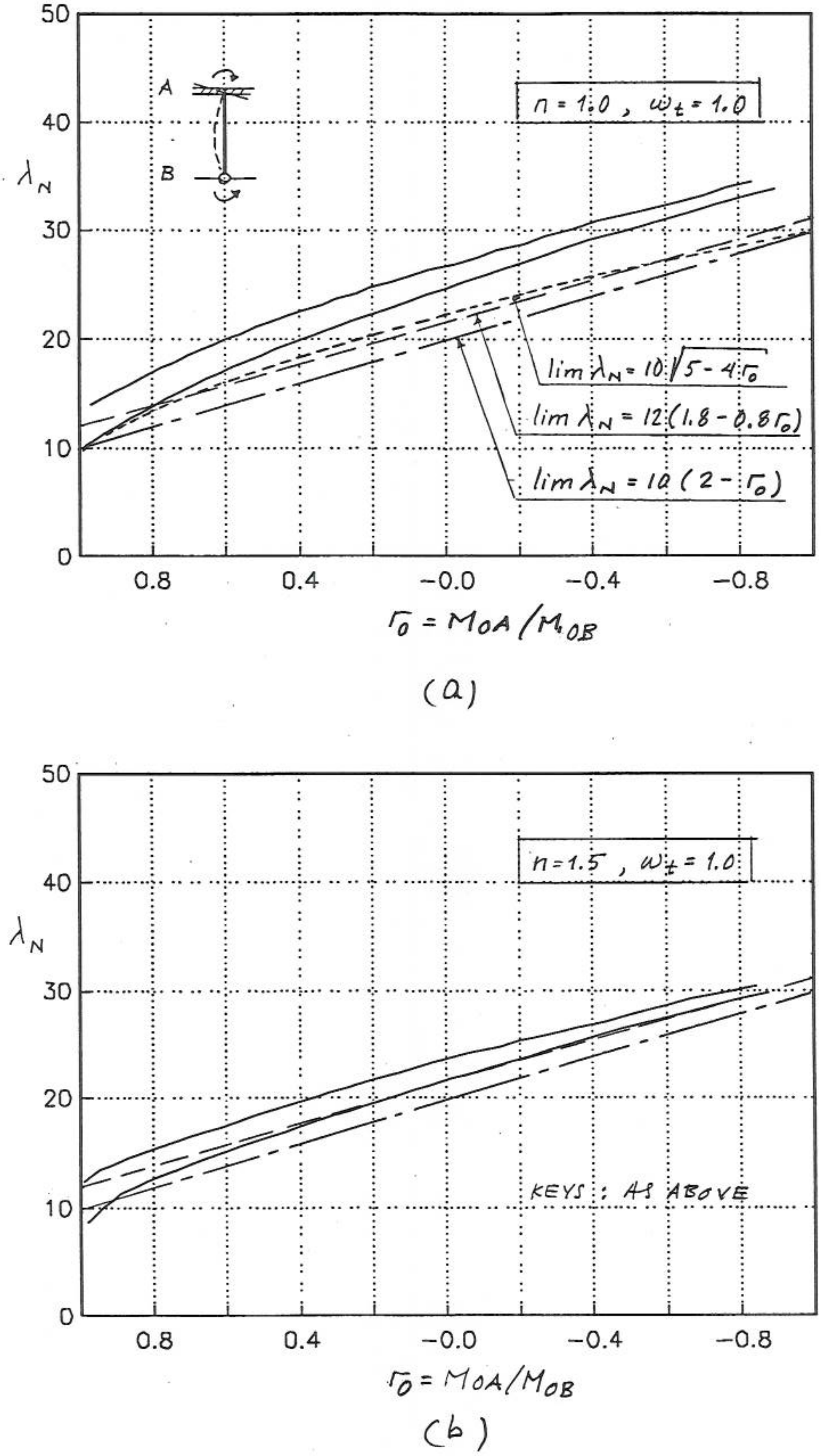


Fig. 9 Slenderness limits for restrained r.c. columns vs. 1st order end moment ratios for $\omega_t = 1.0$ and (a) $n = 1.0$ and (b) $n = 1.5$. Rectangular cross-section with equal and distributed reinforcement in the four faces, $h'/h = 0.8$, $\epsilon_{yd} = 0.0025$, $k_t = 8/3$.