The Symmetry of Monetary Policy

Testing the linearity of the aggregate supply curve on Norwegian data

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PREFACE

Supervisor at the Department of Economics at the University of Oslo was Professor Ragnar Nymoen. The empirical part of the thesis was written as a part of an internship at Norges Bank. Supervisor at Norges Bank was Q. Farooq Akram. Both have shown a keen interest in the project and I would like to thank them for valuable comments and suggestions. I would also like to thank the people at Norges Bank for help and discussions. A special thank to my fiancée, Miss Frida Viken, for her support and patience during the completion of the thesis.

Any remaining errors are my own responsibility.

Oslo, November 2007
Thomas Lystad
EXECUTIVE SUMMARY

An interest rate policy is symmetrical if the nominal interest rate response to equally large positive and negative deviations from the inflation target are of identical magnitude but with opposite signs. The governor of Norges Bank (NoB, The Norwegian central bank), Svein Gjedrem, stated in 1999 that good monetary policy includes a symmetrical interest rate: “Over time it is important that the interest rate is set symmetrically. Such symmetry is necessary to sustain the expectations of nominal stability” (Gjedrem, 1999).

![Figure 1. Norges Bank's expectation of the sight deposit rate in the baseline scenario with fan chart. Per cent. Quarterly figures. 04 Q1 – 08 Q4. Source: Norges Bank, Inflation Report 3/2005.](image)

Norges Bank began its inflation targeting policy about two years after Gjedrem made this statement. The governor has not spoken in terms of a symmetrical interest rate during the period of inflation targeting. Implicitly however, NoB has communicated a symmetrical property of the interest rate by the use of fan charts (see Figure 1). The purpose of the fan chart is threefold: it should provide a forecast of the most likely outcome of the economy. The most likely outcome is represented by the thick black line at the centre of the projections. Second, it should convey the degree of uncertainty surrounding the most likely outcome. The degree of uncertainty is represented by the width of the fans. Lastly, it should provide information on the balance of this uncertainty. The spread around the most likely outcome should indicate the risks related to the level of uncertainty. As illustrated in Figure 1, the risks

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1 This definition of a symmetrical interest rate is model dependant, as will be illustrated in Section 1.10
2 Author’s own translation of: "Over tid er det viktig med symmetri i rentesettingen. En slik symmetri er nødvendig for å opprettholde tilliten til nominell stabilitet." (Gjedrem, 1999)
of an interest rate lower than the most likely outcome is equal to the risk of an interest rate higher than the most likely outcome. The balance of risk is symmetrical.

The optimality of a symmetrical interest rate is based on a simplified model of the economy, often referred to as the linear-quadratic framework. There is however reason to question if the linear-quadratic framework serves as a good model. If any of the assumptions underlying this framework is changed, it will directly impact interest rate setting and a symmetrical interest rate is no longer optimal. The expressed view of a symmetrical interest rate makes thus a tight restriction on monetary policy.

The purpose of this thesis is twofold: it will demonstrate why the property of a symmetrical interest rate / symmetrical fan charts makes a tight restriction on monetary policy. Second, it will test if one of the necessary restrictions for a symmetrical interest rate to be optimal can be rejected on Norwegian data.

The thesis proceeds as follows. The first chapter begins by addressing the questions of what monetary policy is, which variables the central bank wants to stabilise and how it can achieve its goals by using the interest rate as its instrument. With respect to the change towards inflation targeting, the economic rationale when choosing a precise inflation target and a production target, is discussed. This discussion makes the foundation of the economic model derived at the end of the chapter. The model is based on the linear-quadratic framework. The purpose of the first chapter is to illustrate how the linear-quadratic framework leads to an optimal symmetrical interest rate. The second chapter questions the symmetrical interest rate as an optimal result. It thoroughly analyses how altering some of the assumptions made in chapter one impacts the interest rate setting. Specifically, it is the shape of the loss function, the shape of the aggregate supply curve, whether shocks are additive only and the credit channel that are discussed. The third chapter tests if one of the necessary conditions for a symmetrical interest rate to optimal, the linear aggregate supply curve, can be rejected on Norwegian data. The supply curve is taken from one of NoB’s economic models. It will be shown that linearity of the supply curve cannot be rejected. The supply curve has however poor empirical properties, suggesting that the lack of evidence of non-linearity not necessarily is evidence of linearity. Another explanation could be that the model is poorly specified. Estimation is done using Eviews 5.1.
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1 OPTIMAL SYMMETRICAL POLICY

1.1 What is monetary policy?
Monetary policy describes the process of how the government or a central bank manages its instrument to achieve specific goals. Such goals can be to stabilise inflation, production, the exchange rate, employment, consumption, asset prices or other economic variables that the government wants to control. The main instruments that over time have been used in achieving these goals are the money supply, open market operations, setting reserve requirements, trading in foreign exchange markets and/or the interest rate. NoB’s main instrument is the interest rate. (Norges Bank, 2007).

1.2 The transmission mechanism - How the interest rate affects the economy
Commercial banks have large deposits of assets in NoB. NoB controls the interest rate that it gives on these deposits, often termed the key rate. When the NoB raises the key rate, it increases commercial bank’s gain from depositing assets. Commercial banks respond by increasing their interest rate; the interest rate consumers and businesses pay on their loans, and are granted on their deposits. How the interest rate impacts the most important variables in the economy, the transmission mechanism, is illustrated in Figure 2.

An increase in the interest rate leads to a lowering of production through the raised cost of handling debt. In order to meet this increased cost, households reduce their level of consumption. When demand and the level of production are lowered, businesses must meet this cut in income by reducing its costs. A business can reduce costs by several means: cut in staff, cut in wages, lower margins, restructuring, etc. In aggregate, there will be downward pressure on employment as a result from the increased interest rate.
The exchange rate is measured as Norwegian Kroner (NOK) in terms of the foreign currency, i.e. kr/£. A reduced exchange rate is thus equivalent to an appreciation of the NOK. When the interest rate is raised, investors find it relatively more profitable to buy Norwegian financial assets. This leads to an increased demand for NOK, which means that the NOK appreciates. In terms of how the exchange rate is valued (kr/£), an increase in the interest rate leads thus to a lowering of the exchange rate.

A higher interest rate reduces the level of inflation. The lower level of consumption, the lower level of employment and thus the lower level of production, push the rate of inflation down. Also, the lowering of the exchange rate, (stronger NOK), meaning that one can buy more foreign goods per NOK, also leads to a lower rate of inflation as foreign goods become relatively cheaper with the stronger NOK. In addition, the interest rate impacts inflation expectations through workers wage demands, which again impact the current level of inflation.

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3 Inspired by NoB’s illustration from the World Wide Web: http://www.norgesbank.no/english/monetary_policy/affect.html
One important conclusion can be reached from the above discussion: The interest rate impacts inflation, expected inflation, production, the exchange rate and employment in the same direction in the short run (0 – 3 years) i.e. an increase in the interest rate puts downward pressure on both production and inflation, and vice versa. This feature has important implications on how NoB conducts their monetary policy. Understanding these implications requires knowledge of the objective NoB is trying to reach.

1.3 The objective of monetary policy in Norway

In Norway, the goals of NoB are set by the government. The mandate was changed in March 2001 when it was decided that the NoB should pursue an inflation target. The target was set to 2.5%. In the mandate, the NoB is also instructed to create a stable development of production, employment and the value of the NOK (the exchange rate - NOK)\(^4\).

Two interesting aspects of this mandate needs further comment. First, Norway is the twenty-fifth country in the world that has changed its goal of monetary policy towards inflation targeting, New Zealand being the first in 1990. The new trend of inflation targeting countries is a result of the failure of previous regimes, where one tried to mainly stabilise other variables, such as production or the exchange rate.

Second is the amount of variables that is to be stabilised. The mandate specifies that NoB should not only focus on a 2.5% inflation rate, but also the development of production, employment and the exchange rate. The goal of stabilising more than one variable raises an interesting question: What happens if there should be a conflict between stabilising inflation and i.e. the level of production? Norway is currently experiencing such a trade-off situation. Being hit by a supply shock, the level of production is above trend level, while inflation is below its target of 2.5%. Since the interest rate impacts production and inflation in the same direction, both variables cannot be stabilised easily. Increasing the interest rate would dampen the level of production, pushing it towards the trend level. However, an increased interest rate would also dampen the level of inflation, moving it even further away from its target level.

The government has not directed the NoB on how to respond to such offsetting shocks through its mandate. The NoB must thus make a judgement of the relative weight it assigns to

\(^4\) See Appendix 6.3 for the precise defined mandate
stabilising inflation and production. If the central bank were only to focus on stabilising inflation, so called strict inflation targeting, the economy would end up creating an unnecessary high level of variation in production, which would be sub-optimal with regards to the mandate. On the other side, if the NoB solely focused on stabilising the level of production, that would create too high a variation in inflation. Somewhere between these two extreme policies, lies the optimal policy. It is the relative weight that the NoB assigns to stabilising inflation and production that gives the best outcome for the economy. Precisely what that relative weight should be is a question NoB is continuously trying to answer.

1.4 Costs related to production

The total amount of what is produced in a country is measured as Gross Domestic Product (GDP). Norway increased its total amount of production in 2005 with 2.2% from the year before. This type of variation in production happens every year, and economists measure this rate of growth with regards to a so called trend level of growth. This trend level is defined as the level of growth that is “compatible with a stable development in prices and wages” (Bjørnland, 2006, p.6).

Figure 3. The Output Gap. The official output gap by NoB. Annual series.⁵

Figure 3 illustrates the output gap. The output gap is the percentage deviation between the level of production and the trend level of production⁶. If the level of growth is above the trend

⁵ See Appendix 6.1 for a description of the series

⁶ Measuring this trend level and the output gap is not a trivial task. According to Orphanides et al., “the reliability of output gap estimates in real time tends to be quite low. Different methods give widely different estimates of the output gap in real time and often do not even agree on the sign of the gap” (1999, p.24).
level of growth, there is a positive output gap. If the level of growth is however below trend level, there is a negative output gap. These variations around the trend level are called business cycles and they represent a cost to society, in particular due to employment fluctuations\(^7\).

1.4.1 Welfare costs of employment fluctuations\(^8\)

Production is a direct function of employment. When the level of production varies, the level of employment also varies. If there is some element of imperfect competition in the economy (which there always will be, the only market that is close to being perfectly competitive is the foreign exchange market), there will be welfare costs of employment fluctuations.

![Figure 4. The Efficient vs. the Imperfect Level of Employment.](image)

Figure 4 illustrates the deadweight loss the economy experience if there is imperfect competition. The labour market supply curve represents workers marginal rate of substitution between work and leisure. The higher the wage paid, the more willing people are to substitute leisure for work. The “demand – efficient” curve represents the amount of workers that firms are willing to hire for the different values of the real wage, assuming that firms are operating in a perfectly competitive market. \(L^e\) represents the amount of labour that would be hired in such an efficient economy. The “demand – Imperfect” curve represents firms’ demand for

\(^7\) A discussion of the welfare costs due to consumption fluctuations can be found in Appendix 6.8
\(^8\) This section is mostly inspired by Sørensen, et al. 2005. p. 601-605
labour in the case of imperfect competition. $L^m$ is the amount of workers that would be hired in an imperfect economy.

If we further assume that there are labour unions operating in this economy, and they have demanded a real wage, $w$, which is higher than the equilibrium real wage, it will create an equilibrium level of employment $L^1$ which is even lower than the imperfect level of employment. Given the assumptions of imperfect competition and labour unions, $L^1$ is the natural rate of employment. The shaded area represents the deadweight loss in the labour market. It shows the total amount of inefficiency that prevails due to labour unions and monopolistic competition.

Figure 5. The Welfare Effects of Employment Fluctuations.

Figure 5 illustrate how fluctuations around the natural level of employment cause welfare costs in terms of varying levels of deadweight losses. If the economy enjoys a booming period, and employment increases to the level of $L^+$, it would be welfare improving since the level of employment would move towards the efficient level of employment $L^e$. The total welfare gain is represented by the two areas A and B. However, if the economy is going through a recession, pushing the level of employment back to $L^-$, it would be welfare worsening. The total amount of welfare loss equals the areas C and D. Variation in employment, and thus variation in production, is costly to the economy since the welfare gain during booms is less than the welfare loss during recessions: $C + D > A + B$.
1.5 Costs of Inflation\textsuperscript{9}

The costs related to inflation can be divided into three categories. First, there are costs related to the variation of inflation. Second, inflation by itself, both positive and negative, incurs a cost to the economy. The costs related to positive and negative inflation (deflation) is of such different nature that they will be treated separately.

1.5.1 Costs of variation in inflation

- Miscalculations: When i.e. a labour union bargains the wage of its members, it does so in nominal terms, making expectations about what price level it believes will prevail in the coming period of the wage contract. If the inflation rate turns out to be higher than what the union expected, the member workers of that union will suffer a welfare loss due to a decreased real wage.

- Redistribution of wealth: A higher than expected inflation rate redistributes wealth from creditors to debtors. This is due to how a higher inflation rate means a lower rate of real interest. The real interest rate is the real cost of holding money, or in the eyes of debtors, it is the cost they must pay to borrow money. If the real interest rate drops, so does the cost of their loan.

- Credibility loss: NoB is targeting a certain level of inflation. If inflation varies too much around this level, there is a risk that the market will lose faith in the inflation target.

1.5.2 Costs of inflation

- Shoe-leather costs: there are transactions costs related to the handling of money. When the inflation rate rises, holding money becomes more expensive, and the frequency of transactions will increase.

\textsuperscript{9} The following discussion on the costs related to inflation can be found in any standard macroeconomics book, i.e. Sørensen, et al. 2005, Gartner. 2003, Mankiw .2000.
- **Menu costs:** It is costly for a firm to change prices on its products. An increase in inflation forces firms to bear a higher cost related to price changes.

- **Relative price distortions:** Firm’s change prices at different times. When inflation is high, the relative price of goods will differ due to the asynchronous manner in which price changes occur. Inflation thus leads to a distortion of consumer choices as firms with the same costs charge different prices for their products.

- **Loss of competitiveness:** For a given nominal exchange rate, a higher level of inflation leads to higher wage costs and thus a loss of international competitiveness.

### 1.5.3 Costs of deflation – liquidity trap

The economy is in a liquidity trap if it is experiencing deflation (a negative rate of inflation) and the level of the interest rate is low. With an already low interest rate, it is difficult for the central bank to provide the economy with more liquidity as no lender is willing to lower the nominal interest rate below zero. A liquidity trap thus describes a situation where it is difficult to stimulate the economy through monetary policy in a situation where such stimulation is needed. A liquidity trap could impose huge real costs on the economy as a recession is prolonged by the inability of providing economic stimulus.

### 1.6 Which Stabilisation Levels to Target

#### 1.6.1 Choosing a production target

Trend level of production is the level that is associated with a stable development of wages and prices. It is this level of production which the central bank should target. According to Mankiw, one of the most important lessons of macroeconomics is that in the long run, the rate of money growth determines the rate of inflation, but it does not affect the rate of production (p.529, 2000). If the central bank would try to target a higher level than the trend level of production, it would in the long run only lead to a higher level of inflation.
1.6.2 Choosing an inflation target

The inflation target in Norway is set to 2.5%. The inflation target should be set as low as possible as a low inflation target reduces the costs of inflation (i.e. shoe-leather costs, menu costs, relative price distortions, etc). At a zero percent inflation rate, these costs of inflation would be non-existent. There is, however, strong arguments against a zero percent inflation target. First, if the inflation target were zero percent, the risk of the economy going into a liquidity trap (deflation) would be severe. Second, most economies have the property of so-called downward nominal wage rigidity. Simplified, this means that employees, and their unions, are reluctant to accept a cut in the nominal wage. If inflation is positive, and employees accept that their nominal wage is held constant, then they are actually accepting a wage cut, in real terms. Some degree of inflation can thus be beneficial, so as to create flexibility in the labour market. Third, price indices tend to overstate the true rate of inflation. The inflation target in Norway was set to 2.5% so as to accommodate the risks of deflation, the impact of downwards nominal rigidity and the problem of price indices overestimating inflation10.

1.7 The Loss Function

The discussion has so far been on an intuitive level, trying to answer some of the more basic questions related to monetary policy: what it is, how it affects the economy, what variables to stabilise, etc. In deriving what economists call an optimal monetary policy, that is, the monetary policy which is best for the economy, a mathematical model is needed. The rest of chapter 1 is devoted to developing such a model. The starting point of the model is to use some of the results from the discussion so far, and make a mathematical expression describing the preferences of the central bank – the loss function.

The previous discussion was related to the welfare costs of variation in production and inflation, and what the desired level of production and inflation should be. The main two conclusions from that discussion can be used to derive a loss function for NoB:

1. NoB is trying to stabilise production, \( y_t \), around its trend level, \( \bar{y}_t \). There is a welfare loss related to variation in production around this level.

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10 A more thorough analysis can be found in Appendix 6.10
2. NoB has chosen an inflation target of 2.5%. In making a general loss function, the inflation target will be described by \( \pi \). There is a welfare loss when inflation, \( \pi_t \), varies around its target.

Based on these two characteristics a natural loss function for one period would thus be\(^{11}\):

\[
L_t = \lambda (y_t - \overline{y}_t)^2 + (\pi_t - \overline{\pi})^2
\]  

(1)

The subscript \( t \) denotes that it is the loss incurred in period \( t \). The first term \( (y_t - \overline{y})^2 \) gives the variation in production around its trend value. As the variables are expressed in natural logarithms the term \( y_t - \overline{y} \) expresses the percentage deviation from trend. The second term \( (\pi_t - \overline{\pi})^2 \) expresses variation in inflation around the inflation target. Since inflation is measured in percent, the term \( \pi_t - \overline{\pi} \) expresses the percentage point deviation off target. \( \lambda \) represents the relative weight NoB puts on stabilising production versus inflation, i.e. a high \( \lambda \) means that the NoB puts a large relative weight on stabilising production.

Figure 6 illustrates how this loss function looks like when drawn in \( \pi, y \) space. The different curves shows different levels of loss based on the same value of \( \lambda \). If the economy is on target, both with regards to inflation and production, there is no social loss (\( L=0 \)). The \( L=1 \) curve represents all the values of inflation and production that according the loss function gives a total loss of 1 unit. The loss function defines an ellipse. This is due to the relative weight assigned to \( \lambda \), where the loss functions are drawn based on the assumption that the NoB “feels” a greater loss due to inflation variation than production variation (\( 0<\lambda<1 \)). This assumption is realistic for economies with an inflation target. As described above, the loss function holds a property which is of great importance in terms of a symmetrical interest rate:

\(^{11}\)Since the economy adjusts gradually, the total loss should take into account also the discounted expected future losses, so that the total loss becomes: \( \sum_{\tau=0}^\infty \delta^\tau L_{t+\tau} \). See Svensson (2004) for a more detailed exposition.
KEY ASSUMPTION 1: The loss function (1) has the property of being symmetrical, meaning that a positive or negative deviation of equal size creates the same level of loss.

According to the governor of NoB, Mr. Svein Gjedrem, this is a property that fits well with the preferences of all inflation targeting countries: “all inflation targeting countries has the property of symmetry in common: avoiding too low inflation is equally important as avoiding too high inflation” (2005). Mr. Gjedrem is referring to the property of symmetry in inflation. In addition, the loss function above is also symmetrical in production. Avoiding too low production is thus equally important as avoiding too high production.

The symmetry discussed above is often referred to as symmetry of preferences. This is because the loss function is a set of preferences for the central bank, and it is symmetrical with regards to both inflation and production. The main question of this thesis is concerned with another type of symmetry: the symmetry of the optimal policy. How a set of preferences for the central bank, the loss function, leads to an optimal policy, is the topic of the next two sections.

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12 Authors own translation of: "Men felles for alle land med inflasjonsmålstyring er at det er symmetri: Det er like viktig å unngå for lav som for høy inflasjon." (Gjedrem, 2005)
1.8 A Model of the Economy: AS-AD

This section will explain two equations that represent the aggregate demand and aggregate supply side of the economy. The analysis will focus on the variables inflation and production only. Employment (unemployment) is thus assumed to be a direct function of production. This is in accordance with Okun’s law which states that “deviations of income from its potential level are proportional to the difference between the actual and the natural unemployment rate” (Gartner, 2003, p.402). It is also assumed that the economy is closed, which is why the exchange rate is excluded from the analysis. These simplifications are made so as to make the exposition of an optimal symmetrical policy as intuitive and clear as possible.

1.8.1 Aggregate Supply (AS)

\[ \pi_t = \pi_{t-1} + \alpha (y_t - \bar{y}) + u_t \]  

Equation 2 represents the short run aggregate supply side of the economy (AS). It specifies how inflation, \( \pi_t \), is determined by the level of production and by the expected level of inflation. Expectations are assumed to be adaptive. The expected level of inflation is thus equal to the inflation level in the previous period, \( \pi_{t-1} \). \( (y_t - \bar{y}) \) is the output gap in percent. \( \alpha \) is a positive coefficient, meaning that the output gap is positively related with inflation - a positive output gap will put upward pressure on inflation, and vice versa. \( \alpha \) is also known as the inflation-production trade-off. More precisely, the slope of the AS curve indicates that it takes a 1 percent reduction in output to reduce inflation by \( \alpha \) percentage points. Two important key assumptions follows from this discussion:

**KEY ASSUMPTION 2:** The AS curve is linear, meaning that the inflation-production trade off is constant for all levels of production.

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13 The formation of expectations is simplified in this model. The model used by NoB, which will be analysed in Chapter 3, is based on a more complex formation of expectations.
\( u_t \) are the shocks that hits the supply side of the economy\(^{14} \). These shocks can be shocks to the oil price, labour market, capital market, etc. The figure below shows the aggregate supply curve, and how such shocks impact this curve when drawn in \( y_t, \pi_t \) space.

**KEY ASSUMPTION 3:** The economy is hit by additive shocks only. Additive shocks serves in contrast to i.e. multiplicative shocks, which impacts the slope of the curve in addition to the intercept.

![Figure 7. Aggregate Supply.](image)

A higher oil price will increase the general price level (AS \(^{-}\)) since oil is an important component in the production of many goods. When the price of oil increases, so does the production costs, and that will put upward pressure on the general price of products in the economy. A sudden decrease of labour costs, which could be caused by higher competition from Eastern Europe due to greater labour mobility, would have the opposite effect, pushing production costs down. This would in the end decrease the general price level of goods in the economy (AS \(^{+}\)). These examples illustrates how shocks to the supply side of the economy impacts the AS curve in our model. Understanding how these shocks impact the whole economy requires an introduction to the demand side of the economy.

\(^{14}\) Such shocks are often assumed identically and independently distributed (i.i.d.) with zero mean and constant variance.
1.8.2 Aggregate Demand (AD)

\[ y_t - \bar{y} = -\varphi (i_t - \pi_{t-1}) + v_t \]  

Equation (3) represents the aggregate demand side of the economy. More precisely, it shows the equilibrium between the money market and the goods market. \((y_t - \bar{y})\) is the output gap in percent. \(i_t\) represents the instrument that the central bank can control; the nominal interest rate. An increase in the interest rate, nominal or real \((i_t - \pi_{t-1})^{15}\), leads to a lowering of the output gap, all other things equal. This aspect of the model is thus in accordance with the previous discussion of the transmission mechanisms in the economy. \(\varphi\) represents the interest rate sensitivity of production, i.e. the percentage decrease in the output gap as a response to a percentage point increase in the nominal interest rate.

**KEY ASSUMPTION 4:** The impact of the interest rate on aggregate demand is symmetrical, meaning that the interest rate sensitivity of production is equal in absolute terms, \(|\varphi|\), regardless of whether the interest rate is raised or lowered.

\(v_t\) represents the shocks do the demand side of the economy such as a change in government purchases, a change in taxes or a change in how consumers or business view the future.

With reference to how the interest rate affects inflation from the discussion of the transmission mechanism, let's assume that the central bank increases the interest rate when inflation is above target, and decrease the interest rate when inflation is below target. This can be specified by the following interest rate rule:

\[ i_t = \pi_{t-1} + \kappa (\pi_t - \bar{\pi}) \quad 0 < \kappa \]

\(\kappa\) represents how forcefully the central bank reacts to inflation deviations around the target, \(\bar{\pi}\). The higher the value of \(\kappa\), the more forcefully the interest rate will change as a response.

\[15\] Following the assumption of adaptive expectations in the expression of the real interest rate \(r_t = i_t - \pi_t^e\).
to a change in inflation. Using this interest rate rule in the demand function, gives the following aggregate demand relationship, as a function of inflation and production:

$$\pi_t = \bar{\pi} - \beta (y_t - \bar{y}) + \beta \nu_t, \quad \beta = -1/(\varphi \kappa)$$  \hspace{1cm} (4)

The aggregate demand curve slopes downward due to the interest rate response from the central bank. The higher the value of $\kappa$, the lower the value of $\beta$, and the flatter the demand curve. The next section seeks to explain how the central bank can control the total outcome of the economy by how it responds to inflation rate changes, put differently, by which value it assigns to $\kappa$.

Figure 8. Aggregate Demand

1.8.3 The AS-AD Equilibrium

Together, the supply and the demand curves give the complete model for the economy (Figure 9). The intersection between aggregate supply and aggregate demand (AS=AD) represents the stable equilibrium of the economy (A). It is where production is at its natural level and where the inflation level is at its target.

Figure 9 shows how a positive supply shock, i.e. an increase in the oil price, puts upward pressure on inflation, making the AS curve shift upwards (AS $\uparrow$). The new equilibrium (B), when the central bank follows the interest rate rule, is at a point where inflation is higher than the inflation target and where production is lower than the natural level. If the central bank
would like to change the point of equilibrium, it could assign a different weight to how it responds to inflation changes in its interest rate rule by changing the value of $\kappa$.

![Figure 9. The AS – AD Equilibrium](image)

Figure 9. The AS – AD Equilibrium

Figure 10 illustrates how different interest rate rules, depending on how forcefully the central bank wants to react to inflation rate changes, impact the equilibrium position of the economy. The figure also describes the offsetting effects of supply shocks. This was already mentioned in the discussion of the mandate given by the government. Here the focus is how the AS-AD model can describe the trade-off effect of supply shocks.

![Figure 10. AS – AD: Altering the Interest Rate Rule](image)

Figure 10. AS – AD: Altering the Interest Rate Rule

Let’s assume that the central bank would like to react more forcefully to inflation rate changes, by assigning a higher value to $\kappa$. Given the same supply shock, the new interest rate
The Optimal Policy

"In its most general form an economical optimal policy is characterized as an optimal choice among alternative feasible time paths in transforming the economy from a given initial state to a desired final state at the end of the planning horizon." (Kumar, 1969, p. 600). This definition of optimal policy is easy to apply to our model, by combining the preferences of the central bank with the AS-AD model of the economy. This is done in Figure 11:
Figure 11 shows how the economy is affected when hit by a negative supply shock (B) (“negative” refers to how the shock impacts the level of production). The negative supply shock creates a total loss of $L_1$ units. The central bank, can by changing the interest rate rule, “turn” the AD curve, and create a new equilibrium with a lower loss, $L_2$, at point O. The loss is lower since the loss function that goes through point O is smaller than the loss function that goes through point B. Point O is not only better than point B, but in this situation it is also optimal. This is due to the fact that the loss function has the same slope as the AS curve at point O. The loss function is thus tangent to the AS curve. If the loss function would be further reduced, it would not “touch” the AS curve, and such a situation would not be feasible. The optimal policy is thus to reduce the level of loss as much as possible given a state of the economy (represented by the aggregate supply curve (AS (-)).

In deriving the point of optimal policy we are not taking into account whatever shocks that might impact the demand side of the economy. This is due to how the central bank controls the demand side, and not the supply side. It must take the supply side as given. Whatever shocks that might hit the demand side, it can be accounted for in the interest rate setting. Since the point of optimal policy can be deduced without taking the demand side into account, all points that are optimal, given all types of shocks hitting the supply side, can easily be deduced. Figure 12 illustrates how all these optimal points make a straight line in $y_t, \pi_t$ space.
The slope of the optimal policy condition is given by the slope of the AS curve and the relative weight the central bank assigns to stabilising inflation and production. This result is given from the mathematical problem of minimising the loss function with respect to the AS function:

$$\pi_t = \bar{\pi} - \frac{\lambda}{\alpha}(y_t - \bar{y})$$

This is the function of the optimal policy line. It can also be interpreted as the aggregate demand curve of the economy when the central bank follows an optimal interest rate rule. Its slope depends on the relative weight of stabilising output vs. inflation, \(\lambda\), and the slope of the aggregate supply curve, \(\alpha\). This result is intuitive. If a high weight is attached to stabilising production (\(\lambda\) is high), and there is a negative supply shock, it is thus optimal with a relatively high level of inflation and a small level of the output gap.

The interest rate that ensures that the point of optimal policy is reached is the optimal interest rate rule. It is given by inserting the optimality condition into the AD curve (as a function of the nominal interest rate), solving for \(i_t\):

$$i_t = \pi_{t-1} + \left(\frac{\alpha}{\varphi \lambda}\right)(\pi_t - \bar{\pi}) + \left(\frac{1}{\varphi}\right)v_t$$

---

16 The full calculation of this problem can be found in Appendix 6.4
17 This is Røisland og Sveen’s interpretation in their article: “Pengepolitikk under et inflasjonsmål”. They present a similar model, and their article makes an excellent reference
Three important aspects of this optimal interest rate should be noted. First, if there is a shock to aggregate demand, \( v_t \), the interest rate should be set so as to neutralise the shock completely\(^{18} \). This is optimal as there is no trade-off between stabilising output and inflation due to demand shocks.

Second, the coefficient in front of the inflation gap shows how forcefully the interest rate should respond to different economic situations. The interest rate response is thus dependant on the slope of the AS curve, \( \alpha \), the interest elasticity of demand, \( \varphi \), and the relative weight put on output vs. inflation stabilisation, \( \lambda \). In our model, all of these variables are fixed and the coefficient is thus a constant. This leads us to the third and most important feature of the optimal interest rate: given the slopes of the AD and AS curves, and given the relative weight attached to the loss function, the optimal interest rate is symmetrical.

### 1.10 The Symmetry of the Optimal Interest Rate Policy

The property of symmetry in the optimal interest rate is easier appreciated when assuming that the inflation gap coefficient is 0.5, that there are no shocks to the demand side, and the inflation target is set to 2%. The optimal interest rate thus becomes:

\[ i_t = 3.5\% \]
\[ i_t = 2\% \]
\[ i_t = 0.5\% \]

---

\(^{18}\) This is easily seen by inserting the optimal interest rate back into the AD curve, which of course gives the optimal policy line.
This optimal interest rate condition makes a straight line in $i_t, \pi_t$ space, as illustrated in Figure 13. The optimal interest rate is 2% when the economy is at its long run equilibrium and inflation is at its target, (no shocks to either side of the economy). This is the policy neutral interest rate. When there are shocks hitting the economy, the optimal interest rate is symmetric around this value. If the inflation rate is 3%, which is one percentage point above target, the optimal nominal interest rate should thus be 3.5%. If the inflation rate is 1%, which is one percentage point below target, the optimal nominal interest rate should thus be 0.5%. Since the 3.5% and 0.5% interest rate both deviates 1.5 percentage points from the policy neutral rate, they are thus symmetrical. This leads us to a somewhat more precise definition of what symmetrical optimal policy setting really is:

**Definition**\(^{19}\): An optimal interest rate policy is symmetrical if the nominal interest rate response to equally large positive and negative deviations from the inflation target, are of identical magnitude but with opposite signs.

### 1.11 Conclusion

The model developed in chapter 1 will be referred to as the linear-quadratic model\(^{20}\). The linear-quadratic model has illustrated how a symmetrical interest rate can be an optimal policy. Before inflation targeting was introduced, NoB expressed that such symmetric interest rate setting was optimal: “over time it is important that the interest rate is set symmetrically. Such symmetry is necessary to sustain the expectations of nominal stability” (Gjedrem, 1999\(^{21}\)). Currently they are expressing symmetry of interest rate setting by the use of symmetrical fan charts, as illustrated in Figure 14. According to NoB, it is equally likely that any given shock to the economy will push the interest rate higher than the expected path as if it will push the interest rate lower than the expected path.

---

\(^{19}\) This definition cannot be generalised. A symmetrical interest rate should be symmetrical with respect to all variables it is trying to stabilise. The above definition refers to symmetry of inflation only. Implicitly however, due to the structure of the linear-quadratic model, symmetry with respect to inflation leads also to symmetry of production.

\(^{20}\) Linear refers to the economy (AS-AD) and quadratic refers to preferences (the loss function).

\(^{21}\) Authors own translation of: "Over tid er det viktig med symmetri i rentesettingen. En slik symmetri er nødvendig for å opprettholde tilliten til nominell stabilitet.” (Gjedrem, 1999)

The linear-quadratic model developed in this chapter illustrated how such shocks, hitting either the demand or the supply side of the economy, indeed provides such a symmetric response. Given that the linear-quadratic model is a good model of the economy, it is easy to understand NoB in their symmetrical policy setting. There is however reason to question if the model developed so far is a good model. The next chapter is devoted to discussing how altering the key assumptions will impact the optimal interest rate setting. It will be shown that the optimality of symmetrical policy making should represent the exception, rather than the rule.
2 ASYMMETRICAL POLICY AS AN OPTIMAL RESULT

A robust result is one that still holds if some assumptions of the model are changed. This section questions the robustness of the symmetrical optimal policy result. I will systematically go through the implications of changing the four key assumptions made so far: the specific symmetrical shape of the loss function, the linear aggregate supply curve, that all shocks are additive only and the assumption that there is no credit channel.

2.1 Key Assumption 1: Asymmetries in the Loss Function

The loss function we have looked at so far has the property of being symmetrical. It was chosen for two reasons. First, this type of function has become a standard framework in the economic literature to represent central banks’ preferences. Second, its symmetrical property is in accordance with the expressed preferences of NoB.

In contrast to human preferences, which not always need to be justified, nor consistent, the preferences of a central bank should be based on sound economic principles. In chapter 1, care was taken in explaining the economic principles underlying the variables (production and inflation) that are included in the loss function. That the interaction between these variables was symmetrical is a property that was assumed to be optimal, without further discussion. According to Alan Blinder, the former Vice Chairman of the Federal Reserve, the symmetrical property should be given a closer look: “academic macroeconomists tend to use quadratic loss functions for reason of mathematical convenience, without thinking much about their substantive implications. The assumption is not innocuous. …. practical central bankers and academics would benefit from more serious thinking about the functional form of the loss function” (Blinder, 1997, p.6). This section intends to examine the “serious thinking” of economists related to the functional form of the loss function. Economic literature can in this respect be divided into two categories; preference asymmetry of inflation and preference asymmetry of production.

2.1.1 Preference Asymmetry of Inflation

Is it just as good to be five minutes early when catching a train, as five minutes late? This question, a standard analogy to monetary policy, describes a situation where preferences
obviously not are symmetric. With an inflation target of 2.5%, and a symmetrical loss function, one would think that it is just as good that a shock pushes the economy three percentage points upwards to a 5.5% inflation rate, as if a shock would push it three percentage points downwards to a 0.5% rate of deflation (-0.5% inflation). But with the extreme costs to society due to deflationary pressures, should one be indifferent between these two scenarios?

Karagedikli and Lees, two economists at the Reserve Bank of New Zealand, believes that there is room for asymmetry in the loss function: “There is no particular reason to suppose that the costs of below target inflation are of the same magnitude as the costs of above target inflation. Policymakers could form a coherent argument in favour of asymmetric preferences based on the literature on the costs of inflation” (2004, p.7).

Nobay and Peel modelled such asymmetric preferences using a linex loss function (2003)22. This type of loss function was first introduced by Varian in 1975, and further discussed by Zellner (1986). Nobay and Peel’s article however, seems to be the first reference when it comes to modelling preference asymmetry of inflation within a monetary policy framework. The loss function they used was of the form23:

\[ L = \frac{2(e^{\beta(\pi_t - \pi)} - \beta(\pi_t - \pi) - 1)}{\beta^2} + \lambda(y_t - \overline{y})^2 \]  

(8)

This type of loss function deserves a closer look as it quite neatly can be used to test the hypothesis of asymmetric preferences. \( \beta \) is a constant describing preferences of inflation relative to its target. If \( \beta > 0 \), the central bank believes that inflation above target is more costly than inflation below target, and vice versa. The linex function has thus the property that preferences are symmetric if \( \beta \) approaches zero. More precisely, the linex function becomes the symmetric loss function, introduced in equation (1), when \( \beta \) approaches zero24.

22 Ruge-Murcia (2001) makes a reference to the mimeo of this article from 1998
23 The precise loss function used by Nobay and Peel were: \( L = \frac{e^{\beta(\pi_t - \pi)} - \beta(\pi_t - \pi) - 1}{\beta^2} + \lambda(y_t - \overline{y})^2 \). The loss function in equation (7) was altered so as to show the link to the symmetric loss function in equation (1).
24 By L’Hopital’s rule: \( \lim_{\beta \to 0} \frac{2(e^{\beta(\pi_t - \pi)} - \beta(\pi_t - \pi) - 1)}{\beta^2} = \frac{\partial}{\partial \beta} \left( (\pi_t - \overline{\pi}) e^{\beta(\pi_t - \pi) - (\pi_t - \overline{\pi})} \right) \bigg|_{\beta=0} = (\pi_t - \overline{\pi})^2 \)
\[
\lim_{\beta \to 0} L = \lim_{\beta \to 0} \left( \frac{2(e^{\beta(\pi_t - \bar{\pi})} - \beta(\pi_t - \bar{\pi}) - 1)}{\beta^2} \right) + \lambda(y_t - \bar{y})^2
\] 

(9)

\[
= (\pi_t - \bar{\pi})^2 + \lambda(y_t - \bar{y})^2
\]

This property allows researchers to test the symmetry of central banks’ preferences by checking if \( \beta \) is significantly different from zero. Ruge-Murcia tests this hypothesis on the three inflation targeting countries Canada, Sweden and the United Kingdom and he finds “limited but encouraging support for notion of asymmetric preferences” (2003, p27).

Dolado, Dolores and Ruge-Murcia test the linex function on U.S. data, comparing the Burns-Miller (pre-1979) and Volcker-Greenspan (post-1982) regimes at the U.S. Federal Reserve. They find that “the Fed’s inflation preferences during the Volcker-Greenspan regime appear to be asymmetric, in the sense that positive inflation deviations from its target are weighted more heavily than negative ones, even if they are of the same magnitude. In contrast, it is not possible to reject the null hypothesis of quadratic inflation preferences during the Burns-Miller regime” (2004, p.17).

The main question this thesis is concerned with is how such a change of preferences impact the optimal interest rate policy. With this type of loss function, the optimal interest rate in the AS-AD model is given by\(^{25}\):

\[
i_t = \pi_{t-1} - \left( \frac{\alpha}{\varphi \beta \lambda} \right) (1 - e^{\beta(\pi_t - \bar{\pi})}) + \left( \frac{1}{\varphi} \right) y_t
\]

(10)

It might not be straightforward to see how this optimal interest rate is asymmetric. This property is easier appreciated when the optimal interest rate is illustrated as a function of inflation, as can be seen by Figure 15.

\(^{25}\) The full calculation of this problem can be found in Appendix 6.5
Asymmetrical Interest Rate Policy. Preference Asymmetry of Inflation. Inflation above target is assumed worse than inflation below target, i.e. $\beta > 0$.

An interest rate response was defined to be symmetric, in our simplified model, if the nominal interest rate response to equally large positive and negative deviations from the inflation target, are of identical magnitude but with opposite signs, all other things being equal. This is not the case for the optimal interest rate response illustrated in figure 16. Even though there is reason to believe that the different costs to society of inflation deviating from its target should lead to preference asymmetry of inflation, most of the literature focus on asymmetry related to variation in production.

2.1.2 Preference Asymmetry of Production

The discussion in section 1.4 provided good reasoning why society experiences a smaller welfare gain during expansions than the level of loss experienced during contractions. This led to the conclusion that less variation in production is good for society, which again led to a loss function which was strictly increasing in variation of production. But if there is a welfare gain related to production above trend, why is this not reflected in the loss function of the central bank? A growing number of economists seem to believe that central banks are already taking it into account, and they refer to the phenomena as “recession aversion”. Recession aversion means that policy makers care more about falls than increases in output, relative to

---

26 An inflation target of 2% and a policy neutral interest rate of 2% are assumed. If a shock would push the inflation rate down to $\pi_t^{(-)}$, the optimal response would be to decrease the interest rate with one percentage point, to 1%. If however an equally large shock would push the inflation rate up to $\pi_t^{(+)}$, the optimal response would be to increase the interest rate by two percentage points, to 4%. Since these two interest rate responses are not of the same absolute value, these responses are not symmetric.
the natural level. Cukierman argues that recession aversion is not only caused by strict economic welfare arguments, but also by political pressure: “political establishment is more sensitive to the costs of recessions than it is content with expansions. Since, in a democratic society, independent but accountable central banks are not totally insensitive to the wishes of the political establishment some of this asymmetry trickles down into the objective function of the central bank as well” (2003, p.546). The loss function Cukierman presents is of a fairly simple and somewhat familiar character:

\[
L_t = \begin{cases} 
    (\pi_t - \bar{\pi})^2 + \lambda (y_t - \bar{y})^2 & \text{if } y_t < \bar{y} \\
    (\pi_t - \bar{\pi})^2 & \text{if } y_t > \bar{y} 
\end{cases}
\]  
(11)

If production is below trend, \(y_t < \bar{y}\), the loss function coincides with the symmetrical loss function. If however, production is above trend, \(y_t > \bar{y}\), the central bank no longer feels a loss related to production. The loss is then given by variation in inflation only. The intuition behind this loss function is seen more clearly when drawn in \(\pi, y\) space:

![Figure 16. Preference Asymmetry of Production – Recession Aversion](image)

Figure 16 shows how the loss function coincides with the symmetric loss function when production is below trend. When production is above trend, the central bank is indifferent between all levels of production, given the level of inflation. In the words of Mr. Mervin King, the central bank is thus an “inflation nutter” when production is above trend (King, 1997). One property of this loss function is important to point out. That a central bank feels
differently about recessions and expansions do not mean that it is targeting a higher level of production than the natural rate. The central bank is still targeting the natural rate of production \((\bar{\pi})\)\(^\text{27}\). Put differently, the loss function is minimised if inflation is at its target and production is at the natural rate. The difference from the symmetrical loss function lies in how it responds if production is above or below this natural rate.

Figure 17 illustrates how the central bank responds given the AS-AD framework and the loss function given by (11)\(^\text{28}\). The optimal interest rate policy coincides with the optimal interest rate policy derived from symmetrical preferences when inflation is above target. If inflation is below target, meaning that the economy is hit by a negative supply shock, then the interest rate is set so as to reach the inflation target. The optimal interest rate response is thus not symmetrical. It will differ according to the types of shocks that are hitting the economy.

---

\(^{27}\) If the central bank were not targeting the natural rate of production, an inflation bias would arise. This result is based on the seminal papers by Kydland and Prescott, and Barro and Gordon. The most important hypothesis Cukierman presents in his paper is that an inflation bias will occur if there are asymmetric preferences, even when the central bank is targeting the natural rate of production.

\(^{28}\) Calculation of the optimal interest rate policy given recession aversion can be found in appendix 6.6.
2.1.3 Conclusion

One important conclusion can be reached from the above discussion: Preferences that are asymmetric, either with respect to inflation or production, leads to an asymmetrical interest rate setting, everything else equal.

Even though there are strong economic arguments in favour of asymmetrical preferences, such preferences might be too complicated to be operational. According to Svensson, the symmetrical loss function should be chosen due to its simplifying character: “some researchers have argued that asymmetric preferences are relevant in monetary policy and also examined their implications. This would require a more complex loss function. Put differently, a second-order approximation is not enough, and higher-order terms are needed. I find a symmetric loss function for monetary policy very intuitive, especially since these days not only too high inflation but also too low inflation is considered undesirable due to the risk of falling into liquidity traps and deflationary spirals... Furthermore, more complex loss functions and more complicated tradeoffs may be too sophisticated to be both operational and sufficiently verifiable for reasonable accountability” (Svensson, 2002, p. 279).

Bearing in mind how NoB has explicitly and repeatedly expressed a symmetrical character of their preferences, the symmetrical loss function will be used in the remaining analysis.

2.2 Key Assumption 2: The Non-linear AS curve

In the models used so far, it has been assumed that the AS curve was of a linear form. The slope of the AS curve, $\alpha$, was introduced as the parameter describing the inflation – production trade-off, without much further explanation. This trade-off describes an important economic mechanism, and the issue deserves further attention.

The main reason that drives the trade-off is price stickiness. This can easily be illustrated by how labour unions negotiate wage contracts. These contracts last for one year in Norway. In the United States, the average contract time is about three years. When the union negotiates the contracts, and since they are interested in achieving as high a real wage as possible, they must take into account the price level that will prevail in the period of the wage contract. Let’s assume that in the middle of such a wage contract period, the level of inflation unexpectedly jumps. Since the wage contract is negotiated in nominal terms, the unexpected rise in inflation
effectively lowers the real wage. From the firms’ perspective, this means that labour is cheaper. Taking advantage of the lower price, firms higher more labour, which again leads to a higher production of goods and services. This explains the inflation-production trade-off. The linear AS curve however, relies upon the assumption that the inflation-production trade-off is constant for all levels of production. It does not matter at which level of production the economy is at, the trade-off the central bank is facing between inflation and production in booming periods is thus the same as the trade-off it is facing during recessions.

The purpose of this section is to question the assumption that the AS curve is linear. A non-linear AS curve means that the inflation-production trade-off is not constant for all levels of production. The discussion will focus on the convex AS curve.

The AS curve is defined to be convex if there is a positive trade-off between inflation and production, and if this trade-off is increasing with the level of production\(^{29}\). This is illustrated in Figure 18:

![Figure 18. A Convex AS Curve](image)

The main theoretical argument underlying the convex AS curve is due to capacity constraints. When the economy is experiencing a boom, and when production is close to its capacity limit, an increase in demand will mostly impact the level of inflation. An aggregate supply curve describing the property of capacity constraint could be represented by the following function:

\[\frac{\partial^2 \pi}{\partial y^2} < 0 < \frac{\partial \pi}{\partial y} \]

Notice how this definition differs from the definition of a convex curve: \[\frac{\partial^2 \pi}{\partial y^2} < 0 \leq \frac{\partial \pi}{\partial y}\]. This is done so that the linear curve does encapsulate the definition of convex.

\(^{29}\) More formally: \(\frac{\partial^2 \pi}{\partial y^2} < 0 \leq \frac{\partial \pi}{\partial y}\). Notice how this definition differs from the definition of a convex curve: \(\frac{\partial^2 \pi}{\partial y^2} < 0 \leq \frac{\partial \pi}{\partial y}\). This is done so that the linear curve does encapsulate the definition of convex.
The quadratic term of production is what makes this supply curve differ from the linear supply curve. The coefficient of this quadratic term, $\eta$, represents the degree to which the trade-off between inflation and production is increasing as production increases. The function becomes the linear supply curve if $\eta=0$.

If the economy is described by such a supply curve, the central bank’s optimal interest rate rule is given by equation 13. The term in front of the inflation gap is dependant on the level of production. Since in our model, the level of production is uniquely determined by the level of inflation (a quadratic relationship), the interest rate respond is thus asymmetrically with respect to deviations from the inflation target.

\[
\pi_i = \pi_{i-1} + \alpha(y_i - \bar{y}) + \eta \pi_i^2 + \nu_i
\]

(12)

The asymmetrical response is easier appreciated when considering the AS-AD model. For illustration purposes, we assume the extreme case when the central bank is an “inflation nutter”, setting the interest rate so as to stabilise inflation completely. This is illustrated by the perfectly horizontal AD curve in the Figure 19. Two shocks to the supply side, of identical magnitude but with opposite signs, are illustrated. In terms of production, it is much more costly to stabilise inflation during recessions (A), than it is to stabilise inflation during booms (B). This is shown by how much production deviates from trend value at the two equilibriums. The figure shows thus the general principle of how the costs of supply shocks are asymmetrical with a convex AS curve. An optimal interest rate response set to neutralise this asymmetry, must consequently also be asymmetrical.31

\[
i_i = \pi_{i-1} + \left( \frac{\alpha}{\phi \lambda} \right) + \left( \frac{2\eta}{\phi \lambda} \right) \nu_i (\pi_i - \bar{\pi}) + \left( \frac{1}{\phi} \right) \nu_i
\]

(13)

30 See calculation in Appendix: 6.7
31 Assuming symmetrical preferences
Many studies have tried to shed light on what the supply schedule looks like in the short run. Dolado, et al, looked at the shape of the AS curve in both the U.S. and in the Euro area. They could not reject the linear specification for the U.S. market, but in the Euro area (more specifically: Germany, France and Spain) they found significant evidence indicating a convex AS curve, as illustrated in Figure 20.

Dolado et al believe that high nominal wage rigidity in the European labour market can explain why they found evidence of a convex AS curve in the Euro area. The curve then
reflects how workers’ willingness to cut their real wage lowers as production and inflation decreases. The more flexible U.S. market would then also explain why they could not find evidence of a convex AS curve in the U.S.

David Turner, an OECD economist, studied the G7 economies\textsuperscript{32}. In contrast to Dolado et al findings, he found evidence of a convex AS curve in the United States. Also Japan and Canada could better be described by a convex AS curve. Concluding on his results, he pointed out that his findings were sensitive to both the model specification and the reliability of output gap measurements (Turner, 1995).

Estimation on Norwegian data has mainly been done on the wage curve. The wage curve refers to a negative relationship between the real wage and unemployment. Given Okun’s law, and a positive relationship between the real wage and inflation, a convex wage curve is equivalent to a convex supply function. In his Ph.D. thesis, Johansen (1995) reports of a strongly non-linear wage curve based on data from 1964 - 1990. The curve is convex, with a large trade-off when unemployment is low, and an almost flat wage curve for employment rates above 3%. In addition to a nominal wage floor, Johansen mention composition effects as a possible cause of convexity. If the ratio of long-term unemployed increases with average unemployment, and “if the long-term unemployed exert less downward pressure on wages than do the recently laid off workers, the marginal effect from increased average unemployment will be decreasing” (Johansen, 1995, p. 10).

Figure 21. A Concave - Convex AS Curve

\textsuperscript{32} The G7 economies consist of: United States, Japan, Germany, France, Italy, United Kingdom and Canada
The empirical evidence on the shape of the AS curve is however far from conclusive. In the literature, there are mostly three different shapes discussed: the convex, the concave and the concave-convex. Chapter 3, which constitutes the empirical part of this thesis, will test if the linear specification can be rejected on the Norwegian economy.

2.3 Key Assumption 3: Multiplicative Shocks

The optimal interest rate derived from the linear-quadratic model, (equation 6), was based on the assumption that the slope of the AS curve, $\alpha$, did not vary with time. The economy was assumed to only be hit by additive shocks. This assumption is strong.

![Figure 22. Multiplicative Shocks](image)

Empirical evidence suggests that the slope of the AS curve has changed with time. Globalisation seems to be the driving force of this development and it has caused a flattening of the AS curve. Figure 22 illustrates how multiplicative shocks, such as globalisation (AS$^{(+)}$), might impact the AS curve. There are three reasons why globalisation has had this effect. First, globalisation has led to increased competition and thus lessening the scope of raising prices when demand rises. Second, increased trade and investment has made goods prices less sensitive towards domestic demand. Third, labour mobility has increased, making also wages less sensitive towards domestic demand pressures. (Iakova, 2007)

If the slope of the AS curve changes with time, $\alpha_t$, the optimal interest rate is then given by:
\[ i_t = \pi_{t-1} + \left( \frac{\alpha}{\varphi} \right) (\pi_t - \bar{\pi}) + \left( \frac{1}{\varphi} \right) \nu_t \]  

(14)

A varying slope of the AS curve will directly impact the optimal interest rate rule. The symmetry of the interest rate is thus directly impacted by how the central bank views the properties of the potential shocks that might hit the economy.

2.4 Key Assumption 4: The Credit Channel of Monetary Policy

In the AS-AD model it was assumed that an interest rate increase would impact aggregate demand, in absolute terms, by the same amount as an interest rate cut. There are however strong arguments in favour of an asymmetric impact on the demands side. Such a property of the economy is often viewed in terms of the credit channel of monetary policy, and it is illustrated in the figure below.

**Figure 23. The Financial Accelerator Effect**

Figure 23 illustrates how an interest rate cut pushes demand upwards (AD\(^{(+)}\)). It also illustrates how an interest rate increase impacts demand more than the interest rate cut (AD\(^{(-)}\)).

The credit channel of monetary policy is based on a variety of alternative non-monetary assets. It serves in contrast to the money view (which was assumed in the main model) where all non-money assets are assumed homogenous (bonds). One often used rationale explaining
the asymmetric impact is the financial accelerator effect. Small firms are particularly vulnerable to interest rate increases during recessions. They do not have the same ability as large firms to find alternative means of funding, i.e., capital market, internal funding, etc. If balance sheets are already low, an interest rate increase will not only increase the value of money and thus decrease the level of investment, but it will also lower the value of the collateral used by small firms as their source of funding. In turn, this value reduction will force these firms to further reduce their investment. According to Walsh; “a rise in the interest rates may have a much stronger contractionary impact on the economy if balance sheets are already weak, introducing the possibility that nonlinearities in the impact of monetary policy may be important” (p.324, 2003).

2.5 Conclusion

This purpose of this chapter was to question the robustness of the symmetrical optimal policy result. Changing any of the four key assumptions: the specific symmetrical shape of the loss function, the linear aggregate supply curve, that all shocks are additive only and the assumption that there is no credit channel, will directly impact the symmetry of monetary policy. If any of these assumptions do not hold it is not optimal to set the interest rate symmetrically.

![Figure 24. Market expectations of the sight deposit rate in the baseline scenario with fan chart based on interest rate options. Percent. Half-yearly figures. 05 H1 – 08 H2. Based on options prices at 27 October 2005. Source: Norges Bank, Inflation Report 3/2005.](image)

33 See Walsh, chapter 7: The Credit Channel of Monetary Policy
Figure 24 illustrates uncertainty based on market expectations. In contrast to the symmetrical property of the fan charts expressed by NoB (Figure 14), the market does not believe the balance of risk to be symmetrical. That market expectations of uncertainty are asymmetrical could be due to a more complex view of the economy than the linear-quadratic framework given in chapter 1, i.e. if the market believes that the supply curve is non-linear.
3 TESTING THE LINEARITY OF THE SUPPLY CURVE

The model developed in chapter 1 illustrated how the expressed symmetrical interest rate by NoB could be an optimal policy. Chapter 2 questioned this result by loosening some of the strong assumptions made in the linear-quadratic model. The purpose of this chapter is to test if one of the necessary assumptions of a symmetrical interest rate, the linear supply curve, is valid on Norwegian data. The supply curve estimated is taken from one of NoB’s models. The empirical attributes of the model will also be discussed.

3.1 Norges Bank’s Calibrated Supply Curve

As of the third Inflation Report 2005 NoB began publishing the future interest rate path (with uncertainty bands) describing the expected levels of the interest rate which Norges Bank finds most likely (i.e. Figure 14). This was in contrast to previous interest rate paths which were based on market expectations. Developing these new interest rate paths is done by the aid of many economic models in addition to judgement concerning the current economic situation. The core model used in forecasting the economy and developing interest rate paths is commonly known as 1A. The supply side equation (1AS) in that model describes the short run trade-off between production and inflation. It is given by:

\[ \pi_i = \alpha_0 \pi_{i-1} + \alpha_1 \pi + (1 - \alpha_0 - \alpha_1) E_\pi \pi_{i+4} + \alpha_2 x_{i-1} + \alpha_3 \Delta x_{i-1} + \alpha_4 \sum_{i=2}^{5} \beta_i \Delta q_{i-1} + \varepsilon_i \]  

(15)

\( \pi_i, \pi, x_i, q_i \) represents inflation, the inflation target, the output gap and the real exchange rate respectively. \( \alpha \) ’s and \( \beta \) ’s are constants. \( E_i \pi_{i+4} \) denote the expectation of inflation one year ahead (each time period is in quarters). \( \Delta \) indicates the change in a variable between two time periods. \( \varepsilon_i \) is a stochastic residual.

The inclusion of expected future inflation is a standard representation of New Keynesian models. It is normally based on the assumption that agents face restrictions on how often they can change prices and that they are forward looking in their price setting behaviour. Model 1AS differs from New Keynesian models by i.e. the inclusion of lagged inflation and the inflation target. Including these terms can be rationalised by assuming that a proportion of the
firms will adjust their prices based on both the previous periods’ level of inflation and the level of the inflation target. An overriding concern when developing the model was to ensure that “the model has a dynamically stable solution” (Husebø et al, 2003, p. 10). The coefficients on inflation are thus restricted to sum to unity which implies a vertical long-run supply curve. It should be noted that sufficiency of a dynamically stable solution concerns lagged dependant variables. The inflation target defined by equation 15 is in this respect a constant, and its inclusion in the homogeneity restriction does not make a sufficient condition for a dynamically stable solution.

Inflation is a function of the output gap. New Keynesian models are based on real marginal cost being the driving variable of the inflation process as firms set prices as a mark-up on marginal costs. The output gap is used as a measure of marginal cost in 1AS. This is in accordance with the standard sticky price framework without variable capital, where there is an approximate proportionate relation between marginal cost and the output gap (Gali et al, 1999).

The inclusion of differenced output gap does not hold strict theoretical underpinnings. However, Husebø, et al provides two non New Keynesian rationales why this term is included in the supply function: “First, inflation can start to pick up even if the output gap is negative, if the output gap is closing fast. Second, given that estimates of the level of the output gap are uncertain, having the change in the output gap (which is less sensitive to assumptions) in the Phillips Curve make the forecasts for inflation somewhat more robust” (p. 10, 2004).

No theoretical reasoning was given in the model documentation of including the change in the real exchange rate. That the change in the real exchange rate should be included can be argued through the impact on the overall price level from the price level on imported goods. In model 1AS, the changes of the real exchange rate are included from lag two to five. According to Husebø et al. this specification “matches the model properties to recent empirical research on the pass through from exchange rates to prices”.

Model 1AS is calibrated. According to Husebø et al, an overriding concern when calibrating the model was to ensure that the parameters obtained was in accordance with economic theory and available empirical evidence. The calibrated parameter values are given in Table 1. All the calibrated values are between zero and one.
Table 1. Calibrated Parameter Values

<table>
<thead>
<tr>
<th>i</th>
<th>( \alpha_i )</th>
<th>( \beta_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.60</td>
<td>0.2</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>4</td>
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</tr>
</tbody>
</table>

3.2 Generalised Method of Moments

Estimation follows Nymoen and Tveter 2007 and is done by the method of Generalised Method of Moments (GMM). There are particularly two properties of model 1AS which makes GMM a preferred estimator. Firstly, there is an endogeneity problem in 1AS due to the expected future rate of inflation. Secondly, the model holds the property of first-order moving average error terms. This autocorrelation property of the model is found by its rational expectation solution. The procedure is illustrated in Bårdsen et al, (2005, p. 292)\textsuperscript{34}. This section analyses how the GMM estimator deals with these two properties.

Simplifying the notation of 1AS, yields the following expression:

\[
\pi_t = X_t \beta + \epsilon_t \quad t = 1 \ldots T
\]  

\( X_t \) is a \((1\times9)\) matrix representing the explanatory variables, and \( \beta \) is a \((9\times1)\) matrix representing all coefficients in 1A:

\[
X_t = \begin{bmatrix}
\pi_{t-1} & 1 & E_t & \pi_{t+4} & x_{t-1} & \Delta x_{t-1} & \Delta q_{t-2} & \Delta q_{t-3} & \Delta q_{t-4} & \Delta q_{t-5}
\end{bmatrix}
\]

\[
\beta' = \begin{bmatrix}
\alpha_0 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \beta_2 & \alpha_4 \beta_3 & \alpha_4 \beta_4 & \alpha_4 \beta_5
\end{bmatrix}
\]

The endogeneity problem is due to the expectation of future inflation. Modelling expectations requires an assumption about how these expectations are realised. By assuming that expected

\textsuperscript{34} They assume that the feedback channel of output is a function of its own one period lag. This is in accordance with model 1A.
future inflation is determined as a function of future inflation and an error term
\( E, \pi_{t+4} = \pi_{t+4} + \nu_{t+4} \), the level of future inflation becomes endogenous:\(^{35}\)

\[
E[X_t \epsilon_t] \neq 0_{2,1} \quad t = 1 \ldots T
\] (17)

The endogeneity problem requires the use of an instrumental variable estimator such as the GMM. Good instruments, \( Z \), should be uncorrelated with the disturbance term and highly correlated with the variable for which it serves as an instrument\(^{36}\). The additional instruments used by Nymoen and Tveter are \( \pi_{t-2}, \Delta q_{t-1}, \Delta q_{t-6} \) and \( \Delta q_{t-7} \). Applying this additional set of instruments corrects the endogeneity problem as the total set of instruments, \( Q_t \), are uncorrelated with the residual. The orthogonality condition then follows:

\[
E[Q_t \epsilon_t] = E[Q_t (\pi_t - X_t \beta)] = 0_{12,1} \quad t = 1 \ldots T
\] (18)

\( Q_t \) is a \((12 \times 1)\) matrix, where each element represents an instrument:

\[
Q_t' = \begin{bmatrix}
\pi_{t-1} & 1 & x_{t-1} & \Delta x_{t-1} & \Delta q_{t-2} & \Delta q_{t-3} & \Delta q_{t-4} & \Delta q_{t-5} & \Delta q_{t-6} & \Delta q_{t-7}
\end{bmatrix} = [W_t; Z_t]
\]

This total instrument set can be partitioned into two matrices. \( W_t \) is a \((1 \times 8)\) matrix representing the exogenous variables in equation (15) and the constant term. \( Z_t \) is a \((1 \times 4)\) matrix representing the additional instruments used to correct the endogeneity problem.

The empirical mean of the orthogonality condition (18) is given by the \((12 \times 1)\) vector:

\[
g_T = g_T(\pi_t, Q_t, X_t, \beta) = \frac{1}{T} \sum_{t=1}^{T} Q_t (\pi_t - X_t \beta)
\] (19)

The idea underlying the GMM estimator is to choose the value of \( \beta \) which brings the value of \( g_T \) as close to its theoretical counterpart, the zero vector \( 0_{12,1} \), as possible (Biorn, 2007).

---

\(^{35}\) See calculation in Appendix 6.8

\(^{36}\) The set of additional instruments, \( Z \), will be thoroughly discussed in section 3.4
If there are more instruments than endogenous explanatory variables, which there are in this case, then there are more equations (12) than unknown parameters (9). The equation system is overidentified. The GMM estimator resolves this problem by minimising the quadratic form $g_T S_T g_T$, given by (20), where $S_T$ is a $(12 \times 12)$ positive definite weighting matrix:

$$\hat{\beta}_{GMM} = \arg\min[g_T S_T g_T]$$  \hspace{1cm} (20)

The GMM estimator is chosen because it resolved two inherent problems of model 1AS with respect to estimation. First, the endogeneity problem is resolved by the use of instrumental variables. The second issue of estimation was related to the models predictions of autocorrelated error terms. GMM resolves this problem by an appropriate choice of the weighting matrix $S_T$. It can be shown that the optimal choice of $S_T$ (the most efficient estimator) is the inverse of the asymptotic covariance matrix of the empirical moment $g_T$ as given by (see Greene p. 206):

$$S_T^{OPTIMAL} = \left(\text{Asy.Var}(\sqrt{T}g_T)\right)^{-1}$$  \hspace{1cm} (21)

### 3.3 Testing Linearity

Testing whether the trade-off between inflation and the lagged value of the output gap is constant for all levels of the output gap can be done by the following specification:

$$\pi_t = \alpha_0 \pi_{t-1} + \alpha_1 \pi^* + (1 - \alpha_0 - \alpha_1) E_t \pi_{t+4} + \alpha_2 x_{t-1} + \alpha_3 \Delta x_{t-1} + \alpha_4 \sum_{i=2}^{5} \beta_i \Delta q_{t-i} + \delta^* (S^* x_{t-1}) + \epsilon_t$$  \hspace{1cm} (22)

$S$ is a dummy variable taking the value 1 if there is a positive output gap and zero otherwise.

$$S = \begin{cases} 1 & \text{if } x_{t-1} \geq 0 \\ 0 & \text{if } x_{t-1} < 0 \end{cases}$$
\( \delta \) is a coefficient describing the additional trade-off when the output gap is positive\(^{37}\). Figure 25 illustrates equation 2 where \( \delta \) is negative.

**Figure 25. Threshold Model 1**

Table 2 provide the estimates of equation 22 using the sample period 1993q1 – 2006q1. None of the estimates of the extra effect are significantly different from zero, regardless of inflation and output gap measure. The sign of \( \hat{\delta} \) depends on the inflation measure. This indicates that there is little reason to conclude that there is a different trade-off between inflation and production in booms and recessions. The formal test of this hypothesis, the t-test, shows that \( \hat{\delta} \) is not significantly different from zero. It should be noted that autocorrelation of order four and five is significant\(^{38}\) and that autocorrelation makes the t-test unreliable. The problem of autocorrelation will be discussed when analysing the instrument set used.

Non-linearity might arise even though the test of Threshold Model 1 could not reject the linear model. This is due to how Threshold Model 1 only holds one cut-off point. If the true specification of the supply curve was i.e. concave-convex (see Figure 21), the test statistic would not necessarily reject linearity when using Threshold Model 1. Allowing for two cut-off points, instead of one, could reveal potential non-linear shapes that are not handled by Threshold Model 1. The procedure is illustrated in Figure 26 and Figure 27. Two alternative specifications, both with two cut-off points, but with different economic interpretations, are used.

\(^{37}\) Note on spline regressions: making the function continuous requires in general that one allow the intercept to vary for the different segments (Greene, 2003, p. 121). Such manipulation is not necessary as the threshold value above is zero, making the intercept for both segments equal.

\(^{38}\) The autocorrelation test is given in Table 7.
<table>
<thead>
<tr>
<th></th>
<th>CPI</th>
<th>CPI-ATE</th>
<th>CPI-ATED</th>
<th>NB-CAL</th>
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<td>(0.171)</td>
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<td>(0.130)</td>
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<td>$\alpha_i\pi$</td>
<td>0.095</td>
<td>0.214</td>
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<td>(0.912)</td>
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<td>$(1-\alpha_0-\alpha_i)$</td>
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<td>(0.385)</td>
<td>(0.116)</td>
<td>(0.432)</td>
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<tr>
<td>$\alpha_2$</td>
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<td>0.476</td>
<td>0.070</td>
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<tr>
<td></td>
<td>(0.277)</td>
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<td>(0.243)</td>
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</tr>
<tr>
<td>$\alpha_3$</td>
<td>-0.722</td>
<td>-0.060</td>
<td>-0.459</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td>(0.495)</td>
<td>(0.194)</td>
<td>(0.558)</td>
<td></td>
</tr>
<tr>
<td>$\alpha_4\beta_2$</td>
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<td>-0.005</td>
<td>0.019</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>(0.060)</td>
<td>(0.016)</td>
<td>(0.020)</td>
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</tr>
<tr>
<td>$\alpha_4\beta_3$</td>
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<td>0.021</td>
<td>3E-04</td>
<td>0.030</td>
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<td></td>
<td>(0.071)</td>
<td>(0.014)</td>
<td>(0.039)</td>
<td></td>
</tr>
<tr>
<td>$\alpha_4\beta_4$</td>
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<td>0.004</td>
<td>-0.002</td>
<td>0.045</td>
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<tr>
<td></td>
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<td>(0.014)</td>
<td>(0.031)</td>
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<td></td>
<td>(0.029)</td>
<td>(0.013)</td>
<td>(0.024)</td>
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</tr>
<tr>
<td>$\delta$</td>
<td>0.552</td>
<td>-0.052</td>
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<td></td>
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<tr>
<td></td>
<td>(0.443)</td>
<td>(0.085)</td>
<td>(0.202)</td>
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<p>| | | | | |</p>
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<tr>
<td>$R^2$</td>
<td>0.31</td>
<td>0.90</td>
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<tr>
<td>DW</td>
<td>1.77</td>
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<tr>
<td>$\sigma$</td>
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<td>0.70</td>
<td>1.00</td>
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<tr>
<td>JB</td>
<td>0.47</td>
<td>7.33</td>
<td>2.77</td>
<td></td>
</tr>
<tr>
<td>$\chi^{(J)}$</td>
<td>1.09</td>
<td>2.85</td>
<td>1.85</td>
<td></td>
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<tr>
<td></td>
<td>(0.78)</td>
<td>(0.42)</td>
<td>(0.60)</td>
<td></td>
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</tbody>
</table>

Table 2. IAS Estimates – Threshold Model 1. Note: GMM estimates for period 1993q1 - 2006q1. Standard errors are shown in brackets. No homogeneity restrictions made on the coefficients. The estimated equations include two lags of inflation, lagged output gap, lagged change in output gap and seven lags of the change in the real exchange rate. CPI, CPI-ATE and CPI-ATED refer to different inflation measures. Software Eviews 5.1.
Threshold Model 2 provides the supply curve with two cut off points:

\[
\pi_t = \alpha_0 \pi_{t-1} + \alpha_1 \pi + (1 - \alpha_0 - \alpha_1) E_t \pi_{t+1} + \alpha_3 x_{t-1} + \alpha_3 \Delta x_{t-1} + \alpha_4 \sum_{i=2}^{5} \beta_i \Delta q_{t-i} + \delta_1 (S_1 \cdot x_{t-1}) + \delta_2 (S_2 \cdot x_{t-1}) + \varepsilon_t
\]

(23)

\(S_1\) and \(S_2\) are dummy variables which takes the value one if the output gap is respectively higher than \(x^H\) or less than \(x^L\).

\[
S_1 = \begin{cases} 
1 & \text{if } x_{t-1} \geq x^H \\
0 & \text{if } x_{t-1} < x^H 
\end{cases}
\]

\[
S_2 = \begin{cases} 
1 & \text{if } x_{t-1} \leq x^L \\
0 & \text{if } x_{t-1} > x^L 
\end{cases}
\]

The above specification allows the slope of the supply curve to vary depending of the value of the output gap. This is illustrated in Figure 26.

**Figure 26. Threshold Model 2**

**Figure 27. Threshold Model 3**

Threshold Model 2 is not a continuous function. It makes a “jump” at the break off points as the specification force the different line segments to share the same intercept. In terms of economic interpretation, Threshold Model 2 allows the testing of slope variation only. The discontinuous function also holds the property of a costless change in inflation at the threshold values.
Figure 27 illustrates the model when the specification is changed so as to make the function continuous. Threshold Model 3 allows both the slope and intercept of the different line segments to vary. The specification of Threshold Model 3 is given by:

$$\pi_t = \alpha_0 \pi_{t-1} + \alpha_1 \pi_t + (1 - \alpha_0 - \alpha_1) E_t \pi_{t+4} + \alpha_2 x_{t-1} + \alpha_3 \Delta x_{t-1} + \alpha_4 \sum_{i=2}^{5} \beta_i \Delta q_{t-i}$$

$$+ \delta_1 S_1(x_{t-1} - x^H) + \delta_2 S_2(x_{t-1} - x^L) + \varepsilon_t$$

(24)

Table 3 provides the Wald test of a non-linear supply curve for both threshold models. The linear specification cannot be rejected on any of the models, regardless of where in the business cycle the cut-off point is placed.

There is no empirical evidence from either of the Threshold Models supporting a non-linear shape of the supply curve in 1AS. This result stands in contrast to previous research on the supply side of the Norwegian economy. The lack of empirical support of a non-linear shape does however not mean that there is empirical support of a linear shape. It could be that the model at hand is a poor model of the Norwegian economy and that estimation is impacted by the effect of i.e. omitted variables.

<table>
<thead>
<tr>
<th>$(x^L, x^H)$</th>
<th>(-0.5, 0.5)</th>
<th>(-1, 1)</th>
<th>(-1.5, 1.5)</th>
<th>(-2, 2)</th>
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</thead>
<tbody>
<tr>
<td>TM 2</td>
<td>0.08</td>
<td>0.92</td>
<td>0.33</td>
<td>0.72</td>
</tr>
<tr>
<td>TM 3</td>
<td>0.13</td>
<td>0.88</td>
<td>0.26</td>
<td>0.77</td>
</tr>
</tbody>
</table>

Table 3. Wald Test. TM (Threshold Model) $H_0 : \delta_1 = \delta_2 = 0$. Degrees of freedom: (2, 39)

Nymoen and Tveter 2007 question if 1AS is a suitable model for the Norwegian economy. Assessing the empirical validity of the model they conclude that 1AS has low empirical foundation: it lacks empirical consistency. The estimated coefficients were very sensitive with respect to the period of estimation and they often had the wrong sign (in particular the coefficient in front of the change in the output gap, and coefficients in front of the change in the log of the real exchange rate). In addition, Nymoen and Tveter found the estimated model
to hold significant autocorrelation of order four, in contrast with the predictions of 1AS. Such
autocorrelation could stem from an omitted autocorrelated variable.

Two rationales can explain the lack of empirical consistency: either the model at hand is a
poor (or too simple) model. Alternatively, given the data available, the estimation procedure
and the model structure, it is not possible to arrive at empirically consistent results. In the
words of Nymoen and Tveter\textsuperscript{39}: “a direct estimation of the supply side of Norges Bank’s
model gives estimates far from the calibrated values which Norges Bank believes describe the
inflation mechanisms in Norway. This can to some extent be due to the model itself. The
problem of weak identification makes precise estimation difficult. Another interpretation,
supported by the fact that it has been possible to derive other models which are theoretical
consistent and realistic, is that 1AS is too simple in describing inflation in Norway”

The purpose of the next sections is to shed light on the difficulty of attaining empirically
consistent results. It will analyse how sensitive the estimates are towards choice instrument
set and time period of estimation.

### 3.4 Instrument Sensitivity\textsuperscript{40}

An instrument set is defined by two properties. First, it must be uncorrelated with the error
term. Second, it must be highly correlated with the variable for which it serves as an
instrument. A particular instrument set can be tested against both of these criteria.

Testing instrument exogeneity is done by the over-identifying restrictions test, denoted $\chi^{(J)}$.
The null hypothesis of this statistic is that the instruments are exogenous. The hypothesis of
instrument exogeneity could not be rejected in any of the previous regressions. It should be
noted that this test statistic holds low power.

\textsuperscript{39} Authors own translation of: “direkte estimering av tilbudssiden I Norges Banks model gir resultater som er
langt fra de kalibrerte verdiene som Norges Bank anser som dekkende for å beskrive inflasjonsmekanismene i
Norge. Dette kan i noen grad skyldes at selve modellformen, på grunn av svak identifikasjon unndrar seg presis
estimering. En annen tolkning, som støttes av at det har vært mulig å oppnå teoretisk konsistente og realistiske
modeller met et annet utgangspunkt, er at likning (1) er en for enkel modell for inflasjonen i Norge.”
\textsuperscript{40} Sensitivity analysis with respect to the output gap can be found in Appendix 6.2
<table>
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</tr>
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<td>(0.014)</td>
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<td>(0.017)</td>
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<table>
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<th>JB</th>
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</tr>
<tr>
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<tr>
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<tr>
<td>(-)</td>
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<td>2.83 (0.42)</td>
<td>2.88 (0.41)</td>
<td>3.03 (0.39)</td>
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<tr>
<td>(+)</td>
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Table 4. Threshold Modell 2. Output Gap: NB, Inflation Measure: CPI-ATE. GMM estimates for period 1993q1 - 2006q1. Standard errors are shown in brackets. No homogeneity restrictions made on the coefficients.
<table>
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<tr>
<th>$(x^L, x^H)$</th>
<th>$(-0.5, 0.5)$</th>
<th>$(-1, 1)$</th>
<th>$(-1.5, 1.5)$</th>
<th>$(-2, 2)$</th>
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<td>$\alpha_0$</td>
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<td>0.742</td>
<td>0.722</td>
<td>0.715</td>
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<td>(0.108)</td>
<td>(0.109)</td>
<td>(0.119)</td>
<td>(0.128)</td>
</tr>
<tr>
<td>$\alpha_1\bar{\alpha}$</td>
<td>0.326</td>
<td>0.253</td>
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<td>0.245</td>
</tr>
<tr>
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<td>(0.896)</td>
<td>(0.099)</td>
<td>(0.116)</td>
<td>(0.117)</td>
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<tr>
<td>$(1 - \alpha_0 - \alpha_1)$</td>
<td>0.280</td>
<td>0.137</td>
<td>0.163</td>
<td>0.171</td>
</tr>
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<td>(0.383)</td>
<td>(0.143)</td>
<td>(0.158)</td>
<td>(0.164)</td>
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<tr>
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<td>0.057</td>
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<td>0.049</td>
</tr>
<tr>
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<td>(0.026)</td>
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<tr>
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<td>-0.128</td>
</tr>
<tr>
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<td>(0.492)</td>
<td>(0.224)</td>
<td>(0.222)</td>
<td>(0.234)</td>
</tr>
<tr>
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<td>-0.002</td>
<td>-0.005</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
<td>(0.014)</td>
<td>(0.015)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>$\alpha_4\beta_3$</td>
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<td>0.026</td>
<td>0.024</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>(0.050)</td>
<td>(0.013)</td>
<td>(0.014)</td>
<td>(0.014)</td>
</tr>
<tr>
<td>$\alpha_4\beta_4$</td>
<td>0.089</td>
<td>0.005</td>
<td>0.003</td>
<td>0.005</td>
</tr>
<tr>
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<td>(0.068)</td>
<td>(0.015)</td>
<td>(0.016)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>$\alpha_4\beta_5$</td>
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<td>-1E-04</td>
<td>7E-04</td>
<td>-2E-04</td>
</tr>
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<td>(0.029)</td>
<td>(0.013)</td>
<td>(0.013)</td>
<td>(0.014)</td>
</tr>
<tr>
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<td>-0.009</td>
<td>-0.012</td>
<td>-0.004</td>
</tr>
<tr>
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<td>(0.057)</td>
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<td>(0.023)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>$\delta_2$</td>
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<td>-0.014</td>
<td>-0.013</td>
<td>-0.021</td>
</tr>
<tr>
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<td>(0.063)</td>
<td>(0.019)</td>
<td>(0.023)</td>
<td>(0.024)</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.36</td>
<td>0.90</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>DW</td>
<td>1.88</td>
<td>1.46</td>
<td>1.47</td>
<td>1.43</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.93</td>
<td>0.70</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>JB</td>
<td>2.34</td>
<td>10.1</td>
<td>9.64</td>
<td>7.81</td>
</tr>
<tr>
<td>$\chi^{(J)}$</td>
<td>1.76</td>
<td>2.87</td>
<td>2.87</td>
<td>2.99</td>
</tr>
<tr>
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<td>(0.62)</td>
<td>(0.41)</td>
<td>(0.41)</td>
<td>(0.39)</td>
</tr>
</tbody>
</table>

Testing instrument relevance can be done by the use of an auxiliary regression. Preferably, this test should be performed regressing the endogenous explanatory variable on all exogenous and all instrumental variables. As expected inflation is not observable, this test is performed using the observed level of inflation as the endogenous variable:

\[ \pi_{t+4} = W_t \delta + Z_t \gamma + u_t \]  

(25)

\( W_t \) is a \((1 \times 8)\) matrix representing the exogenous variables in equation (15) and the constant term. \( Z_t \) is a \((1 \times 4)\) matrix representing the additional instruments used to correct the endogeneity problem. The coefficients in front of the instrumental variables (\( \gamma \)) should not be zero if the instruments are correlated with expected future inflation. Testing instrument relevance can thus be done by the following F-test:

\[
\begin{align*}
F - \text{test} : & \\
H_0 : \gamma = 0 & \\
H_1 : \gamma \neq 0
\end{align*}
\]

This test has low power as the expected rate of future inflation is not observable. A rule of thumb when checking for weak instruments says that if the null hypothesis cannot be rejected, with an F-statistics well below 10, then the instrument set is poor. The F-statistic obtained using CPI-ATE is 0.78 with a p-value of 0.54, indicating that the instrument set used previously is weak.

With respect to proper estimation of \(1A\), two natural questions arise: how sensitive are the estimates of \(1A\) towards the choice of instrument set, and how to go about in finding an optimal set of instruments?

Answering the first question, testing instrument sensitivity, can be done by removing one of the instrumental variables from the instrument set previously used. Not using the two period lag of inflation as an instrument, yields the following set of instruments:

\[
Z_1 = [\Delta q_{t-6} \Delta q_{t-7} ]
\]

(26)
Table 6 provides the estimates when using this set of instruments. All coefficients are positive, which is in line with the calibrated values by NoB. The change of sign in some of the coefficients indicates that estimation is sensitive towards the choice of instrument set. $Z_t$ does not pass the test of instrument relevance with an F-statistic well below 1. This indicates that one poor set of instruments has been replaced by another poor set of instruments. A more precise conclusion would thus be that estimation is sensitive towards poor choice of instrument sets.

It is difficult to rationalise the choice of one of these set of instruments and not the other. Optimally, economic theory should underlie the choice of instrumental variables. The next section will use economic theory to rationalise why the variables included in the proposed instrument sets are thought to be correlated with the expected future rate of inflation.

3.4.1 Expected Inflation and the Term Structure

The expected future level of inflation can be related to the term structure of interest rates. Financing an investment can be done in two ways, both which should yield the same expected payoff. The investment can be financed by a long term loan, with $n$ terms to maturity and a fixed interest rate per period equal to $i_t^F$. The total amount to be paid after $n$ periods is then equal to $(1+i_t^F)^n$. Alternatively, it can be financed with short term debt. Assuming that the investor is risk neutral, the expected total amount to be paid after $n$ periods equals the multiplication of current and expected future interest rates, given by the right hand side of (27):

$$
(1 + i_t^F)^n = (1 + i_t) \times (1 + E_t i_{t+1}) \times (1 + E_t i_{t+2}) \times \cdots \times (1 + E_t i_{t+n-1})
$$

(27)

The possibility of arbitrage ensures that the expected value of the two means of financing should be equal. Taking logs on both sides, rearranging and using the definition of nominal interest rates $i_t = r_t + E_t \pi_{t+1}$, (where $E_t \pi_{t+1} = E_t p_{t+1} - p_t$), yields:

$$
i_t^F = \frac{1}{n} \sum_{j=0}^{n} E_t r_{t+j} + \frac{1}{n} E_t \pi_{t+n}
$$

51
Making the assumption that the real interest rate is constant, $\bar{r}$, setting $n=4$, yields the following expression:\footnote{\[E_t\pi_{t+4} = E_t p_{t+4} - p_t\]}

\[i_t^F = \bar{r} + \frac{1}{n} E_t \pi_{t+4}\] (28)

According to (28), any variation of the fixed four period interest rate, $i_t^F$, is caused by variation in the expected future level of inflation. $i_t^F$ thus satisfies one of the criteria of good instruments: economic theory suggests that it is highly correlated with the expected rate of future inflation. The other criterion, that it is uncorrelated with the error term, is ensured by using a lagged value of the fixed interest rate. The following instrument set is thus defined:

\[Z_2 = i_{t-1}\] (29)

Table (6) shows the estimation results using $Z_2$ as instrument set. The change of instrument set alters the sign of many of the coefficients with respect to previous estimates. In particular, the constant term and the estimate of the trade-off between inflation and production become negative.

Testing the relevance of this instrument set, it is clearly rejected as an instrument which is highly correlated with future inflation. This test-result starkly contrasts the predictions of the economic rationale described above. It should be noted that two important assumptions were made when developing the relationship between $i_t^F$ and $E_t \pi_{t+4}$. First, investors were assumed to be risk neutral. Second, the real interest rate was assumed to be constant. If any of these assumptions are too strong, meaning that the variation in either risk premium or the real interest rate, $\eta$, could account for the variation in the one year interest rate $i_t^F$, it could explain why $Z_2$ failed the test as a valid instrument.
<table>
<thead>
<tr>
<th></th>
<th>$Z_1$</th>
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<th>$Z_3$</th>
<th>NB</th>
</tr>
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<td>(0.064)</td>
<td></td>
</tr>
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</tr>
<tr>
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<td>(0.215)</td>
<td>(0.156)</td>
<td>(0.122)</td>
<td></td>
</tr>
<tr>
<td>$1-\alpha_0-\alpha_4$</td>
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<td>0.439</td>
<td>0.125</td>
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</tr>
<tr>
<td></td>
<td>(0.317)</td>
<td>(0.159)</td>
<td>(0.073)</td>
<td></td>
</tr>
<tr>
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</tr>
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<td>(0.040)</td>
<td>(0.020)</td>
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</tr>
<tr>
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<td>(0.148)</td>
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<tr>
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<td>-0.022</td>
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<td>(0.016)</td>
<td>(0.013)</td>
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<table>
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<tr>
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<th>$\sigma$</th>
<th>JB</th>
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<td>Ø</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>(5.36)</td>
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</table>

Table 6. Instrument Sensitivity. GMM estimates for period 1993q1 - 2006q1. Standard errors are shown in brackets. No homogeneity restrictions made on the coefficients.
Table 7. Autocorrelation Test – Ljung-Box P values $H_0 : \rho_1 = \rho_2 = \cdots = \rho_k = 0$. The $Z$ column refers to the regression in Table 2 using CPI-ATE. The other columns relates to Table 6.

<table>
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<tr>
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<td>0.08</td>
<td>0.75</td>
</tr>
<tr>
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<td>0.05</td>
<td>0.00</td>
<td>0.39</td>
</tr>
<tr>
<td>5. order</td>
<td>0.01</td>
<td>0.08</td>
<td>0.00</td>
<td>0.50</td>
</tr>
</tbody>
</table>

3.4.2 Rational Expectation Hypothesis

A second approach in developing an economic link between expected future inflation and an instrument set is the rational expectation hypothesis. Given that model 1A is the correct model for the economy, the rational expectation hypothesis states that people use all the available information to make the best possible forecasts of future inflation. In particular, all available information includes the structure of the economy as given by model 1A. More formally:

$$E_t \pi_{t+4} = E[\pi_{t+4}|I_{t-1}]$$  \hspace{1cm} (30)

The expectation of future inflation is based on the information set available at the end of period $t-1$ ($I_{t-1}$). This information set, as specified by model 1A, contains six variables in addition to the lagged variables already specified in the supply equation. Forming expectations about future inflation should according to the rational expectation hypothesis be done based on the additional information these variables provides. The additional set of instruments should thus contain these six variables: the nominal 3 month interest rate ($i_{3m}$), the real one year interest rate ($r_{12m}$), the real three year interest rate ($r_{36m}$), the nominal interest rate ($i$), the real exchange rate ($q$) and the foreign output gap ($y_{gap}^F$).

$$Z_3 = [i_{3m_{t-1}} \ r_{12m_{t-1}} \ r_{36m_{t-1}} \ i_{t-1} \ q_{t-1} \ y_{gap_t}^F]$$  \hspace{1cm} (31)

The estimation results using this instrument set is given in Table 6. In contrast to any of the other instrument sets, this is the only one which has not clearly been rejected as being relevant. Two coefficients are estimated with the wrong sign; however, neither of these
estimates is significantly different from zero. The estimate of the output gap is clearly significant and identical to the calibrated value. The estimates on past inflation, the inflation target and expected future inflation are close to the calibrated values, but insignificant.

Table 7 provides the Ljung Box p-values of the autocorrelation test. The previous noted problem of autocorrelation of order four is not an issue using the instrument set based on the rational expectation hypothesis. The model predicts autocorrelation or order one. The hypothesis that there is no autocorrelation cannot be significantly rejected. This could indicate that the coefficient in front of expected future inflation is zero. It could also be that the data material is not sufficient in determining the level of autocorrelation. The lack of evidence of autocorrelation is in this respect not evidence of lack of autocorrelation.

3.5 Parameter Stability

![Parameter Stability Graphs]

Figure 28. Recursive estimation. Instrument set Z. 95% confidence interval. Estimates on data from 1993q1 ending 2000q2 to 2006q1. NB represents the calibrated value by Norges Bank.
This section will analyse how the estimates on 1AS varies over time. The analysis is based on recursive estimation where estimation is first done on the interval from 1993q1-2000q2. Then one quarter is added to the sample and the estimation is rerun on the sample from 1993q1 – 2000q3. This procedure is repeated, by adding one quarter of data each time, until the whole data sample is reached (1993q1-2006q1). Testing parameter stability is done using two different instrument sets \( Z \) and \( Z_3 \).

Figure 29. Recursive estimation. Instrument set \( Z_3 \). 95% confidence interval. Estimates on data from 1993q1 ending 2000q2 to 2006q1. NB represents the calibrated value by Norges Bank.

Recursive estimation illustrates how sensitive estimation is towards time period regardless of which instrument set is used. Many of the estimated parameters change significantly as more data is added. In addition, the calibrated values by Norges Bank are not within the 95% confidence intervals, which they should be if the model is well specified. Some variables even
show the wrong sign, indicating an opposite effect of what the model predicts, i.e. expected future inflation and the change in the real exchange rate.

3.6 Conclusion

Testing the core model which NoB uses in its forecasting of the Norwegian economy does not reject a linear shape of the supply curve. However, the empirical attributes of model 1A does not support its qualifications as a good economic model. The estimated parameters are highly sensitive towards choice of instrument set and time period of estimation. It should thus be noted that the linearity tests do not hold full power if the model is misspecified.
4 Conclusion

NoB expresses a symmetrical property of the interest rate by the use of symmetrical fan charts. This thesis was concerned with the optimality of the symmetrical interest rate policy. Using a simple AS-AD model, it has been illustrated how the assumptions of a quadratic loss function and a linearised model of the economy underlies the optimality of symmetrical monetary policy. There are strong economic arguments against these simplifying assumptions, making the optimality of symmetrical fan charts questionable. Indeed, the market believes that uncertainty should be asymmetric (Figure 24). However, when testing the assumption of a linear aggregate supply curve, it could not be rejected on Norwegian data.
5 References

http://www.uio.no/studier/emner/sv/oekonomi/ECON5120/h07/undervisningsmateriale/ECO N5120_H07_Note08.pdf

http://folk.uio.no/hildecb/teaching/Monetary1.pdf


6 Appendix

6.1 Data Description

6.1.1 Main series

CPI: Consumer Price Index (quarterly, unadjusted). FPAS_HIST: QUA_PCPI

CPI-ATE: Consumer Price Index Adjusted for Taxes and Energy Prices. FPAS_HIST: QUA_PCPIJAE (quarterly, unadjusted)

CPI-ATED: Consumer Price Index Domestic Sources. FPAS_HIST: QUA_PCPIJAEI (quarterly, unadjusted)

NB: Output Gap. FPAS_HIST: QUA_GAP_Y (quarterly - note that the quarterly series is converted from annual series. Last 8 quarters is set by judgement to avoid end-of-series converting problem)

REX: Real Exchange Rate. FPAS_HIST: QUA_QI44. The 44 countries included in the index with weights (per thousand) in parentheses are: Argentine (1.5), Austria (10.5), Australia (4.5), Bangladesh (0.6), Belgium (23.1), Brazil (5.3), Canada (19.4), Chile (1.4), China (22.7), Colombia (1.5), Czech Republic (3.0), Denmark (74.5), Finland (35.5), France (42.6), Germany (141.3), Great Britain (102.4), Greece (1.2), Hong Kong (4.9), Hungary (1.8), India (3.3), Indonesia (2.2), Ireland (13.0), Iceland (2.8), Italy (42.4), Japan (47.9), Malaysia (2.9), Morocco (1.0), Netherlands (46.3), Pakistan (1.2), Philippines (0.7), Poland (6.8), Portugal (8.2), Russia (18.3), Singapore (8.8), South Africa (2.2), South Korea (11.4), Spain (17.5), Sri Lanka (0.3), Switzerland (13.6), Sweden (165.2), Taiwan (7.6), Thailand (3.1), Turkey (3.4) and the USA (72.3).

Series are available upon request.
6.1.2 Inflation

The level of inflation can be measured by different means. Inflation is the increase in the overall level of prices. On quarterly data, the yearly inflation rate is given by:

\[ \pi_t = 100(p_t - p_{t-4}) \]  

(32)

The price level, \( p_t \), are commonly measured by the consumer price index. The purpose of this index is to measure the increase in prices of goods typically consumed by households. The development of the inflation rate based on the consumer price index (CPI) is illustrated in Figure 30. Figure 30 also illustrates the development of two other inflation measures. One is the measure used by NB in its inflation targeting. This measure is based on the consumer price index adjusted for taxes and energy prices (CPI-ATE). The effects of taxes and energy are excluded as they are thought to be temporary only and thus difficult to control by monetary policy. The third inflation measure is based on the domestically driven price index adjusted for taxes and energy (CPI-ATED). The difference between CPI-ATE and CPI-ATED is thus the impact on inflation from imported goods.

Figure 30. Inflation. Quarterly data, seasonal adjusted

\[ ^{42} \text{Denoted in natural logarithm} \]
For testing purposes, the six years of experience with inflation targeting is a short period of time. However, the Norwegian economy has experienced low and stable inflation since the NOK was made floating after being pegged to the EURO’s precursor ECU in 1992. Equation 1 is of this reason estimated on a sample period beginning the first quartile 1993. This is in accordance with Nymoen and Tveter’s estimation.

### 6.1.3 The Output Gap

\[ x_t = y_t - \overline{y}_t \]  \hspace{1cm} (33)

The output gap, \( x_t \), is the percentage deviation between the actual level of production, \( y_t \), and the trend level of production, \( \overline{y}_t \). There are great uncertainties associated with the output gap measure. Firstly, potential output (\( \overline{y}_t \)) is not directly observable and must be estimated. Such estimation is connected with a high degree of uncertainty. Alternative detrending methods can be used to arrive at the trend level of production. As there is no objectively correct value of the output gap this paper will analyse if the impact from using different detrending methods can explain the lack of theoretical consistency observed when estimating IAS. The official output gap used by NoB makes the core output gap series in this analysis. Sensitivity towards the use of this series is analysed through the use of two other output gaps series, based on two different detrending methods. The first part of this section will outline the rationale underlying these detrending methods and compare these data series with the official view of NoB.

The second cause of output gap uncertainty relates to the measurement of the current state of the economy (\( y_t \)). Figures of national accounts are revised, often extensively. This uncertainty is often termed data uncertainty. The second part of this section will illustrate the impact on the output gap due to data uncertainty.
Trend uncertainty - the Hodrick Prescott Filter HP

The HP filter is a univariate method of detrending. The trend level of production is attained by minimising a weighted sum of the deviation of production from its trend level \((y_t - \bar{y}_t)\) and the change in the trend growth rate \((\bar{y}_{t+1} - \bar{y}_t) - (\bar{y}_t - \bar{y}_{t-1})\):

\[
\bar{y}_t = \arg \min \left[ \sum_{t=1}^{T} (y_t - \bar{y}_t)^2 + \lambda \sum_{t=2}^{T-1} ((\bar{y}_{t+1} - \bar{y}_t) - (\bar{y}_t - \bar{y}_{t-1}))^2 \right]
\]  

The smoothing parameter, \(\lambda\), provides the weight given to these conflicting objectives. In the extreme case where \(\lambda=0\), the trend level of production equals the actual level of production. In the opposite case, if weight is only given to minimising the change in the trend growth rate (\(\lambda\) tend to infinity), the increase in the trend level becomes constant. No theoretical reasoning underlies the choice of the smoothing parameter. Hodrick and Prescott found that a value of the smoothing parameter \(\lambda=1600\) gave a good fit on quarterly U.S. data. According to Norges Bank the smoothing parameter should be given a weight of 20000 to give a good fit on Norwegian quarterly data (Inflation Report 2/2004). Figure 31 illustrates the difference between the official view on the output gap by NoB (NB) and the output gap using the HP filter (HP) with a smoothing parameter of 20000.

![Figure 31: The Output Gap (%). Official and HP-filtered. HP filtering on quarterly GDP at fixed prices (seasonally adjusted) (\(\lambda=20000\)).](image-url)
In addition to the fact that there is no objectively correct value of the smoothing parameter, there are two further concerns when detrending using the HP filter. The first is the problem of imprecise estimates at the end points. It is caused by how the end of a time series does not hold information about the future. As the HP filter is two sided, and since it is constructed so as to minimise the change in trend growth, which includes the next period level of trend, the endpoints will be based on one side information only. Figure 31 illustrates a clear deviation between the official level of the output gap and the constructed HP series at the endpoints. The end point problem could explain this discrepancy. The second cause of concern is due to how the HP filter cannot immediately capture structural breaks in trends. A one-time shift in the trend level, caused by i.e. economic reforms, will only gradually be estimated by the HP filter as it assigns positive weight to minimising the change in the trend growth rate43.

**Trend Uncertainty - the Production Function Method**

The production function method (PF) of detrending is a multivariate method. It relies on the assumption that the level of production can be described by Cobb-Douglas technology and thus determined by the aggregate capital stock \( (k_t) \), aggregate labour stock \( (l_t) \), and total factor productivity \( (a_t) \) (all variables are denoted in logs). Detrending these components in the production function yields the following expression for the trend level of production:

\[
\bar{y}_t = \zeta_0 + \zeta_1 \bar{l}_t + \zeta_2 \bar{k}_t + \bar{a}_t
\] (35)

The upper bar denotes the trend levels. \( \zeta \)'s are constants. The trend levels of the capital stock, the total number of hours worked and total factor productivity is estimated by the use of HP-filter. Figure 32 compares the official output gap (NB) with the output gap retrieved when using the production function method of detrending44.

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43 Bjørnland et al, 2006
44 The data the production function method is the same as used by Bjørnland, et al. A special thanks to Anne Sofie Jore for making this series available.
Figure 32: The Output Gap (%) – Official and PF-filtered. PF filtering on quarterly data. Data

The advantage of the production function method is that it holds theoretical underpinnings. It is however subject to the same critique as the HP-filter since HP filtering is used in detrending the level of capital, number of hours worked and total factor productivity (Bjørnland et al). In addition, it restricts technology to follow a Cobb-Douglas form, which may be a strong assumption.

Data Uncertainty

Figures of the national account are often revised. Figure 33 illustrates the output gap between 1993q1 and 2002q1 based on information in 2002q1 and 2006q1. Both series are detrended by the same method, using HP-filter with a smoothing parameter of 20000.
There are clear discrepancies between the levels of the output gap depending on what information set is used to create the output gap series. At some years, i.e. 1998, the two series show different sign of the output gap. Figure 33 thus illustrates that there is great uncertainty surrounding the output gap depending on which information set it is based upon. This paper will analyse how this uncertainty impacts the estimates of IAS.

### 6.1.4 The Real Exchange Rate

The real exchange rate indicator used is I-44. The nominal exchange rate underlying the real exchange rate is based on a weighting of Norway’s 44 most important trading partners. The foreign consumer price level is based on a weighting of 25 of the 44 most important trading partners. Figure 34 illustrates the development of the real exchange rate.

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45 See Appendix 6.1.1
Using I-44 as the real exchange rate indicator may cause biased estimates. The calibrated coefficients of the change in the real exchange rate were based on available empirical evidence. In particular, the calibration was influenced by empirical findings on the pass through from exchange rate changes to prices. The foreign price index, of which I-44 is based upon, will not necessarily illustrate foreign inflation on imported consumption goods. This is due to how imported goods differ from the goods which make up this index. In addition, this index does not reflect changes caused by a changing import pattern from high- to low-cost countries. In turn, foreign prices could thus explain more of the observed level of deflation on imported goods than assumed only using foreign price indexes. As the real exchange rate is based upon foreign price indexes, the real exchange rate explains less of the observed deflation on imported goods than previously assumed (IR. 1/2004). This empirical observation was taken into account when calibrating the model. Estimating the supply curve using I-44 as the exchange rate indicator will not take this effect into account. The estimates will thus be biased. The size of the bias, however, is uncertain. It is affected by the weight assigned to the new empirical evidence on the exchange rate pass-through, and how the change in the exchange rate pass-through is thought to impact the supply side of the economy. These issues are not further discussed in the model documentation.
6.2 Output Gap Sensitivity

Uncertainty relating to the output gap can be caused by which detrending method is used and what information the measure of production is based upon. This section will analyse these two uncertainties separately.

6.2.1 Trend uncertainty

Table 8 provides estimates when using the three alternative detrending methods: the official output gap by NoB (NB), detrending using the HP-filter (HP) and detrending using the production function method (PF). The fourth column provides the calibrated values in model 1A (NB-CAL).

The significance of lagged inflation and the three period lag of the change in the real exchange rate are robust to a change in the output gap measure. The estimated coefficient of the output gap does not hold the same robustness and it becomes insignificantly different from zero when using HP and PF. Even though insignificant, it does hold the correct sign for all detrending methods. The same does not apply for the change in the output gap. Its estimated coefficient is consistently estimated with the wrong sign for all output gap measures. Neither of these estimates however is significantly different from zero.
<table>
<thead>
<tr>
<th></th>
<th>NB</th>
<th>HP</th>
<th>PF</th>
<th>NB-CAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_0)</td>
<td>0.732</td>
<td>0.810</td>
<td>0.826</td>
<td>0.600</td>
</tr>
<tr>
<td></td>
<td>(0.105)</td>
<td>(0.078)</td>
<td>(0.090)</td>
<td></td>
</tr>
<tr>
<td>(\alpha_1,\pi^*)</td>
<td>0.247</td>
<td>0.233</td>
<td>0.175</td>
<td>0.125</td>
</tr>
<tr>
<td></td>
<td>(0.118)</td>
<td>(0.190)</td>
<td>(0.175)</td>
<td></td>
</tr>
<tr>
<td>(1-\alpha_0-\alpha_1)</td>
<td>0.154</td>
<td>0.080</td>
<td>0.100</td>
<td>0.350</td>
</tr>
<tr>
<td></td>
<td>(0.143)</td>
<td>(0.168)</td>
<td>(0.175)</td>
<td></td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>0.048</td>
<td>0.035</td>
<td>0.024</td>
<td>0.070</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.041)</td>
<td>(0.031)</td>
<td></td>
</tr>
<tr>
<td>(\alpha_3)</td>
<td>-0.114</td>
<td>-0.037</td>
<td>-0.016</td>
<td>0.100</td>
</tr>
<tr>
<td></td>
<td>(0.213)</td>
<td>(0.022)</td>
<td>(0.018)</td>
<td></td>
</tr>
<tr>
<td>(\alpha_4,\beta_2)</td>
<td>-0.003</td>
<td>0.008</td>
<td>0.003</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.018)</td>
<td></td>
</tr>
<tr>
<td>(\alpha_4,\beta_3)</td>
<td>0.025</td>
<td>0.035</td>
<td>0.032</td>
<td>0.030</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.018)</td>
<td>(0.016)</td>
<td></td>
</tr>
<tr>
<td>(\alpha_4,\beta_4)</td>
<td>0.005</td>
<td>0.014</td>
<td>0.011</td>
<td>0.045</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>(0.021)</td>
<td>(0.022)</td>
<td></td>
</tr>
<tr>
<td>(\alpha_4,\beta_5)</td>
<td>0.002</td>
<td>-0.001</td>
<td>0.003</td>
<td>0.045</td>
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<tr>
<td></td>
<td>(0.013)</td>
<td>(0.016)</td>
<td>(0.016)</td>
<td></td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.90</td>
<td>0.89</td>
<td>0.89</td>
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<tr>
<td>DW</td>
<td>1.46</td>
<td>1.50</td>
<td>1.51</td>
<td></td>
</tr>
<tr>
<td>(\sigma)</td>
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<td>0.70</td>
<td>0.70</td>
<td></td>
</tr>
<tr>
<td>JB</td>
<td>8.81</td>
<td>6.61</td>
<td>5.36</td>
<td></td>
</tr>
<tr>
<td>(\chi^{(J)})</td>
<td>2.86</td>
<td>2.39</td>
<td>2.46</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.41)</td>
<td>(0.49)</td>
<td>(0.48)</td>
<td></td>
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</table>

Table 8: Estimates for model 1AS Output Gap variable Output Gap variable: NB, HP, PF. Note: GMM estimates for period 1993q1 - 2006q1. Standard errors are shown in brackets. No restrictions made on the coefficients. The estimated equations include two lags of inflation, lagged output gap, lagged change in output gap and seven lags of the change in the real exchange rate. CPI, CPI-ATE and CPI-ATED refers to different inflation measures.
Table 9 provides the Ljung-Box p values of the autocorrelation test. Autocorrelation of order 1, which the model predicts, is significant using all detrending methods. Autocorrelation of order 4 and 5 is also significant using all detrending methods, in contrast to the predictions of the model.

<table>
<thead>
<tr>
<th></th>
<th>NB</th>
<th>HP</th>
<th>PF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Order</td>
<td>0.09</td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td>2. Order</td>
<td>0.16</td>
<td>0.12</td>
<td>0.19</td>
</tr>
<tr>
<td>3. Order</td>
<td>0.28</td>
<td>0.24</td>
<td>0.34</td>
</tr>
<tr>
<td>4. Order</td>
<td>0.01</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>5. Order</td>
<td>0.01</td>
<td>0.03</td>
<td>0.02</td>
</tr>
</tbody>
</table>

Table 9: Autocorrelation Test – Ljung-Box P values $H_0: \rho_1 = \rho_2 = \cdots = \rho_k = 0$

### 6.2.2 Data Uncertainty

This section will analyse the impact of data uncertainty on the estimates of the short run trade-off between inflation and production. The period of estimation is 1993q1 – 2002q1. Over this sample period the data are continuously revised from 2002q1 – 2006q1 (2004q2 and 2004q4 are missing). Figure 35 illustrates how the estimates on the trade-off between production and inflation, $\hat{\alpha}_{2,v}$, change as the data is revised. The subscript $v$ denotes vintage, meaning that the period of estimation (1993q1 – 2002q1) is held constant.

The estimates vary extensively, from the lowest estimate in 03q1 of 0.055 to the highest in 2004q3 of 0.138. The error bars provides a 95% confidence interval around the point estimates of $\hat{\alpha}_{2,v}$. The calibrated value used by NB, represented by the solid line, is always within the 95% confidence interval. Figure 36 thus indicates that the calibrated value by NB is robust towards updated information on the output gap.
Figure 35. Output Gap Sensitivity – Data Uncertainty. GMM point estimates for period 1993q1 - 2002q1 using real time data from 2002q1 – 2006q1. 95% confidence intervals. 2004q2 and 2004q4 are missing. No restrictions made on the coefficients. The estimated equations include two lags of inflation, lagged output gap, lagged change in output gap and seven lags of the change in the real exchange rate. CPI-ATE and HP-QuarterlyM\textsuperscript{46} are the respective inflation and production measures.

The estimation results reported in Table 8 and Table 9 does not indicate that the choice of output gap measure can explain the observed empirical inconsistency. The autocorrelation problem remains regardless of output gap measure and many of the variables provides estimates that are not significantly different from zero.

6.3 The regulation on monetary policy

(\url{http://www.dep.no/filarkiv/132945/regnorgesbank.pdf})

Established by Royal Decree of 29 March 2001 pursuant to Section 2, third paragraph, and Section 4, second paragraph, of the Act of 24 May 1985 no. 28 on Norges Bank and the Monetary System

\textsuperscript{46} HP-QuarterlyM is obtained by data on GDP mainland at fixed prices with HP detrending using $\lambda = 20000$. 

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§ 1.

Monetary policy shall be aimed at stability in the Norwegian krone’s national and international value, contributing to stable expectations concerning exchange rate developments. At the same time, monetary policy shall underpin fiscal policy by contributing to stable developments in output and employment.

Norges Bank is responsible for the implementation of monetary policy. Norges Bank’s implementation of monetary policy shall, in accordance with the first paragraph, be oriented towards low and stable inflation. The operational target of monetary policy shall be annual consumer price inflation of approximately 2.5 per cent over time.

In general, the direct effects on consumer prices resulting from changes in interest rates, taxes, excise duties and extraordinary temporary disturbances shall not be taken into account.

§ 2.

Norges Bank shall regularly publish the assessments that form the basis for the implementation of monetary policy.

§ 3.

The international value of the Norwegian krone is determined by the exchange rates in the foreign exchange market.

§ 4.

On behalf of the State, Norges Bank communicates the information concerning the exchange rate system ensuing from its participation in the International Monetary Fund, cf. Section 25, first paragraph, of the Act on Norges Bank and the Monetary System.
This regulation comes into force immediately. Regulation no. 0331 of 6 May 1994 on the exchange rate system for the Norwegian krone is repealed from the same date.
6.4 Deriving the Optimal Interest rate Rule

6.4.1 The Model

Loss function: \[ L_t = \lambda (y_t - \bar{y})^2 + (\pi_t - \bar{\pi})^2 \]

AS: \[ \pi_t = \pi_{t-1} + \alpha (y_t - \bar{y}) + u_t \]

AD: \[ y_t - \bar{y} = -\phi (i_t - \pi_{t-1}) + \nu_t \]

6.4.2 The Derivation

\[ \min_{y_t, \pi_t} L_t \] \[ \Rightarrow \min_{y_t, \pi_t} \lambda (y_t - \bar{y})^2 + (\pi_t - \bar{\pi})^2 \]

s.t. AS \[ \pi_t = \pi_{t-1} + \alpha (y_t - \bar{y}) + u_t \]

The Lagrangian \((V)\) gives:

\[ V = \lambda (y_t - \bar{y})^2 + (\pi_t - \bar{\pi})^2 - \theta (\pi_t - \pi_{t-1} - \alpha (y_t - \bar{y}) - u_t) \]

This yields the following first order conditions:

I. \[ \frac{\partial V}{\partial y_t} = 2\lambda (y_t - \bar{y}) + \theta \alpha = 0 \]

II. \[ \frac{\partial V}{\partial \pi_t} = 2(\pi_t - \bar{\pi}) - \theta = 0 \]

Together these give the first order condition for the optimal policy, or the optimal AD curve:

\[ \pi_t = \bar{\pi} - \frac{\lambda}{\alpha} (y_t - \bar{y}) \]

Inserting this into the AD relationship, solving for \(i_t\), gives the nominal interest rate that must be set to achieve the optimal solution at different levels of inflation:

\[ i_t = \pi_{t-1} + \left( \frac{\alpha}{\phi \lambda} \right) (\pi_t - \bar{\pi}) + \left( \frac{1}{\phi} \right) \nu_t \]
6.5 Deriving the optimal interest rate with the linex loss function

6.5.1 The Model

Loss function:
\[ L = \frac{2(e^{\beta(\pi_t - \bar{\pi})} - \beta(\pi_t - \bar{\pi}) - 1)}{\beta^2} + \lambda(y_t - \bar{y})^2 \]

AS:
\[ \pi_t = \pi_{t-1} + \alpha(y_t - \bar{y}) + u_t \]

AD:
\[ y_t - \bar{y} = -\varphi(i_t - \pi_{t-1}) + v_t \]

6.5.2 The Derivation

Minimizing \( L_t \) subject to AS
\[ \min_{y_t, \pi_t} L_t \Rightarrow \min_{y_t, \pi_t} \frac{2(e^{\beta(\pi_t - \bar{\pi})} - \beta(\pi_t - \bar{\pi}) - 1)}{\beta^2} + \lambda(y_t - \bar{y})^2 \]
subject to \( \pi_t = \pi_{t-1} + \alpha(y_t - \bar{y}) + u_t \)

The Lagrangian \( V \) gives:
\[ V = \frac{2(e^{\beta(\pi_t - \bar{\pi})} - \beta(\pi_t - \bar{\pi}) - 1)}{\beta^2} + \lambda(y_t - \bar{y})^2 - \theta(\pi_t - \pi_{t-1} - \alpha(y_t - \bar{y}) - u_t) \]

This yields the following first order conditions:

I. \[ \frac{\partial V}{\partial y_t} = 2\lambda(y_t - \bar{y}) + \theta\alpha = 0 \]

II. \[ \frac{\partial V}{\partial \pi_t} = \frac{2(e^{\beta(\pi_t - \bar{\pi})} - 1)}{\beta} - \theta = 0 \]

Together these give the first order condition for the optimal policy:
\[ (y_t - \bar{y}) = \left( \frac{\alpha}{\beta\lambda} \right)(1 - e^{\beta(\pi_t - \bar{\pi})}) \]

Inserting this into the AD curve, solving for \( i_t \), gives the nominal interest rate that must be set to achieve the optimal solution at different levels of inflation:
\[ i_t = \pi_{t-1} - \left( \frac{\alpha}{\varphi\beta\lambda} \right)(1 - e^{\beta(\pi_t - \bar{\pi})}) + \left( \frac{1}{\varphi} \right)v_t \]
6.6 Deriving the optimal interest rate with recession aversion

6.6.1 The Model

Loss function: \[ L_t = \begin{cases} \frac{1}{2} (\pi_t - \bar{\pi})^2 + \lambda (y_t - \bar{y})^2 & \text{if } y_t < \bar{y} \\ (\pi_t - \bar{\pi})^2 & \text{if } y_t > \bar{y} \end{cases} \]

AS: \[ \pi_t = \pi_{t-1} + \alpha (y_t - \bar{y}) + u_t \]

AD: \[ y_t - \bar{y} = -\varphi (i_t - \pi_{t-1}) + v_t \]

6.6.2 The Derivation

\[
\begin{align*}
\min_{\pi_t, \alpha, \lambda} L_t & \quad \Rightarrow \\
\text{s.t. } & \quad \min_{\pi_t, \alpha, \lambda} \begin{cases} \frac{1}{2} (\pi_t - \bar{\pi})^2 + \lambda (y_t - \bar{y})^2 & \text{if } y_t < \bar{y} \\ (\pi_t - \bar{\pi})^2 & \text{if } y_t > \bar{y} \end{cases} \\
\pi_t = \pi_{t-1} + \alpha (y_t - \bar{y}) + u_t & \quad \text{st. } \pi_t = \pi_{t-1} + \alpha (y_t - \bar{y}) + u_t
\end{align*}
\]

A) If \( y_t > \bar{y} \):

\[
\Rightarrow \min_{\pi_t, \alpha, \lambda} (\pi_t - \bar{\pi})^2 + \alpha (y_t - \bar{y}) + u_t - \bar{y})^2
\]

\[ y_t - \bar{y} = \frac{1}{\alpha} (\bar{\pi} - \pi_{t-1} - u_t) \]

\[ i_t = \pi_{t-1} - \left( \frac{1}{\varphi \alpha} \right) (\bar{\pi} - \pi_{t-1} - u_t) + \left( \frac{1}{\varphi} \right) v_t \]

The optimal interest rate is thus independent of the current inflation rate.

B) If \( y_t < \bar{y} \) : derivation follows from the main model, which gives:

\[ i_t = \pi_{t-1} + \left( \frac{\alpha}{\varphi \lambda} \right) (\pi_t - \bar{\pi}) + \left( \frac{1}{\varphi} \right) v_t \]
6.7 Deriving the optimal interest rate with non-linear AS curve

6.7.1 The Model

Loss function:
\[ L_t = \lambda(y_t - \bar{y})^2 + (\pi_t - \bar{\pi})^2 \]

AS:
\[ \pi_t = \pi_{t-1} + \alpha(y_t - \bar{y}) + \eta \nu_t^2 + u_t \]

AD:
\[ y_t - \bar{y} = -\varphi(i_t - \pi_{t-1}) + \nu_t \]

6.7.2 The Derivation

\[ \min_{y_t, \pi_t} L_t \Rightarrow \min_{y_t, \pi_t} \lambda(y_t - \bar{y})^2 + (\pi_t - \bar{\pi})^2 \]

s.t. AS
\[ \pi_t = \pi_{t-1} + \alpha(y_t - \bar{y}) + \eta \nu_t^2 + u_t \]

The Lagrangian \( V \) gives:
\[ V = \lambda(y_t - \bar{y})^2 + (\pi_t - \bar{\pi})^2 - \theta(\pi_t - \pi_{t-1} - \alpha(y_t - \bar{y}) - \eta \nu_t^2 - u_t) \]

This yields the following first order conditions:

I. \[ \frac{\partial V}{\partial y_t} = 2\lambda(y_t - \bar{y}) - \theta \alpha + 2\theta \eta \nu_t = 0 \]

II. \[ \frac{\partial V}{\partial \pi_t} = 2(\pi_t - \bar{\pi}) - \theta = 0 \]

Together these give the first order condition for the optimal policy:
\[ (y_t - \bar{y}) = -\frac{\alpha}{\lambda}(\pi_t - \bar{\pi}) - \frac{2\eta}{\lambda}(\pi_t - \bar{\pi})y_t \]

Inserting this into the AD curve, solving for \( i_t \), gives the nominal interest rate that must be set to achieve the optimal solution at different levels of inflation:
\[ i_t = \pi_{t-1} + \left( \frac{\alpha}{\varphi \lambda} \right)(\pi_t - \bar{\pi}) + \left( \frac{2\eta}{\varphi \lambda} \right)(\pi_t - \bar{\pi})y_t + \left( \frac{1}{\varphi} \right)\nu_t \]
6.8 The Simultaneity problem

Given the system of equations:

\[ E_{t+4} = \pi_{t+4} + \nu_{t+4} \]

\[ \pi_t = \alpha_0 \pi_{t-1} + \alpha_i \pi_t + (1 - \alpha_0 - \alpha_i) E_{t+4} + \alpha_2 x_{t-1} + \alpha_3 \Delta x_{t-1} + \alpha_4 \sum_{j=2}^5 \beta_j \Delta q_{t-j} + \epsilon_t \]

Inserting (1) into (2) gives:

\[ \pi_t = \alpha_0 \pi_{t-1} + \alpha_i \pi_t + (1 - \alpha_0 - \alpha_i)\pi_{t+4} + \alpha_2 x_{t-1} + \alpha_3 \Delta x_{t-1} + \alpha_4 \sum_{j=2}^5 \beta_j \Delta q_{t-j} + (1 - \alpha_0 - \alpha_i) \nu_{t+4} + \epsilon_t \]

Which implies endogeneity:

\[ \text{cov}[\pi_{t+4}, (1 - \alpha_0 - \alpha_i) \nu_{t+4} + \epsilon_t] = \text{cov}[\pi_{t+4}, (1 - \alpha_0 - \alpha_i)(E_{t+4} - \pi_{t+4}) + \epsilon_t] = \text{cov}[\pi_{t+4}, -(1 - \alpha_0 - \alpha_i)\pi_{t+4}] = -(1 - \alpha_0 - \alpha_i) \sigma^2_{\pi_{t+4}} \neq 0 \]

6.9 Welfare costs of consumption fluctuations

The value of all that is produced in an economy is the real income of the economy. When there is variation in the amount that is produced, there will also be variation in the real income, which again will lead to variation in consumption, i.e. if real income decreases, consumption will also decrease. The marginal utility of consumption is assumed to be negative, i.e. a consumer will feel greater pleasure from the first ice-cream he eats on a warm summer day, than what the level of added pleasure will be if he decides to eat a second one. The assumption of decreasing marginal utility leads thus to consumers wanting to smooth their consumption path, i.e. during a one week summer holiday it is better to enjoy one ice-cream a day, than seven the first day, and none the rest of the holiday. Enjoying one ice-cream a day gives the least variation in consumption. If monetary policies can effectively stabilise
the level of production, and thus the level of consumption, it would therefore improve the welfare of the consumer⁴⁷.

6.10 Choosing an inflation target

Mr. Gjedrem wanted a 2% inflation target in 2001 when the mandate was changed. The government however, went for a 2.5% target after the major union LO had given its approval. Mr. Gjedrem believed that this target was too high. (DN, 20.06.2001). The economic rationale when deciding on an inflation target is the topic of this section.

As previously discussed, inflation in itself is costly to society due to factors such as Shoe-leather costs, menu costs, relative price distortions, etc. These costs are reduced if the level of inflation is reduced. It is therefore optimal to set the inflation target as low as possible. At a zero percent inflation rate, these costs of inflation would be non-existent. There is however strong arguments against a zero percent inflation target.

First, if the inflation target were zero percent, the risk of the economy going into a liquidity trap (deflation) would be severe. Due to the huge costs and difficulties of stabilising inflation when experiencing deflation, it is important to avoid the possibility of liquidity traps.

Second, most economies have the property of so called downward nominal wage rigidity. Simplified, this means that employees, and their unions, are reluctant to accept a cut in the nominal wage. If inflation is positive, and employees accept that their nominal wage is held constant, then they are actually accepting a wage cut, in real terms. Some degree of inflation can thus be beneficial, so as to create flexibility in the labour market.

Third, price indices tend to overstate the true rate of inflation. In a report by an Advisory Commission to the U.S. senate in 1996 found that “the U.S. consumer price index overestimates inflation by 1.1 percentage points a year” (Lequiller, 1997, p.1). This

⁴⁷ The debate of whether capital markets can be effectively used to smooth consumption when production varies is omitted. The purpose of the paragraph is to give the reader an economic rationale to why variation in production can be costly to society.
overestimation was mostly caused by the difficulties of measuring the effect of new products entering the market (ibid). Another problem with price indices is how to exclude the temporary effects on the consumer price index (CPI) that does not impact the development of inflation over time. In April 2006 the CPI increased 3.8% from the previous year. This increase was strongly impacted by the rapid change in price levels of gas and electricity. Gas and electricity represents goods which prices are highly variable, a variation the central bank cannot control. Excluding gas and electricity prices, controlling for taxes (CPI-ATE), gave a core inflation rate of 2.5%. According to the central bank governor, one should be cautious when judging core inflation based on CPI-ATE as it most likely overestimates the effect of gas and electricity. The reason for this is twofold. First, Norges Bank assumes that the tax increase only impacts consumers. In reality, however, it is likely that the effect of a tax increase is shared between consumers and producers, depending on the price sensitivity of supply and demand. Second, the increase in gas and electricity prices is probably not just caused by temporary factors, but they are also likely to be caused by other, more lasting factors (Gjedrem, 7. June. 2006). These lasting shocks should not be excluded by the price index since they impact the trend development of inflation, which is what the central bank wants to control. As price indices tend to overestimate the true inflation rate, the central bank should not try to target a zero percent inflation rate. Doing so would effectively be a deflation target.

The costs related to inflation moves in favour of an inflation target as low as possible. Risks of deflation, downwards nominal rigidity and the problem of price indices overestimating inflation are factors that show the advantage of having some level of trend inflation in the economy. The above discussion provides some reasoning to why the inflation target should not be as high as 10% or 15%. But it has not showed why one should choose an inflation target of 2% and not 2.5%, which was what the central bank governor Gjedrem wanted. Obviously, he believed that 2% was more than sufficient to deal with the risks of deflation, downwards nominal rigidity and overestimating price indices. Mr. Gjedrem was not alone on his position. Other inflation targeting countries, such as Canada, Israel, UK and Sweden focus on achieving lower than 2.5% inflation. Some of these countries however do not have explicit targets, but their policy is aimed at keeping inflation within a band of e.g. 1 – 3 %. The policies of the different inflation targeting countries can be seen from Figure 6.
Many countries focus on achieving a low level of inflation. 15 of the 20 inflation targeting countries aims at always keeping inflation lower than 4%. Brazil, Columbia, South Africa, Hungary and the Philippines however, seem to allow for a bit more inflation. The different aims of monetary policy amongst inflation targeting countries raise a natural question: Does economic literature not give a precise answer to where an inflation target should be, or is it the specific economic structure of each country that makes different inflation targets optimal? The short answer is that it is a bit of both.

Economists disagree about what the optimal inflation target is. They even disagree whether or not one should have an inflation target (Mankiw, 2000). As the previous discussion illustrated, economic literature states that there are costs related to inflation, and one should try to minimise these costs. Norway is however in somewhat of a unique situation. The country enjoys great wealth from its oil and gas extraction, and this wealth will be gradually phased into the economy. One could thus argue that a higher inflation target allows for a bit more flexibility when in the process of enjoying this wealth. This is due to how wealth consumption puts pressure on the domestic inflation rate.

In an on-line question-and-answer session on Norwegian financial news dn.no, Mr. Gjedrem toned down the importance of where an inflation target should be. In response to a question of why Norway sat its inflation target to 2.5%, and not 2% which is the target rate at Sveriges Rigsbank and the European Central Bank, he answered: “The government sets the target rate. The 2.5% target was based on the fact that Norway experienced a 2.5% inflation rate during the nineties. Adding to that, England and some other countries did have a 2.5 % target at that
time. Whether the inflation target is 2.5% or 2% is not of great significance” (Gjedrem, 2006)48.

48 Authors own translation of: ”Det er regjeringen som setter målet. Bakgrunnen for 2,5 % var at vi hadde hatt en inflasjon på 2,5% på 90-tallet. Dessuten hadde England og enkelte andre land et mål på 2,5% den gangen. Det betyr ikke mye om vi har et mål på 2,5% eller 2%”