Climate Change and Electricity Transmission

An empirical study with indication of some theoretical consequences

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Executive summary

In an economy like the Norwegian, where most of the energy is supplied by hydro power plants and the outdoor temperatures are very fluctuating, the domestic supply and the demand for energy can be expected to depend on climatical phenomena such as precipitation and temperature. This thesis will indicate how a change in the statistical distribution of the demand for electricity transmission capacity with the neighbouring countries, having their energy supply less directed by the weather, is likely to be affected by some of the consequences of global warming.

The initial statistical model estimates the chosen climatical variables’ effect on net import. It turned out that the regression model of monthly net import on its first lag and lagged precipitation, explaining supply, and an aggregate of monthly temperatures, explaining demand, fitted the sample data well. However, the demand for transmission capacity depends on the distribution of transmission at points in time, and not the monthly average of net import. At all points in time the transmitted quantity will equal the absolute value of net import, so the estimated net import distribution were used to infer the transmission demand distribution.

When the marginal costs of meeting the demand are different from the benefits, the optimal level of the transmission capacity depends on the demands’ volatility as well as its expectation. By the so-called “Newsboy model”, optimal capacity would be at a level where the probability of demand not being met equals the ratio of marginal costs to marginal benefits. When these are exogenously given and the demand distribution is known, the optimal capacity will equal the mean demand plus a fraction of its standard deviation depending on the cost benefit ratio.

However, if the costs or benefits are non-linear functions of capacity it gets more complicated. A by-product of higher transmission capacity is that the losses for a given quantity of electricity will fall since the capacity utilisation falls. This will increase the average benefit of transmission since more of the transmitted energy becomes available to the consumers. However, what matters for the optimal capacity is the marginal benefit, which is less likely to change.
The other part of the marginal benefit of the ability to meet demand is the reduction in the penalty for not meeting demand. This is defined as the quantity that would be desirable to transmit, multiplied by the price difference, and shared equally between the markets separated by limited transmission capacity. At both sides of the border the price level is determined by the generator with the highest marginal costs. When these costs vary there would be efficiency gains by generating more at the low cost side, less at the other; and transmitting the difference. A marginal increase in transmission capacity would allow a marginal reduction in the production by the high cost side and a marginal increase at the low cost sides; reducing the price difference as well as the size of the bottleneck. Thus the marginal benefit of transmission would be an increasing function of capacity and the newsboy solution will understate the optimal capacity.

The conclusion of the empirical study was that the monthly transmission demand standard deviation increased less than proportionately with the standard deviations in the explanatory climatical variables. Without having any estimate of the likely order of magnitude of their future increase, an eventual increase the cost of keeping the transmission at its optimal level cannot be estimated. However, the direction of the capacity cost change is unambiguous, if the weather becomes more volatile, net electricity import gets more volatile as well. Hence the optimal transmission capacity will increase unless the expected net import falls sufficiently to compensate it. Nevertheless, the global warming is likely to cause a more humid and warmer climate, and according the signs of the coefficients in the net import equation this will reduce energy demand and increase its supply. Consequently the total effect of the climate change on the costs of keeping the transmission capacity at its optimal level remains undetermined.
Preface

The background for this thesis is a hypothesis that the domestic electricity supply and demand depends on climatical phenomena such as precipitation and temperature, which are likely to be affected by the potential global warming. The estimated model relies on the assumption that the variation in electricity demand can be explained by temperature variations in the largest cities, and that the hydro power supply varies with the precipitation over the regions with the largest dams. It turned out that the mentioned climatical variables explained most of the transmission demand volatility, but their influence on its mean could not be identified. The hypothetical climate change is likely to get many consequences. This project is limited to the cost of increasing the transmission capacity with our neighbouring countries in order to handle larger fluctuations between energy supply and demand.

I would like to acknowledge my supervisor Finn Førsund for his guidance and inspiring comments, and Jan F. Foyn at Nordpool for giving me access to their ftp-server. Without supervision and access to this impressive database this project would have remained an idea.

Any errors and mistakes are my responsibility.
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1. Introduction

1.1 Future distributions of temperature and precipitation

As a response to the potential problem of global climate change the Intergovernmental Panel on Climate Change were established in 1988. In one of their published reports, IPCC (2001), I came across the following two figures:
As the changes in statistical distribution are not yet documented, I will only discuss the theoretical consequences if they do.

1.2 Models of temperature dependent electricity demand

Henley and Peirson have published three papers on the topic in Britain. In their 1997 paper they tested different non-linear models using household data and concluded the demand’s responsiveness to the temperature is greatest when it is between 10 and 20°C. When the temperature is below the lower threshold heating is capacity constrained, and at temperatures above the higher demand increases with temperature. This might be because of the use of alternative fuels at very low temperatures which cannot be fired up gradually, and because refrigeration equipment consumes more energy at higher ambient temperatures.
Unsurprisingly, the most extreme temperatures occurred less frequent than the temperatures near the mean, making the shape of the tails of the polynomial less reliable as it was based on less information than the middle. They also found income and price effects.

There are important differences between the determinants of Norwegian and British temperature dependent electricity demand that makes the temperature dependence unlikely to be identical. The temperature varies over a wider range in Norway, and the mix of energy sources is different. Electricity accounted for 79 per cent of the Norwegian households’ energy consumption, fire wood 15 percent and fuel oil five per cent\(^1\). The UK domestic consumption on “energy supplied basis” were 2% coal, 1 per cent “other solid fuels,” 75% gas, and 23% electricity\(^2\). Thus, the temperature dependent electricity demand in Norway is unlikely to have the exact same functional shape.

Døhl (1999) found a similar relationship for Norway, where the temperature explained short term variations around a long term trend.

1.3 Models for the supply of hydro- and thermal power

According to Førsund (2004) the supply of hydropower depends on the total usable inflow during a scarcity period, which usually is a year. The electricity price will be such that the available water just covers the sum of domestic demand and net export. In the real world where neither precipitation nor temperature is known in advance, the electricity price will fluctuate within the years as information is revealed.

Higher precipitation volatility will make extremely dry or wet years more likely and increase the electricity exchange volatility. The long-term variation between years is relevant for hydropower plants since the reservoirs have insufficient capacity to compensate all of the precipitation variation between years. According to St. meld. nr 11 (1995-96), the annual variation in precipitation causes up to 40 TWh fluctuation in generation, which is relatively high compared with the 112 TWh annual production.

\(^1\) http://www.ssb.no/emner/01/03/10/husenergi/
\(^2\) http://www.dti.gov.uk/energy/images/icon_excel.gif
The prices vary between areas with mostly hydropower and regions with mostly thermal power when the transmission capacity between them is limited. Assuming both of the regions are competitive; the supply curve in the hydropower region is determined by ranking the power plants by increasing opportunity value of their available water, and in the thermal power region it is determined by ranking the plants by falling efficiency. The supply curve in the hydropower region will vary with the annual precipitation, whereas the supply curve in the thermal power region is independent of the weather. Assuming a downward sloping electricity demand curve, limited import and export capacities, and variable cost of transmission (losses); domestic prices will be higher in dry years, and lower in wet. In foreign electricity markets, where hydropower is not such a dominating energy source, the price does not fluctuate between years for the same reason. This gives an opportunity to reduce the domestic price variation between years by importing in dry and export in wet.

1.4 The electricity transmission demand distribution

I intend to investigate how the expected future changes in the statistical distributions of temperature and rainfall can be expected to alter the net import and transmission demand distributions.

According to NVE (1999) the Norwegian electricity market has been vertically separated into competitive generators, and regulated monopolistic transmission companies since 1991. Thus one of the requirements of efficient demand allocation can be believed to be satisfied. The generators, large consumers and electricity retailers submit their hourly prices and expected quantities to Nordpool, who arranges them into supply and demand curves, and determines the spot price as the level where the curves intersect. The functioning of the competitive market between the generators requires that the transmission lines are not congested so electricity demand can be allocated efficiently to the generators with the lowest water values.

As explained above; the prices in the hydro- and thermal power regions are not likely to follow the same paths unless there is unlimited transmission capacity. Theoretically it will be optimal to export hydropower at full capacity when the foreign price level exceeds the home price, and vice versa. If the foreign price path fluctuates around the fairly stable water value, the domestic hydropower generators can be expected exploit the price difference in excess of
the transmission tariff by importing to meet their supply responsibilities when the foreign prices are relatively low (during the night) and save their water for periods when it is relatively high (during the day). Nevertheless, the observations of hourly averages of electricity transmission sourced from the Nordpool ftp server, are not constantly at the capacity constraints but fluctuating between them. This might be because the capacity constraint varies over time if other bottlenecks appear between the generators and consumers than the cables physically crossing the borders. Furthermore, even if they were either importing or exporting at full capacity at all points in time; the hourly averages would be in-between if the direction of the energy flow changed within the hours.

The transmission tariffs in Norway are set by the Norwegian Water Resources and Energy Directorate (NVE)\(^3\). Ideally they should reflect long-run marginal costs. However, the networks consist of long-lived investments subject to economies of scale so the long run marginal costs are not easy to define. These economies of scale make network operators natural monopolies, which need to be regulated for economic efficiency. The Directorate regulate the transmission companies’ revenues such that their operating and depreciation costs are covered, and provide a reasonable return on capital given efficient operation, utilisation, and development of the grid.

The usable inflow and temperature paths at individual households are obviously exogenous variables; however they cannot be measured directly so the observed proxies may need to be treated as endogenous. Other exogenous variables varying between the regions or over time, like the real prices of substitute energy sources, household incomes and the number of daylight hours, may be needed to improve the consistency of the model.

1.5 Theoretical consequences of increased demand volatility

Today, consumers can chose between fixed price contracts of lengths varying between a month and a year, allowing them to share parts of the price risk with the generator\(^4\). A side effect of these contracts is that the reduction in price volatility will increase the quantity volatility since it reduces the short term demand elasticity.

\(^3\) http://www.nve.no
\(^4\) http://www.tussa.no
I have come across the following theoretical consequences of demand volatility when surveying literature for this project:

- Assuming inelastic short term demand, and stationary mean, a volatility increase would increase the probability of extremely high demand to occur, requiring higher investment in capacity in order to keep the probability of rationing at the optimal level. This theoretical cost can be estimated as the cost of the increase in optimal capacity as explained in the “newsboy model.” Ravindran et al. (1987)

- If the transmission companies had been regulated by a price cap, an asymmetric price regulation that limits the upside if the demand turns out to be higher than expected, but not the downside if demand turns out to be lower, there will be a real option value of delaying investment until actual demand is revealed, which will increase with volatility. Doobs (2004)

- If more energy needs to be transmitted in order to even out the increased supply and demand fluctuations, and transmission losses increasing approximately with the square of the transmitted energy, the increased cost of losses will be a consequence of the distribution change. Giancoli (1987)

Of the mentioned costs; only the first will be considered as the influence on expected quantity transmitted could not be identified, and the Norwegian transmission tariff regulation is symmetric.
2. Theory

2.1 Econometric theory

The plots in Figure 3 below of monthly import – export from Nordpool Elspot Flow (SFLO) and consumption – production from Production Operating Norway (PONO), show that the net import one month is correlated with its value the preceding month. In econometric terms, net import is likely to follow an autoregressive (AR) process.

Figure 3: Monthly net import

2.1.1 Decomposition of the net import variance

My hypothesis is that the volatility of net import can be explained by demand and supply variations caused by variations in temperature and precipitation. This statistical relationship will be used to decompose the net import variance by applying the definition of conditional variance,

\[ \text{Var}[y] = \text{Var}_x [E[y|x]] + E_x [\text{Var}[y|x]], \]
where $y$ represents the endogenous variable, $x$ the variables explaining it. $\text{Var}_x[\text{E}[y|x]]$ is the regression variance and $\text{E}_x[\text{Var}[y|x]]$ the residual variance. Equivalently it is written as total sum of squares = regression sum of squares + error sum of squares. The ratio of regression variance to residual variance is called the coefficient of determination, a frequently used measure of how well a model fits its sample.

2.1.2 Tests of heteroscedasticity, autocorrelation and normality

The model’s disturbance term is basically the sum of the impact of its omitted explanatory variables. In a time series where the omitted variables are likely to follow a seasonal pattern, the dynamic properties of the disturbances need to be tested to see if the assumptions for the estimation of the parameters are satisfied.

Heteroscedasticity in the form of persistence of squared disturbances, called Auto Regressive Conditional Heteroscedasticity (ARCH) effects, is likely to cause imprecise estimates of the regression coefficients, called inefficiency. A more serious problem in time series with lags of the dependent variable is persistence of the level of the disturbances, called autocorrelation, because it makes the disturbance and the explanatory variable correlated as explained in 2.1.3. This correlation makes it impossible to distinguish statistically the effect of the disturbance and the explanatory variable and the estimators will be inconsistent.

The relatively small sample size, around 70 months, makes it uncertain that the disturbances are normally distributed. The Chi square, F and t-tests of the hypothetical properties of the disturbances and coefficients require a normal distribution, and an improper assumption of normally distributed disturbances might lead to wrongly acceptance or rejection of the hypotheses about their statistical properties.

The PC Give tests statistic for ARCH effects is obtained from an auxiliary regression of the squared disturbances on a given number of its own lags. The test statistic, $TR^2$, is the number of observations multiplied with the coefficient of determination. This statistic is larger the higher the probability that the squared residuals are correlated. PC Give reports both the Chi square distributed statistic and its F approximation, which is assumed to be better behaved in small samples. The first parameter in the critical value for the F distribution is the number of lags (restrictions) in the auxiliary equation, and the second the degrees of freedom. The
parameter, \( s \), in the \( \chi^2(s) \) critical value is the number of imposed restrictions, i.e. the number of lags in the auxiliary equation. A variable follows a \( \chi^2(s) \) distribution if it is a sum of \( s \) squared variables that individually are independently standard normal distributed. The ratio of two Chi square distributed variables divided by their degrees of freedom follows an \( F \) distribution. Green (2000)

In PC Give, all reported test statistics are followed by the probability of observing a larger statistic under the null hypothesis. The null hypothesis is accepted at a five percent significance level if the chance of observing a larger test statistic in a sample where it actually is zero is between one and five per cent, and accepted at the one per cent level if it is smaller than one per cent. The opposite conclusion is made with the same confidence if the probabilities of observing a larger test statistic are 95 and 99 per cent respectively.

2.1.3 Autocorrelation tests and consequences of its presence

The Durbin Watson statistics indicating autocorrelation are automatically generated by PC Give. However, according to Green (2000) the DW method is not effective in models with lagged dependent variables, as the DW will be biased towards 2, i.e. towards not detecting autocorrelation. In single equation autoregressive time series models, the Portmanteau test is stronger. According to the PC Give manual, their Portmanteau statistic for a given number of lags, \( s \), is the squared sample size, \( T \), times the auxiliary equation sum of squared correlation estimates, \( r_j \), divided by its degrees of freedom.

\[
LB(s) = T^2 \sum_{j=1}^{s} \frac{r_j^2}{T-j}, \tag{2.2}
\]

Under the assumptions of the test, LB(s) is asymptotically distributed as \( \chi^2(s - n) \), where \( n \) is the lag length of the dependent variable in the main equation. The Lagrange multiplier (LM) test of autocorrelation is analogous to the ARCH effect test, but without squaring the disturbances in the auxiliary regression. Also in this case the test statistic, the square coefficient of determination, \( R^2 \), times the sample size, \( T \), will be modelled as \( F \) and \( \chi^2 \) distributed under the null hypothesis.
In an equation with lagged dependent variables, error autocorrelation will result in inconsistent estimators. If for example the error term, $\epsilon_t$, follows a first order autoregressive process, $\epsilon_t = \rho \epsilon_{t-1} + u_t$, where $u_t$ is white noise, it can be demonstrated that the disturbance will be correlated with the explanatory variables in a model with lagged dependent variables. The following resembles the equation for net electricity import $y_t$ depending on its own lag and other explanatory variables represented by $x_t$.

\[(2.3)\quad y_t = \alpha_0 + \alpha_1 y_{t-1} + \beta_1 x_t + \epsilon_t.\]

The following explanation of why autocorrelation causes inconsistent coefficients is a modification of an example in Green, W. (2000).

\[(2.4)\quad \text{Cov}(y_{t-1}, \epsilon_t)\]

Inserting the autoregressive equation for the error term;

\[= \text{Cov}(y_{t-1}, \rho \epsilon_{t-1} + u_t)\]

the white noise $u_t$ is uncorrelated with everything that precedes it,

\[= \rho \text{Cov}(y_{t-1}, \epsilon_{t-1})\]

and since the process is stationary;

\[= \rho \text{Cov}(y_t, \epsilon_t)\]

Inserting the equation for $y_t$;

\[= \rho \text{Cov}[\alpha_0 + \alpha_1 y_{t-1} + \beta_1 x_t + \epsilon_t, \epsilon_t],\]

and applying the definition of a covariance of a sum,

\[= \rho \{\alpha_1 \text{Cov}(y_{t-1}, \epsilon_t) + \beta_1 \text{Cov}(x_t, \epsilon_t) + \text{Cov}(\epsilon_t, \epsilon_t)\}\]

since $x_t$ is measured without error and thus exogenous and uncorrelated with the disturbance $\text{Cov}(x_t, \epsilon_t) = 0$

\[= \rho \{\alpha_1 \text{Cov}(y_{t-1}, \epsilon_t) + \var(\epsilon_t)\} = \rho \alpha_1 \text{Cov}(y_{t-1}, \epsilon_t) + \rho \sigma_u^2\]

Therefore by $\text{Cov}(y_{t-1}, \epsilon_t) = \rho \text{Cov}(y_t, \epsilon_t)$ and the stationarity assumption; the covariance can be expressed explicitly by the regression parameters;

\[(2.5)\quad \text{Cov}[y_{t-1}, \epsilon_t] = \frac{\rho \sigma_u^2}{(1 - \rho \alpha_1)(1 - \rho^2)}\]

For the least squares estimator we can use the general result for the coefficient estimator
In order to express the limiting estimator explicitly in terms of the equation parameters, an expression for the denominator needs to be developed as well.

\[
\text{Var}[y_t] = \alpha_1^2 \text{Var}[y_{t-1}] + \beta_1^2 \text{Var}[x_t] + \text{Var}[\epsilon_t] + 2\alpha_1 \text{Cov}[y_{t-1}, \epsilon_t] + 2\beta_1 \text{Cov}[x_t, \epsilon_t]
\]

Since the process is stationary, \(\text{Var}[y_t] = \text{Var}[y_{t-1}], \text{Var}[x_t] = \sigma_x^2\) and \(\text{Var}[\epsilon_t] = \frac{\sigma_u^2}{1 - \rho^2}\)

Collecting terms to express the variance as a function of the moments,

\[
\text{Var}[y_t] = \frac{\beta_1^2 \sigma_x^2 + \frac{\sigma_u^2}{1 - \rho^2} + 2\alpha_1 \frac{\rho \sigma_u^2}{(1 - \rho \alpha_t)(1 - \rho^2)}}{1 - \alpha_1^2}
\]

rearranging it,

\[
\text{Var}[y_t] = \frac{\beta_1^2 \sigma_x^2 (1 - \rho^2)(1 - \rho \alpha_t)}{(1 - \rho^2)(1 - \rho \alpha_t)(1 - \alpha_1^2)} + \frac{\sigma_u^2 (1 - \rho \alpha_t)}{(1 - \rho^2)(1 - \rho \alpha_t)(1 - \alpha_1^2)} + 2\alpha_1 \frac{\rho \sigma_u^2}{(1 - \rho \alpha_t)(1 - \rho^2)(1 - \alpha_1^2)}
\]

The variance and covariance in terms of the disturbance moments can be inserted into the estimator to demonstrate its inconsistency;

\[
p \lim a = \alpha_1 + \frac{\rho \sigma_u^2}{(1 - \rho \alpha_t)(1 - \rho^2)}
\]

\[
p \lim a = \alpha_1 + \frac{\rho \sigma_u^2 (1 - \alpha_1^2)}{\beta_1^2 \sigma_x^2 (1 - \rho^2)(1 - \rho \alpha_t) + \sigma_u^2 (1 + \alpha_1 \rho)}
\]

\[
p \lim a = \alpha_1 + \frac{\rho \sigma_u^2 (1 - \alpha_1^2)}{\beta_1^2 \sigma_x^2 (1 - \rho^2)(1 - \rho \alpha_t) + \sigma_u^2 (1 + \alpha_1 \rho)}
\]
Therefore least squares is inconsistent unless $\rho$ equals zero

One explanation of autocorrelation in time series models are misspecification in the form of omission of variables that are correlated across periods. The obvious treatment would be to identify and include them if available. If not, the inconsistency caused by the autocorrelation could be amended using instrumental variables. This method requires that the potential instruments can be proved to be correlated with the endogenous explanatory variable and uncorrelated with the disturbance.

### 2.1.4 Modelling persistent squared disturbances

If the null hypothesis of no ARCH effects is rejected, the model can be made more efficient using the PC Give volatility package. According to Hamilton (1999) the ARCH model is basically a set of two equations modelled simultaneously; one for the statistical relationship we want to investigate and an autoregressive equation of the squared disturbances from the main equation.

The simplest form of the model is the ARCH(1),

$$y_t = \beta x_t + \epsilon_t$$  \hspace{1cm} (2.10)

$$\epsilon_t^2 = u_t^2(\alpha_0 + \alpha_1 \epsilon_{t-1}^2),$$  \hspace{1cm} (2.11)

where $u_t$ is white noise

It follows that under the hypothesis of no autocorrelation,

$$E[\epsilon_t | x_t, \epsilon_{t-1}] = 0,$$

so that $E[\epsilon_t | x_t] = 0$, and

$$E[y_t | x_t] = \beta x_t,$$

but

$$\text{Var}[\epsilon_t | \epsilon_{t-1}] = E[\epsilon_t^2 | \epsilon_{t-1}] = E[u_t^2 | \epsilon_{t-1}](\alpha_0 + \alpha_1 \epsilon_{t-1}^2) = \alpha_0 + \alpha_1 \epsilon_{t-1}^2$$

so $\epsilon_t$ is conditionally heteroscedastic with respect to $\epsilon_{t-1}$. If the process is stationary such that the residual’s unconditional variance exists, it can be calculated using the definition of conditional variance.
(2.12) \( \text{Var}[\varepsilon_t] = \text{Var}[E[\varepsilon_t \mid \varepsilon_{t-1}]] + E[\text{Var}[\varepsilon_t \mid \varepsilon_{t-1}]] = \alpha_0 + \alpha_1 \text{Var}[\varepsilon_{t-1}] \)

if the distribution is stationary then
(2.13) \( \text{Var}[\varepsilon_t] = \text{Var}[\varepsilon_{t-1}] = \frac{\alpha_0}{1 - \alpha_1} \)

The most efficient way of solving this set of regression equations is numerically by a non-linear maximum likelihood.

If the ARCH effects die out gradually a GARCH model with a small number of terms is likely perform better than an ARCH model with many. In the GARCH model, the disturbance, \( \varepsilon_t \), conditional on information available at time \( t \), \( \Psi_t \), is assumed to be
\( \varepsilon_t \mid \Psi_t \sim \text{N}[0, \sigma_t^2] \)

The equation for the conditional variance will be,

(2.14) \( \sigma_t^2 = \alpha_0 + \delta_1 \sigma_{t-1}^2 + \delta_2 \sigma_{t-2}^2 + \cdots + \delta_p \sigma_{t-p}^2 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 \varepsilon_{t-2}^2 + \cdots + \alpha_q \varepsilon_{t-q}^2 \)

or in lag operator polynomials

(2.14b) \( \sigma_t^2 = \alpha_0 + D(L) \sigma_{-1}^2 + A(L) \varepsilon_t^2 \)

If the process is stationary, indicated by no unit roots in the lag polynomials, then the unconditional variance exists and will be the following:

(2.15, 2.16) \( \text{Var}[\varepsilon_t] = \alpha_0/[1 - A(1) - D(1)] \), and \( \text{Cov}(\varepsilon_t, \varepsilon_s) = 0 \) for all \( s \neq t \)

### 2.1.5 Tests for normality and consequences of its absence

The PC Give tests for normality are based on Doornik and Hansen (1994), who employ a small sample correction, and adapt the test for the multivariate case. The test is basically to check whether the main indicators of normality, the residuals’ skewness and excess kurtosis,
are significantly different from what would have been observed in a normally distributed simulated sample.

Their test approximately $\chi^2(2)$, statistic is; $e_2 = z_1^2 + z_2^2$ where $z_1$ and $z_2$ denote the transformed skewness and kurtosis. The correction is supposed to create statistics much closer to standard normal in small samples.

If the disturbances are non-normal, the parameters could be estimated by the pseudo maximum likelihood method. White (1982) and Weiss (1982) showed that maximizing the same maximum likelihood function as if it were correct produces consistent estimators despite the misspecification. However, the variances of the disturbances and slope parameters need to be adjusted in order to get the true probability of observing a larger value of the test statistic under the null hypothesis than what was actually observed.

2.1.6 Measurement errors

The degree day measure defined in chapter three is an aggregate measure of monthly temperatures at representative weather stations. It serves as a proxy for a continuous measure of the temperature path outside individual households, which cannot be measured directly. Similarly, the aggregation of precipitation measured at representative meteorological stations is a proxy for the individual dams’ unobservable usable inflow. In this case the observations themselves might not contain sufficient information to determine the effect of the latent variables, and additional information from instrumental variables could be required to get consistent estimators.

The most important consequence of measurement errors is the same as for autocorrelated disturbances; the disturbance is correlated with the explanatory variable so their effect cannot be distinguished. The presence of autocorrelation makes lags of the regressors’ obviously ineffective as instruments as they will also be correlated with the disturbance.

The reservoir levels, or their change from the preceding month, are likely to be correlated with the latent usable inflow, and perhaps uncorrelated with their measurement error. An instrument variable model could be made by defining the reservoir change or level as instruments and declaring the precipitation endogenous (because of its measurement error).
A lagged variable cannot be declared endogenous in PC Give, but it is possible to create a new variable of lagged observations and declare it as endogenous.

2.2 Theoretical consequences of increased weather volatility

2.2.1 Inferring transmission demand distribution from net import moments

The model is based on the assumption that the temperature path explains local electricity demand and the history of net inflow explains local supply. The quantity transmitted at any point in time is the difference between local supply and demand. The difference between average supply and demand will obviously underestimate the transmission since the direction of the transmission can fluctuate within the period the average is taken over. The error will be smaller the shorter the interval, and at points in time when flow can only go in one direction, the transmitted quantity equals the absolute value of import.

Since the supply cannot be expected to respond to precipitation at points in time, only its history since the water can be stored in dams, the observations need to be aggregated into longer series. This aggregation makes it impossible to identify transmission demand directly from observed local supply and demand, so only the conditional distribution of net import can be identified. The emphasis will be on the impact of the weather on the transmission volatility and not its level, so the expected values of net import and transmission demand will not modelled.

Once the statistical relationship between the net import and the weather variables is estimated, the second moment can be decomposed in order to see how it is affected by the changes in the moments of precipitation and temperature. The response of the unconditional volatility to changes in the future moments of temperature and precipitation will be illustrated by plotting the net import standard deviation for various values of them.

An estimated model makes it possible to decompose the net import volatility into two parts; the regression volatility which is the variation explained by precipitation and temperature, and the unexplained residual volatility. Defining net electricity import as import, $Y$, minus export, $X$, and transmission as import plus export, the second moment of the transmission quantity can be inferred from the second moment of net import after writing both of them as
functions of the unobservable import and export moments, and use the two equations to eliminate them.

Variance of net import, \( Y - X \);

\[ (2.17) \quad \text{Var}(Y + (-1)X) = 1^2 \text{Var}(Y) + (-1)^2 \text{Var}(X) + 2 \cdot 1(-1) \text{Cov}(X,Y) \]

\[ = \text{Var}(Y) + \text{Var}(X) - 2 \cdot \text{Cov}(X,Y) \]

Analogously for transmitted quantity, \( Y + X \);

\[ (2.18) \quad \text{Var}(X + Y) = 1^2 \text{Var}(Y) + 1^2 \text{Var}(X) + 2 \cdot 1 \cdot 1 \text{Cov}(X,Y) = \text{Var}(Y) + \text{Var}(X) + 2 \text{Cov}(X,Y) \]

so the net import variance can be calculated by combining equations 17 and 18 above and cancelling the unobservable moments of gross import and export, \( \text{Var}(Y) \) and \( \text{Var}(X) \).

\[ (2.19) \quad \text{Var}(Y - X) = \text{Var}(Y + X) - 4 \text{Cov}(X,Y) \]

The covariance between export, \( X \), and import, \( Y \), is demonstrated to be zero by applying the definition of covariance and the physical constraint that the energy can only flow one way through a given connection at any point in time.

\[ (2.20) \quad \text{Cov}(X,Y) = \frac{1}{n-1} \sum_{i=1}^{n} (X - \mu_X)(Y - \mu_Y) \]

Separating the sum applying that \( X = 0 \) if \( Y > 0 \) and vice versa;

\[ (2.21) \quad \text{Cov}(X,Y) = \frac{1}{n-1}( -\mu_Y \sum_{i=1}^{n} (X - \mu_X) |_{y=0} + \frac{1}{n-1}( -\mu_X \sum_{i=1}^{n} (Y - \mu_Y) |_{x=0}, \]

inserting the empirical means, \( \mu_X = \frac{1}{n-1} \sum_{i=1}^{n} X \) and \( \mu_Y = \frac{1}{n-1} \sum_{i=1}^{n} Y \), shows that the covariance would have to be zero. Consequently, when the covariance between import and export is zero at points in time, the second moments of net import and transmission must be equal.
2.2.2 Conversion from monthly to hourly moments

Conversion of the monthly demand moments to hourly will be necessary since the measure of transmission capacity is the maximum effect (MW) at points in time, and not monthly averages of energy transmission (MWh/month). The hourly moments will also be a simplification, but it is as close to a time path it is possible to get.

If necessary, the conversion of the mean would simply be to divide the monthly average by the number of hours in a month. Conversion of the second moment is slightly more complicated as it requires the covariance structure between all hours within each month. The simplest model of the covariance structure is to model the hourly transmission as following an AR(1) process. Then the correlation coefficient, \( \rho \), is obtained from the following regression where \( x_t \) is the hourly export,

\[
(2.22) \quad x_t = \rho x_{t-1} + u_t
\]

Solving this recursively, the statistical relationship between two hours’ transmission an arbitrary length of time apart, can be calculated as follows.

\[
(2.23) \quad x_t = \rho (\rho x_{t-2} + u_{t-1}) + u_t
\]

\[
= \rho^2 x_{t-2} + \rho u_{t-1} + u_t
\]

etc

\[
= \rho^n x_{t-n} + \rho^{n-1} u_{t-(n-1)} + ... + \rho^2 u_{t-2} + \rho u_{t-1} + u_t
\]

where the impact of lagged variables decline with the lag length when \( \rho < 1 \).

Inserting

\[
x_i = \rho^i x_j + \rho^i j^{-1} u_{j-1} + ... + \rho^2 u_{t-2} + \rho u_{t-1} + u_t; \ i > j
\]

into,
(2.24) Cov\( (x_i, x_j) = E[(x_i - E[x])(x_j - E[x])] \)

This gives an expression for the covariance between two observations an arbitrary length, \( i - j \), apart provided the hourly export covariance structure actually follows an AR(1) process.

Inserting \( x_i \) as a function of \( x_j \), omitting the zero terms \( E[u_t] \) and \( Cov(x_s, u_t) \) when \( s \neq t \).

(2.25) Cov\( (x_i, x_j) = E[(\rho^{ij}x_j + \rho^{ij-1}u_j + \ldots + \rho^2u_{i-2} + \rho u_{i-1} + u_i - E[x_j])(x_j - E[x_j])] \)

\[ = \rho^{ij} E[x_j^2] - \rho^{ij} E[x_j E[x]] - E[x_j E[x]] + E[x]^2 \]

\( E[x_i] = E[x_j] = E[x] \), and \( E[x_j^2] = E[x^2] \) when the distribution is stationary

\[ = \rho^{ij} E[x^2] - \rho^{ij} E[x]E[x] - E[x]^2 + E[x]^2 \]

\[ = \rho^{ij} Var[x] \]

Covariance between hour number \( i \) and \( j \) when their individual variances are identical and equal to \( \sigma^2_h, \sigma_{ij} = \sigma^2_h \rho^{ij}, i > j \). The monthly variance can be expressed as a function of the hourly variances using the definition of a variance of a sum; and sum all the hourly variances and covariances within each month. Assuming the correlation, \( \rho \), between two adjacent months and hourly variance is stationary; the relation between the estimated monthly variance, \( \sigma^2_m \), and \( \sigma^2_h \), is the following in an \( M \) hour month:

(2.26) \[ \frac{\sigma^2_m}{\sigma^2_h} = \sum_{j=1}^{M} \sum_{i=1}^{M} \rho^{|j-i|} \]

The assumption that the correlation matrix is symmetric makes it possible to avoid the absolute value signs,

\[ = M + 2 \sum_{j=1}^{M-1} \sum_{i=1}^{M-j} \rho^j \]
where $M$ is the sum of elements on the diagonal, and the double sum the elements at its sides.

The sums of correlation coefficients can be simplified as it is a finite geometric series,

$$
\sum_{j=1}^{N} \rho^j = \frac{\rho(1-\rho^N)}{1-\rho},
$$

provided the process is stationary so the correlations, $\rho$, are smaller than one.

$$
\frac{\sigma_m^2}{\sigma_h^2} = M + 2 \sum_{j=1}^{M-1} \frac{\rho(1-\rho^{M-j})}{1-\rho}
$$

setting the non-indexed parameters outside the sum,

$$
= M + 2 \frac{\rho}{1-\rho} \sum_{j=1}^{M-1} (1-\rho^{M-j})
$$

splitting up the sum,

$$
= M + 2 \frac{\rho}{1-\rho} \left( (M-1) - \rho^M \sum_{j=1}^{M-1} \rho^{-j} \right)
$$

applying the finite sum formula again,

$$
= M + 2 \frac{\rho}{1-\rho} \left( (M-1) - \rho^M \frac{\rho^{-1}(1-\rho^{M-1})}{1-\rho^{-1}} \right)
$$

collecting terms,

$$
= M + 2 \frac{\rho}{1-\rho} \left( (M-1) - \frac{\rho^{M-1}-1}{1-\rho^{-1}} \right)
$$

Solving with respect to $\sigma_h^2$,

$$
(2.27) \quad \sigma_h^2 = \frac{\sigma_m^2}{M + 2 \frac{\rho}{1-\rho} \left( (M-1) - \frac{\rho^{M-1}-1}{1-\rho^{-1}} \right)}
$$
Thus if the hourly variance is stationary, it will be proportional to the monthly. If it still holds that the hourly variance is a fixed fraction of the monthly even if the actual hourly covariance structure is more complicated than above; a slightly easier approach without estimating its exact covariance structure could be used. Then the assumed constant ratio of hourly to monthly sample moments could used to scale down the decomposed monthly variance to the level the hourly would have been if the monthly changed.

2.2.3 The newsboy model

In the model outlined in Ravindran et al. (1987) there are two deterministic equations for the net benefit of investment conditional on whether demand has exceeded capacity or not. If demand, \( x \), has not exceeded capacity, \( Q \), the net benefit will equal the demanded quantity times its value, \( s \), less the fixed, \( a \), and variable costs, \( c \), of capacity.

\[
B(Q|x) = sx - a - cQ \quad \text{if } x \leq Q
\]

If demand exceeds capacity there will be a penalty of \( p \) per unit unmet demand. E.g. due to allocation inefficiency if the lowest cost generator becomes unavailable when the lines are congested.

\[
B(Q|x) = sQ - p(x - Q) - a - cQ \quad \text{if } x > Q
\]

Expected net benefit if the demand is modelled as following a continuous distribution \( f(x) \);

\[
E[B(Q)] = \int_0^Q sx f(x)dx + \int_Q^\infty ((s + p)Q - px) f(x)dx
\]

If we assume for simplicity the parameters do not depend on the chosen capacity, net expected benefit can be maximised with respect to capacity applying Leibniz’s rule to the two parts of the equation individually.

\[
\frac{dE[B(Q)]}{dQ} = (s + p)\int_0^\infty f(x)dx - c = 0
\]
rearranging and applying the definition of the cumulative distribution function, $F(Q)$

\[
(2.32) \quad \int_{Q}^{\infty} f(x)dx = 1 - F(Q) = \frac{c}{s + p}
\]

When the transmission demand is following a normal distribution, the optimal capacity can be expressed explicitly using the inverse cumulative standard normal distribution, which can be converted to non-standard using the sample moments. Finally the optimal capacity will be as follows provided the fixed investment costs can be expected to be covered:

\[
(2.33) \quad Q = \mu + \sigma \Phi^{-1}\left[1 - \frac{c}{s + p}\right], \text{ where } \mu \text{ and } \sigma \text{ are the moments of the demand.}
\]

Thus, holding the mean constant, the optimal capacity will be a linear function of the transmission demands’ standard deviation. An estimate of the cost of increased weather volatility will be the cost of the increase in optimal capacity, which is directly proportional to the increase in the demand standard deviation.
3. Data and estimation method

The final data set consists of monthly observations of temperatures, precipitation, and electricity production and consumption from January 1999 till October 2004.

3.1 Weather data

The data describing the weather are downloaded from the Norwegian Meteorological Institute\(^5\) eKlima service. This service does unfortunately not give away time series for all stations, so I have chosen the ones believed to be most representative. Populations for the cities, published by Statistics Norway, are used as weights for the temperature aggregate, and reservoir capacities, published by the Norwegian Water Resources and Energy Directorate, as weights for the precipitation observations.

3.1.1 Construction of weighted degree day measure from city temperatures

The area near the border between the two Norwegian Elspot areas are not densely populated so approximating the border by the closest county borders might not lead to an important error. Using county borders, NO1 represents about 74 per cent of the population, and about 73 per cent of the reservoir capacity measured in energy-units and NO2 about 26 and 27 per cent respectively.

The map in Figure 4 below of the Nordic Elspot regions was copied from www.statnett.no 30.08.2004.

\(^5\) http://met.no/
Oslo/Bærum, Stavanger and Bergen represents about 29 percent of the population of NO1, and Ålesund, Trondheim, Bodø and Tromsø represent about 25 per cent of the population in NO2. The temperatures are measured daily and will be aggregated to monthly degree day measures applying the formula outlined below.

Assuming the daily energy demand increases linearly by $a$ energy units per degree the temperature falls within a limited interval, the monthly energy demand would be proportional to a weighted sum of them. Houses with modern winter insulation are getting some heat from electric appliances, humans, radiation through windows etc, giving an upper bound, $u$, for the temperature range where heating demand depend on the outdoor temperature. Similarly a lower threshold, $l$, would be a consequence of capacity constraints in heating appliances and installed wiring. Ignoring any causes of nonlinearities in demand, the contribution to energy demand a day $i$ with temperature $x_i$ would be:

$$
\begin{align*}
  a & 0, & \text{if } x_i \geq u \\
  a(u - x_i), & \text{if } l \leq x_i < u \\
  a(u - l), & \text{if } x_i < l
\end{align*}
$$

Graphically it can be illustrated as in Figure 5 below;
Figure 5 Hypothetical energy demand vs temperature

A more accurate relationship might be modelled by allowing different slopes for varying temperature intervals. Total energy demand during an $m$ day month with temperature $x_i$ a given day would be:

\[(3.1) \quad E = a \sum_{i=1}^{n} (u - l)1_{x_i < l} + (u - x_i)1_{l \leq x_i < u}\]

Where $1_{l \leq x_i < u}$ is an indicator function.

The demands sensitivity to the degree days defined above, $a$, will be a coefficient in the net import regression. The upper and lower thresholds, $u$ and $l$, are coefficients that would need to be determined by the model using trial and error for values to see which ones give the best fit unless a suitable Maximum Likelihood could be formulated.

The histogram below illustrates that the daily temperatures are slightly skewed.
3.1.2 Aggregation of precipitation to a representative measure

Aggregate winter and summer precipitation in the counties having the largest reservoir capacities are used to represent the consequences of the precipitation variance for the energy supply within each Elspot region. According to NVE, the counties with the largest shares of NO1 reservoir capacity in descending order are; Hordaland (18%), Sogn & Fjordane (15%), Rogaland (13%), The largest in descending order in the NO2 area is; Nordland (45%), Møre & Romsdal (19%), and Sør-Trøndelag (14%). The representative precipitation will be the weighted by these relative shares.

Precipitation is aggregated to monthly sums of daily precipitation measured in mm at the representative stations, and the aggregate for the Elspot region is simply the average of the representative station in the representative counties weighted by their reservoir capacities. As
for the degree day measures; using sums of the daily observations within each month instead of average will account for parts of the varying lengths of the months.

![Precipitation histogram](image)

**Figure 7: Daily precipitation**

The histogram in Figure 7 of the daily sample precipitation shows that its distribution is asymmetric, with wet days occurring less frequently the wetter they are.

### 3.2 Electricity data

#### 3.2.1 Transfer losses

In alternating current circuits there are two types of energy loss; fixed and loss varying with the current. The loss that does not vary with the current occurs in the fluctuating magnetic field in the iron core of the transformers. This is relatively small in comparison to the loss caused by current flowing through transmission lines, cables and transformer windings and causing them to heat up. DTI (2003)
According to Ohm’s law the variable loss of energy is proportional to the cables’ resistance, which depends linearly on their length and the square of the transmitted current, and non-linearly on their temperature. Electrical energy is defined as current times its voltage, so for constant voltage; transmission losses will be proportional to the square of the transmitted energy for a given line.

According to NordPool, Grid losses are treated as “electricity consumption” by the transmission system operator (TSO)\(^6\). When the reported quantities are the energy supply to the grid, the electricity “consumed” by the grid operator needs to be subtracted from imports as it is unavailable for household consumption. Thus the observed net import would overstate the actual net import unless it is adjusted for the transmission losses.

Statnett publishes expected marginal losses for each hub intended to calculate tariffs for transmission companies. The published marginal losses can be negative if an operator can be expected to be relieving the grid if it supplies to a hub in a deficit area, or transports energy away from a hub in a surplus area. These losses are always symmetric, if energy demand leads to positive marginal losses of a given percentage, energy supply would lead to negative losses of the same magnitude but opposite sign.

However, the marginal losses will overstate the average losses that become unavailable to the consumers since they increase more than proportionally with the transmitted quantity. Unfortunately, I have not come across any useful transmission loss time series.

**3.2.2 Production and consumption from Nordpool**

The two most relevant data sets I have come across that can be used to infer the net electricity import are downloaded from the Nordpool ftp server. “Operating data Norway” contains Nordic production and consumption on and off from 1996 onwards, with the relevant series continuously from January 1999 on. Another dataset called “Elspot flow” contains import and exports between Elspot regions from mid 1999 onwards for NO1 and end 2000 for NO2. Both of them are organized with observations as hourly rates measured in MWh/h for each hour. Only the longest data set that could not be separated into NO1 and NO2 turned out to have sufficient number of observations for the estimations.

The files are updated once a day, after the price has been set. The series had to be aggregated and organized into monthly observations, and Visual Basic and Excel pivot table reports turned out to be handy. Some of the Elspot areas have merged and split over the years, so I have attempted to aggregate them into the same as the present in order to get longer continuous time series. Transmissions between the areas abroad are omitted because of the lack of weather data for the Elspot areas outside Norway. From the beginning of the sample until the end at October 2004 there are 70 months, the summer months do however not show much variation in the degree day measure and may not contribute to the identification of the temperature coefficients; however they are be kept in the sample to give more precise estimates of the other.

### 3.3 Other explanatory variables

Income growth can be expected to affect consumption multiplicatively and perhaps with a lag if the consumers prefer to even out short term variations in their consumption. From the beginning of the sample in January 1999 until the end in September 2004, GNP\(^7\) deflated by the CPI\(^8\) grew by 23 per cent. Over the four years up to September 2004, annual electricity consumption fell by 3.5 per cent, which could indicate that the effect of income growth is dominated by other factors during the length of the sample.

Norsk Petroleumsinstitutt\(^9\) publishes annual averages of recommended retail hating oil prices from 1991 on, and from 2000 on they have continuously published all price changes and the dates when they occurred. Using the consumer price index as deflator and the average price in 1999 as base, the real price had risen by 31 per cent in September 2004.

Investment and scrapping of domestic electricity generation capacity might also influence net import demand, but will be omitted as an explanatory because of its infrequent variation in the relatively short sample.

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\(^7\) [http://www.ssb.no/emner/09/01/knr/](http://www.ssb.no/emner/09/01/knr/)

\(^8\) [http://www.ssb.no/emner/08/02/10/kpi/](http://www.ssb.no/emner/08/02/10/kpi/)

\(^9\) [www.np.no](www.np.no)
3.4 Summary

Indexing the monthly import, export, degree days, and precipitation at 100 in January 1999 shows that production follows precipitation with a lag and degree days follows the same cycle as consumption. It also shows that production is higher than consumption when precipitation has been relatively high during the preceding months and vice versa.

![Graph showing indexed plot of the most relevant variables]

Figure 8: Indexed plot of the most relevant variables

Furthermore, as the variables show some persistence in levels the process can be expected to follow an autoregressive process. A regression of net import, i.e. consumption-production that is consistent with the plots above is the following;

\[(3.2) \quad (1 - L)(\text{Consumption} - \text{Production}) = \beta_0 + L \beta_1 \text{Precipitation} + \beta_2 \text{Degree days}\]

where L is a lag operator and the $\beta_i$'s are the coefficients to be estimated.
4. Empirical analysis

The empirical analysis will attempt to test my hypothesis that parts of the statistical variation in the energy transmission demand can be explained by variations in climatic conditions such as snow- or rainfall and temperature.

4.1 Local electricity demand

Henley and Peirson (1997) concluded that the daily electricity demand in the UK is a non-linear function of the temperature. It is minimal at outdoor temperatures slightly less than normal indoor temperature, and it increases as it gets either colder or warmer. Døhl (1999) has found a similar relationship for Norway. He suggested that temperature variations explain short term demand variations around a long term trend, and the long term trend depends on income and relative prices. The emphasis in this project will be on the volatility, i.e. variations around the long term trend, so the likely influence of any long term explanators will only be modelled if necessary to get consistent estimates of the short term coefficients. Furthermore, the sample size is fairly short so it might not contain enough variation in the long term explanators to identify their likely effect.

The closest substitute for electricity used for heating in Norway is heating-oil. The demand for electricity could not be expected to be affected much by price fluctuations in the short run unless as a fairly large share of the consumers are able to switch between these two energy sources. Including real heating oil price as an explanatory variable will reduce the already small sample for the NO region by a year, and the estimation of the coefficient itself reduces the degrees of freedom further.

The standard deviation of a coefficient is inversely proportional to the variation in the corresponding variable so a plain temperature average, which evens out temperature variations within each month, will be an inefficient explanator of electricity demand variations. In order to capture most of the effects of its variations, the degree day measure defined in chapter three will be used instead. Looking at the time plot of consumption in Figure 8, chapter three; electricity demand could be expected to follow an autoregressive
process. However, when including the mentioned degree day measure, which is likely to explain parts of the seasonal variations; the autoregressive coefficient gets a low value and the t-probability reported by PC Give is near \( \frac{1}{2} \), and thus inconclusive. This t-probability basically says that the probability of observing the same t-value, a measure of the coefficient estimate precision, would have a fifty-fifty chance of being larger or smaller by coincidence in a random sample. After trying various upper and lower thresholds for the degree day measure it turned out that temperatures in the range between -10 and +17 Celsius gave the best fit. An OLS regression of Norwegian consumption on the degree day measure resulted in the following equation:

\[
\text{NOcons} = 6.81 \cdot 10^6 + 9254 \times \text{degree days}, \quad R^2 = 0.943578
\]

(SE): \( 1.05 \cdot 10^5 \) (276)

![Figure 9 Consumption versus weighted degree days](image)

As can be seen from the histogram in Figure 6, chapter three, most of the temperature observations are within the range where electricity demand is most sensitive to temperature variations, so the error from choosing a too narrow band is likely to be larger than the error from choosing a too wide one.
The Chi-square and the F-tests for the disturbance properties are based on that they are normally distributed, so it might be useful to test the probability that they are first. The $\chi^2$ distributed test statistic of normality is higher the less likely the disturbances are to follow a normal distribution. The test concluded there was only a ten per cent chance of observing a larger statistic under the null hypothesis that the disturbances actually were normally distributed. The consequence of non-normality is that the tests assuming normality based on them could be misleading.

Both the Chi-square and F-tests for error autocorrelation resulted in larger test statistics than almost certainly would be observed under the null hypothesis of no autocorrelation. The main consequence of autocorrelation in an equation without any lagged endogenous variables is inefficiency of OLS relatively to GLS. GLS is a regression on variables that are transformed by applying known- or consistently estimated correlation coefficients. The indicated error autocorrelation is illustrated in the Figure 10 below showing the residual persistence.

![Figure 10: Regression residuals from consumption versus degree days](image)
The null hypothesis of no ARCH effects was rejected by the F-test, as there was only a 0.15 per cent chance of observing a larger test statistic. Furthermore, the Chi-square test of the null hypothesis of no variable dependent heteroscedasticity using squares was rejected at one per cent significance level. The equation below for squared consumption residuals is obtained by the general to specific procedure.

\[(4.2) \quad \mu_t^2 = 1.286 \cdot 10^6 \text{ degree days}^2 \]
\[(\text{SE}): \quad (2.25 \cdot 10^5)\]

When regressing the squared residual on its first lag; both the lag and the intercept remained significant:

\[(4.3) \quad \mu_t^2 = 1.212 \cdot 10^{11} + 0.3854 \mu_{t-1}^2 \]
\[(\text{SE}): \quad (4.63 \cdot 10^{10}) \quad (0.113)\]

![Figure 11: Illustration of the ARCH effect](image-url)
Despite the time- and variable dependent heteroscedasticity and the autocorrelation, the reasonably good fit shows that inefficiency is not a serious problem when estimating the effects of degree days on electricity consumption.

A possible explanation of why the electricity demand disturbances would be correlated with the squared degree day measure could be the linear approximation of the relationship that the mentioned studies proved to be nonlinear. The linear approximation makes the expected demand further away from the actual at extreme temperatures, where the error from the linearization will be greatest.

An explanation of the disturbance autocorrelation and ARCH effects could be that the disturbance term includes omitted variables that vary seasonally, for example that it is darker during the winter increasing demand for illumination, or higher prices reducing it relative to what the temperature explains. If the electricity demands’ sensitivity to changes in temperature, its coefficient, varies seasonally but is constrained to be constant; the unexplained part of the variation will vary seasonally. Furthermore, the extent of the measurement error might vary seasonally if the difference between the average temperatures at the representative stations and the temperature paths at the households who are actually consuming electricity for heating vary between the seasons.

### 4.2 Local electricity supply

The model is based on the assumption that the electricity supply in a given area is depending on the history of production and usable inflow. Aggregated precipitation over representative stations is used as a proxy for the unobservable inflow, which can be expected to result in the common measurement error problem; inconsistent estimators. Furthermore, the correlation between latent usable inflow and precipitation observed at representative stations might vary between the different regions if for example the reservoir capacities are different. If extreme inflow to the smallest reservoirs spill over and rainfall after extremely dry periods get absorbed, the available amount of energy is likely to be non-linearly depending on precipitation.

The usable inflow is the sum of rainfall, melted ice and snow that finally reach the reservoirs, with melting dominating during the spring and rainfall during the autumn. Winter
precipitation can be expected to explain inflow with longer lags than summer rain, and it probably matters less when the snow fell during the winter as it cannot flow from the watershed into the reservoir until it melts. Thus usable inflow from precipitation is not unlikely to depend on the season it fell as well as its quantity. Bye (2002) has a more detailed explanation of how the usability of precipitation varies within the year. Furthermore, the magnitude of a possible measurement error could be expected to vary seasonally.

In order to keep the model parsimonious, aggregated precipitation and lagged production for fairly long periods need to be used to explain electricity supply. The sample size made it necessary to aggregate summer and winter precipitation into one single measure, as the number of winter month observations is quite small. A quarter of the precipitation falls as winter precipitation, and it is only converted into usable water during the spring months. If the sample were larger it might have been possible to identify individual slopes for precipitation falling in different seasons.

As suggested in chapter three, monthly electricity generation can be expected to be autoregressive as the history of production and usable inflow determines the quantity of available water, and thus its value relative to the market price. After trying different combinations of lagged production and precipitation it turned out that the first lags of production and precipitation gave the best fit. A theoretical reason for not including more than the first precipitation lag is that information about earlier precipitation lags is already contained in the lagged production variable through the autoregressive structure.

An AR(1) of production on precipitation lagged one month resulted in the following equation,

\[(4.4) \quad \text{NOprod} = 0.7167 \times \text{NOprod}_1 + 1.092 \times 10^6 + 1.641 \times 10^4 \times \text{precipitation}_1, \quad R^2 = 0.74804\]

\[(\text{SE}): \quad (0.0655) \quad (6.54 \times 10^5) \quad (3.36 \times 10^3)\]

From the F test of ARCH effects for one lag it was hard to tell with any significance whether or not there was any effect, it was estimated that there was a 47 per cent probability of observing a larger test statistic. The evidence of error autocorrelation was far stronger, the Chi square and F-tests concluded there was only a 2.44 and 2.76 per cent chance respectively
of observing a larger statistic under the null hypothesis. In this equation, which includes a lagged endogenous variable, error autocorrelation causes inconsistent coefficients and consequently the residuals based on them will be inconsistent as well. Thus the estimated residual correlation coefficients cannot be used to transform the variables in order to make the residuals better behaved. A plot of residuals against time illustrates this misspecification of the model.

![Production Residuals vs Time](image)

**Figure 12: Residuals from the production versus precipitation regression**

Also in this case the most likely explanation of the error autocorrelation and ARCH effects could be omitted variables fluctuating seasonally.

As explained in chapter one, the electricity prices in areas dominated by thermal plants depends on the efficiency of the marginal power plant, and will thus follow a the same predictable pattern as the load. When the transmission capacity is limited, the relative price difference with the neighbouring countries is likely to follow seasonal variations as well. Thus generators can be expected to use their water when the prices abroad are high in order to export even though the precipitation the preceding month was not particularly wet and vice versa. Furthermore, if the price sensitivity varies through the year, an electricity
generator with large enough market share to take advantage of it might optimize its production schedule with respect to it.

Another source of seasonal variance is the reservoir filling cycle; the generators know almost certainly that the usable inflow increases every spring. When their capacity is limited they need to export the energy in their reservoirs at the end of the winter in order to avoid flooding at the end of the spring, thus their production might be high relative to the annual average even if the precipitation in the preceding period was not.

The consequences of the mentioned error autocorrelation and measurement errors, inconsistency, could be avoided using instrumental variables. The sample correlation between monthly precipitation and reservoir levels\(^{10}\), published by SSB, were about 0.4. Testing whether or not the instrument is correlated with disturbances from a consistent regression is not possible, since none are available.

As testing whether the instruments can be used is impossible, considering how the instrument and the error are likely to be related in the real world might be a more feasible possibility. In this case the measurement error could possibly be correlated with the potential instrument, for example if the usable inflow from a given amount of precipitation depends on the reservoir level, which is not unlikely when less of the inflow is usable the higher the probability of flooding, it would be useless as an instrument.

4.3 Estimation of transmission demand between regions

The demand for transmission is a consequence of the distance between the areas where electricity is demanded and where it is generated. A small part of the supply is lost in transmission and is accounted for as “consumption” by the transmission line operator. This will lead to an overestimation of net import, unless it is subtracted, since parts of it never become available to the consumers. Unfortunately, I have not come across any time series of total or average losses, only expected marginal losses published by Statnett. Total losses cannot be estimated from marginal losses due to the non-linear relationship between losses and transmitted quantity.

\(^{10}\) http://www.ssb.no/emner/10/08/vannmag/
The so-called water value model, outlined in Førsund (2004), is much used to explain the hydro power price level during a water scarcity period. In simplest deterministic model, the reservoir is assumed to be partly filled in the beginning of the scarcity period with no further inflow during the period. In this model the water value, \( \lambda(W_t) \), is constant for the whole scarcity period, \( t \), and is by definition the price level that equates the available water, \( W_t \), to the sum of the period’s deterministic home, \( H_t \), and net export demand, \( X_{t}^{\text{EI}} \). Thus, the net export during the scarcity period will be given by the following energy balance equation;

\[
X_{t}^{\text{EI}} = W_t - H_t
\]

A unexpected increase in the supply of usable water will reduce the price of local energy, but due to the inelastic short term demand the local consumption changes only up to a point, making the excess energy desirable to export and vice versa. Colder weather causes the demand curve to shift, increasing the demand for a given price level, instantly increasing the demand for import (or reducing the export). The relative prices will not be modelled explicitly; simply assumed to be doing their job allocating the electricity demand to the available generators with the lowest opportunity cost.

In addition to the availability of data, the optimal length of the time series will be constrained by inconsistency due to omitted variables that may vary in the very long term, such as technological progress, capital (for example heat pumps and improved winter insulation) that substitute for parts of the energy demand, energy supplied locally from new sources, changes demography and price sensitivity etc. Constrained by limited resources and lacking sufficiently long time series for all the relevant variables, time series that potentially are long enough to detect long term trends or even possible climate changes will not be applied. Thus the model will be limited to estimating the statistical relation between the present distributions of the two mentioned weather measures and the present distribution of net import demand fluctuations.

The coefficients estimated by some of the mentioned procedures are reported in Table 4.1 below. In the model of net import, the indications of autocorrelation and ARCH effects were far lower than in the equations for supply and demand individually. The F-test of error autocorrelation at one lag showed that there was an 85 per cent chance, and the Chi square 86 per cent, of observing a larger test statistic under the null hypothesis of no error.
autocorrelation. Similarly, the F-test of ARCH effects indicated that there would be an 85 per cent chance of observing a larger test statistic under the null hypothesis of no ARCH(1) effects. The disturbance normality test showed that there was a 96.5 per cent chance of observing a larger test statistic under the null hypothesis of normally distributed errors so it could be accepted at five per cent confidence level.

F- and Chi-square tests for variable dependent heteroscedasticity using squares and cross products indicated that the probability of observing higher statistics in a sample without heteroscedasticity were 0.38 and 0.26 per cent respectively. As explained in chapter two; the consequence of heteroscedasticity in the form of residual variance depending on the level of the explanatory variables is inefficient estimators. The insignificance of the intercept may or may not be a consequence of the disturbance heteroscedasticity. It might have been caused by misspecifying the disturbances as heteroscedastic, or simply an insufficient number of observations to identify it.

Including the variables that could be expected to account for the seasonally varying omitted variables, price difference, price ratio, heating oil price, heating oil price ratio, season dummies, reduced the probability of observing higher statistics for all the mentioned tests under their null hypotheses. A large increase in the coefficient standard deviations indicated that the loss of degrees of freedom matters more for the coefficient precision than a possible improvement in residual behaviour.

If the conditional variance structure could be identified, the heteroscedasticity could be modelled by applying Feasible Generalised Least Squares, which basically is to weigh the observations by the inverse conditional standard deviation. An attempt to transform the equation variables by dividing them by the conditional standard deviation did not remove the problem of variable dependent heteroscedasticity and did not give any meaningful result. Green (2000) says it remains uncertain whether FGLS corrections are any better than OLS in small samples with known heteroscedasticity but unknown parameters. FGLS is clearly better asymptotically, but in small to moderately sized samples the variance incorporated by the estimated variance parameters may offset the gains to GLS.
Table 4.1: Regression results

<table>
<thead>
<tr>
<th>Method</th>
<th>Lagged precipitation</th>
<th>Lagged Net import</th>
<th>degree days</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endogenous precipitation and degree days with lagged reservoir levels and lagged degree days as instruments</td>
<td>Coefficient -4 810</td>
<td>0,77</td>
<td>1663</td>
<td>-47274</td>
</tr>
<tr>
<td></td>
<td>Std.Error 6 115</td>
<td>0,08</td>
<td>1134</td>
<td>405600</td>
</tr>
<tr>
<td>As above but no intercept</td>
<td>Coefficient -3747</td>
<td>0,80</td>
<td>1241</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Std.Error 1833</td>
<td>0,07</td>
<td>543</td>
<td></td>
</tr>
<tr>
<td>Endogenous precipitation with lagged reservoir levels as instrument</td>
<td>Coefficient -4 366</td>
<td>0,78</td>
<td>1566</td>
<td>-81772</td>
</tr>
<tr>
<td></td>
<td>Std.Error 5 363</td>
<td>0,07</td>
<td>574</td>
<td>418400</td>
</tr>
<tr>
<td>As above but no intercept</td>
<td>Coefficient -4 707</td>
<td>0,79</td>
<td>1564</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Std.Error 1593</td>
<td>0,06</td>
<td>456</td>
<td></td>
</tr>
<tr>
<td>Endogenous degree days with lagged degree days as instruments</td>
<td>Coefficient -4 747</td>
<td>0,79</td>
<td>1333</td>
<td>69795</td>
</tr>
<tr>
<td></td>
<td>Std.Error 2 051</td>
<td>0,06</td>
<td>458</td>
<td>199100</td>
</tr>
<tr>
<td>As above but no intercept</td>
<td>Coefficient -4 326</td>
<td>0,79</td>
<td>1393</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Std.Error 1461</td>
<td>0,06</td>
<td>453</td>
<td></td>
</tr>
<tr>
<td>OLS</td>
<td>Coefficient -5 505</td>
<td>0,78</td>
<td>1678</td>
<td>40833</td>
</tr>
<tr>
<td></td>
<td>Std.Error 1970</td>
<td>0,06</td>
<td>388</td>
<td>196900</td>
</tr>
<tr>
<td>OLS no intercept</td>
<td>Coefficient -5 194</td>
<td>0,78</td>
<td>1691</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Std.Error 1269</td>
<td>0,06</td>
<td>380</td>
<td></td>
</tr>
<tr>
<td>GARCH(1, 0)</td>
<td>Coefficient -5 505</td>
<td>0,78</td>
<td>1678</td>
<td>40833</td>
</tr>
<tr>
<td></td>
<td>Std.Error 1241</td>
<td>0,06</td>
<td>372</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Robust.StE 1252</td>
<td>0,05</td>
<td>313</td>
<td>9</td>
</tr>
<tr>
<td>GARCH(0, 1)</td>
<td>Coefficient -5 505</td>
<td>0,78</td>
<td>1678</td>
<td>40833</td>
</tr>
<tr>
<td></td>
<td>Std.Error 1241</td>
<td>0,06</td>
<td>372</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Robust.StE 1252</td>
<td>0,05</td>
<td>313</td>
<td>9</td>
</tr>
</tbody>
</table>

Having in mind the 15 per cent chance the residuals are following an ARCH(1) process it was estimated as if it were one. Modelling the disturbance using the PC Give volatility package as if it were following a GARCH(1, 0) or GARCH(0, 1) process did not change the coefficients in the main equation, but reduced the standard deviation of the intercept estimate enough to make it significant. The coefficient in the jointly determined auxiliary equation for
the lagged conditional disturbance variance was low and uncertain, and the coefficient in the lagged squared disturbance equation was practically zero. Omitting the insignificant constant term from the OLS equation only changed the disturbance test statistics slightly; it still tested positively for variable dependent heteroscedasticity at one percent significance, positively for normally distributed errors at five per cent level, no ARCH effects at 25 per cent level, and no autocorrelation at 14 and 13 per cent for the Chi square and F- tests respectively.

The coefficients from the individual regressions of consumption and production are not directly comparable with the values above as simply subtracting production from consumption would result in lagged production instead of lagged net import on the right hand side.

The improved behaviour of the residuals from the net import model relatively to the individual import and export models could possibly indicate that the temperature explains parts of the seasonal variation in production, or that precipitation explains parts of the seasonal variation in consumption. Even though the degree day- and precipitation measures are for different areas and one period apart, each of them might follow a seasonal pattern that is fairly correlated with what it would have been if it were measured for the same period and area as the other. The degree day measure could explain an increase of electricity supply because snow and ice melts when it gets warmer and increases the usable inflow. The precipitation could explain consumption if it serves as a proxy for the seasonal variable darkness, which increases the electricity demand for illumination.

4.4 Interpretations of the regression result

As it is impossible to test whether the instruments meet both of the requirements to be instrumental variables and because of their high coefficient standard deviations relatively to the plain OLS model, they will not be used in the next chapter. The volatility models are not justified by the tests for ARCH effects, but for some reason gave far lower estimate of the intercept than the plain OLS. However, they will not be used fearing the improved efficiency might be due to a misspecification. Nevertheless, the constant term falls out of the equation for the decomposition of the variance, so it might not be critical to get an exact value.
Expected net import can be calculated by inserting the expected values for the explanatory variables into the equation;

\[ (4.6) \ E[\text{net import}] = 0.78 \ E[\text{net import}_1] + 1691 \ E[\text{degree days}] - 5194 \ E[\text{precipitation}_1] + E[u] \]

If the net import distribution is stationary, the expected values at different points in time will be equal;

\[ (4.7) \ E[\text{net import}] = 7713 \ E[\text{Degree days}] - 23689 \ E[\text{precipitation}] \]

The sample first moments are:

\[
E[\text{Degree days}] = 310, \\
E[\text{Precipitation}] = 103, \\
E[\text{Net import}] = -139000
\]

Inserting the regressors’ sample moments in the equation results in \( E[\text{netimp}] = -54005 \) MWh, which is not consistent with the sample monthly net import mean at -139000. This might not come as a surprise knowing the high standard deviations of the intercept term and net import. The empirical mean is the average of 70 observations, and the net import standard deviation is about hundred times larger than its mean.

Similarly, the explanatory variables effect on the stationary net import variance can be estimated using the formula for the variance of a sum.

\[
(4.8) \quad s^2_{\text{net}} = 0.78^2 \cdot s^2_{\text{net}_1} + 1691^2 \cdot s^2_{\text{dd}} + (-5194)^2 \cdot s^2_{\text{pc}_1} + 2 \cdot 1691 \cdot 5194 \cdot s_{\text{dd}} \cdot s_{\text{pc}} \cdot \text{Corr}(\text{DD}, \text{PC}_1) \\
+ s_{\text{net}} \left( 2 \cdot 0.78 \cdot 1691 \cdot s_{\text{dd}} \cdot \text{Corr}(\text{NI}_1, \text{DD}) + 2 \cdot 0.78 \cdot 5194 \cdot s_{\text{pc}} \cdot \text{Corr}(\text{NI}, \text{PC}) \right) \\
+ \text{Var}[u]
\]

Where \( s_{\text{NI}} \) is net import standard deviation, \( s_{\text{PC},1} \) is lagged precipitation standard deviation, \( s_{\text{DD}} \) is degree day standard deviation, and \( \text{Var}[u] \) the residual variance.
This quadratic equation in the standard deviation for net import can be solved using the following equation:

\[(4.9) \quad a s^2 + bs + c = 0 \quad \leftrightarrow \quad s = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

where

\[a = (1 - 0.78^2),\]
\[b = -(2 \cdot 0.78 \cdot 1691) \cdot s_{DD} \cdot Corr(NI_1, DD) + 2 \cdot 0.78 \cdot 5194 \cdot s_{PC} \cdot Corr(NI, PC)\]
\[c = -(1691^2 s_{DD}^2 + (-5194)^2 s_{PC}^2 - 2 \cdot 1691 \cdot 5194 \cdot s_{DD} \cdot s_{PC} \cdot Corr(DD, PC_1) + Var[u])\]

Inserting the sample moments in the equation for the stationary standard deviation results in net import standard deviation of 1.1 MWh, which is consistent with the sample moment.

In order to illustrate the sensitivity of the net import standard deviation on changes in the degree day and precipitation standard deviations, a plot holding everything else constant could be useful. The plot below is made in Excel holding the precipitation and temperature correlation calculated from the sample constant at 0.4.
Figure 13: Net import standard deviation

The model result indicates that net import standard deviation can be expected to be more sensitive to changes in degree day- than precipitation standard deviation; a ten per cent increase in degree day standard deviation leads to a 2.2 per cent increase in net import standard deviation. The same increase in the precipitation standard deviation leads to 1.5 per cent increase, and an increase of both to a 3.6 per cent increase, slightly less than the sum since the variables are positively correlated and have the opposite effect on net import. The correlation between temperature and precipitation has the physical explanation that cold air cannot contain as much humidity as warmer, so less will precipitate from it when the pressure or temperature falls. Giancoli (1988).
5. Theoretical implications of the volatility increase

5.1 Change of net import distribution

Chapter four concluded that the net monthly import was not unlikely to be an autoregressive linear function of the period’s degree-days and lagged precipitation. The explanatory variables’ effect on the net import’s mean is straightforward and their effect on its unconditional (long term) standard deviation were calculated by applying the definition of variance of a sum, which is the following second-order equation in net import standard deviation.

\[
\text{StDev}^2[y_t] = \beta_1^2 \text{StDev}^2[y_{t-1}] + \beta_2^2 \text{StDev}^2[x_t] + \beta_3^2 \text{StDev}^2[z_{t-1}] \\
+ \text{StDev}[x_t] \text{StDev}[y_{t-1}] \text{Corr}[x_t, y_{t-1}] \\
+ \text{StDev}[x_t] \text{StDev}[z_{t-1}] \text{Corr}[x_t, z_{t-1}] \\
+ \text{StDev}[z_{t-1}] \text{StDev}[y_{t-1}] \text{Corr}[z_{t-1}, y_{t-1}] \\
+ \text{Var}[u_t]
\]

where \( y_t \) is net import, \( x_t \) is degree days, and \( z_t \) is precipitation.

If the variance is stationary as well, \( \text{StDev}[y_t] = \text{StDev}[y_{t-1}] \), then the square equation can be solved the usual way.

5.2 The theoretical impact on optimal capacity

The transmitted quantity will be the sum of the export and import, not the difference as the net export. As explained in chapter two, the distributions of net import and transmitted quantity will have the same variance. Their means will obviously be different, but since the mean cannot be identified and falls out of the final equation if it is held constant it will not be considered further.

The hypothetical change in the demand distribution changes the optimal capacity estimated by the probabilistic one period “newsboy” model described in chapter two. This model basically says that the probability of demand exceeding capacity should equal the ratio of the marginal capacity cost to the marginal benefit of having enough. Unless this ratio is
symmetric, with equal marginal benefits and costs; optimal capacity will be different from mean demand.

Application of the newsboy model requires the distribution of transmission demand at points in time, but the estimated distribution is for the net monthly energy transmission. This raises two problems, first the distribution of monthly transmission needs to be converted to hourly, and secondly the transmitted quantity is censored demand since not all demand is met. Green (2000) explains how the unobservable demand moments could have been inferred from the assumed distribution function for the censored variable and the moments of the observed variable if both moments were identified.

5.2.1 Parameters in the probabilistic model

The tree parameters for capacity cost, benefit per unit of met demand, and penalty per unit unmet demand, \( c, s, \) and \( p \), must be measured in the same unit for the ratio between them to make sense. Since the economic benefit of transmitting, and the penalty for not being able to meet demand are rates of cost per unit of time and electric effect, the cost of capital must be expressed as a continuous rate as well. Furthermore, to make the parameters time independent all units must be expressed in real terms. These parameters are unfortunately hard to measure directly. In order to make a crude numerical example and see how consistent the model is with the real world, Statnett’s most recent investment, Nea-Järpstrømmen, will be investigated. An easier approach would be applied to the other regions, simply assuming the present capacity is optimal with respect to the present parameters and assume the cost/benefit ratio remain constant when the demand distribution changes.

Cost of capital of Statnett’s most recent project

According to Statnett (9/2004), their next planned 420 kV transmission line Nea Järpstrømmen will cost 155 MNOK for their quarter of the overhead line with the remaining paid by Svenska Kraftnät. The line replaces a scrapped 300kV line so the cost will be attributed to the total capacity and not only its increment. According to Sand (2003), transmission lines have an economic lifetime of 30 years, discounted by the ministry at a 6 per cent annual nominal rate, and annual operating costs a small proportion of the initial investment. The report does not say the capacity of the line, but the old line’s capacity is
600MW, and new 420 kV AC lines have normally around 1000 MW capacity, so this value will be applied in the following crude numerical example.

The cost of capital as a rate per unit of time could be modelled as the sum of the real interest rate on the initial investment continuously compounded, depreciation, and real maintenance costs measured as a rate per unit of time.

The continuous cost of capital including both depreciation and interest, \( c \), could be defined as the continuous cash flow for the project’s life time with the same present value as the initial expenditure, \( C \).

\[
(5.2) \quad C = \int_{0}^{T} ce^{-rt} dt
\]

When the project’s life time is \( T \) years, the cost expressed as a cash flow rate would be;

\[
(5.3) \quad c = \frac{C}{\int_{0}^{T} e^{-rt} dt} = \frac{C}{\frac{1}{r} \left[ e^{-rT} - 1 \right]}
\]

If the annually compounded real interest rate is 3.50 per cent the continuously compounded interest rate is 3.44 per cent p.a. Inserting values from the capacity investment mentioned above gives the continuous cost of \( c = 33 \) MNOK/year for the installations 30 years life time. The cost of capital measured as an hourly cash flow will be NOK 3780/h, or NOK 3.78/MWh if its capacity is 1000MW.

**Benefit per unit transmitted**

An estimate of the benefit per unit transmitted could be the tariff to the transmission company which is supposed to cover its long run marginal costs. If the regulation were efficient the long run marginal cost would equal the marginal benefit. This tariff consists of two parts; a variable part that covers the marginal transmission loss depending on load and the electricity price, and a fixed residual part covering the rest of the transmission
companies’ costs. Using numbers from Statnett\textsuperscript{11}; the 2004 residual tariff is 6 kr/MWh and the energy tariff is the system price at the time of transfer multiplied the expected marginal loss, which is published by Statnett for around ten weeks ahead. In order to keep the model simple, the system price and the marginal losses are set equal to their unconditional expectations. The published loss estimates are separate for day and night and symmetric with marginal losses from demand equal to the negative of marginal losses from supply and truncated at ±10 per cent. After analysing hourly data from Nordpool, it turned out that 50-56 per cent of total import and 79-86 percent of total export occurs during the day. This makes sense in regions exporting hydropower and importing thermal power, as the cost of thermal power is more sensitive to intra day demand variations than hydro power. Valuing marginal losses at system price like Statnett does in its tariff; the average variable component is about NOK 6/MWh. Thus the total average transmission tariff used as a proxy for the marginal benefit of transmission, $s$, will be around NOK 12/MWh.

\textit{Marginal economic cost of unmet demand}

The economic bottleneck cost is defined in Statnett (8/2004) as half the price-difference between the areas where it would be desirable to transfer electricity, times the size of the bottleneck. This could be interpreted as if both sides of the border share equally the cost of inefficient demand allocation caused by the bottleneck, indicated by a difference marginal production costs in excess of the transmission tariff. Prices in the different Elspot areas are obtained from the Nordpool ftp-server, and for simplicity I will use half the mean price difference conditional on that a bottleneck has occurred. Half of the average absolute price difference between NO2 and SE, conditional that it is larger than the average transmission tariff calculated above; is NOK 23.9/MWh.

\textbf{5.2.2 Applying the estimated parameter values into the newsboy model}

Inserting the values for NO2 outlined above into the right hand side of the model, it would be optimal to set capacity at a level where the probability of demand exceeding capacity is around ten per cent. In the hourly price data there is a price difference larger than the average transmission cost between NO2 and Sweden 16 per cent of the time. This deviation is not horribly large considering the discontinuous steps of capacity investment as well as the models’ other unrealistic simplifications and the inaccuracy of the data.

\textsuperscript{11} \url{http://home.statnett.no/tapssatser/}
The tests of the net import model in chapter four accepted the hypothesis of normally distributed disturbances at five per cent confidence level. Since transmission demand is a linear combination import and export, it can be considered normally distributed like the net import. Then optimal capacity, \( Q \), can be expressed explicitly as in equation (2.33);

\[
Q = \mu + \sigma \Phi^{-1} \left[ 1 - \frac{c}{s + p} \right], \text{ where } \mu \text{ and } \sigma \text{ are the demand moments}
\]

When the cost benefit ratio is 1:10, it would be optimal that demand is met 90 per cent of the time. In order to expect that, capacity should exceed the mean demand by 128 per cent of the standard deviation.

If the prices can differ with the average transmission tariff, about NOK12 /MWh, before it is considered desirable to transmit more; capacity is constrained 35.2, 19.3, and 15.6 per cent of the time respectively at the DK1<->NO1, SE<->NO1 and SE<->NO2 connections respectively. If transmitted quantity at the maximum capacity indicates congestion; transmission demand is rationed 25.4, 26.7, 0.6 and per cent of the time. These deviations might be reasonable if the maximum capacity is not constant, but depending on other bottlenecks. Furthermore, the marginal transmission tariff varies with the electricity system price, and the hourly means do not reach extreme values as often as the pointwise transmission they are aggregated from.

At the connection with Denmark, which is the one with highest cost/benefit ratio, optimal capacity would equal mean demand plus 38 per cent of the standard deviation, at the connection between south of Norway and Sweden it would be optimal to have capacity equal to mean demand plus 87 per cent of the standard deviation. Using the fraction of time capacity it would be desirable to transmit between northern Norway and Sweden instead of the crudely estimated cost/benefit ratio for Nea-Järpstrømmen it would be optimal to have capacity exceeding expected demand by 101 per cent of the standard deviation.

\textit{Inferring optimal capacity for the remaining connections}

For the connections between south of Norway with Sweden and Denmark I have not come across the costs of capacity. However, assuming the capacities are optimal initially, the only
unknown variable in the expression for the optimal probability of meeting demand, \( \Phi = 1 - \frac{c}{s + p} \), is the marginal cost \( c \). Half the price difference between NO and DK conditional on being over NOK 12/MWh is 39, between NO1 and SE is 23.9, and between NO2 and SE is 22.5. Setting the benefit of transmission equal to the estimated NOK 12/MWh, lacking a more precise number, the estimated capacity cost is NOK 14/MW for the connection with Denmark, and about 7.5 for the connection between south of Norway and Sweden.

5.2.3 Likely errors of considering the parameters constant

As opposed to newspapers the electricity price is endogenous. Thus the price differences between the Elspot areas are not likely to be constant, but increasing with the size of the bottleneck. The plot below of net transfer versus price difference shows how the price difference increases as the transmitted quantity approaches the capacity constraints. This would make the “penalty” increase more than proportionally with the quantity of unmet demand. The more often a connection is congested in the same direction during the period the average net import is taken over, the closer it will be to the capacity constraint. The price difference will be endogenous to the size of the bottleneck in the sense that would be higher the more it would be desirable to transmit before the marginal electricity generating costs at both sides of the connection became equal. So even if the probability of bottlenecks occurring remains exogenously constant at the cost/benefit ratio; increased volatility would probably increase the size of them when they occur.
Another issue that makes the application to transmission lines more complicated is the fact that increased transmission capacity will reduce the transmission losses for a given transmitted quantity. According to Ohm’s law, the voltage drop is proportional to the resistance and the current, and by the definition of electric energy the loss would be the voltage drop times the current. Thus the energy loss, electric current multiplied by its voltage drop, would increase with the square of the current and thus with the square of the transmitted energy. The transmission losses will increase the temperature of the conductor which again increases its resistance. Hence, increasing the lines’ capacity by upgrading the voltage or reducing its resistance will thus reduce the losses for a given transmitted quantity of energy. In a more realistic model, benefit per unit of transmitted energy would have been a non-linear function of demand, and the optimal capacity would require a numerical solution.

Since both of them are increasing functions of demand; this simplification will bias the optimal probability of demand exceeding capacity.
5.3 Expected cost of the capacity change

When the cost/benefit relationship remains constant, the cost of increased capacity becomes directly proportional to the increase in standard deviation. Multiplying the change in optimal capacity with, $c$, the unit capacity cost gives the cost increase as function of changing standard deviation for a given connection.

\[
(5.4) \quad c\Delta Q = c\Delta \sigma \Phi^{-1} \left[ 1 - \frac{c}{s + p} \right]
\]

Dividing by the standard deviation change, $\Delta \sigma$, gives the cost per unit of standard deviation increase, which could be more useful while the expected increase in volatility remains unknown.

\[
(5.5) \quad \frac{c\Delta Q}{\Delta \sigma} = c\Phi^{-1} \left[ 1 - \frac{c}{s + p} \right]
\]

Multiplying the cost estimates with the estimates of capacity per unit standard deviation results in the following cost per unit increase in standard deviation for the three connections.

**Table 2: Costs of increased net import standard deviation**

<table>
<thead>
<tr>
<th>Connection</th>
<th>Cost per unit increase in net import standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO2&lt;-&gt;SE</td>
<td>3.40</td>
</tr>
<tr>
<td>NO1&lt;-&gt;SE</td>
<td>6.50</td>
</tr>
<tr>
<td>NO1&lt;-&gt;DK1</td>
<td>5.30</td>
</tr>
</tbody>
</table>

If the TSO behaves rationally in the sense that it always keeps capacity at the optimal level, and the weather volatility changes slowly relative to the investments lifetime, the loss from increasing weather volatility will be reflected in the cost of keeping the transmission capacity at the optimal level.

If the price difference is endogenous to the size of the bottlenecks, it is not unlikely that higher weather volatility lead to higher electricity price variability. The income effect of
volatile prices will reduce the risk-averse consumers’ utility, and the increased transmission capacity costs alone will understate the actual costs.

Furthermore, the estimated standard deviation understates the transmission demand volatility because of the censoring. All demand in excess of capacity is observed as if it were equal to it, and since the mean demand is lower than the censoring limit the average observed squared deviations from the mean will understate the actual.
6. Conclusion

The intention of this project was to investigate how the statistical distribution of electricity transmission demand in an economy relying mostly on hydro power can be expected to be affected by potential climatical changes.

It turned out that a regression of monthly net import on its first lag and lagged precipitation, explaining supply, and an aggregate of temperatures, explaining demand, fitted the sample data well. However, the demand for transmission capacity depends on the distribution of transmission at points in time, and not the monthly average of net import. At all points in time the transmitted quantity will equal absolute value of net import, so the estimated net import distribution were used to infer the transmission demand distribution.

When the marginal costs of meeting the demand are different from the benefits, the optimal level of the transmission capacity depends on the demands’ volatility as well as its expectation. By the so-called “Newsboy model”, optimal capacity would be at a level where the probability of demand not being met equals the ratio of marginal costs to benefits. When these are exogenously given and the demand distribution is known, optimal capacity will equal the mean demand plus a fraction of its standard deviation depending on the cost benefit ratio.

However, if the costs or benefits are non-linear functions of capacity it gets more complicated. A by-product of higher transmission capacity is that the losses for a given quantity of electricity will fall when the capacity utilisation falls. This will increase the average benefit of transmission since more of the transmitted energy becomes available to the consumers. However, what matters for the optimal capacity is the marginal benefit, which is less likely to change.

The other part of the marginal benefit of the ability to meet demand is the marginal reduction in the penalty for not meeting demand. This is defined as the quantity that would be desirable to transmit, multiplied by the price difference, and shared equally between each of the markets separated by limited transmission capacity. At both sides of the border the price
level is determined by the generator with the highest marginal costs. When these costs vary between the separated markets there would be efficiency gains by generating more at the low cost side; and less at the other and transmitting the difference. A marginal increase in capacity would allow a marginal reduction in the production by the high cost side and a marginal increase at the low cost sides; reducing the price difference as well as the size of the bottleneck. Thus the marginal benefit of transmission would be an increasing function of capacity and the newsboy solution will understate the optimal capacity.

The conclusion of the empirical study was that the monthly transmission demand standard deviation increased less than proportionately with the standard deviations in the explanatory climatical variables. Without having any estimate of the likely order of magnitude of their future increase, an eventual increase the cost of keeping the transmission at its optimal level cannot be estimated. However, the direction of the capacity cost change is unambiguous, if the weather becomes more volatile, net electricity import gets more volatile as well. Hence the optimal transmission capacity will increase unless the expected net import falls sufficiently to compensate it. Nevertheless, the global warming is likely to cause a more humid and warmer climate, and according the signs of the coefficients in the net import equation this will reduce energy demand and increase its supply. Consequently the total effect of the climate change on the costs of keeping the transmission capacity at its optimal level remains undetermined.
References


All the web sites listed in the footnotes contained the relevant information at time of writing, but I cannot guarantee that they will in the future.