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Monetary policy and nominal rigidities under low inflation

By
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Monetary policy and nominal rigidities
under low inflation

by

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Abstract
In most European countries, nominal wages are given in collective agreements or individual employment contracts, and the employer cannot unilaterally cut wages, even after the expiration of a collective agreement. Ceteris paribus, workers have a stronger bargaining position when they try to prevent a cut in nominal wages. If inflation is so low that some nominal wages have to be cut, workers’ stronger bargaining position requires higher unemployment in equilibrium. The upshot is a long run tradeoff between inflation and unemployment for low levels of inflation. The prediction that low inflation involves higher unemployment in Europe but not in the US is consistent with previous empirical findings.

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1 Introduction

In recent years, a number of countries have adopted explicit inflation targets for monetary policy. While there is general agreement as to the notion that monetary policy must ensure low inflation, several economists have argued that if inflation is too low, downward rigidity of nominal wages may lead to higher wage pressure, and higher equilibrium unemployment (see eg Tobin, 1972, Holden, 1990, 1994, and Akerlof, Dickens and Perry, 1996, 2000). Against this view, two main counterarguments have been put forward (eg King, 1999). First, labour markets are likely to adapt to low inflation, so that possible downward nominal wage rigidity is likely to be mitigated or disappear over time. Secondly, downward nominal wage rigidity, even if it exists, is unlikely to be empirically relevant, because due to positive productivity growth there is room for considerable growth in nominal wages even under low inflation.

The objective of this paper is to provide a theoretical framework for evaluating these arguments. The basis for the framework is the institutional feature of European labour markets pointed out Holden (1994) and MacLeod and Malcomson (1993). Nominal wages are given in contracts, either collective agreements or individual employment contracts, and can as such only be changed by mutual consent. European labour laws stipulate that the employer cannot unilaterally cut nominal wages, even after expiration of the collective agreement. Incorporating this feature in a non-cooperative bargaining model, I show that workers’ bargaining position is stronger when they try to prevent a cut in nominal wages. If inflation is so low that some nominal wages should

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1 Low inflation may also limit the scope for expansionary monetary policy as the nominal interest rate cannot be negative, cf Keynes (1936).
have been cut, workers’ stronger bargaining position requires higher unemployment in equilibrium.

The crucial question is then how much inflation that is required to avoid nominal wage rigidity inducing higher equilibrium unemployment. Productivity growth reduces the required rate of inflation, by creating room for nominal wage growth even at constant prices. However, other factors work in the other direction. To allow for changes in relative wages without nominal wage cuts, average nominal wages must grow. If workers can inflict large costs on the firms by use of other types of industrial action (eg work-to-rule), nominal wages may increase even when nominal wage rigidity is binding. Furthermore, if aggregate nominal demand is stochastic, a negative demand shock may induce excess unemployment via binding nominal wage rigidity, even if inflation on average is above the ex post minimum level necessary to avoid nominal wage rigidity. The excess unemployment arising due to binding nominal wage rigidity will not be recovered under a positive demand shock.

The key implication of the model is the existence of a long run tradeoff between unemployment and inflation. This feature is derived without any money illusion, ad hoc asymmetries, or irrational behaviour. Thus, my paper is not subject to the critique of models with downward nominal rigidities provided in Ball and Mankiw (1994). In a menu cost model with endogenous price setting, Ball and Mankiw showed that nominal adjustment that is asymmetric under positive trend inflation, may turn symmetric under zero trend inflation. Moreover, zero inflation is optimal in the model of Ball and Mankiw. My model shows that the costs of inflation need not be related to whether nominal rigidities are asymmetric or symmetric. Too low inflation enhances the bargaining
position of the workers, and this involves higher long run unemployment even if the model treats nominal adjustments in a symmetric way.

Other recent popular explanations of nominal wage rigidity have generally appealed to money illusion or fairness considerations (i.e. that workers and managers view a cut in nominal wages as unfair). I do not wish to contend the existence of such effects (cf e.g. documentation in Shafir, Diamond and Tversky, 1997, or Bewley, 1998), rather, I think that fairness considerations and legal effects in many situations may re-enforce each other. While the main macroeconomic implications of nominal wage rigidity probably does not depend on the cause of the rigidity, understanding the reasons for nominal wage rigidity seems crucial to evaluating to what extent rigidities will disappear in a zero inflation economy.

The legal argument for nominal wage rigidity suggests important variation across countries. In the US, the legal framework is different from the European, and it is generally a lot easier for employers to cut nominal wages. This is consistent with studies showing very strong downward nominal wage rigidity in Sweden and Italy (in Sweden in spite of soaring unemployment and several years of close to zero inflation), while nominal wage cuts are more widespread in the US (see section 7). I argue that this difference makes it more costly (in terms of unemployment and lost output) to aim for a very low rate of inflation in most European countries than in the US. This is consistent with the empirical findings of Bullard and Keating (1995).

The remainder of the paper is organised as follows. In section 2, I present important institutional features of the wage setting in Western Europe and the US. The basic model is provided in sections 3 and 4. Section 5 derives the equilibrium of the
model. Some implications for the choice of monetary policy is presented in section 6. In section 7, key parts of the model are evaluated against available empirical evidence. Section 8 concludes. All proofs are in the appendix.

2 **Nominal rigidity in the wage setting process**

The crucial assumption in the model, and the source of the nominal rigidity, is that the nominal wage of the old contract affects the parties’ disagreement point in the wage bargaining and thus also the outcome of a contract renewal. To motivate this assumption, this section provides a crude picture of some elements of the institutional setting of wage determination in many Western European countries and the US.

Most workers in Europe are hired in permanent jobs. The general principle is then that the prevailing terms of employment are interpreted as a legal contract, and may as such only be changed by mutual consent. In England, the difficulty the firm might face when trying to cut the wage was shown by *Rigby v Ferodo Limited* [1988] ICR 29 (HL), where the employer had unilaterally proposed a 5 per cent wage reduction. The employee had continued to work, without accepting the wage cut. The court held that the wage reduction was in breach of contract, so that the employee could claim the arrears of pay wrongfully withheld from him under the contract (McCullen, 1992).

To reduce wages, the employer must persuade the employee to accept the wage cut. One possibility is to threaten to lay off the employee temporarily or permanently unless he accepts a wage cut. In principle, the employer can terminate the employment contract and offer a new contract with lower pay. However, this procedure may be risky.

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2 This section draws upon the country chapters in Blanpain (1994), Holden (1994), Malcomson (1997), and private communications with Stein Evju.
In some countries, courts may interpret a job offer by the firm at a lower pay as evidence that the initial dismissal was unwarranted, unless the wage reduction could be justified by the economic situation of the firm. The feasibility of this approach depends crucially on the employment protection legislation in the country. In countries with weak employment protection legislation, like the UK, enforcing a cut in nominal wages is likely to be a lot easier than in countries like Germany, Italy and Sweden.

For workers covered by collective agreements, ie the large majority of employees in Western Europe (in 1994, bargaining coverage in Western European countries was with considerable uncertainty assessed to be in the interval 70 - 98 per cent, with two exceptions, UK 47 per cent and Switzerland 50 per cent, OECD, 1997, table 3.3), additional issues arise. Collective agreements are usually of finite duration. However, unless a work stoppage has been initiated, it is in most countries a well established practice that production continues under the terms of the old agreement until a new agreement is reached, even after the old agreement has expired (holdout), cf. Cramton and Tracy (1992) and Holden (1994). This implies that the union must agree to a change in the terms of the agreement, ie. the employer may not lawfully unilaterally change the terms of the agreement, even after the agreement has expired.

Again, the employer has a variety of measures that can be used to persuade or threaten unions/workers to accept a nominal wage cut. Workers can be laid off temporarily or permanently, possibly in connection with a plant closure, or the firm can use lock-out. If such threats are credible, workers may voluntarily accept a wage cut. However, as will be argued below, it may often be unprofitable for the firm to actually implement the threat, in which case the threat will not be credible.
The employer can also unilaterally terminate the collective agreement, following specific legal procedures and after some time delay. However, termination of the agreement is risky, as the agreement also regulates work. Furthermore, in many countries the terms of the agreement are in this event considered to be included in the individual employment contracts. Thus, consent by the employees is still required, as discussed above.

Usually, the remuneration also consists of more "flexible" parts, like bonus schemes and fringe benefits that give the employer some scope for reducing remuneration even within the existing contract. I address this issue in the formal model. For now, observe that contractual and labour regulations may also limit the flexibility associated with other types of remuneration than fixed pay. Lebow et al (2000) show that US firms are able to circumvent some, but not all the wage rigidity by varying benefits.

In the US, there are much less restrictions on employers’ possibility of unilaterally cutting the wage. For individual workers, the basic presumption is that employment is at will, implying that either party may terminate the employment relationship for any reason, or for no reason at all. Furthermore, if the employer announces a wage cut, the employee's continuance in service is considered to constitute acceptance, in contrast to the situation in Europe (Malcomson, 1997). However, contracts and specific circumstances may prevent employment-at-will, making it more difficult for the employer to cut wages.

In the formal model, I consider a collective bargaining framework and assume that the employer’s measure to enforce a nominal wage cut is the use of lock-out threats,
while threats of dismissing workers are neglected. Presumably, these threats work in similar fashions.

3 The model

We consider a standard monopolistic competition, macro framework. The economy consists of a large number K symmetric firms, each producing a different good (alternatively, firms may be thought of as industries, each consisting of several firms that produce an identical product under Bertrand competition). There is one union in each firm, with 1/K members, where total labour supply in the economy is normalized to unity. The wage level is set at firm level, in a bargain between union and firm. In each firm j, there is an old nominal wage contract $W_{j,t-1}$, where subscript $-1$ indicates the previous contract period.

The assumption that the old contract is in nominal terms is consistent with the large empirical prevalence of nominal contracts (see Gottfries, 1992, for a possible explanation). Note also that this does not rule out indexation at given dates during the contract period. For example, at the expiration of a two-year contract, the wage is given in nominal terms even if there has been indexation after one year of the contract.

The model considers one contract period, which is divided into three stages.

Stage 1. The central bank CB sets the total money stock $M > 0$.

Stage 2. In each firm, union and the firm bargain over the real wage.

Stage 3. Each firm sets price and employment level.
All agents are fully aware of how the economy works, so they can predict what other agents will do at the same and later stages of the model. The assumption that money is set before wages implies that the credibility of monetary policy is not an issue. Each wage setter has a negligible effect on aggregate variables and treat them as exogenous, so there is no strategic interaction between wage setters and the CB.

Each firm $j$ has a constant returns to scale production function $Y_j = N_j$, where $Y_j$ is output and $N_j$ is employment. The real profits of the firm are

\begin{equation}
\pi_j = \frac{(P_j Y_j - W_j N_j)}{P},
\end{equation}

where $P_j$ is the price of output, $W_j$ is the nominal wage in firm $j$, and

\begin{equation}
P = \left(\frac{1}{K} \sum_j P_j^{1-\eta} \right)^{\frac{1}{1-\eta}} \quad \eta > 1,
\end{equation}

is the aggregate price level. The demand function facing each firm is

\begin{equation}
Y_j = \left(\frac{P_j}{P}\right)^{\eta} D/K, \quad \text{where } D = \Theta M/P. \quad (3)
\end{equation}

The utility of the union in firm $j$ is assumed to be a function of the real wage level $W_j/P$ and the employment level of the firm:

\[^3\text{As is well known, (3) can be derived in an optimising framework with CES utility functions defined over consumption and holdings of real money stock, where } \eta \text{ and } \Theta \text{ are parameters in the utility function.}\]
(4) \[ U_j = \frac{(W_j}{P} - \beta_j R) N_j. \]

R is the average real income of all workers in the economy,

(5) \[ R = R(u, \frac{W}{P}) \equiv (1 - u) \frac{W}{P} + uB, \]

where \( u = 1 - N \) is the aggregate rate of unemployment, \( N = \sum_j N_j \) is aggregate employment,

(6) \[ W = \left( \frac{1}{K} \sum_j W_j^{\eta - 1} \right)^{1/(\eta - 1)} \]

is the aggregate nominal wage level and \( B > 0 \) is the payoff for the unemployed.

(Existence of equilibrium requires that \( B \) is not too large, otherwise unemployment will exceed 100 percent.) The parameter \( \beta_j > 0 \) captures the bargaining qualifications of the union in firm \( j \) relative to the average income in the economy; a higher \( \beta \) indicates higher qualifications. \( \beta_j \) is assumed to take on two different values, \( \beta^U \) and \( \beta^D \), where \( \beta^U > \beta^D \).

The motivation for including \( \beta \) is to generate changes in relative wages in a convenient way, with simple algebra. The same qualitative results could be derived with more general utility functions.

Equilibrium in this model is a situation where, for given values of \( M, \Theta \) and \( W_{j,-1} \), there is Nash equilibrium in prices in stage 3, and wages are given by a subgame perfect equilibrium (SPE) in the bargaining model in stage 2. To find the equilibrium, we start by analyzing stage 3. The first order condition of the profit maximization problem is
\begin{equation}
P_j = \nu W_j, \quad \text{where } \nu = \eta/(\eta-1) > 1. \tag{7}
\end{equation}

As profits are concave in $P_j$, the first-order condition (7) is sufficient to ensure a unique maximum, constituting Nash equilibrium in the price setting game. From (3) and (7) we obtain the labour demand

\begin{equation}
N_j = (\nu W_j/P)^\eta D/K \tag{8}
\end{equation}

The indirect payoff functions of the union and the firm, as functions of $W_j/P$, $D$ and $R$, can be found by substituting out from (7) and (8) in (1) and (4)

\begin{equation}
\pi_j = \pi(W_j/P, D) = (\nu-1)(W_j/P)^{1-\eta\nu} D/K, \tag{9}
\end{equation}

\begin{equation}
U_j = U(W_j/P, \beta_j R, D) = (W_j/P-\beta_j R) (W_j/P)^{\eta\nu} D/K. \tag{10}
\end{equation}

\section{The wage setting}

The standard approach in models of union bargaining, eg Layard, Nickell and Jackman (1991) is to employ the Nash bargaining solution, where the disagreement points are specified as players' payoffs during a strike. In this setting the wage of the old nominal contract is irrelevant. Under the interpretation of Binmore, Rubinstein and Wolinsky (1986) of the Nash bargaining solution as the limit case of the Rubinstein (1982) alternating offers bargaining game, this involves the implicit assumption that a strike is an automatic consequence of there being a delay in reaching an agreement in the bargaining. However, this assumption is in sharp contrast with reality, as in many countries it is frequently the case that the old contract expires before a new agreement is
reached, and without a strike being initiated. In this case production continues under the terms of the old contract while the parties are bargaining (holdout). To explore the effect of the old wage contract, we need a model that allows for holdout as well as strike and lockout. I adopt an extension of the Rubinstein (1982) model similar to Holden (1994,1999) (see Figure 1).

There are two bargaining rounds preceding the Rubinstein alternating offers bargaining game starting in round 3, which decides the payoffs from round 3 on. In round 1a, the firm makes an offer, which the union may accept or reject (1b). Acceptance ends the bargaining, while upon a rejection, the union choose whether it wants to strike (1c). If it chooses to strike, the game proceeds directly to the alternating offers game in round 3. If the union does not strike, the game proceeds to round 2a, where the union makes an offer, which the firm may accept or reject (2b). Acceptance ends the bargaining; after a rejection, it is the firms' turn to choose whether to initiate a lock-out (2c). Then the game proceeds to round 3. If a work stoppage (ie strike or lock-out) has been initiated, both parties receive (for simplicity) zero payoffs from round 3 on until an agreement is reached. If no work stoppage has been initiated, production continues under the terms of the old contract while the parties are bargaining (holdout). As a convention, players do not initiate a work stoppage if they can get the same payoff under the existing contract.

If a work stoppage has occurred, the payoffs of the parties when normal production is resumed are assumed to be $\lambda^F \pi(W_j/P, D)$ and $\lambda^U U(W_j/P, R, D)$, where $0 < \lambda^F, \lambda^U < 1$. Thus, a work stoppage involves non-negligible costs to the parties, irrespective of how soon an agreement is reached after a work stoppage has been initiated (fixed costs; Holden, 1994). While not modelled explicitly, these costs may arise because there
is a time delay before production is resumed, or because the occurrence of a work stoppage may have an adverse effect on the reputation of the firm, thus reducing productivity/profitability in the future. Furthermore, if the model is extended to allow for risk aversion and uncertainty as to the payoffs during a conflict, so that initiating a work stoppage involves a non-negligible probability of a lengthy conflict, and the wage outcome is uncertain, the fixed costs may be interpreted as the amount that the parties are willing to give up so as to avoid risk (a similar argument is formalised in Holden, 1999).

In equilibrium, the agreement will be reached in round 1 or 2, and there will be no costly dispute. However, to find the SPE outcome, we must analyse the game backwards. As of round 3, we have the Rubinstein alternating offers bargaining game. Binmore, Rubinstein and Wolinsky (1986) show that in the limit when the time delay between offers converges to zero, the outcome is given by the Nash bargaining solution (assuming for simplicity that the players have equal discount factors). If one of the parties has initiated a work stoppage, the outcome of the wage bargaining is thus given by

\[
\frac{W_j}{P} = \arg \max \lambda^F \pi \left( \frac{W_j}{P}, D \right) \lambda^U U \left( \frac{W_j}{P}, \beta_j R, D \right)
\]

Substituting out for (9) and (10), the first order condition can be solved for

\[
\frac{W_j}{P} = \omega^\nu (\beta_j R) = \frac{2(\eta - 1)}{2(\eta - 1)} \beta_j R
\]
However, if none of the parties has initiated a work stoppage, there will be a holdout in round 3. During a holdout, parties are bound to observe the details of the old contract. However, the contract is rarely so specific that it covers all aspects that determine the payoffs of the parties. Workers may inflict a cost on the firm by a variety of different industrial actions (see eg Blanpain, 1994), for example by strictly adhering to the working rules (work-to-rule) so as to reduce efficiency and profits. On the other hand, remuneration to the workers may also consist of some elements that are at the discretion of management, so that management may reduce the effective remuneration of the workers even under the existing contract. Again, I will choose a very simple specification. Let the payoffs of the parties during a holdout be \((1-\phi_j)\pi(W_{j,-1}/P, D)\) and \((1-\varepsilon_j)U(W_{j,-1}/P, \beta_j R, D)\), where \(\phi_j\) and \(\varepsilon_j\) are parameters satisfying \(0 < \phi_j, \varepsilon_j < 1\), reflecting that a holdout is costly to both parties. The real wage outcome of a wage negotiation where holdout threats prevail in the bargaining is given by.

\[
W_j/P = \arg \max [\pi(W_j/P, D) - (1-\phi_j)\pi(W_{j,-1}/P, D)]
\]

\[
[U(W_j/P, \beta_j R, D) - (1-\varepsilon_j)U(W_{j,-1}/P, \beta_j R, D)]
\]

For analytical tractability, I use linear approximations to the true payoff functions, and the outcome of the wage setting under holdout threats is then on the form (cf appendix).

\[
W_j/P = (1+\kappa_j)W_{j,-1}/P, \quad \text{where } \kappa_j = (\phi_j - \varepsilon_j)/2.
\]
(14) allows for a simple interpretation: A holdout will lead to higher money wages ($\kappa_j > 1$) if and only if a holdout is more costly to the firm than to the union, ie. $\phi_j > \epsilon_j$. The common assumption is that a holdout is more costly to the firm than to the union (Moene, 1988, Holden, 1989, 1997, and Cramton and Tracy, 1992). Note that it is straightforward to show that the qualitative results would hold even without using the linear approximation, but the simple and easily interpretable form of (14) would be lost.

Consider now the choice of the parties whether to initiate a work stoppage in round 1 or 2. Clearly, no party will initiate a work stoppage, leading to a costly dispute, if he/she can obtain higher payoff by renegotiation under a holdout. To formalise this intuition, let $\omega^L$ and $\omega^S$ be two critical values for the real wage implied by a holdout, defined by the following equations

\[
\begin{align*}
(15) & \quad \pi(\omega, D) = \lambda^F \pi(\omega^N, D) \\
(16) & \quad U(\omega^S, \beta_j R, D) = \lambda^U U(\omega^N, \beta_j R, D)
\end{align*}
\]

If $(1+\kappa_j)W_{j-1}/P \leq \omega^L$, the firm will prefer holdout leading to a new agreement on $(1+\kappa_j)W_{j-1}/P$ to a work stoppage. Likewise, if $(1+\kappa_j)W_{j-1}/P \geq \omega^S$, the union prefers a holdout to a work stoppage. Substituting out for (9), (10) and (12) in (15) and (16) shows that $\omega^L$ and $\omega^S$ are functions of $\beta_j R$: $\omega^L(\beta_j R)$ and $\omega^S(\beta_j R)$. From the fact that $\partial \pi / \partial (W_j / P) < 0$, $\partial U / \partial (W_j / P) > 0$ and $\lambda^U, \lambda^F < 1$, it is immediate that $\omega^S(\beta_j R) < \omega^N(\beta_j R) < \omega^L(\beta_j R)$ for all $\beta_j R$. The intuition is that to avoid the costs of a work stoppage, the firm will accept a higher wage, and the union a lower wage, than the wage that would obtain if an agreement were reached after a work stoppage.
Proposition 1

There exists a unique SPE outcome \( \omega_j^*(W_{j,-1}/P, \beta_j R) \) to the wage bargaining in firm \( j \), given by

(i) If \((1+\kappa_j)W_{j,-1}/P < \omega^S(\beta_j R)\), \( \omega_j^* = \omega^S(\beta_j R) \) (the union threatens to strike)

(ii) If \((1+\kappa_j)W_{j,-1}/P \in [\omega^S(\beta_j R), \omega^L(\beta_j R)]\), \( \omega_j^* = (1+\kappa_j)W_{j,-1}/P \) (holdout threats prevail)

(iii) If \((1+\kappa_j)W_{j,-1}/P > \omega^L(\beta_j R)\), \( \omega_j^* = \omega^L(\beta_j R) \) (the firm threatens lock-out)

Thus, if \((1+\kappa_j)W_{j,-1}/P\) is within the interval \([\omega^S, \omega^L]\), no player can credible threaten to stop work. However, the players may inflict a cost at the opponent while observing the old contract, and the outcome of the bargaining in this situation is an agreement on the new real wage \((1+\kappa_j)W_{j,-1}/P\) (case (ii)). But if the old wage is outside this interval (cases (i) and (iii)), the player who is disadvantaged by the old contract would obtain higher payoff by initiating a work stoppage than he would get from a holdout, thus threats to initiate a work stoppage are credible. The opponent will then concede to a new wage agreement that gives the threatening player the payoff that he would have gotten if a work stoppage took place. In equilibrium, the threats will not be carried out.

In effect, the player who wants to renegotiate the contract by use of work stoppage threats has a weaker bargaining position than the opponent. To raise the wage above the outcome from a holdout, the union must threaten to call a costly strike, and the costs associated with calling a strike weaken the potency of this threat. Correspondingly, the costs that the firm incurs by initiating a lock-out weaken the potency of lock-out threats.
It seems plausible that $\kappa_j$ varies among firms and industries. In firms where neither party has much scope in inflicting a cost to the opponent under the contract, $\kappa_j$ can be set to zero, and a holdout would lead to constant nominal wages, consistent with the spike at zero nominal wage change often observed in empirical studies (see references in section 7 below). However, if $\kappa_j > 0$, nominal wages will increase even when nominal wage rigidity is binding. This is consistent with evidence suggesting a floor on nominal wage growth at the central wage negotiations in the manufacturing sectors in the Nordic countries (Holden, 1998).

Note that if there are legal or other restrictions on the firms’ use of a lock-out (as in several European countries), this will weaken the employers’ lock-out threats so that $\omega^L$ would increase and the interval widen (formally, this can be captured by reducing $\lambda^F$ in (15), or by assuming that the parties’ payoffs during a work stoppage are relatively less favourable to the firm during a lock-out; both modifications leading to an increase in $\omega^L$).

As the old contract affects the bargaining outcome, the parties should ideally take into consideration that the bargaining outcome will affect future wage negotiations. This is neglected in the present model. However, in Holden (1997), I analyse an infinite-horizon version of a similar model, where agents take into consideration how the bargaining outcome in one period affects subsequent negotiations. There it is shown that this feature does not affect the qualitative results, only dampens the magnitudes.

5 Equilibrium

We now turn to the equilibrium of the whole economy, cf. the following Proposition.
**Proposition 2**

For given values of \( M, \Theta \) and \( W_{j-1}, j = 1, \ldots K \), there exists a unique equilibrium to the economy, where the outcome of the wage setting is given by Proposition 1, and the price setting is given by (7).

Then turn to the characteristics of the equilibrium. For sake of comparison, let us first consider equilibrium in the simpler case where all firms are identical \( (\beta_j = 1 \text{ for all } j) \), and there are no fixed costs of initiating a work stoppage \( (\lambda^F = \lambda^U = 1) \); the bargaining outcome is given by the Nash bargaining solution (12) \( \omega^N(R) \) irrespective of the wage of the old contract. This case corresponds directly to a simplified version of the standard model of eg. Layard, Nickell and Jackman (1991). There is a unique equilibrium level of unemployment, \( u^N \), given by the intersection of the price curve \( W/P = 1/\nu \) (given from (7), aggregating over firms) and the wage curve \( \omega^N(R) \) (cf Figure 2), ie \( u^N \) is given by \( 1/\nu = \omega^N(R) \) where \( R = (1-u^N)1/\nu + u^Nb \) from (5). The intuition is that if unemployment is higher than \( u^N \), unions will be too weak to obtain the real wage implied by the price setting; if unemployment is below \( u^N \), unions will be so strong that they obtain a too high real wage. In this equilibrium, the money stock only determines nominal variables.

Then consider the more general model, including fixed costs of initiating a work stoppage and allowing for differences in \( \beta \). Consider first an equilibrium where strike threats are used in all firms. The equilibrium requirement that price setting is consistent with wage setting is thus (using (6) and Proposition 1, and simplifying the RHS)
The RHS of (17) is strictly decreasing in $u$, thus (17) determines a unique rate of unemployment $u^S$. Likewise, in the lock-out case, a unique equilibrium rate of unemployment $u^L$ is given by

\[
\frac{1}{v} = \frac{W}{P} = \left( \frac{1}{K} \sum_j \left( \omega^S (\beta_j R)^{1-\eta} \right)^{1-\eta} \right)^{1/(1-\eta)}
\]

$u^L > u^S$ follows from the fact that $\omega^L > \omega^S$. The situation is illustrated in Figure 2 (wage dispersion among firms is neglected in the Figure).\(^4\) The price setting requires an outcome along the horizontal line, and the wage setting requires an outcome between the curves $\omega^L$ and $\omega^S$. As will be shown below, it turns out that all unemployment rates in the range between $u^L$ and $u^S$ are consistent with equilibrium.\(^5\) Note also that the difference between $u^L$ and $u^S$ is larger, the larger the difference between $\omega^L$ and $\omega^S$.

To explore the model further, it is necessary to be more specific about productivity growth and relative wages. Let $\alpha$ denote the growth on labour productivity from the previous contract period. Given a condition like (7) in the previous contract period, the real wage then was $(W/P)_1 = 1/(1+\alpha v)$. Regarding relative wages, it is

\[^4\] Figure 2 may be unfamiliar to some readers; see Blanchard (2000), page 119 for a textbook treatment of the standard case with a unique equilibrium.

\[^5\] A similar range is also derived in Holden (1994) and (1997). McDonald (1995) surveys other theories of a range of equilibria.
shown in the appendix that the relative wage, denoted $1 + \mu > 1$, is the same irrespective of whether strike or lock-out threats prevail in the wage bargaining, i.e. that $\omega^S(\beta^UR)/\omega^S(\beta^DR) = 1 + \mu$. I assume that the same relative wages applied in the previous contract period, thus ruling out many tedious sub cases but not affecting the qualitative results. For simplicity, I also set $\kappa_j = \kappa$. We have the following Proposition.

**Proposition 3**

There is a trade-off between unemployment and inflation:

There exist critical values $M^S$ and $M^L$, and corresponding inflation rates $z^S$ and $z^L$, where $M^S > M^L$, and $z^S > z^L$, such that

(i) \quad If $\theta M > M^S$, strike threats prevail in the wage bargaining in all firms, inflation $P/P_{-1} - 1 > z^S$, and the rate of unemployment, $u = u^S$.

(ii) \quad If $\theta M \in [M^L, M^S]$, holdout threats prevail in at least some firms, inflation $P/P_{-1} - 1 \in [z^L, z^S]$, and the rate of unemployment $u \in [u^S, u^L]$.

(iii) \quad If $\theta M < M^L$, lock-out threats prevail in the wage bargaining in all firms, inflation $P/P_{-1} - 1 < z^L$, and the rate of unemployment, $u = u^L$.

The intuition of case (i) is as follows. To ensure the low unemployment equilibrium, $u = u^S$, strike threats must prevail in the wage setting in all firms. As all unions can obtain a nominal wage $(1+\kappa)W_{j,-1}$ by a holdout, strike threats must give at least this wage. In particular, this must be the case for unions that face a reduction in their relative wage by
\((1+\mu),\) ie average nominal wage growth must be at least \((1+\kappa)(1+\mu) - 1.\) Deducting productivity growth, inflation must at least be \(z^S = (1+\kappa)(1+\mu)/(1+\alpha) - 1.\)

Likewise, for \(u = u^L\) and lock-out threats prevailing in all firms, lock-out threats must not give a higher wage than \((1+\kappa)W_{j,-1}\), even in a firm where the relative wage increases by \((1+\mu)\). Thus, the average nominal wage growth cannot be higher than \((1+\kappa)/(1+\mu)\). Deducting productivity growth, inflation cannot be higher than \(z^L = (1+\kappa)/[(1+\mu)(1+\alpha)] - 1.\)

The monetary policy determines which regime prevails. If money growth is sufficiently high to involve inflation greater than \(z^S\), (which is equivalent to \(\Theta M > M^S\)), the economy will be in the “strike” regime. If money growth is so low that inflation is below \(z^L\), \(\Theta M < M^L\), a “lock-out” regime prevails. The critical values for the money growth rates depend on the situation in the previous contract period, because possible changes in the rate of unemployment implies that the real money stock may also change, so that inflation may deviate from the money growth rate in the short run.

The upshot is a Phillips curve as displayed as figure 3 (note that the relationship between inflation and employment is not necessarily smooth and symmetric like figure 3). Inflation above a critical rate \(z^S\) is associated with low unemployment, inflation below a critical rate \(z^L\) is associated with high unemployment, and inflation at intermediate rates are associated with intermediate levels of unemployment. In figure 3, \(z^S\) is assumed positive and \(z^L\) negative, but this depends on the size of the parameters \(\kappa\) (the money wage growth under holdout threats), \(\mu\) (the change in relative wages) and \(\alpha\) (the rate of productivity growth). Thus, whether these critical values are positive or negative is an empirical question, which is likely to vary among countries and over time. For example,
in a low inflation era, firms have an incentive to choose a more extensive use of flexible
types of remuneration, which may increase \( \varepsilon \), thus reducing \( \kappa, z^S \) and \( z^L \).

Proposition 3 suggests an important difference between the US and most
European countries. In the US, where cutting nominal wages is relatively easy, the \( \omega^L \)
and \( \omega^S \) curves are close, and the range for possible equilibrium rates of unemployment,
\( u^L - u^S \), is probably small. In this case aiming at low inflation will involve only a small
increase in unemployment. In many European countries where unilateral nominal wages
cuts are more difficult to enforce, bargaining coverage is high, and employment
protection legislation strong, the range is likely to be larger, and consequently low
inflation will involve a greater increase in unemployment.

6 Monetary policy

The existence of a long run tradeoff between inflation and unemployment implies that the
monetary policy has long run effects on unemployment. In discussing this, let us also take
into consideration that in practice the central bank does not have perfect control over
aggregate nominal demand. Available evidence suggests a lag of six to nine months in
the effect of a shift in interest rates on output (Clarida, Gali and Gertler, 1999), and
monetary policy decisions are based on information about the economy that is only
available after some time delay. To capture this in a simple fashion, I assume that the
parameter \( \Theta \) in the demand function (3) is stochastic, and realized after the determination
of the money stock. However, I assume that \( \Theta \) is realized before wages are set, to avoid
the well-known issues related to shocks under wage rigidity, and thus sharpen the focus
on the novel aspect regarding the effect of the nominal wage of the previous contract. Let
\( \Theta \) be distributed over the support \([1/(1-\theta), 1- \theta]\), ( \( 0 < \theta < 1 \)), with expected value \( E\Theta = 1 \).
In this situation there is not only a long run tradeoff between inflation and unemployment, there is also a tradeoff between variability of inflation and unemployment. If the money stock is set to ensure an outcome along one of the vertical lines in figure 3, where unemployment is \( u^L \) or \( u^S \), all the variability in \( \Theta \) will induce variability in the price level, and thus in inflation. In contrast, if the money stock is set at an intermediate value, aiming at a value in the middle of the interval \((u^L, u^S)\), variability in \( \Theta \) will have less effect on the price and inflation level, at the expense of inducing variability in output and employment.

What will the CB do? Consider first the extreme case where the central bank has lexicographic preferences, with low unemployment being the first priority. In this case, the CB will set \( M = M^S(1+\theta) \), ensuring that the economy ends up in a “strike” regime with unemployment at its minimum value \( u^S \) even if the demand parameter \( \Theta \) takes its lowest possible value \( 1/(1+\theta) \). However, if \( \Theta \) in this situation takes its expected value unity, inflation is

\[
\frac{P}{P_t} - 1 = z^S (1 + \theta) \equiv \frac{(1 + \kappa)(1 + \mu)(1 + \theta)}{1 + \alpha} - 1
\]

Thus, the result of stochastic demand is higher inflation than as given by Proposition 3. If the CB aims at a lower rate of inflation, say \( z^S \), unemployment will exceed \( u^S \) any time a negative demand shock \( \Theta < 1 \) occurs. Note that in contrast to a traditional model with short run linear Phillips curve around the natural rate of unemployment, negative and
positive demand shocks will not cancel out: if a positive demand shock takes place, unemployment does nevertheless not become lower than $u^*$.\footnote{If I were to add shocks after the wage setting (as is common in models with staggering of wage contracts), this would induce additional dynamics and it would be possible to have unemployment below the minimum equilibrium rate on a temporary basis.}

In the more general case, where the CB attaches costs to both inflation and unemployment, the choice will depend on the weight given to each. Will the CB choose the optimal tradeoff between inflation and unemployment?

If the CB is assigned an inflation target, and thus must set expected inflation equal to this target, this is not an issue in this model. Expected inflation is strictly increasing in the nominal money stock, thus the money stock is uniquely determined by the inflation target. Consequently, the CB will choose the appropriate monetary policy if the society chooses the appropriate inflation target. However, if the CB is allowed to set the inflation target itself (like the European Central Bank), the situation may be different. Assume for the sake of argument that the CB has the same inherent preferences over inflation and unemployment as society in general. However, in addition the CB is concerned about its own reputation for ability, ie whether outsiders think that the CB is capable of its job. If outside observers take high inflation variability as a sign of bad monetary policy, the CB has an incentive to choose a low target for inflation, which is associated with low inflation variability and thus easier to fulfill. While this will involve more output variability, the CB will not necessarily be blamed because output and unemployment are usually reckoned to be less controllable by the monetary policy than is the rate of inflation.
7 Empirical relevance

A number of recent studies have found evidence for the existence of downward nominal wage rigidity, cf. Fehr and Goette (2000) for Switzerland, Beissinger and Knoppik (2000) for Germany, Dessy (1999) for Italy, Christofides and Leung (1999), and Fortin and Dumont (2000) for Canada, Holden (1998) for the manufacturing sectors in the Nordic countries, Agell and Lundborg (1999) for Sweden, Kimura and Ueda (1997) for Japan, and Altonji and Devereux (1999) and Lebow, Saks and Wilson (2000) for the US. An exception to this picture is Smith (2000), who finds little evidence for downward nominal wage rigidity in the UK. In general these studies find that the rate of inflation affects the distribution of wage changes, which is consistent with the model here, but not with standard models with overlapping wage contracts where the rate of inflation is irrelevant.

Regrettably different methods and data make it difficult to compare the degree of downward nominal wage rigidity across countries. However, the studies show that money wage rigidity is much stronger in Sweden and Italy than in the UK and the US, which is consistent with the explanation of the present paper, in light of the much stronger employment protection legislation and higher coverage rates of collective agreements in Sweden and Italy. Indeed, Lebow, Saks and Wilson (2000, Table 2), based on data from the US Employment Cost Index for the period 1981-1999, report that 15 percent of the observed nominal wage and salary changes are negative. In contrast, Agell and Lundborg (1999), based on survey evidence among managers in Swedish firms with a total of 187 000 employees, find that nominal wage cuts were virtually absent in the 1990s, in spite of soaring unemployment and several years with close to zero inflation.
The prediction that low inflation may lead to lower output and employment in many European countries but not in the US is consistent with evidence in Bullard and Keating (1995). Studying the long run relationship between inflation and output in 58 countries over the period 1960-90, Bullard and Keating find 16 countries that have experienced permanent shocks to both inflation and the level of output. Of these 16 countries, Bullard and Keating find a positive and significant long-run response of the level of real output to a permanent inflation shock for the four European countries with the lowest rates of inflation (Germany, Austria, Finland and the UK, neglecting Cyprus where the positive coefficient is insignificant due to a very large confidence interval). However, for the US, which incidentally also had low inflation, the permanent shock to inflation had no significant permanent effect on output (the point estimate being close to zero).

More recently, Wyplosz (2001) have found some preliminary evidence for France, Germany, the Netherlands and Switzerland that unemployment is higher for very low rates of inflation. Fehr and Goette (2000) find that wage sweeps-up caused by nominal rigidity are strongly correlated with unemployment in their study of data from the Swiss Labor Force Survey 1991-1998.

In the model presented here, there are constant returns to labour and real wages are given by (7), thus not affected by cyclical fluctuations. This is consistent with empirical evidence generally indicating that real wages are acyclical (or slightly procyclical). However, under the perhaps more realistic case that there is decreasing returns to labour, demand shocks under nominal wage rigidity along a downward sloping labour demand curve involves countercyclical real wages, and this has been raised as a
key objection against macroeconomic models with nominal wage rigidity. However, as pointed out by Spencer (1998), technology shocks may induce procyclical behaviour of real wages, and this would also be the case in the model presented here. Spencer shows that US postwar data indicates that a positive demand disturbance is associated with a temporary decline in real wages, consistent with a model with nominal wage rigidity.

The paper is also consistent with the findings of Ball and Cecchetti (1990) (and others) that inflation is more variable and less predictable when it is higher, and with the analysis of different monetary policy responses by Ball (1999), who finds that too strict monetary policy in the 1980s and 90s in some European countries has led to a long-lasting increase in unemployment.

8 Concluding remarks

Does close to zero inflation involve higher unemployment due to the existence of downward nominal wage rigidity? I address this issue in a model incorporating the institutional feature of European labour markets that nominal wages are a part of a contract, either a collective agreement or an individual employment contract, and can as such only be changed by mutual consent. I show that workers’ cet. par. have a stronger bargaining position when they try to prevent a cut in nominal wages, and the stronger bargaining position of the workers implies higher unemployment even in the long run.

The nominal rigidity emphasised here is different from the stickiness in models with staggered wage contracts. Here, nominal rigidity is the consequence of old nominal contracts affecting the outcome of contract renegotiations. Thus, the rigidity may last longer than the duration of the contract periods, even without any staggering of contracts. More importantly, the macroeconomic implications differ. Standard models with
staggering yield rigidity in the price level, a tradeoff between inflation variability and output variability (Taylor, 1979), but no long run tradeoff between inflation and unemployment. Indeed, the rate of inflation per se is irrelevant. In the present model, the nominal rigidity is crucially related to the rate of inflation, and there is a long run tradeoff between inflation and unemployment for low rates of inflation.

Some economists (eg. King, 1999) have argued that downward nominal wage rigidity is unlikely to be empirically relevant, because positive productivity growth leaves room for growth in nominal wages even at constant prices. However, as shown in the present paper, there are also several factors that work in the other direction. To allow for changes in relative wages without nominal wage cuts, average nominal wages must grow. If workers can inflict large costs on the firms by use of other types of industrial action (eg work-to-rule), nominal wages may increase even when nominal wage rigidity is binding. Furthermore, imperfect control of aggregate nominal demand implies that inflation on average must be above the minimum level necessary to avoid nominal wage rigidity, so as to avoid nominal wage rigidity if a negative demand shock takes place. Unlike cyclical fluctuations around a constant mean, the higher unemployment when nominal wage rigidity binds will not be recovered by lower unemployment in subsequent periods, because higher demand “than necessary” will only lead to higher inflation, and not lower unemployment.

The key predictions of the model are in line with existing empirical evidence. The evidence for downward nominal wage rigidity surveyed in section 7 can be explained by the present model, but not by traditional models of overlapping wage contracts. Furthermore, different labour market institutions, among other things a stronger legal
position when resisting nominal wage cuts, higher bargaining coverage and stricter employment protection legislation in many European countries than in the US, would suggest that downward nominal wage rigidity is more prevalent in Continental Europe than in the US. This is consistent with empirical evidence that downward nominal wage rigidity is much stronger in Sweden and Italy than in the UK and the US. The model suggests that the stronger downward nominal wage rigidity in Europe implies that pursuing zero inflation is more costly (in terms of higher unemployment) in Europe than in the US, which is consistent with evidence indicating that low inflation is associated with lower output in some European countries but not the US (cf references in section 7).

In practice, there are costs associated with the level and variability of both inflation and unemployment. The optimal choice of an inflation target should take all these costs into consideration. However, if the CB is allowed to set the inflation target itself (like the European Central Bank), additional issues may arise. If outside observers take high inflation variability as a sign of bad monetary policy, the CB has an incentive to choose a low target for inflation, which is associated with binding nominal rigidities and low inflation variability and thus easier to fulfil. The cost can be higher unemployment.

An additional concern in the European Monetary Union is the interplay between asymmetric shocks and downward nominal wage stickiness. In countries experiencing positive demand shocks, a tight labour market may lead to considerable inflation, while downward rigidity will dampen or prevent wage cuts in countries with a slacker labour

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7 However, I do not claim that the ECB deliberately has chosen low inflation at the cost of permanently higher unemployment. According to the mainstream view, which presumably is shared by the ECB, there are no long run unemployment costs associated with choosing a low inflation target.
market. The problem is exacerbated by the Balassa-Samuelson effect, as cross country variation in the relative productivity growth of traded vs non-traded sectors implies that inflation must differ among countries, cf Sinn and Reutter (2000).

The prediction of the paper may seem inconsistent with combination of low inflation and apparent wage moderation in many European countries since the mid 1990s, cf. Pochet and Fajertag (2000). However, the wage moderation must also be seen in light of the fact that persistent high unemployment in many countries has lead to steps towards deregulation of labour markets, as well as the conclusion of social pacts explicitly aimed at wage moderation (cf. Pochet and Fajertag, 2000).

An obvious response to my analysis is that labour laws could be changed, so as to obtain greater nominal wage flexibility. However, this response raises several issues. The legal importance of the existing nominal wage contract is a consequence of standard contract law, implying that contracts between two parties can only be renegotiated by mutual consent. MacLeod and Malcomson (1993) and Holden (1999) show that this feature may play an important role in inducing efficient levels of investment, by preventing one player from reaping the return of the investment of the other player by demanding a renegotiation of the contract. Furthermore, restrictions on the employer’s right to unilaterally cut nominal wages seem a key ingredient if employment protection legislation is to be effective; changes in labour laws are thus likely to be resisted by insiders.

These objections do not imply that nothing will change. As alluded to above, one would expect firms to change remuneration systems in a low inflation era, so as to increase nominal flexibility. In the present model this would involve a reduction in $\kappa$, ie
the nominal wage increase under holdouts, and thus reduce the inflation bias. Secondly, one would expect low inflation to lead to increased use of temporary employments contracts (Holden, 2001), a tendency that has taken place in many European countries over the last decades. However, the extensive nominal wage rigidity in Sweden and Switzerland documented by Agell and Lundborg (1999) and Fehr and Goette (2000), even after years of close to zero inflation and high unemployment, shows that rigidities may be highly persistent.
Appendix

Derivation of (14), the outcome of the wage bargaining when holdout is costly

Using linear approximations to the true payoff functions, ie.

\[ \pi(W_{j,-1}/P, D) = \pi_w W_{-1}/P \text{ and } U(W_{j,-1}/P, \beta_j R, D) = U_w W_{-1}/P, \]

the Nash bargaining solution (14) reads (omitting subscript indicating firm)

\[ \frac{W}{P} = \arg \max \left[ \left( \frac{W}{P} - \frac{W_{-1}}{P} \right) \pi_w + \phi_j \pi(W_{j,-1}/P, D) \right] \]
\[ \left[ \left( \frac{W}{P} - \frac{W_{-1}}{P} \right) U_w + \epsilon_j U(W_{j,-1}/P, \beta_j R, D) \right]. \]

The first order condition can be rearranged to

\[
\frac{W}{P} = \frac{W_{-1}}{P} + \frac{1}{2} \left( \frac{\phi_j \pi(W_{j,-1}/P, D)}{\pi_w} - \frac{\epsilon_j U(W_{j,-1}/P, \beta_j R, D)}{U_w} \right)
\]

which can be reduced to (14) (invoking the same linear approximations). QED

Proof of Proposition 1

The proof follows well-known procedures, so only a sketch is provided.

Case (i), \((1 + \kappa_j)W_{j,-1}/P < \omega^S\): The equilibrium path is that the firm offers \(\omega^S\), which is immediately accepted by the union. The union will not accept a lower wage, because it would be better to reject and initiate a strike. The union will not reject \(\omega^S\), because it cannot obtain higher payoff, as the firm will reject any demand \(W/P > \omega^S\).

Case (ii), \(\omega^S \leq (1 + \kappa_j)W_{j,-1}/P \leq \omega^L\): The equilibrium path is that any offer different from \((1 + \kappa_j)W_{j,-1}/P\) is rejected, and no player initiates a work stoppage. Any deviation will inflict a loss at the deviating player.

Case (iii), \((1 + \kappa_j)W_{j,-1}/P > \omega^L\): There are two alternative equilibrium paths, leading to the same outcome. One path is that the firm offers \(\omega^L\), which the union accepts. The other is that the firm offers less, is rejected by the union, and then the union offers \(\omega^L\) which the
firm accepts. The firm will accept \( \omega^L \) because it cannot obtain more by rejecting. The union will not accept less, as it can obtain \( \omega^L \).

QED

The relative wage is 1 - \( \mu \) irrespective of type of threats in the wage setting

This feature follows directly from the property that both \( \omega^L \) and \( \omega^S \) are linear functions of \( \beta_j R \). To show this, note that substituting out for \( \pi \) using (9), (15) can be solved for

(20) \[ \omega^L (\beta_j R) = (\lambda^F)^{1-\eta} \omega^N (\beta_j R) = k^L \beta_j R \quad \text{where} \quad k^L \equiv (\lambda^F)^{1-\eta} \frac{2\eta-1}{2(\eta-1)} \]

To verify the same property for \( \omega^S \), observe that on the assumption that \( \omega^S (\beta_j R) = k^S \beta_j R \), (16) reads (substituting out for (10)) \( (k^S \beta_j R - \beta_j R)(k^S \beta_j R)^\eta = \lambda (k^N \beta_j R - \beta_j R)(k^N \beta_j R)^\eta \). (\( k^S \) must be greater than unity to make the LHS positive.) This equality can be reduced to \( (k^S-1)(k^S)^\eta = \lambda (k^N-1)(k^N)^\eta \), which determines \( k^S \) uniquely independently of \( \beta_j R \) (in the appropriate interval), validating the assumption \( \omega^S (\beta_j R) = k^S \beta_j R \).

QED

Proof of Proposition 2

First observe that from Proposition 1, we have the nominal wage in firm \( j \) as a continuous function of \( W_{j,-1}, \beta_j R \) and \( P \):

(21) \[ W_j = W(W_{j,-1}, \beta_j R, P). \]

By substituting out recursively for (5), the definition of \( u \), and (6) – (8), we see that the nominal wages in each firm are continuous functions of the nominal wages in each firm.

(22) \[ W_j = h(W_1, \ldots, W_K; \beta_j, M) \quad j = i, \ldots, K. \]

For a given \( M \), \( W_j \) is clearly bounded above, so we can restrict attention to values of \( W_j \) in the interval \([0, z]\), where \( z \) is an arbitrary and very large number. Let \( Z \) be the
associated K-dimensional set \([0, z] \times \ldots \times [0, z]\) (which is compact and convex), and \(W = (W_1, \ldots, W_K)\), be an K-dimensional vector. (22) is then equivalent to

\[
(23) \quad W = H(W; \beta, M),
\]

where each component of \(H\) is equal to \(h\) (and thus continuous) and \(H\) is a mapping from \(Z\) into \(Z\). We can invoke Brouwer’s fix point theorem, which ensures that there exists a fix point \(W^*\) so that \(W^* = H(W^*; \beta, M)\), which constitutes an equilibrium in the model.

Then turn to uniqueness. Denote the equilibrium \((W_1^*, \ldots, W_K^*)\). Suppose that there is another equilibrium \((W_1', \ldots, W_K')\), and let the associated equilibrium levels of the real money stock be \((M/P)^*\) and \((M/P)'\). It is immediate that \((W_1', \ldots, W_K')\) cannot involve the same relative wages as \((W_1^*, \ldots, W_K^*)\) (i.e. that \(W_j' = gW_j^*\) for all \(j\), where \(g\) is a constant different from unity). To see this, let \(g > 1\), so that \(P' > P^*\) and \((M/P)' < (M/P)^*\). From (3), this implies that employment is lower in all firm, implying that aggregate unemployment is higher, which is inconsistent with \(W_j' > W_j^*\) (under holdout threats, the nominal wage would have been the same; under strike or lock-out threats, the real wage would have been lower).

Then suppose that \((W_1', \ldots, W_K')\) involves different relative wages than \((W_1^*, \ldots, W_K^*)\). As the average real wage is the same, due to the price setting, it follows that for at least one firm \(j\), \((W_j/P)' > (W_j/P)^*\), and for at least one firm \(i\), \((W_i/P)' < (W_i/P)^*\). However, this requires that \(R' > R^*\), and \(R' < R^*\), which is clearly inconsistent.

**Proof of Proposition 3**

To the equilibrium levels of unemployment \(u^S\) and \(u^L\) there are associated unique equilibrium values \((M/P)^S\) and \((M/P)^L\), with associated values \(M^S\) and \(M^L\) (recall that the equilibrium price level is uniquely given, cf Prop. 2). As nominal wages are increasing in the money stock in the strike case, it is clear that if strike threats are used in all firms for \(\theta M = M^S\), then it is also used for all \(\Theta M > M^S\). And correspondingly, lock-out threats are used for all \(\Theta M < M^L\).

(i) If strike threats is to prevail in all firms, we know from Proposition 1 that we must have \(W_j > (1 + \kappa)W_{j-1}\). In particular, this must hold in a firm where there has been a
negative shift in the wage aspirations of the workers, from $\beta^U$ last period to $\beta^D$ in the current period. This requires that (let $\omega^i_r = \omega^i(\beta^i R)$, $i = S, L; r = U, D$)

$$\omega^{SD} P > (1 + \kappa) \omega^{SU} P_{-1} = (1 + \kappa) \omega^{SU} / (P_{-1} (1 + \alpha)),$$

where the latter equality is due to the increase in real wages due to productivity growth. Rearranging (24) leads to

$$\frac{P}{P_{-1}} > (1 + \kappa) \left( \frac{\omega^{SU}}{\omega^{SD}} \right) \frac{1}{1 + \alpha} = \frac{1 + \kappa}{1 + \mu} = 1 + z^S.$$

Correspondingly, in the lock-out case (ii), we know from Proposition 1 that we must have $W_j < (1 + \kappa) W_{j-1}$. In particular, this must hold in a firm where there has been a negative shift in the wage aspirations, from $\beta^D$ last period to $\beta^U$ in the current period. This requires that

$$\omega^{LU} P < (1 + \kappa) \omega^{LD} P_{-1} = (1 + \kappa) \omega^{LD} / (P_{-1} (1 + \alpha)).$$

Rearranging (26) leads to

$$\frac{P}{P_{-1}} < (1 + \kappa) \left( \frac{\omega^{LD}}{\omega^{LU}} \right) \frac{1}{1 + \alpha} = \frac{1 + \kappa}{(1 + \mu)(1 + \alpha)} = 1 + z^L. \quad \text{QED}$$
References:


Fehr, E. and L. Goette (2000). Robustness and real consequences of nominal wage rigidity. Institute for Empirical Research in Economics, University of Zurich, WP 44.


**Figure 1.** The wage bargaining

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Figure 2. With no fixed costs of a work stoppage, $\lambda^F = \lambda^U = 1$, the unique outcome to the wage setting is $w^N(1-u)$, and the unique equilibrium rate of unemployment $u^N$. $w^H(1-u)$ ($H = N, S, L$) is defined by imposing symmetric wages, ie $W/P = \omega^H((1-u)(W/P) + ub)$ and solving for $W/P$ as function of $(1-u)$ and $b$ ($b$ is constant, and is suppressed in the notation). With fixed costs, all unemployment levels in the range $[u^L, u^S]$ are consistent with equilibrium.

Figure 3. The long run trade-off between employment and inflation under productivity growth, changes in relative wages, and incomplete labour contracts.