A Survival Analysis of Consumer Prices in Norway 1975–2004

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Preface

This thesis marks the end of my master degree in Economics at the University of Oslo and was written during a student internship at Norges Bank’s Research Department. I would like to thank Norges Bank for economic funding and inspiring working conditions. I would also express my gratitude to my supervisor, Fredrik Wulfsberg at Norges Bank, for invaluable help while working with this thesis. All remaining errors and weaknesses are my own responsibility.
Summary

In this thesis I estimate hazard functions of price observations using observed retail prices for Norway in the period 1975–2004. The hazard function gives the probability that we observe a price change in month $t$, given that the price has been constant for $t-1$ months.

Time-dependent and state-dependent models of price setting behavior are two common theories in macroeconomics used to explain nominal price rigidities. Time-dependent price setting models are characterized by exogenous timing of price readjustments, and two widely used time-dependent models are given in Calvo [1983] and Taylor [1980]. If price setters are homogenous the resulting hazard function in the Calvo model will be constant for all duration lengths. If prices are set as in the Taylor model, the hazard function should have a spike of value one at the contract renewal and be zero for all other duration lengths. Further, several contracts with different lengths in the economy result in several spikes in the hazard function, and aggregation of heterogenous price setters in the Calvo model results in a downward sloping hazard function (Álvarez, Burriel, and Hernando [2005]).

In state-dependent models of price setting, the timing of price readjustments is endogenous. One theory of state-dependent pricing is the existence of a cost attached to the action it takes to change the price. The timing of price changes under state-dependent models depends on economic conditions like inflation. Nakamura and Steinsson [2008] shows that menu cost models can give rise to a multitude of different shapes of the hazard function.

In order to find evidence on state-dependent and time-dependent models, I estimate hazard functions based on a data set of monthly retail price observations in Norwegian firms in the period 1975–2004, using an estimator called the Kaplan-Meier hazard rate. The Kaplan-Meier hazard rate is a nonparametric method and is estimated from the number of price changes observed for each duration lengths divided by number of price spells “at risk”. One problem with applying the Kaplan-Meier hazard rate is that it assume that the sample is homogeneous. I find, however, clear evidence of heterogeneity across different outlets. E.g. Energy items are characterized by frequent price changes (as much
as 68 per cent of the price spells have duration equal to one month in this sector), while Services are characterized by infrequent price changes (only 35 per cent of the price spells have duration equal to one month). In order to take account of this heterogeneity I have constructed one hazard rate for each item, and then aggregated by taking the mean. This implies that each item is weighted equally instead of each price spell. Another problem with survival analysis, is the problem of censored price spells. Censoring occurs when the start or end of a price spell is not observed, and therefore implies that we do not know the exact duration of a price spell that is censored. However, I find that adjusting for left-censored price spells do not alter the hazard rate very much.

I find that the aggregate hazard rate shows evidence on a negative duration dependence, and with spikes every 12th month. This means that overall the probability of observing a price change is lower the longer the price have been constant, but that the probability of observing a price change is high the 12th, 24th and 36th month with constant price. Álvarez, Burriel, and Hernando [2005] shows that aggregation of several Calvo and Taylor models result in a hazard function with a negative duration dependence, and this can therefore be one possible explanation of why the aggregated hazard rate has a negative duration dependence. I also investigate how the hazard function changes from a period with high and volatile inflation (1974–1989) to a period of low and stable inflation (1990–2004). I find that the hazard function from the period with high and volatile inflation is higher than the hazard function from the period of low and stable inflation. One implication of this is that the period with high and volatile inflation is characterized with more frequent price changes than the period with low and stable inflation. I also find large differences in estimated hazard rates for different product categories. E.g. the hazard rate for the HICP product type Energy is high and volatile, while the hazard rate for the HICP product type Service is low and with regular spikes every 12th month.

All calculations are done in Stata and the thesis is written in LaTeX.
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1 Introduction

In this thesis I estimate hazard functions of price observations using observed retail prices for Norway in the period 1975–2004. The hazard function gives the probability that we observe a price change in month $t$, given that the price has been constant for $t - 1$ months.

Time-dependent and state-dependent models of price setting behavior are two common theories in macroeconomics used to explain nominal price rigidities. Time-dependent price setting models are characterized by exogenous timing of price readjustments. Since the timing of price readjustments is exogenous it will be independent of the gap between the existing price and the price that would have been with fully flexible nominal prices. Two widely used time-dependent models are given in Calvo [1983] and Taylor [1980]. In the model given in Calvo [1983] the firms receive a signal when they can change its price and the probability of receiving a signal is independent of last time the firm received a signal. With a constant fraction of the firms receiving a signal every month the hazard function will be constant through time. The main idea in the model proposed by Taylor [1980]\footnote{The model in Taylor [1980] is a model that tries to explain wage rigidities, but it can also be used on prices.} is that firms set prices based on contracts, and that the firms are unable to change the price during the contract period. If prices are set as in the Taylor model the hazard function should have a spike of value one at contract renewal, and be zero for all other duration lengths. Further, several contracts with different lengths in the economy results in several spikes in the hazard function.

In state-dependent models of price setting, the timing of price adjustments is endogenous. One theory of state-dependent pricing is the existence of a cost attached to the action it takes to change the price. The cost can be to find information about the state of the economy before the new price is decided, or it can be more physical costs like printing out new price-labels. The timing of price changes under state-dependent models depends on economic conditions like inflation. When the gap between the price that would have been without price rigidities and the existing price is large enough to cover the cost at-
tached to the price change, the firm will change the price. Nakamura and Steinsson [2008] shows that menu cost models can give rise to a multitude of different shapes of the hazard function.

In order to find evidence on state-dependent and time-dependent models, hazard functions are estimated based on a data set that consists of monthly retail price observations in Norwegian firms in the period 1975–2004. I emphasize two common problems when survival analysis is applied in economics, namely the problem of censoring (see section 2.2) and the problem of aggregating heterogenous firms (see section 3). Álvarez, Burriel, and Hernando [2005] shows that aggregation of several Calvo and Taylor models result in a hazard function with negative duration dependence.2 I also investigate how the hazard function changes from a period with high and volatile inflation (1974–1989) to a period of low and stable inflation (1990–2004), and how the hazard function differs across different sectors.

The structure of the thesis is as follows: Section 2 explains the data set and the problem of censoring. Section 3 gives an introduction to the topic survival analysis and document heterogeneity in duration between different COICOP divisions and HICP product types. Section 4 explains the Kaplan-Meier estimate of the hazard and survival function. Empirical results of the Kaplan-Meier method are also presented in this section. Section 5 concludes.

## 2 Data

The data are collected by Statistics Norway (SSB) in order to compute the Norwegian Consumer Price Index (CPI).3 The sample consists of more than 14 million price observations from the period January 1975 to December 2004.4 Each observation is marked with an item number, outlet number, month, year, and retail price. Each observation is also flagged for item substitution or quality change if the product has changed from one month

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2This result is elaborated in Appendix B.
3Information about collection method is given in Statistisk sentralbyrå [2001]
4Wulfsberg [2009] gives a more detailed description of the data.
to the next month, and if the item was on sale. 12 per cent of observed price changes are related to sales and I treat these in the same way as price changes that are not marked for sales.\footnote{Wulfsberg [2009] shows that the frequency of price changes is almost the same independent of whether sale observations are included or not.} 9,626 of the price observations are marked for that the item has been discontinued, and these observations have been dropped.

The main classification method used in the CPI is the Classification of Individual Consumption by Purpose (COICOP). At the most aggregated level there are 12 COICOP divisions\footnote{These divisions are further divided into 58 COICOP groups, and at the most disaggregated level 119 COICOP classes (see United Nations [2000]).} (see Table 1). Each observation is also classified by HICP product types as either Unprocessed food, Processed food, Energy, Non energy industrial food, or Services (see Table 2). I present hazard and survival functions for each COICOP division and each HICP product type in section 4.

2.1 Price spells and trajectories

In duration analysis it is practical to organize the data where one observation represents a price spell, which is defined as a period where the price is constant and continuously observed, see Cleves, Gould, and Gutierrez [2004] for how this is done in stata. One price observation is then a sequence of price quotes, $P_{ikt}$, where $i$ is an index that gives the item, $k$ is an index for outlets selling item $i$, and $t$ is a time index. It is possible to follow one specific item sold in one particular outlet through time and identify price spells and price trajectories. A price spell is a period where the price of an item is constant and continuously observed. $T_{ikj}$ denotes the price spell duration, where $j = 1, \ldots, N_{ik}$ and $N_{ik}$ denotes the number of observed spells for item $i$ sold in outlet $k$. The duration is from the first month a particular price is observed and including the months with constant price and where the price is observed in every month. The price trajectory is identified by $(T_{ik1}, T_{ik2}, \ldots, T_{ikN_{ik}})$, where the length of a trajectory, $L_{ik}$, is the number of months between the first and the last observation and can include missing observations between

\footnote{Some items are very specific defined, such as the item Bicycle, DBS Logic LTC, 24 gear, while other items are more loosely defined such as the item Color TV, 28\".}
price spells. In the data set there are 4,392,840 price spells and 433,660 price trajectories. The average number of price spells per price trajectory is 10.1, and the median number of price spells is seven. The mean and median length of a trajectory is 40.7 and 37 months. The length of the price trajectories varies from one month to 327 months.

Examples of two price trajectories for four items are illustrated in Figure 1. For the item *Hot dogs, vacuum-packed*, the price trajectory given by the solid blue line is 18 months long (from September 1989 to February 1991), consists of seven price spells, and there are no missing observations. The price trajectory given by the red dashed line is 54 months long (from September 1990 to February 1995), consists of 34 price spells, and has two missing observations (in November 1990 and May 1992). Figure 1 illustrates how different the price trajectories can be for different items, and for the same item sold in different outlets. We see that the price trajectory for the item *Petrol (gasoline), unleaded*
95 oct., self-service is characterized by many and short price spells (prices are observed monthly so that shorter price spells are not observed), while the item T-shirt, cotton is characterized by fewer and longer price spells. The two outlets that sell the item Single room, w/bathroom and breakfast, weekend rate have very different price setting behavior although they sell the same item. The outlet represented by the solid blue line has only increased the price during the period it is observed, while the outlet represented by the red dashed line has both increased and decreased the price several times.

In the data set there are 1,135,352 periods of missing price observations, and more than 70 per cent of the price trajectories contain missing observations. Figure 2 gives the distribution of the length of periods with missing observations. More than 60 per cent of the periods only last one month and 77 per cent are at most two months long. The existence of missing observations implies that longer price spells are often split into shorter ones possibly creating a downward bias for the estimated duration. To mitigate this bias we impute missing periods that are at most two months long if either (a) the price spell before or after the missing observation has a duration of at least three months when there is only one missing observation, or (b) four months when there are two missing observations (this ensures that we do not impute values for items that change their prices often, like Petrol (gasoline), unleaded 95 oct., self-service (see Figure 1), and thereby create longer price spells). If the first observation after a period of missing observations is marked for item substitution or quality change no imputation is carried out. If one of the above criteria is met I apply one of the two rules:

1. If \( P_{ikt-1} = P_{ikt+1} \), i.e. the price observation the month before is equal to the price observation the month after the missing observation, I assume that there has been no price change, and impute for \( P_{ikt} = P_{ikt-1} = P_{ikt+1} \).

2. If \( P_{ikt-1} \neq P_{ikt+1} \), i.e. the price observation the month before is different from the price observation the month after the missing observation, I impute the price by extrapolating the shortest spell.
Imputation of a single missing observation leads to 308,569 fewer price spells, and imputation of spells with two missing observations leads to 36,395 fewer price spells. After imputation the data set consists of 4,047,875 price spells. The average number of price spells per price trajectory is 9.33, and the median number of price spells is six. The distribution density of the remaining periods with missing observations is given in Figure 2.

Table 1 gives the number of price observations, items, price spells and price trajectories in each COICOP division. The numbers in parenthesis gives the percentage share of price observations, items, price spells and price trajectories. COICOP division 1 *Food and non-alcoholic beverages* is by far the largest group with 58 per cent of the price observations and 69.5 per cent of the spells, while COICOP division 10 *Education* is the smallest group with only 2,618 price observations and 291 spells. Table 2 gives the number of price observations, items, price spells and price trajectories for each HICP product type. *Processed food* has the largest share of price observations, but *Unprocessed food* has the largest share of price spells.
Table 1: Number of price observations, items, spells and trajectories by COICOP division, column percentages below.

<table>
<thead>
<tr>
<th>COICOP division</th>
<th>price obs.</th>
<th>Number of items</th>
<th>spells</th>
<th>traj.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Food and non-alcoholic beverages</td>
<td>8,442,474</td>
<td>202</td>
<td>2,806,189</td>
<td>253,673</td>
</tr>
<tr>
<td></td>
<td>(58.5)</td>
<td>(20.2)</td>
<td>(60.5)</td>
<td>(58.8)</td>
</tr>
<tr>
<td>2 Alcoholic beverages, tobacco and narcotics</td>
<td>307,755</td>
<td>43</td>
<td>58,648</td>
<td>8,597</td>
</tr>
<tr>
<td></td>
<td>(2.1)</td>
<td>(1.5)</td>
<td>(1.5)</td>
<td>(2.0)</td>
</tr>
<tr>
<td>3 Clothing and footwear</td>
<td>1,245,464</td>
<td>140</td>
<td>244,025</td>
<td>39,236</td>
</tr>
<tr>
<td></td>
<td>(8.6)</td>
<td>(12.6)</td>
<td>(6.0)</td>
<td>(9.1)</td>
</tr>
<tr>
<td>4 Housing, water, electricity, gas and other fuels</td>
<td>202,540</td>
<td>30</td>
<td>44,371</td>
<td>9,862</td>
</tr>
<tr>
<td></td>
<td>(1.4)</td>
<td>(1.1)</td>
<td>(2.3)</td>
<td></td>
</tr>
<tr>
<td>5 Furnishings, household equipment and routine household maintenance</td>
<td>1,686,407</td>
<td>146</td>
<td>373,186</td>
<td>50,437</td>
</tr>
<tr>
<td></td>
<td>(11.7)</td>
<td>(9.2)</td>
<td>(11.7)</td>
<td></td>
</tr>
<tr>
<td>6 Health</td>
<td>217,448</td>
<td>52</td>
<td>39,126</td>
<td>6,235</td>
</tr>
<tr>
<td></td>
<td>(1.5)</td>
<td>(1.4)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 Transport</td>
<td>750,617</td>
<td>114</td>
<td>173,318</td>
<td>16,771</td>
</tr>
<tr>
<td></td>
<td>(5.2)</td>
<td>(4.3)</td>
<td>(3.9)</td>
<td></td>
</tr>
<tr>
<td>8 Communication</td>
<td>21,173</td>
<td>17</td>
<td>6,202</td>
<td>803</td>
</tr>
<tr>
<td></td>
<td>(0.2)</td>
<td>(0.2)</td>
<td>(0.4)</td>
<td></td>
</tr>
<tr>
<td>9 Recreation and culture</td>
<td>536,257</td>
<td>129</td>
<td>98,532</td>
<td>15,786</td>
</tr>
<tr>
<td></td>
<td>(3.7)</td>
<td>(2.4)</td>
<td>(3.6)</td>
<td></td>
</tr>
<tr>
<td>10 Education</td>
<td>2,570</td>
<td>7</td>
<td>291</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>(0.0)</td>
<td>(0.0)</td>
<td>(0.0)</td>
<td></td>
</tr>
<tr>
<td>11 Restaurants and hotels</td>
<td>214,849</td>
<td>44</td>
<td>31,544</td>
<td>6,731</td>
</tr>
<tr>
<td></td>
<td>(1.5)</td>
<td>(0.8)</td>
<td>(1.6)</td>
<td></td>
</tr>
<tr>
<td>12 Miscellaneous goods and services</td>
<td>814,931</td>
<td>100</td>
<td>162,879</td>
<td>23,179</td>
</tr>
<tr>
<td></td>
<td>(5.6)</td>
<td>(4.0)</td>
<td>(5.4)</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>14,442,485</td>
<td>1,114</td>
<td>4,038,311</td>
<td>431,363</td>
</tr>
<tr>
<td></td>
<td>(100)</td>
<td>(100)</td>
<td>(100)</td>
<td>(100)</td>
</tr>
</tbody>
</table>

Note: The total row is not equal to the total amount of price observations, items, spells and trajectories since 12,058 of the price observations are not identified by a COICOP division number or by a HICP product type number.

Table 2: Number of price observations, items, spells and trajectories by HICP product type, column percentages below.

<table>
<thead>
<tr>
<th>HICP product type</th>
<th>price obs.</th>
<th>Number of items</th>
<th>spells</th>
<th>traj.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Unprocessed food</td>
<td>3,751,595</td>
<td>150</td>
<td>1,683,569</td>
<td>120,749</td>
</tr>
<tr>
<td></td>
<td>(26.0)</td>
<td>(13.5)</td>
<td>(41.7)</td>
<td>(28.0)</td>
</tr>
<tr>
<td>2 Processed food</td>
<td>4,998,634</td>
<td>185</td>
<td>1,181,268</td>
<td>141,521</td>
</tr>
<tr>
<td></td>
<td>(34.6)</td>
<td>(16.6)</td>
<td>(29.3)</td>
<td>(32.8)</td>
</tr>
<tr>
<td>3 Energy</td>
<td>128,421</td>
<td>15</td>
<td>73,451</td>
<td>3,471</td>
</tr>
<tr>
<td></td>
<td>(0.9)</td>
<td>(1.3)</td>
<td>(1.8)</td>
<td>(0.8)</td>
</tr>
<tr>
<td>4 Non energy industrial goods</td>
<td>4,271,438</td>
<td>501</td>
<td>865,239</td>
<td>125,755</td>
</tr>
<tr>
<td></td>
<td>(29.6)</td>
<td>(45.0)</td>
<td>(21.4)</td>
<td>(29.2)</td>
</tr>
<tr>
<td>5 Services</td>
<td>1,292,397</td>
<td>263</td>
<td>234,784</td>
<td>39,867</td>
</tr>
<tr>
<td></td>
<td>(8.9)</td>
<td>(23.6)</td>
<td>(5.8)</td>
<td>(9.2)</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>14,442,485</td>
<td>1,114</td>
<td>4,038,311</td>
<td>431,363</td>
</tr>
<tr>
<td></td>
<td>(100)</td>
<td>(100)</td>
<td>(100)</td>
<td>(100)</td>
</tr>
</tbody>
</table>

Note: See Table 1
2.2 Censoring

Another prominent feature of the data is censored price spells.\(^8\) Censoring occurs when the start or end of a price spell is not observed. For price spells that are censored it is therefore only possible to quantify the minimum length, and not the exact length of the price spell. There are three different types of censoring: left-censoring, interval-censoring and right-censoring.\(^9\)

Left-censoring is when the initial price change starting a new spell occurred before the item was under observation. The first price spell in a price trajectory is always left-censored, since it is not possible to know whether the first observation in a price spell is a price change when the price the month before is not observed. The first price spell in the blue price trajectory for the item *Hot dogs, vacuum-packed* in Figure 1 on page 4, starts in September 1989 and ends in October 1989. Since we do not know the price before September 1989, all we can say is that this price spell is at least two months long.

Right-censoring occurs when the length of the price spell cannot be exactly quantified since the price change ending the price spell occurs when the item is not longer under observation. The last price spell in a price trajectory is always right-censored. The last price spell in the price trajectory given by the solid blue line for the item *Hot dogs, vacuum-packed* in Figure 1 on page 4 is only one month long, and since we do not know what happens with the price after this month it is right-censored. Longer spells are more likely to be right-censored than shorter spells, which means that censoring entails a downward bias in the estimation of the duration of price spells (see Figure 3 on page 12).

There is also a possibility that a price spell is both left-censored and right-censored. This means that we do not observe a price change for a sequence of continuously observed price observations.

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\(^9\) In survival analysis one distinguish between censoring and truncation. Censoring is when we know for sure that a price change has occurred when the item was not under observation, while truncation is when we do not know whether there has been a price change or not. Often, when a price spell is censored, we cannot be sure that there has been a price change before the first observation, or after the last observation. I follow convention in the literature and do not distinguish between censored and truncated price spells.
Table 3: Censored price spells.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of spells</td>
<td>%</td>
<td>No. of spells</td>
<td>%</td>
<td>No. of spells</td>
<td>%</td>
</tr>
<tr>
<td>Left and Right</td>
<td>618,690</td>
<td>15.3</td>
<td>318,265</td>
<td>13.5</td>
<td>300,425</td>
<td>17.8</td>
</tr>
<tr>
<td>Left</td>
<td>623,610</td>
<td>15.4</td>
<td>364,389</td>
<td>15.4</td>
<td>259,221</td>
<td>15.3</td>
</tr>
<tr>
<td>Right</td>
<td>623,610</td>
<td>15.4</td>
<td>338,911</td>
<td>14.4</td>
<td>284,699</td>
<td>16.8</td>
</tr>
<tr>
<td>Non</td>
<td>2,181,966</td>
<td>53.9</td>
<td>1,336,200</td>
<td>56.7</td>
<td>845,766</td>
<td>50.1</td>
</tr>
<tr>
<td>Total</td>
<td>4,047,876</td>
<td>100.0</td>
<td>2,357,765</td>
<td>100.0</td>
<td>1,690,111</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Table 3 gives the number of price spells by type of censoring for the whole data set. We see that censoring is common, however, 54 per cent of the price spells in the whole data set are not censored. Table 3 also shows the extent of censoring for two subperiods, where one period goes from January 1975 to December 1989 and the other goes from January 1990 to December 2004. The first price spell in a price trajectory in the period 1975–1989 is always left-censored, but the last price spells in this period do not need to be right-censored if the price trajectory continue into the next period. This explains why we have more left-censored price spells than right-censored price spells in the first period. Similarly the first price spell in a trajectory in the period 1990–2004 do not need to be left-censored, while the last price spell in a trajectory will always be right-censored. Almost 57 per cent of the spells in the first period is not censored, and 50 per cent in the last period is not censored.

Looking at the extent of censoring across products Table 4 shows that some COICOP divisions have a larger fraction of their price spells censored. E.g. in COICOP division 3 Clothing and Footwear only 36 per cent of the spells are not censored, while in COICOP division 7 Transport as much as 62.8 per cent of the spells are not censored. Table 5 shows the fraction of censored price spells by product type. For Energy products 75.2 per cent of the spells are not censored, while Non energy industrial goods have 44 per cent of the spells not censored.

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10 For the price spells that starts before January 1990 and ends after January 1990 the price spell is put in the period where the longest part of the spell belongs. E.g. if a spell start in October 1989 and end in February 1990 it belongs to the first period. In 2,811 cases where the number of months before and after January 1990 is exactly the same, I have chosen to put the price spell in the first period.
Table 4: Censored price spells by COICOP division.

<table>
<thead>
<tr>
<th>Censoring</th>
<th>COICOP division 1</th>
<th></th>
<th>COICOP division 2</th>
<th></th>
<th>COICOP division 3</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of spells</td>
<td>%</td>
<td>No. of spells</td>
<td>%</td>
<td>No. of spells</td>
<td>%</td>
</tr>
<tr>
<td>Left and Right</td>
<td>375,013</td>
<td>13.4</td>
<td>7,329</td>
<td>12.5</td>
<td>64,987</td>
<td>26.6</td>
</tr>
<tr>
<td>Left</td>
<td>415,069</td>
<td>14.8</td>
<td>10,232</td>
<td>17.4</td>
<td>45,673</td>
<td>18.7</td>
</tr>
<tr>
<td>Right</td>
<td>415,069</td>
<td>14.8</td>
<td>10,232</td>
<td>17.4</td>
<td>45,673</td>
<td>18.7</td>
</tr>
<tr>
<td>Non</td>
<td>1,601,038</td>
<td>57.0</td>
<td>30,855</td>
<td>52.7</td>
<td>87,692</td>
<td>36.0</td>
</tr>
<tr>
<td>Total</td>
<td>2,806,189</td>
<td>100.0</td>
<td>58,648</td>
<td>100.0</td>
<td>244,025</td>
<td>100.0</td>
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<table>
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<tr>
<th>Censoring</th>
<th>COICOP division 4</th>
<th></th>
<th>COICOP division 5</th>
<th></th>
<th>COICOP division 6</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of spells</td>
<td>%</td>
<td>No. of spells</td>
<td>%</td>
<td>No. of spells</td>
<td>%</td>
</tr>
<tr>
<td>Left and Right</td>
<td>8,679</td>
<td>19.5</td>
<td>78,616</td>
<td>21.0</td>
<td>3,478</td>
<td>8.9</td>
</tr>
<tr>
<td>Left</td>
<td>7,172</td>
<td>16.2</td>
<td>61,804</td>
<td>16.6</td>
<td>6,344</td>
<td>16.2</td>
</tr>
<tr>
<td>Right</td>
<td>7,172</td>
<td>16.2</td>
<td>61,804</td>
<td>16.6</td>
<td>6,344</td>
<td>16.2</td>
</tr>
<tr>
<td>Non</td>
<td>21,348</td>
<td>48.1</td>
<td>170,962</td>
<td>45.8</td>
<td>22,960</td>
<td>58.7</td>
</tr>
<tr>
<td>Total</td>
<td>44,371</td>
<td>100.0</td>
<td>373,186</td>
<td>100.0</td>
<td>39,126</td>
<td>100.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Censoring</th>
<th>COICOP division 7</th>
<th></th>
<th>COICOP division 8</th>
<th></th>
<th>COICOP division 9</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of spells</td>
<td>%</td>
<td>No. of spells</td>
<td>%</td>
<td>No. of spells</td>
<td>%</td>
</tr>
<tr>
<td>Left and Right</td>
<td>17,617</td>
<td>10.2</td>
<td>1,009</td>
<td>16.3</td>
<td>20,746</td>
<td>21.1</td>
</tr>
<tr>
<td>Left</td>
<td>23,445</td>
<td>13.5</td>
<td>1,203</td>
<td>19.4</td>
<td>17,776</td>
<td>18.0</td>
</tr>
<tr>
<td>Right</td>
<td>23,445</td>
<td>13.5</td>
<td>1,203</td>
<td>19.4</td>
<td>17,776</td>
<td>18.0</td>
</tr>
<tr>
<td>Non</td>
<td>108,811</td>
<td>62.8</td>
<td>2,787</td>
<td>44.9</td>
<td>42,234</td>
<td>42.9</td>
</tr>
<tr>
<td>Total</td>
<td>173,318</td>
<td>100.0</td>
<td>6,202</td>
<td>100.0</td>
<td>98,532</td>
<td>100.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Censoring</th>
<th>COICOP division 10</th>
<th></th>
<th>COICOP division 11</th>
<th></th>
<th>COICOP division 12</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of spells</td>
<td>%</td>
<td>No. of spells</td>
<td>%</td>
<td>No. of spells</td>
<td>%</td>
</tr>
<tr>
<td>Left and Right</td>
<td>53</td>
<td>18.2</td>
<td>7,535</td>
<td>23.9</td>
<td>29,749</td>
<td>18.2</td>
</tr>
<tr>
<td>Left</td>
<td>51</td>
<td>17.5</td>
<td>5,795</td>
<td>18.4</td>
<td>27,508</td>
<td>16.9</td>
</tr>
<tr>
<td>Right</td>
<td>51</td>
<td>17.5</td>
<td>5,795</td>
<td>18.4</td>
<td>27,508</td>
<td>16.9</td>
</tr>
<tr>
<td>Non</td>
<td>136</td>
<td>46.8</td>
<td>12,419</td>
<td>39.3</td>
<td>78,114</td>
<td>48.0</td>
</tr>
<tr>
<td>Total</td>
<td>291</td>
<td>100.0</td>
<td>31,544</td>
<td>100.0</td>
<td>162,879</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Note: See Table 1

A third and different concept of censoring is interval-censoring, which occurs when we have missing observations between price spells in a price trajectory. Despite imputation of short periods of missing observations (see previous section), many interval censored spells still remain. In contrast to left-censored and right-censored price spells, we know both the minimum and maximum length of interval censored spells. For example the second price
Table 5: Censored price spells by HICP product type.

<table>
<thead>
<tr>
<th>Censoring</th>
<th>Unprocessed food</th>
<th>Processed food</th>
<th>Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of spells</td>
<td>%</td>
<td>No. of spells</td>
</tr>
<tr>
<td>Left and Right</td>
<td>194,345</td>
<td>11.5</td>
<td>187,997</td>
</tr>
<tr>
<td>Left</td>
<td>236,833</td>
<td>14.1</td>
<td>188,468</td>
</tr>
<tr>
<td>Right</td>
<td>236,833</td>
<td>14.1</td>
<td>188,468</td>
</tr>
<tr>
<td>Non</td>
<td>1,015,558</td>
<td>60.3</td>
<td>616,335</td>
</tr>
<tr>
<td>Total</td>
<td>1,683,569</td>
<td>100.0</td>
<td>1,181,268</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Censoring</th>
<th>Non energy industrial goods</th>
<th>Services</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of spells</td>
<td>%</td>
</tr>
<tr>
<td>Left and Right</td>
<td>183,094</td>
<td>21.2</td>
</tr>
<tr>
<td>Left</td>
<td>150,486</td>
<td>17.4</td>
</tr>
<tr>
<td>Right</td>
<td>150,486</td>
<td>17.4</td>
</tr>
<tr>
<td>Non</td>
<td>381,173</td>
<td>44.0</td>
</tr>
<tr>
<td>Total</td>
<td>865,239</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Note: See Table 1

spell given by the solid blue line for the item *T-shirt, cotton* in Figure 1 on page 4 were at most 13 months long, and it is recorded with 12 months. With missing observations during a price trajectory the last price spell before the missing observation is right-censored, while the first price spell after the missing observation is left-censored. When we know both the minimum and maximum length of the spell we have more information than we have about the spells that are censored because of entry and exit of items in the data set. However, except from when imputation is carried out, I do not take advantage of this information when hazard and survival functions are calculated below.

Censoring is not independent of duration. As shown in Figure 3 the share of censored spells increases for longer duration. As the share of left-censored price spells is more or less constant for all durations, longer price spells are more likely to be right-censored than shorter price spells. The high share of left- and right-censored price spells for durations from one to three months can be explained by missing values in trajectories with short price spells (the cases where we have chosen to not impute for missing values, see section 2.1).

\[11\] In this case we know that the spell is censored and not truncated since we know that there has been a price change when the item was not under observation.
3 Survival Analysis

How long do firms keep the price on their products constant? And is it more likely that the price will change the longer it has been constant? Further, how does the likelihood of a price change depend on economic factors like inflation? And does the likelihood varies between different items?

All these questions have in common that the dependent variable is duration and not a variable at a fixed point in time. Figure 4 gives the distribution density of duration\textsuperscript{12} of the price spells in the whole sample and the two subperiods. Almost half of the price spells have duration equal to one month in the whole sample. A larger share of the price spells have duration smaller than four months in the period 1975–1989, than in the period 1990–2004. The density of duration in the two subperiods are equal for duration between four and six months, while a larger share of the price spells have duration larger than six months in the subperiod 1990–2004.

Figure 5 gives the distribution of duration in every COICOP division, and we see heterogeneity in price setting behavior between the different COICOP divisions. In COICOP

\textsuperscript{12}Duration is measured without taking into account that some of the price spells are censored (see section 2.2).
division 1 Food and non-alcoholic beverages above 50 per cent of the price spells have duration equal to one month, and 92 per cent of the spells are six months or shorter. In both COICOP division 7 Transport and COICOP division 8 Communication a large share of the spells have a duration equal to one month. In COICOP division 10 Education only 22 per cent of the spells lasts for one month, and as much as 47 per cent of the spells have a duration equal to 12 months or more. The rest of the COICOP divisions show more similar pattern with 30 to 40 per cent of the spells equal to one month, and about 10 per cent of the spells equal to 12 months or longer. Figure 6 gives the distribution of duration by HICP product type, and it reveals heterogeneity in price setting behavior between the different HICP product types. Both Unprocessed food and Energy has more than 60 per cent of their price spells equal to one month and more than 96 per cent are at most six months long. Processed food, Non energy industrial goods and Services all have less than 40 per cent of their price spells equal to one month, and Services has more than 13 per cent of the price spells equal to 12 months or longer.

The duration of a price spell, \( T \), is a stochastic variable, with stochastic properties defined by its distribution function (see Figure 4–6). The fact that the dependent variable is duration and not a variable at a fixed point in time leads to problems in classical
Figure 5: Distribution density of duration by COICOP division.

Figure 6: Distribution density of duration by HICP product type.

Note: Truncated at 12 months
regression analysis. This can be illustrated with an example where duration of a price spell is the dependent variable and the inflation rate is an exogenous variable. Since the inflation rate is a variable that change over time, it will also change during the time a price remains constant. One problem will therefore be how to measure exogenous variables that change over the duration length. If we decided to use the inflation rate at the last month of the duration, it is not necessarily true that this is a good specification of the inflation rate for the whole duration length. In classical regression models we also often impose the restriction that the residuals are normally distributed, but when the dependent variable is duration this is not a good specification. Furthermore, normally distributed residuals do not account for the censored nature of the data (see section 2.2), and they will also not take into account that it only makes sense with positive values on the dependent variable. In order to account for the problems raised above, it is appropriate to use survival analysis instead of classical regression models (Kiefer [1988]).

In survival analysis the distribution function is instead described by what is called the survival and hazard functions. I will now define these functions following Kalbfleisch and Prentice [2002].

$T$ is a discrete non-negative random variable denoting the time to a price change, that is $T$ take values $t_1 < t_2 < \ldots$ where $t_1$ is duration equal to one month. $T$’s probability function is defined as

$$f (t_j) = \Pr \left(T = t_j\right), \quad j = 1, 2, \ldots, \quad (1)$$

and is the unconditional probability of observing a price change after $t_j$ months. $T$’s cumulative distribution function is defined as

$$F (t) = \Pr \left(T \leq t\right). \quad (2)$$

$F (t)$ is a monotone, non-decreasing function of time, and gives the probability of a price change during $t$ months. The survival function, $S(t)$, gives the probability that there is
Figure 7: Example of Hazard functions.

See A.3 for definition of the Exponential, Weibull and Loglogistic model.

no price change for at least \( t \) months, and is defined by

\[
S(t) = 1 - F(t) = \Pr(T \geq t) = \sum_{j|t_j \geq t} f(t_j). \tag{3}
\]

The survival function is a monotone, non-increasing function of time. The hazard function for duration equal to \( t_j \) months is defined as the probability of a price change at \( t_j \), given that there has not been any price changes for \( t_{j-1} \) months, and is defined by

\[
h_j = Pr(T = t_j \mid T \geq t_{j}) = \frac{f(t_j)}{S(t_j)}, \quad i = 1, 2, \ldots, \tag{4}
\]

where \( S(t_j^-) = \lim_{t \to t_j^-} S(t) \). The hazard function is therefore a rate between the unconditional probability of a price change in month \( t \) and the probability of no price change for at least \( t \) months. In our case, the hazard function measures how likely it is to observe a price change for a given duration length.

The hazard function can be constant, decreasing, increasing, monotonic or non-monotonic. The blue line in Figure 7 gives an example of a constant hazard function. In this case the hazard function has no memory, i.e. the probability of a price change is independent
of duration \((h_j = h)\). A decreasing hazard function, as given by the red line in Figure 7, means that the longer duration the less likely it is that we will observe a price change (negative duration dependence). An increasing hazard function, illustrated by the green line in Figure 7, means that we have a positive duration dependence. The yellow line in Figure 7 gives an example of an non-monotonic hazard function. In this case the probability of a price change increases for durations smaller than four months, but for durations larger than four months the probability of observing a price change decreases with duration. Furthermore, the hazard function is also a measure of how often the prices adjust in the economy. A high value of the hazard function means that the price adjust frequently (an evidence of flexible prices), while a low value means that the price adjust more rarely (an evidence of sticky prices).

The cumulative hazard function is defined by

\[
H(t) = \sum_{j=1}^{\infty} h_j
\]  

and measures the total amount of risk that has been accumulated up to time \(t\), i.e. the number of times we would expect to observe price changes over a given time period.

Note that if we know the hazard function, we can derive \(H(t), S(t), F(t)\) and \(f(t_j)\). The survival function at each point in time is the probability that the price remains constant for at least \(t\) months. The value of the survival function at one month is therefore the same as \((1 - h_1)\). Similarly, the survival function at two months is equal the probability that we do not observe a price change for at least two months, and this is the same as \((1 - h_1) \cdot (1 - h_2)\). This gives that the survival function is linked to the hazard function by the following relationship

\[
S(t) = \prod_{j: h_j \leq t} (1 - h_j).
\]
The probability function defined by the hazard function is

\[ f(t_j) = h_j \prod_{l=1}^{j-1} (1 - h_l). \quad (7) \]

This can be seen from observing that \( \prod_{l=1}^{j-1} (1 - h_l) = S(t_j) \).

The advantage of analyzing the hazard rate instead of the probability function is that the two models on time-dependent pricing proposed by Calvo [1983] and Taylor [1980], and state-dependent models with menu cost, are all models with assumptions about the shape of the hazard function. An assumption in time dependent models is that the hazard function is exogenous, in contrast to state-dependent models where the hazard function is endogenous.

4 Kaplan-Meier estimates of the hazard rate

In this section I will estimate the hazard function. There are three main methods to quantify the hazard function: nonparametric models, semiparametric models and parametric models. Nonparametric models makes no assumptions about the functional form of the hazard function, i.e. we do not impose any restrictions on whether it is increasing, decreasing, constant, or a mixture over time. Neither do nonparametric models take account for what is causing price changes. The empirical hazard and survival function is derived by only using the observed price changes for our items in the years 1975–2004. I focus on a nonparametric model, namely the Kaplan-Meier estimate. I discuss semiparametric models and parametric models in appendix A.

Let \( t_{i1} < t_{i2} < \ldots < t_{ik} \) be observed duration lengths for each item \( i \), where \( t_{i1} \) is duration equal to one month. \( d_{ij} \) is the number observed price changes for item \( i \) at duration equal \( j \) months, and \( m_{ij} \) is the number of price spells that ends as right-censored at \( t_{ij} \). \( n_{ij} = (m_{ij} + d_{ij}) + \cdots + (m_{ik} + d_{ik}) \) denotes the number of price spells “at risk” for each item and each duration lengths. The Kaplan-Meier estimate of the hazard function
is
\[ \hat{h}_{ij} = \frac{d_{ij}}{n_{ij}} \quad i = 1, 2, \ldots, l \quad \text{and} \quad j = 1, 2, \ldots, k. \quad (8) \]

Equation (8) states that the hazard rate for each item and for each duration length, \( t_{ij} \), can be estimated from the number of price changes observed at \( t_{ij} \) relative to the number of price spells with duration at least equal to \( t_{ij} \). Since this is only a ratio it is often called the Kaplan-Meier hazard rate, a convention I will follow. Equation (8) can only be used when the data is recorded in discrete time like our monthly price observations. It can be shown that \( \hat{h}_{ij} \) maximizes the likelihood function over all survival functions (see Kalbfleisch and Prentice [2002, chap. 1.4]).

From equation (6) and equation (8) we get the Kaplan-Meier estimate of the survival function (also called the Kaplan-Meier product-limit estimate)
\[ \hat{S}(t) = \prod_{j \mid t_{ij} \leq t} \left( \frac{n_{ij} - d_{ij}}{n_{ij}} \right). \quad (9) \]

A shortcoming of the Kaplan-Meier survival function is that once the estimator is zero, it remains zero regardless of future activity.\(^{13}\) The rest of this section gives the results of the Kaplan-Meier hazard rate and survival function when they are applied on a data set that makes the foundation of the Norwegian CPI in the period 1975–2004 (see section 2 for a description of the data set).

**Heterogeneity**

The Kaplan-Meier hazard rate and survival function assumes that the sample is homogeneous. In section 3 we observed heterogeneity in the distribution of duration between the different COICOP divisions and HICP product types (see Figure 5 and 6 on page 14). To take account of this heterogeneity I first construct hazard rates for each item and then aggregate by taking the mean of the hazard rates over all duration lengths. This implies

\(^{13}\)A brief overview of another common nonparametric model, the Nelson-Aalen estimate, is given in appendix A.1.
that the hazard rate is calculated by weighting each item equally.\textsuperscript{14} The hazard rate given by the blue line in the left panel in Figure 8 is calculated in this way. The shape of the hazard rate shows a weak negative duration dependence, at least for the first 12 months. Furthermore, there are distinct spikes at 12, 24 and 36 months indicating that many outlets change prices on a regular basis as in time-dependent models of price setting with Taylor contracts equal to one, two or three years.

Alternatively, the hazard rate given by the red line in Figure 8 is calculated by pooling price spells, i.e. giving each price spell the same weight by adding up all price spells at risk and number of price changes for each duration lengths. The pooled hazard rate has a stronger negative duration dependence than the hazard rate given by the blue line, particularly for durations smaller than one year. One reason for this difference is that calculating the hazard rate first at item level and then aggregate adjusts for over-represented small duration lengths. A large share of the price spells in COICOP division 1 \textit{Food and non-alcoholic beverages} has a short duration (see Figure 5 on page 14). This division accounts for 69.5 per cent of the total number of price spells in the sample, while the share of items is only 26.9 per cent of the whole sample (see Table 1 on page 7).

\textsuperscript{14}Baumgartner et al. [2005] apply a weighting scheme where each price spell is weighted with the inverse total number of price spells that is observed for that item, and in addition each item is also attached to its CPI weight. This strategy adjust for the large share of short price spells, but it does not take into account for the observed heterogeneity.
Another reason may be that aggregation of heterogenous firms (almost always) gives a decreasing hazard function (see Álvarez, Burriel, and Hernando [2005], and two stylized examples in appendix B). Figure 8 also shows that spikes are smaller in the pooled hazard rate.

The difference between the two calculation methods for the survival function is shown in the right panel of Figure 8. Consistent with the corresponding hazard rates, the survival function by item given by the blue line, is above the pooled survival function given by the red line. The probability of no price change after one month is 82 per cent, and 19 per cent after one year in the case where each item is given the same weight. The corresponding pooled estimates are 67 and 10 per cent. This means that heterogeneity produce a downward bias in the survival function when polling. I thus proceed with calculating the average item specific hazard rate.

Censoring

In Figure 9 the hazard rate for the whole data set is calculated with two different strategies that deals with censored price spells. The first strategy deal with left-censored price spells. Left-censored price spells produce a downward bias in the estimation of the hazard rate since these price spells are not recorded as “at risk” prior to the first observation even
though they most likely should have been (remember that the first observation in a left-censored price spell not necessarily corresponds to a price change). This means that the denominator in the hazard rate (see equation (8)) is systematically too low when left-censored price spells are included. In order to take account of this problem it is common to exclude the left-censored price spells (see e.g. Nakamura and Steinsson [2008]). Using this approach yields the hazard rate given by the green line in Figure 9. This is a standard procedure in duration analysis. Exclusion of the left-censored price spells shifts the hazard rate upward by 1.1 per cent on average.

The second strategy deals with right-censored price spells. Following Baudry et al. [2007], I compute the hazard rate excluding half of the right-censored spells (a total amount of 621,946 price spells) from the total number of price spells at risk for each duration lengths. This procedure adjusts for the fact that some price spells are only recorded as “at risk”, and never recorded as a price change, even though they would have been if they was still observed. The resulting hazard rate is given by the blue line in Figure 9. The hazard rate shifts upward by 10.5 per cent on average. Furthermore, it shows a more constant duration dependence and less distinct spikes at the annual frequencies, but the semi annual spikes are more pronounced.

Since the resulting hazard rates is quite similar when all price spells are included and when adjusting for left-censored price spells, I have chosen to include all price spells when hazard rates are calculated below. I have also chosen to not adjust for right-censored price spells since it is not clear why one should choose this strategy.

**High versus low inflation period**

In the period 1975–1989 average yearly inflation was 8.42 per cent with a standard deviation of 2.73, and in the period 1990–2004 average yearly inflation was 2.32 per cent with a standard deviation of 1.08. The inflation was, thus, much higher and more volatile in the first period, than in the last period. In Figure 10 the solid blue line is the hazard

---

15Excluded price spells are randomly drawn.
16Source: OECD.Stat, monthly CPI observations.
Figure 10: Kaplan-Meier hazard rates (left) and survival functions (right) for the periods 1975–1989 and 1990–2004.

Rate (left) and the survival function (right) for the period 1975–1989, and the red dashed lines gives the hazard rate and survival function for the period 1990–2004. The hazard rate for the first period is on average 6.2 per cent above the hazard rate for the last period reflecting that prices changed more often in the high-inflation period. That the conditional probability of a price change is higher in the period with high and volatile inflation is in line with state-dependent models of price setting.\textsuperscript{17} We see that the spikes at 12, 24, and 36 months are more distinct in the first period than in the last period. The hazard rate from the first period has also a spike at every 6th month which is absent in the last period. The spikes are consistent with price setting behavior as given by Taylor contracts. The differences in the two hazard rates may imply that the Taylor contracts have changed from the first period to the second period. Consistent with the evidences from the hazard rates, we see in the right panel of Figure 10 that the survival function for the period 1990–2004 is above the survival function for the period 1975–1989.

Product categories

Hazard rates and survival functions by COICOP division are given in Figure 11. COICOP division 8 Communication and COICOP division 10 Education are excluded when hazard

\textsuperscript{17}The relationship between frequency of price changes and inflation is not unambiguous in state-dependent models of price setting.
Figure 11: Kaplan-Meier hazard rates (left) and survival functions (right) by COICOP division.

rates and survival functions are calculated at COICOP division level because of few observations. The shape and the value of the hazard rates differ across the divisions, which reflects heterogeneity in price setting behavior. COICOP division 1 Food and non-alcoholic beverages has the highest probability (29 per cent) for a price change given that there has been no price change for one month. The hazard rate for COICOP division 1 also has a
clear negative duration dependence, which means that the probability of a price change decreases the longer the price remains constant. Further, there are no regular spikes in this hazard rate. The hazard rate for COICOP division 2 Alcohol beverages, tobacco and narcotics is characterized by many and distinct spikes, where the probability of a price change given constant prices for one year is almost 70 per cent. The hazard rate shows first an increasing and then decreasing duration dependence. Both COICOP division 3 Clothing and footwear and COICOP division 5 Furnishings, household equipment and routine household maintenance shows a relatively low and flat hazard rate. This can be an evidence of price setting behavior as given in the Calvo model, where each firm in the two divisions face the same probability of changing the price each month. The average probability of observing a price change in these two COICOP divisions is respectively 8.3 and 10 per cent. The hazard rate for COICOP division 6 Health has a weak negative duration dependence for durations up to 18 months, but for durations larger than 18 months it shows a positive duration dependence, which means that for long durations the probability of observing a price change is increasing. COICOP division 7 Transport has two very distinct peaks at 12 and 24 months, and it is therefore common in this sector to review the price once a year or every second year. This is an evidence of price setting behavior as in Taylor contracts with duration equal to one or two years. COICOP division 9 Recreation and culture has a low and flat hazard rate which is an evidence that the price setting is sticky in this sector. Further, the two peaks after 12 and 24 months are an evidence on that the outlets review their prices on yearly basis. The probability that an outlet in this division keeps the price constant more than one month is 89 per cent, the probability that it remains fixed after six months is 42 per cent, and the probability that the price is fixed for more than 12 months is 16 per cent. Both COICOP division 11 Restaurants and hotels and COICOP division 12 Miscellaneous goods and services shows more or less a constant duration dependence.

For the HICP product categories, Figure 12 shows that the hazard rate for Unprocessed food is high for duration equal to one month and then declines rapidly to duration equal to 18 months, and flattens out for larger durations. This can be an evidence on price
Figure 12: Kaplan–Meier hazard rates (left) and survival functions (right) by HICP product type.

setting behavior that corresponds to the Calvo model with heterogeneous firms as shown in Appendix B. The hazard rate for Energy items is in general high for durations smaller than one year and this is an evidence on that prices are adjusted frequently in this sector. Further, we see that the hazard rate is volatile with many distinct spikes. The three other HICP product types, Processed food, Non energy industrial goods and Services, have a lower hazard rate and a more constant duration dependence. Further, we see that Processed food has a distinct spike for duration equal 12 months, and that Services has distinct spikes every 12th month, which is an evidence on that outlets in this sector change their prices on a yearly basis, as in the Taylor model with contracts equal to one, two or three years. Therefore, the prices are most sticky for Services, and most flexible for Energy and Unprocessed food.

Figure 13–15 give the different hazard rates in each COICOP division, and for each of
Figure 13: Kaplan–Meier hazard rates (left) and survival functions (right) for COICOP division 1–3.

The two subperiods. COICOP division 4 Housing, water, electricity, gas and other fuels, COICOP division 6 Health, and COICOP division 11 Restaurants and hotels are excluded because of few observations, especially in the period 1975–1989. The two hazard rates for COICOP division 1 Food and non-alcoholic beverages shows a different pattern in the
two subperiods. The hazard rate for the period 1975–1989 shows first a negative duration dependence, but for durations larger than 18 months the duration dependence is positive. The hazard rate for the period 1990–2004 shows a negative duration dependence for all duration lengths. The hazard rate for COICOP division 5 Furnishing, household equipment and routine household maintenance in the period 1975–1989 also first shows a negative duration dependence for short durations, and a positive duration dependence for larger durations, while the hazard rate for the period 1990–2004 shows a negative duration dependence for all duration lengths. The hazard rate for the first subperiod also shows distinct spikes every 6th months while there is no evidence of spikes in the hazard rate for the last period. In fact, all COICOP divisions show more distinct spikes in the first period than in the last period. It therefore seems like it was more common for the outlets
to review the price setting at an yearly basis in the period 1975–1989 than in 1990–2004.
It is not possible to say whether time-dependent pricing was a more used price setting
rule in the period 1975–1989 than in 1990–2004. Even though we observe a more regular
price-setting behavior in the first period, it can be that the outlets still reviewed their
prices on a regular basis, but that they concluded that it is optimal to leave the price
unchanged. When comparing survival functions between the two subperiods we see that
the survival function for the last period in general lies above the survival function for the
first period, at least for long durations. The gap between the two survival functions for
COICOP division 12 Miscellaneous goods and services is especially high. In the last period
the probability of no price change for 12 months is 36 per cent, while it is only 16 per cent
in the first period.
Figure 16–17 gives hazard rates and survival functions for HICP each product types in each of the periods. The hazard rate for the first period is in general above and with more distinct spikes than the hazard rate for the last period.
Figure 17: Kaplan-Meier hazard rates (left) and survival functions (right) for HICP product type 4 & 5.

Other studies

Dhyne et al. [2005] summarize three common features on hazard rates estimated in different European countries:\(^{18}\) (a) Hazard rates for price changes computed from the full sample of price spells display an overall decreasing pattern in all countries; (b) Hazard rates are also characterized by local modes at durations of 12, 24 and 36 months, indicating that a fraction of firms review their prices on an annual basis; (c) Hazard rate for duration one month is typically quite high, reflecting the share of price spells with very short durations (mainly oil products and unprocessed food retailers). We see that these common features also suits the Norwegian data.

\(^{18}\)The hazard rates that are compared are estimated on CPI data from Austria, Belgium, Finland, France, Italy, Luxembourg and Netherlands.
5 Conclusion

I have estimated the Kaplan-Meier hazard rate for a large data set of consumer prices. I started with a data set where each observation recorded a time and the measurements taken at that point in time for each item in one outlet. In order to calculate the hazard rate I had to convert this data set into a duration format, where each observation records a price spell.

The method used in this thesis is based on a graphical representation of the Kaplan-Meier hazard rate and survival function, and my results are based on visual inspection of these graphs. Although, time-dependent and state-dependent models of price setting behavior are mainly theories with restrictions on the hazard rates, I have also chosen to include the survival functions since these also provide relevant information about price setting behavior.

Section 4 revealed a large amount of heterogeneity in the price setting behavior across COICOP divisions and HICP product types, and calculating hazard rates based on the different product categories resulted in different levels and shapes of the hazard rates. Further, this section also showed that this heterogeneity has a large impact on the shape of the aggregated hazard rate. If we do not take account of the observed heterogeneity, the aggregated hazard rate shows a stronger negative duration dependence, than if we adjust for heterogeneity.

I also found some evidence on that the level of inflation in the economy has an impact on how often firms change their prices. The hazard rate calculated from retail prices in the first period is in general above the hazard rate from the last period. This result is in line with state-dependent models of price setting. A second interesting result is that the hazard rate from the first period displayed more distinct spikes than the hazard rate from the last period. This result can be interpreted that some firms changed their prices as in Taylor contracts, and that the Taylor contracts has changed during the two periods. It is however not possible with this investigation to say whether prices are more flexible in the first period than in the last period. It can be that firms reviewed their prices as often in
the last period as in the first period, but that they concluded that the gap between the existing price and the optimal price was not large enough to change the price.

This thesis has only used a nonparametric method when calculating hazard rates. It is therefore limiting how strong conclusions one can draw, especially when it comes to state-dependent models of price setting behavior. It is therefore interesting to estimate hazard functions based on parametric methods, but this is beyond the scope of this thesis.

References


Appendix

A  Further on Survival Analysis

A.1  Nonparametric model: Nelson-Aalen

The Nelson-Aalen cumulative hazard function at any time $t$ is given by

$$
\tilde{H}(t) = \sum_{r \mid t_r \leq t} \frac{d_r}{R_r}
$$

(A.1)

where $R_r$ is the number of price spells with duration at least equal to $t_r$, $d_r$ is the number of price spells with duration exactly equal to $t_r$ months, and the sum is over all the duration lengths where we observe price changes less than or equal to $t$. This is an estimate of the cumulative hazard function introduced in equation (5). By using the theoretical
relationship in equation (6) one can convert the Nelson-Aalen estimate of the cumulative hazard function to an estimate of the survival function. Both $\hat{H}(t)$ and $\hat{S}(t)$ are consistent estimators.

In small samples the Kaplan-Meier estimator is superior when estimating the survival function, and the Nelson-Aalen estimator is superior when estimating the cumulative hazard function, but in large samples the two methods are equivalent, and this is independent of whether it is the cumulative hazard function or the survival function that you want to estimate (Cleves, Gould, and Gutierrez, 2004).

A.2 Semiparametric models

In semiparametric models no parametric form of the survival function is assumed, but the covariates are specified. As seen below, when we use the Cox proportional hazard regression model, we run a regression based on the vector $x_j\beta$, where we put information we find important for the hazard rate in this vector and parametrize it via the size of the $\beta$'s.

The Cox proportional hazard regression model

The hazard rate under Cox proportional hazard regression model is assumed to be

$$h(t|x) = h_0(t) \exp(x\beta)$$

(A.2)

where the regression coefficients $\beta$ are to be estimated from data. $h_0(t)$ is the baseline hazard function, and it is left unestimated. Since the baseline hazard function is unestimated we do not impose any restriction and the shape of the hazard function: it can be increasing, decreasing, constant, monotonic or non monotonic. $x$ are the specified covariates.

Even tough we do not impose any restrictions on the shape of the hazard function over time, we do assume that if we regress several hazard function for different subjects
in the data, the hazard functions will be proportional to another. In our case, if we want to compare the hazard function regressed on the price spells in the period 1975–1989 with the hazard function regressed on price spells from 1990–2004 we need to assume that the two hazard functions have the same shape over time and that the only thing that differs are the size of the hazard function. This would mean that

\[
\frac{h(t|\mathbf{x}_{1975-1989})}{h(t|\mathbf{x}_{1990-2004})} = \frac{h_0(t) \exp(\mathbf{x}_{1975-1989}\beta_x)}{h_0(t) \exp(\mathbf{x}_{1990-2004}\beta_x)} = \frac{\exp(\mathbf{x}_{1975-1989}\beta_x)}{\exp(\mathbf{x}_{1990-2004}\beta_x)}
\]  

where \(\mathbf{x}_{1975-1989}\) are the covariates from the period 1975–1989 and \(\mathbf{x}_{1975-1989}\) are the covariates from the period 1990–2004, is a constant. Other subjects could be each COICOP division or product type, or it could be to regress one hazard rate for each item number.

### A.3 Parametric models

In the third group, parametric models, we make assumptions about the functional form of the survivor function. This is done by making assumptions about the distributions error term. In standard OLS the assumption is that the error terms are normally distributed, but the error terms are not normally distributed in survival analysis, and we therefore need to assume some other distribution. Parametric estimation is appropriate when you do have an idea of what the baseline hazard looks like. To check the result of parametric estimation it can be useful to compare the result with the result from using the Cox method for estimating the hazard function. If there is a large deviation between the results of the two hazard functions, the parametric method is most likely not useful.

**Exponential**

In this case the hazard function is given by

\[
h(t) = \lambda
\]
and it is constant. This means that the conditional probability of a price change in a given short interval of time is the same regardless of when the observation is made. A constant hazard function is typical for a process where future actions have no impact on the hazard function today. The survivor function is given by

\[ S(t) = e^{-\lambda t} \]

**Weibull**

The Weibull hazard function assumes that

\[ h(t) = \lambda t^{p-1} \]

The hazard function is monotonically increasing or decreasing depending on \( p \). It is increasing for \( p > 1 \), decreasing for \( p < 1 \), and equal to the exponential hazard function if \( p = 1 \). The survivor function is

\[ S(t) = e^{-\lambda t^p} \]

This model would allow the estimated cumulative hazard to increase at an increasing rate, but it is restrictive in the sense that the hazard function is necessarily monotonic decreasing or increasing.

**Lognormal**

\[ f(t) = \frac{p}{t} \phi\{p \ln(\lambda t)\} \]

where \( \ln -t \) is normally distributed with mean equal \(-\ln \lambda \) and standard deviation equal to \( \frac{1}{p} \).

\[ S(t) = \Phi\{-p \ln(\lambda t)\} \]

This is a non-monotonic hazard function that first increase and then decrease.
Loglogistic

\[ h(t) = \frac{\lambda p (\lambda t)^{p-1}}{1 + (\lambda t)^p} \]

where \( \ln t \) has a logistic distribution with mean equal to \( -\ln \lambda \) and variance equal to \( \frac{\pi^2}{3p^2} \).

This hazard function permits non-monotonic behavior as long as \( p > 1 \). It first increase and then decrease. The corresponding survivor function is

\[ S(t) = \frac{1}{1 + (\lambda t)^p}. \]

B  Examples of aggregation of heterogenous price setters

Below, I show that aggregation of time-dependent models like those of Calvo [1983] and Taylor [1980] with heterogenous firms (almost always) gives a decreasing aggregated hazard function, following Álvarez, Burriel, and Hernando [2005]

Case of two Calvo agents

Assume first that the aggregate economy consists of two groups of Calvo agents with sizes \( s_1 \) and \( s_2 \), and define \( \lambda = \frac{s_1}{s_1 + s_2} \). The density function for each of the two groups is

\[ f^i_C(t) s_i = (1 - \theta_i) \theta_i^{l-1} s_i \quad \text{(B.1)} \]

and the aggregate density function is given by the weighted sum

\[ f_C(t) = (1 - \theta_1) \theta_1^{l-1} \lambda + (1 - \theta_2) \theta_2^{l-1}(1 - \lambda). \quad \text{(B.2)} \]

The survival function for each of the two groups is given by

\[ S^i_C(t) s_i = \theta_i^{l-1} s_i, \quad \text{(B.3)} \]
Figure 18: Hazard function of two different Calvo agents and the corresponding aggregated hazard function (left) and one Calvo and one Taylor model (right).

The hazard function for each of the two groups is given by

\[ h^i_C(t) = (1 - \theta_i), \]  

and the aggregated hazard function is given by\(^{19}\)

\[ h_C(t) = \beta(t) (1 - \theta_1) + [1 - \beta(t)] (1 - \theta_2) \]  

where

\[ \beta(t) = \left[ 1 + \left( \frac{\theta_2}{\theta_1} \right)^{t-1} \left( \frac{1-\lambda}{\lambda} \right) \right]^{-1}. \]

Assume that there is an economy that consists of one group of firms that keeps their prices constant of an average of three months, and the other group keeps their price constant of an average of 12 months. The corresponding hazard functions will be \( h^1 = \frac{1}{3} \) and \( h^2 = \frac{1}{12} \). We further assume that the size of the group is 0.5. The two hazard functions

\(^{19}\)The aggregated hazard function is calculated from equation (4).  \( h_C(t) = \frac{f_C(t)}{S_C(t)} = \frac{(1-\theta_1)^{t-1} \lambda + (1-\theta_2) \theta_2^{t-1} (1-\lambda)}{\theta_1 \lambda + \theta_2 (1-\lambda)} = \beta(t) (1 - \theta_1) + [1 - \beta(t)] (1 - \theta_2) \)
given by equation (B.5) is given by the blue and red line to the left in Figure 18. The green line gives the corresponding aggregated hazard function calculated from equation (B.6). We see that the aggregation of two constant heterogenous hazard functions leads to an downward sloping hazard function, and also that the aggregated hazard function converges asymptotically to the one with longest average price duration.

One group with Calvo agents and one group with Taylor contracts

Assume now that the aggregate economy consists of two groups of firms with sizes \( s_1 \) and \( s_2 \), where group 1 adjust their price according to a Calvo model and group 2 adjust their prices according to a Taylor contract of length \( J \). The density, survival and hazard function for group 1 is given by equations (B.1), (B.3) and (B.5). The density function for group 2 is given by

\[
f_2^T(t) = \begin{cases} 
0 & \text{for } t \neq J \\
1 & \text{for } t = J
\end{cases}
\] (B.7)

and the aggregate density function in this economy is given by

\[
f_{CT}(t) = \begin{cases} 
(1 - \theta_1) \theta_1^{t-1} \lambda & \text{for } t \neq J \\
(1 - \theta_1) \theta_1^{t-1} \lambda + (1 - \lambda) & \text{for } t = J
\end{cases}
\] (B.8)

The survival function for group 2 is given by

\[
S_2^T(t) = \begin{cases} 
1 & \text{for } t \leq J \\
0 & \text{for } t > J
\end{cases}
\] (B.9)

with corresponding aggregate survival function

\[
S_{CT}(t) = \begin{cases} 
\theta_1^{t-1} \lambda + (1 - \lambda) & \text{for } t \leq J \\
\theta_1^{t-1} \lambda & \text{for } t > J
\end{cases}
\] (B.10)
The hazard function for group 2 is given by

\[
h_T^2(t) = \begin{cases} 
0 & \text{for } t \neq J \\
1 & \text{for } t = J 
\end{cases} 
\]  

(B.11)

and the aggregate hazard function takes the following form

\[
h_{CT}(t) = \begin{cases} 
\alpha(t)(1 - \theta_1) & \text{for } t < J \\
\alpha(t)(1 - \theta_1) + [1 - \alpha(t)] & \text{for } t = J \\
(1 - \theta_1) & \text{for } t > J 
\end{cases} 
\]  

(B.12)

where \( \alpha(t) = \frac{s_1^{\theta t - 1}}{s_1^{\theta t - 1} + s_2} \). To the right in Figure 18 the aggregated hazard rate is illustrated for \( s_1 = 0.94, s_2 = 0.06 \), average duration of price spells in group 1 is 12 months and the Taylor contract faced by group 2 is 12 months. We see that aggregation of these two models will give a downward sloping hazard function until the time of contract renewal, while after the contract renewal the hazard function is constant and equal the hazard function given by the Calvo model.