The Evaluation Semantics – A Short Introduction

The purpose of the following is to come to more clarity concerning what entities we need commit to in order to provide adequate semantics for modal logics and related logics for e.g. counterfactuals. The theses defended have as consequence that we do not need possible worlds as first order objects, and certainly not pluralities so utterly implausibly suggested by David Lewis. As there is a plethora of points of view to make sense of what "possible worlds" **are**, e.g. modal fictionalism, it seems pertinent to point out certain simplifications that can be made in order to avoid some befuddlements which are prone to put philosophical discourse in disrepute. The view defended here is that we only need valuation-attributes, which may be regarded as states or properties of propositions (sentences) and, in the quantificational case, also of ordered pairs formed from elements of and sets from a given domain of discourse and linguistic items.

We first point out that we may presuppose a kindred semantics for classical propositional logic by letting valuations, or valuation-attributes, be states or properties of propositional variables of the formal language. For V a valuation and p a propositional variable, we have Vp or not Vp. For the recursive clauses we presuppose V~p iff not Vp and V(p∧q) iff Vp and Vq. It is straightforward to verify that α is a tautology iff Vα for any valuation-attribute fulfilling the imposed constraints. A slight advantage with this approach over typical approaches is that we need not postulate functions that have truth or falsity, or more conventionally e.g. 0 or 1, as values when applied to formulas. The slight advantage lies in the fact that the denotations of "truth" and "falsity" remain obscure and the invocation of e.g. "0" and "1" seem arbitrary, whereas copulation as presupposed in the suggested framework here does not involve the postulation of such arbitrary or obscure entities.

We move on to modal logics and first consider *the propositional case*: Let Greek letters α, β, γ in the following stand for formulas in the object language of some modal propositional logic. Standardly, a formula α is seen as valid on a frame <W,R> iff V(α,w)=1 for all models <W,R,V> based on <W,R> and every w∈W. W is thought of as a set of *possible worlds*, and $R \subset W^2$ is the *accessibility relation* on W. Instead of V(α,w)=1, many prefer to write wVα. This latter notation suggests that the "possible worlds" may be seen as functions from the value *assignment* V of a modal model to a compounded *valuation-property* wV. This hints that we instead of postulating a set of *possible worlds*, may restrict ourselves to what we call an *evaluation frame* <**E**,**R**>, where the *evaluation* **E** is a set of *valuation-attributes* (often we just write *valuations*) V, V'... and $\mathbf{R} \subset \mathbf{E}^2$ the accessibility relation on **E**. We think of the evaluation frames as our *models*, and are interested in validity relative to various models, i.e. evaluation frames, with various restrictions on **R**. In order to remind that we are making use of the evaluation semantics and not a standard possible worlds semantics we continually use the term "evaluation frame" instead of "model".

Notice that what we henceforth think of as valuations in the evaluation semantics can neither be identified with possible worlds nor with valuation-assignments (often just called *valuations)* relative to frames. Instead, what we take as valuations in the evaluation semantics is rather some sort of hybrid, if you like.

For V∈**E** we write Vα iff α holds according to valuation V. We require:

Valuation-consistency	If V∈**E** then not both Vα and V~α
Valuation-completeness	If V∈**E** then either Vα or V~α
Valuation-disjunctivity	If V∈**E** then V($\alpha \vee \beta$) iff Vα or Vβ
Valuation-apodicticity	If V∈**E** then V□α iff ($\prod$V')(VRV'→V'α)

In the last sentence we used another arrow for the material implication, and a capital Greek Π for the universal quantifier. This is used only occasionally here to underline that we are stating this in the metalanguage, and will be relaxed henceforth.

Exercise: Given only Valuation-consistency, Valuation-completeness, Valuation-disjunctivity and Valuation-apodicticity, show: a) Valuation-conjunctivity, i.e. for v∈**E**, V(α∧β) iff Vα and Vβ. b) Valuation-negativity, i.e. that for V∈**E**, V~α iff **not** Vα. c) Valuation-implicativity, i.e. that for V∈**E**, V(α⊃β) iff Vα only if Vβ. d) Valuation-hypodicticity[1], i.e. that for V∈**E**, V◊α iff there is a V' such that VRV' and V'α.

Result: A fully adequate semantics for modal propositional logics is induced, and soundness and completeness considerations carry over with few or none alterations. Restrictions on **R** induce different logics as in standard frameworks. This is left as an exercise here.

A note on modal correspondence is apt as an aside here. Typically correspondence results make appeals to a set theoretic apparatus (with lots of set theoretic comprehension) in order to obtain a correspondence between e.g. the modal schema □α⊃α and the reflexivity of R. In our context this is not the way to go about. Instead we presuppose *linguistic comprehension principles* which put constraints on our evaluations. Let e.g. ב(V,V') be any first-order condition on the valuations V and V' in R and =. We then presuppose that for all valuations V there is a β in the formal language so that for all V', V'β iff ב(V,V'). Let us with this consider the correspondence for e.g. reflexivity. It is elementary that □α⊃α is valid in an evalution frame **ER**=<**E**,**R**> when **R** is reflexive. Suppose □α⊃α holds at all valuations V∈**E**. We then have V(□α⊃α) for all V∈**E**. By exercise 1c), V□αonly if Vα. Spelled out this gives that for all V'(V**R**V'⇒V'α) only if Vα. By linguistic comprehension there is a β so that for all V', V'β iff V**R**V'. As α in the previous sentence is schematic, we substitute and obtain that for all V'(V**R**V'⇒V'β) only if Vβ. By linguistic comprehension this reduces to for all V'(V**R**V'⇒V**R**V') only if V**R**V. As the antecedent is a predicate logical tautology, we obtain V**R**V. In consequence, □α⊃α is valid in an evaluation frame **ER**=<**E**,**R**> iff **R** is reflexive. Other correspondence results are established similarly. The accommodation of second order conditions on valuations requires more linguistic comprehension to obtain suitable correspondence results. Linguistic comprehension for first order conditions will hold in all evaluations on account of the compactness of first order logic.

The quantificational case: We now let an *evaluation-model* be a triple <**D**,**E**,**R**>, where **D** is the domain of discourse, **E** the evaluation and **R** the accessibility-relation on **E**. **E** is again a set of valuations V, V'…., but now also on pairs as formed in the following. We assume x, y, z, x', y', z', x''… etc to be our variables and F, G, H,… etc. as predicates. We concentrate on the monadic situation as the generalization to the n-adic case is obvious. Valuation-consistency, Valuation-completeness, Valuation-disjunctivity and Valuation-apodicticity are still maintained as requirements. For d∈**D**, we may have V<x,d> or not. If V<x,d>, we write V(x)=d, as the relation is assumed to be functional. For a predicate F and D' a subset of **D**, we may have V<F,D'> or not. Again we write V(F)=D', as the relation on such pairs is again assumed to be functional. For relations, n-tuples are invoked as is standard. If x is a variable we write V(x)V' to signify that V and V' at most differ in that there is one d∈**D** so that V<x,d> and not V'<x,d>. We now impose plenist constraints on evaluations:

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[1] The term "hypodicticity" is a neologicism; the term is philologically and etymologically reasonable and has its roots in the Greek term ύποδειξις (hypodeixis).

(1) Valuation-plenism: If $V \in \boldsymbol{E}$, x is a variable and $d \in \mathbf{D}$, $\exists V'(V' \in E \ \& \ V(x)V' \ \& \ V'(x)=d)$.
(2) Barcan-plenism: $\forall V', V''(V\mathbf{R}V' \& V'(x)V'' \rightarrow \exists V'''(V(x)V''' \& V'''\mathbf{R}V''))$
(3) Converse-Barcan-plenism: $\forall V', V''(V(x)V' \& V'\mathbf{R}V'' \rightarrow \exists V'''(V\mathbf{R}V''' \& V'''(x)V''))$

For formulas of monadic quantified modal logic we have

Valuation-atomicity: VFx iff $Vx \in VF$
Valuation-generality: $V\forall x\alpha$ iff for all V' s.t. V(x)V', $V'\alpha$.

Result: With the plenist constraints (1), (2) and (3), quantificational modal logics with the Barcan formula and its converse are straightforwardly accounted for. It is noteworthy that the Barcan-formula and its converse in the Evaluation Semantics are validated by imposing constraints which ensure that the evaluations considered are appropriate full, as stated by (2) and (3), and not by impositions concerning the domain. Completeness and soundness considerations carry over to our contexts with slight alterations. The constraints (1)-(3) may be adjusted to justify logics as in Kripke's approach with an underlying free logic. Notice that although only one domain is presupposed each valuation V may be assigned a sub-domain $dom(V)=\{d:d \in \mathbf{D} \& \text{ there is a variable x s.t. } V(x)=d\}$. If one wants, one may consider quantifier clauses relative to valuation specific sub-domains either in place of or as a supplement to Valuation-generality if one e.g. wants more than one style of quantifier. It is arguable that the evaluation semantics fits most naturally with the point of view that all objects exist eternally and necessarily, but more restrained quantifiers may supplement in as far as one seeks descriptions of more local concepts of reality.

The evaluation semantics is at least as flexible as standard possible worlds semantics, and as adequately, or perhaps better, accounts for modal discourse. Or at least so I argue. Details showing soundness and completeness are here left out as exercises and for more comprehensive forthcoming accounts.

Philosophical comments: It is obvious that there may be many set theoretic accounts of the semantics for modal logics, and I fully agree with Kripke in that there is no mathematical substitute for philosophy. However, attention to details is important. The evaluation semantics suggests that we indeed only need commit ourselves to *properties* (or, if one prefers to think more extensionally, *sets*) of sentences (propositions) and such pairs as have been invoked to semantically account for modal logics. (This is in deviation from the project of accounting for properties and propositions in terms of possible worlds, but that project has seemed misguided from its onset.) In the same way as there is no problem in accepting the existence of non-instantiated properties as *having won 30 Olympic gold medals*, there seems to be no ontological problem with accepting the existence of valuation-attributes which do not valuate propositions in accordance with how things really are. Notice well that our valuation-attributes should not be confused with *Ersatzworlds* as earlier suggested in the more philosophically bent literature. Residual perplexities concerning type are shared with standard approaches and not discussed here.

One could advantageously think of our valuation attributes as maximal consistent *states* which the world could be in. With such a terminology one is at liberty to think that the states invoked constitute second order analogues of so-called *possible worlds* in standard approaches. However, to my ears it sounds better to say that a statement/proposition/sentence is necessarily true if it holds in all possible states than to say that it holds in all (accessible) possible worlds. This is, one might say, perhaps because I am mundane. More seriously: I think it is true that our world could have been somewhat different from what it actually is. In the terminology which suggests itself from the Evaluation Semantics we may express this by saying that our world could have been in a different possible state. But I do not know how an adherent to possible world talk can express the idea that our world could have been in a different possible state, for to say that a different possible world could have obtained does not seem to assign a property to our world. Of course, here I have assumed that our world itself can be taken as a member of the domain **D**. It is unclear that adherents to standard possible worlds talk can assume naturally that there is one unique world to be presupposed semantically and that this world itself can be a member of the domain presupposed.

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