Number processing in bilinguals

Modelling numerical cognition: Language effects on number processing

Martin M. T. Brierley

Masteroppgave i spesialpedagogikk
Utdanningsvitenskapelig fakultet
Institutt for spesialpedagogikk

UNIVERSITETET I OSLO

Våren 2009

Abstract

The goal of this literature review is to investigate if there is any reason to suspect that bilingualism in itself in can affect numerical cognition. The assumption is that this is not the case, meaning the other factors than bilingualism itself - such as language comprehension or other cultural educational or socioeconomic factors must account for any differences in mathematics performance between various bilingual and monolingual populations.

This thesis is an attempt to assess the current level of knowledge on the subject. Various models of numerical cognition are reviewed and are subject to a theoretical discussion, in which their general merits and ability to predict language effects on number processing are considered in light of relevant research.

Existing general models of numerical cognition are not designed from the outset to accommodate, and therefore make predictions on, interaction between multiple languages and number processes. Based on the reviewed research and models, a very tentative general model of bilingual numerical cognition is outlined. Predictions on the effects of bilingualism on number processing are discussed. Pedagogical implications of bilingual numerical cognition are considered briefly. Some thoughts on the possible benefits of expanding this field of knowledge in general and for special needs education concludes the thesis.

Contrary to expectation, available evidence suggests that language does affect numerical cognition, although the effect is limited to processing of exact numbers. There is also evidence that aspects of number processing in bilinguals, particularly retrieval of verbal arithmetic facts such as tables memorized by rote, varies across languages. However, while these effects are important from a descriptive perspective in cognitive psychology, they are likely to be relatively insignificant from a more normative, educational perspective, as compared to other factors that affect mathematics performance in bilinguals, such as issues related to comprehension.

Acknowledgements

Firstly, I would like to thank my mentor Guri A. Nortvedt for enthusiastic guidance, humorous scientific chit-chat and loans from her personal library. I would also like to thank my father David and my friend Knut for useful comments on language and content respectively. Last, but not least, I would like to thank Janne-Charlotte for enduring my reclusive behaviour, mood swings and erratic sleep pattern during the last two months.

Carpe Noctem!

Nesodden, May 30th, 2009

Martin Brierley

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1 Introduction

In Norway, as in other western countries, the debate on language issues in education continually resurfaces in the popular media, provoking no end of interest. Opinions are rife as professed and self-professed experts on the subject, teachers, politicians, writers, organizations, parents and just about everyone else contribute to the discourse. However, the debate seems to have two contradictory aspects to it. While both concern bilingual education, they generally appear to be considered separate issues:

On one hand there is the familiar debate on the problems that are faced by (or posed by, according to perspective) language minorities in society in general and in school in particular - and what is to be done about it (e.g. Arbeiderpartiet, 2009; Clemet, 2002; Skaar, 2006). On the other hand, there is a debate which seems to be rooted in the need for the corporate sector to do business abroad and therefore to employ people who master additional languages, in order to remain competitive in a globalized marketplace (e.g. Engström & Tessem, 2007; Ernes, 2007).

The European Union, while having no official educational policy of its own, nevertheless recommends that its school-age citizens should learn at least two languages in addition to their own (Espeland, 2009). Many pupils from language minorities, particularly those whose first language is not a minority language internationally (e.g. Turkish, Arab, or Spanish), should in theory be attractive additions to the workforces of the countries in which they reside. However, the fact remains that many groups of language minority pupils do not perform adequately in school, as measured by average grades or average test results, when compared to majority pupils (Hvistendal & Roe, 2004; Marks, 2005; Rönnberg & Rönnberg, 2001).

The nature of these challenges themselves, and particularly the actions required to rectify this situation, are subjects of much controversy. One common assumption regarding language minority pupils is that their difficulties primarily due to difficulties in comprehending the majority language. Accordingly, popular school-level interventions are diverse programs whereby the language minority pupil is either denied the use of his or her mother tongue during school hours, in order to ensure maximum exposure to the majority language, as well as various "softer" approaches where the language minority pupil receives various amounts of tutoring in his or her first language (Baker, 2006).

When it comes to mathematics and understanding numbers, there seems to be a popular perception that it is less dependent upon language than other school subjects and that it therefore should somehow "even the field" between minority and majority (Rönnberg & Rönnberg, 2001). However, this does not seem to be the case. Pupils from language minorities worldwide do not necessarily perform better in mathematics than in other subjects. On the contrary - quite often, mathematics seem to be one of the more challenging subjects (Marks, 2005; Rönnberg & Rönnberg, 2001).

A reason for the perception of mathematics as an "universal language" may be the reliance of mathematics on the common Arabic numeral system and international symbols (Rönnberg & Rönnberg, 2001). However, in reality Arabic numbers are nothing more than a notation system that has been accepted internationally (although hardly universally) and which has been implemented into a number of languages as the preferred system for the notation of quantity. Also, many mathematical terms are words that are "borrowed" from the "host language", but which take on a different use and meaning in the mathematical context. This jargon or "mathematic language" is often called the mathematics register (Lee, 2006; Rönnberg & Rönnberg, 2001). One characteristic of the mathematics register is that it demands a much higher level of precision than the common language, as just one misplaced word may alter the mathematical problem completely. Its use is also characterized by a lack of extra words that would otherwise be required in order to form proper sentences (Lee, 2006). Accordingly, language must logically play an important part in number comprehension and mathemathics education.

When observing the considerable controversy around the subject of educating pupils who use multiple languages, it is tempting to ask if this controversy may be due lack of knowledge. Is our understanding of the mechanisms underlying bilingualism inadequate, giving birth to unfounded assumptions and misconceptions? If some more basic knowledge of the invidual cognitive components of bilingualism was attained, could this clarify some basic premises and thereby lead to a more fruitful debate on the issue - and eventually to better educational models?

When specific groups of pupils exhibit problems in adapting to mainstream schooling, it often becomes the task of the special needs educator to examine the individual situation and to attempt to rectify the problem. Special needs educators should therefore expect to come into contact with issues related to minorities and bilingualism. Accordingly, the field of special needs education in Norway has been expanded to incorporate research on this issue (Lie, 2004).

The decision was therefore made to examine one particular aspect of the relationship between bilingualism and mathematics. In keeping with the idea of examining as basic factors as possible, it was decided to focus on if and how language affects use and manipulation of number. If it should prove to be the case that language does affect number processing, it would imply that monolinguals and bilinguals are cognitively different.

Most of the literature on language effects on mathematics education deals with monolingual or bilingual pupils' comprehension of the mathematics register, the structure of mathematical problems and what can be done to facilitate it (e.g. Barwell, 2005; Lee, 2006; Miura & Okamoto, 2003; Rönnberg & Rönnberg, 2001). As little learning can take without language comprehension, it is only natural to assume that this is the major factor in the relationship between language and mathematics. Indeed, it would be reasonable to assume that it can account for all of the difference between bilinguals and monolinguals that can reasonably be attributed to language factors and that no cognitive difference exists between the group.

The questions that form the basis for this thesis are therefore: To what extent does language affect number processing? Are there any potential differences in bilingual numerical cognition and if so, are there any implications for mathematics education of bilinguals? The assumption is that there are no differences amongst bilinguals and monolinguals and therefore that only external factors such as comprehension of the teaching language and the mathematics register affects bilingual numerical cognition.

How can one investigate these issues? A complicating factor is that the mind is a complex and poorly understood aspect of humanity (Uttal, 2005). Fortunately, over the last 30 some models and theorethical frameworks have emerged from the field of cognitive psychology, which attempt to model the interaction of different processes related to numerical cognition (e.g. Campbell & Clark, 1992; Campbell & Epp, 2004; Cipolotti & Butterworth, 1995; Dehaene, 1992; McCloskey, 1992). While theoretical models can provide a basis for understanding interaction between language and number processing, most of these models do not explicitly consider the general impact of more than one language. Therefore, not only the question of these models' general suitability and their postulations of language effects must be considered, but also their suitability as the basis for a model that does make predictions on the interaction between multiple languages and number processes.

This thesis is a literature review which is divided into five chapters; the first being this introductory chapter. The second chapter considers both some general methodological challenges in writing literature reviews, as well as some specific methodological concerns that pertains to the topic of this thesis. While this is a subject that is seldom elaborated upon in this type of thesis, there seems to be little reason to suggest that it should therefore somehow be less important than for other types of research. The third chapter discusses some central concepts in the domain of numerical cognition and considers three central theoretical perspectives, or models, on the phenomenon. The fourth chapter considers these models' relative merits as predictors of language effects on numerical cognition. In the last chapter, language effects on numerical cognition are discussed. Also, how a very tentative model of bilingual numerical cognition can be considered, as well as the pedagogical implications of this model, as well and future directions.

In the following sub-chapters some terms that are central to this thesis will be discussed, namely the concepts of bilingualism, numerical cognition and models of cognition.

1.1 Bilingualism

The meaning of the word is *bilingual* is deceptively simple: It is derived from the latin words *bis linguae*, meaning "two languages". It refers to someone who uses, or has the ability to use, two languages. However, it is less clear what sort of criteria that should be used in order to determine the language status of a given person: At what level of proficiency in the two languages can one be labeled a bilingual (Baker, 2006; Engen & Kulbrandstad, 2004)?

For example, pupils in most schools in Europe are taught at least one language in addition to their own. Does that mean they are bilingual? As noted previously, the word "bilingual" can take on quite different meanings according to the context in which it is used. Technically, a child who has a Norwegian mother and an English father is bilingual. As is a child of deaf parents who both speaks Norwegian and Norwegian sign language (NTS). However, when the term is used in the popular media, it usually refers to children of immigrant parents from specific countries (Karrebæk, 2006).

The impromptu definition above, "someone who uses, or has the ability to use, two languages" is is in reality two alternate definitions: One which refers to the use of languages ("someone who uses") and one which refers to ability ("has the ability to use"). Various criteria have been suggested for both definitions. Ability-related criteria have ranged from having the ability to understand utterances in another language, to having native-like command of two languages (Engen & Kulbrandstad, 2004). Criteria for use can span from alternately using two languages (Engen & Kulbrandstad, 2004), to someone who meets and has use for two or more languages in their daily lives (Karrebæk, 2006).

However, no current definition is entirely satisfactory and to create a definition that can be used to determine exactly who is and who is not bilingual, may ultimately prove futile (Baker, 2006). In the context of this thesis, however, the bilinguals that are of interest are primarily those who command both languages well enough that comprehension problems do not obscure any cognitive challenges related to the interaction between multiple languages and number processes.

1.2 Numerical cognition

Numerical cognition, or how numbers are processed mentally, has been the subject of research both on humans and animals for a long time (Zorzi, Stoianov & Umiltà, 2005). There are at least three features which set numbers apart from other domains of cognition: Firstly, numbers represent a very specific aspect of reality, namely numerosity. Secondly, numbers are the object of several other specific processes such as comparison, estimation and calculation. Thirdly, numbers can be represented in several different formats, such as different types of digit systems, written and spoken number words, etc. (Noël, 2001). The term *numerosity* is used to refer specifically to a measurable numerical quantity (Dehaene, 1992).

The basic understanding of magnitude and simple arithmetic ability is fundamental to humans - infants can sum and subtract some numerosities even before knowing number words (McCrink & Wynn, 2004). Even animals, such as pigeons, can compare and perform approximate mathematical operations on sets of objects (e.g. Brannon, Wusthoff, Gallistel & Gibbon, 2001). However, there has been much controversy regarding skilled arithmetic performance in humans, especially regarding the nature of underlying mental representations of number (Zorzi et al., 2005). This issue is obviously important to the subject of this thesis, and will therefore be discussed more thoroughly in chapter 3.

1.3 Models of cognition: Mind vs. Brain

The models discussed in this thesis mainly derive from the fields of cognitive psychology and cognitive neuropsychology. Cognitive psychology investigates the nature of the mental processes that underlie cognitive abilities, such as language understanding and production, information storage, object recognition, social interaction, etc. (Coltheart, 2001). Cognitive neuropsychology is a branch of cognitive psychology that relies on studies of the impaired mind for the acquisition of knowledge on the normal brain. It is not a variant of neuropsychology, because while neuropsychology is concerned with the functions of the physical brain, cognitive neuropsychology investigates the functional processes of the mind (ibid.). This distinction is important because of the mysterious nature of the connection between mind and brain: We know that the mind is created by the brain, but we yet to understand how (Uttal, 2005). Consequently, studies of the physical brain, while fruitful from a neurological or neuropsychological perspective, have only limited practical significance to disciplines such as psychology and educational research (Cubelli, 2009).

Studies of the mind, on the other hand, are mostly based on behavioural data and can form the basis for clinical endeavours such as rehabilitation and assessment of practical ability (Coltheart, 2001). A couple of points should be kept in mind: Models in cognitive psychology represent systems that have no tangible substrate, they are not models of actual mechanisms. They are also incomplete representations of the systems they represent, as the human mind is much too complex to model in its entirety (Uttal, 2005). Consequently, cognitive models must reduce reality into comprehensible simplifications of very specific processes, whilst ignoring most of the factors that affect the complete system (Willingham, 2009). Further aspects of cognitive neuropsychological research and its suitability for educational inferences will be discussed in chapter 2.5.

2 Method

This thesis is a theoretically oriented literature review. A theoretical review is, according to Cooper (1998), a text in which one hopes to present theories that offer to explain a particular phenomenon and compares their *"breadth, internal consistency, and the nature of their predictions"* (p. 4). In addition, theoretical reviews often contain descriptions of experiments and may contain reformulations or integrations of notions from the theories presented (ibid).

Information on such theories and experiments must necessarily come from the existing body of scientific literature. In literature reviews, the process of searching through and selecting texts from this body can therefore be considered equivalent to the process of collecting empirical data in empirical studies (ibid.). However, the process of reviewing literature is not exclusive to theoretical reviews. It has significant similarities to historical research, where one gathers information from text material of the era one wishes to study. Indeed, a historic source may be defined as any source that was already in existence prior to the educational researcher's decision to utilize it. In other words, everything other than new empirical data generated during the research process can be considered to be historic sources (Fuglseth, 2006; Tveit, 2002).

Gall, Gall and Borg (2007) emphasizes that any type of educational study should contain a thorough literature review, in order to show that the author has a good overview of the field in which he is operating and that he is up to date on recent developments, so that these can be taken into consideration. Unfortunately, the literature review is often slighted - sometimes to the detriment of the value of the empirical portion of the research (ibid.). Befring (2007) notes that the methods used are rarely subject to critical discussion in educational research that is based on text analysis. He suggests that this may, at least in part, be due to a general lack of emphasis on methodic issues in historical research - and that this is evident through the lesser amount of publications that detail principles and norms for this kind of research, compared to the extensive body of literature on empirical research methods.

According to Fuglseth (2006), some of the more important issues in this respect are issues related to source evaluation and hermeneutics, or - in more common terms - assessing the quality of sources and issues related to their interpretation. Also, there is the issue of how to find and obtain access to sources of interest. In this respect, the challenges are clearly somewhat different in historical research and theoretical reviews: In the former any text is interesting as long as it can be used to shed some light on the subject of interest - indeed, historical sources need not be texts at all, although they mostly are (Tveit, 2002). In the latter, only literature adhering to strict scientific standards are of value. Therefore libraries, indexes of dissertations and theses and scientific databases become the primary points of access for the researcher (Ary, Jacobs & Razavieh, 1996).

The issues mentioned here will be examined more closely in the following sub-chapters. First, some common norms for source evaluation in historic research are discussed. Secondly some specific issues regarding the use of research reports as sources are considered. Thirdly the procedures used to find relevant sources for this thesis are explained, and thereafter the role of hermeneutics are briefly considered. Finally some consideration will be given to an issue that is specifically related to the theories discussed within this thesis, namely the validity of inferences from studies in cognitive psychology and cognitive neuropsychology.

2.1 Source evaluation

Source evaluation is the process whereby the investigator attempts to determine how trustworthy source material is and its relevance and scientifical soundness (Befring, 2007). By its nature, historical research is one discipline that to a large extent relies upon compiling evidence from already existing source material. It is therefore only natural that this field of research is where many guiding principles of source evaluation originates. It stands to reason that the value of a historical study primarily relies upon the researcher's ability to judge the authenticity and validity of his sources (Gall et al., 2007). The same can be said for the theoretical review. Indeed, as Mertens and McLaughlin (2004) point out, the critical analysis of sources is an important part of the research process of any literature review. Indeed, as noted earlier: Since any research paper should contain a thorough review, source evaluation is important to any educational research. It can be useful to divide the concept of source evaluation into two areas: Internal and external criticism (Ary et al., 1996).

2.1.1 External criticism

External criticism is leveled at circumstances pertaining to the creation of the text (Ary et al., 1996; Gall et al., 2007; Tveit, 2002). Gall et al. (2007) suggests that in order to evaluate a document on the external level, one should examine aspects such as the genuinity of the source, who the author is, where it was written, when it was written and under which conditions it was written. This is because it is in the interest of the investigator to get as close as possible to the actual event and because the circumstances surrounding the creation of a text can affect its content: If, for example, a text was created under pressure from a third party, there is little doubt that its contents would be radically different to what they would have been if the author created it of his own free will. One frequently used example of a situation where research may be affected by the conditions surrounding it is research that is comissioned or financed by parties that have an interest in its outcome.

Of course, not all principles of historical research are equally applicable to a theoretical review, since the theoretical review is based on research literature, which is expected to conform to criteria that are quite different from common texts. As far as articles published in many scientific journals are concerned, there are often some safeguards already in place, e.g. a panel that will screen articles prior to publishing, in order to prevent avoid forgery, ensuring that credit is give to the correct authors, that correct bibliographical information is provided and so forth. However, multiple authors are often credited with writing a single article. This can make it difficultto determine how much of a text and which parts of a text each author is responsible for (Gall et al., 2007).

In many journals, there are also mechanisms to help the editorial staff determine the quality of submissions such as a peer-review system. In spite of such safeguards there has been examples of fraudulent research even in internationally renowned journals. One such example is the infamous Jon Sudbø case, in which a respected name in cancer research was discovered to have fabricated all his data material (see e.g.

Dahlberg, 2007).

While one seldom can be absolutely certain of the genuinity of a text, one can, according to Gall et al. (2007) create hypotheses on the fraudulent nature the text and examine it with these hypotheses in mind. Viewing the text from such a perspective may lead to the discovery of information within the text that that makes the hypothesis untenable, thereby building confidence in the text. Also, the reliability of one's own text can be improved by expressing any doubts about the quality of a source (ibid.). However, it is not hard to spot one difficulty regarding this proposition: Finding the confidence to question senior researchers in a field may prove challenging; especially to students or junior researchers.

Tveit (2002) adds that one should also consider the closeness of the source to the event it describes and the author's use of his sources. The author of the source text did, of course, face the same methodic challenges as far as souce evaluation is concerned, as the reviewer does. Sources are commonly categorized according to their distance in time and space to the event they describe. These guidelines are equally applicable for evaluating the source author's use of sources and for evaluating the importance of the source in relation to the review.

The most common way of classifying sources in in educational research it to divide sources into two categories: Primary and secondary sources (e.g. Ary et al., 1996; Fuglseth, 2006; Gall et al., 2007; Mertens & McLaughlin, 2004). Primary sources are texts that are authored by an actual witness to an event, while secondary sources recount the event as reported in another text, which may be either a primary source, or another secondary source. While these categories may suffice for research that primarily relies on empirical data rather than a literature review, theory-heavy texts like the present thesis may benefit from a more nuanced approach. Tveit (2002) uses the terms "førstehåndskilder" and "annenhåndskilder" (*first-hand sources* and *second-hand sources* - author's translations), to refer to the source's distance from the event it is describing; while the terms primary source and secondary source instead refer to the investigator's use of the sources. In the following, these concepts will be elaborated upon.

A first-hand source is an actual report on an event by one or more people who were present, or in other words, where *"only the mind of the observer intrudes between the original event and the investigator"* (Ary et al., 1996, p. 490). In educational research this usually means a book or journal article which describes a study of, an opinion on, or a theory about a phenomenon, that is written by the actual person or persons who performed the research, formed the opinion, or conceived the theory. First-hand sources are preferable and use of such sources increases the credibility of the text which is based upon them. However, the situation may arise where the first-hand source is not available to the investigator. For instance, if one wants to refer to writings by Plato, the original documents simply do not in exist any more. However, this fact has not kept scores of researchers from writing texts based upon Plato's texts.

A second-hand source is a source that describes events not directly observed or experienced by the author, or as Ary et al. (1996) conceptualizes it, where one or more minds intrude between the observer and the investigator. A typical example of a second-hand source in research literature is an article in which an author makes a point which is attributed to someone else. It is, however, a first-hand source concerning the author's own comments.

Even translations of texts are regarded as second-hand sources (Tveit, 2002). While Lev Vygotsky's pivotal works in developmental psychology are available in their original form, they are of little use unless one can read Russian. In a translation, the text has passed through the mind of the translator. While most translators can be trusted to try to keep as closely as possible to the original text, they still have to make a number of linguistic choices in order to create a clear text in another language: Obviously, direct translation seldom produces acceptable text. Such decisions can affect the text in a number of ways and may therefore subtly affect the meaning conveyed by it. Still, scores of writings have been based upon Vygotsky's work, without the authors having taken the time and effort to learn Russian. So while second-hand sources do not automatically increase the credibility of the text in which they are used, when extra thought is put into their evaluation and some consideration is put into the way they are used, they can be perfectly acceptable sources.

Primary source is a term which refers, in Tveit's (2002) terms, more to the status assigned to the source by its user, than to the source itself. A primary source is considered more valuable and is therefore more central to the arguments presented in a text than a secondary source - irrespective of whether it is a first-hand or second-hand source. Ideally, a primary source is also a first-hand source. However, this need not always be the case, as with the above example: When basing one's work upon the writings of Lev Vygotsky without being able to read Russian, one must decide to use a second-hand source as a primary source. Obviously, other criteria in source evaluation should then be considered extra carefully. Importantly, a primary source should always be the best source available to the researcher.

Secondary sources are sources deemed less valuable than primary sources and mainly used for more peripheral points in the text (Tveit, 2002). They are not sufficient for making important points or underpinning central arguments that could endanger conclusions drawn if sources were proven to be inadequate. Since less emphasis is put on them, they need not pass as strict evaluations as primary sources. However, such sources still can be important to an author, as their use can save a lot of time and resources, compared to identifying, locating and retrieving the primary source for every minor point in a text.

2.1.2 Internal criticism

Internal criticism entails considering the quality of text itself (Ary et al., 1996; Gall et al., 2007; Tveit, 2002). One should consider the internal consistency of the text, if the arguments in it are logical and if there are contradictions or other elements that subtract from the overall impression (Befring, 2007). Also, one should consider the accuracy and worth of the statements within. This makes internal criticism a somewhat more complex task than external criticism (Gall et al., 2007). There are two overarching theories on how to should judge the truth or falsity of a statement are the correspondence theory and the coherence theory (Tveit, 2002):

Correspondence theory forms the basis of empirical research. According to this theory, a statement is true if it corresponds with reality - i.e. if it corresponds with that which is observable. Applied to research literature, this would mean that a statement is true if it corresponds to the (empirical or historical) data presented (Tveit, 2002). In other words, convincing arguments can be put forwards to the effect that it is backed by the evidence presented it should be assumed to be true. One can also consider the realism of the events described in a text: The likelihood that the described events could have occured given the circumstances, the apparent reliability of the information presented, etc. (Gall et al., 2007).

Coherence theory, on the other hand, postulates that a statement is true if it is consistent with what else is known on the subject (Tveit, 2002). Applied to research literature, this means that the statement is true if it consistent with other statements or theories on the subject. A problem with the coherence theory is that if one, in an empirical study, finds something that is radically opposed to commonly accepted knowlendge, one may reject the findings, even if they are true. On the other hand, if one keeps to correspondence theory only, there is a risk of endorsing research results that are false.

As a result, it is good practice to take both theories into account, and both consider if a statement both corresponds with the data it is based upon and what else is commonly known on the subject. One should therefore attempt to consult multiple sources on the same subject. Replicated studies are an advantage in this respect, but with some types of studies, particularly case studies, replication can prove difficult. This point will be adressed in chapter 2.5, in which some types of studies incorporated into this review will be discussed.

2.2 Research reports as sources

A few issues concern articles that contain reports of empirical research which are not equally relevant relevant to other types of source texts in literature reviews or historical studies. Since a large part of the sources for this thesis are research reports, this sub-chapter is devoted to such issues.

One issue is that one must not only consider the quality and origins of the text itself, as described in the previous sub-chapters. It is also necessary to judge the quality of the research that it describes (Gall et al., 2007). To this end, one must examine the text for descriptions on how the research procedures were carried out. Something about the preparatory work is often mentioned in the introduction to a report, most reports will also have a method chapter describing the procedures that were followed. However, there are often restrictions on report lengths in journals, which can lead to the omission of details that would have been useful in evaluating the validity of the research presented, which can cause the reader to have to make assumptions or guesses about aspects of the study (ibid.).

Mertens & McLaughlin (2004) suggest a number of critical questions that can help in analyzing the data analysis and interpretation of data in empirical studies (see pp. 201-202). These will not be listed in detail here. However, the central points concern such factors as evaluation of the statistical procedures used in quantitative studies, the regularity of the data and the degree to which peer researchers have been consulted for alternative perspectives on its interpretation in quantitative studies, and if alternative explanations are accounted for.

Another issue pertaining to research reviews is the degree to which the studies that are reviewed are representative of studies on the subject in general (Cooper, 1998). Particularly in well-known international journals access to publication of papers is tightly regulated. Therefore, a great deal of the research done is never published in such journals, making it difficult to access for the investigator. There may be features common to published reports that are less common in research in general. For example it is not likely that research projects in general produce statistically significant results as frequently as indicated by the number of such results presented in journal articles (ibid.). In addition, investigators usually only have access to a limited number of retrieval channels, which may skewer the selection of research reports (ibid.).

All in all, it is improbable that research retrieved for a review is going to be representative of all research conducted on the subject. Cooper therefore suggests that when reviewing sources, the investigator should try to search as broadly as possible and should attempt to imagine what inaccessible studies might have said and how they might differ from the ones retrieved. Naturally, this is not something which is easily accomplished. However, most importantly, Cooper argues that the investigator should keep account of the search process and that the review should feature this account. This makes the search process open to criticism and replication by the reader, in the same manner that research procedures should be laid open to criticism and replication in empirical research reports. As a consequence, this process is the subject of the next sub-chapter.

2.3 Literature search

The first challenge facing the prospective author of a scientific text is to get an overview of the status in the field in which he is interested. What is the current level of knowledge on the issue? What has already been said on the particular subject (Gall et al., 2007)? Even determining where to start can sometimes be a daunting task, but fortunately there are publications that can provide the needed starting points, in the form of handbooks and other collections of writings that provide overviews of the field, reviews of trends and disscussions in the field and research syntheses (Ary et al., 1996). Such sources can be termed "preliminary sources" (Gall et al., 2007). Examples of such books used during the work on this thesis are the *Handbook of Child Psychology, sixth edition* (edited by Damon & Lerner, published in 2006 by John Wiley & sons, Hoboken) and *The Handbook of Mathematical Cognition* (edited by Campbell, published in 2005 by Psychology Press, Hove). While such compilations may be a good starting point, they rarely provide insight into the very latest developments in the field. Even if they are quite recent, the process of compiling, publishing and distributing such a book takes time, and so a full overview can not be made without extensive forays into the world of scientific journals.

Gathering journal articles can be an exhausting process, although the event of computerized databases have made this task somewhat easier than it used to be (Mertens & McLaughlin, 2004). The event of digital full-text articles has further improved it. Recent development in the field continue the trend of making scientific literature more accessible and to this end the library at the University of Oslo provides the X-port service (http://x-port.uio.no/), which is a search engine capable of performing meta-searches in a number of scientific databases.

A meta-search is a search that is conducted in several databases simultanously or in rapid succession without requiring further input from the user, greatly reducing the amount of labour involved. By selecting a X-port category such as "utdanningsvitenskap" (educational research) or "psykologi" (psychology) and "fagdatabaser" (scientific databases), one can select any number of databases from a list and the system will search them according to the specified keywords, all the while marking articles which are available in full text to be downloaded, or providing library data information for those that are not.

This convenient service has been used extensively during the research for this thesis, both initially, as a tool to get an overview and later, in order to locate articles on a specific subject, search for other works by specific authors or groups of authors, or find articles referred to by other authors. Its coverage is extensive and includes numerous databases such as ERIC, PsychINFO, EMBASE and MEDLINE. In order to test its coverage, searches were conducted directly in some databases such as ProQuest Psychology and the ISI Web of Science. However, these did not turn up results that were not available through X-port as well. Also, during work on this thesis, X-port never failed to locate a specific article when queried. This suggests that it does cover the field well enough for information retrieval for a project such as this thesis. It was also noted that articles often are present in more than one database anyway, especially if they have been influential in the field or are written by renowned authors.

The majority of the initial database searches were necessarily of a general nature, the aim being to get an overview of recent developments in the field. Therefore, the most important keywords used were language-related ("language", "speech", "bilingual", etc.), maths-related ("maths", "mathematics", "numeracy", etc.) and brain-related ("brain", "mind", "cognition", etc.). Since the aim was to get an overview of current status in the field, a time frame of 15 years was initially decided upon. Later on, as the search progressed towards more specific subjects, the time frame was expanded, so as to also be able to find older, but important articles.

After obtaining search results, abstracts and bibliographical data were retrieved for the more interesting-looking articles, in order to perform rudimentary quality control by considering such factors as the number of times they had been cited, what journal they had been published in, who their authors were and which institutions their authors were associated with. Fairly quickly some author names started to appear more often than others, suggesting that some researchers whose fields of interest lay particularly close to the subject of this thesis, whose names could then be looked up in journal databases, library databases, and on the Internet in general. Often, researchers have their own homepages, on they explain their fields of interest, place of work and list their bibliographies. Such homepages can therefore prove quite useful in finding information on the author, locating additional work, other authors in the same sphere of interest, etc.

When conducting meta-searches, limiting the searches is paramount. Otherwise one may get an unwieldy number of search results. One way of limiting the searches in scope is to limit the type of articles requested. Research reports were, of course, included. As were articles that presented theoretical models based on empirical evidence. Articles that commented on, or brought new perspectives, to other research reports were also included as the debate following the publication of controversial articles can be very interesting from a theoretical point of view.

Naturally, as work progressed searches became increasingly specific, due to the fact that one develops an increasingly clear idea of what one needs of further information: Either in terms of writings by specific author, a specific subject, perspective - or a book or article that has been referred to in another text which needs to be investigated further for purposes of verification of a statement, or to provide additional information.

2.4 Hermeneutic considerations

When an article, book chapter, or other text has been located, evaluated, judged to be relevant and trustworthy and its potential use determined, the only matter remaining is how the content of text is interpreted by the investigator. This subject is the matter of this sub-chapter.

The term "hermeneutics" originated in theology, as term decribing techniques for an important endeavour in that particular field, namely the interpretation of bible texts (Hjardemaal, 2002). Since, the term has been adopted by other frequent interpreters of text, such as historians and law researchers and expanded to mean the interpretation of a text in general (Befring, 2007). The process of text interpretation and its use in a new text can be divided into three stages: Firstly the semantic meaning of the words are decoded. Then one attempts to extract the general meaning, or message of the text as a whole. Finally one expresses the general meaning in one's own words, in creating the new text (Kjeldstadli, 1999). The first step of the process merely requires command of the language in which the text is written, so that meaning can be attached to the writing. Apart from any language-based misunderstandings that may occur at this stage, it is mainly in the second and third step that the original message is prone to distortion. It is therefore good reason to be mindful of factors that may colour one's interpretations of the sources.

Many of these factors are related to what is often termed preconceptions (Alvesson & Sköldberg, 1994; Wormnæs, 2006). Preconceptions are based on our earlier experiences and serve as "spectacles" through which reality is observed. They are useful because they allow us to quickly assess everyday information based upon previous experience. However, they are less useful when trying to view information objectively, since they are not necessarily rational, or applicable to all contexts. If, for example, one is observing a child solving an arithmetic task and making off-hand assumptions about the reasoning behind a particular strategy used by the child, these may prove to be quite different from the child's reasoning, as described by the child itself.

The process of text interpretation is often described as a circle, wherein the reader gains an increasingly good understanding of smaller parts of the larger message. This in turn resulting in better understanding of the meaning of the whole message, which in turn leads to an even better understanding of the part. This is often termed the "hermeneutic circle". For each transfer from part to whole and vice versa, the understanding passes through, is modified by, and may in turn slightly modify the preconceptions involved (Alvesson & Sköldberg, 1994).

How we perceive the language of a text, as well as our feelings, mood, wishes, interests, values, attitudes and theoretical perspectives all contribute to form the basis for our understanding of a text (Wormnæs, 2006). Wile it is neither realistic nor desirable to rid oneself of all prior influence, Alvesson and Sköldberg (1994) notes that reflecting upon one's preconceptions in a systematic way can go a long way in improving the validity of interpretations. One should therefore attempt to clarify which of one's beliefs, values and experiences that are relevant to the topic being investigated, as being conscious of one's preconceptions may negate some of their impact on the interpretation of information (Gall et al., 2007).

2.5 Validity of inferences in cognitive psychology

The data from which models in cognitive psychology are constructed mostly originate from studies of cognitively normal people who perform a task such as solving an arithmetic problem (Coltheart, 2001). Another important source of data is studies in cognitive neuropsychology investigating people who are cognitively impaired, due to e.g. brain damage, disease or disability (McCloskey, 1992). In both cases, inferences are based upon measured factors such as the error rates or error types that subjects produce during completion of tasks, differences in completion speeds, imagery from brain imaging techniques, or behaviour that the participants exhibit during task resolution. A few points concerning such inferences will now be discussed.

Single-patient studies have undoubtedly provided useful information in a variety of scientific disciplines. Such studies can be especially useful when the individual studied exhibits unusual traits (Befring, 2007). This type of study is quite common in cognitive neuropsychology, as one investigates the impairment of a brain damaged or otherwise cognitively impaired person. While a description of an impairment can be interesting by itself, it is not uncommon for researchers in cognitive neuropsychology to attempt to extrapolate from their findings onto the general, cognitively normal population. Such inferences are evident in many cognitive neuropsychological studies (e.g. Cipolotti & Butterworth, 1995; McCloskey, Sokol & Goodman, 1986), which raises the question: Given that every human is an individual, how can one claim that one human can represent all of humankind in matters of cognition?

The basis for this endeavour is the assumption that architectures of cognition are universal (Coltheart, 2001). Another example of such logic can be found in the field of medicine: Since humans for the most part are anatomically similar to each other, a bone found in one human can generally be expected to appear in all other humans. However, since cognitive mechanisms are not directly observable, inferences must be based on the observation of behaviour that is somehow qualitatively different from similar behaviour in the cognitively normal. The assumption is then that cognitive impairments can reveal features of the normal mind by way of their absence. The psychological term for such an inference is a dissociation (Coltheart, 2001 ; McCloskey, 1992). However, in order to strengthen the case for such an inference, one should ideally be able to also exhibit the opposite case, thereby making it a double dissociation (Noël, 2001). As an illustration of this logic one can imagine a stereo system that ceases to play music through the left speaker. If the speaker itself and all the cables are intact, this would suggest that inside the system there are separate circuits for left and right sound production. If one subsequently came across a stereo system that would play through the lesft, but not the right speaker, this would be a double dissociation, and would make a stronger case for the inference.

Another point concerning single-case studies is the question of replication. Since the most interesting cases often will be the most unusual ones, it goes without saying that replicating such a study can be quite impossible. If a study is done on a patient with a one-in-a-million injury, it is unlikely for other researchers to come across a similar case. If the investigator doubts a finding in such a case, there is in reality only two possibilities that are open: Either to reject the assumption of the universality of cognitive processes or reject the research methods used (Coltheart, 2001). It therefore falls upon the original researcher to conduct as sound an investigation as possible, so as to minimize the possibility of errors in the study.

Group studies are common in cognitive psychology, the rationale being that one can infer something about underlying cognitive processes by observing behaviour in cognitively normal subjects (Coltheart, 2001). Of course this requires studying more than a few subjects in order to eliminate interference from abnormal subjects, strengthening the case for extrapolating the results to the people in general. However, one notable feature of such studies is that the number of participants often is relatively small (see e.g. Campbell & Xue, 2001; Spelke & Tsivkin, 2001). One reason for this may be the fact that to be able to extract information of the necessary detail level, one must often have subjects complete fairly large sets of tasks, so that the necessary level of measurement sensitivity is achieved. This does, of coures, increase the cost of the study in time and financial resources, thereby limiting its size. Recently, brain imaging techniques have also been employed in group studies, in order to make inferences not only on the basis of behaviour, but also on brain activation. The employment of such techniques also increases complexity of the investigation and the cost per subject further.

Commonly, results are (often implicitly) generalized to a very large population. For example a study of language switching effects on arithmetic involving 26 Chinese-English bilingual students might bear a title such as "language switching in Chinese-English bilinguals" (example from Campbell, Kanz & Xue, 1999), implying generalization to all Chinese-English bilinguals on the planet, regardless of languages known, age and education level. However, the same logic apply to this type of studies as with single patient studies - the assumption that cognitive structures are in principle universal. Also, contrary to single-case studies, replication of group studies is quite possible if they are well conducted and described. Therefore, studies that fully or in part replicate such a study or otherwise supports its results can strengthen the case for the validity of inferences.

Studies using brain imaging techniques face their own set of validity issues, according to the technical limitations of different imaging technologies and how image results can reasonably be interpreted. All studies that employ brain imaging techniques that are referenced to in this thesis used functional Magnetic Resonance Imaging (fMRI) technology. The purpose of fMRI is to allow one to localize brain activity (Desmond & Chen, 2002). The main reason for conducting fMRI studies in neuropsychology is to examine how different tasks affect brain activity. While fMRI, for this purpose, is a great improvement over previously available techniques of brain imaging, it still has shortcomings that may affect validity. Since only a part of the foundation for this thesis rests upon such studies, just a few points related to them will be discussed. For a review of additional factors that may affect the validity of inferences from fMRI images, see Desmond and Chen (2002).

One technically limiting factor is resolution. For each scan, the operator has to set the required resolution, both in time and space. fMRI works by taking a series of two-dimensional snapshots of the brain, that contrasts blood oxygen levels, the logic being that increased brain activity leads to greater demand for oxygenated blood (Desmond

& Chen, 2002). The snapshots are horizontal and vertical "slices" of the brain, in effect creating a three-dimensional grid. The spacing between these slices can be anywhere from 5-7 mm in the brain in general and 1-4 mm in designated areas. Time between snapshots can from 1-5 seconds (Desmond & Chen, 2002). Needless to say, as thoughts move through the brain's millions of cells at ligthning speeds, much can happen in the space of a few seconds - or in between two slices. This limits how specific information that can be gleaned from such a study and thereby the precision of interpretations.

A factor that potentially affects interpretation is *deactivation.* In order to determine which areas are activated during a particular task, it is necessary to btain a baseline reading from the subject so as to be able to compare the "normal" activation patterns to the activation patterns during the tasks (Venkatraman, Siong, Chee & Ansari, 2006). The commonly accepted interpretation of increased brain activity in an area, is that this area plays a part in performing the cognitive task at hand. However, researchers often also observe *deactivation* in some areas, compared to the baseline - an occurence that is not well understood (Raichle et al., 2001; see also Gusnard & Raichle, 2001). As a consequence of this, observed deactivations are not often discussed in the research report (Venkatraman et al., 2006). This is questionable, as even if it should be shown to have no impact on the validity of the study, it may conceal findings that could turn out to be important at a later date.

2.5.1 Neuroscience and education

As noted in the previous sub-chapter, this thesis in special needs education is based on cognitive psychology and cognitive neuropsychology. Consequently a question that needs to be addressed is: Is this a useful combination? Can and should fields related to cognitive science, and particularly to neuroscience, influence educational research in general and special needs education in particular?

According to Willingham (2009), this question is pointless, because even neuroscience has already influenced education at various points in recent history. One of the examples used by Willingham to underline this point is particularly relevant to special needs education: Fairly recent neuropsychological research into the structures that support phonology in the brain, has supported the theory that dyslexia is primarily a phonological disorder, rather than a visual perceptual disorder. This was an ongoing debate in educational research to which neuropsychology contributed towards a resolution. Willingham does, however, point out some challenges to the relationship between these fields of research. He has named these challenges the *goals problem*, the *vertical problem* and the *horizontal problem*.

The goals problem arises from the differing goals of the two scientific disciplines. As with most natural sciences, neuroscience is descriptive, while educational research, like many social sciences, is normative in nature. Neuroscience will therefore never be able to provide the prescriptive solutions requested by educational researchers, which limits the utility of cooperation (Willingham, 2009). To counter this point, one could argue that while neuroscience by necessity is descriptive and while educational research's strong connection to the profession of teaching (with all its political saliency) means that normative research results are often expected from it, this does not necessarily mean that all educational research must be normative, nor that educational research could not in any way benefit from research of a descriptive nature - either in educational research itself, in cognitive psychology or neuropsychology, or in other fields. Also, symbiotic relationships between descriptive and normative diesciplines are not uncommon, the relationship between physics and engineering being but one example.

The vertical problem refers to the difference in levels of analysis used in the two fields. Willingham argues that neuroscience's detailed approach to the study of cognition means specific functions and processes are studied in isolation, disregarding as many factors as possible, so as to keep models simple enough to comprehend. The highest level of analysis in this field is therefore the interaction of function or the mapping of functions onto brain areas. In educational research, the very lowest level of analysis is the complete individual and analysis can be carried out on significantly higher levels; such as school, community, or country. Accordingly, one could argue that not only do analysis levels not overlap, there is quite some distance between the lowest analysis level of educational research and the highest analysis level of neuroscience.

A few counterarguments to this point can be made. Firstly, the lowest level of educational research is not necessarily the complete human individual. Indeed, a fair amount of research is concerned with lower levels of analysis, such as individual strategies or particular behavioural features, such as subvocal speech (e.g. Corkum, Humphries, Mullane & Theriault, 2008; Ostad & Sørensen, 2007). In order for such efforts to be feasible, it is necessary to limit the number of factors taken into account. This does not automatically make such research pedagogically irrelevant.

Secondly, the continuing specialization in and subdivision of research fields, especially within educational psychology and special needs education, is also lowering levels of analysis. As researchers in special needs education examine ever more specific and narrowly defined disorders, interfacing with neuropsychology can be expected to become increasingly fruitful. One example is the very pedagogically relevant educational research conducted into specific language impairment (SLI) and its subcategories (se e.g. Leonard, 1998), to which neuroscience has provided, and quite likely will continue to provide, valuable information.

The horizontal problem concerns the utility of neurological knowledge for educational purposes. As Willingham states, knowledge that e.g. the intraparietal sulcus contributes to number sense is not very relevant to educators and may even be misleading. The latter point is underlined by Mason (2009), who looks with concern upon the number of "neuromyths" that are prevalent amongst teachers, partly due to the misinterpreted neurological concepts propagated by the commercial, non-scientific "brain-based learning" industry. However, Cubelli (2009) points out that it is not necessarily the theories on the physical brain, nor neuropsychological theories on mappings between the physical brain and cognitive functions that are most relevant to education, but the theories of mind that are derived from e.g. cognitive psychology and neuropsychology. These overarching theories of cognitive functions should, Cubelli notes, contribute to improving practice and to increase the effectiveness of the teaching experience.

All in all, one should not blindly expect neuroscience to provide the answers (Willingham, 2009). But neither should brain-related research be disregarded as irrelevant - especially fields such as cognitive neuropsychology can have profound implications for education, rehabilitation, treatment and assessment of people with various disorders (Coltheart, 2001; Willingham, 2009).

3 Numbers and the mind

In the following sub-chapters, some important terms relating to numerical cognition will be discussed, followed by reviews of the three dominating models (or theoretical frameworks) for understanding human numerical cognition. The aim is to evaluate current theoretical platforms for use as a foundation on which to base hypotheses on interaction between language and number in bilingual number processing.

The process by which children come to learn number concepts and mathematical understanding has been a hotly debated issue through much of the history of research on this issue (Geary, 2006). For several decades the Piagetian, constructivist view was the dominating force in developmental psychology (Newcombe, 2002). According to this view, children construct the cognitive mechanisms needed for cognitive tasks from their experiences and so no level of inherent, biological mechanisms to facilitate such knowledge is expected to be present at the moment of birth (Geary, 2006). That a view that leaves such ample grounds for optimism regarding what can be achieved through educational means has had such a large impact on educational thinking is hardly surprising.

This is not so much the case with the rival, nativist view. In the strictest sense such a view postulates that all behaviour observed in a species must have a basis in its biological features. Since complex cognitive processes such as arithmetic and language are unique characteristics of humankind, they must be handled by biological neural components that are unique to humankind (Newcombe, 2002). If this were true, it could mean that children with difficulties in solving arithmetic problems might have such problems because parts of their brain was underderdeveloped. The implications for education would leave little reason for optimism: Changing innate biological features is not normally within the power of educators, so at most one could hope to teach children with such deficiencies some alternative coping strategies or techniques that might limit the impact of it upon the child's future life.

However, neither nativism or constructivism in the purest sense seems to be fully compatible with a number of findings in cognitive research. Meaningful theoretical frameworks for understanding complex processes in the human mind must therefore be found somewhere in between these two extremes (Newcombe, 2002). It must be accepted that both experience and evolved mechanisms motivate and drive human behaviour (Uttal, 2005).

It is tempting to assume that a rigid, nativist view underlies much of the increasing amount of cognitive research that is conducted, as the search for connections between brain and behaviour continues, fuelled by technological advances that allow it to be pursued by ever more sophisticated means. Geary (2006) categorizes several notable names in psychological research, such as Brian Butterworth and Stanislas Dehaene as neo-nativists, as they attempt to tie aspects of number processing to specific locations in the human brain (see e.g. Butterworth, 1999; Dehaene, 1997). However, neither author proposes that mathematical understanding is "pre-programmed" into the brain, although both present compelling arguments to the effect that some basic processes involved in numerical cognition may be innate - and quite possibly not restricted to humans. However, Dehaene (1997) remarks that higher level cognitive functions, such as reading and mathematics, were developed too recently in human history in order for us to have evolved specific brain areas devoted to them. Some of the basics of number processing will be considered in the following sub-chapters.

3.1 Components of number processing

In order to create a hypothesis on the cognitive architecture underlying a higher-level cognitive process, one must first break the larger process down to what one believes to be the individual stages that make up that process. According to Campbell & Epp (2005), the act of solving a simple arithmetic problem can for instance be broken down into the following overarching stages:

1. Converting the stimuli into appropriate internal codes
2. Retrieving or calculating the answer
3. Producing the answer

These stages are, of course, over-simplified - but will suffice as an example for now.

With respect to the first stage, stimuli (in the form of numerosity) can be produced and received in a number of formats. Those that have garnered most interest from researchers are also the ones that are most language-related: Spoken words, written words and written numbers. In the first case, numbers are perceived audibly, while in the other two they are perceived visually. The format of the numbers in the first two cases fall within the category of *number words* (Brysbaert, 2005), as they form part of our normal vocabulary of words. Written numbers can be termed *visual Arabic numbers* (Dehaene, 1992) - a term inspired by the Hindu-Arabic origin of the numeral system. They are distinct from number words because they are not connected to a specific language, and so they do not have their own verbal equivalents - each language associates them with their own number words. Numerosities can, of course, also be perceived by other means, such as:

- Visual magnitudes, e.g. a number of objects
- Audible signals, e.g. three knocks at the door
- Tactile information, e.g. feeling a number of objects

As will become clear in the course of this thesis, the visual magnitude format has also been important to the study of numerical cognition. The latter two have, however, not garnered much interest. In a school setting, an arithmetic task given to a pupil will usually involve stimuli in the language-related formats. If, for example, the task is given in Arabic numeral format, and the teacher demands a verbal answer, the processing stages commence with the conversion of the stimuli into mental representations of numerosity, or codes. The concept of codes, or internal, mental representations of numerosities is fundamental to many theories of numerical cognition and much of the debate surrounding models of numerical cognition has revolved around the number and nature of such codes (for reviews, see e.g. Campbell & Epp, 2005; Noël, 2001).

The three stages outlined above imply three sets of cognitive systems: One set for encoding the perceived magnitudes into the appropriate internal code or codes, one set for the retrieval or computation of the answer and of course one set that mirrors the first set, which converts numerosities from the codes back to responses which allow us to produce the answer in any format we know (Campbell & Epp, 2005).

An example will illustrate this better: If a teacher gives a pupil a task, for instance the arithmetic problem "3 x 7 = " presented on the blackboard as the expression and requests an oral answer to that task, the process could be divided into the following three steps:

1. The pupil reads the problem on the blackboard. The visual Arabic number stimuli is then converted to an internal code in the brain so that it can be accessed by the calculation processes.

2. The problem can now either be calculated, or the answer retrieved if it was previously learned by rote. In this example, we will assume that the pupil is familiar with multiplication tables. Therefore the answer, 21, is retrieved rather than calculated anew.

3. The answer is converted from the an internal code to verbal output, upon which the pupil speaks the answer out loud.

While this example illustrates the logic of building a model, a proper model of numerical cognition would, of course, have to be able to make specific predictions on the nature of internal codes and how individual subsystems relate to these and communicate with one another. The three overarching stages must therefore be broke down to a number of specific processes and steps which can explain findings in behavioral and other studies of how humans use and manipulate numerosities.

Next, some basic number processes are discussed, as these concepts are important to the discussion in the following chapters. Firstly a group of basic processes that can be termed quantification processes are considered and thereafter the concept of arithmetic facts storage and retrieval.

3.1.1 Quantification processes

Extensive psychological research has resulted in a number of terms to describe how humans determine quantities. Counting, subitizing and estimation are basic processes that can be termed "quantification processes", as they are processes whereby the brain assesses quantities (Dehaene, 1992). However, borders between these concepts are not clearly established and the terms therefore tend to overlap. Indeed, there is some controversy regarding wether they should be considered separate processes at all (see e.g. Mandler & Shebo, 1982; Revkin, Piazza, Izard, Cohen & Dehaene, 2008). These terms and the controversies surrounding them will now be discussed.

Counting is arguably the most immediately obvious number process. Arising from a need to quantify and categorize the world around us, counting is common to all types of societies, although the type and extent of numerical words, visual aids and notations employed to this end vary considerably, both historically and in different cultures, different points in history (Butterworth, 1999; Geary, 2006).

Subitizing may or may not be related to counting. The term was coined to describe the rapid, precise and seemingly effortless enumeration of small sets of objects (Mandler & Shebo, 1982; Revkin et al., 2008). This apparent feature of humankind has puzzled psychologists for almost 100 years, still the question of what cognitive mechanism that underlies it remains debated (Revkin et al., 2008). In the meantime, there has been a number of studies of infants and animals, whose results suggest that the ability to enumerate small sets of objects is an innate feature that is not limited to humans (Dehaene, 1997). A classic study of numerosity in animals by Meck and Church (Meck & Church, 1983), resulted in a theory which attribute this ability to a quick, non-verbal counting mechanism. This theory served as a basis for further animal experiments by Gallistel and Gelman (1992), who generalized their findings to humans. They suggested that this non-verbal counting mechanism serves as a foundation for the later acquisiton of verbal counting skills.

One difficulty of demonstrating that different processes underlie subitizing and counting, is that if subitizing is a separate process from counting that only applies to small numerosities, one should be able to identify the precise point at which subitizing stops and counting takes over. This point has been placed anywhere between 3 and 6 by different researchers at different points in time (Mandler & Shebo, 1982). However, reaction time studies - where participants are shown a collection of objects (e.g. a cluster of dots) for a short time and then asked to estimate the quantity as quickly as possible, show that there is a significant drop in reaction time once the quantity increases to 3 or 4 (dependent on the design of the individual studies), suggesting that the threshold where other processes take over is somewhere in the vicinity of these numbers (Dehaene, 1992; Mandler & Shebo, 1982).

Estimation in combination with approximate calculation may be the number processes that are most frequently employed in our everyday lives, as we try to estimate how much to pay for the pair of trousers that are on sale for 30% off, to judge wether we can afford it, how much is left in our wallet and so on (Siegler & Booth, 2005). In contrast to subitizing, estimation seems to be a learned process, closely linked to mathematical proficiency (Geary, 2006; Siegler & Booth, 2005). However, estimation has also been the subject of some controversy. Like subitizing, it can be viewed as a form of counting in which a faster and less accurate method is employed than in normal, precise counting (Dehaene, 1992).

In their classic study in numerical cognition, Mandler and Shebo (1982) tasked participants with estimating numbers of different objects on paper while timing their responses and noting errors. They found that their participants could rapidly and precisely enumerate magnitudes up to 3. From 4 and up to 6 or 7 reaction times increased sharply and the precision decreased somewhat - the participants started to produce errors. From 7 and up reaction times did not drop as much, but precision decreased significantly and the higher the numbers, the less precise were the estimations. They therefore suggested that subitizing, counting and estimation are separate cognitive processes and that the determining factor of which process was employed was the size of the numerosities involved: When faced with numerosities no greater than 3, the participants subitized. From 4 to 6 or 7 they counted and from 7 and up the quantities were too time-consuming to count under the speed requirements imposed by the researchers, so the participants resorted to estimation.

In line with these and other findings, it has been hypothesized that the process of estimation follows Weber's law of stimuli and perception (also called the Weber-Fechner law): It becomes increasingly inaccurate, logarithmically proportional to the size of the numerosity involved (Dehaene, 1992; Dehaene, 1997; Izard & Dehaene, 2008; Revkin et al., 2008). This means that as the number of objects to be quantified increases, the number estimated will be increasingly further off the mark. Also, when estimating the answer to a mathematical problem, thereby performing approximate calculation, the same effect of number size on accuracy has been observed. Also a "distance effect" has been observed, which means that the accuracy of approximate calculation also decreases with the distance, numerosity-wise, between the numbers involved (Dehaene, 1992). In other words, estimating 42 + 56 generally produces a more accurate result than 25 + 71.

While arguments have been presented to the effect that subitizing is a separate process from counting, this does not necessarily mean that it is a separate process from estimation. Since estimation increases in precision with lower magnitudes, it is tempting to assume that numbers from 1 to 3 can be estimated unfailingly and that subitizing therefore is nothing more than estimation of small numerosities. However, this hypothesis has been questioned by a recent study by Revkin and colleagues (2008), who administered a number of timed numerosity comparison and naming tasks to 18 young adult subjects. Their results showed that there was no logarithmic regularity to the variations in error rates as a function of the variations in magnitudes in the stimuli, when the numerosities involved were 8 or less. For numerosities above 10 the error rates did, as expected, follow Weber's law. While this does not disprove that the estimation process may extend to small numerosities, it does add to the evidence that there is a separate cognitive process available for enumerating small magnitudes.

3.1.2 Arithmetic facts

Arithmetic facts or number facts are often defined as simple arithmetic problems that are stored in memory and the answer to which can then be directly retrieved without calculation (Zaunmüller et al., 2009). It therefore seems to be geared towards easing or avoiding the effort of computation. Knowledge of arithmetic facts is fundamental to arithmetic competence, not only because it increases the speed whereby one can solve simple arithmetic problems, but also because retrieving simple arithmetic facts improve performance on more complex arithmetic tasks (Holmes & McGregor, 2007).

Committing arithmetic facts to memory is often termed *rote learning* (Dehaene, 1992), or in more common terms "learning by heart". This strategy for learning mathematics has has traditionally been actively encouraged by school systems in many countries, at least for certain types of arithmetic operations (McCloskey & Macaruso, 1995). Most notable is arguably multiplication, as most of us can attest to having spent endless school hours memorizing multiplication tables.

Some terms important to this thesis have now been clarified. The following sub-chapters contain overviews over the three most well-known theoretical frameworks for understanding mathematical cognition.

3.2 The Modular Theory

In 1985, McCloskey, Caramazza and Basili proposed a new theoretical framework for research on cognitive processes. They proposed that not only data from cognitively normal subjects, but also from cognitively impaired subjects such as brain damaged patients should serve as the foundation upon which to generate theories of cognitive processes in the general population. This was proposed under the assumption that cognitive impairments should logically reflect the functioning of normal cognitive systems in which components have been damaged (McCloskey et al., 1985).

In keeping with this proposition, they developed a schematic model of the relationship between cognitive processes involved in number processing and calculation. As functional brain imaging technology was not available at the time, it is primarily based upon behavioural data from case studies. One particular case is discussed in order to show how the model can explain the strange impairments in numerical cognition that sometimes appear in brain-damaged patients. This model and the ideas upon which it was based, became quite influential in the field (Cipolotti & Butterworth, 1995).

McCloskey (1992) further expounded the model and its empirical foundation. As can be seen from fig. 1, the model consists of four major parts: Numeral Comprehension Mechanisms, Calculation Mechanisms, Numeral Production Mechanisms and the Abstract Internal Representation. In the model, components of number processing are organized in the following way (McCloskey, 1992; McCloskey et al., 1985; see also McCloskey & Macaruso, 1995):

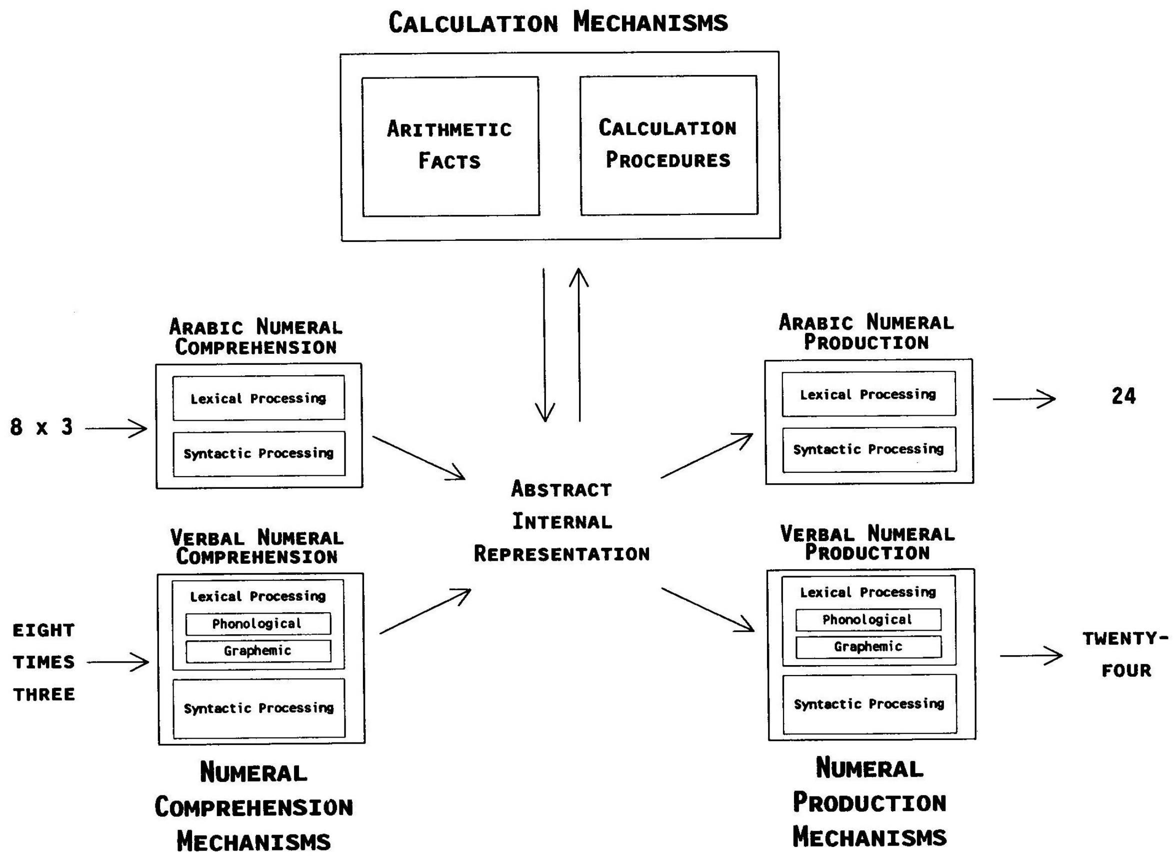

Figure 1: The major processing components of the Modular Theory (from McCloskey, 1992, p. 113)

The Abstract Internal Representation is, according to McCloskey (1992), the mental number representation format in which the processing mechanisms, or modules, communicate with one another. This is also where semantic information, or meaning, connected to the numerosity is retrieved. In this representation, or code, numbers are represented as powers of ten; so that e.g. the number 21 is represented as {2}10EXP1, {1}10EXP0, or in other words: Two times ten to the power of one, plus one time ten to the power of zero.

Numeral Comprehension Mechanisms handle input and are divided into two parts. One for Arabic numbers and one for number words. Each mechanism contains a module for lexical processing and one for syntactic processing (e.g. McCloskey, Caramazza & Basili, 1985; McCloskey, 1992), so that spoken or written numbers are recognized. The comprehension mechanisms encode the numerosity into the Abstract Internal Code for retrieval of semantic information and further processing by other modules (ibid.). Encoding or comprehension mechanisms for numerosities perceived in formats not related to language, such as visual magnitudes, are notably absent from this model - it is strictly limited to the processing of stimuli presented in speech or writing. While language does provide a large part of the stimuli that activate processes in numerical cognition, especially in formalized settings, such as schools, the lack of processes to handle input in other formats limits the model's utility as a general model of numerical cognition somewhat.

Production Mechanisms mirror comprehension mechanisms and are responsible for producing numerical output - i.e. converting numerosities from the abstract internal code into the desired output format for expression in a modality of language (McCloskey et al., 1985; McCloskey, 1992).

Calculation Mechanisms are where the actual mathematical processing takes place. McCloskey (1992) suggests that this mechanism is further divided into two modules, each responsible for an aspect of calculation: One is responsible for the storage and retrieval of arithmetic facts, the other contains calculation procedures, which allows one to calculate answers to arithmetic problems that can not be retrieved from the arithmetic fact storage. Amongst the cases discussed by McCloskey, Caramazza and Basili (1985) is the case of one M. W., a highly educated person who suffered brain damage. As a result he could not retrieve arithmetic facts directly. This was demonstrated by asking him to (amongst a number of other tasks) calculate 7 x 7. He then performed the easier computations of 7 x 10 and 7 x 3 and subsequently subtracted one from the other, to come up with the correct answer. In addition to demonstrating his difficulties in retrieving arithmetic facts as compared to calculating the answers, This also demonstrated that he understood the principle of multiplication perfectly, excluding the possibility of impairments in the mechanisms for calculation.

One major criticism of the Modular Theory is the lack of an asemantic transcoding route (McCloskey, 1992; Noël, 2001). As noted, in the Modular Theory, semantic information is retrieved prior to any further processing. Therefore, the model predicts that one cannot process or in any way manipulate numbers mentally without connecting them to their semantic meaning. Most controversially, numbers cannot even be repreated verbally without such retrieval taking place (Noël, 2001).

An asemantic transcoding route would enable numbers to pass directly from the input systems to the output systems, postulating that people can receive numbers and output them - in the same format, or a different one - while recognizing the numbers but not their meaning. However, as Noël (2001) remarks, providing evidence in the form of a double dissociation in this case might prove challenging. This is because it would require demonstrating both cases where the asemantic transcoding route can be assumed to be preserved when the semantic route is damaged, as well as cases that suggest that the asemantic route is damaged while the semantic route is preserved.

While showing that someone can transcribe numbers from one format (e.g. number words) to another (e.g. Arabic numerals), without being able to attribute meaning to the numbers is quite possible, to show that someone only can transcode numbers by attaching meaning to them could prove difficult. The reason is that it could be challenging to prove that transcoding only proceeded by the semantic route and not by the asemantic route.

3.2.1 The Multiroute Model

Cipolotti and Butterworth (1995) investigated the cognitive impairments of S. A. M., who suffered from an unknown progressive neurological degenerative condition. Through an extensive battery of tests, Cipolotti and Butterworth found that S. A. M. could understand numbers and number words, both oral and written. He could also compute mathematical problems and produce the answers to them. However, he could not read numbers or number words from paper and immediately speak them out loud, neither could he, when asked, directly write down numbers or number words that were spoken to him.

This does not necessarily provide the other end of the double dissociation with respect to asemantic transcoding. Since *all* transcoding was impaired in S. A. M., and number output therefore had to occur through calculation, the case does not provide a clear separation of semantic and asemantic routes. However, it does show that transcoding and computing processes are separate, something that cannot easily be accommodated by the Modular Theory - since in that model both tasks involve the Abstract Code. Therefore, Cipolotti and Butterworth decided to add an asemantic transcoding route to the model, in the process dubbing it the "Multiroute Model", inspired by similar models of language processes.

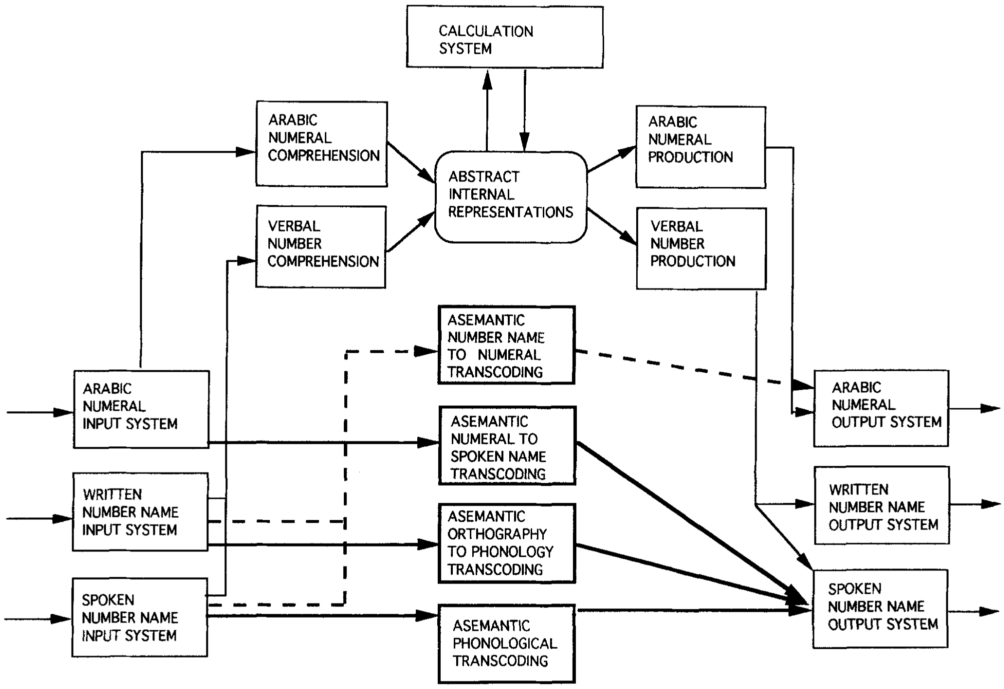

Figure 2: The Multiroute Model (from Cipolotti & Butterworth, 1995, p. 386)

While dissociations in cognitive neuropsychology may have difficulty providing evidence for an asemantic transcoding route, a number of studies of cognitively normal subjects have provided results that point towards asemantic transcoding. One of the more novel approaches was one used by Fias (2001). He considered a phenomenon termed the *SNARC effect*, SNARC stands for "Spatial-Numerical Association of Response Codes" and is a term coined to describe an intriguing fact that has been observed in the behaviour of subjects in cognitive studies: Namely that right-handed subjects tend to respond more quickly to number tasks involving large numbers with their right than with their left hand (Dehaene, 1992).

Fias (2001) reasoned that since the SNARC effect seems to be related to the numerical value of the stimuli, it must signify that the mind accesses semantic information (in this case number value) for the numbers. He consequently hypothesized that if a non-semantic transcoding pathway was used for a task, the subjects would not exhibit a SNARC effect during that task - their response times would not be affected by nuber size using either hand. His subjects were therefore given a number word parity judgement task, which was hypothesized to require semantic access in order to determine if the numbers were even or odd, as well as a simple number word naming task. Consistently, the right-handed subjects responded quicker to the parity judgement task with their right hand as the numerosities increased, while no such difference was noted for the transcoding task.

A series of experiments conducted by Ratinckx, Brysbaert and Fias (2005) expanded these findings to the domain of Arabic numerals. The experiments vere performed on separate groups of first-year psychology students and consisted of a variety of timed two-digit Arabic number naming tasks specifically designed to probe for the existence of asemantic transcoding. This group of reseachers had previously defended McCloskey's (1992) view that there is no asemantic transcoding and conducted some studies that provided evidence to this effect (e.g. Brysbaert, 1995). However, Ratinckx et al.'s (2005) findings combined with the findings of Fias (2001) forced them to reinterpret the results of their earlier studies and eventually reconsider their position.

3.3 The Encoding-Complex Hypothesis

A radically different theoretical framework for understanding numerical cognition was proposed by Campbell and Clark, as they disagreed with several underlying premises of the Modular Theory (Campbell & Clark, 1988; see also Campbell, 1994; Campbell & Clark, 1992; Campbell & Epp, 2004; Campbell & Epp, 2005). They formed the Encoding-Complex Hypothesis upon examining data from a study by McCloskey, Sokol and Goodman (McCloskey et al., 1986). McCloskey et al. investigated two brain-damaged patients by having them perform a large number of number naming tasks. They then used their findings to make inferences on the nature of verbal number production. However, Campbell and Clark (1988) noted that there seemed to be additional patterns to the errors produced by one of the participants, H. Y., that were not addressed by the authors. When making errors in number naming, i.e. speaking the wrong number word when confronted with Arabic number stimuli, H. Y. seemed to prefer number words that corresponded to Arabic numbers related to the correct number, either by visual similarity (e.g. 5 instead of 8), parity (even or odd, e.g. 14 instead of 16), or numerical proximity (closeness in value, e.g. 15 instead of 16). When McCloskey and colleagues' data were re-analyzed in light of this hypothesis, Campbell and Clark found that the effect of these three variables together accounted for 50% of the variability in error percentage across stimuli numbers.

The main characteristic that sets the Encoding Complex Hypothesis apart from other models is that it does not assume that cognitive functions are organized in an additive way (ibid.). The Modular Theory assumes that there is a particular sequence in which cognitive processes must occur, so that each process is added to the next in a chain of events. It also assumes that the use of one cognitive sequence excludes the use of another. For example, in the Multiroute Model number naming must either proceed via the semantic or the non-semantic Arabic number to number word transcoding route. The routes are mutually exclusive (Campbell & Clark, 1992; Campbell & Epp, 2004; Campbell & Epp, 2005).

Finding the Modular Theory's linear processing chain incapable of explaining the apparent interference from related numerosities in the case of H. Y., Campbell and

Clark (1988) postulated that any number of processes and transcoding pathways can be active simultaneously, even for the same number task, thereby potentially interfering with one another. This interference account H. Y.'s errors, as well as for some common types of errors in arithmetic, such as operand intrusion errors (Campbell & Clark, 1988; Campbell & Clark, 1992; Campbell & Epp, 2004). Operand intrusion errors are where one of the numbers in an arithmetic problem erroneously makes it into the answer, so that one sometimes responds 17 when asked to calculate 2 x 7. In that example some calculation or fact retrieval has obviously taken place, since the answer is only around 0,5 times off the mark. However, one of the operands, 7, is carried over to the answer. This can be interpreted as meaning that at least two processing routes are active simultaneously, and that the error is due to insufficient inhibition of the less suitable route (ibid.).

Campbell & Clark (1988; 1992) also opposed the idea of a sentral, semantically mediated representation of number and instead proposed that the mind represents numerosities in multiple ways, each corresponding to an input format such as visual input through Arabic numbers, verbal input through speech and so forth. They further suggested that in the non-impaired mind, previous experience with a type of task contributes to the integration of the processes involved in it, strengthening the transcoding pathways between the cognitive processes involved. Some processing routes which have proven to be more suitable gradually come to be activated more strongly for some types of tasks. They are, however, still prone to some interference from other, weaker routes. Campbell and Clark use the term *encoding-retrieval integration* for this aspect of cognition. An impaired mind, where some of the networks are damaged, may have to rely on other, weaker routes that are more prone to interference. This would explain the increased number of number-naming errors by H. Y., as well as the recorded error patterns.

These assumption is not only quite radical as compared to the Modular Theory, but also have interesting pedagogical implications, as it suggests that the integration of processes can be systematically improved by external stimulation, such as practicing h a type of task.

The human brain is arguably the most intricate organ ever evolved and there can be little doubt of the immense complexity of higher-level cognitive functions. The flexibility of the Encoding-Complex Hypothesis is therefore quite appealing, as it seems to be able to accomodate almost any observable pattern of behaviour related to numerical cognition. However, as McCloskey (1992) and Fias (2001) points out, it does not lend itself easily to predictions, such as of the effects of loss of function due to brain damage. The abstract nature of the hypothesis means that one cannot easily form hypotheses of how manipluating one component will affect the performance of others. In order for it to generate testable predictions, the hypothesis needs to be constrained, by further specifying and defining the concrete processes involved in numerical cognition and their interaction, thereby developing it into a more specific model (McCloskey, 1992; Fias, 2001).

3.4 The Triple-Code Model

Inspired by the Encoding-Complex Hypothesis, another approach to understanding numerical cognition was proposed by Dehaene (1992, see also Dehaene & Cohen, 1995). Accordingly, this model has some features that bear similarity to features of the Encoding-Complex Hypothesis. As with the Encoding-Complex Hypothesis, a major difference in this model, as opposed to the Modular Theory, is that it is not a model of connected processing modules as such, but rather a model of interaction between internal representations of number. However, Dehaene imposed a number of restrictions. For instance, the notion that numerosities are represented internally in different ways, which correspond to "surface formats" or formats which numerosities are perceived and transmitted in was kept, but the Triple-Code Model limits the number of such codes to three: One each for modalities of language, object group recognition and Arabic number recognition - hence the name of the model. The codes are termed the analogue magnitude code, the visual Arabic number form and the auditory verbal word frame (Dehaene, 1992; Dehaene & Cohen, 1995; see also Dehaene, Piazza, Pinel, & Cohen, 2003).

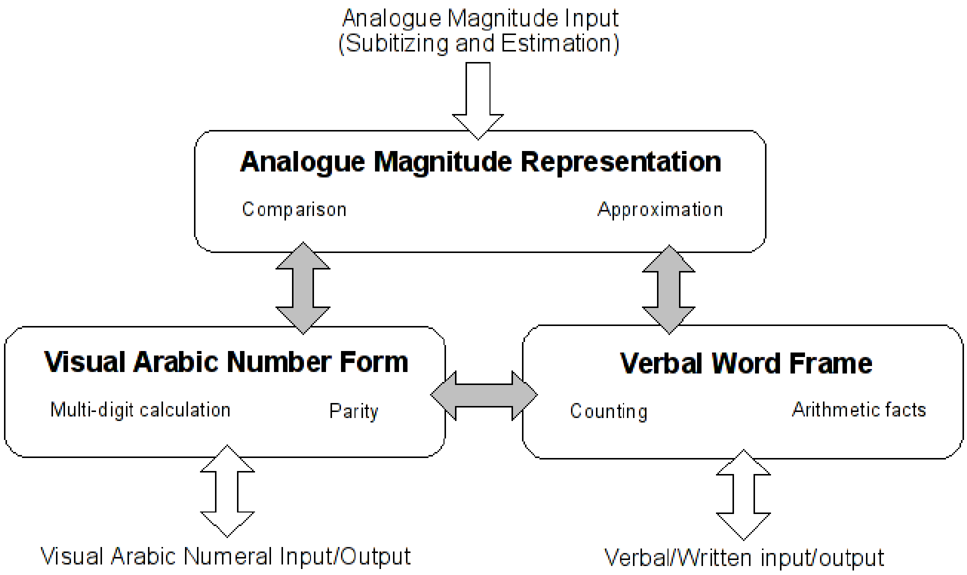

Figure 3: A simplified visualization of the Triple-Code Model (after Dehaene & Cohen, 1995)

The model was first conceived as a functional cognitive model, but later an attempt was made to map the mental codes onto physical brain areas, thereby turning a model of mind into a tentative model of the neurological cicuits involved in numerical processing (see Dehaene & Cohen, 1995; Dehaene et al., 2003). However, as noted in chapter 2.5.1, anatomical postulations are of limited interest to educational research as these do not necessarily affect the functional aspects and therefore have limited pedagogical implications. However, the anatomical postulations have inspired a number of researchers to test the anatomical postulations using brain imaging technology, which has uncovered new knowledge that is also applicable to the functional postulations (e.g. Dehaene et al., 2003; Schmidthorst & Brown, 2004; Venkatraman et al., 2006). Some implications of such studies will be discussed later.

In fig. 3, the codes themselves are represented by the boxes, while the arrows represent transcoding pathways. Dehaene (1992) assumes that each code subserves specific input and output processes and that different types of numerical processing and memory is mediated by different codes. Therefore, numerosities must be converted into the appropriate codes before processing. In the non-impaired mind, numerosities are assumed to be converted directly between the codes as required.

However, multiple conversions is predicted to reduce processing speed (ibid.). Note the direct transcoding path beween the visual Arabic number form and the verbal word frame, which provides the asemantic transcoding path lacking in the Modular Theory. Dehaene (1992) goes on to tentatively specify which number processes are mediated by which codes and these postulations will now be considered:

The analogue magnitude representation subserves subitizing and approximate input by way of estimation (ibid.). It also mediates number comparison and approximate calculation processes. With it, the Triple-Code introduces a feature not present in the Modular Theory-based models - namely input processes for numerosities not conveyed by way of a modality of language (written or spoken numbers). The processes postulated to be mediated by it are therefore predicted not to be tied to language.

Importantly, Dehaene (1992, see also Dehaene & Cohen, 1995) assumes that the analogue magnitude code is the only code which mediates access to semantic information - a considerable restriction of the role of semantics compared to the Modular Theory. The assumption is that numerosities in this code are represented on a mental "number line", which is directional and roughly follows the direction of writing. Furthermore, it becomes increasingly more imprecise as magnitudes become larger according to Weber's law, thereby accounting for the distance effect (Dehaene 1992; Dehaene and Cohen, 1995; see also Izard & Dehaene, 2008).

The visual Arabic number form subserves visual Arabic numeral input and output processes (Dehaene 1992). Within the code, numbers are represented as strings of digits on a visuo-spatial sketchpad, so that the number "24" is recognized as consisting of the two separate digits 2 and 4 (ibid.). It is, however, postulated to mediate processes related to visualization of Arabic numbers, such as parity judgement and precise multi-digit calculation. Note, however, that in Fias' (2001) study, which was discussed in the previous sub-chapter, parity judgement was assumed to involve semantic information and this task type was shown to be subject to the SNARC effect. This is expected by Dehaene (1992), who considers the visual Arabic number form to be the primaty mediating factor in parity judgement, but that the process still requires activation of the magnitude code.

The name and nature of the Arabic number form implies that in order to generate hypotheses on language and number process interaction in users of additonal numeral systems, such as Khmer or Mandarin numerals, the model may need to be expanded with additional codes subserving the additional input and output processes, although this possibility is not explicitly discussed in the articles describing the model.

The verbal word frame subserves, according to Dehaene (1992), verbal and written input and output processes and the processes it mediates is therefore directly linked to language use. It is created and manipulated using processes also associated with language processing. In it, analogues of number word sentences (e.g. "forty-three") are represented. Number words in written or spoken form are recognized as such by processes within this code. The code is not postulated to mediate any semantic information retrieval. It is, however, assumed to mediate storage of learned, automated mathematical processes such as counting and arithmetic facts, which are therefore predicted to be dependent upon language (ibid.). This prediction, combined with the assumption that multi-digit operations are mediated by the Arabic number form, combines to postulate that all exact number processing is dependent upon language. This notion has been the subject of considerable controversy (Campbell & Epp, 2004) and will therefore be discussed in more detail in chapter 4.

4 Language and number processing

In the previous chapter three models or theoretical frameworks for understanding numerical cognition were reviewed. In this chapter their predictions regarding language effects on numerical cognition are discussed and relevant research is considered.

4.1 Language effects in the Modular Theory

One immediately apparent feature of the Modular Theory (see McCloskey, 1992; McCloskey & Macaruso, 1995; McCloskey, Caramazza & Basili, 1985; McCloskey, Sokol & Goodman, 1986), is that it does not model any input or output processes that are not language based. Conceivably, any number of additional comperehension and output processes could be added to the model in order to account for additional languages or numeral systems, however, this aspect is not explicitly discussed by McCloskey and colleagues.

The only process that is directly related to language is the comprehension process - as soon as input is comprehended, it is converted into the Abstract Internal Representation, from which it can be accessed by calculation mechanisms or output encoding processes (McCloskey, 1992). The model therefore does not predict any performance differences for arithmetic or number naming tasks between different input and output formats (languages or numeral systems) which cannot be attributed to differences in comprehension. Accordingly, the role of language in cognitive number processing is assumed to be quite restricted.

Research on bilinguals and number processing suggest, however, that other factors than comprehension may play a part. Frenck-Mestre and Vaid (1993) tested 15 american exchange students at a French university in mental arithmetic tasks, while varying input format between Arabic numbers and the two languages. They found significant variations in response times according to input format, both between Arabic numbers and languages, as well as between languages. Studies by amongst others Campbell et al. (1999), Bernardo (2001) and Spelke and Tsivkin (2001) have also found considerable difference between response times and error patterns between formats, when comprehension problems were not an issue. Additionally, a study by Meuter and Allport (1999), that was replicated and extended by Campbell (2005), found that the response time on number naming tasks increase more when bilingual subjects are required to switch their output from their second language to the first, than vice versa. This finding is surprising; one would expect the opposite to be true, as Meuter and Allport indeed did. Also, the Modular Theory is incapable of explaining this apparent contradiction.

Also, the postulation of a central unit for number processing means that the Modular Theory predicts strong transfer of learning between formats. Any type of mathematics ability should transfer from practice in one format to the next, so as long as comprehension is not an issue, number tasks learned in one format should generalize to all other formats. However, not only have studies by amongst others Campbell et al. (1999), Spelke and Tsivkin (2001) and Venkatraman et al. (2006) found that performance on number tasks in bilinguals vary with both format and task type. In addition, both Venkatraman et al. (2006) and Spelke & Tsivkin (2001) trained their bilingual subjects in unfamiliar task types in both languages and observed that the subjects did not perform as well on some types of tasks in the language in which they were not trained, regardless if this was their first or second language. Neither the Modular Theory nor the Multiroute model seems to be able to account properly for any of these findings and are therefore in their current state not well suited to make predictions on bilingual number processing performance

4.2 Language effects in the Triple-Code Model

Compared to the Modular Theory, the Triple-Code Model's predictions on the effects of language upon numerical cognition is markedly different. The Triple-Code Model is, as noted in chapter 3.4, characterized by a significantly different distribution of number processes. The number processes are thought to be distributed across different codes, rather than to be collected into a central calculation unit (Dehaene, 1992). Therefore, communication between different processes requires transcoding of numerical information (ibid.). The implication is that if a task requires the employment of processes that are mediated by another code than the one corresponding to the input format, transcoding between the codes must take place. This is also true if stimuli need to be converted to another format for output, e.g. from Arabic numeral to verbal number name. Since different input and output formats are linked to different codes, an important prediction of the Triple-Code Model is that number processes will vary in sensitivity to linguistic factors according to which codes they are mediated by.

The analogue magnitude code is assumed by Dehaene (1992) and Dehaene & Cohen / 1995) to be the only code not subserving an input system that is associated with an aspect of language - as it subserves input processes that enumerate "analogue magnitudes" in the form of groups of objects. It is held to mediate approximate enumeration and calculation processes, such as estimation, subitizing and comparison. These processes are therefore held to be fully language-independent. Therefore the model predicts that people who lack linguistic concepts of number can still perform some types of number processing. Several studies have demonstrated cases of people with impaired language abilities due to aphasia, developmental disorders, or other cognitive impairments who nonetheless have preserved number processing abilities (for reviews, see e.g. Butterworth, 1999; Dehaene, 1997). Also, there are studies that have demonstrated that both pre-verbal children (e.g. McCrink & Wynn, 2004) and animals (e.g. Meck & Church, 1983) have rudimentary number processing abilities.

The two remaining codes can be considered to be dependent upon language in one form or the other: The verbal word frame, which subserves input and output in the form of number words and the Arabic visual code, which subserves written Arabic numeral input and output (Dehaene 1992, Dehaene & Cohen, 1995). These codes mediate the remaining number processes, such as Arabic digit operations, parity judgement, counting and arithmetic facts learned by rote memorization such as multiplication tables (ibid.). In other words these codes between them mediate all the processes necessary for exact calculation.

While the independence of some number processing from language is fairly well established, the idea that other number processes are dependent upon language is open to contention (see e.g. Noël, 2001; Gelman & Butterworth, 2005). Some interesting studies that have been conducted in the time since the Triple-Code Model was conceived, are studies of indigenous Amazonian peoples, whose languages are extremely lacking in number words: The Mundurukú (Pica et al., 2004) and the Pirahã (Frank et al., 2008; Gordon, 2004). The Mundurukú do not have specific linguistic concepts of magnitudes larger than 5, while the Pirahã only have words for 1 and 2 and do not use even those consistently. Still, studies of both people show that the tribespeople have well-developed estimation and comparison skills, enabling them to successfully estimate and compare the numerosities of large quantities of objects. However, precise calculation seems to be beyond their abilities. These findings seem to provide support for Dehaene's (1992) postulation that out of a variety of cognitive functions related to numerosities, some are mediated by language and some are not.

It should be noted that the fact that the Amazonian tribes do not calculate precisely and possess only rudimentary language-based concepts of number, does not necessarily mean that these two factors have a causal relationship. For instance, it may be that their inability to calculate stem from a lack of practical necessity for calculation or from a lack of emphasis on calculation in their culture, rather than the lack of linguistic concepts (Gelman & Butterworth, 2005).

However, whatever the ontogenetic status of calculation in relationship to language may be, there are, as noted in the previous sub-chapter, several studies that demonstrate that switching of language as well as other formats affects at least some types of number processing performance. Spelke & Tsivkin (2001) tested Russian students at an American university who were trained in unfamiliar number tasks detected a difference in performance that favoured the language in which they had received training - but only for exact calculation tasks. In approximate calculation tasks no difference was detected. Unfortunately, the different parts of the study was conducted on different groups of subjects and the number of participants in each part was quite low (less than ten for each experiment). However, Venkatraman et al. (2006) also conducted a bilingual training study on 20 subjects, which replicated Spelke & Tsivkin's findings and even detected a variance in approximate tasks as well - thereby providing support for the notion of language effects upon number processes.

Accordingly, while being able to predict language effects on numerical cognition, the Triple-Code is unable to account for the differences in performance across languages in bilinguals, due to the fact that it does not model interaction between more than one language and number processes.

4.3 Language effects in the Encoding-Complex Hypothesis

In an attempt to demonstrate how a specific, functional model can be constructed on the basis of the Encoding-Complex Hypothesis and in so doing address criticism from amongst others McCloskey (1992) and Fias (2001), Campbell and Epp (2004) combined the structure of the Triple-Code Model with the underlying assumptions of the Encoding-Complex Hypothesis, in order to create a model that would provide a theoretical framework which could accomodate findings from an earlier study by Campbell and colleagues (1999). This endeavour is of particular interest to the subject at hand, because the participants in the study were bilinguals. Being Chinese-English, they also used two numeral systems, Mandarin and Arabic. The model therefore takes additional formats that are relevant to bilinguals into account that are not considered in the other models.

The participants in Campbell et al.'s (1999) study were 26 Mandarin or Cantonese-English bilingual university students. They were of Chinese origin, so Chinese was their first language, but they studied subjects in English in Canada, meaning that they spoke good English. They were tested on a number of simple arithmetic tasks, comparison tasks and number naming tasks, with stimuli presented in either Arabic or Mandarin numerals. Output was varied between Chinese or English. In general, significant variation in response times and error rates between stimuli and output formats were found, consistent with earlier findings by Frenck-Mestre and Vaid (1993). Also, performance varied according to task type across formats - some types of tasks were more efficient in some format combinations than others (Campbell et al. 1999, Campbell & Epp, 2004).

Campbell and Epp (2004) then developed a schematic of the interaction between codes and number processes in the Chinese-English bilinguals (fig. 4). Because of the previously mentioned similarities between the some of the postulations of the Encoding-Complex Hypothesis and the Triple-Code Model, they decided to adapt the Triple-Code Model to accomodate the additional language and numeral system, as well as those underlying assumptions of the Encoding-Complex Hypothesis that are not present in the Triple-Code Model - such as variability of encoding-retrieval integration strength and the simultanous activation of multiple routes for each processing task (ibid.).

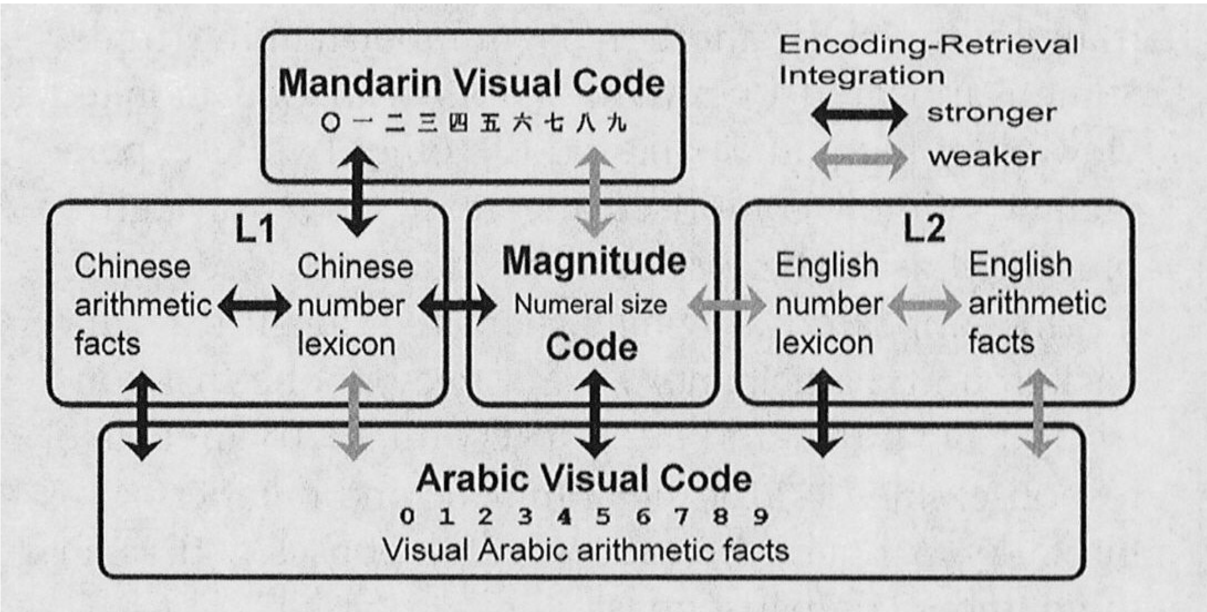

Figure 4: An Encoding-Complex model of number processing in Chinese-English bilinguals (from Campbell & Epp, 2004, p. 231)

In the Triple-Code Model, numerosities can be converted directly from any code into any other code. However, as can be seen from fig. 5, Campbell and Epp departed from this principle upon installing the additional verbal and number codes.

Specifications of interactions between the arithmetic fact retrieval and lexical information in each language code were also added, in order to explicitly illustrate number fact retrieval. These associative pathways enables the model to illustrate the level of encoding-retrieval integration of the number fact storage in each language to other processes.

In line with the Encoding-Complex Hypothesis' postulations of variable strength in transcoding and associative pathways, the arrows have different colours according to strength. Naturally, the Encoding-Complex Hypothesis assumes that the level of integration varies on a continuous scale and not in graded steps. However for the sake of simplicity, only two colours were used in the depiction: Grey arrows illustrate weak integration of the interfacing processes, black arrows represents strong integration (ibid.).

The magnitude code in Campbell end Epp's model corresponds very much to Dehaene's (1992) definitions of the analogue magnitude representation. Functionally, it subserves visual magnitude input processes and mediates access to semantic information, comparison, approximation, subitizing and estimation - all the while representing numbers on a logarithmically compressed number line. Campbell and Epp also agree with Dehaene's notion that the brain's ability for such representation serves as the foundation for human numerical ability.

The language-dependent codes (L1 and L2) mediate, according to Campbell and Epp, the lexical and syntactic information connected to each number and the arithmetic fact storage and retrieval processes. Therefore they function in much the same way as the verbal word frame of the Triple-Code Model (Dehaene, 1992; Dehaene & Cohen, 1995). Campbell and Epp's model does not predict any direct pathways between the two language-dependent codes. Therefore, conversion between these codes can proceed along two paths: Via the magnitude code, thereby involving retrieval of semantic information, and via the or the Arabic visual code, involving visualization of the corresponding Arabic numbers.

An increasing body of research has investigated Arabic to verbal number transcoding (e.g. Cipolotti & Butterworth, 1995; Cohen, Dehaene & Verstichel, 1994; Fias, 2001; Ratinckx et al., 2005). However, there has been little research investigating transcoding between verbal codes as compared to transcoding between other formats. Therefore, while it does not seem unlikely that translation of number words between languages could involve visualisation of a numeral instead of, or in addition to, semantic retrieval, this organization of codes remains tentative.

The the Chinese-English bilinguals' transcoding pathway between the processes of the English (L2) verbal code and the magnitude code is deemed to be weak, as they would have had more experience dealing with practical day-to-day estimation and approximation tasks in their native language (Campbell & Epp, 2004). In addition, as they were not assumed to have had much use for direct translation from mandarin numerals into English number words, no direct connection is assumed to have formed between these two codes. Arabic numerals, which are extensively used in China in addition to the Chinese numerals, were expected to be strongly integrated with both language codes, but with different processes within each code. In the English language code the strong integration is to the number lexicon, while in the Chinese code it is to the artihmetic facts storage. This is founded on the findings that the subjects named Arabic numbers more rapidly in English, while still performing simple arithmetic more rapidly when presented in Chinese (ibid.).

Assumed lack of practice in translating Mandarin number words to English and vice versa serve as the reason for the lacking direct transcoding pathway between the two verbal codes. Transcoding between languages must therefore proceed via either the magnitude code or the Arabic visual code. Indeed, as the Encoding Complex Hypothesis postulates multiple simultaneous activations, it should proceed through both simultaneously. The stronger route would usually be the more suitable route, while the weaker route would interfere unless properly inhibited. Both strength and inhibition is, as previously noted, experience-based (Campbell & Clark, 1988; Campbell & Clark, 1992; Campbell & Epp, 2005).

The Arabic and mandarin visual codes subserve numeral input and output processes in their respective number formats, in the same way as the visual Arabic code in the Triple-Code Model. As with the verbal codes, no direct connection between the visual Arabic and visual mandarin code is modeled, indicating that the participants in the study were assumed to either access semantic information in the magnitude code when converting from Arabic to mandarin numerals and vice versa, or employing an asemantic transcoding process through the Chinese verbal code. However, this prediction is not explicitly discussed in Campbell and Epp's article and the relationship between other number representations than the Arabic code does not seem to have been investigated in other studies, leaving this organization somewhat speculative for now.

At this point the dynamic nature of the Encoding-Complex Hypothesis should be quite evident. However, Campbell & Clark also tentatively suggest that arithmetic facts are spread accross the codes according to which code they are relevant to (Campbell, 1994; Campbell & Clark, 1988). For example, based on tentative inferences from two studies that were designed to test various other hypotheses (Kashiwagi, Kashiwagi & Hasegawa, 1987 and McNeil & Warrington, 1994), Campbell and Epp (2004) speculate that, given enough experience, simple arithmetic tasks in Arabic number format might also be stored in memory which is mediated by the visual Arabic code. This assumption, is clearly in need of further investigation, as the topic of centralized vs. distributed and language dependant mediation of number facts is far from resolved and findings have been conflicting (Noël, 2001). This topic will be the subject of further discussion in chapter 5.5.

5 Number processing in bilinguals

In the previous two chapters, a series of models of mathematical cognition have been reviewed and their predictions of language effects on number processing have been discussed. The model that contains the most specific predictions for bilingual numerical cognition is Campbell and Epp's (2004) Encoding-Complex model. It is, however, constructed in order to explain the findings of one particular study of Chinese-English bilingual students (Campbell et al., 1999), and no attempt is made to expand it into a general model of bilingual mathematical cognition. Therefore the question is: What evidence is there that Campbell and Epp's Encoding-Complex model can serve as a basis for a more general model of number processing in bilinguals?

Even if the Encoding-Complex model is not a general model of number processing, it is, as previously stated, based upon two general theoretical frameworks of numerical cognition: The Encoding-Complex Hypothesis and the Triple-Code Model (Campbell & Epp, 2004). It therefore demonstrates how models can be constructed to account for interactions between multiple formats in numerical cognition based upon such a flexible theoretical foundation. As a consequence a number of assuptions of a more general nature are built into it.

In the following sub-chapters some of the assumptions on basic number processes will be discussed in light of research on the matter. Assuming that some features of the Encoding-Complex model can be used as a basis for a more general model, three more questions follow: What could such a model hypothetically look like? What would it predict in terms of pedagogical consequences? And finally, what areas need further investigation in order to expand the theorethical basis for such models, thus furthering knowledge on the issue? Discussion of these questions will form the concluding sub-chapters of this thesis.

5.1 Effects of format and skill

The feature that to the greatest extent sets the Campbell and Epp's (2004) model apart from other models of numerical cognition, is the stipulation of variations in encoding-retrieval integration, which originated in the Encoding-Complex Hypothesis. In addition, there is the idea of two extra mental representations or codes, as compared to the Triple-Code Model (see Dehaene, 1992; Dehaene & Cohen, 1995).

While there are relatively few studies of number processing in bilinguals, many seem to provide support for these notions. Some countries naturally provide good environments in which to pursue such research. Amongst these are the Philippines, which have two official languages (Filipino and English) and therefore has a large number of bilinguals. A. B. I. Bernardo (of the De La Salle University in Manila) has conducted a number of studies investigating different aspects of bilingualism and mathematics (e.g. Bernardo, 2005; Bernardo & Calleja, 2005). In one study he examined processing of addition tasks in 21 Filipino-English bilinguals (Bernardo, 2001). His subjects were asked to perform simple additions, presented in three formats: Filipino number words, English number words and Arabic numbers. First they were presented with an addition task, then they were presented with an answer and were asked to determine if this answer was correct or not. Their responses were timed and their error rates noted. The subjects were pupils at a local English-language High School and therefore had more experience doing mathematics in English. Indeed, they stated that they preferred doing arithmetic tasks in this language.

Bernardo noted that the pupils processed the tasks both faster and more precisely in English, their second language. This not only suggested differing underlying patterns in number processing between the two languages, but also that the stronger verbal number code need not be the bilinguals' first language, if another language is used for learning and practicing arithmetic. This is consistent with the notion of access to arithmetic facts being mediated by a language code. Also, it suggests that the integration of processes differ in strength and efficiency as determined by prior experience with tasks utilizing that code, thereby providing support for the notion that encoding-retrieval integration varies with experience in a type of task in a specific format.

Further evidence for the dynamic nature of cognitive number processes have been provided by Spelke and Tsivkin (Spelke & Tsivkin, 2001). As previously noted, they studied Russian-English bilinguals who performed numeric operations, arithmetic equations, numerical fact questions and non-numerical fact questions in both languages. Importantly, they trained their subjects in unfamiliar types of tasks and tested them in both the newly familiar types and in new, unfamiliar types - thereby examining the effect of training across formats. Consistent with, amongst others, Dehaene (1992, 1997), Campbell and Epp (2004), Bernardo (2001), Gordon (2004) and Frank and colleagues' (2008) postulations of language-independent processing of approximate numbers, Spelke and Tsivkin found that their subjects tended to solve problems involving approximate numbers and non-numerical facts equally well regardless of language and regardless of which language they were trained for the tasks in. The subjects also readily generalized learned procedures to other tasks of a similar type.

However, when tasks involved exact numbers, the subjects performed much better in the language in which they had received training for that type of task and they also retrieved information involving exact numbers more efficiently. This not only adds weight to Campbell et al. (1999) and Bernardo's (2001) interpretations of their findings thereby providing support for the Encoding-Complex Hypothesis' postulation of experience effects on integration of cognitive processes, but also demonstrates that such integration is flexible and that the encoding-retrieval integration of processes involved in specific task types can change fairly rapidly with training.

5.2 Counting

Studies of indigenous people in the Amazon region that have been discussed previously point towards the necessity of number words for counting, since these people have a very limited number of words for exact number and are unable to perform exact calculation (Frank et al., 2008; Gordon, 2004; Pica et al., 2004). However, earlier studies of the Oksapmin in Papua New Guinea (e.g. Saxe, 1982) show that those people can count to 27 by pointing to different body parts. They also have number words, but these are equal to the names for the body parts. Accordingly, while their counting system is clearly more limited than in industrialized cultures, it is significantly more advanced than that of the Amazonian Pirahã and Mundurukú tribes, who could only count to around two and five respectively (Gordon, 2004; Pica et al., 2004).

A possible alternative explanation for the Amazonian tribes' lack of ability to count is therefore that other factors than language itself mediate the preciseness and complexity of enumeration systems. It could, for example, be argued that there is little reason to expect that a culture will develop a more complex number system, featuring a larger amount of number words, than it has practical use for (Gelman & Butterworth, 2005).

This does, however, not necessarily mean that extensively developed verbal concepts of numbers are not required for the use and communication of large numbers in societies in which this is necessary. Presumably one will sooner or later run out of body parts or other immediately available visual representations. Frank et al. (2008), who went to the Amazonian region and retested a sample of Pirahã tribespeople on the tasks used by the initial study of these people by Gordon (2004), suggests that the verbal concept of exact number can be viewed as a cognitive technology - a tool which enables one to conduct more precise enumeration of larger quantities and therefore also enables more advanced mathematical operations than what is otherwise possible.

Consequently, while it is likely that other cultural factors than language can influence the relationship between counting and mental representation of exact numerosity, it is equally likely that language is the most dominant mediating factor in advanced cultures.

5.3 Subitizing and estimation

There seems to be a growing consensus that functions such as subitizing, estimation and approximate calculation employ language-independent cognitive processes (Campbell & Epp, 2004). These processes are, by both Campbell and Epp (2004) and Dehaene (1992) forwarded as being mediated by an analogue magnitude code - a mental representation of visual magnitudes that are not conveyed by a modality of language (written or spoken numbers). As previously noted this is further supported by behavioural studies of both monolingual subjects (e.g. Frank et al., 2008; Gordon, 2004) and bilinguals (e.g. Bernardo, 2001; Dehaene, Spelke, Pinel, Stanescu & Tsivkin, 1999; Spelke & Tsivkin, 2001). There is even research indicating that animals have some abilities for recognition of quantities (e .g. Meck & Church 1983), which implies that these cognitive processes at least in part are of a quite basic and primitive nature.

Dehaene, Spelke, Pinel, Stanescu and Tsivkin (1999) produced both behavioural and brain imaging evidence to this effect. However, while the behavioural part of that study was conducted on bilinguals, the brain imaging part was not, therefore this study does not provide conclusive evidence. However, Venkatraman et al. (2006) conducted a combined behavioural and brain-imaging study on 20 Chinese-English bilinguals, who were trained in two unfamiliar task types in one language and then performed tasks of that type in both languages, as well as untrained tasks. While behavioural studies have not shown significantly increased reaction times for estimation tasks across languages, Venkatraman and colleagues found that larger brain areas were activated when approximate tasks were conducted in the untrained language, suggesting that a greater effort was involved. They also detected an increase in reaction times, comparable to the difference for exact arithmetic.

A significant difference had not been detected in other studies on the subject, such as the study by Spelke and Tsivkin (2001) which was discussed earlier. Venkatraman et al. suggested that the reason for this was that Spelke & Tsivkin's approximation tasks were simpler, requiring shorter overall response times. Venkatraman and colleagues tentatively attribute the difference in performance and brain activation to difficulties in converting the problems into an abstract, language-independent code.

This is a likely explanation, as the stimuli were presented in either English or Chinese number word format, thereby requiring transcoding into the magnitude code for processing. Based upon Campbell and Clark's (1988; 1992) Encoding-Complex Hypothesis on number processing one would expect that the strength of the transcoding pathway between the language code and the magnitude code would have increased for the trained, but not for the untrained language. Accordingly, the

Encoding-Complex Hypothesis, as well as Campbell and Epp's (2004) Encoding-Complex model for numerical cognition in Chinese-English bilinguals predicts this effect - without thereby assuming that approximate number processes are language-dependant.

5.4 Arithmetic facts

Arithmetic facts are, by Dehaene (1992) and Campbell & Epp (2004) primarily thought to be mediated by verbal mental representations of number. The necessity of language for such operations does, however, remain controversial (Campbell & Epp, 2004). As Gelman and Butterworth (2005) point out, it is one thing to maintain that language facilitates aspects of number use, but another thing to hold that it provides its causal underpinning.

On one hand, a number of studies have shown effects of language upon arithmetic fact performance (e .g. Bernardo, 2001; Campbell, 2005; Campbell et al., 1999; Frenck-Mestre & Vaid, 1993; Venkatraman et al., 2006). In Spelke and Tsivkin's (2001; see also Dehaene et al., 1999) previously mentioned study, their bilingual subjects consistently performed better on exact arithmetic tasks in the language in which they had been trained. Their study is particularly interesting in this context, because they compared the performance of their subjects on tasks requiring arithmetic facts with their performance on approximation tasks and non-numerical facts, in both languages. All task types were new to the subjects. They were trained in some tasks in one language and some in the other, and also performed untrained tasks. While performance on all types tasks benefited from training, the arithmetic fact training primarily benefited task performance in the language of training. It did not generalize to similar types of facts in the other language, to the same extent as training in approximate tasks did. This suggests that at least certain types of arithmetic facts are indeed tied more to language than other number processes, and can therefore be conceptualized as being stored and retrieved within a linguistically mediated mental code. However, by demonstrating that training on approximate tasks and arithmetic facts lead to increased performance on tasks involving these across languages, Spelke and Tsivkin's study also suggests the storage of relevant arithmetic facts in the nonlinguistically mediated analogue magnitude code.

A mentioned previously, Campbell and Epp (2004) speculates that the Arabic number code may also mediate some form of memory for arithmetic facts. Dehaene (1992) attributed multi-digit operations to the Arabic number codes, which might be interpreted as suggesting that this code may mediates access to some facts and procedures for such operations. Case studies of patients who retain exact calculating ability in the face of impairment of language (e.g. Cappelletti, Butterworth & Kopelman, 2001; see also Gelman & Butterworth, 2005) and people with developmental disorders such as autism, who nevertheless excel in mathematics (see e.g. Dehaene, 1997) have traditionally been seen as arguments to the effect that number facts are part of a semantically mediated central code (see e.g. Noël, 2001). However, such findings can also be explained by arithmetic facts stored in the Arabic Visual Code. In summary, it would seem that language (either in the form of number words or Arabic numerals) plays a part as a medium for acquisition of arithmetic fact knowledge, the relationship between formats and arithmetic facts needs further investigation.

To summarize the discussion in this chapter so far, two of the questions posed in the introduction can now be considered: Does language affect number processing? And if so, to what extent? Current evidence seems to suggest that the impact of language upon numerical cognition varies with both input and output format, level of experience with the type of task across formats, as well the types of number processes involved in a particular type of task. Evidence therefore seems to favour the notion that language has a larger effect on the processing of exact number than processing for approximate number, when approximate numbers are not presented in a language-dependant format.

5.5 Modelling bilingual numerical cognition

In this sub-chapter, a tentative model of number processing in bilinguals is proposed and its predictions are considered in light of research on bilingual numerical cognition. As discussed in the previous sub-chapter there is some evidence that a number of the features of Campbell and Epp's (2004) Encoding-Complex model of numerical cognition in Chinese-English bilinguals are applicable to bilinguals in general, although some of the predictions of the model are in need of further investigation. Therefore, the model depicted in fig. 5 is merely a slightly modified version of Campbell and Epp's model. Similarities and differences to this model will therefore form part of the discussion, as will questionable aspects and limitations of the model.

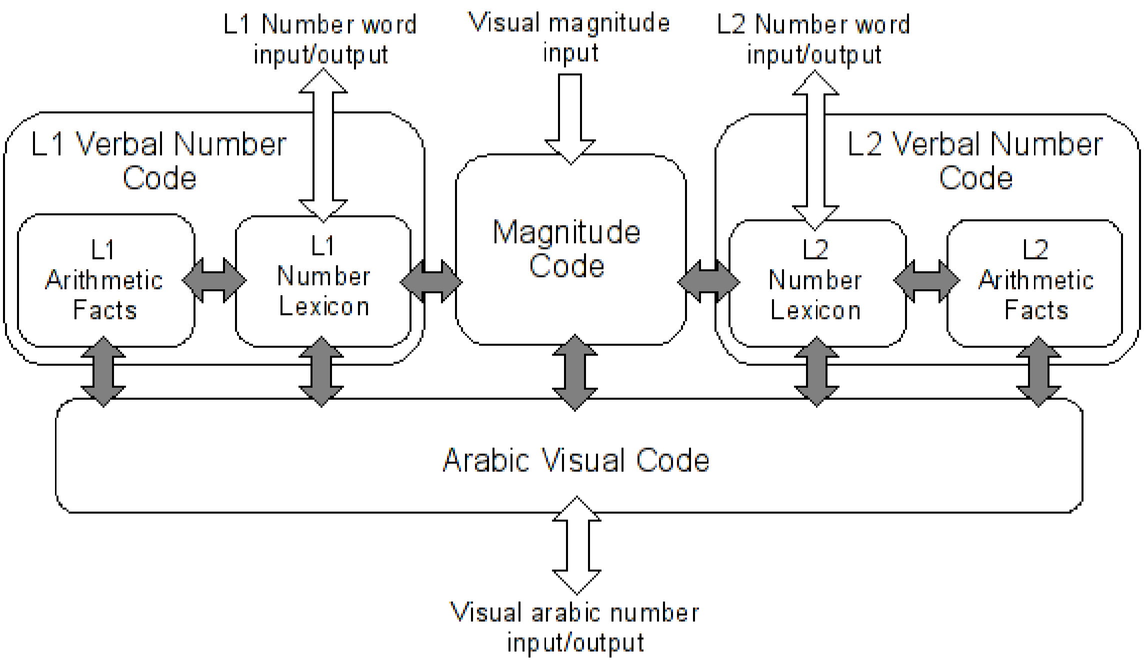

Figure 5: A tentative, generalized version of Campbell and Epp's (2004) Encoding-Complex model.

Transcoding and associative pathways are displayed as grey arrows in fig. 5. Transcoding pathways are arrows that connect codes, thereby illustrating conversion between the internal representations, while associative pathways are internal retrieval paths within a code (Bernardo, 2001; Campbell & Epp, 2004). In the illustration all arrows are grey, whereas in Campbell and Epp's (2004) model, they were colour-coded according to their expected strengths (see fig. 4). As discussed in chapter 3.3, the Encoding-Complex Hypothesis the transcoding patways will vary in strength according to previous experience with particular types of tasks in particular types of input- and output formats, thereby accounting for the effects of experience upon performance (Bernardo, 2001; Campbell & Clark, 1988; Campbell & Clark, 1992; Campbell & Epp, 2004). However, in order for the model in fig. 5 to be a general model, no transcoding pathway strengths can be modeled, as the encoding-retrieval integration levels will vary from bilingual group to bilingual group, and even from individual to individual. The general model can therefore only serve as a template from which to create specific models in order to understand the performance of groups or individuals and can therefore only make predictions of a very general nature. The more information that is gathered from for instance a particular group of bilinguals, the better the model is able to make specific predictions upon manipulating features such as the codes and encoding-retrieval pathways.

Also, as postulated by Campbell & Clark (1998; 1992; Campbell & Epp, 2004; 2005), for any given task, any number of routes can be activated simultaneously and routes that are not inhibited sufficiently will interfere with more suitable routes, accounting for typical errors made during different types of number tasks, such as operand intrusion errors in simple arithmetic.

Number codes: The most obvious omission compared to Campbell & Epp's model is the Mandarin Number Code. Mandarin numerals, being specific to Chinese speakers, are not used by most bilinguals. Arabic numerals, on the other hand, are. While also other groups of bilinguals also use additional numeral systems, Campbell and Epp's model is the only current model of bilingual and binumeral number processing. As this model's postulation of this code is based upon just one study and describes only that particular group of bilinguals, there is currently no evidence that additional number codes in other bilingual populations (such as Khmer numerals in Thai or Cambodian bilinguals) would relate to the other codes in the same way as the Mandarin number code relates to the other codes in Campbell and Epp's model.

Considering how the Encoding-Complex Hypothesis holds that the integration of representations and processes will vary with task-based experience, there is good reason to believe that any additional non-Mandarin number code would not relate to the other codes in the same way as the Mandarin code. The reason is as follows: The

Famount of experience that one has with each task type and format combination will necessarily be affected by the relative functions of the different numeral systems and languages in the society in which one lives, as was indeed expected to be the case with the Chinese-English bilinguals in Campbell and Epp's model (2004). Therefore people from other countries, speaking other languages are unlikely to feature the exact same interactions between codes.

However, Campbell and Epp clearly demonstrate how additional codes can be added to the Encoding-Complex Hypothesis in order to construct hypotheses and explain findings. As information on other bilingual populations becomes available, the model can be adjusted accordingly and adapted to explain findings in these groups.

As per Dehaene's (1992) postulations, the number codes are assumed to mediate operations related to the visualization of numerals, such as multi-digit arithmetic operations and parity judgement. As noted in chapter 3.2.1, available evidence suggests that parity judgement requires access to the semantic information as well, suggesting that this is a process that involves more than one codes. It is possible that the Arabic Visual Code could also mediate some memory for arithmetic facts (Campbell & Epp, 2004). Such facts could be visual recollection of Arabic-number based arithmetic facts learned by rote, for instance an expression such as "3 × 3 = 9". However, while evidence favours the view that arithmetic facts are stored separately from other skills and can be selectively impaired (Zaunmüller et al., 2009), the question of fact types and format dependence clearly requires further investigation.

Verbal codes: As in Campbell and Epp's model there is one verbal code for each language, each organized in a similar fashion vis á vis the magnitude and Arabic codes. In fig. 5, two Verbal Number Codes are depicted: L1 (for language 1) and L2 (for language 2). Many people around the world do, of course, use more than two languages. As noted previously, the flexibility of the Encoding-Complex Hypothesis allows for the addition of codes as necessary in order to account for additional input and output modalities. However, two verbal codes suffice to illustrate the relationship between verbal codes. For simplicity's sake only two Verbal Number Codes are therefore displayed.

As discussed in chapter 4.3, the lack of a direct transcoding pathway between the verbal codes means that one must either abstract the meaning of a number word in one language, or visualize the Arabic number, in order to find the appropriate number word in another language. This assumption is based on Campbell and colleagues' (1999; see also Campbell & Epp, 2004) observation that the Chinese-English bilinguals' response times for number naming was disproportionately high when naming mandarin numerals in English, compared to other transcoding tasks. This finding suggests that no direct transcoding took place between these two formats (Campbell & Epp, 2004). However, it does not necessarily suggest this exact ordering of codes and connecting pathways. The lack of a direct verbal transcoding pathway should lead to increased response times for tasks presented in one language requiring response in the other, than e.g. for tasks presented in Arabic format requiring response verbally. However, the limited amount of research on the matter means that this aspect of bilingual number processing has not yet been properly investigated and the relationship between the verbal codes must therefore remain speculative.

As noted in chapters 4.2 and 5.4, there is little question that language affects numerical cognition and especially arithmetic facts related to exact calculation. While there is debate whether this is due to language being a necessity for the development of exact calculation processes, or just a facilitating medium (e.g. Gelman & Butterworth, 2001), the effects of language upon number processing is behaviourally observable, and therefore this distinction does not necessarily affect any practical implications.

In accordance with Dehaene (1992) and Dehaene & Cohen's (1995) thoughts, as well as with several studies that have demonstrated language effects upon processing of exact number (e.g. Bernardo, 2001; Campbell, 1994; 2005; Campbell et al., 1999; Dehaene et al., 1999; Frenck-Mestre & Vaid, 1993; Spelke & Tsivkin, 2001; Venkatraman et al., 2006), the Verbal Number Codes are therefore expected to mediate verbal arithmetic facts, such as addition and multiplication tables, as well as counting.

The magnitude code: Consistent with Dehaene's (1992) model, and the findings of a number of previously discussed studies regarding the language independence of approximate aspects of number processing (e.g. Bernardo, 2001; Dehaene et al., 1999;

Dehaene et al., 2003; Pica et al., 2004; Frank et al., 2008; Spelke & Tsivkin, 2001), the magnitude code is assumed to mediate magnitude comparison and approximate calculation processes, while interfacing with approximate visual magnitude input processes through subitizing and estimation of quantities. Because it mediates semantic information, it also serves as the "nexus" of the model and is presumed to handle much of the transcoding between other codes (Campbell & Epp, 2004). This postulation is probably the least controversial, as the debate on the issue of representations in the field of cognitive psychology mainly revolves around to which degree there are *other* representations than a central, semantic number representation - not *if* there is a central, semantic number representation (see e.g. McCloskey, 1992; Noël, 2001).

Input and output processes are depicted as white arrows in fig. 5. As with Dehaene's (1992) model, three categories of input/output processes are modeled. Verbal number-word input and output, visual Arabic input and output and visual magnitude input processes. The verbal number word input and output processes subserve the written and spoken modality of language, but not numerals. The visual Arabic input and output processes subserve written Arabic numbers and the visual magnitude input subserves non-linguistic visual number input, through the processes of subitizing and estimation, as modelled by Dehaene (1992) and Dehanene and Cohen (1995).

In conclusion, while several aspects of Campbell and Epp's (2004) Encoding Complex models seem well-founded and applicable to bilinguals in general, and while aspects, particularly of Dehaene's (1992) Triple-Code Model are becoming increasingly well investigated, the model depicted in fig. 5 needs to remain very tentative at this stage. A number of specific features of the modeled interaction between additional languages and the core representations of the Triple-Code Model needs to be investigated. Also particular aspects of numerical cognition needs to be investigated in bilingual studies designed for this particular purpose needs to be conducted before a true, functional model of bilingual numerical cognition can be presented.

In the next sub-chapter, pedagogical implications of the discussion in this chapter as a whole are considered, and in the last sub-chapter some thoughts on future lines of research, as well as on the interaction between educational research and cognitive psychology are aired.

5.6 Pedagogical implications

Two of the three questions put forward in the introduction have hitherto been discussed: To what extent does language affect number processing and are there any potential differences in bilingual numerical cognition? Since evidence seems to favour the idea that language to a certain extent does affect number processing and that there consequently are some differences in bilingual numerical cognition, the last question also needs consideration: Are there any implications for the mathematics education of bilinguals?

Effects of format and skill: One postulation of the model in fig. 5 is the increasing level of integration between processes, according to amount of task-based experience. The notion that number processing efficiency varies as a function of input format, task type and task-based experience is, as noted earlier, supported by an increasing body of studies (Bernardo, 2001; Campbell, 1994; Campbell et al., 1999; Frenck-Mestre & Vaid, 1993; Meuter & Allport, 1999; Spelke & Tsivkin, 2001; Venkatraman et al., 2006). In other words, there is a "format × task type × experience"-interaction. This means bilinguals' performance in e.g. simple arithmetic tasks are predicted to vary according to which language or format the task is presented in and according to experience with similar tasks in that particular combination of input and output formats (language one, language two, numeral system).

Furthermore, e.g. Spelke & Tsivkin's (2001) study of Russian-English bilinguals shows that for language-dependant number-fact tasks, the performance of the participants improved significantly in a matter of days, especially for tasks presented in the language in which they were trained. This suggests that integration of processes is relatively flexible and can change fairly rapidly. It can therefore be assumed that that the encoding-retrieval integration of processes can be altered by pedagogical means, so as to form integrations of transcoding and associative pathways that are suitable for the desired types of tasks in the desired input and output formats. As amongst others Campbell and Clark (1988; 1992) predict, integration of suitable paths and inhibition of unsuitable ones should continually increase with practice.

Subitizing, estimation and comparison are, as discussed in chapter 5.3, generally held to be language-independent number processes. In their studies, both Spelke and Tsivkin (2001) and Venkatraman et al. (2006) demonstrated that their subjects' performance on approximate calculation tasks also improved with experience. However, Spelke and Tsivkin noted that contrary to experience in arithmetic fact tasks, experience with approximation tasks seemed to generalize across task types and languages. While Venkatraman and colleagues found a performance difference in approximation tasks presented in the trained, as opposed to the untrained language

This indicates that the language of training is less relevant to the utilization of such experiences. Furthermore, it is consistent with Dehaene's (1992) and Dehaene & Cohen's (1995) postulation of approximate processes being mediated by a language-independent magnitude code, as the information would then be more readily accessible across languages. While the integration of verbal codes and each language code (see fig. 5) are predicted to vary sligtly according to which language is most frequently associated with everyday approximation tasks (see chapter 5.3), the pedagogical implication is that training in approximation tasks should be readily transferred from one language to the next.

Effects of task type: As noted in chapter 5.3, subitizing, estimation and approximation is expected to be mediated by a language-independent format, and therefore, be less language-dependant than other number processes. Arabic number processing is expected to be mediated by a separate Arabic Visual Code, which interfaces with all language codes in bilinguals from societies where Arabic numbers are common (as shown in fig. 5). While not language-independent as such, Arabic numbers are therefore expected to be relatively equally accessible across languagees. However, numerosities presented in number word format and arithmetic facts are expected to be mediated by the Verbal Number Code in which they were learned, and are therefore expected to be more demanding cognitively than numerosities presented in other formats. This is predicted to be especially true for bilinguals, when arithmetic facts and mathematical tables are stored in one code but used in another, or where they are scattered across verbal codes.

The type of mathematical task in school that are likely to be most affected by this are mathematical word problems - or math word problems for short. Math word problems can be defined as *"verbal descriptions of problem situations wherein one or more questions are raised the answer to which can be obtained by the application of mathematical operations to numerical data available in the problem statement"* (Verschaffel, Greer & De Corte, 2000, p. ix). They are, in other words, mathematical tasks that are presented as a blocks of text, rather than mathematical expressions.

Not only is the processing of number words in a number of studies consistently shown to be more cognitively challenging than numerals (Campbell & Epp, 2005). Also, problems featuring number words could, according to the model in fig. 5, involve a number of additional transcodings compared to tasks presented as Arabic numbers. In addition to any arithmetic facts that can be retrieved from the Verbal Number Code that corresponds to the language in which the problem was presented, the numerosities involved must be converted into the magnitude code for retrieval of semantic information, as well as into the Arabic Visual Code for multi-digit calculations or for output as arabic numbers. The increased complexity of cognitive interactions for this type of problem is predicted to be particularly true of bilinguals, if most of the bilingual's arithmetic facts is based in the other language so that the code in which the relevant arithmetic facts and tables are stored does not correspond to the language in which the task is presented. In addition to the transcodings described above, arithmetic facts must then be retrieved from the other code, further adding to the number of transcodings involved and increasing the chance of interference from more of the less suitable encoding-retrieval pathways.

As math word problems are more difficult than e.g. problems presented in Arabic numerals of the same arithmetic complexity, they must be justified by other pedagogical means than promoting raw computing efficiency. A common justification for math word problem use is that it promotes real-world problem solving skills (Verschaffel et al., 2000). Therefore, the degree to which this is true is a more important pedagogical factor than its cognitive efficiency.

Arithmetic facts: As discussed in chapter 5.4, arithmetic facts and their relationship to language are controversial and different types of arithmetic facts may be mediated by different internal representations of number. Nevertheless, several studies of bilingual subjects point to significant performance differences between languages for arithmetic facts (e.g. Dehaene et al., 1999; Frenck-Mestre & Vaid, 1993; Spelke & Tsivkin, 2001; Venkatraman et al., 2006). Even studies in which the subjects were trained for specific types of tasks in specific languages have demonstrated a marked impact on response times and error rates for tasks given in the untrained language, even if the training periods were relatively short, spanning just a couple of days (Spelke & Tsivkin, 2001; Venkatraman et al., 2006). Therefore, consistent with Campbell and Epp's (2004) and Dehaene's (1992) models, and with Holmes and McGregor's (2007) rote memory learning study, this model assumes that important arithmetic facts, such as tables and other information and procedures learned by rote, are stored in and mediated by, the verbal codes. The pedagogical implication is that for maximum efficiency in arithmetic fact retrieval and therefore in exact arithmetic processing, bilinguals should learn arithmetic facts in the language in which they will be used.

Practical implications: At this point, however, it is prudent to put the above pedagogical implications into a practical perspective. Firstly, it should be noted that response time differences shown by studies of language effects on numerical cognition are measured in milliseconds. The studies frequently involve a large number of tasks, often hundreds, in order to be able to provide sufficient data to create measurable and reliable results (see e.g. Bernardo, 2001; Campbell 2005; Spelke & Tsivkin, 2001). This means that for all intents and purposes, such factors as the speed difference and error rates between for example exact number word tasks in a bilingual person's first and second language is not necessarily likely to be noticeable in everyday life.

Secondly, the abovementioned pedagogical implications are only true as long as there are no other factors present that impair the bilingual's performance, such as cultural factors or language comprehension problems. Several studies have shown significant effects for factors such as the socioeconomic status (e.g. Marks, 2005), language comprehension problems (for reviews, see e.g. Ellerton & Clements, 1991; Rönnberg & Rönnberg, 2001) and problems related to the comprehension of the mathematics register (for a review, see e.g. Rönnberg & Rönnberg, 2001; or see Lee, 2006), particularly with respect to minority pupils. It is therefore likely that the presence of such other factors will overshadow the impact of bilingualism upon number processing completely. Bernardo (2005) notes that *"although language may be an important factor that shapes the cognitive processes of bilinguals, we should not stretch the importance of this factor in bilingual cognition"* (p. 423).

Therefore, while language effects on numerical cognition is an issue, that issue is quite separate and probably less important to in the practical education of bilinguals than issues of e.g. language effects on interaction, communication and learning.

5.7 Concluding remarks and future directions

In the introduction to this thesis, three questions were posed: To what extent does language affect number processing? Are there any potential differences in bilingual numerical cognition and if so, are there any implications for mathematics education of bilinguals?

Whether due to its necessity for exact number processing, or merely its ability to facilitate it, available evidence suggests that language does affect number processing, although it affects different processes to varying degrees and that bilingual numerical cognition does differ from monolingual numerical cognition in terms of the number of representations involved, potentially making it substantially more complex. This means that the initial assumption, made during the introduction to this thesis - namely that there are no differences in numerical cognition between monolinguals and bilinguals, is untenable. However, while language affects numerical cognition, that issue is quite separate from, and probably not practically relevant to, the education of biliguals compared to other culture-related factors affecting that may affect bilingual mathematics performance.

Humans' high level of understanding and use of number is arguably one of the main characteristics that sets us apart from animals. To attempt to understand how it relates that other important characteristic, language, is therefore a very worthwhile task. As noted on several occasions, particularly at various points in chapter 5.5, research on numerical cognition is nowhere near complete and research on bilingual numerical cognition even less so. Also, as with all science, once new findings and theories are presented, previous research must be re-evaluated and new research conducted in order to examine and validate the new thoughts and findings.

While models of numerical cognition are nothing new, a true general model for numerical cognition is - as should be evident from the discussion of the merits and various shortcomings of the tentative model put forward in chapter 5.5 - some way off. A true general model should take into account both every conceivable stimuli and output format, account for the effects of task-based experience, and be specific as to the relationships between processes and codes. Models based on the Encoding-Complex Hypothesis and the Triple-Code Model may prove fruitful, but needs expansion, specification and confirmation - something that requires considerable research effort.

Some examples of issues that need further investigation have been mentioned at various points in this thesis. On such issue is the organization of codes in models of bilingual numerical cognition. While there have been a number of studies investigation the relationship between codes in the Triple-Code Model, pertaining to monolinguals (e.g. Dehaene et al., 2003; Schmidthorst & Brown, 2004; Holmes & McGregor, 2007), the relationship between these and additional codes in people who use more than one language and/or numeral system have not yet been sufficiently investigated. An experiment such as a bilingual study in which the participants name numbers presented in two languages and Arabic numerals verbally in both languages, while reaction times, error rates and error types are recorded, is an example of a study type that could perhaps help clarify this issue.

Another issue is the question of the nature of arithmetic fact storage and retrieval, and its relationship with representational codes. Are arithmetic facts mediated by verbal codes only, are different types of facts mediated by different codes, or are all facts mediated by a central, language-independent code?

The ontogenetic aspects of the relationship between language and number is in need of further investigation. As noted in chapter 5.3, there is still considerable debate on the dependence of exact calculation upon language. While the practical effects of language upon numerical cognition are more important pedagogically than the ontogenetic relationship between the two, and this issue therefore may be of less relevance to educational research, the issue is undoubtedly of significance psychologically, as it concerns a very basic aspect of human cognitive development and function.

Eventually, a detailed enough model should be achieved that it can be genuinely useful, not only for theoretical purposes, but also for clinical purposes; e.g. in assessment of cognitive function in people with developmental disorders or brain damage. Indeed, such assessment tools for brain damaged patients already exist for various aspects of language, based upon models of linguistic cognition (see e.g. Kay, Lesser & Coltheart, 1992; Swinburn, Porter & Howard, 2004). Conceivably, in time, even tests based upon a model such as a refined version of the Encoding-Complex Hypothesis could be made in order to examine the strength of encoding-retrieval integration as a factor of input and output format and task type, thereby guiding pedagogical efforts in the teaching of e.g. groups of pupils that need special needs education. This is but one example of the utility of crossing borders between scientific disciplines, in this case cognitive psychology and educational research,

Lastly, an interesting study that may be opening up new perspectives on numerical cognition is was conducted by posed by Tang and colleagues (2006; see also Cantlon & Brannon, 2006). They gathered brain imaging data from native Chinese and native English-speakers in China. The study revealed that different brain areas in the two groups were activated for simple arithmetic tasks - even when they were presented in the common format of Arabic numerals. This suggests that not only number words, who differ from one language to the next, but also common formats may be represented differently in the brain in people from different cultures. Therefore, language and other cultural factors may have an even more defining effect number processing than previously expected. Also the question whether the Amazonian tribes' lack of proficiency in exact calculation (see chapter 5.2) is due to their language use or to other practical or cultural It is, however, but one study. Accordingly, there is good reason to continue the investigation into how significant language and also how other cultural factors are to the acquisition of numerical competence and to mathematical performance. Such investigations could possibly require contributions not only from cognitive psychology, but also from other fields of research that regularly deal with cultural factors, such as social anthropologists.

The ultimate goal for any research should be to increase our understanding of ourselves and the world in which we live and also to use that knowledge to better human condition. Arguably, the most important venues for passing on knowledge from one generation to the next are formalized institutions of learning, in which the modern person spends most of his or her childhood, and also - in many countries - his or her youth and early adulthood. As bilingualism becomes ever more common, understanding its nature and pedagogical consequences should become ever more important to educators in order that a proportional increase in educational challenges related to bilingualism is to be avoided. To this end, any contribution from any field of science should be welcomed with open arms.

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