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Table of Contens

Understanding Gödel's Ontological Argument

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In 1970 Kurt Gödel, in a hand-written note entitled "Ontologischer Beweis", put forward an ontological argument for the existence of God, making use of second-order modal logical principles. Let the second-order formula $P(\mathbf{F})$ stand for "the property F is positive", and let "God" signify the property of being God-like. Gödel presupposes the following definitions:

Def. 1. $\mathrm{God}x \equiv (\mathbf{F})(P(\mathbf{F}) \supset \mathbf{F}x)$
(*x* is God if, and only if, for all **F**, *x* has **F** if **F** is positive.)

Def. 2. $\mathbf{F}\mathrm{Ess}x \equiv \mathbf{F}x \;\&\; (\mathbf{G})(\mathbf{G}x \supset \Box(y)(\mathbf{F}y \supset \mathbf{G}y))$
(An object *x* has a property **F** essentially if, and only if, any property **G** which x has is such that necessarily, anything which has **F** also have **G**.)

Def. 3. $\mathbf{NE}x \equiv (\mathbf{F})(\mathbf{F}\mathrm{Ess}x \supset \Box(\exists y)\mathbf{F}y)$
(*x* has the **NE**-property if, and only if, *x* is such that any property which *x* has essentially is such that it is necessary that something has that property.)

Gödel next presupposes the following axioms:

A1 $P(\mathbf{F}) \equiv \sim P(\sim\mathbf{F})$
(A property **F** is positive if, and only if, the property of not being **F** is not positive.)

A2 $[P(\mathbf{F}) \;\&\; \Box(x)(\mathbf{F}x \supset \mathbf{G}x)] \supset P(\mathbf{G})$
(If **F** is a positive property and it is necessary that for all *x* which have **F** that they also have **G**, then also **G** is a positive property.)

A3 $P(\mathrm{God})$

A4 $P(\mathbf{F}) \supset \Box P(\mathbf{F})$

A5 $P(\mathbf{NE})$

Jordan Howard Sobel has in the article *Gödel's Ontological Proof*[1] shown that Gödel's assumptions lead to modal collapse, in that all truths turn out to be necessary truths. C. Anthony Anderson later showed, in *Some Emendations of Gödel's Ontological Proof*,[2] that a version of Gödel's argument can be rectified. Anderson retains only half of A1 (the implication from the left side to the right), and A2 and A4 remain unchanged. A3 and A5 are changed due to the fact that Anderson presupposes some other definitions than the ones suggested by Gödel.

What I shall now do is to take the property of being God-like as primitive and presuppose the following definition:

D $P(\mathbf{F}) \equiv \Box(x)(\mathrm{God}x \supset \mathbf{F}x)$
(**F** is a positive property iff it is necessarily the case that anything which is God-like has the property **F**.)

We can show that D is logically equivalent under second-order S4, including certain instances of comprehension, with the union of Gödel's Def. 1, A2, A3 and A4. Some would prefer to think of D as an axiom and not a definition.

I shall briefly indicate how we may prove the claim of the previous paragraph. That D entails A2, A3 and A4 is quite immediate. D entails A2 because of the transitivity of strict implication. D entails A3, because we have that $P(\mathrm{God}) \equiv \Box(x)(\mathrm{God}x \supset \mathrm{God}x)$, but here the right hand side is a theorem, and so the left hand side is a theorem. D entails A4, under S4, because of the characteristic S4 principle.

In order to prove that D entails Gödel's Def. 1., i.e. that $\mathrm{God}x \equiv (\mathbf{F})(P(\mathbf{F}) \supset \mathbf{F}x)$, we consider the two relevant cases (directions). Case (1): $D \;\&\; \mathrm{God}x \;\&\; (\exists\mathbf{F})(P(\mathbf{F}) \;\&\; \sim\mathbf{F}x)$. Let the **property F** be a witness for the third conjunct, i.e. for the existential second order statement. We then have $D \;\&\; \mathrm{God}x \;\&\; P(\mathbf{F}) \;\&\; \sim\mathbf{F}x$. But the three first conjuncts jointly entail that $\mathrm{F}x$, which contradicts the fourth conjunct. Case (2): $D \;\&\; \sim\mathrm{God}x \;\&\; (\mathbf{F})(P(\mathbf{F}) \supset \mathbf{F}x)$. But we have already established that D entails that $P(\mathrm{God})$, and so the third conjunct entails that $\mathrm{God}x$, which contradicts the second conjunct. This establishes that D entails Gödel's Def. 1.

We next need to show that Gödel's Def.1 + his A2-4 entails D. Case (1): Assume A2 + A3, and that $\Box(x)(\mathrm{God}x \supset \mathrm{F}x)$. Given A2 and A3, it follows that $P(\mathbf{F})$. So Gödel's assumptions entail that $\Box(x)(\mathrm{God}x \supset \mathbf{F}x) \supset P(\mathbf{F})$ is a theorem. Case (2): Suppose Def. 1 and A4 and that $P(\mathrm{F})$ and $\mathrm{God}x$. Given Def. 1, it follows that $\mathrm{F}x$, i.e. we will have that it is a theorem that $P(\mathbf{F}) \supset (x)(\mathbf{G}x \supset \mathbf{F}x)$. By necessitation and distribution, we have that $\Box P(\mathrm{F}) \supset \Box(x)(\mathbf{G}x \supset \mathbf{F}x)$ is

[1] In 'On Being and Saying', *Essays for Richard Cartwright*, MIT Press 1987.
[2] In *Faith and Philosophy*, 7 (3) 1990, pp. 291-303.

a theorem. Given A4, this means that $P(\mathbf{F}) \supset \Box(x)(Gx \supset \mathbf{F}x)$ is a theorem. Combining the cases, this establishes that the definition D is a theorem, under the presupposed machinery of second order S4, of the system Def. 1 + A2-4.

Taken together, the considerations above establish that the definition D is logically equivalent under second-order S4, including the presupposed comprehension-principles, with the conjunction of Gödel's Def. 1, A2, A3 and A4.

Given D, we may in various ways introduce definitions and axioms which are sufficient in order to derive the main theorem that there is a God. The following approach is one which has the desirable feature of being somewhat similar in character to Gödel's Def. 2 and Def. 3:

D2: $\mathbf{MCP}(\mathbf{F},x) \equiv (\mathbf{F}x \;\&\; P(\mathbf{F}) \;\&\; (\mathbf{G})((\mathbf{G}x \;\&\; P(\mathbf{G})) \supset \Box(y)(\mathbf{F}y \supset \mathbf{G}y))$
(**F** is a maximal composite of object *x*'s positive properties iff *x* has **F** and **F** is positive and all positive properties **G** which *x* has are such that it is necessarily the case that all objects which have **F** also have **G**.)

D3 $\mathbf{N}x \equiv (\mathbf{F})(\mathbf{MCP}(\mathbf{F},x) \supset \Box(\exists y)\mathbf{F}y)$
(*x* has the **N**-property iff *x* is such that if **F** is a maximal composite of *x*'s positive properties then it is necessary that some object *y* has the property **F**.)

It now turns out, and this is left as an exercise, that only the following two axioms are needed in order to derive the crucial theorem, presupposing second-order B:

Ax. 1: $P(\mathbf{F}) \supset \sim P(\sim \mathbf{F})$

Ax. 2: $P(\mathbf{N})$

The proof I have developed does not lead to modal collapse, as does Gödel's original proof. We know this because the principles presupposed in Anderson's emendation of Gödel's proof logically entails, though are not entailed by the principles in the ontological proof which I have offered for consideration. And Anderson has shown that his assumptions do not lead to modal collapse. If we add an axiom stating that if someone is God-like then it is a positive property to be identical with her, we obtain a monotheistic result.

There has been disagreement and controversy over the question as to which of the principles presupposed by Gödel, or rather presupposed in amended versions of Gödel's argument, should be taken to be epistemologically problematic as seen from the point of view of those who have more non-theistic leanings. By arguing from substitution-instances with the definiens

of D in the suggested axioms Ax. 1 and Ax. 2, we respectively obtain the principles that *it is possible that something is God-like*, and that *it is necessarily the case that if something is God-like then it is necessary that something is God-like.* Given the plausibility of D and the fact that D is logically equivalent under second-order S4 with Gödel's Def. 1 + A2-4, it is suggested that those with non-theistic leanings should question the plausibility of Ax. 1 or Ax. 2.

I have later been able to improve upon the result reported here. By making use of a result by Petr Hájek,[3] which he made we aware of at the Liblice-conference, and presupposing certain recursive definition-clauses for *divine* (positive) and *godly being*, we may show that even Ax. 2 is eliminable if we presuppose a reasonable second order comprehension principle for the predicate *godly being*. And so it in fact turns out that, modulo the (reasonable, I think) logical apparatus presupposed, only Ax. 1 is needed in order to derive the theistic conclusion. I hope to be able to publish this improved result, alongside with certain remarks, in a future paper.

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[3] Hájek, Petr 1996: "Magari and Others on Gödel's Ontological Proof", in: Ursini and Agliano, editors: *Logic and Algebra*, Marcel Dekker, Inc., pp. 125-136.