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Cost-effective Abatement of Ground-level Ozone in Cities and for Larger Regions: Implications of Non-monotonicity

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Cost-effective Abatement of Ground-level Ozone in Cities and for Larger Regions: Implications of Non-monotonicity

by

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Abstract: Ground-level ozone concentrations have adverse effects e.g. on human health and crops. Ozone is not emitted, but atmospheric reactions involving nitrogen oxides (NOx) and volatile organic compounds (VOCs) cause its formation. However, functions that relate precursor emissions to ozone concentrations are typically neither *convex* nor *monotonic*. Others have shown that corner solutions, where only one emission type is abated, can be cost-effective since the ozone-formation is *non-convex*. This paper shows that the *non-monotonicity* implies an even more radical abatement strategy in some cases: the optimal amount of NOx emitted can be larger than the amount emitted in no-control.

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Key words: Ground-level Ozone, Cost-effective Abatement, Non-monotonicity, Transboundary Air Pollution.

JEL classification: Q25, H77

1. Introduction

Stratospheric ozone protects the earth against dangerous ultraviolet radiation from the sun. Ground-level ozone, however, is the main component in smog, and it has adverse effects on human health, animals, crops, trees and building materials (Harrison, 1996; Heyes and Schüpp, 1995). Ozone is not emitted, but atmospheric reactions involving nitrogen oxides (NOx) and volatile organic compounds (VOCs) cause its formation. However, for a given amount of VOCs, increased NOx concentrations have a declining marginal effect on ozone concentrations. This implies that functions that relate the amounts of precursor emissions, of NOx and VOCs, to ozone concentrations are *non-convex*.[1]

The practical relevance of different types of non-convexities are discussed in Burrows (1986), while the implications of non-convexities on cost-effective abatement policies are discussed in Repetto (1987). Repetto points out that ozone formation is non-convex in New York, and that contour curves for ozone are convex and almost parallel to the isocosts in the optimal solution. Consequently, many emission vectors are almost optimal and "corner solutions", where only one emission type is abated, are potentially cost-effective.

This paper shows that the cost-effective abatement strategy can be even more radical in some cases. The reason is that the ozone-function not only is non-convex, it is also *non-monotonic* since increased NOx emissions give reduced ozone concentrations in some cases.[2] If the optimal solution also have this local property, then the optimal amount of NOx emitted is larger than amount emitted in no-control, implying negative abatement. The purpose of this paper is to

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[1] Basically, a function *f(x)* is convex if, for any two points x', x'' in the domain, the condition $\theta f(x') + (1-\theta) f(x'') \geq f(\theta x' + (1-\theta) x'')$ is satisfied for all $0 < \theta < 1$. See for instance Sydsaeter and Hammond (1995) for an exact definition.

[2] If the value for the function *f(x)* always is increased when the value of *x* is increased, then *f(x)* is a monotonic (or monotonically increasing) function. See for instance Chiang (1984).

illustrate these implications of non-monotonicity formally rather than to formulate actual regulatory strategies.

In USA there has been severe problems with smog in Metropolitan areas, for instance in Los Angeles and New York. Originally ozone was thought of as a local pollutant, and controls focused on reducing hydrocarbon emissions (Farrel *et al.*, 1999), which is a type of VOC. However, ozone is the substance for which national ambient air quality standards (NAAQS), established in title I of the 1990 Clean Air Act Amendments (CAAA), is most widely violated (Repetto, 1987), and the NAAQS for ozone were violated within 27 states in 2001 (EPA, 2001). In such states, certain emission reducing technologies for stationary sources of NOx and VOCs are required by title I. In addition, VOC emissions must be reduced by 15%, compared to 1990 levels, within six years (Heninger and Shah, 1998). Some of the regulations within title I also address the long-range transport of ozone and precursors. Krolewski and Mingst (2000) provides an overview of NOx regulations in USA, while Henderson (1995) investigates the effects of the non-attainment status on air quality.

In addition to the technological controls on NOx in USA, emissions have been traded under the NOx budget (title I) among 12 states during the summer season for the last two years, while the Acid Rain Program under title IV in CAAA allows emission averaging. Emission averaging means that NOx emissions per unit electricity produced at a plant can be larger than required by the standard, as long as the average emission-factor over all plants owned by the company doesn't violate the standard.

The classic Los Angeles type of urban smog is not experienced in Europe (Harrison, 1996). Still, for 1990 emissions, the WHO health guideline criteria for ozone exposure will typically be exceeded some days over the year in many European Countries (Amann *et al.*, 1998b). The European policy towards ground-level ozone has followed a dual track of (a) national emission ceilings for NOx and VOCs, without any trade between or within[3] countries, and (b) legislation on technological standards. The obligations are signed upon in protocols within the UN/ECE Convention on Long-Range Transboundary Air Pollution (LRTAP), see UN/ECE (1996, 1999a). The parties within LRTAP are well aware of the long-range transport of ozone and precursors, and the RAINS model is used to calculate cost-effective abatement policies for all Europe. See Alcamo and Hordijk (1990) for a documentation of the RAINS model (without ozone module), and Amann *et al.* (1999) for recent scenarios prepared for the convention. However, RAINS cost curves are mainly based on technological abatement strategies, and only for positive abatement. The type of effects considered in this paper is therefore ruled out in RAINS simulations. See Cofala and Syri (1998) for a documentation of NOx abatement cost curves used in the RAINS model, and Klimont *et al.* (2000) for a documentation of VOC cost curves.

The rest of this paper is organised as follows. Section 2 gives a brief overview of ground-level ozone formation, while a city model for cost-effective abatement policies, consistent with the early approach towards ground-level ozone in USA, is developed in section 3. A regional model that accounts for long-range transport of ozone and precursors is developed in section 4. Different types of cost-effective solutions are identified for both models, and implications of non-monotonicity are illustrated. Section 5 concludes the analysis.

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[3] Klaassen and Nentjes (1997) did not find any evidence for trades where permissions to emit were transferred from one party to another in exchange for money within European countries. However, many European countries have used market instruments to reduce emissions (UN/ECE, 1999b).

2. Ground-level Ozone Formation

Ozone is not emitted, but atmospheric reactions involving NOx and VOCs cause its formation. The following illustration of ozone formation is based on Cleveland and Graedel (1979) and Heyes and Schöpp (1995).

Sunlight (*v*) split nitrogen dioxides (NO_2) into nitrogen monoxides (NO) and a single oxygen atom (O)

$$NO_2 + hv \rightarrow NO + O\,, \tag{1.1}$$

where *h* is Planck's constant. The single oxygen atom (O) reacts with a oxygen molecule (O_2) and produces ozone (O_3)

$$O + O_2 \rightarrow O_3\,. \tag{1.2}$$

However, ozone reacts with nitrogen monoxides so that ozone is removed again

$$O_3 + NO \rightarrow O_2 + NO_2\,, \tag{1.3}$$

and a steady-state ozone concentration in a relatively unpolluted environment is given by the ratio

$$O_3 = k\frac{NO_2}{NO}\,, \tag{1.4}$$

where *k* is a constant. However, volatile organic compounds in effect competes with ozone in (1.3), so that nitrogen monoxides are transformed into nitrogen dioxides without the consumption ozone, and this leads to increased net ozone production. The ozone formation is therefore effective if there are stable concentrations of both NOx and VOCs, in addition to sunlight. On the other hand, if the concentration of NOx is large, the ozone-generating effect of VOCs is reduced. Increased NOx emissions can therefore give reduced ozone formation in cases where the initial concentration of NOx is large, see figure 1.

(Figure 1 about here).

Figure 1 is taken from Amann *et al.* (1998a). It shows typical contour curves (isopleths) for mean ozone concentrations in Europe. "Factor" 1 gives 1990 emissions for all European countries, and it is assumed that all countries reduce their emissions by 40% at factor 0.6, etc. The non-monotonic effect of NOx emissions on ground-level ozone is illustrated by the upward-sloping parts of contour-curves in the figure at the right.

Similar effects of increased NOx emissions are also reported elsewhere, for instance in Cleveland and Graedel (1979), Glasson (1981) and in van Ierland and Schmieman (1999). According to Glasson (1981) there are also considerable evidence to suggest that part of the ozone-reduction experienced in Los Angeles is caused by increased NOx emissions. For VOCs, increased emissions give increased ozone values in figure 1. However, others have found a slight non-monotonicity also for hydrocarbons (a type of VOC), see Repetto (1987). The spatial aspect is also important for ground-level ozone. For instance, increased NO emissions give reduced ozone concentrations close to the emitting source, see (1.3). However, since this produces NO_2, ozone concentrations will typically increase at locations further away from the emitting source, see (1.1) and (1.2). See Cleveland and Graedel (1979) for a discussion of this subject.

3. The City Model

A model for reduced ground-level ozone concentrations in a city is developed in this section. Different types of cost-effective solutions, which minimise total abatement costs for NOx and VOCs, subject to a constraint on ground-level ozone, will be identified. At first it is assumed that emissions are constrained by their no-control amounts, but this assumption will be relaxed. Since a city has a limited extension and since we only consider abatement possibilities for sources within the city, the long-range atmospheric transport of ozone and precursors between different regions are not modelled explicitly.[4] Others, for instance Repetto (1987) and Kim *et al.* (1998), have used similar approaches. They study ozone reductions in the New York Metropolitan region and in California's San Joaquin Valley respectively. The concentration of ground-level ozone in the city is given by

$$O_3 = f\left(e^{nox}, e^{voc}\right) \tag{2}$$

where e^{nox} and e^{voc} are emitted amounts of NOx and VOCs in the city, while O_3 is the concentration of ground-level ozone.[5] The slopes of contour curves, which are combinations of NOx and VOC emissions such that ozone concentrations are constant, are given by

$$\left.\frac{de^{voc}}{de^{nox}}\right|_{dO_3=0} = -\frac{f_n}{f_v}, \tag{3}$$

where f_n and f_v are finite partial derivatives of the *f*-function with respect on e^{nox} and e^{voc} respectively. Based on figure 1, it is assumed that increased VOC emissions always give increased ozone concentrations. However, for a given amount of VOCs, increased NOx emissions are assumed to give increased ozone concentrations only if the emitted amount of NOx is smaller than a certain value, $\beta\left(e^{voc}\right)$, which corresponds to a point where a contour curve is horizontal in figure 1. If the emitted amount of NOx is larger than $\beta\left(e^{voc}\right)$, it is assumed that additional NOx emissions give reduced ozone concentrations, so that

$$f_v > 0,\ f_n \begin{cases} > 0 \mid e^{nox} < \beta\left(e^{voc}\right) \\ = 0 \mid e^{nox} = \beta\left(e^{voc}\right) \\ < 0 \mid e^{nox} > \beta\left(e^{voc}\right) \end{cases} . \tag{4}$$

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[4] The specification in (2), which gives the ozone concentration, allows exogenous long-range transport. Such exogenous terms are, however, suppressed by the notation.

[5] Ozone concentrations can be measured in different units. See for instance Amann *et al.* (1998b) and Kim *et al.* (1998). These differences are, however, not important for the present paper.

The abatement of NOx and VOC emissions are defined by

$$\begin{aligned} r^{nox} &\equiv e_{nc}^{nox} - e^{nox} \\ r^{voc} &\equiv e_{nc}^{voc} - e^{voc}, \end{aligned} \tag{5}$$

where e_{nc}^{nox}, e_{nc}^{voc} >0 are emitted amounts of NOx and VOCs in no-control, defined by the amounts that would be emitted in a market economy in the absence of governmental controls on NOx and VOC emissions. Minimal abatement cost, over all emitting sources in the city, is given by

$$C = g\left(r^{nox}, r^{voc}\right); \quad e_{nc}^{nox} \geq r^{nox} \geq 0,\ e_{nc}^{voc} \geq r^{voc} \geq 0\,, \tag{6}$$

while marginal abatement costs for NOx (g_n) and VOCs (g_v), which are the partial derivatives of (6) with respect on r^{nox} and r^{voc} respectively, are given by

$$g_n \begin{cases} = 0 & | \quad r^{nox} = 0 \quad \Leftrightarrow \quad e^{nox} = e_{nc}^{nox} \\ \in \langle 0, \infty \rangle & | \ e_{nc}^{nox} > r^{nox} > 0 \ \Leftrightarrow \ 0 < e^{nox} < e_{nc}^{nox} \\ = \infty & | \quad r^{nox} = e_{nc}^{nox} \quad \Leftrightarrow \quad e^{nox} = 0 \end{cases}, \tag{7.1}$$

$$g_v \begin{cases} = 0 & | \quad r^{voc} = 0 \quad \Leftrightarrow \quad e^{voc} = e_{nc}^{voc} \\ \in \langle 0, \infty \rangle & | \ e_{nc}^{voc} > r^{voc} > 0 \ \Leftrightarrow \ 0 < e^{voc} < e_{nc}^{voc} \\ = \infty & | \quad r^{voc} = e_{nc}^{voc} \quad \Leftrightarrow \quad e^{voc} = 0 \end{cases}. \tag{7.2}$$

It is also assumed that g_{nn} and g_{vv} are strictly positive. This implies that marginal abatement costs for NOx are increased if the abated amount of NOx increases partially, while marginal costs for VOCs are increased if VOC abatement increases. In addition, marginal abatement costs are zero for zero abatement, positive for positive abatement and infinite if the emitted amount is zero. Isocosts are combinations of NOx and VOC emissions such that abatement costs are constant, and the slope of an isocosts is given by

$$\left.\frac{de^{voc}}{de^{nox}}\right|_{dC=0} = -\frac{g_n}{g_v} \leq 0\,. \tag{8}$$

The cost-effective solution for reduced ground-level ozone concentrations is defined by the feasible emission vector (e^{nox}, e^{voc}) that gives least costs among those feasible emission vectors that doesn't violate the target for ground-level ozone concentrations. The optimisation problem is therefore given by

$$\begin{aligned} & min_{e^{nox},e^{voc}} g\left(e_{nc}^{nox} - e^{nox}, e_{nc}^{voc} - e^{voc}\right) \\ & s.t. \quad f\left(e^{nox}, e^{voc}\right) \le \alpha \\ & s.t. \quad 0 \le e^{nox} \le e_{nc}^{nox},\ 0 \le e^{voc} \le e_{nc}^{voc}, \end{aligned} \tag{9}$$

where the definitions of abatement are substituted into (a) the arguments of the cost function and (b) the constraints on abatement, and α is the target for ground-level ozone. The Lagrangian function is given by

$$\begin{aligned} L = -g\left(e_{nc}^{nox} - e^{nox}, e_{nc}^{voc} - e^{voc}\right) + \lambda\left(\alpha - f\left(e^{nox}, e^{voc}\right)\right) \\ + \mu_{upper}^{nox}\left(e_{nc}^{nox} - e^{nox}\right) + \mu_{lower}^{nox} e^{nox} \\ + \mu_{upper}^{voc}\left(e_{nc}^{voc} - e^{voc}\right) + \mu_{lower}^{voc} e^{voc}, \end{aligned} \tag{10}$$

where λ is the shadow price on the ozone-restriction, while $\mu_{upper}^{nox}, \mu_{lower}^{nox}, \mu_{upper}^{voc}, \mu_{lower}^{voc}$ are shadow prices for upper and lower restrictions on NOx and VOC emissions respectively. The first-order (Kuhn-Tucker) conditions for an optimal solution are given by[6]

$$g_n - \lambda f_n - \mu_{upper}^{nox} + \mu_{lower}^{nox} = 0, \tag{11.1}$$

$$g_v - \lambda f_v - \mu_{upper}^{voc} + \mu_{lower}^{voc} = 0, \tag{11.2}$$

$$\lambda\left(\alpha - f\left(e^{nox}, e^{voc}\right)\right) = 0, \tag{11.3}$$

$$\begin{aligned} \mu_{upper}^{nox}\left(e_{nc}^{nox} - e^{nox}\right) = 0, \mu_{lower}^{nox} e^{nox} = 0, \\ \mu_{upper}^{voc}\left(e_{nc}^{voc} - e^{voc}\right) = 0, \mu_{lower}^{voc} e^{voc} = 0, \end{aligned} \tag{11.4}$$

$$\alpha \ge f\left(e^{nox}, e^{voc}\right), e_{nc}^{nox} \ge e^{nox} \ge 0, e_{nc}^{voc} \ge e^{voc} \ge 0, \tag{11.5}$$

$$\lambda, \mu_{upper}^{nox}, \mu_{lower}^{nox}, \mu_{upper}^{voc}, \mu_{lower}^{voc} \ge 0. \tag{11.6}$$

First we consider the case where the constraint on ozone is non-binding, defined by $\alpha > f\left(e^{nox}, e^{voc}\right)$. Then, by (11.3), $\lambda = 0$. Suppose $g_n > 0$ in this case. Then, by

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[6] See for instance Sydsaeter and Hammond (1995).

(11.1) and (11.6), $\mu_{upper}^{nox} > 0$, and then $e^{nox} = e_{nc}^{nox}$ by (11.4). But this is a contradiction since (7.1) implies $e^{nox} < e_{nc}^{nox}$ when $g_n>0$. Consequently, $\lambda = 0$ implies $g_n=0$ and $e^{nox} = e_{nc}^{nox}$ from (7.1). By symmetry, VOC emissions are also equal to no-control values if $\lambda = 0$. Obviously, resources are not spent on reducing emissions if the emitted amounts in no-control gives ozone-concentrations that doesn't exceed the targeted amount.

In the following it is assumed that no-control emissions violates the restriction on ozone concentration. Consequently, $\lambda > 0$ and, by (11.3), $\alpha = f\left(e^{nox}, e^{voc}\right)$. It is also assumed that *some* amounts of both NOx and VOCs can be emitted without violation of the restriction.[7] The *optimal* solution is also characterised by strictly positive emitted amounts since marginal abatement costs for a substance is infinite if the emitted amount is zero[8], and by (11.4) this implies $\mu_{lower}^{nox} = \mu_{lower}^{voc} = 0$. Also, since $f_v>0$, it follows from (11.2) and (11.6) that $g_v > 0$, implying $e^{voc} < e_{nc}^{voc}$ from (7.2), which in turn imply $\mu_{upper}^{voc} = 0$ from (11.4). In total, the first order conditions can be simplified to

$$g_n - \lambda f_n - \mu_{upper}^{nox} = 0, \tag{12.1}$$

$$g_v - \lambda f_v = 0, \tag{12.2}$$

$$\alpha = f\left(e^{nox}, e^{voc}\right),\ \ \lambda > 0, \tag{12.3}$$

$$\mu_{upper}^{nox}\left(e_{nc}^{nox} - e^{nox}\right) = 0, \tag{12.4}$$

$$\mu_{upper}^{nox} \geq 0, e_{nc}^{nox} - e^{nox} \geq 0. \tag{12.5}$$

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[7] In the 1980s, several states in the Northeastern USA recognised that it was impossible for them to attain the ozone standard with in-state controls alone (Farrell *et al.*, 1999).

[8] Suppose that zero NOx is emitted in the optimal solution. Then g_n is infinite by (7.1) and $\mu_{upper}^{nox} = 0$ by (11.4). Then, by (11.1), $\lambda f_n = \infty$. Since f_n by assumption only takes finite numbers, λ must be infinite. Then, by (11.2), g_v must be infinite also (the lower constraint can also be positive, but then the emitted amount is zero and g_v is infinite), implying zero VOC emissions from (7.2). But in this case both NOx and VOC emissions are zero in the optimal solution, and this contradicts the assumption that some amounts of NOx and VOCs can be emitted without violation of the restriction.

First we consider the case where $\mu_{upper}^{nox} = 0$. By (12.1) and (12.2) the optimal solution satisfies

$$-\frac{g_n}{g_v} = -\frac{f_n}{f_v}, \tag{13}$$

in this case, and, by (3) and (8), this implies that an isocost and a contour curve for ozone are tangents to each other. This is a standard finding in economics: relative costs are balanced against relative benefits in the optimal solution. Also, by (12.3), the value of the contour curve for ozone that goes through the optimal point is the targeted amount of ozone. Figure 2 illustrates the optimal solution. Isocosts take increasing values from no-control towards the origin, while contour curves take increasing values upwards. The vector $\left(e^{nox^*}, e^{voc^*}\right)$ gives the cost-effective solution.

(Figure 2 about here).

In figure 2, both NOx and VOCs are abated in the optimal solution. It has been shown that VOC abatement always is strictly positive. However, the abated amount of NOx in the optimal solution can be equal to zero even if $\mu_{upper}^{nox} = 0$. In that case, $e^{nox} = e_{nc}^{nox}$ such that g_n=0 by (7.1). But then, by (12.1), f_n must be zero too. This is a special case where the slope of the contour curve for ozone for the value α by coincidence is equal to zero exactly at e_{nc}^{nox}, see figure 3.

(Figure 3 about here).

Now we consider the case where $\mu_{upper}^{nox} > 0$. By (12.4) this implies $e^{nox} = e_{nc}^{nox}$, and g_n=0 by (7.1). In this case, the abated amount of VOCs in the optimal solution is given implicit by $\alpha = f\left(e_{nc}^{nox}, e^{voc}\right)$. In addition, by (12.1), f_n<0 since g_n=0 and $\lambda, \mu_{upper}^{nox} > 0$. Therefore, from (3), the optimal solution is located on the upward-sloping part of the contour curve for ozone. This case is illustrated in figure 4.

(Figure 4 about here).

The intuition for this result is that costs are increased if we slide down the contour curve for ozone from the optimal point in figure 4, while positions to the right for the optimal point are out of the domain for the cost function. The corner solutions for abatement, illustrated in figures 3 and 4, are possible only because the ozone function is non-convex.[9] Consequently, this analysis supports the informal argument in Repetto (1987): corner-solutions can be optimal as a consequence of the non-convex ozone-function.

However, in Wolfgang (1999) it is shown that it is incorrect to restrict emissions by their no-control values. Firstly, it is obviously technological *feasible* for a firm to emit more than the amounts that are optimal in the absence of

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[9] See footnote 1 for the definition of a convex function. Let *x′* and *x″* be two points on the contour curve for ozone for the value of the constraint. Then $f(x') = f(x'') = \alpha = \theta f(x') + (1-\theta) f(x'')$. A straight line between two points on the contour curve, for instance in figure 4, gives larger values for ozone since contour curves are convex and increasing upwards. This implies that $\theta f(x') + (1-\theta) f(x'') < f\left(\theta x' + (1-\theta) x''\right)$, so that the ozone-function is non-convex in this domain. However, consider a hypothetical case where the ozone-function is strictly convex in the whole domain. In this case contour curves must be concave. We also assume that increased NOx emissions gives a partial increase in ozone for low NOx concentrations. The contour curves are therefore falling in their whole domain in this hypothetical case. Suppose now that the optimal amount of NOx emissions is equal to the no-control value in this hypothetical case. Then, since the contour curves for ozone are falling and isocosts are horizontal at no-control emissions for NOx, the contour curve for the value of the constraint crosses an isocost from above in the optimal solution. Consequently, there exist points in-between the isocost and the contour curve that gives fewer costs and less ozone concentrations than the optimal solution, and this is a contradiction. This implies that, given the assumptions about the cost function and the partial effect of increased NOx

governmental controls. Secondly, it can be *optimal* for a profit maximising firm to emit an amount larger than no-control if emissions taxes are negative. From a social point of view negative abatement and emission subsidies are contra-intuitive for most cases. However, in the case of non-monotonic pollution functions, the emitted amounts in the optimal solution can be larger than no-control. It is easy to see this if we prolong the isocost in figure 4 by imagination such that it crosses the contour curve for ozone. Clearly, locations in-between the imaginary line-segment and the contour curve gives less abatement costs and smaller ozone concentrations, compared to the point where the curves cross each other. The optimal amount of NOx emitted is therefore larger than the no-control value in that case.

The restrictions on maximum emissions are dropped in the following. Also, since the global minimum for abatement costs by construction must be located at the no-control emission vector, marginal abatement costs are assumed to be negative for negative abatement. See Wolfgang (1999) for a discussion. In total, marginal abatement costs are assumed to take the same sign as the abated amount so that[10]

$$\begin{aligned} sign(g_n) &= sign\left(e_{nc}^{nox} - e^{nox}\right) \\ sign(g_v) &= sign\left(e_{nc}^{voc} - e^{voc}\right). \end{aligned} \tag{14}$$

The new first order condition for NOx is given by

$$g_n - \lambda f_n = 0, \tag{15}$$

and all optimal solutions satisfies (13) and (12.3). Suppose that the optimal solution is located on the upward-sloping part of the contour curve for ozone, implying

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emissions at low NOx concentrations, corner solutions for abatement are possible only if the ozone-function is non-convex.

[10] An alternative formulation is

$g_n > 0 \Leftrightarrow e^{nox} < e_{nc}^{nox}, g_n = 0 \Leftrightarrow e^{nox} = e_{nc}^{nox}, g_n < 0 \Leftrightarrow e^{nox} > e_{nc}^{nox}$,

$g_v > 0 \Leftrightarrow e^{voc} < e_{nc}^{voc}, g_v = 0 \Leftrightarrow e^{voc} = e_{nc}^{voc}, g_v < 0 \Leftrightarrow e^{voc} > e_{nc}^{voc}$.

$f_n<0$ from (3). Then, by (15), $g_n<0$, and from (14) it follows that $e^{nox} > e^{nox}_{nc}$. Consequently, if the optimal solution is located on the upward-sloping part of the contour curve, the amount of NOx emitted in the optimal solution is larger than the no-control value. This case is illustrated in figure 5.

(Figure 5 about here).

In the optimal solution, the isocost and the contour curve for ozone are tangents to each other, and the amount of ozone is exactly the targeted amount. These necessary conditions for optimality are, however, not sufficient for optimality. But if, in addition, the Lagrangian function is concave over the whole domain, then a stationary point for the Lagrangian function is an optimal solution, see Sydsaeter and Hammond (1995). The condition for local maximum is less strict: if the boarded Hessian matrix is positive at the stationary point under consideration, then this is a local maximum. Figure 6 illustrates cases where the necessary conditions for cost minimisation are satisfied even though the stationary points fail to be optimal solutions.

(Figure 6 about here.)

The diamond on the right-hand side in figure 6 is a local cost-minimum. However, the circle at the left gives fewer costs and smaller ozone concentrations. The diamond on the bottom of the contour curve for ozone shows a local cost-maximum since locations in-between the isocost and the contour curve for ozone, in the close neighbourhood of that point, gives fewer costs and smaller ozone concentrations.

4. The Regional Model

In this section, a model for reduced ground-level ozone concentrations at several locations within a large region is developed. A cost-effective solution minimises total abatement costs in the region, subject to the constraints on ground-level ozone at all locations under consideration. Obviously, the atmospheric transport of substances must be accounted for in this model. Models for cost-effective abatement policies that account for ozone formation and atmospheric transport of precursors have also been used by others, see for instance Amann *et al.* (1998a) and van Ierland and Schmieman (1999).

Suppose there are n emitters in total (the units can for instance be states in USA and countries in Europe), while there are restrictions on ozone concentrations at m locations. Let the index and set of emitters be i and I respectively, such that $i \in I = \{1,...,n\}$, and let the index and set of locations be j and J respectively, such that $j \in J = \{1,...,m\}$. The cost-effective solution for this problem minimise the sum of abatement costs, subject to the constraints on ground-level ozone concentrations,

$$\begin{aligned} & min_{e_i^{nox},e_i^{voc}} \sum_{i\in I} g_i\left(e_{nc\ i}^{nox} - e_i^{nox}, e_{nc\ i}^{voc} - e_i^{voc}\right) \\ & s.t. \quad f_j\left(e_1^{nox}, e_1^{voc}, ..., e_n^{nox}, e_n^{voc}\right) \le \alpha_j, \qquad j = \{1,...,m\}, \end{aligned} \tag{16}$$

where g_i is the cost function for emitter i, $e_{nc\ i}^{nox}$ and $e_{nc\ i}^{voc}$ are strictly positive no-control emissions of NOx and VOCs respectively for emitter i, e_i^{nox} and e_i^{voc} are actual emissions, f_j is the ozone function for location j, and $\alpha_j > 0$ is the restriction on the ozone concentration at location j. First order conditions for NOx and VOCs for an emitter are given by

$$g_{n_i} = \sum_{j\in J} \lambda_j f_{n_i}^j \ , \tag{17.1}$$

$$g_{v_i} = \sum_{j \in J} \lambda_j f_{v_i}^j, \tag{17.2}$$

where g_{n_i}, g_{v_i} are marginal abatement costs for NOx and VOCs for emitter *i,* and $f_{n_i}^j$, $f_{v_i}^j$ are partial derivatives of the ozone function for location *j* with respect on emitter *i*'s NOx and VOC emissions respectively,[11] and the shadow prices on ozone restrictions at various locations are given by $\lambda_j \geq 0$. It is assumed that the partial derivatives of the cost functions satisfies

$$\begin{aligned} sign\left(g_{n_i}\right) &= sign\left(e_{nc\ i}^{nox} - e_i^{nox}\right) \\ sign\left(g_{v_i}\right) &= sign\left(e_{nc\ i}^{voc} - e_i^{voc}\right) \end{aligned}, \tag{18}$$

cf. equation (14).

For locations sufficiently remote from the emitter under consideration, it is assumed that the partial derivatives of the corresponding ozone-functions, with respect on NOx and VOC emissions from this particular emitter, is zero. However, for the remaining locations it is assumed that $f_{v_i}^j > 0$ while $f_{n_i}^j$ can take both signs. If all binding constraints on ozone concentrations are sufficiently remote from the emitter under consideration, then, by (17.1), (17.2) and (18), emitted amounts in the optimal solution are given by no-control values for that emitter. In the following it is assumed that all emitters are sufficiently close to influence the concentration of NOx, VOCs and ozone in at least one location *j* where $\lambda_j > 0$. Then, by (17.2), $g_{v_i} > 0$, and by (18), $e_i^{voc} < e_{nc\ i}^{voc}$. This means that the emitted amount of VOCs in the optimal solution is less than the no-control value for every emitter.

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[11] The intended interpretation of the units in *I* in this paper is European countries or states in USA. The analysis could, however, easily be generalised to account for the specific location for every emitting source. Little is gained by this in the present paper, and that generalisation would also require an investigation of special cases where single sources emit either NOx or VOCs or both in no-control, and the analysis would be somewhat unfocused. The emitting units are countries in the RAINS model too.

The sign of the right-hand side in (17.1) is ambiguous since $f_{n_i}^{j}$ can take both signs. If increased NOx emissions from a particular emitter give increased ozone concentrations at some locations, and the shadow prices for ozone are large enough for these locations, then the right-hand side in (17.1) is strictly positive, implying $g_{n_i} > 0$ and $e_i^{nox} < e_{nc\ i}^{nox}$ by (18). However, if increased NOx emissions for the emitter under consideration give reduced ozone concentrations at locations where the shadow prices for ozone are large, so that the right-hand side in (17.1) is negative, then $g_{n_i} < 0$ and $e^{nox}{}_i > e_{nc\ i}^{nox}$ by (18). Consequently, the amount of NOx emitted is larger than the no-control value if the sum of the λ-weighted marginal effects of increased NOx emissions on ozone concentrations is negative, and vice versa. Since the right-hand sides in (17.1) and (17.2) are emitter-specific, the optimal solution can be characterised by reduced NOx emissions (compared to no-control) for some emitters while emissions are increased for other emitters. Also, negative abatement for NOx is most likely to occur for emitters that are close to high-NOx areas where increased NOx concentrations gives reduced ozone concentrations. The reason is that an emitter has relatively larger influence on NOx concentrations at locations close to the emitter compared to locations far away.

Figures 2, 3 and 5 illustrate optimal solutions also for the regional model if the label "$O_3 = \alpha$" is replaced by the function

$$W_i = \sum_{j \in J} \lambda_j^* f_j \left(e_1^{nox^*}, e_1^{voc^*}, \ldots, e_i^{nox}, e_i^{voc}, \ldots, e_n^{nox^*}, e_n^{voc^*} \right), \tag{19}$$

where each variable in the vector $\left(\lambda_{j \in J}{}^*, e_{k \in I}^{nox^*}, e_{k \in I}^{voc^*} \right)$ is the optimal solution for the variables in the corresponding vector $\left(\lambda_{j \in J}, e_{k \in I}^{nox}, e_{k \in I}^{voc} \right)$. Equation (19) can, somewhat inaccurately, be interpreted as the value of ground-level ozone concentrations in the optimal solution, as a function of the emitted amounts for emitter *i*. The right-hand side in (17.1) divided by the right-hand side in (17.2), multiplied by −1, gives the slope of a contour curve for the W_i-function. Consequently, in an optimal solution, the isocost curve for an emitter *i* has the same slope as a contour curve for the W_i-function.

5. Conclusions

Others have studied implications of *non-convexities* in ozone formation, and it is indicated that corner solutions for abatement are potentially cost-effective. The purpose of this paper has been to illustrate implications of *non-monotonicity* in ozone formation, rather than to formulate actual regulatory strategies. Two models for cost-effective reductions of ground-level ozone have been developed in this paper: a city model and a regional model. However, both models gave the same kinds of cost-effective solutions.

The main findings in this paper are as follows. The optimal amount of VOC emitted is less than the no-control value in the city model. For NOx, however, the emitted amount in the optimal solution is less than the no-control value only if the optimal solution is located on the downward-sloping part of the contour curve for ozone. If, on the other hand, the optimal solution is located on the upward-sloping part of the contour curve, the optimal amount of NOx emitted is larger than the no-control value. In the regional model, VOC abatement is strictly positive for every emitter, while NOx abatement is strictly positive for a particular emitter if the contour curve for the "ozone-value function" is downward-sloping for this emitter in the optimal solution. If, on the other hand, the contour curve for the "ozone-value function" is upward-sloping for a particular emitter, then the emitted amount of NOx is larger than the no-control value for this emitter.

It would be trivial to extend the model in this paper and show that the optimal emission vector in principle can be implemented by appropriate emission taxes under standard assumptions. If the optimal amount of NOx emitted is larger than no-control for a unit (country or state), then the optimal emission tax for NOx is negative for this unit. For the regional model, such emission taxes must be emitter-specific, for instance country- or state-specific, while only one NOx tax rate and one VOC tax rate is required for the city model. If the deposition pattern differs substantially for sources within a country, then the optimal tax structure must of course account for these differences too.

The assumption that marginal abatement costs are zero if the abated amount of the substance under consideration is zero, is important for the outcomes of the analysis. If something else is assumed, then negative abatement does not necessarily imply an optimal solution on the upward-sloping part of the contour curve for ozone, and vice versa. However, unless negative abatement is ruled by an inaccurate assumption, the optimal solution for NOx emissions can in general be larger than the emitted amount in no-control.

Both NOx and VOC emissions have other adverse effects than the formation of ground-level ozone. For instance, NOx emissions have influence on acidification, eutrophication and on the densities of fine particles (Lükewille *et al.*, 2001). In principle, a cost-effective policy must account for all environmental effects and constraints. Positive NOx abatement can therefore be optimal when all environmental constraints are accounted for, even if negative NOx abatement is optimal in a particular optimisation that only accounts for the constraints on ground-level ozone. Still, the targets for ground-level ozone may dominate the optimal abatement strategies, at least for some emitters.

Suppose, however, that the optimal amount of NOx emitted is larger than the no-control value for the short-run target, but less than no-control value for the long-run target. Suppose also that there are additional cost elements for *changed* emissions due to frictions in the economy. In this case, the optimal dynamic control of NOx emissions may imply positive NOx abatement even in the short-run.

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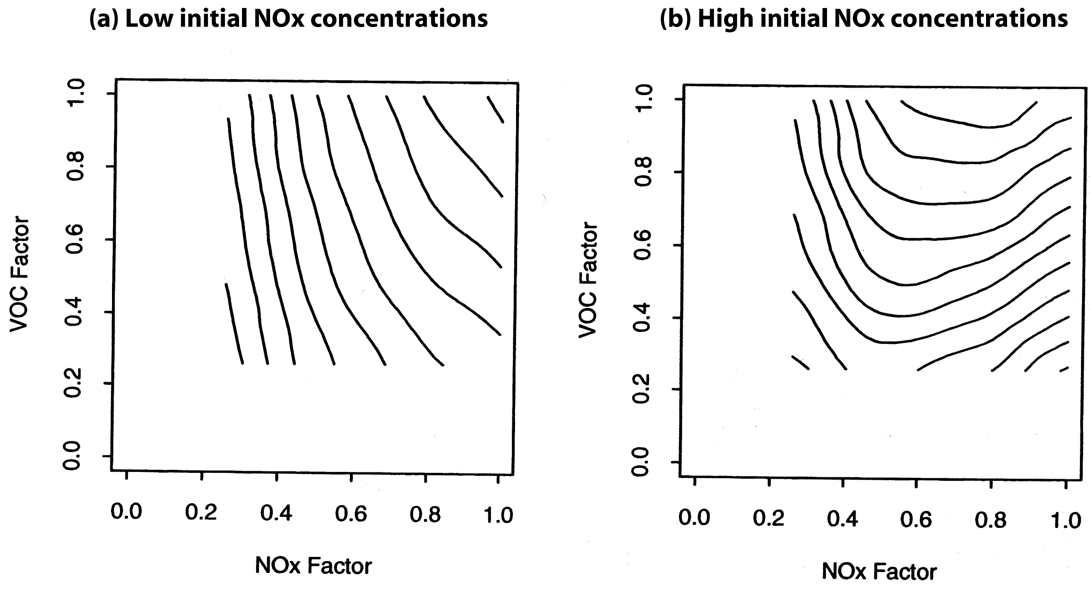
Figure 1:Typical patterns of ozone behaviour in Europe.



(a) Low initial NOx concentrations

(b) High initial NOx concentrations

Source: Amann et al. (1998a).

The explaining text after (a) and (b) is added.

Figure 2: Strictly positive abatement in a cost-effective solution.

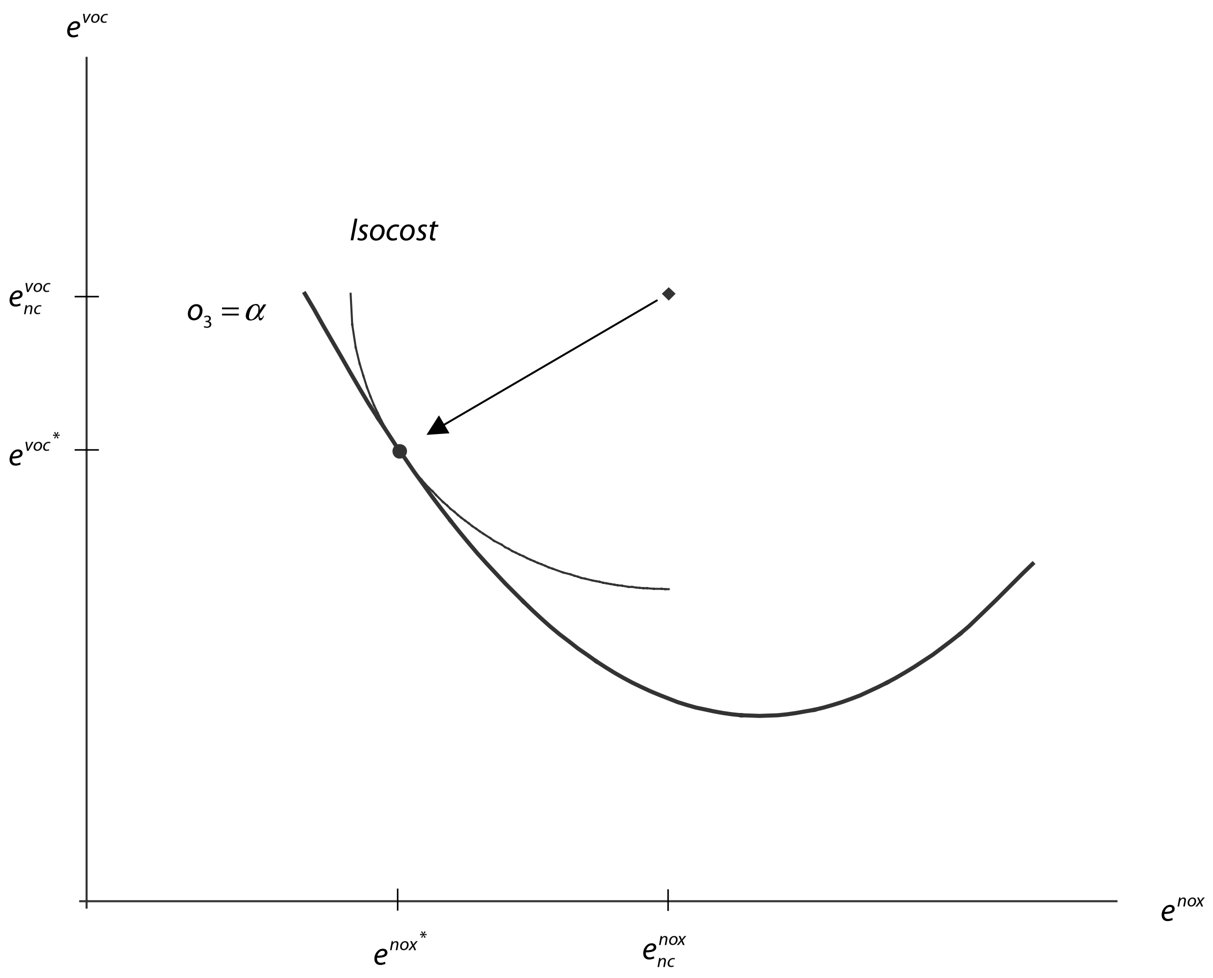


Figure 3: Zero abatement for NOx in a special case where the contour curve for ozone for the value of the constraint by coincidence is horizontal exactly at the no-control amount for NOx.

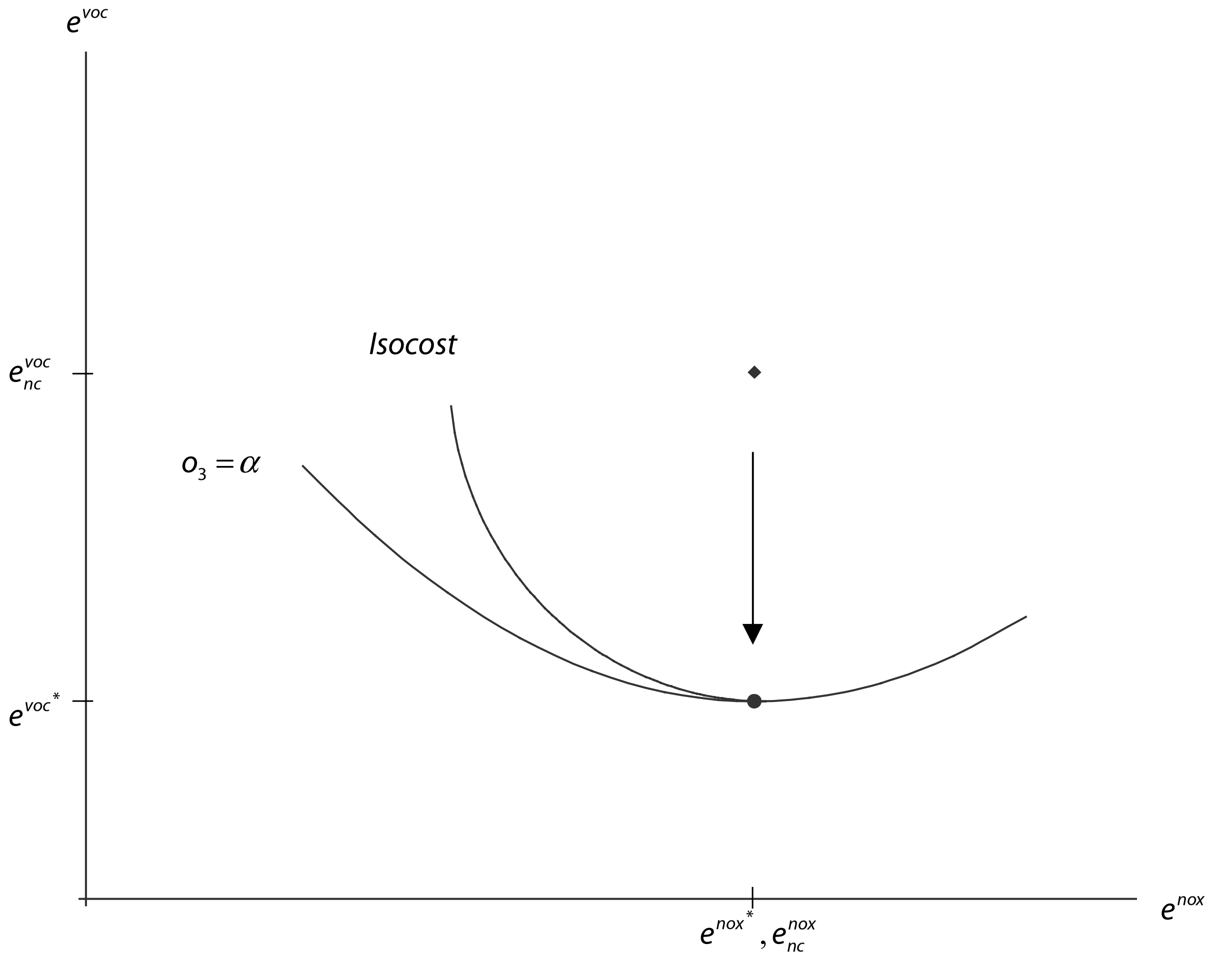


Figure 4: Corner solution for NOx abatement when the upper constraint for NOx emissions is binding.

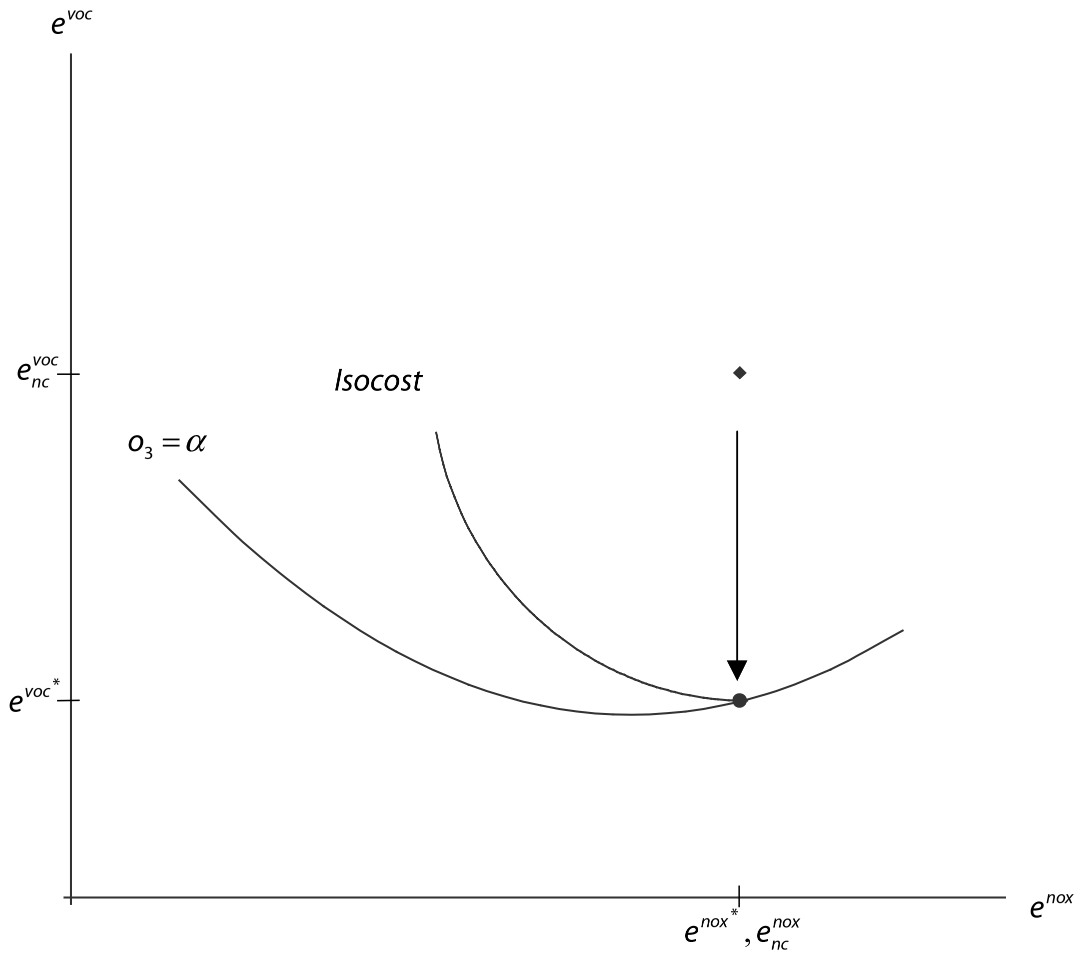


Figure 5: An optimal solution on the upward-sloping part of the contour curve for ozone implies negative NOx abatement.

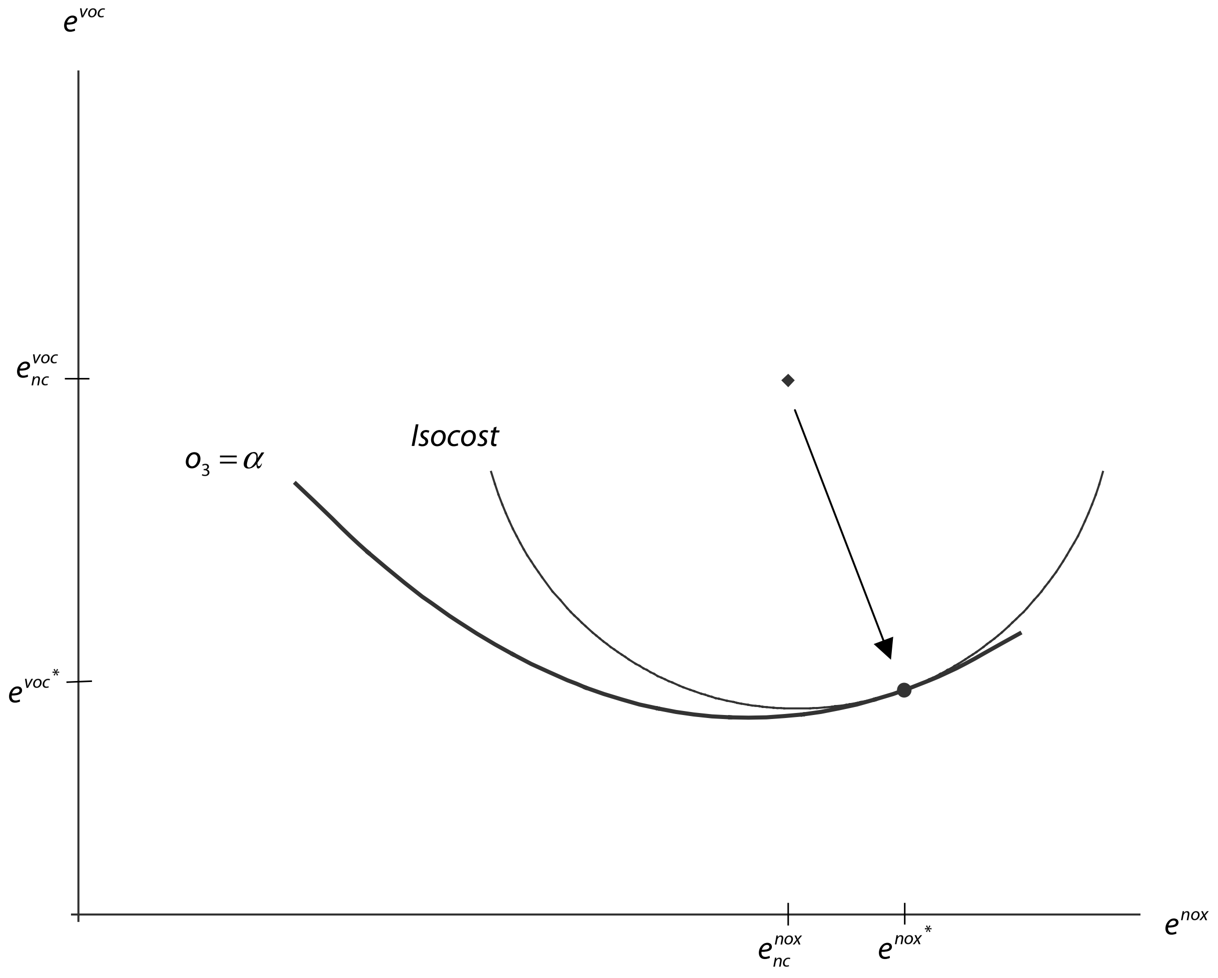


Figure 6: Cases where stationary points fail to be cost-effective solutions.