The Evaluation Semantics for Modal Logics

The purpose of the following is to come to more clarity concerning what entities we need commit to in order to provide adequate semantics for modal logics and related logics for e.g. counterfactuals. The thesis defended is that we do not need possible worlds as first order objects, and certainly not pluralities so utterly implausibly suggested by David Lewis. As there is a plethora of points of view to make sense of what "possible worlds" **are**, e.g. modal fictionalism, it seems pertinent to point out certain simplifications that can be made in order to avoid some befuddlements which are prone to put philosophical discourse in disrepute. The view defended here is that we only need valuation-attributes, which may be regarded as properties of propositions (sentences) and, in the quantificational case, also of ordered pairs formed from elements of and sets from a given domain of discourse and linguistic items.

The propositional case: Let Greek letters α, β, γ stand for formulas in the object language of some modal propositional logic. Standardly, a formula α is seen as valid on a frame $<W,R>$ iff $V(\alpha,w)=1$ for all models $<W,R,V>$ based on $<W,R>$ and every $w \in W$. W is thought of as a set of *possible worlds*, and $R \subset W^2$ is the *accessibility relation* on W. Instead of $V(\alpha,w)=1$, many prefer to write $wV\alpha$. This latter notation suggests that the "possible worlds" may be seen as functions from the value *assignment* V of a modal model to a compounded *valuation-property* wV. This hints that we instead of postulating a set of *possible worlds*, may restrict ourselves to what we call an *evaluation frame* $<\boldsymbol{E}, \mathbf{R}>$, where the *evaluation* $\boldsymbol{E}$ is a set of *valuation-attributes* (often we just write *valuations*) V, V'... and $\mathbf{R} \subset \boldsymbol{E}^2$ the accessibility relation on $\boldsymbol{E}$. We think of the evaluation frames as our *models*, and are interested in validity relative to various models, i.e. evaluation frames, with various restrictions on **R**. In order to make clear that we are making use of the evaluation semantics and not a standard possible worlds semantics we continually use the term "evaluation frame" instead of "model".

Notice that what we henceforth think of as valuations in the evaluation semantics can neither be identified with possible worlds nor with valuation-assignments (often just called *valuations)* relative to frames. Instead, what we take as valuations in the evaluation semantics is rather some sort of hybrid, if you like.

For $V \in \boldsymbol{E}$ we write $V\alpha$ iff α holds according to valuation V. We require:

V-consistency	If $V \in \boldsymbol{E}$ then not both $V\alpha$ and $V{\sim}\alpha$
V-completeness	If $V \in \boldsymbol{E}$ then either $V\alpha$ or $V{\sim}\alpha$
V-disjunctivity	If $V \in \boldsymbol{E}$ then $V(\alpha \vee \beta)$ iff $V\alpha$ or $V\beta$
V-apodicticity	If $V \in \boldsymbol{V}$ then $V\Box\alpha$ iff $(\Pi V')(VRV' \rightarrow V'\alpha)$

In the last sentence we used another arrow for the material implication, and a capital Greek Π for the universal quantifier. This is used only occasionally here to underline that we are stating this in the metalanguage, and will be relaxed henceforth.

Exercise 1: Given only V-consistency, V-completeness, V-disjunctivity and V-apodicticity: a) Show V-conjunctivity, i.e. that for $v \in \mathbf{E}$, $V(\alpha \wedge \beta)$ iff $V\alpha$ and $V\beta$. b) Show V-negativity, i.e. that for $V \in \mathbf{E}$, $V{\sim}\alpha$ iff **not** $V\alpha$. c) Show V-distributivity, i.e. that for $V \in \mathbf{E}$, if $V(\alpha \supset \beta)$ and $V\alpha$ then $V\beta$. d) Show V-hypodicticity[1], i.e. that for $V \in \boldsymbol{V}$, $V\Diamond\alpha$ iff there is a V' such that VRV' and $V'\alpha$.

[1] The term "hypodicticity" is a neologicism; the term is philologically and etymologically reasonable and has its roots in the Greek term ὑποδειξις (hypodeixis).

Result: A fully adequate semantics for modal propositional logics is induced, and soundness and completeness considerations carry over with few or none alterations. Restrictions on **R** induce different logics as in standard frameworks.

The quantificational case: We now let an *evaluation-model* be a triple <D,E,R>, where D is the domain of discourse, E the evaluation and R the accessibility-relation on E. E is again a set of valuations V, V’…., but now also on pairs as formed in the following. We assume x, y, z, x’, y’, z’, x’’… etc to be our variables and F, G, H,… etc. as predicates. We concentrate on the monadic situation. V-consistency, V-completeness, V-disjunctivity and V-apodicticity are still maintained as requirements. For d∈D, we may have V<x,d> or not. If V<x,d>, we write V(x)=d, as the relation is assumed to be functional. For a predicate F and D’ a subset of D, we may have V<F,D’> or not. Again we write V(F)=D’, as the relation on such pairs is again assumed to be functional. For relations, n-tuples are invoked as is standard. If x is a variable we write V(x)V’ to signify that V and V’ at most differ in that there is one d∈D so that V<x,d> and not V’<x,d>. We now impose plenist constraints on evaluations:

(1) Variability-plenism: If V∈E, x is a variable and d∈D then there is a V’∈E such that V(x)V’ and V’<x,d>.
(2) Barcan-plenism: ∀V’,V’’(VRV’&V’(x)V’’→∃V’’’(V(x)V’’’&V’’’RV’’))
(3) Converse-Barcan-plenism: ∀V’,V’’(V(x)V’&V’RV’’→∃V’’’(VRV’’’&V’’’(x)V’’))

For monadic atomic formulas of quantified modal logic we have VFx iff there is a d∈D so that V(x)=d and a subset D’ of D so that V(F)=D’, and d∈D. The n-adic cases and recursive clauses are obvious and omitted here.

Result: With the plenist constraints (1), (2) and (3), quantificational modal logics with the Barcan formula and its converse are straightforwardly accounted for. Completeness and soundness considerations carry over with slight alterations. Barcan-plenism and converse-Barcan-plenism may be omitted to justify logics as in Kripke’s approach with an underlying free logic. The evaluation semantics is precisely as flexible as standard possible worlds semantics, and as adequately accounts for modal discourse. Or at least so I argue.

Philosophical comments: It is obvious that there may be many set theoretic accounts of the semantics for modal logics, and I fully agree with Kripke in that there is no mathematical substitute for philosophy. However, attention to details is important. The evaluation semantics suggests that we indeed only need commit ourselves to *properties* (or, if one prefers to think more extensionally, *sets*) of sentences (propositions) and such pairs as have been invoked to semantically account for modal logics. (This is in deviation from the project of accounting for properties and propositions in terms of possible worlds, but that project has seemed misguided from its onset.) In the same way as there is no problem in accepting the existence of non-instantiated properties as *having won 30 Olympic gold medals*, there seems to be no ontological problem with accepting the existence of valuation-attributes which do not valuate propositions in accordance with how things really are. Notice well that our valuation-attributes should not be confused with *Ersatzworlds* as earlier suggested in the more philosophically bent literature. Residual perplexities concerning type are shared with standard approaches and not discussed here.