The demand for cash in Norway

By

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Preface

This thesis was written during my time as a student assistant at Norges Bank, Cashier's Department. The work was included in their project on cash demand. I want to thank Norges Bank both for the opportunity to work in a supporting and motivating environment, and adapting the data material used in the thesis. I will especially take this opportunity to thank Thomas Kjørstad and Olaf Gresvik for their support and cooperation during the work with my thesis. Finally, I am very grateful to Terje Skjerpen who has been an excellent supervisor and a source of inspiration. His massive support and constructive critical feedback during the work have been invaluable.

Oslo, February 2005
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Summary

The purpose of this paper is to provide an econometric model which can be used to explain and determine the demand for cash in Norway, and to use this model to forecast the demand for cash in the period 2004.3-2007.4.

From a central banks perspective, there is particularly one reason to be interested in the demand for cash. The central bank is the issuer of notes and coins. Since one of its objectives is to satisfy the demand for cash, it is of the central banks interest to ensure that they supply the society with notes and coins in the most efficient way. Obviously this is also of great importance for the society. For the central bank to do this, an efficient amount of notes and coins must be produced. In order to be able to plan the production in the best way, it is therefore of interest to investigate closer what determines the demand for cash. Further, one may also argue that it is of interest to investigate the demand for cash for monetary policy purposes. The reason for this is that the amount of cash held for transaction purposes is closely related to domestic spending, and hence to domestic price developments.

The demand for cash increased steadily year by year until 1999, and has decreased somewhat since then. In the same period the use of alternative payment instruments, such as payment cards, increased rapidly. One would therefore expect that the use of cash should have been reduced dramatically, something that has not yet happened. This paper studies the determinants of cash demand in Norway during the period 1980-2004. A thorough discussion of the different determinants for cash demand is carried out. The starting point for the discussion is the concept of money and its role, the evolution of the payment system and theoretical models of money demand. Since most of the theoretical models of money demand focus on a quite narrow concept of money, it is believed that some of these models are also relevant when investigating the demand for cash. This seems to especially be the case for models considering the private households demand for money. Based on this discussion, empirical models of demand for real cash are developed. Rather than estimating the theoretical models themselves, they are used to determine which explanatory variables that should be considered when an empirical model is specified.

In the empirical analysis, quarterly data and a general-to-specific approach are used. For this analysis I have used the software programs PcGive 10.1 and TSP 4.5. Due to problems with autocorrelation in the residuals and lack of data, a VAR model seems not appropriate to use

for estimating the demand for cash. Instead a single equation equilibrium correction model is used. Considering different initial models, eliminating insignificant variables and applying different statistical tests, suggests that there are two competitive, but also quite similar, models of relevance. Both models have only private point-of-sale consumption and real deposit interest rate as significant explanatory variables in addition to a linear trend, seasonal variables and impulse dummies. The rapid evolution in the payment system seems to have a negative effect on the demand for cash, which may dampen the increase in the demand for real cash. This effect is represented in the models by a negative linear trend.

The estimation results show that the two models short-run effects differ slightly, while the long-run effects are quite similar. The long-run elasticity for consumption was found to be approximately 0.63 and the semi-elasticity for the interest rate was found to be approximately 0.02 in both models. These results differ compared to the results obtained by Fischer, Köhler and Seitz (2004) for the Euro area.

The explanatory variables were found to be weakly exogenous with respect to all parameters in the structural equation for real cash, validating a single equation approach. In addition to this, the highly significant equilibrium correction term suggests that an equilibrium correction model is appropriate.

In order to choose between one of the two competitive models, parameter stability and ex post forecasting properties are investigated. However, this does not lead to a clear conclusion of which model to choose. Both models seem to have parameters that are reasonable stable. On the other side, both models have some problems when it comes to forecasting in the sample period 2002.1-2004.2. An explanation for this may be that this forecasting period was a rather turbulent period for the Norwegian economy. The main reason for this was probably that the economy needed some time to adjust to the change in the monetary policy regime in March 2001. Since the model selection analysis gives no clear suggestion of which model to choose, both models are treated on an equal basis.

For given evolution paths of the exogenous variables, ex ante forecasts for the period 2004.3-2007.4 are obtained by the two models. In addition to this forecasts produced by a simple AR(8) model are also considered. The forecasts produced by the two competitive models that are developed, suggest that the demand for real cash will rise for the next couple of years, and then slowly decrease from the end of 2006. In contrast to this the simple AR(8) model suggests that the demand for cash will increase throughout the whole forecasts period. In order to make the forecasts more robust for potential breaks, forecasts with intercept correction are conducted. Finally, I produce forecasts under alternative assumptions of the evolution paths of the exogenous variables.

Contents

1. Introduction

Notes and coins are issued by Norges Bank and are legal tender in Norway. The currency component of the monetary aggregates is the narrowest definition of money analysed by central banks. It compromises notes and coins in circulation held by non-Monetary Financial Institutions. The total amount of currency in circulation (cash)[1] is controlled by the monetary authorities. Central banks are in principle able to decide precisely the amount of currency they put into circulation and know the exact amount of outstanding currency. On the other hand, after the currency is put into circulation, central banks are not able to track notes and coins as it circulates. What characterizes cash is that it enables immediate final settlement without an intermediary. Consequently there is little direct statistical information on where the currency circulates, who holds the currency and for which reasons currency is held.

Furthermore the amount of outstanding cash in Norway increased steadily year by year in the sample period until 1999, and has decreased somewhat since then. In the same period the use of alternative payment instruments, such as payment cards, increased rapidly. One would therefore expect that the use of cash should have been reduced dramatically, something that has not yet happened. A possible explanation for this might be that cash is commonly used in the unregistered (illegal) economy.

From a central banks perspective, there is particularly one reason to be interested in the demand for cash. The central bank is the issuer of notes and coins. Since one of its objectives is to satisfy the demand for cash, it is of the central banks interest to ensure that they supply the society with notes and coins in the most efficient way. Obviously this is also of great importance for the society. For the central bank to do this, the correct amount of notes and coins must be produced. In order to be able to plan the production in the best way, it is therefore of interest to investigate closer what determines the demand for cash. Further, one may also argue that it is of interest to investigate the demand for cash for monetary policy purposes. The reason for this is that the amount of cash held for transaction purposes is closely related to domestic spending, and hence to domestic price developments.

[1] I will use the terms currency in circulation and cash as synonyms. They both stand for notes and coins in circulation.

The purpose of this paper is to provide an econometric model which can be used to explain and determine the demand for cash in Norway, and to use this model to forecast the demand for cash for the period 2004.3-2007.4.

In the year 2007, Norges Banks will close down their printing press for bank notes.[2] Norges Bank needs such a model for forecasts in their planning procedure when making future orders for banknotes. On the other hand, this paper concentrates on the total demand for cash, and does not focus on the demand for different denominations of coins and banknotes.

Most papers in the literature on cash focus on the relation between currency holdings and the illegal economic activity, or on the influence of financial innovations on cash demand. Very few of them focus on developing empirical models for the demand for cash. The main reasons for this are probably lack of relevant data and the lack of explicit theories for the demand for cash. In the following I will give a brief summary of some of the main findings in the literature on cash.

There are particularly two papers, which have been the main sources of inspiration for my analysis. These are both papers that focus on modelling the demand for cash. The first is Fischer, Köhler and Seitz (2004) who analyses the demand for currency in circulation in the euro area since the beginning of the 1980s. They estimate the demand for euro legacy currencies in total, and for small and large denominations within a VAR cointegration framework. The second is Drehmann and Goodhart (2000). Here international comparisons of currency demand equations are done. They use panel data to estimate the total demand for cash, as well as for high- and low-value notes. Furthermore, an important paper on cash usage, which however does not concentrate on estimating the demand for cash, is Rogoff (1998). He focuses on the underground demand for euro notes. The paper is primarily written in response to, and to criticize, the ECB decision of issuing large value euro notes.

Another interesting study is Humphrey, Kim and Vale (2001). They studied the effects of using electronic payment instruments on the efficiency of the payment system, using Norwegian data from the period 1989-1995. Their starting point is that different payment

[2] Coins are produced by Det Norske Myntverket AS since 2003.

services have different production costs. The results show that payment users are quite sensitive to relative prices that reflect the user costs.

From earlier works at Norges Bank, two papers on cash are relevant. These papers are however, not directly relevant for the analysis that I have done, but serve as inspiration sources. Humphrey, Kaloudis and Øwre (2000) estimated the share of cash used in consumer transactions at points of sale in Norway in the period 1980-1999. The results showed that this share declined over the period as a result of the increased use of debit cards. This paper also contains forecasts for cash use in the future and estimates for total cash use in connection with both legal and illegal activities. Gresvik and Kaloudis (2001) tried to estimate the transaction motivated demand for cash in all sectors of the Norwegian economy. They find that only 37 % of the total amount of outstanding currency can be explained by this motive in the year 2000. They conclude that the rest of this amount stems from the unregistered economy.

My thesis is structured as follows: Chapter 2 gives verbal discussion of money and the payment system. It discusses the concept of money, its role, interpretation and functions, with a particular view of money interpreted as cash. Further a brief explanation of the payment system, its role and development follows. Chapter 3 provides a brief summary of some of the major theories for the demand for money. In addition, a theoretical model which is especially derived for the demand for currency is presented. The model is based on the models in Rogoff (1998) and Fischer, Köhler and Seitz (2004). The econometric theory, method and tests that are used in the empirical part of the thesis are explained in Chapter 4. In Chapter 5, two cash demand functions for the Norwegian economy are estimated. Chapter 6 presents some forecasts for the period 2004.3-2007.4 for the demand for cash in Norway. The empirical models from Chapter 5 are used in addition to a simple univariate model.[3] Finally, Chapter 7 draws some conclusions.

2. Money and the payment system; its functions and development

2.1 Introduction

In Norway banknotes and coins are issued by Norges Bank. They are legal tender and can be used for all payment transactions where payer and payee meet. That is, both parts have the

[3] I have used the software programs PcGive 10.1 (see Hendry and Doornik (1996)) and TSP 4.5 (see Hall and Cummins (1999)) in the empirical analysis.

right to demand settlement in notes and coins.[4] Cash is a mean of payment, and is therefore also a part of the payment system. In the last decades most payment systems in modern market economies have undergone major changes. Basically, the main part of these changes has been the replacement of paper-based instruments by the use of electronic based payment instruments. The Norwegian payment system is among the most efficient and well-developed payment systems in the world. Extensive use of electronic services, particularly internet banking and payment cards, provides customers with a wide range of services and has reduced bank's costs. According to Norges Bank (2004a), the key factors in achieving an efficient payment system in Norway have been an infrastructure that facilitates economies of scale, prices that reflect the production cost of services and active competition on the supply side.

2.2 What is money?

If you ask any man or woman in the street, "what is money?" the typical response would probably consist of the person in question taking out his/her wallet and showing a colourful piece of paper with some numbers printed on it. However, an economist will show considerably less confidence if confronted with the same question. Instead of formulating a straight answer he or she will propose a number of functions performed by this elusive thing called "money". In other words, instead of designating what money is, economists describe what money does, see Heijdra and van der Ploeg (2002). John Hicks formulated this as:

> *"Money is what money does. Money is defined by its functions."* Hicks (1967)

In broad terms we can distinguish between three major functions of money, see McCallum (1989):

- Money as a medium of exchange in economic transactions
- Money as a medium of account
- Money as a store of value

[4] However, the Norges Bank Act does include the provision that no one is obliged to accept more than twenty five coins and notes of each denomination in any one payment.

The first of the three major functions of money can be explained by comparing two different hypothetical economies. Assume that the first economy is a direct barter economy. That is, exchange of goods is carried out by means of direct barter of goods and services. The second economy features monetary exchange, where the special medium of exchange commodity is money. Comparing two such economies, we can clearly see why money is used in modern economies. In a barter economy each part of a transaction must agree on a medium of exchange that the other part accepts. In contrast, in a monetary economy, the parts settle in a generally accepted medium of exchange. Money can also be divided into smaller values that can be divided or added up as required. Hence money helps to facilitate transactions. Therefore a monetary system provides simplifications to a barter economy. The efficiency gain is that transactions can be made easier, which in turn may allow for a better division of labour, a more efficient economy and higher level of wealth, Norges Bank (2004a). A more thorough discussion of this is made in Heijdra and van der Ploeg (2002).

The second major function of money is that they serve as a medium of account. For an economy with N different commodities, there are $N(N-1)/2$ distinct relative prices. If all goods are expressed in terms of money and money is thus the medium of account, then only N different (absolute) prices for the different commodities need to be recorded. Thus if we consider an economy consisting of 100 different commodities (including money as one of the commodities), the number of relative prices are 4950. However, in a monetary economy it is natural to express the prices of each of the 99 other commodities in terms of money. Therefore market participants do not need to know all the 4950 relative prices they need only knowledge of the 99 money prices of the commodities. This simplification reflects the advantage of having a common unit of account.

A third function of money is to serve as a store of value. In a monetary economy, money can be used to buy goods, not only today but also in the future. A stock of money can therefore be said to represent “future purchasing power”, Heijdra and van der Ploeg (2002). In the future, money can be exchanged for goods, which can be consumed or used in the production process. Hence, money can therefore be used as a store of value. However, there are many other assets, such as bonds, company shares etc. which typically outperform money in this role, because they yield a positive rate of return which money does not.

We have seen above that a pure monetary economy helped to facilitate transactions compared to a pure barter economy. However, other generally accepted mediums of exchange existed before the appearance of money in the economy. Especially metals such as gold and silver have some of the same qualities as money. This made them particular suitable for use as a medium of exchange and also as a store of value. First, the quality is uniform and relatively easy to identify. Second, the value reflects the quantity and purity of the metal in question. Third, the metal is divisible and could therefore be adjusted to transactions of different values. Finally the value of the metal does not diminish over time and the metal is easy to transport due to its relatively high value for low volumes. However, the fact that money (interpreted as currency) has no other value in itself other than being the medium of exchange, makes it more suitable than gold and silver for this purpose. Metals such as gold and silver have a commodity value, for example they can be used for decorations and as jewelry, while coins and paper (banknotes) have no such value. Coins and banknotes can also be produced at extremely low costs, while extraction of gold and silver is a costly, time demanding and a complicated process. Extractions of these metals are also limited to a few specific geographic areas. On the other hand, gold and silver have an advantage compared to money (interpreted as currency). They can be used as a common medium of exchange all around the world. Most countries have specific currencies, which mean that to carry out a transaction in a foreign country, the money needs to be changed to the specific currency used in this country. This problem is however likely to have had less relevance in the last decade and in the future. For most European countries a specific reason for this is the introduction of a common currency, namely the Euro.

When Norges Bank was founded in 1816, the Bank was given the sole right to issue banknotes and coins. First the Central Bank operated with the silver standard, in the notion that banknotes were guaranteed to be exchanged for par value in silver. In 1874 the silver standard was replaced by the gold standard, which was finally phased out in 1931. Today the value of a banknote or coin is based on the confidence in the issuer. This means that confidence in the central bank and the government is essential for the value of cash issued by the central bank and deposits held in the central bank accounts (central bank money).

2.3 How to measure money?

As pointed out above, it can often be problematic to give a precise economic definition or explanation of what money is. If this is the case, it is also problematic to measure money. A fundamental question is; "What is, and what is not sufficiently liquid to be called money?"[5] To avoid this problem, monetary aggregates are defined. However, the definition of these aggregates may differ between countries and over time. In this paper, the main focus is on currency in circulation. This is an extreme narrow definition of money. When economists normally refer to money, they often use broader definitions of money. It can therefore be useful to give an explanation of different monetary aggregates. I will here focus on the definitions used by ECB and Norges Bank. For the monetary aggregates M0 and M1, the same definition is used, but as will be seen below, they differ when it comes to the broader monetary aggregates.

2.3.1 The monetary base M0

From the perspective of commercial banks, a perfect substitute to currency in circulation, are reserves that these banks hold with the central bank. If a bank is in need of currency, it can always convert its reserves with the central bank into currency at negligible transaction costs. On the other hand, if a bank's cash balances are too high, it can always exchange them for reserves. So, there is a perfect substitutability between reserves and currency. This concept of money is called the monetary base, and is defined as:

M0 = currency in circulation + the value of bank's reserves at the central bank.

2.3.2 The money stock M1

From the perspective of a private individual, an almost perfect substitute to currency in circulation is sight deposits. Overnight deposits can be transferred into currency at very low cost and almost at every time, due to the existence of cash dispensers. The sum of these two substitutes is often referred to as the money stock M1, and is defined as:

M1 = currency in circulation + overnight deposits.

[5] By liquid, we understand: means that can directly be used as means of payments, or means that quickly and easily can be converted into means of payments.

2.3.3 Broader monetary aggregates M2 and M3

The monetary stock M1 is often considered the most relevant monetary aggregate for theoretical models of money demand. Such models usually assume a quite narrow definition of money. However, when it comes to monetary policy most central banks usually use broader monetary aggregates, such as M2 and M3. Norges Bank puts most weight on M2, when they perform their monetary policy. In contrast, the European Central Bank (ECB) uses a "reference value" for the money stock M3 as a "pillar" of its stability oriented monetary policy strategy. Norges Bank defines their broad monetary aggregate M2 as follow.

M2 = M1 + the money-holding sector's other bank deposits (in NOK and foreign currency) except restricted deposits (bank saving with tax credit etc.), incl. certificates of deposit.[6]

The ECB defines their broader aggregates as follows.

M2 = M1 + deposits with agreed maturity up to 2 years[7]
+ deposits redeemable at notice up to three months.[8]
M3 = M2 + repurchase agreements[9]
+ money market fund shares/units and money market paper
+ debt securities issued with maturity up to 2 year.[10]

An illustration of the quantities of the monetary aggregates in Norway is given in Figure 1. As mentioned above, Norges Bank does not focus on the broad monetary aggregate M3, and it is therefore not illustrated in Figure 1.

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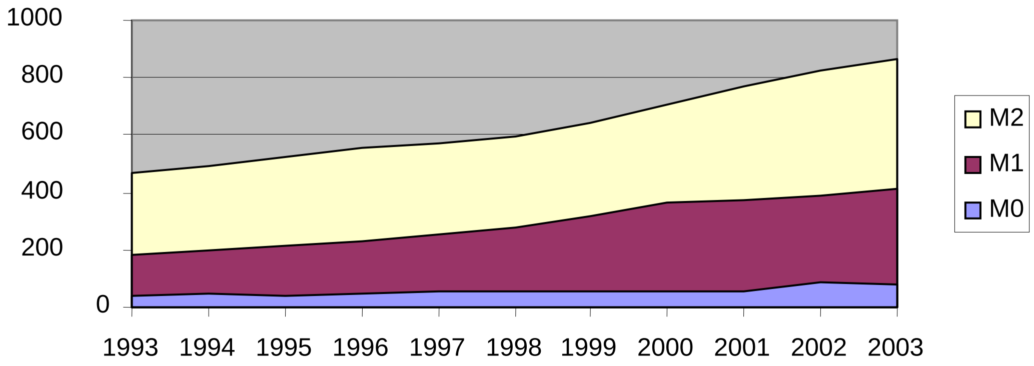
[6] By money holding sector is here meant: local government, non-financial corporations, households and other financial corporations (i.e. excluding banks and state lending institutions).
[7] Consists mainly of time deposits with a given maturity which, depending on national practices, may be either not convertible prior to maturity or convertible only subject to a penalty, see ECB (1999, 2001) for more details.
[8] Consists of savings deposits for which the holder has to respect a fixed period of notice (up to three months) before being able to withdraw funds. Also here it might be some cases where it is possible to withdraw a certain fixed amount in a specific period or an earlier withdrawal subject to a penalty payment, see ECB (1999, 2001) for more details.
[9] An agreement whereby an asset is sold but the seller has a right and an obligation to repurchase it at a specific price on a future date or on demand, see ECB (1999, 2001) for more details.
[10] It represents a promise to make regular payments for a specified period of time (here up to two years). See ECB (1999, 2001) for more details.

Figure 1: Monetary aggregates 1993-2003. Annual average in billions of NOK



Source: Norges Bank

From the definitions of these broader monetary aggregates, it is obvious that not all assets that are included can be regarded as perfect or close substitutes to currency in circulation. Further it might also be problematic that these monetary aggregates do not coincide with the definition of money that is often used in theoretical models. However, M2 and M3 have statistical advantages, that is the demand for M2 and M3 are more stable than the demand for narrow money (M0 or M1). This makes the broader monetary aggregates more appropriate to use as leading indicators for the development in the price level. A more thorough explanation of this can be found in ECB (1999, 2001).

2.4 The role and functions of the payment system

As pointed out in Norges Bank (2004a), it is of great importance for all well-developed countries to have an efficient payment system. But what does it actually mean that a payment system is efficient? According to Norges Bank, what characterizes an efficient payment system is that transactions have to be effected quickly, securely and at low costs.[11] Most individuals and companies use the payment system every day in one way or another. Payments are often made by cash or other payment instruments that provide access to money in an account. Therefore an efficient and secure transfer of means of payment is essential for the execution of different capital transactions, the settlement of foreign exchange rate and in connection with the implementation of monetary and fiscal policy, Norges Bank (2004b).

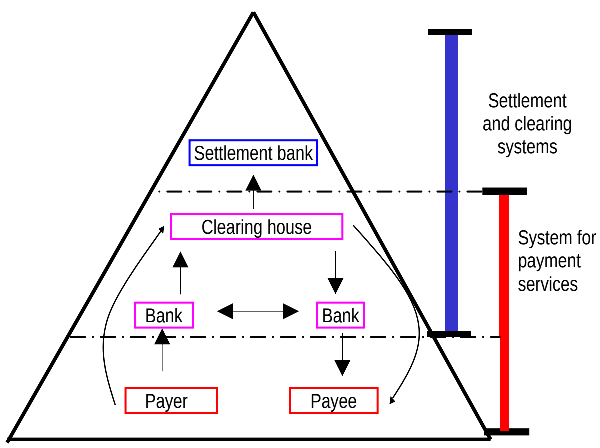
[11] This is Norges Banks definition of an efficient payment system, and is more thoroughly explained in Norges Bank (2001) and Norges Bank (2004b).

In Norway, retail payments are executed using cash and/or deposit money. Cash (notes and coins) is issued by Norges Bank and is legal tender in Norway. Cash therefore represents a claim on Norges Bank. On the other hand, deposit money represents a claim on banks. Claims are settled by making assets available for payee.

The Norwegian payment system consists primarily of systems of transferring deposit money from payer to payee by means of various payment instruments such as giros, payment cards and cheques. Banks play an important role in the payment system since they have a statutory monopoly on receiving deposits from the public, and bank deposits provide the basis for most payment transactions. Cash payments do not involve any payment transfer services since the final settlement is made without an intermediary.

The payment system may be divided into two main parts; system of payment services and the interbank system (see Figure 2). Payment services involve the use and provision of card services, cheques, giros and cash to execute retail payments. The interbank system is a system for clearing and settlement of transactions between banks.[12] Payment transactions go through the entire system before reaching the payee, unless the payer and payee have accounts in the same bank, or a bank acts as a settlement bank, which is the case for among others DnB NOR. However, since this paper focus on demand for cash, I will only look at the system for payment services and not the interbank system.

Figure 2: The Payment system in Norway



Source: Norges Bank

[12] An overview of the Norwegian interbank system is given in Norges Bank (2004b)

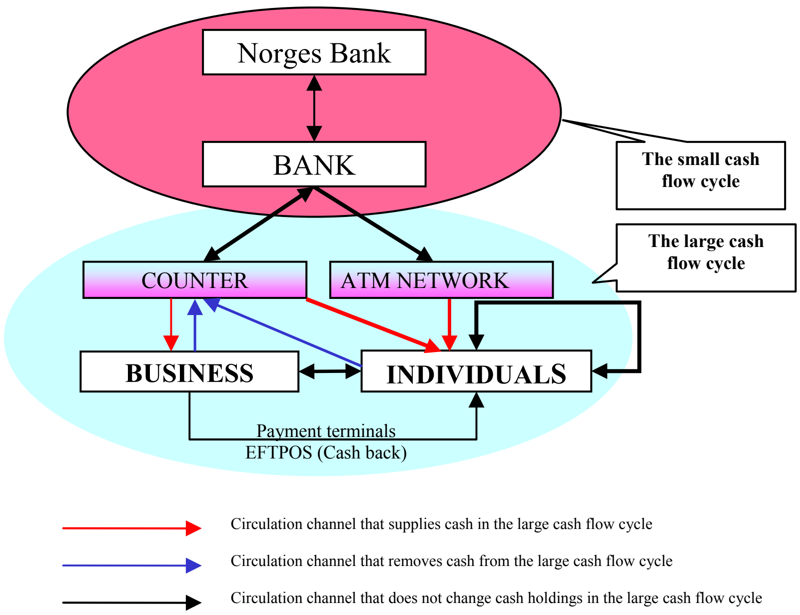
2.5 Payment services: Cash and its alternatives

As mentioned, coins and banknotes are issued by Norges Bank and both payer and payee have the right to demand settlement in notes and coins. What characterizes cash is that it enables immediate final settlement without an intermediary. This property makes cash an attractive and flexible mean of payment, especially for small-value payments. A study by Gresvik and Kaloudis (2001) indicates that about a third of the total cash balance is used as payment for registered trade in goods and services. The remaining two thirds are used for non-registered purposes. Such purposes are either legal such as hoarding and private transactions, or illegal activities (tax evasion, crime etc.). Before going into more detail about cash and its alternatives it can be of some interest to take a closer look on how cash is distributed to the public and how it circulates, in other words: the cash flow cycle.

2.5.1 The cash flow cycle

To acquire cash and convert deposit money into cash, the infrastructure is vital. The most important aspects of the cash systems infrastructure are illustrated in Figure 3 below.

Figure 3: The cash flow cycle



Source: Norges Bank

Figure 3 gives a description of the cash flow cycle, which can be divided into two main parts, the small cash flow cycle and the large cash flow cycle.

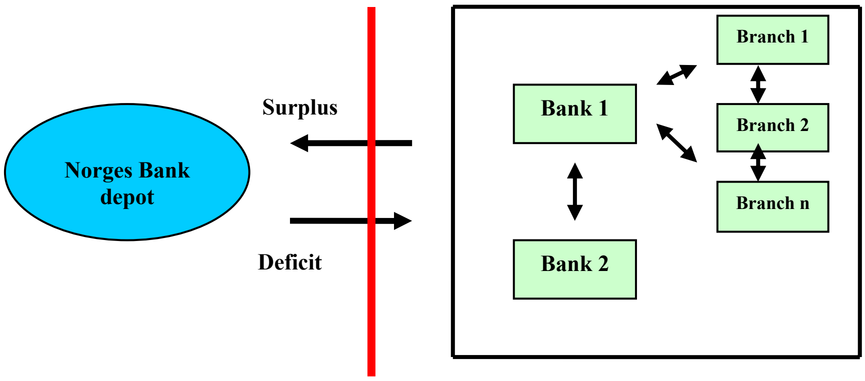
The small cash flow cycle between the central bank and commercial banks comprises the system for producing, replacing, processing and supplying cash. Banks turn in excess cash holdings to Norges Bank and requisition cash when needed (this is explained in more detail in the next section). Nearly all banks have accounts in Norges Bank in connection with cash deposits and withdrawals; see Norges Bank (2001).

In the large cash flow cycle, banknotes and coins circulate. The circulation is primarily between banks, enterprises and households/individuals. There are three main channels of how cash is supplied and put into the system. An individual can either acquire cash by going to the bank, from cash dispensers (ATMs) or in shops with payment terminals (cash back). This is illustrated in Figure 3. The red arrows show cash that enters the large cash flow cycle through withdrawals from ATMs and bank/post offices. Both withdrawals and cash back in connection with goods purchases represent conversion from deposit money to cash. However, cash back in connection with goods purchases does not supply new banknotes of coins to the large cash flow cycle because the cash is already supplied to the shops. The opposite case is illustrated by the blue arrows. They show cash on its way out of the large and into the small cash flow cycle, in other words enterprise and household sectors' payment of cash to banks. Finally the black arrows represent transactions that keep cash in circulation among members of the public.

2.5.2 What is currency in circulation?

The cash flow cycle explains how people acquire cash and how deposit money can be converted into cash. However, since this paper focus on notes and coins in circulation, it is important to distinguish between cash that circulates and cash that are held by Norges Bank. This relates to the discussion in Section 2.3.1 (on the monetary base M0). As explained there, a commercial bank has a perfect substitute to notes and coins in circulation, which is to place cash reserves in Norges Bank. Since banks are risk averse and earn interest on their cash reserve at Norges Bank, they want to minimize their cash holdings. Therefore they place their excess cash holdings in cash reserves at Norges Bank. This cash is withdrawn from circulation. To get the cash back in circulation again, the bank withdraw money from their reserves (held at Norges Bank). The banks then distribute the cash to their branches and into the large cash flow cycle. The red line in Figure 4 illustrates the distinction between cash that circulates and cash that are not in circulation.

Figure 4: Cash in circulation



Source: Norges Bank

2.5.3 The use of cash in different type of transactions

As mentioned above, cash enables immediate final settlement without an intermediary, which makes cash especially attractive as a mean of payment for small-value payments. As motivated in Humphrey, Kim and Vale (2001), the demand for payment instruments are derived from their use in transactions. They divide transactions into four different types:

1. Point-of-sale payments for food, clothing, transportation, entertainment and other retail purchases.
2. Bill payments for rent, mortgage, utilities, insurance, etc.
3. Disbursements for payroll, retirement, social benefits, etc.
4. Financial payments for interbusiness transactions, bank funding, government securities, foreign exchange, stock and commodity market transactions, etc.

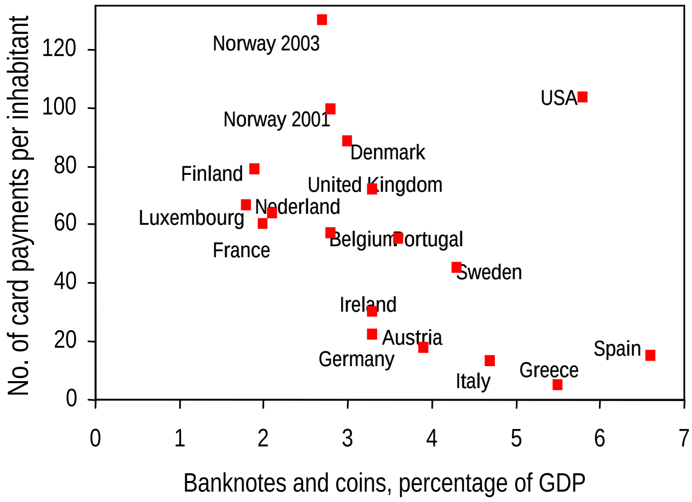
Due to the rapid evolution of the payment system in Norway, it is likely that use of cash is only appropriate for the first type of these transactions, namely point-of-sale transactions. The alternative to cash in point-of-sale transactions is use of deposit money.[13] Especially the use of payment cards can be considered as a substitute for cash in point-of-sale transactions.

[13] See Section 2.3.2 on the money stock M1.

2.6 Cash and the payment system: trends and developments

To illustrate roughly the main trends in the Norwegian payment system, we consider three figures. Figure 5 shows the share of cash in relation to GDP and the number of card payments per inhabitant in some European Union countries, the US and Norway. As we can see, most European Union countries have a higher share of cash in relation to GDP than Norway. There is also a tendency for countries with widespread card usage to have a lower cash share. However, this does not necessarily indicate cause and effect. In addition to the existence of alternatives to cash, cash holdings are also influenced, among other things, by the velocity of cash circulation.[14]

Figure 5: Banknotes and coins in circulation and payments by cards, international comparison 2001

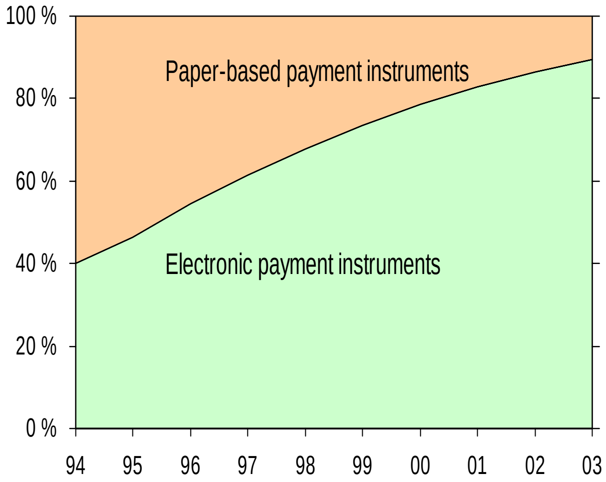


Source: ECB, CPSS (BIS) and Norges Bank

Further, as can be seen from Figure 6, the use of electronic payment instruments has grown rapidly through the last decade. In 1994 about 40 percent of all transactions where electronic based, and in 2003 electronic based transactions had risen to almost 90 per cent of all transactions. However, be aware that not all electronic payment instruments are substitutes to cash.[15] Furthermore, the figure gives an illustration of the development in the payment system.

[14] The concept velocity will be explained in Section 3.3.
[15] In fact, as discussed later, it is likely that payment cards are the only true substitute to cash.

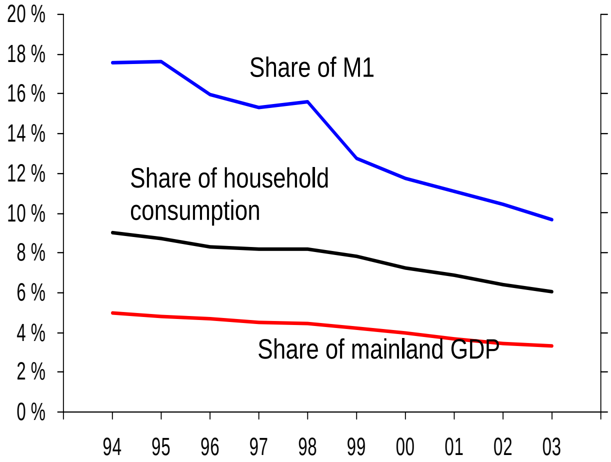
Figure 6: Use of paper-based and electronic payment instruments 1994-2003. Transactions in per cent



Source: Norges Bank

The declining ratio of notes and coins in circulation to the monetary aggregate M1, to household consumption and to mainland GDP, may also reflect the increased use of cashless payment instruments.

Figure 7: Value of notes and coins in circulation as a share of M1, household consumption and mainland GDP 1994-2003. Per cent



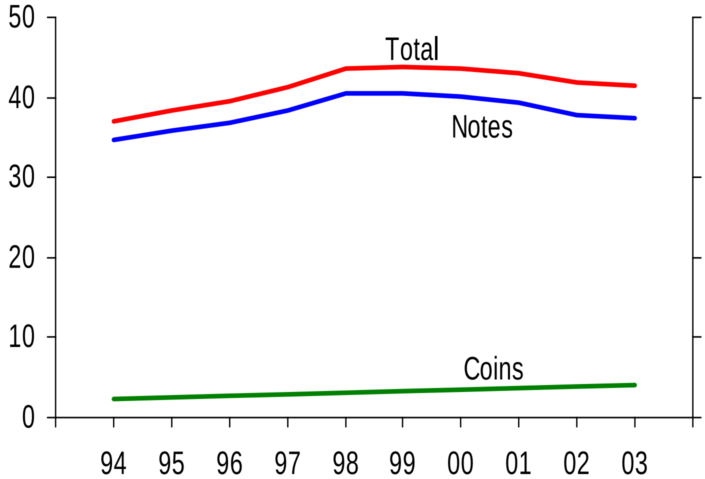
Source: Statistics Norway og Norges Bank

2.6.1 Cash

The use of cash and deposit money for transaction purposes is influenced by both trends in household consumption and the costs involved in acquisition and development of alternative means of payment.

The total nominal value of notes and coins in circulation increased every year in the period 1980-1999, even though the use of alternative payment services such as credit and debit cards increased rapidly. In 1999 the total value of cash in circulation (average value per year) reached its maximum at 43.8 billion NOK.[16] Thereafter the average value per year has decreased slightly. In 2003, the total value of cash in circulation was 41.6 billion NOK, with banknotes accounting for 37.5 billion NOK and coins for 4.1 billion NOK. This is illustrated in Figure 8.

Figure 8: Cash in circulation 1994-2003. Annual average. In billions of NOK



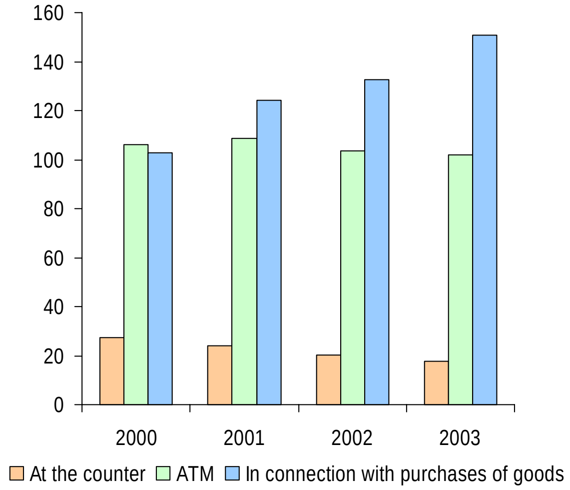
Source: Norges Bank

One aspect that is likely to have an influence on the use of cash for transactions purposes is the access to cash. As explained in Section 2.5.1. there are three main channels of how cash is distributed to the public. The development in these main channels (in addition to the development in alternative payment instruments which is discussed below) is therefore likely to have an influence on the use of cash. In the last decade the number of bank branches has declined slowly. The number of ATMs increased steadily until 2002 and has slowly been reduced since then. However, the main development which has made cash much more accessible over the last decade is the introduction of cash back from payment terminals. Cash back in connection with good purchases has become an important channel for acquiring cash. In 2000 this accounted for around 43 per cent of all cash withdrawals in Norway. This amount

[16] NOK = Norwegian Kroner

had increased to almost 56 per cent in 2003. An illustration of the number of cash withdrawals divided into the three channels (for acquiring cash) is given in Figure 9.

Figure 9: Number of cash withdrawals in Norway 2000-2003. In millions of withdrawals



Source: Norges Bank

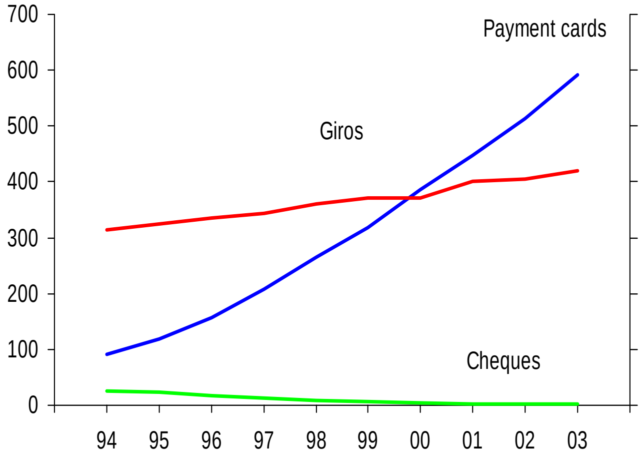
Another aspect that is also likely to have an influence on the use of cash is the illegal economy. The importance of cash for transactions in the illegal economy is a familiar phenomenon. As mentioned above, cash enables immediate final settlement without an intermediary, which makes cash perfectly suitable for use as mean of payment in illegal activities. There exist several papers in the literature, concerning the use of cash in illegal activities. A good illustration of this is given in Rogoff (1998). However because of lack of information and data about the illegal economy it will not be further elaborated in this paper.[17]

2.6.2 Alternatives to cash

Since the beginning of the 1980s, the increasing use of cashless payment instruments may have contributed towards reducing the demand for cash for transaction purposes. In particular use of payment cards is a substitute to cash payments since they are mainly used for small and medium-value purchases. As can be seen from Figure 10, the use of payment cards has increased considerably in Norway in the second half of the 1990s.

[17] However, as can be seen in the chapters below, I will discuss variables that may capture the effect of the underground economy on the use of cash.

Figure 10: Payment instruments 1994-2003.
Millions of transactions

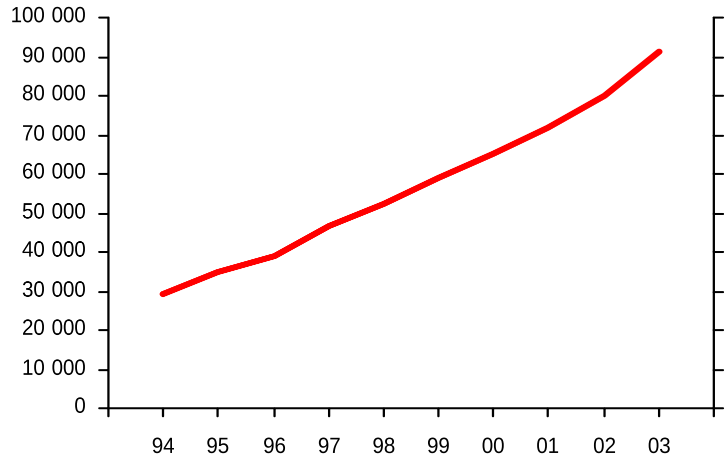


Source: Norges Bank

Figure 10 also shows the development in use of giros and cheques as payment instruments. The use of giro has increased significantly over the last ten years. However, giro is often not considered as a substitute to cash, and is therefore not further discussed in this paper.[18] As can also be seen the use of cheques have declined rapidly over the last decades, and now only accounts for about 0.1 per cent of all cashless payment transactions in Norway. The main reason that payment cards now can be considered as a substitute for cash is the rapid increase in point-of-sale terminals that accept payment cards. This has also lead to a rapid increase in the widespread of payment cards. There are few other countries where payment cards are used more often than in Norway. At the end of 2003 the total number of payment cards issued to Norwegian citizens where more than 6.9 million.[19] This is more than twice as much as in 1994. Further there were over 91 000 point-of-sale terminals (EFTPOS[20] payment terminals) that accepted payment cards in just under 60 000 locations at the end of 2003, which also is more than twice as many as in 1994. An illustration of this is given in Figure 11.

[18] For a detailed discussion on the development in giro payments in Norway, see Norges Bank (2004b)
[19] This is the number of "pure" cards (excluding oil companies' cards). That is, many of these cards are so-called double cards. This means that the card for example is both a bank card and a VISA. For a further discussion and explanation of different types of payment cards, see Norges Bank (2004a).
[20] EFTPOS = Electronic Funds Transfers at Point of Sale

Figure 11: No. of EFTPOS payment terminals 1994-2003



Source: Norges Bank

3. Theory of money demand

3.1 Introduction

In the literature there exist several different theories about the demand for money. Most theories are closely related to functions and the definition of money. Thus, the demand for money will be different depending on the combinations of assets that are included in the specific monetary aggregate. This means that a theory for money demand, using a broad monetary aggregate (M3), might be inappropriate when considering a more narrow monetary aggregate (M1). In this paper the focus is on demand for cash, leading us to a very narrow definition of money. Most of the literature on demand for money focuses on a broader definition of money. There exists very little literature considering demand for currency, and the literature that exists is mainly empirically oriented. However, some theoretical work has been done. Further the standard theoretical approaches to the demand for money do focus on a narrow concept of money, usually defined as the money stock M1. Thus, in the basic models it is assumed that money is a non-interest-bearing asset. This indicates that using basic models can be applicable for estimating demand for currency.

In this section I will first give a brief theoretical explanation of how economists motivate that people holds money. I will here go deeper into the functions and properties of money. Further, I will briefly explain two models of money demand, which often appears in economic textbooks. Finally, I will present a theoretical model for currency demand, based on models in Rogoff (1998) and Fischer, Köhler and Seitz (2004)

3.2 The properties of money

Why do people hold money, even though it does not pay interest? This is a fundamental question in macroeconomic theory. In this section I will try to explain economists thought of the determinants of the quantity of money demanded by private agents. Private agent decision of their demand for money will typically give rise to a choice problem. This choice problem involves the following consideration: for a given amount of wealth, individuals will normally wish to hold only a fraction of it in form of money. The remainder is held in the form of other assets such as bonds, stocks, houses etc. Every individual then has the opportunity to hold more money at a present time if it chooses to hold less of its wealth in other assets.

Cash pays no interest, while there are other assets that pay positive interests. So why are people then willing to hold money at all? The reason that people hold money is that it helps facilitate transactions. So the role of money as a medium of exchange is of great importance when considering why people want to hold a fraction of their wealth in money. This gives rise to an optimization problem – that of balancing the expected transactional benefits of holding an additional unit of money against the cost of doing so, which is the extra interest forgone.

From this tradeoff, the main characteristics of the demand for money by an individual at a point in time can be deduced. We can structure it as the following three characteristics, see McCallum (1989).

> 1. Since the purpose of holding money is to facilitate planned transactions, more money will be held the greater is the volume of transactions planned.
>
> 2. Since it is the real quantities of goods and services that people care about, the relevant quantity of money demanded will be expressed in real (price deflated) terms. That is, the behavioral relationship to be studied relates real money balances demanded to real transactions planned.

3. Since the drawback to holding money – the cost – is the interest sacrificed, the (real) quantity willingly held will be smaller the higher is the rate of interest on alternative assets.

We can now formally express these three properties in terms of a function relating the quantity of real money demanded by a representative person at time t, M_t/P_t, to his planned real spending during period t, Y_t, and the prevailing rate of interest on some relevant asset, i_t. If we now let L denote the function, we can assume that the person's money demand behavior satisfies

$$\frac{M_t}{P_t} = L(Y_t, i_t). \quad (1)$$

For this equation to satisfy the three characteristics above, L must be increasing in Y_t and decreasing in i_t. In other words we have to assume that $L_1(Y_t, i_t) > 0$ and $L_2(Y_t, i_t) < 0$.[21]

3.3 Quantity theory

As mentioned above, there exist many different theoretical models considering private agents demand for money. The quantity theory of money can be regarded as the earliest explanation of the demand for money. Its starting point is the so-called equation of exchange, which was developed by Fisher (1911).

$$M_t V_t \equiv P_t T_t. \quad (2)$$

Here M_t is the quantity of money, V_t is the transactions velocity, P_t is the price level, and T_t is the volume of transactions. The transactions velocity is the ratio of total transactions to money balances. Since it is difficult to find reliable statistical information on the volume of transactions, and it is assumed that the volume of transactions moves more or less in parallel with real GDP, the standard form of the quantity theory is:

$$M_t V_t \equiv P_t Y_t. \quad (3)$$

[21] $L_j(Y_t, i_t)$ denotes the partial derivative with respect to argument j.

The income velocity of money, V_t, measures the number of times the stock of money is turned over per period in financing the periodical flow of income. It is therefore equal to the ratio of GDP to the money stock.

Both (2) and (3) are identities. For this to be a theory of money demand, it requires some assumption about the velocity. Early proponents of the quantity theory assumed that V would be a rather stable variable that is determined mainly by institutional features of a country's payments system and by payment habits of its inhabitants. Therefore the velocity is regarded as a constant. The quantity theory can then be formulated as a theory of the demand for money.

$$\frac{M_t}{P_t} = \frac{1}{\overline{V}} Y_t, \qquad (4)$$

where $\overline{V}$ denotes a constant velocity of money. This equation gives us two important elements for any theory of money demand, see Bonfinger (2001). It shows that the demand for money:

1. Is proportional to the amount of real transactions, represented by the GDP

2. Is proportional to the price level (the elasticity of nominal money with respect to the price level is one). An interpretation is that the demand for money is actually a demand for real money.

A newer and more sophisticated approach is followed by Milton Friedman; see for example Friedman (1956). Here the velocity is no longer a constant but is assumed to be a function of the real interest rate. The demand for real money can then be written as:

$$\frac{M_t}{P_t} = \frac{1}{V(r_t)} Y_t. \qquad (5)$$

Today the quantity theory still plays an important role in economics, however now more as a building block for more complex approaches, see Bonfinger (2001).

3.4 Baumol-Tobin inventory model

Another, also quite simple model that is often presented in most textbooks on monetary theory (see e.g. Bonfinger (2001)) is the Baumol-Tobin inventory model, which was developed by Baumol (1952) and Tobin (1956). This is a model where only the transaction motive is taken into account, that is, people hold money because they want to use it for payments. This model applies mainly to a narrow definition of money. In the model the assets are restricted to non-interest-bearing money and interest-bearing bonds. Further it is assumed that the transfer for money into bonds is associated with fixed transactions costs (*c*). The model derives the optimal money holdings for a household that receives a monthly income which is completely spent on the purchase of goods and services (*PY*). It is assumed that the purchases are spread evenly in time throughout the month. Every month the household receives the income in form of a cheque. In order to credit this to its account, the individual must go to the bank. At the beginning of the month the money stock (*M*) of each household is therefore given by

$$M = PY \,. \tag{6}$$

Further the household is now confronted with the following decision problem. It can convert parts of the money stock into bonds, which will give an interest income. However there are also transaction costs for buying bonds. In this model both the optimum number of exchange transactions and the optimum average money holdings are determined. The average money holdings during the month is given by

$$M = \frac{PY}{2n} \,, \tag{7}$$

where *n* is the number of transactions. Further the total monthly costs of the payment services (PS) are

$$PS = OC + TC = Mi + (n-1)c \,. \tag{8}$$

Equation (8) says that the total costs of the payments services (*PS*) is equal to the opportunity costs of holding money (*OC*) and the monthly transaction costs (*TC)* for converting money into bonds. The opportunity cost of money is given by average money holdings (*M*) multiplied with the interest rate on bonds (*i*). The monthly transaction cost is given by *(n-1)* times the transaction cost *c*, where it is assumed that the transaction costs are independent of the amount that is transferred and they are only required for the exchange of money into bonds. By solving (7) for n and inserting this result into (8) gives us

$$PS = Mi + c\left(\frac{PY}{2M}\right) - c\,. \tag{9}$$

Cost minimization is then obtained by differentiating PS with respect to M and setting the resulting equation equal to zero. The first order condition of the optimization problem, which represents the optimal real money stock, is then given by:

$$\left(\frac{M}{P}\right)^{*} = \sqrt{\frac{c_R Y}{2i}}\,. \tag{10}$$

Here $c_R = \frac{c}{P}$ represents real transactions costs.

The main results of the Baumol-Tobin model are that the demand for non-interest-bearing money depends:

1. Positively on the real income. But in contrast to the quantity theory, the relationship is not proportional.

2. Negatively on the interest rate of bonds.

3. Positively on the real transactions costs. In fact when the transactions costs become very small, the demand for non-interest bearing money goes to zero.

Even though the Baumol-Tobin model seems to be relevant for describing a narrow money aggregate (and then also currency demand), its assumptions seem highly artificial and almost unrelated to reality (see McCallum (1989)). It is therefore preferable to focus on a different model that is more general than the Baumol-Tobin model. In the next section I will therefore focus on such a model. The model I will present is a version of a more general model. This version is especially derived for analyzing the demand for currency.

3.5 A theoretical model of currency demand

The empirical literature on currency demand has already recognized the importance of currency use in the underground economy. However, the theoretical treatments of money demand and optimal inflation taxation have remained curiously oblivious to this possibility, see Rogoff (1998). The model that is presented, is a model that in addition to the effects included in Equation (1), tries to capture the presence of tax evasion or underground economic activities on the demand for currency. The model concentrates on a specific theoretical model, the so-called money-in-the-utility-function model. It captures the role of money as a store of values as well as a medium of exchange. The model is based on the models in Rogoff (1998) and Fischer, Köhler and Seitz (2004).[22] The aim of the model is to derive which variables that should be incorporated in an empirical currency demand model.

Consider an economy where the domestic currency is the sole legal tender. Lifetime individual utility, *U*, is given by the discounted sum of per period utility functions of the representative agent. This is represented in Equation (11)

$$U = \sum_{t=0}^{\infty} \beta^t u(C_t, \frac{M_{t+1}}{P_t}) . \qquad (11)$$

In Equation (11) C_t is consumption in period *t*, M_{t+1} denotes the nominal currency stock held between period *t* and *t+1* and P_t is the price level. It is assumed that u is strictly concave with $u_i > 0$ and $u_{ii} < 0$,[23] where i = 1, 2. The parameter β is a discount factor that is positive but smaller than unity. Thus the household has positive time preferences and "discounts the future" in the sense that the utility it receives in *t* from a given C_t and $\frac{M_{t+1}}{P_t}$ combination is greater than from the same combination if planned for *t+1*: if $C_{t+1} = C_t$ and $\frac{M_{t+2}}{P_{t+1}} = \frac{M_{t+1}}{P_t}$, then β < 1 implies that $u(C_t, \frac{M_{t+1}}{P_t}) > \beta u(C_{t+1}, \frac{M_{t+2}}{P_{t+1}})$.

[22] Fischer, Köhler and Seitz (2004), consider a slightly different model. They consider a small open economy, where some of the demand for the domestic currency comes from abroad. However, since only a very small fraction of the total Norwegian currencies in circulation is held abroad (Statistics Norway has estimated it to be about 0.7 per cent of the total currencies in circulation, see Gresvik and Kaloudis (2001)), I will not focus on this.

[23] u_{ii}, denotes the double derivative, with respect to argument i.

In this model people hold currency because real balances are an argument of the utility function. This approach underpins an implicit assumption that agents gain utility from both consumption and leisure. Real currency balances enters the utility function directly because it allow agents to save time in conducting their transactions. Therefore the model capture currency's role as a store of value and as a medium of exchange.

In order to make the model as simple as possible, the individual is in each period endowed with a fixed and time-invariant gross real income, *Y*. It also faces a proportional tax on earned income at a notational rate τ. The tax rate is notional in that the agent can reduce its effective tax rate by holding a higher level of real currency levels, *M/P*. This should capture the idea that using currency helps avoiding the detection of income by tax authorities. Thus, net real taxes paid by the individual are:

$$\tau g\left(\frac{M_{t+1}}{P_t Y}\right) \quad \text{with } g(0) = 1,\ g'(.) < 0,\ \ g''(.) > 0 \text{ and } \lim_{M/PY \to \infty} g\left(\frac{M_{t+1}}{P_t Y}\right) \geq 0$$

In other words, the value of *g* must lie in the interval [0,1]. Our assumptions on the tax evasion technology implies that the individual's budget constraint in nominal terms for any time *t* may be written as

$$\frac{B_{t+1}}{(1+r_t)} + \frac{M_{t+1}}{P_t} + C_t = B_t + \frac{M_t}{P_t} + Y\left(1 - \tau g\left(\frac{M_{t+1}}{P_t Y}\right)\right). \qquad (12)$$

Here, B_t is the real value of one-period bond holdings that mature at the beginning of period *t*, denominated in units of time-*t* consumption. r_t represents the real interest rate and is the rate of return on bonds held from period *t* to period *t+1*.[24] Note finally that the individual begins period *t* with asset stocks B_t and M_t. In (12) the left hand side totals expenditures on consumption and bonds during *t*, plus currency balances held at the end of the period. Similarly the right hand side in (12) totals the resources available to the household from current income, currency that is brought into the period and bonds purchased (loans made) in the past, in other words sources of wealth. To assure a bounded budget set, we have to assume

[24] This differs from Rogoff (1998) and Fischer, Köhler and Seitz (2004), who both assume a constant real interest rate

that $1-\frac{P_t}{P_{t+1}}\frac{1}{(1+r_t)}=1-\frac{R_{m,t}}{R_t}=1-\frac{1}{\pi_{t+1}(1+r_t)}=\frac{i_{t+1}}{1+i_{t+1}}\geq 0$.[25] That is, the real gross return on money held from t to $t+1$ ($R_{m,t}$) must be less or equal to the real gross return on bonds held from t to $t+1$ (R_t). Otherwise agents would be able to make infinite profits by choosing arbitrarily large money holdings financed by issuing bonds. In other words, the nominal interest rate i_{t+1} cannot be negative.[26]

Further, the household's object is for a given price system $\{r_t\}_{t=0}^{\infty}$, $\{P_t\}_{t=0}^{\infty}$ and initial money M_0 and bonds B_0, to maximize (11) by choosing a sequence $\{\mathrm{C}_t, \mathrm{M}_{t+1}, \mathrm{B}_{t+1}\}_{t=0}^{\infty}$ subject to the budget constraint (12). We can then formulate the following Lagrangian:

$$L=\sum_{t=0}^{\infty}\beta^t\left\{u(C_t,\frac{M_{t+1}}{P_t})-\lambda_t\left(\frac{B_{t+1}}{1+r_t}+\frac{M_{t+1}}{P_t}+C_t-B_t-\frac{M_t}{P_t}-Y\left(1-\tau g(\frac{M_{t+1}}{P_tY})\right)\right)\right\}. \tag{13}$$

Maximization of (13) gives us the following first order conditions:

$$\frac{\partial L}{\partial C_t}=\beta^t u_C(C_t,\frac{M_{t+1}}{P_t})-\beta^t\lambda_t=0, \qquad \forall t, \tag{14}$$

$$\frac{\partial L}{\partial M_{t+1}}=\frac{1}{P_t}\beta^t u_{M/P}(C_t,\frac{M_{t+1}}{P_t})-\beta^t\lambda_t\frac{1}{P_t}\left(1+\tau g'(\frac{M_{t+1}}{P_tY})\right)+\beta^{t+1}\lambda_{t+1}\frac{1}{P_{t+1}}=0, \forall t, \tag{15}$$

$$\frac{\partial L}{\partial B_{t+1}}=-\beta^t\lambda_t\frac{1}{(1+r_t)}+\beta^{t+1}\lambda_{t+1}=0, \qquad \forall t \tag{16}$$

and

$$\frac{\partial L}{\partial \lambda_t}=C_t+\frac{B_{t+1}}{(1+r_t)}+\frac{M_{t+1}}{P_t}-B_t-\frac{M_t}{P_t}-Y\left(1-\tau g(\frac{M_{t+1}}{P_tY})\right)=0, \qquad \forall t. \tag{17}$$

Acknowledging that (14) is an identity in time and rearranging yields:

---

[25] I have here defined $\pi_{t+1}=\frac{P_{t+1}}{P_t}$. Further the last equality follows from the Fisher identity $1+i_{t+1}=\frac{P_{t+1}}{P_t}(1+r_t)$.

[26] Ljungqvist and Sargent (2000) give a formal and detailed explanation of this condition.

$$u_C(C_t, \frac{M_{t+1}}{P_t}) = \lambda_t, \qquad \forall t. \quad (18)$$

This means that the following must hold:

$$u_C(C_{t+1}, \frac{M_{t+2}}{P_{t+1}}) = \lambda_{t+1}, \qquad \forall t. \quad (19)$$

Further, if we rearrange (16) we get:

$$\beta\lambda_{t+1} = \frac{\lambda_t}{(1+r_t)} \Leftrightarrow \frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{\beta(1+r_t)}, \qquad \forall t. \quad (20)$$

Dividing (18) by (19), inserting for equation (20) and rearranging give us

$$u_C(C_t, \frac{M_{t+1}}{P_t}) = (1+r_t)\beta u_C(C_{t+1}, \frac{M_{t+2}}{P_{t+1}}), \qquad \forall t. \quad (21)$$

Finally if we now insert (18) and (19) into equation (15) and rearrange, we get the following equation:

$$\frac{1}{P_t} u_C(C_t, \frac{M_{t+1}}{P_t})\left[1 + \tau g'(\frac{M_{t+1}}{P_t Y})\right] = \frac{1}{P_t} u_{M/P}(C_t, \frac{M_{t+1}}{P_t}) + \frac{1}{P_{t+1}} \beta u_C(C_{t+1}, \frac{M_{t+2}}{P_{t+1}}), \quad \forall t. \quad (22)$$

Condition (21) is the standard consumption-Euler equation. It states that at a utility maximum the consumer cannot gain from shifts of consumption between periods. Equation (22) determines the allocation of income between money and consumption. I will now take a closer look at the equation: $\frac{1}{P_t}\left[1 + \tau g'(\frac{M_{t+1}}{P_t Y})\right]$ is the quantity of current consumption a person must forgo to raise real balances by one unit,[27] and $u_C(.)$ is the marginal utility of that

[27] This expression is less than $1/P_t$. The reason for this is that when holding a larger amount in terms of money (currency), one pays less tax (tax evasion). Therefore a person must forgo a smaller amount of current consumption than in a case without such taxes, to raise real balances by one unit.

consumption. On the right-hand side, the first term is the marginal utility the agent gets from having one extra currency unit to conduct transactions. Breaking down the second term on the right hand side, $1/P_{t+1}$ is the quantity of consumption the individual will be able to purchase in the period *t+1* with an extra currency unit, $\beta u_C(.)$ is the marginal utility of date *t+1* consumption, discounted to period *t*.

Combining (21) and (22) yields:

$$\frac{u_{M/P}(C_t, \frac{M_{t+1}}{P_t})}{u_C(C_t, \frac{M_{t+1}}{P_t})} - \tau g'(\frac{M_{t+1}}{P_t Y}) = 1 - \frac{P_t}{P_{t+1}(1+r_t)} = \frac{i_{t+1}}{1+i_{t+1}}, \qquad \forall\, t. \quad (23)$$

Equation (23) characterizes the optimal solution to the maximization problem. The equation has two endogenous variables. The solution to the optimization problem involve that the paths for C_t, M_t, B_t and λ_t are functions of τ, Y and the paths for r_t and P_t. Under certain parametric functions and together with a transversality condition it will be possible to calculate the explicit solutions.[28] This will give a demand function for real balances which will be increasing in y and decreasing in nominal interest rate i_t. The one important difference compared to the previous models in this chapter, is that money (currency) demand also depends positively *on the marginal tax rate* τ.

4. Econometric methodology

4.1 Introduction

In the previous chapter I discussed some theoretical models. In the next chapters the focus will be on the empirical investigation of the demand for currency in circulation. Often there is a gap between the theoretical model and the empirical model specification. One reason for this is that econometricians not only have to estimate the coefficients in a theoretical model, but also have to worry about the validity of the estimators. In other words, they must worry about serial correlation, multicollinearity, heteroscedasticity, simultaneity and so forth. Further some dynamic models such as the model in Section 3.5 do not have closed form

[28] This transversality condition says that the initial financial wealth plus the present discounted value of disposable income must be equal to the present discounted value of expenditure on consumption and on “renting” real balances. See Obstfeld and Rogoff (1996) for a detailed explanation.

solutions.[29] Therefore it is difficult to give such models an empirical representation that coincides perfectly with the theoretical representation. It may therefore be argued that it is appropriate to use the theoretical models as references to which variables that should be included in the econometric specification of the model.

Further, in this case, the focus is on demand for currency in circulation. As mentioned in Chapter 3, there exist few theories that consider such a narrow definition of money. This feature combined with the fact that the development in the payment system have increased rapidly over the last decades, may lead us searching for “new” variables which are not represented in theoretical models, but are believed to influence the demand for currency in circulation. This is discussed in Chapter 5.

The aim of this chapter is to give a clear overview of the econometric methodology that is used in the empirical investigation of demand for currency in circulation. I will first shortly motivate and explain the choice of methodology. Then I will give a brief explanation of the different steps in this procedure, explaining different statistical tests and statistical properties that are relevant for the empirical analysis presented in the next chapters.

4.2 General-to-specific methodology

The empirical investigation described in the next chapters, is based on the so-called general-to-specific methodology.[30] Basically the general-to-specific approach can be summarized in the following list of points, which the investigator should follow and work through, see Hendry and Krolzig (2001).

- Start from a general dynamic statistical model (frequently this is a VAR model), which captures essential characteristics of the data. In other words a model which is congruent with data evidence.

- Eliminate statistically insignificant variables by standard testing procedure to reduce its complexity.

[29] The money demand function was not given on explicit form.
[30] This methodology is also often called the LSE methodology, and was developed by British econometricians at the London School of Economics in the 1950s and 1960s, see Gilbert (1986).

- Implement diagnostic tests to check the validity of the reductions to ensure congruency of the final model.

- The model should encompass a wide range of other models. That is, it should be able to explain other models results.

- Check the reliability of the model on sub-samples. This could for example be done by checking the in-sample forecasting properties of the model.

The fact that the data generating process is uncertain and there exist few theoretical models for currency in circulation, makes the general-to-specific approach particularly appropriate to use in this case. However, the approach has some problems/limitations which make it difficult to stick fully to the list presented above. Especially one problem in relation to the use of a VAR model is of great concern. In fact this is a general problem concerning such class of models and was first pointed out by Christopher A. Sims who wrote that:

> *"If every variable is allowed to influence every other with a distributed lag of reasonable length, without restrictions, the number of parameters grows with the square of the number of variables and quickly exhausts the degree of freedom"*.
> Sims (1980, p. 16)

This is a problem that is relevant in this paper. My aim was to start out with a "broad" VAR model, but due to problems with residual misspecification tests, especially due to autocorrelation, another type of model had to be considered. In my case, I have chosen to focus on a "broad" single equation specification. This gives a slightly different modelling procedure than I first had in mind. This is especially the case when considering whether the variables are cointegrated or not. In a VAR approach, this is usually done by applying the maximum likelihood based method used by Johansen (1988, 1991). There exist other methods which consider cointegration in a single equation approach. In my case, I will focus on the method presented in Banerjee, Dolado and Mestre (1998). This method is explained later in this chapter.

4.3 Integrated variables and unit root test

The first step in the process is to specify a general statistical model. By general is meant a "broad" model existing of all the variables that are believed to influence the variable of interest. But before we can specify such a broad model, we want to assure that our variables are transformed such that they are at least weakly stationary. A process $\{y_t\}$ is said to be weakly stationary if the first and second order moments of the process exist and are time-invariant. In other words:

$$
\begin{aligned}
E(y_t) &= \mu < \infty && \text{for all } t \in T \\
E[(y_t - \mu)(y_{t-h} - \mu)] &= \gamma(h) < \infty && \text{for all t and h.}
\end{aligned} \tag{24}
$$

Many economic variables exhibit stochastic trends, such as GDP and consumption, and are therefore not stationary. In other words they have at least one unit root. However, in many cases stationarity can be achieved by simple differencing. A non-stationary series is said to be integrated of order *d*, denoted *I(d)*, if it becomes stationary after being differenced *d* times. When specifying a dynamic economic model we would like to have equations of only stationary variables on each side of the sign of equation. In other words we want to have a balanced equation. An important aspect is that in an equilibrium-correction model (will be explained below) the mapping to stationarity is obtained partly by differencing the variables and partly by a linear combination of level variables that cointegrates. The concept of cointegration will be explained in Section 4.5.

In the empirical investigation one would like to test whether the variables have at least one unit root (non-stationary) or not (stationary). There exist a number of such unit root tests. However, I will only concentrate on one of them, namely the augmented Dickey-Fuller (ADF) test.

4.3.1 Augmented Dickey-Fuller (ADF) test

The ADF test, see Dickey and Fuller (1981), takes the following equation as a starting point.

$$\Delta y_t = \alpha + (\beta - 1) y_{t-1} + \sum_{i=1}^{s} \gamma_i \Delta y_{t-i} + \varepsilon_t \tag{25}$$

It is a one-sided test, which tests H_0: $\beta - 1 = 0$ against H_1: $\beta - 1 < 0$. In other words the null hypothesis is that the variable contains a unit root. A significant test statistic would then reject the null hypothesis and suggest stationarity. So if H_0 is rejected, the variable is said to be stationary. If H_0 is not rejected, there is at least one unit root and the variable is non-stationary. Now, taking the first difference of the variable, we can apply the test procedure again. This time the null hypothesis is that the first difference of the variable has a unit root. If H_0 is rejected, it can be concluded that the variable is *I(1)*. If H_0 can not be rejected the same procedure can be applied again, testing now whether the variable is *I(2)* or not. However, it is important to be aware of that the test is sensitive to the lag length. A maximum lag length of the variable that is tested should therefore be decided in advance. In my case, I have chosen the maximum lag length to be 5. The strategy is then to choose the specification with the highest significant lag length. Alternatively the lag length can be chosen by applying some information criteria, for instance the Akaike information criterion (AIC). Further, it is also important to notice that the critical values and the test statistic are sensitive to inclusions of a constant, or a constant and trend.[31]

4.4 Residual misspecification tests

In this section I will give a brief explanation of the residual misspecification tests that are calculated by the software programme PcGive 10.1. This software programme is the one that is used in the empirical investigation presented in the next chapters. A further discussion and explanation of these tests can be found in Hendry and Doornik (1996).

4.4.1 Error autocorrelation test (AR 1-5 test)

This is a Lagrange multiplier (LM) type of test for r^{th} order residual autocorrelation. It is distributed as $\chi^2(r)$ in large samples, under the null hypothesis of no autocorrelation. However due to better small sample properties an F-form of the test is suggested by Harvey (1981) and is therefore used. In my case, I have chosen a lag length of 5 lags. The test is based on a regression of the following equation:

$$\hat{\varepsilon}_t = \sum_{i=1}^{5} \beta_i \hat{\varepsilon}_{t-1} + u_t , \qquad (26)$$

[31] The critical values PcGive uses are not influenced by inclusion of seasonal dummies.

where $\hat{\varepsilon}_t$ represents the residuals based on the OLS regression of the structural equation. The following hypothesis is tested: H_0: $\beta_i = 0$, where i = 1,…,5 (no autocorrelation) against its alternative H_1: at least one of the β-parameters is different from zero.

4.4.2 Autoregressive conditional heteroscedasticity test (ARCH 1-4 test)

This is an LM type of test for ARCH-effects. It is based on the test outlined in Engle (1982). It tests the null hypothesis of no ARCH against its alternative where ARCH-effects are present. In my case, where a lag length of 4 is chosen, this can be represented by testing H_0: $\gamma_i = 0$, where i = 1,…,4 against H_1: at least one of the γ-s is different from zero in the model:

$$\hat{\varepsilon}_t^2 = c + \sum_{i=1}^{4} \gamma_i \hat{\varepsilon}_{t-i}^2 + \eta_t \qquad (27)$$

The test statistic is asymptotically $\chi^2(4)$-distributed in large samples under H_0, but again an F-form of the test statistic is to be preferred due to better small-sample properties.

4.4.3 Normality test

This is a test, testing whether the skewness (3rd moment) and kurtosis (4th moment) of the residuals corresponds to that of a normal distribution, against its alternative of non-normal residuals. The test statistic is based on the test statistic in Bowman and Shenton (1975), but with some slight modifications. The test reported in PcGive is fully described in Doornik and Hansen (1994). The test statistic is $\chi^2(2)$-distributed under H_0.

4.4.4 Test for Heteroscedasticity using squares

This test is based on White (1980). It involves an auxiliary regression of $\{\hat{\varepsilon}_t^2\}$ on the original regressors (x_{it}) and all their squares (x_{it}^2).[32] The null hypothesis is unconditional homoscedasticity and the alternative is that the variance of the $\{\varepsilon_t\}$ process depends on the

[32] If there is a large number of observations relative to the number of variables in the regression, PcGive also reports a second test for heteroscedasticity. This is a test based on an auxiliary regression of the squared residuals $\{\hat{\varepsilon}_t^2\}$ on all squares and cross-products of the original regressors.

regressors (x_{it}) and their squares (x_{it}^2). Again the F-form of the test is applied, due to its better small sample properties.

4.4.5 Regression specification test (RESET)

The RESET test, based on the test in Ramsey (1969), is a test for misspecification of the functional form. It tests the null hypothesis of correct specification of the model against the alternative that powers of $\hat{y}_t$ such as $(\hat{y}_t^2, \hat{y}_t^3, ...)$ have been omitted. It tests to see if the original linear form is incorrect, by adding powers of linear combinations of the right hand side variables.

4.5 Equilibrium correction model (ECM)

Given that the variables are non-stationary, that is they are not I(0), it is of interest to investigate whether they are cointegrated or not. Let us assume that we have two variables y_t and x_t, and that they are both integrated of order 1. Hence, the first differences of these variables are stationary. However there might exist a linear combination of the two variables y_t and x_t in levels that is stationary. If we assume that we have the following equation:

$$y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \beta_2 y_{t-1} + \varepsilon_t \quad where\ \varepsilon_t \sim \mathrm{NID}(0, \sigma_\varepsilon^2). \tag{28}$$

Then without loss of generality this model can be transformed to:

$$\Delta y_t = \alpha + \beta \Delta x_t + \gamma(y_{t-1} - \delta x_{t-1}) + \varepsilon_t, \tag{29}$$

where $\beta = \beta_0, \quad \gamma = \beta_2 - 1, \quad and \quad \delta = -\dfrac{(\beta_0 + \beta_1)}{\beta_2 - 1}, \quad (provided\ \beta_2 \neq 1).$

Equation (29) is often called an equilibrium-correction equation, where the term $(y_{t-1} - \delta x_{t-1})$ is called an equilibrium-correction term (ECM). The ECM models have been of great interest for applied econometricians because of their ability to distinguish between short-run (immediate) and long-run effects of changes in the variables in a simple way. The ECM-term represents the long-run solution (equilibrium) of the model. That is what economists often refer to as "steady state". The coefficient in front of the ECM-term may therefore be interpreted as an adjustment coefficient.[33] It tells us something about how the model reacts if we are in disequilibrium. β represents the short-run (immediate) effect of a change in Δx_t.

4.6 Test for cointegration in a single equation framework

In this section I will, as mentioned above, focus on the test developed in Banerjee, Dolado and Mestre (1998). They have proposed a test for cointegration in a single equation framework under the assumptions that the regressors are weakly exogenous for the parameters of interest.[34] The test is based upon the ordinary least squares coefficient of the lagged dependent variable in an equilibrium correction model augmented with lags of the regressors. In other words the procedure depends upon the significance of the error-correction term in the ECM re-parameterization of the model. This is a one-sided test and can be represented by testing the following hypothesis: H_0: $\gamma = 0$ (no cointegration) against H_1: $\gamma < 0$ for the linear counterpart of Equation (29).

In Banerjee, Dolado and Mestre (1998), limit distributions of the standardized coefficient and t-ratio versions of the ECM test under the null hypothesis of no cointegration is obtained and critical values are provided. The limit distributions do not depend upon nuisance parameters, but depends on the number of regressors and whether a deterministic trend is included or not (in addition to a constant) in the model.

4.7 The concept of weak exogeneity and testing for weakly exogenous regressors

In this paper, the focus is on finding a single equation model which gives the best possible representation of what determines the demand for real currency in circulation. However, a problem that often occurs in such single equation models is that not all the regressors can be considered as exogenous variables. Therefore one often considers a VAR model, which is a reduced form system, derived from a simultaneous system of equations. But as explained above, in this paper only a single equation model is investigated. If this single equation model is estimated conditional on the regressors, and the true process is determined in a simultaneous system of equations, the results from such estimation may be misleading. This is

[33] However, if lagged values of the dependent variable ($\Delta y_{t-1},\ldots, \Delta y_{t-p}$) are included in the model, some carefulness about the use of the concept adjustment coefficient is needed. Since the adjustment process then also depends on the parameters that are in front of these lagged dependent variables.

[34] The concept weak exogeneity is explained in Section 4.7.

because we then implicitly assume that knowledge of the processes generating the explanatory variables would carry no relevant information to the parameters of interest.

However, sometimes it might be the case that some of the equations in such system have regressors that are so-called weakly exogenous. The concept of exogeneity is studied in detail in Engle, Hendry and Richard (1983).[35] They refer to a variable as exogenous with respect to a particular parameter, if knowledge of the process generating the exogenous variables (denoted as the marginal processes) contains no information about that parameter. That is, if we consider a money demand equation, knowledge of the process generating consumption contains no information about the parameters in front of the exogenous variables in the structural equation. The consumption variable is then said to be weakly exogenous with respect to its parameter in the money demand function. If we are only interested in the money demand equation, the opposite case, where money demand is weakly exogenous for the parameters in the consumption function, do not necessarily have to hold. A more formal definition of weak exogeneity is provided by Engle, Hendry and Richard (1983).[36]

In our case, where the model which is considered is an equilibrium-correction model, one has to pay attention to both weak exogeneity for the short-run and the long-run (equilibrium-correction term) parameters. I will now give a brief explanation of what is meant by weak exogeneity in an equilibrium-correction model. A more detailed explanation can be found in Boswijk and Urbain (1997).

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[35] They distinguish between three different concepts of exogeneity; weak, strong and strictly. However, I will only focus on one of them, namely the concept of weak exogeneity.

[36] This can shortly be explained as follow. Let $x_t = (y_t, z_t)'$ be generated by the process with conditional density function $D(x_t | X_{t-1}, \lambda)$, where X_{t-1} denotes the history of the variables x: $X_{t-1} = (x_{t-1}, x_{t-2}, ..., X_0)$ and X_0 denotes the initial values for x_0, x_{-1} etc. Then let the parameters $\lambda \in \Lambda$ be partitioned into (λ_1, λ_2) to support the factorization $D(x_t | X_{t-1}, \lambda) = D(y_t | z_t, X_{t-1}, \lambda_1) D(z_t | X_{t-1}, \lambda_2)$.

Definition: z_t is weakly exogenous with respect to ψ over the sample period, where ψ denotes the parameter of interest, if and only if there exist a re-parameterization with $\lambda = (\lambda_1, \lambda_2)$ such that

(i) ψ is a function of λ_1 alone,

(ii) $[(y_t | z_t ; \lambda_1)(z_t ; \lambda_2)]$ operates as a sequential cut.

From a vector equilibrium-correction model (VECM) it is possible to partition the left hand side variable as $(y_t, z_t')'$ so that y_t is a scalar time series and z_t is a k-dimensional vector time series ($k = n - 1$). This gives the following conditional and marginal model:

$$\begin{aligned} \Delta y_t &= \phi_1' \omega_{1t} + \lambda(y_{t-1} - \kappa' z_{t-1}) + \xi' u_{2t} + u_{1.2,t} \\ \Delta z_t &= \Gamma_2 \omega_{2t} + \alpha_2 (y_{t-1} - \kappa' z_{t-1}) + u_{2t} \end{aligned} . \quad (30)$$

Here $\omega_{1t} = H'(\Delta z_t', \Delta x_{t-1}', ..., \Delta x_{t-p+1}')'$ is the vector of included differenced regressors in the structural equation, with (unrestricted) parameter vector ϕ_1. H is a known design-matrix, which is included to ensure identification of the structural equation. For a more detailed explanation of this, see Boswijk and Urbain (1997). Further $\omega_{2t} = (\Delta x_{t-1}', ..., \Delta x_{t-p+1}')'$ is the corresponding regressor vector in the marginal model, with associated coefficient matrix $\Gamma_2 = (\Gamma_{21}, ..., \Gamma_{2,p-1})$. Finally $\xi = \Sigma_{22}^{-1} \sigma_{21}$, where Σ_{22}^{-1} and σ_{21} are parts from the covariance matrix of the structural model.[37] In the conditional equation $u_{1.2,t}$ denotes the error in the equation for Δy_t after conditioning on Δz_t. The following null hypotheses of weak exogeneity can be formulated as:

$$\begin{aligned} &H_0^{\xi}: \quad \xi = 0; \quad \text{(orthogonality)} \\ &\mathrm{H}_0^{\alpha}: \quad \alpha_2 = 0; \quad \text{(no equilibrium correction)} \\ &\mathrm{H}_0 = H_0^{\xi} \bigcap \mathrm{H}_0^{\alpha}: \quad \xi = 0, \alpha_2 = 0 \end{aligned}$$

Here H_0^{ξ} represents the null hypothesis of weak exogeneity for the short-run parameters. H_0^{α}, is the null hypothesis of weak exogeneity for the long-run parameters and finally H_0 represents the joint null hypothesis of weak exogeneity both for the short-run and the long-run parameters.

When testing for weak exogeneity of the regressors the test presented in Boswijk and Urbain (1997) is used. They suggest the following procedure for testing whether the regressors are

[37] $\Sigma = \begin{bmatrix} \sigma_{11} & \sigma_{21}' \\ \sigma_{21} & \Sigma_{22} \end{bmatrix}$ is the covariance matrix of the structural model. A more detailed explanation of this and the relationship between the structural and reduced form model is given in Boswijk and Urbain (1997).

weakly stationary or not. First a structural (conditional) model and a marginal model should be estimated by (multivariate) least squares. Then the residuals $\hat{u}_{2t}$ from the marginal model and the lagged disequilibrium errors $\hat{v}_{t-1} = (y_{t-1} - \hat{k}z_{t-1})$ from the structural (conditional) model should be constructed. Next, calculate LM_1, the variable-addition test statistic for the significance of $\hat{u}_{2t}$ in the structural equation, and the statistic LM_2 for the significance of $\hat{v}_{t-1}$ in the marginal model. If exogeneity for the long-run parameters is maintained, and only exogeneity for the short-run parameters is to be tested, then LM_1 is to be compared by critical values from $\chi^2(k)$ distribution. If one wishes to test $\alpha_2 = 0$ while maintaining the orthogonality assumption $\sigma_{21} = 0$, LM_2 should be compared with $\chi^2(k)$ quantiles. Finally the joint hypothesis is tested by comparing $LM = LM_1 + LM_2$ with critical values from the $\chi^2(2k)$ distribution.[38]

4.8 Parameter stability and Chow tests

One criterion that should be fulfilled when selecting a model based on the general-to-specific approach is that the model should contain reasonably constant parameters. Parameter stability is of especially great interest if the model is to be used for forecast purposes. Therefore recursive estimates (with ± 2 standard errors) and sequences of Chow tests (scaled by their 1 % critical values) will be considered. It will be appropriate to give a short explanation of the different Chow tests for structural changes. Further, in the PcGive output sequences of different variants of the Chow tests are computed. I will therefore give a brief explanation of how these sequences are computed. A more thorough explanation of these tests can be found in Maddala and Kim (1998).

4.8.1 Chow break point test

According to Hendry and Doornik (1996), this is considered as the main test of parameter constancy. Assume that we have one particular point in time (T), where we want to check whether a break has occurred or not. The idea of the test is then: estimate (separately) the model for the whole period and also for the sub-period before the break. Then compare whether the full sample estimation results differ much from the sub-sample estimation results. The test statistic can be written in the form:

[38] This assumes that the two tests have equal degrees of freedom. This is however not necessarily the case. If the degree of freedom differ in the two tests, the joint hypothesis will be $\chi^2(k_1 + k_2)$-distributed.

$$\lambda_{BP} = \frac{(RSS_{T+H} - RSS_T)/H}{RSS_T/(T-k)} \sim_{asymp} F(H, T-k) \quad under\ H_0. \tag{31}$$

Here RSS_{T+H} is the residual sum of squares from the estimation of the whole period, and RSS_T is the residual sum of squares from the estimation of the sub-period.[39] The statistic is used to test the null hypothesis of no break in the parameters against the alternative hypothesis of changes in at least one parameter. The test statistic is asymptotically F distributed under H_0, and the null hypothesis is rejected if λ_{BP} is too large, compared to the relevant critical value.

4.8.2 Forecast test

This is a test which follows from a 1-step ex-post forecast analysis, comparing residual variances within the estimation and forecast periods. The test statistic is given by:

$$\xi = \sum_{t=T+1}^{T+H} \frac{e_t^2}{\hat{\sigma}_u^2} \sim_{asymp} \chi^2(H) \quad under\ H_0. \tag{32}$$

In Equation (32) $e_t = y_t - x_t'\hat{\beta},\ where\ t = T+1,...,T+H$ and represents the 1-step forecast error.[40] Further $\hat{\sigma}_u^2$ represents the variance of the forecast error.[41] The null hypothesis is here no structural change in any parameter between the estimation and sample periods. Rejection of the null hypothesis is not crucial for model selection, but may imply that the model under study will not provide very accurate ex ante predictions, see Kiviet (1986). An approximate F-equivalent test statistic can be written as:

$$\lambda_F = \frac{1}{H}\xi \sim_{asymp} F(H, T-k) \quad under\ H_0. \tag{33}$$

The three variants of the tests that are briefly described below are the ones that are computed by PcGive.

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[39] Where $RSS_{T+H} = \sum_{t=1}^{T+H} \hat{\varepsilon}_t^2$ and $RSS_T = \sum_{t=1}^{T} \hat{\varepsilon}_t^2$.

[40] $\hat{\beta}$ is estimated using data up to period *T*.

[41] Under the assumption that $\text{var}(\hat{\beta}) \approx 0$.

4.8.3 Break-point Chow test (N↓ test)

This is a sequence of break point F-tests, where the test statistic is $F(T - t + 1,\ t - k - 1)$-distributed for $t = M, \ldots, T$. It is called N↓ because the number of forecasts goes from $N = T - M + 1$ to 1. The test statistic is given by:

$$\lambda_d(t) = \frac{(RSS_T - RSS_{t-1})(t - k - 1)}{RSS_{t-1}(T - t + 1)} \sim_{asymp} F(T - t - 1, t - k - 1) \quad under\ H_0. \qquad (34)$$

Where $t = M, \ldots, T$. The null hypothesis of no break in the parameters is tested against the alternative hypothesis of changes in at least one parameter.

4.8.4 Forecast F-tests (N↑ test)

This is a sequence of forecast F-tests, which are $F(t - M + 1,\ M - k - 1)$ distributed for $t = M, \ldots, T$. It is called N↑ since the forecast horizon increases from M to T. The model is tested over 1 to $M - 1$ periods against an alternative hypothesis that allows any form of change over M to T. The test statistic can be represented as:

$$\lambda_u(t) = \frac{(RSS_t - RSS_{M-1})(M - k - 1)}{RSS_{M-1}(t - M + 1)} \sim_{asymp} F(t - M + 1, M - k - 1) \quad under\ H_0, \qquad (35)$$

where $t = M, \ldots, T$.

4.8.5 1-step Chow test

This is a sequence of 1-step forecast tests that are $F(1,\ t - k - 1)$-distributed under the null hypothesis of constant parameters, for $t = M, \ldots, T$. The test statistic is given by:

$$\lambda_1(t) = \frac{(RSS_t - RSS_{t-1})(t - k - 1)}{RSS_{t-1}} \sim_{asymp} F(1, t - k - 1) \quad under\ H_0, \qquad (36)$$

where $t = M, \ldots, T$. It is important to be aware of that normality of the dependent variable is needed for this statistic to be distributed as an F-statistic.

4.9 Forecasting

In the empirical analysis, I will distinguish between two types of forecasts. The first type of forecast is so-called ex post forecasts (in-sample forecasts). These types of forecasts are used as a model selection/evaluation criterion. What is meant by ex post forecasts is simply that one uses a model to make forecasts when the observations are already known. That is, instead of estimate a model on the whole data sample, one retains some observations (for example the eight last observations when quarterly data are used) for ex post forecasts. By comparing these ex post forecasts with their respective actual values, one gets an impression of the forecasting properties of the models.

The second type of forecasts is denoted ex ante forecasts (out-of-sample forecasts). Ex ante forecasts are often referred to as genuine forecasts, and represent what one normally thinks of as forecasts. In other words, they are forecasts made for periods beyond the data sample period. This type of forecasts is essential for the analysis made in Chapter 6.

I will now give a brief explanation of some forecasting measures and tools that I will use in the next chapters.

4.9.1 Conditional expectation (optimal predictor)

Under the assumption that the first two moments exists, an h-step forecast can be calculated as the conditional expectation, given by $\hat{y}_{t+h|t} = E\left[y_{t+h} \middle| Y_t, \hat{\theta}\right]$. Here, $Y_t = (y_t, y_{t-1}, y_{t-2}, ...)$ contains all the past information and $\hat{\theta}_t = \hat{\theta}(Y_t)$ are parameter estimates given the information up to time *t*. It can be shown that such forecasts are unbiased, see Clements and Hendry (1998) for proof.

4.9.2 Mean square forecast error (MSFE)

Further it can be shown (see for example Clements and Hendry (1998)), that no other predictor conditional on Y_t alone has a smaller mean square forecast error, in other words it is an efficient predictor. The mean square forecast error is often used as a measure of forecast accuracy, and for an h-step forecast it can be written as:

$$M_h\left[\hat{y}_{t+h} \middle| Y_t, \hat{\theta}_t\right] = E\left[(y_{t+h} - \hat{y}_{t+h})^2 \middle| Y_t, \hat{\theta}_t\right]. \tag{37}$$

Since the conditional expectation both is unbiased and has the minimum MSFE, it is often referred to as an optimal predictor.

The PcGive output reports the mean and the standard deviation of the forecast errors. In addition to these, it presents two other measures of forecast accuracy, the root mean square error (RMSE) and the mean absolute percentage error (MAPE). A very short explanation of these measures follows below.

4.9.3 Root mean square error (RMSE)

This is simply the root of the mean square error. The model is estimated up to period T. 1-step forecasts are then made for the periods $T+1,\dots,T+H$. The RMSE may therefore be written as:

$$RMSE = \left[\frac{1}{H}\sum_{t=T+1}^{T+H}(y_t - f_t)^2\right]^{1/2} \tag{38}$$

Here, H represents the forecast horizon, y_t the actual value and f_t the forecast for period t ($t= T+1,\dots,T+H$).

4.9.4 Mean absolute percentage error (MAPE)

This can be presented as

$$MAPE = \frac{100}{H}\sum_{t=T+1}^{T+H}\left|\frac{y_t - f_t}{y_t}\right| \tag{39}$$

Again, H represents the forecast horizon, y_t the actual value and f_t the forecast for period t ($t = T+1,\dots,T+H$). For a more detailed explanation of RMSE and MAPE, see Makridakis, Wheelwright and Hyndman (1998).

4.9.5 Intercept correction (setting the forecast back on track)

This is a mechanism that forecasters often use and is often referred to as setting the model “back on track”. This is simply done by adding the residual of the current period to the next period's forecast for 1-step ahead forecasts. Hence, the adjustment to the forecast is based only on the forecast error in the current period. I will now give a short explanation of this mechanism.

Assume that the data generating process (DGP) is given by:

$$y_t = \psi y_{t-1} + v_t . \tag{40}$$

Then a forecast for period T+1, based on information from period *T* is:

$$\hat{y}_{T+1} = \hat{\psi} y_T , \tag{41}$$

where $\hat{\psi}$ represents the estimated parameter value. The forecast error is then given by:

$$\hat{e}_{T+1} = y_{T+1} - \hat{y}_{T+1} = (\psi - \hat{\psi}) y_T + v_{T+1} . \tag{42}$$

The forecast from setting the model "back on track" is then:

$$\hat{y}^*_{T+1} = \hat{\psi} y_T + e_T , \tag{43}$$

with a forecast error $\hat{e}_{T+1}$ given by:

$$\hat{e}^*_{T+1} = y_{T+1} - \hat{y}^*_{T+1} = (\psi - \hat{\psi}) y_T + v_{T+1} - e_T = \hat{e}_{T+1} - e_T = \Delta \hat{e}_{T+1} \tag{44}$$

In this way, resetting the forecast has the property of inducing the difference in the original forecast error. It is also possible to apply the method of intercept correction when exogenous variables are included (see Chapter 6). For a more thorough explanation of intercept correction, see Clements and Hendry (1998).

5. Estimating the demand for cash in Norway

5.1 Introduction

In this chapter, I will give a presentation of the first part of the empirical investigation of the demand for cash. I will first give a description of the data used, and how the different variables are constructed. Then I will present econometric models of cash demand, derived by using this data and the econometric methods explained in the previous chapter. This chapter is linked to Chapter 3 in the way that most of the variables that are used in the empirical investigation are strongly connected to the variables that are present in the theories for money demand, outlined in Chapter 3. However, the econometric models that are derived in this chapter are not empirical representations of these theoretical models.

5.2 Data

The empirical analysis is conducted using quarterly seasonally unadjusted data. The data period spans from 1980.1 – 2004.2. The variable to be explained is the total real currency holdings.[42] That is, the variable to be explained does not only contain real balances held by private households, but the total holdings of real balances in the economy. In other words it is the total demand for real cash that is the variable of interest. Most of the data in the analysis is provided by Statistics Norway and databases in Norges Bank. Detailed data definitions and sources are provided in Appendix I.

As motivated in previous chapters, there is no theoretical model which gives a clear specification of which variables that should be included or not. However, there exist some theoretical models that give suggestions to variables that at least should be considered in such a model specification. Further some other variables, which are not included in the theoretical models, are believed to have an influence on the demand for currency in circulation. Chapter 2 gave an indication on which variables that may be appropriate to include in the empirical model.

In general and according to the arguments put forward in chapters 2 and 3, there are three potential influences on the evolution of cash balances. Our regressors can therefore roughly be divided into three categories: general macroeconomic variables, technological (financial innovation) variables and informal economy activity variables, which mainly try to capture the effect of the illegal underground economy.

5.2.1 Macroeconomic variables

(i) Real interest rate

The inclusion of this variable is based on the theoretical argument provided in Chapter 3. It represents an opportunity cost term, and is included in order to capture the effect that interest is sacrificed when holding money (cash). We would therefore expect a negative effect of this

[42] It is important to notice that the real currency holdings are calculated by deflating nominal currency holdings by the same price index as (POS) consumption (see explanation of this price index below).

variable on the demand for cash holdings. The interest rate used in this analysis, is a weighted average of deposit rates in different banks.[43] This particular interest rate variable is chosen because it is believed that deposit money is the only "true" substitute to cash holdings. Therefore the opportunity cost of holding cash is expected to be best captured by using the banks deposit rate. Since real cash holdings are modelled, it may be appropriate to use the real interest deposit rate. The real deposit rate is constructed by subtracting the changes in consumer price index from the nominal deposit rate.

(ii) Real private (point-of-sale) consumption

This variable is included for theoretical reason. It represents the transaction variable explained in Chapter 3. There are several reasons for using a specific private consumption variable instead of GDP or a more general consumption variable. First of all, it is clear that in a modern developed economy, such as Norway, cash is not used for all transactions. Typically cash is only used in so-called point-of-sale (POS) transactions. Further it is likely that private households are the main demander of cash. We wish to construct a variable that captures the transaction motive of holding cash. It is therefore believed that a "narrow" private-consumption-variable, which only consists of private point-of-sale (POS) transactions, is the most appropriate variable to capture this effect. A detailed explanation of how this consumption variable is constructed is given in Appendix II. We would expect a positive effect on the cash holdings of this consumption variable. Again, in order to capture the real cash holdings, this POS-consumption variable is deflated. However, this variable is not deflated by the general consumer price index. Instead it is deflated by a specific price index, which only includes prices of the consumption goods included in the POS variable.[44] An explanation of how this consumer price index is constructed is given in Appendix II.

5.2.2 Financial innovation variables

It is often argued that advances in payments technology have resulted in a substitution of non-cash payments for cash, see for instance Snellman, Vesela and Humphrey (2000). There are different ways to capture such financial innovation effects. Some authors argue that this effect will be captured by including a linear trend in the regressions, see for example Fischer, Köhler and Seitz (2004). Others use more direct measures of financial innovations, such as the

[43] It includes commissions on utilised NOK loans excl. non-accrual loans to non-financial enterprises, households and local government at end of quarter.

[44] This specific consumer price index is constructed by my supervisor Terje Skjerpen.

numbers of ATM's, bankcards or EFTPOS terminals, see for example Drehmann and Goodhart (2000). Especially in a structural analysis of cash demand, these figures are informative, but as argued in Fischer, Köhler and Seitz (2004) for an econometric analysis they have the disadvantage to only be available on an annual basis. In other words, there is lack of quarterly information on these variables. Another aspect is that the innovation process has been so rapid, that it can be argued that it is difficult to distinguish the effect of the different variables. For example cheque was a substitute for cash until the end of the 1980s but, as seen in Chapter 2, the use of cheques can now more or less be ignored. Some of the variables are also believed to be highly correlated, in other words multicollinearity may be a problem.

However, keeping in mind the aspects above, together with the fact that a general-to-specific methodology is used, makes me consider three financial innovation variables in addition to a linear time trend. I will consider, as Drehmann and Goodhart (2000) did, the number of ATMs and EFTPOS terminals as regressors. In addition to these two variables, I will also consider a self-constructed variable. This is the price of using alternative payment instruments to cash. The three variables are described below.

(i) The number of ATMs

It can be argued that the effect of ATMs on cash demand may be ambiguous. On the one hand, ATMs should decrease the transaction costs of money holdings. That is, the cost of acquiring cash is now lower, and according to the Baumol-Tobin theory of transaction demand for money, this implies a decrease in money holdings. On the other hand, more ATMs imply that cash is more available. In other words, cash is a more suitable substitute for non-cash payments. This has a positive effect on the demand of cash. Therefore, it can be argued that the effects of ATMs may be ambiguous.

(ii) The number of EFTPOS terminals

Again, the effect of EFTPOS terminals on cash demand may be argued to be ambiguous. However, it may be argued that it is only ambiguous over a part of the sample. First, more EFTPOS terminals imply that it is easier to make point-of-sale payments using payment cards. Obviously this has a negative effect on the demand for cash. But on the other side, due to the introduction of cash back in connection with good purchases, it can be argued that EFTPOS terminals also may have a positive effect on cash demand. Cash back makes cash more available. However, as mentioned in Chapter 2, cash back was first introduced at the end of 1992.

Further, there is lack of quarterly information for the variables of the number of ATMs and EFTPOS terminals. First of all, only annual data are available. This means that I had to convert the annual data series to quarterly series. I have been in contact with BBS,[45] and they have confirmed that there exists no systematic seasonal pattern for the placement of ATMs and EFTPOS terminals. I have therefore made an assumption of equal changes within the calendar year. In other words I have applied a so-called linear interpolation.[46] In addition to this, there is lack of information on the annual data series. The observations for 1980 and 1981 in the annual data series for ATMs are based on backward estimations from a previous study by Gresvik and Kaloudis (2001) in Norges Bank. The same is the case for the observations from 1982-1986 in the annual data series for EFTPOS terminals.

(iii) Price on the use of alternative payment instruments

When making a point-of-sale payment in for example a grocery store, one has two relevant alternatives. One can pay by using cash or by using card.[47] But if you choose to pay with card, the bank charges a fee. How large the fee is, depends on which private bank the cardholder use. But usually it is around 2 NOK. It might therefore be relevant to consider whether such prices have an effect on the use of cash. I have therefore tried to construct such a price variable. Both card and cheque are substitutes to cash, but the use of these two alternatives have changed significantly over the estimation period. The costs of using these two alternatives are also different. I have therefore constructed the following weighted real price variable:

$$P_{alt} = \frac{\left(P_{card} \times T_{bankcard}\right) + \left(P_{cheque} \times T_{cheque}\right)}{T_{card} + T_{cheque}} \frac{1}{CPIC} \ .$$

Here P_{card} is the average price of a transaction when using bankcard as a payment instrument, and $T_{bankcard}$ denotes the total number of bankcard transactions. The cheque-variable is defined

45 BBS = Bankenes Betalingssentral AS, that is the banks payment and clearing house.

46 This means that if the change in a variable from one year to the next is 100, the change from the previous quarter is 25.

47 Cheque is usually no longer a relevant alternative to cash in point-of-sale transactions. However, it was an obvious alternative to cash in the 1980s.

in a similar way. Further in the denominator T_{card} denotes the total number of all card transactions, and *CPIC* denotes the POS consumption price index.[48] A problem with this price variable is that there is some lack of information for the different components of the variable. First, the number of transactions using card and cheques is only available on an annual basis. However, BBS has monthly data available for card transactions for the years 2000-2003. In the construction of the price variable, I have therefore assumed that the transaction variables (for both card and cheque) follow the same seasonal pattern as in these three years. Second, there is some lack of information about the prices of use of cheque and cards in the 1980s. Therefore some of the numbers for these prices are estimates. A more detailed explanation of this is given in Appendix III. Intuitively, this variable should have a positive effect on cash holdings. The more expensive it is to use alternative payment instruments in a transaction, the more will individuals prefer to use cash.

5.2.3 Illegal economy variables

Tanzi (1982) argued that the high amount of cash outstanding could be due to the demand for cash in the underground economy. It is now commonly known that the illegal economy has a substantial impact on cash demand. It is likely that the majority of the payments in the illegal economy are settled with cash. This would mean that the development of the size of the shadow economy over time should be considered as determinants for cash holdings, see Dotsey (1988).

(i) The share of tax and pensions to GDP

In order to capture the effect of the illegal economy, I have included the share of taxes and pensions in GDP as a potential determinant of currency demand. This ratio should have a positive effect on money holdings. Tanzi (1982) argues that the reason for this is that citizens try to evade taxes by shifting part of their economic activity to the illegal economy, where paying with cash is the common practice. Rogoff (1998) confirms this result and develops a theoretical model of currency demand, where a marginal tax rate is included. The model presented in Chapter 3 is based on the model presented in Rogoff (1998).

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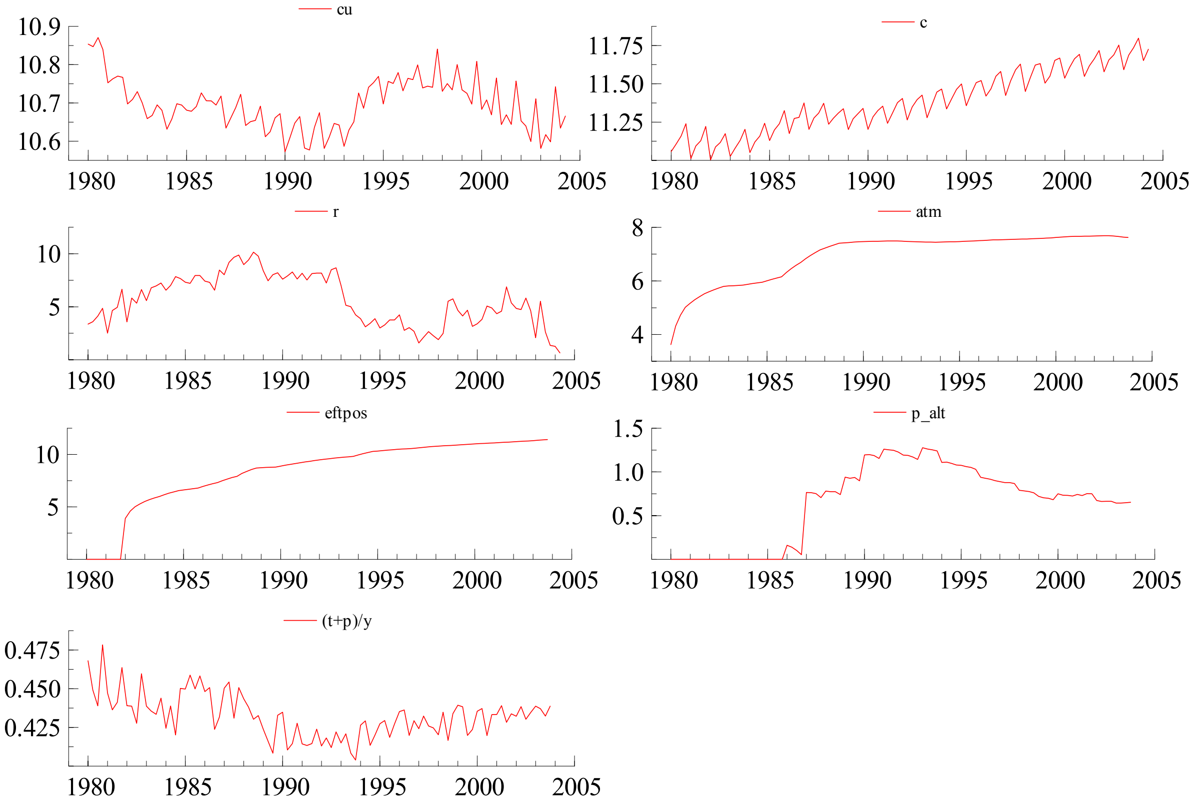
[48] It is important to distinguish between $T_{bankcard}$ and T_{card}. The reason for this is that there exist several types of payment cards, and they are all substitutes to cash, but it is only prices connected to the use of bankcards (the others are free to use). For a more detailed explanation of these variables, see Appendix III.

There may also be other variables included, that can capture the effect of the shadow economy on cash holdings. For example Drehmann and Goodhart (2000) include crime variables. However, I have not included such variables in my analysis. This is due to the fact that it is not evident how one should measure such variables and the fact that there exists no quarterly information. Further Rogoff (1998) also argues that the influence of crime on cash holdings is ambiguous. He argues that criminals will probably use cash intensively, but on the other hand the threat of mugging will deter cash holdings among law-abiding individuals.

5.3 Econometric analysis of cash demand

In this section, I will present the empirical results of modelling total real cash demand in Norway. As mentioned above, seasonally unadjusted data for the period 1980.1 – 2004.2 are used. The estimation period is 1980.1 – 2001.4. The rest of the sample is then used for ex post forecasting purposes, which may be considered as a model selection criterion. Since some of the series show seasonal fluctuation, such as consumption and cash demand, three centred dummies are included in the model specification. These are labelled S_{1t}, S_{2t} and S_{3t}, where S_{it} is related to quarter *i*. Centred seasonal dummies behave as normal seasonal dummies, except that they add up to zero over the calendar year. This means that S_{it} has value 0.75 in quarter *i* and -0.25 in the remaining three quarters. The reason that centred seasonal dummies are used is that they do not affect the asymptotic distribution of the test statistic used to test for cointegration. The variable to be modelled is demand for currency in circulation (cash). The variables are specified above. I will denote them as: real currency holdings (cu), real consumption (c), real interest rate (r), number of ATMs (atm), number of EFTPOS terminals (eftpos), price of using alternative payment instruments (p_{alt}) and finally the share of tax and pensions to GDP ($(t+p)/y$). Further, all variables are in logarithms, except the interest rate and the ratio of tax and pension to GDP, which are untransformed. A graphic illustration of the data series are given in Figure 12 below.

Figure 12: Time series plots of the variables



The first step in the empirical analysis, involves performing stationarity tests, which in my case consists of running ADF tests. As explained in Section 4.3, we want to specify a model which consists of only stationary variables on each side of the equation.[49] ADF tests are therefore performed in order to detect which transformations of the variables that are needed in order to obtain a balanced equation. The results of these tests are presented in Table 1 below.

[49] If the variables in levels are *I(1)*, an equation in stationary variables involves differences of the variables plus a possible linear combination of the variables in levels that are stationary.

Table 1: Unit root tests (ADF-test)

Variable	Test specification	Lag-length	t-value
cu	T	4	-2.294
c	T	5	-3.315
r	C	2	-1.672
atm	T	5	-2.129
eftpos	T	0	-4.947**
(t+p)/y	C	2	-2.106
p_{alt}	C	4	-1.542
Δcu	C	3	-3.595**
Δc	C	4	-2.739
Δr	C	1	-5.452**
Δatm	C	4	-1.773
Δeftpos	C	0	-7.374**
Δ(t+p)/y	C	1	-11.09**
Δp_{alt}	C	3	-3.605**

Notes: C = constant, T =trend + constant. All specifications include centred seasonal dummies.

If constant is included (C): 5% critical value = -2.90. 1% critical value = -3.51.

If trend + constant is included (T): 5% critical value = -3.46. 1% critical value = -4.07.

The critical values are those used by PcGive.

Significant outcomes at a 1 per cent level are indicated by **.

As can be seen from Table 1, most of the variables appear to be *I(1)*, at least at a 5 % significance level. This means that the variables will be transformed into first differences in the analysis.[50] But as can be seen from Table 1, there are some exceptions. It may be claimed that real consumption (*c*) is not *I(1)*. However it is very close to be considered as an *I(1)* variable at the 5 % significance level. In fact if I apply the ADF-test to the whole sample, which is 1980.1 - 2004.2, the hypothesis that real consumption is an *I(1)* variable can not be rejected. I will therefore treat real consumption (*c*) as an *I(1)* variable in the following analysis. Further the number of EFTPOS terminals (*eftpos*) can be considered as trend stationary. However, I will treat this variable as *I(1)* . It can also be seen that the number of ATMs is neither *I(0)* nor *I(1)*. This may be problematic, but I will argue empirically that there are reasons for excluding this variable from the analysis.

The next step is then to specify a model. My initial intention was to use a VAR approach. I started out by first specifying a broad VAR model. The model included all the variables

[50] It is important to notice that the equilibrium correction model contains both variables in levels and in first differences. This was explained in Chapter 4.

described above, where I considered *(t+p)/y*, *atm*, *eftpos* and p_{alt} as exogenous variables. A trend, constant and centred seasonal dummies were also included in the model specification. Since quarterly unadjusted data were used, one had to expect that at least a VAR model of order 4 needed to be specified, in order to rule out problems with autocorrelation. In fact in my case even a VAR model of order 4 was insufficient, for this purpose. As pointed out by Sims (1980) (see Chapter 4), this obviously give rise to problems with too few degrees of freedom. In fact, when only having around 90 observations, very few variables can be included in a VAR specification. I therefore specified a narrower model, considering only real cash, consumption and real interest rate in addition to a linear trend, centred seasonal dummies and a constant, as variables. But even this model, resulted in problems with lack of degrees of freedom, since at least a VAR model of order 6 was needed in order to get rid of autocorrelation in the residuals.[51] Since this paper only focuses on cash demand and the fact that a VAR approach seems inappropriate to use, I have chosen to focus on a single-equation approach. This approach was explained in detail in Chapter 4.

As explained in Chapter 4, a general-to-specific methodology is used. A problem here is to decide how many lags of the different variables to include in the general model specification. I have chosen to look at two different general models. In both model specifications I have chosen to include 4 lags of the consumption and interest rate variables, 1 lag of all the other explanatory variables, centred seasonal dummies, linear trend and a constant.[52] In addition to these terms an equilibrium correction term is included. What differs in the two specifications is that in the first specification 4 lags of the dependent variable (cash) were included, and in the second specification 8 lags were included. Further, checking the correlation matrix between the regressors, it turns out that the numbers of ATMs and the numbers of EFTPOS-terminals are highly correlated.[53] Due to possible problems with multicollinearity, I will therefore specify models where only one of these variables is included as a regressor. In fact one may argue that it is better to include *eftpos* as a regressor than *atm*. A reason for this is that *atm* can not be considered as *I(1)*, which means that including *atm* as a regressor yields an unbalanced equation. But for the time being I will consider both.

[51] In fact, it is not even clear if a VAR model of order 6 gets rid of the problem with autocorrelation. The AR 1-5 test (explained in Chapter 4) had a p-value of 0.0946.
[52] The linear trend has value 1,2,3,... with value 1 occurring for the first observation, 1980.1.
[53] The level variables have a correlation coefficient of 0.94.

Further, the variable p_{alt} gives a contra-intuitive effect on the demand for cash. That is, it seems to have a negative effect on cash demand. In Section 5.1.2 I argued that there is data problems involved in the construction of this variable. Bearing these things in mind, I have therefore disregarded the effect of this variable on the demand for cash.[54]

The discussion above, suggests four variants of the two different model specifications. That is two different variants of the two different model specifications. The difference between the two variants is whether *atm* or *eftpos* is used as regressors in the general model specification. The next step will then be to eliminate statistically insignificant variables by standard testing procedure. Usually when starting out with a general model, there are more than one statistically insignificant variable. When applying the procedure, I have consequently eliminated the variable with the highest p-value. After a variable is eliminated, one then has to estimate the resulting model. The procedure is then applied again, that is the variable with the highest p-value is eliminated and the model is re-estimated. This procedure is continued until the model contains only significant variables. If all these t-tests were independent, a sequence of n single tests would each have had a significance level of α/n, such that the total level would have been α. However, since these t-tests are not independent, this would not be the right significance level to use. In fact, it is not possible to calculate an exact significance level of such a sequence of t-tests, when the tests are not independent. There are different approaches of how to deal with this problem. One suggestion is to use a low significance level for the individual t-tests. In my case, I have chosen a significance level of 2 per cent in the specific t-tests.

When applying this elimination procedure it turns out that neither *atm* nor *eftpos* are significant variables in the maintained models. It also turns out that whichever of these variables one chooses in the general specification, it has no impact on the final model when the general-to-specific procedure is used. I have therefore the following two candidates as the maintained model for the demand of cash (with t-values in parentheses):

[54] However, I will strongly recommend a closer look on the effect of this variable in future applications.

Model 1: (Effective sample: 1981.2-2001.4)

$$\Delta cu_t = \underset{(2.14)}{1.886} - \underset{(-3.38)}{0.0025} trend_t + \underset{(6.34)}{0.5333} \Delta cu_{t-4} + \underset{(2.99)}{0.3445} \Delta c_t + \underset{(3.48)}{0.3629} \Delta c_{t-3}$$

$$- \underset{(-3.86)}{0.0113} \Delta r_t + \underset{(3.39)}{0.0107} \Delta r_{t-2} - \underset{(-5.34)}{0.4990} cu_{t-1} + \underset{(3.10)}{0.3191} c_{t-1} - \underset{(-4.33)}{0.0101} r_{t-1}$$

$$- \underset{(-2.40)}{0.0717} S_{1t} - \underset{(-3.16)}{0.0757} S_{2t} - \underset{(-3.28)}{0.0671} S_{3t} + \underset{(2.25)}{0.0493} D_{1993.4,t} + \underset{(2.06)}{0.0416} D_{1997.4,t} + \hat{\varepsilon}_{1t}.$$

This can also be written as:

$$\Delta cu_t = \underset{(2.14)}{1.886} - \underset{(-3.38)}{0.0025} trend_t + \underset{(6.34)}{0.5333} \Delta cu_{t-4} + \underset{(2.99)}{0.3445} \Delta c_t + \underset{(3.48)}{0.3629} \Delta c_{t-3}$$

$$- \underset{(-3.86)}{0.0113} \Delta r_t + \underset{(3.39)}{0.0107} \Delta r_{t-2} - \underset{(-5.34)}{0.4990} [cu_{t-1} - \underset{(-2.13)}{0.6395} c_{t-1} + \underset{(2.48)}{0.0202} r_{t-1}]$$

$$- \underset{(-2.40)}{0.0717} S_{1t} - \underset{(-3.16)}{0.0757} S_{2t} - \underset{(-3.28)}{0.0671} S_{3t} + \underset{(2.25)}{0.0493} D_{1993.4,t} + \underset{(2.06)}{0.0416} D_{1997.4,t} + \hat{\varepsilon}_{1t}.$$

Here $\hat{\varepsilon}_{1t}$ denotes the residual from the OLS estimation. The t-values of the long-run coefficients are calculated by applying the delta-method, see Kmenta (1997, p. 486). A detailed explanation of the calculations is given in Appendix IV. Further, it can be seen from the equations above that the seasonal effects are almost identical. In other words, the model suggests that it is only the fourth quarter that has a different seasonal pattern for the demand for cash.

Model 2: (Effective sample: 1982.2-2001.4)

$$\Delta cu_t = \underset{(2.92)}{2.03} - \underset{(-4.48)}{0.0029} trend_t + \underset{(4.05)}{0.3435} \Delta cu_{t-4} + \underset{(2.93)}{0.2894} \Delta cu_{t-8} + \underset{(3.70)}{0.2291} \Delta c_t - \underset{(-3.41)}{0.0107} \Delta r_t$$

$$- \underset{(-5.93)}{0.5163} cu_{t-1} + \underset{(4.33)}{0.3228} c_{t-1} - \underset{(-4.46)}{0.0101} r_{t-1} - \underset{(-2.64)}{0.0520} D_{1993.3,t} + \underset{(2.17)}{0.0440} D_{1997.4,t} + \hat{\varepsilon}_{2t}.$$

This can also be written as:

$$\Delta cu_t = 2.03 - 0.0029 trend_t + 0.3435 \Delta cu_{t-4} + 0.2894 \Delta cu_{t-8} + 0.2291 \Delta c_t - 0.0107 \Delta r_t$$
$$(2.92) \quad (-4.48) \quad (4.05) \quad (2.93) \quad (3.70) \quad (-3.41)$$

$$-0.5163[cu_{t-1} - 0.6252 c_{t-1} + 0.0196 r_{t-1}] - 0.0520 D_{1993.3,t} + 0.0440 D_{1997.4,t} + \hat{\varepsilon}_{2t}$$
$$(-5.93) \quad (-2.71) \quad (2.66) \quad (-2.64) \quad (2.17)$$

Again, the t-values for the long-run coefficients are calculated by applying the formula in Kmenta (1997, p. 486). It is here worth mentioning that in this model, there are no significant seasonal effects.

Some general diagnostics and tests for the two models are presented in tables 2 and 3.

Table 2: Empirical measures and diagnostic tests (effective sample: 1981.2/1982.2-2001.4)

	Model 1	**Model 2**
Sigma	0.018	0.019
RSS	0.023	0.024
R^2	0.892	0.885
R^2-adj	0.884	0.877
DW	2.00	1.93
Mean(Δcu_t)	0.000053	0.000754
Var(Δcu_t)	0.002559	0.002625

Notes: Sigma represents the estimated standard error of the regression. RSS denotes residual sum of squares. R^2-adj represents R^2 relative to difference and season, see Hendry and Doornik (1996, p. 239).

Table 3: Residual misspecification tests (effective sample: 1981.2/1982.2-2001.4)

Statistics (p-value)	**AR 1-5 F(5,65)**	**ARCH 1-4 F(4,62)**	**Norm $\chi^2(2)$**	**Hetero F(21,48)**	**RESET F(1,69)**
Model 1	0.13075	0.53182	1.7047	0.75998	0.26472
	[0.9848]	[0.7128]	[0.4264]	0.7576]	[0.6086]
Model 2	0.48059	0.81425	3.4816	0.82250	1.6771
	[0.7894]	[0.5211]	[0.1754]	[0.6661]	[0.1998]

Notes: P-values are given in parentheses.

Table 3 presents residual misspecification tests for the two models. The interpretation of the tests was explained in the previous chapter. Obviously the two non-nested models seem to give statistically acceptable results. The models seem to be free of autocorrelation and ARCH-effects. We can not reject the null-hypothesis of normality, nor do we find any evidence of mis-specified functional form (RESET-test).

I will now take a closer look at the two models. Both models have only consumption and real interest rate as regressors in addition to a constant, a linear trend and lagged variables of the dependent variable. The linear trend is significant in both models. As explained in Section 5.1.2 one can argue that this linear trend captures the effect of financial innovations on the demand for cash. The fact that the coefficient in front of the trend is negative in both models strengthens this argument. On the other hand, the seasonal dummies are only significant in Model 1. In Model 1 a third lag of relative change in consumption and a second lag of change in the real interest rate are significant regressors. These are not significant in Model 2, but lag 8 of the dependent variable (change in cash) is present in Model 2. Two impulse dummies are included in both models. In both models, some large residuals are present in the two last quarters of 1993 and the first quarter of 1994. This is the reason for including the dummies in 1993.3 (Model 2) and 1993.4 (Model 1). These large residuals may be associated with the introduction of cash back. Cash back was, as mentioned above, introduced in 1992, but it is likely that noticeable effects first appeared in 1993. However an event that is more likely to have had a significant impact on the cash holdings, is the change in the monetary policy in December 1992. After massive pressure on the Norwegian currency (NOK), Norges Bank changed their monetary policy from a fixed rate, to a float regime. Obviously this had an impact on the Norwegian economy in the periods after, which can clearly be seen from the massive fall in the real interest rate within the year 1993. Although interest rate is already included in the model, it can be argued that these changes have had a significant impact on other parts of the economy that are not modelled. These may in turn have influenced the demand for cash. Introducing a dummy for 1997.4 can be defended since 1997 was a year when the use of payment cards increased very rapidly. It is then likely that this effect is particular strong in the 4^{th} quarter, due to seasonal effects for transactions.

Both models are so-called equilibrium correction models. These models make it convenient to study both short- and long-run effects. It can be seen from the two models that the short-run (immediate) effects of changes in consumption and interest rate differ slightly. The short-run transaction elasticity is 0.34 in Model 1, but only 0.23 in Model 2. We can interpret this as the immediate demand for cash increases by 0.34 (Model 1) or 0.23 (Model 2) per cent when consumption increases by one per cent ceteris paribus. Further, the short-run interest semi-elasticity in the two models is approximately 0.01 in both models. In other words if the interest rate increases by one percentage point, the immediate effect on the demand for cash, is a decrease by 0.01 per cent.

Furthermore, the two models are even more similar when it comes to the long-run effects. In fact the long-run coefficients of the two variables are almost identical, so is the parameter in front of the equilibrium term. The derived cointegration relationships are for Model 1: $cu_t = 0.6395\ c_t - 0.0202\ r_t$, and for Model 2: $cu_t = 0.6252\ c_t - 0.0196\ r_t$. As we can see, both models have long-run transaction elasticity (consumption coefficient) of approximately 0.63. We can interpret this as the long-run cash demand increases by 0.63 percent if consumption increases by one per cent ceteris paribus. The long-run interest rate semi-elasticity is approximately 0.02 in both models. That is if the interest rate increases by one percentage point ceteris paribus, then the long-run cash demand decreases by 0.02 per cent.

It is of substantial interest to compare these results with the results reported by Fischer, Köhler and Seitz (2004). They have investigated the demand for real cash in the euro area using a cointegration approach. Their derived cointegration relationship is: $cu_t = -8.06 + 1.05\ c_t - 0.04\ r3_t + 0.41e_t$. Here c denotes real private expenditure, $r3$ denotes the three-month money market rate and e denotes the real effective exchange rate of the euro. In their analysis both consumption and interest rate have stronger long-run effects on the demand for cash than obtained in this thesis.

A possible explanation for this may be that Norway has a more developed payment system than the Euro area as a whole. One would therefore expect that the relative use of cash is less in Norway than in the Euro area. On the other side this does not necessarily mean that consumption and interest rate have less marginal effect on the demand for cash. However, one should be careful when interpreting the differences in these results, since they must be viewed in the light of the fact that Fischer, Köhler and Seitz (2004) use a broader consumption variable and a different interest rate variable than I have applied.

Furthermore the highly significant equilibrium correction term (t-value of respectively -5.34 and -5.93) in the two models, gives the impression that an equilibrium correction model is appropriate. In fact comparing the t-values with the non-standard critical values provided by Banerjee, Dolado and Mestre (1998) strengthens this assertion.[55] However, this test relies on the assumption that the regressors are weakly exogenous with respect to the parameters in the real cash demand equation. Results from the weak exogeneity tests, as outlined in Boswijk and Urbain (1997) (and explained in Chapter 4), are given in Table 4.

Table 4: Tests for weak exogeneity (p-values in parentheses)

	Model 1	**Model 2**
LM_1 χ^2 (2)	3.6375 [0.1622]	0.38841 [0.8235]
LM_2 χ^2 (2)	0.4334 [0.8052]	2.1842 [0.3355]
$LM=LM_1+LM_2$ $\chi^2(4)$	4.0709 [0.3965]	2.5726 [0.6317]

Notes: LM_1 represents the test statistic when testing for weak exogeneity for the short run parameters. LM_2 represents the test statistic when testing for weak exogeneity for the long run parameters. Finally LM represents the joint test statistic. Critical values for the joint test: 5 % critical: 9.49, 1 % critical: 13.28.

Table 4 shows that for both models consumption and interest rate are clearly weakly exogenous for the parameters in the structural equation for cash. In light of the tests for weak exogeneity and under the assumption of only one cointegrating relation, one can defend estimating the structural equation with OLS.

The next step will be to check the parameter stability in the two competing models. Sequences of the Chow tests as outlined in Chapter 4 (scaled by their 1 % critical values) and recursive estimates (with ± 2 standard errors) are displayed in the figures below.

[55] When a constant and trend is included and the number of regressors is 2, the 1% critical value is -4.60 and the 5 % critical value is -3.98 for a sample size of 100 observations. One should notice that the sample I have used contains less than 100 observations (that is around 80 observations).

Figure 13: Recursive test statistics for **Model 1**

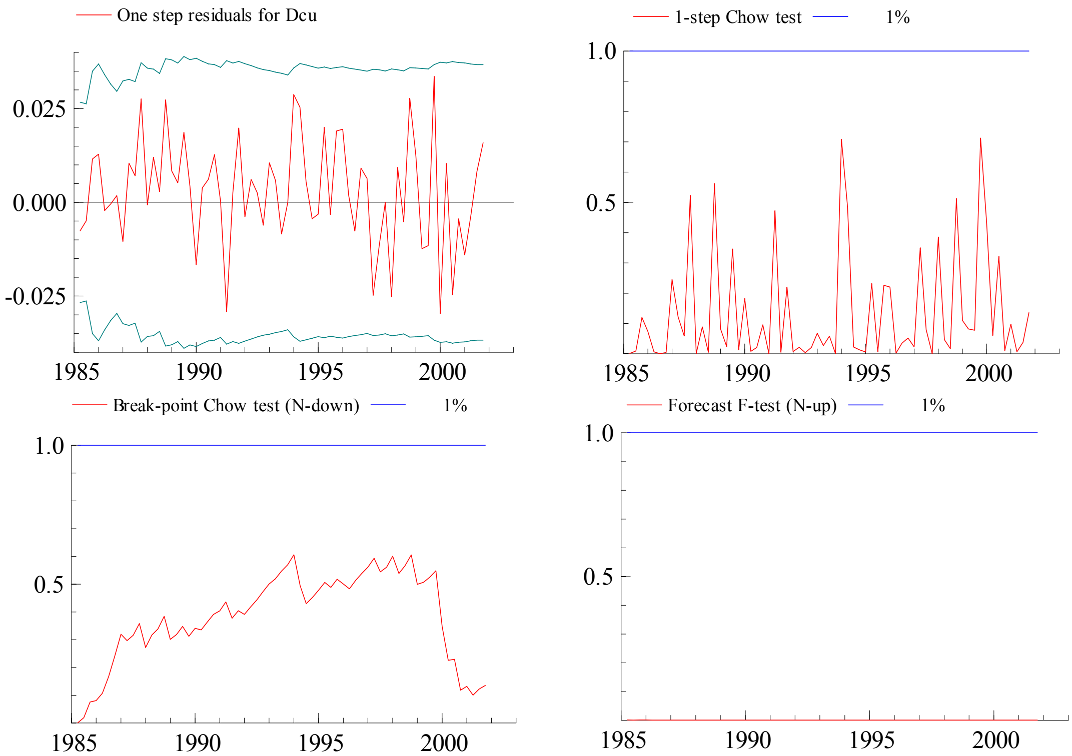
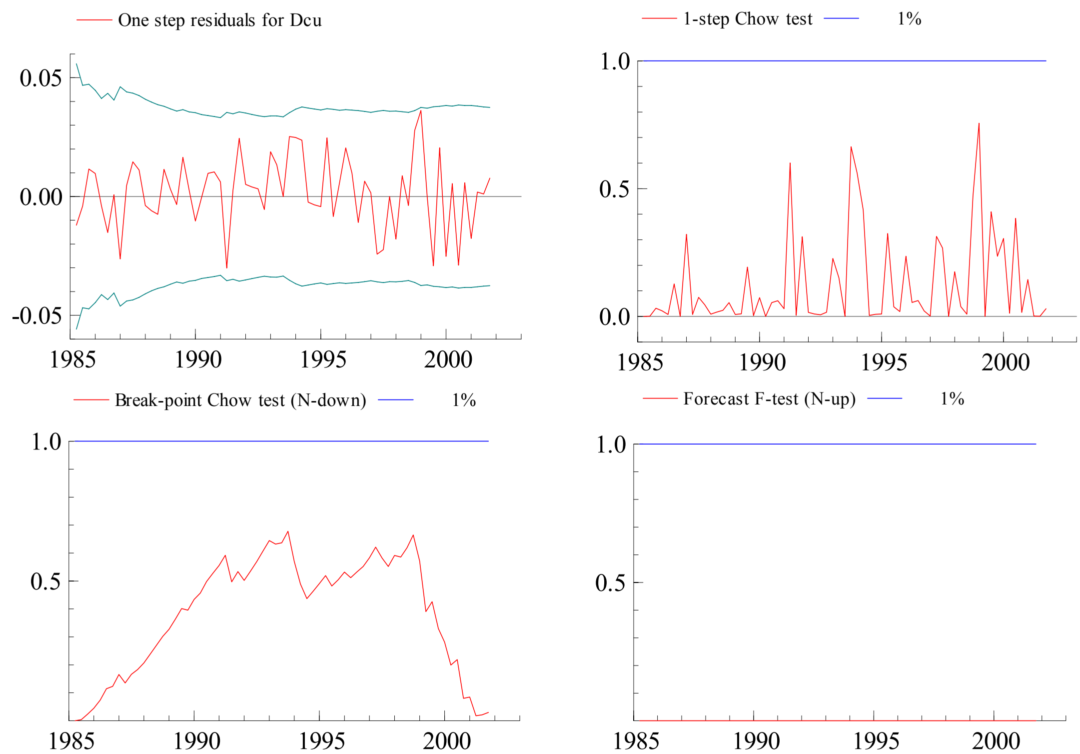


Figure 14: Recursive test statistics for **Model 2**



From figures 13 and 14, one can conclude that the structural equation shows no sign of being mis-specified.

Further, it is of interest to check the recursive estimates of the parameters in the two competing models.

Figure 15: Recursive estimates of parameters in **Model 1** ± SE

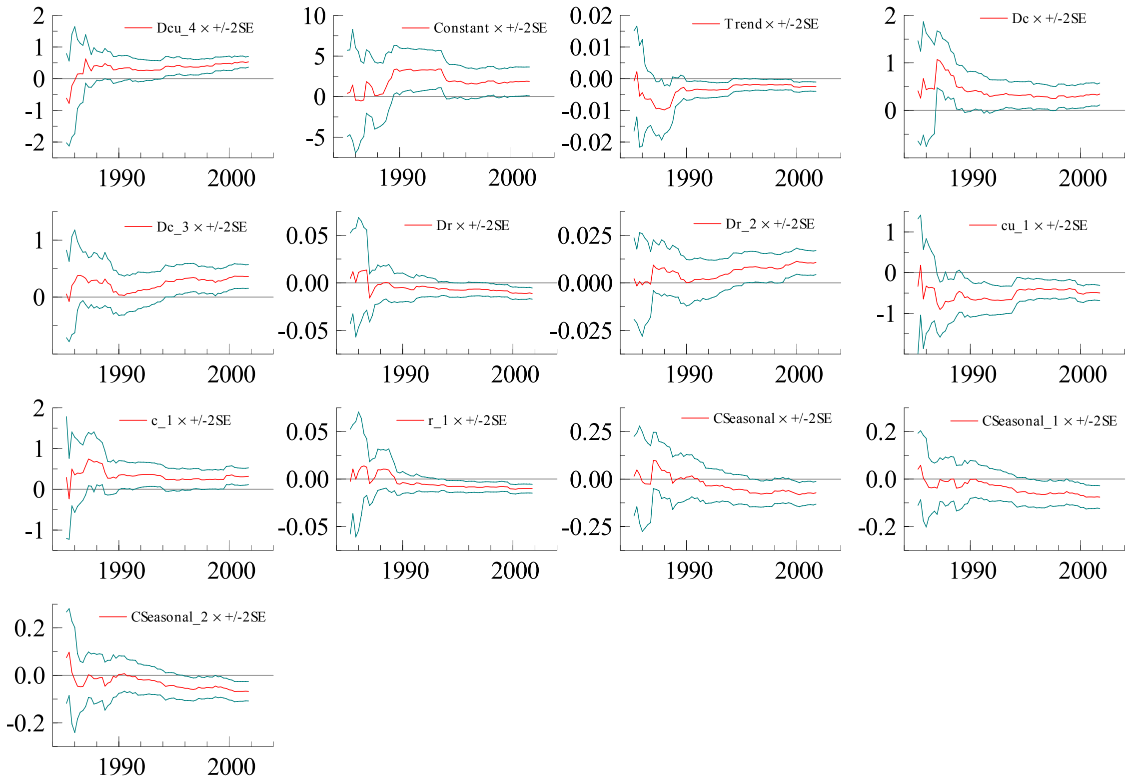
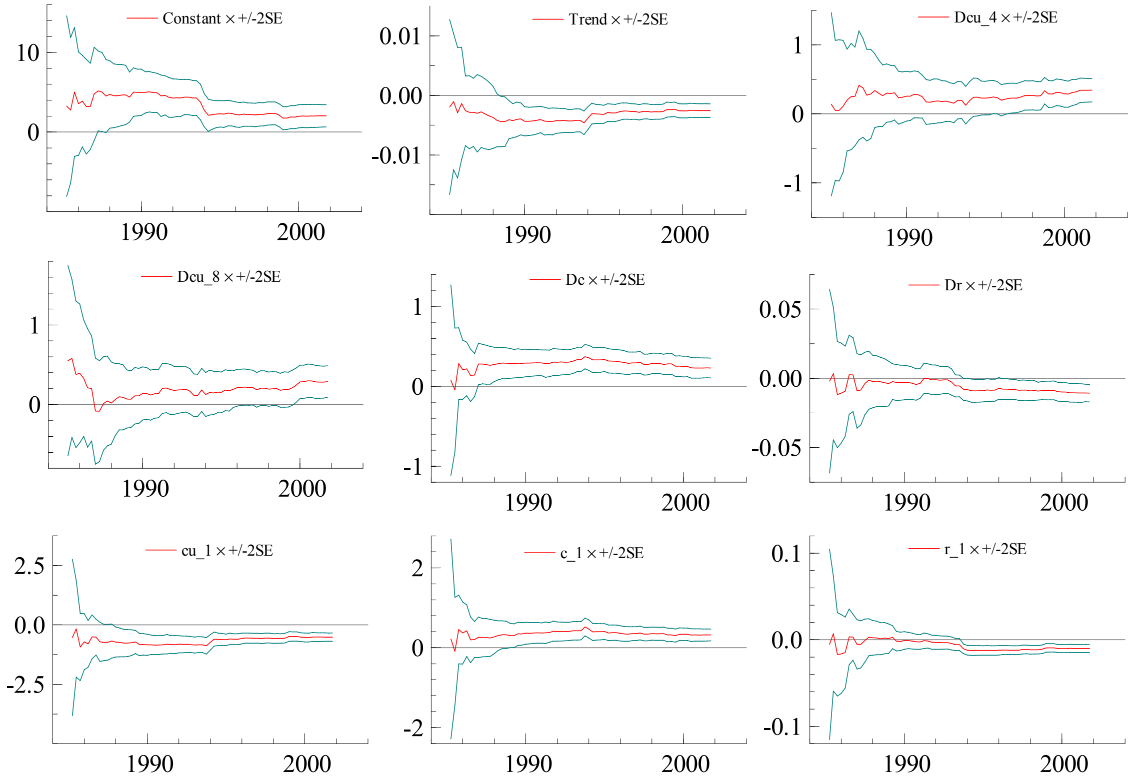


Figure 16: Recursive estimates of parameters in **Model 2** ± SE



As can be seen from the figures above, both the long- and short-run parameters of the models seem to be reasonably invariant over time. Especially this holds for Model 2, which seems to have slightly more stable parameters than Model 1.

A final step in the modelling selection procedure will be to check the forecasting properties of the models within the sample (ex post forecast properties). Information on this is presented in Table 5 and the graphs below. I have here used the derived ex post forecasts for the two competing models for the period 2002.1-2004.2. In the figures also the 5 per cent forecast error bands are provided.

Table 5: Summary measures for ex post forecast errors

	Model 1	**Model 2**
RMSE	0.0509	0.0491
MAPE	102.94	127.41

Figure 17: Actual and fitted values and ex post (in-sample) forecasting outcomes for **Model 1**

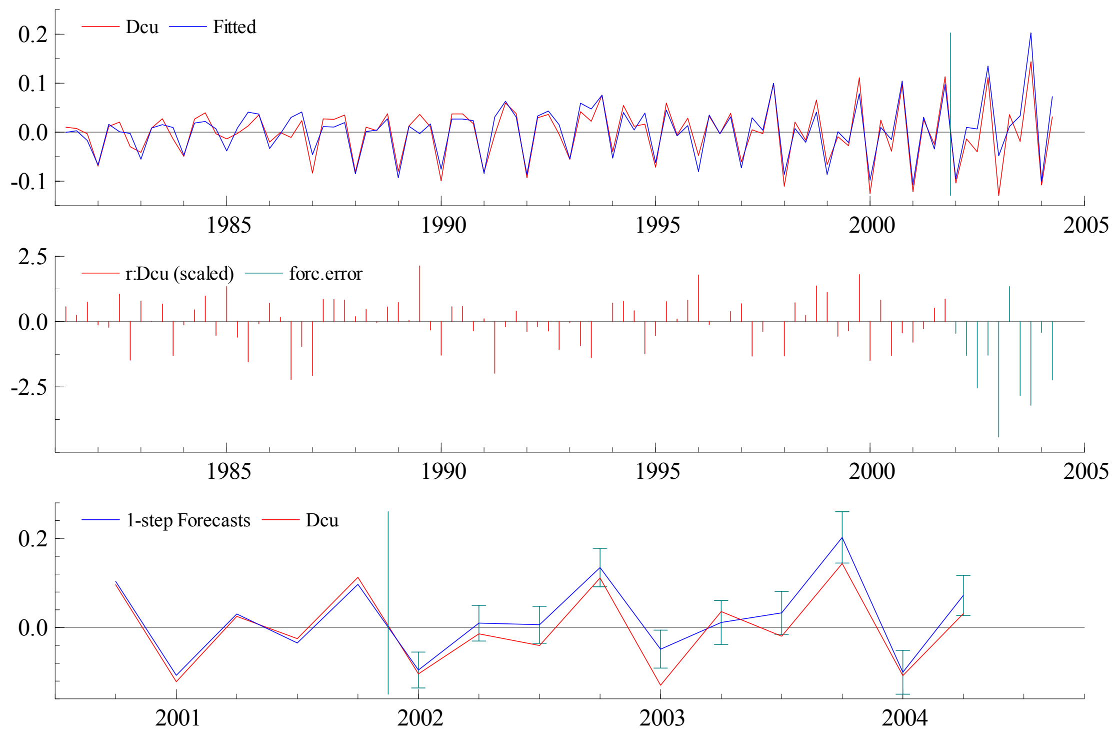
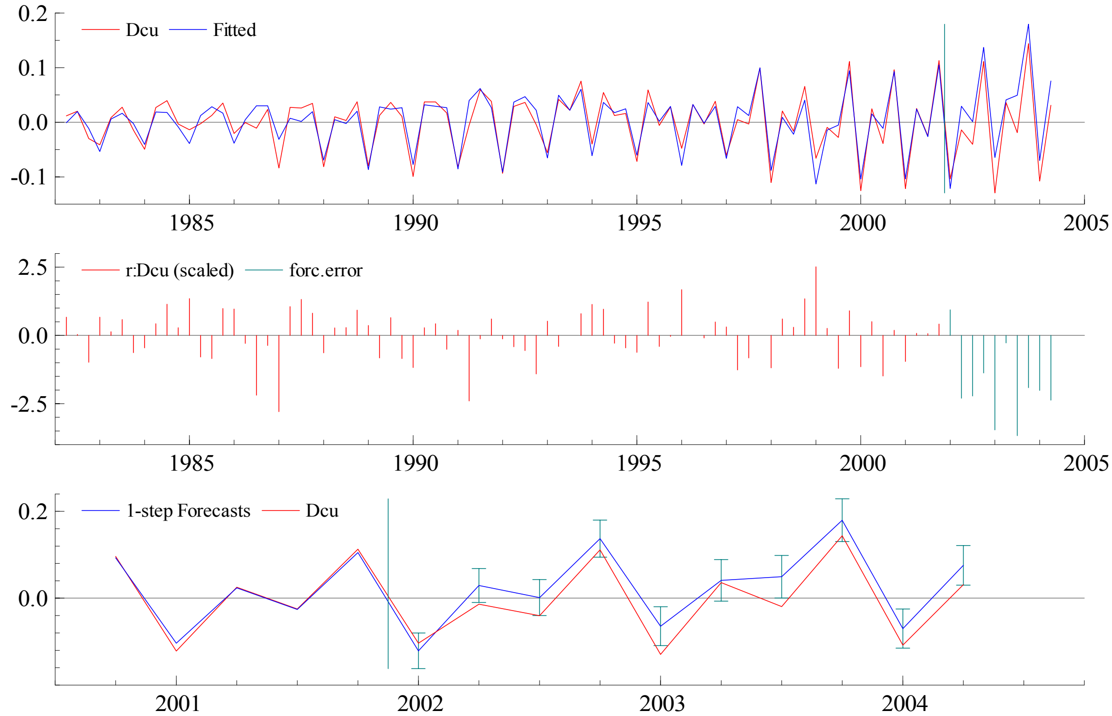


Figure 18: Actual and fitted values and ex post (in-sample) forecasting outcomes for **Model 2**



It can clearly be seen from figures 17 and 18 above, that both models have some problems when it comes to forecasting the demand for real cash. Both models seem to over-predict the true cash demand process. This may be problematic when using the model for ex ante forecasting. It should be noted that some of the forecasts are outside the 5 per cent forecast error bands. Especially the models have difficulties in forecasting the demand for real cash in 2003. On the other hand, it can be argued that this is a turbulent period for the Norwegian economy. In March 2001, Norges Bank changed their monetary policy to an inflation targeting regime. Obviously this had effects on the Norwegian economy. It is believed that the market needed some time to adjust to the new policy regime. Especially the interest rate changed in this period. The years 2001 and 2002 were characterized by a high interest rate. The interest rate then started to fall rapidly in the late 2002 and the decrease continued through the year 2003. Thereafter it seems that the interest rate has been stabilized. However, these changes in the interest rate are taken into account by the models. On the other hand, the models do not capture the fact that these interest rate changes may have affected other non-modelled aspects of the economy. In other words, effects on variables omitted from the structural equation, but which in turn may have an influence on the demand for real cash.

This may therefore be an explanation for the poor forecasting results of the models. I tried to estimate the two models over the shorter sample 1980.1-1999.4, and made forecasts for 2000.1-2001.4. In this case the results where quite good. Further I have estimated the model over the whole sample period (1980.1 – 2004.2) (see next chapter). If I then include an impulse dummy 2003.1, the estimation results seem to be reasonably good.

In summary, both Model 1 and Model 2 seem to give satisfying results when estimating the demand for real cash. One may claim that they both seem to be congruent models for the demand for real cash. However, only one of them can represent the “true” data generating process. The models are quite similar, and give similar results. This makes it difficult to differentiate between which of the two models that gives the best representation of the “true” data generating process of the demand for cash. As a final step in the model selection procedure, I tried to apply different encompassing tests (see Section 4.2), in order to investigate whether one of the models encompassed the other. However these tests gave no clear answer to which model to choose. I will therefore continue to consider both models in the following chapter.

6. Forecasting the demand for cash in Norway

6.1 Introduction

In this chapter I will present some ex ante forecasts for the demand for cash in Norway. For this purpose I have considered three models. The first two models are the ones explained in the previous chapter. In addition to these a simple autoregressive time series model is considered. This last model can be regarded as a benchmark model.

6.2 Models

In the previous chapter, Model 1 and Model 2 were estimated using the sample 1980.1-2001.4. In this way one could retain some observations for ex post forecasts. As argued, this was a step in the model selection procedure. However, now the interest is not on model selection, but on how the selected model(s) performs when it comes to forecasting the demand for cash. In order to produce ex ante forecasts, it is therefore suggested to re-estimate the selected model(s) using all available data. Ideally the estimated coefficients based on the estimations using the whole sample and the sub-sample (1980.1-2001.4) should not differ too much. The results from estimating Model 1 and Model 2 using the sample 1980.1-2004.2 are given below (with t-values in parentheses). Results from the diagnostic tests and the residual misspecification tests are given in Appendix V.

Model 1: (Effective sample 1981.2-2004.2)

$$\Delta cu_t = \underset{(2.10)}{1.903} - \underset{(-2.40)}{0.0015} trend_t + \underset{(8.27)}{0.5852} \Delta cu_{t-4} + \underset{(2.50)}{0.2854} \Delta c_t + \underset{(3.37)}{0.3501} \Delta c_{t-3}$$

$$- \underset{(-2.49)}{0.0060} \Delta r_t + \underset{(3.44)}{0.0083} \Delta r_{t-2} - \underset{(-5.66)}{0.3539} cu_{t-1} + \underset{(2.02)}{0.1750} c_{t-1} - \underset{(-3.94)}{0.0067} r_{t-1}$$

$$- \underset{(-2.27)}{0.0678} S_{1t} - \underset{(-3.34)}{0.0787} S_{2t} - \underset{(-3.45)}{0.0720} S_{3t} + \underset{(2.90)}{0.0618} D_{1993.4,t} + \underset{(2.78)}{0.0566} D_{1997.4,t} - \underset{(-2.80)}{0.0587} D_{2003.1,t} + \hat{\varepsilon}_{1t}$$

Model 2: (Effective sample 1982.2-2004.2)

$$\Delta cu_t = 1.59 - 0.0017 trend_t + 0.3780 \Delta cu_{t-4} + 0.3804 \Delta cu_{t-8} + 0.1383 \Delta c_t - 0.0058 \Delta r_t$$
$$\quad (2.38) \quad (-3.51) \quad (4.54) \quad (4.08) \quad (2.44) \quad (-2.27)$$

$$- 0.3490 cu_{t-1} + 0.1990 c_{t-1} - 0.0065 r_{t-1} - 0.0415 D_{1993.3,t} + 0.0588 D_{1997.4,t} - 0.0385 D_{2003.1,t} + \hat{\varepsilon}_{2t}.$$
$$(-5.61) \quad (3.20) \quad (-3.62) \quad (-2.04) \quad (2.83) \quad (-1.77)$$

If one compares the estimation results for these two models, with the estimation results in Chapter 5 (when using the sub-sample period 1980.1-2001-4), it can be seen that the results are quite similar. However, some differences occur. A reason for this may be that an impulse dummy for 2003.1 is included in both models, when the estimation period was extended to cover the whole sample. The reason for including such a dummy is that 2003.1 was a quarter with unusually high inflation, stemming from the unusually high electricity prices that winter. As can be seen from the estimation results above, this impulse dummy is more significant in Model 1 than in Model 2. The reason for not excluding the impulse dummy in Model 2 (which is not very significant), is that the model then seems to get some problems with parameter stability (Chow tests).

In addition to these two models, a simple AR(8) model in Δcu_t is estimated. Insignificant lags are excluded by applying the same procedure as for Model 1 and Model 2 in Chapter 5. The resulting model is denoted as Model 3. The reason for including such a model in the forecasting analysis is that such simple univariate models have a tendency to outperform more sophisticated models, when it comes to forecasting properties. This model may therefore be considered as a benchmark model, and it is of great interest to compare the forecasts produced by Model 1 and Model 2 with the forecasts obtained by this simple model. The results from estimating Model 3 using the sample 1980.1-2004.2 is given below (with t-values in parentheses). Diagnostic tests and residual misspecification tests related to this model are given in Appendix V.

Model 3: (Effective sample 1981.2-2004.2)

$$\Delta cu_t = 0.0002 - 0.2251 \Delta cu_{t-1} - 0.1572 \Delta cu_{t-2} + 0.4231 \Delta cu_{t-4} - 0.2251 \Delta cu_{t-7} + 0.3734 \Delta cu_{t-8} + \hat{\varepsilon}_{3t}.$$
$$(0.07) \quad (-3.13) \quad (-2.22) \quad (4.40) \quad (-3.06) \quad (3.70)$$

It is not convenient to produce ex ante forecasts for equilibrium correction models in PcGive. However, it is always possible to transform the equilibrium correction model to a model in levels. This is therefore done for the two equilibrium correction models, but also for the simple univariate model. The models in level form are given as:

Model 1:

$$\begin{aligned} cu_t =\ & 1.903 - 0.0015 trend_t + 0.6461 cu_{t-1} + 0.5852 cu_{t-4} - 0.5852 cu_{t-5} + 0.2854 c_t - 0.1104 c_{t-1} \\ & + 0.3501 c_{t-3} - 0.3501 c_{t-4} - 0.0060 r_t - 0.0007 r_{t-1} + 0.0083 r_{t-2} - 0.0083 r_{t-3} - 0.0678 S_{1t} \\ & - 0.0787 S_{2t} - 0.0720 S_{3t} + 0.0618 D_{1993.4,t} + 0.0566 D_{1997.4,t} - 0.0587 D_{2003.1,t} + \hat{\varepsilon}_{1t} \end{aligned} ,$$

Model 2:

$$\begin{aligned} cu_t =\ & 1.59 - 0.0017 trend_t + 0.6510 cu_{t-1} + 0.3780 cu_{t-4} - 0.3780 cu_{t-5} + 0.3804 cu_{t-8} - 0.3804 cu_{t-9} \\ & + 0.1383 c_t - 0.0607 c_{t-1} - 0.0058 r_t - 0.0007 r_{t-1} - 0.0415 D_{1993.3,t} + 0.0588 D_{1997.4,t} \\ & - 0.0385 D_{2003.1,t} + \hat{\varepsilon}_{2t} \end{aligned}$$

and

Model 3:

$$\begin{aligned} cu_t =\ & 0.0002 + 07749 cu_{t-1} + 0.0679 cu_{t-2} - 0.1572 cu_{t-3} + 0.4231 cu_{t-4} - 0.4231 cu_{t-5} - 0.2251 cu_{t-7} \\ & + 0.1483 cu_{t-8} - 0.3734 cu_{t-9} + \hat{\varepsilon}_{3t} \end{aligned} .$$

It is important to notice that what is done here is a pure transformation. This means that the two different representations of the models are identical. When applying this transformation one therefore has to impose some parameter restrictions, which ensure that the transformed models are equal the untransformed models. A problem is that it seems like PcGive is not able to re-estimate the transformed models with the imposed parameter restrictions. This is problematic, since we want to make ex ante forecasts using the exact models selected from the model selection criteria. To avoid this problem, the forecasts have been produced using another software program, namely TSP 4.5 (see Hall and Cummins (1999)).

6.3 Forecasts

A general problem, when performing ex ante forecasts using a single equation model where exogenous variables are present, is that one needs to make some assumptions of how these exogenous variables will evolve in the future. This problem is present when obtaining forecasts using Model 1 and Model 2. In order to obtain forecasts using Model 1 and Model 2, I have decided to base the evolution of the exogenous variables of interests, on forecasts produced by Statistics Norway and Norges Bank. These forecasts are given in Appendix VI. It is however important to be aware of that these forecasts are not exactly the same as the ones obtained by Statistics Norway and Norges Bank. I have here assumed that the price index for POS consumption follows the same evolution as the forecasts produced by Statistics Norway for the general Consumer price index. Furthermore POS consumption follows the same evolution as the forecasts obtained by Statistics Norway on private consumption, and the nominal banks deposit rate follows the same evolution as Norges Banks forecasts for the folio rate, published in Norges Bank (2004c).[56]

Further, it is now possible to produce forecasts for currency in circulation conditioning on the assumed evolution of the exogenous variables. I have chosen to perform h-step ahead forecasts up until 2007.4 for all three models. Further I have transformed the forecast values from logs back to levels by using the anti-log operator. The forecasts are presented in Table 6 (forecasts without intercept correction).

[56] The forecast for the Consumer price index and POS consumption, are consistent with the calculations that are made in Statistics Norway (2004). However, one should notice that the forecasts published in Statistics Norway (2004) are on an annual basis. Further the consumption variable is different, since it covers consumption both by households and NPISHs.

Table 6: Ex ante forecasts for demand for real cash (in billions of 2001- NOK)

Forecast period	Model 1	Model 1	Model 2	Model 2	Model 3	Model 3
	Intercept correction					
	No	Yes	No	Yes	No	Yes
2004.3	43.978	43.165	43.147	42.609	41.247	41.669
2004.4	49.406	47.911	49.315	48.304	47.227	48.089
2005.1	45.277	43.566	44.019	42.888	41.623	42.672
2005.2	45.914	43.957	45.859	44.526	43.249	44.669
2005.3	47.082	44.440	45.850	44.207	41.379	43.243
2005.4	51.925	48.753	51.772	49.772	47.567	50.197
2006.1	46.652	43.762	45.507	43.714	41.981	44.703
2006.2	46.002	43.197	46.656	44.825	43.818	47.010
2006.3	46.898	43.831	46.359	44.274	41.548	45.207
2006.4	50.790	47.572	51.305	48.964	47.954	52.776
2007.1	45.079	42.390	44.552	42.579	42.009	46.701
2007.2	43.924	41.457	45.589	43.658	44.156	49.553
2007.3	45.008	42.439	45.109	43.096	41.560	47.327
2007.4	48.498	45.885	49.391	47.282	48.133	55.467

Since Norges Bank issues banknotes and coins, it is also of interest to present the forecasts for the nominal cash demand. These are simply constructed by multiplying the forecasts for real cash demand by the forecasts for the POS price index.

Table 7: Ex ante forecasts for the nominal demand for cash (in billions of NOK)

Forecast period	Model 1	Model 1	Model 2	Model 2	Model 3	Model 3
	Intercept correction					
	No	Yes	No	Yes	No	Yes
2004.3	44.843	44.014	43.995	43.447	42.058	42.488
2004.4	50.624	49.092	50.531	49.495	48.391	49.275
2005.1	46.486	44.730	45.195	44.033	42.735	43.812
2005.2	47.346	45.328	47.289	45.914	44.597	46.063
2005.3	48.486	45.766	47.218	45.525	42.613	44.533
2005.4	54.061	50.758	53.901	51.819	49.524	52.262
2006.1	48.726	45.708	47.531	45.658	43.847	46.691
2006.2	48.328	45.380	49.014	47.091	46.033	49.387
2006.3	49.294	46.070	48.727	46.535	43.671	47.516
2006.4	54.041	50.616	54.589	52.097	51.023	56.153
2007.1	48.174	45.300	47.610	45.502	44.893	49.907
2007.2	47.298	44.642	49.091	47.011	47.547	53.359
2007.3	48.539	45.769	48.649	46.477	44.820	51.040
2007.4	52.919	50.068	53.893	51.592	52.521	60.523

Unfortunately TSP is not able to calculate the uncertainty for these forecasts. Generally, there are four sources of forecast uncertainty for such single equation equilibrium correction models, used in this analysis. These are:

(i) Error uncertainty.

(ii) Estimation uncertainty.

(iii) Uncertainty connected to the evolution of the exogenous variables (they are all unknown and need to be forecasted).

(iv) Uncertainty connected to the fact that the postulated model does not coincide with the "true" data generating process. This may in turn also lead to prediction bias.

If PcGive could have been used, the forecast uncertainty which would have been calculated, would only have taken the error uncertainty into account. This means, that the forecast uncertainty reported by PcGive is too small. The reason to this is that it neglects the three other uncertainty aspects denoted above.[57] It can therefore be argued that the cost of performing the forecasts using the software program TSP is not of great importance. On the

[57] However, it is generally not clear, how one should deal with the uncertainty source (iv).

other side, it would have been ideal to be able to calculate the true uncertainty of the forecasts of the three models under the assumption that the model is correct.

The forecasts above are ex ante forecasts. It is therefore at this stage not possible to compare the forecasts with the actual values. On the other side, it is of interest to compare the different forecasts produced by the three different models. As can clearly be seen from the forecast results above, the three models yield different forecasts. However, the forecasts produced by Model 1 and Model 2 are more similar than the ones obtained by Model 3. On the other side the two former models are also more similar, and one would therefore expect that the forecasts produced by these models are quite similar. It should also be noted that forecasts produced by Model 1 have a tendency to give a higher value of cash demand than Model 2, especially for the first two years. Further, forecasts produced by Model 3 suggest lower values of cash demand than both Model 1 and Model 2. As argued above, the reason for including Model 3 is that such univariate time series models have a tendency to outperform more sophisticated models, at least with respect to short-run forecasts. One would therefore like to get forecasts results from the two other models (Model 1 and Model 2) that do not differ too much from the forecast results obtained by Model 3.

Further, it can be seen from Figure 17 and Figure 18 in Chapter 5 that when producing ex post forecasts for Model 1 and Model 2, they had a tendency to over-predict the actual values of currency in circulation. This, together with the fact that the forecast results from Model 1 and Model 2 differ considerably from the ones obtained by Model 3, may suggest that Model 1 and Model 2 have a tendency to over-predict the demand for cash.

6.4 Forecasts when applying intercept correction

Intercept correction, is the so-called set the model "back on track" mechanism, which was explained in Section 4.9.5. Forecasts are produced applying intercept correction to the three models discussed above. This is a procedure which is often used by applied forecasters. The reason for this is that it is seen as a possible counter to structural breaks and parameter changes. The concept of intercept correction is thoroughly discussed in Clements and Hendry (1998, 1999).

The ex ante forecasts produced, are so-called h-step ahead forecasts. Intercept correction forecasts are made after setting the model “back on track” in 2004.2. This means that for each of the three models, the residual obtained in 2004.2 is added to the estimated intercept. In other words the fitted values for the demand for cash are identical in the three models, and they are also identical to the actual value of currency in circulation. Forecasts for the demand for cash are then produced by the three different models. The results are presented in tables 6 and 7 above and in Figure 19 and Figure 20 below.

Figure 19: Forecasts for real cash with intercept correction

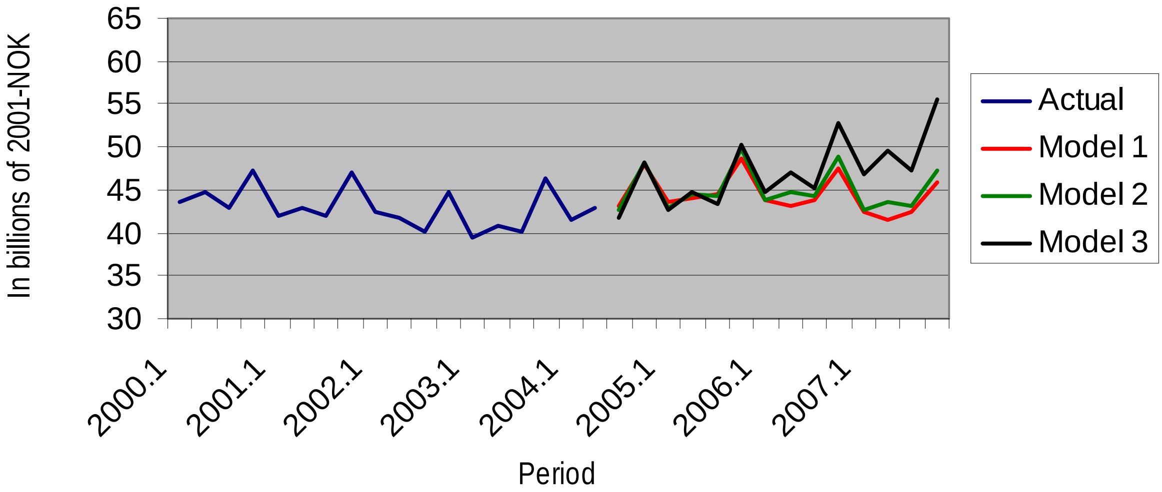
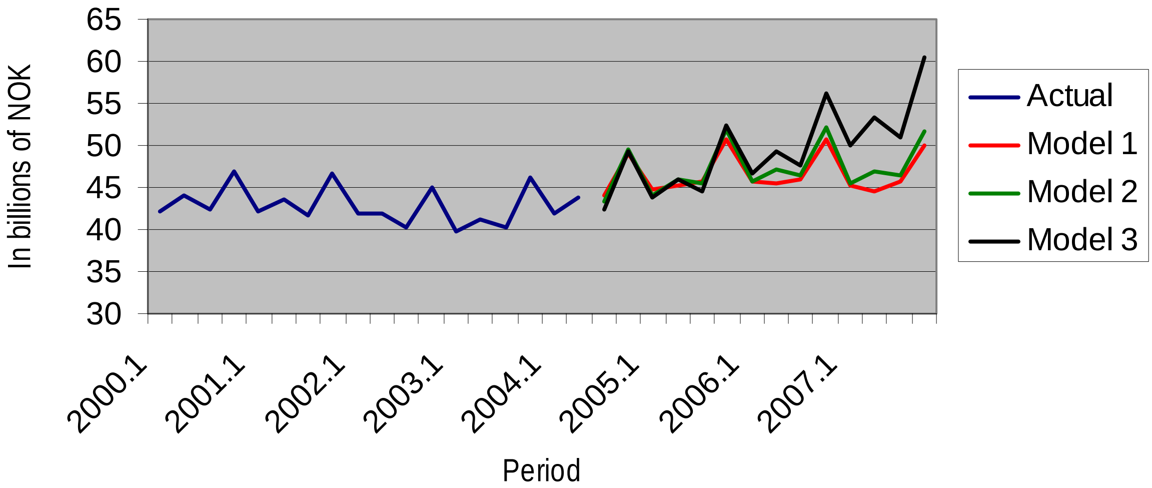


Figure 20: Forecasts for nominal cash with intercept correction



It is now of interest to compare these forecasts. First, it can be seen that for the first 6 quarters, the three models produce reasonable similar forecasts. In fact, Model 1 and Model 2 give quite similar forecasts for the whole forecast period. On the other hand, forecasts produced by Model 3 differ much from the ones obtained by Model 1 and Model 2 for the last periods. However, it seems like the forecasts produced by Model 3 are "out of track". This assertion is even strengthened, if one compares these forecasts with the forecast results without intercept correction. Further comparison shows that Model 1 and Model 2 seem to produce lower forecasts when intercept correction is applied. This might dampen the suspicion that these models over-predict the "true" demand for cash.

6.5 Forecasts when using alternative forecasts for the exogenous variables

The forecasts produced by Model 1 and Model 2 above are all forecasts that relies on a specific evolution of the exogenous variables. As explained above, I have chosen to rely on forecasts produced by Norges Bank and Statistics Norway, for the evolution of the exogenous variables. In the following I will denote these forecasts as "reference-forecasts". Furthermore, it is of interest to investigate how the forecasts produced by the different models change, if the evolution of one of the exogenous variables changes. In this way, one can get an impression of how large the forecast uncertainty is, when basing the forecasts on specific evolution paths of the exogenous variables. I have therefore chosen to consider two specific alternative cases for the evolution of the exogenous variables.

First, I will consider a case where consumption follows a different evolution path, than the forecasts produced by Statistics Norway. In this case I have assumed that the evolution of the consumption variable follows the same path as the forecasts produced by an AR(8)-process for consumption.[58] The specific model is given in Appendix VI. All other exogenous variables are assumed to follow the same path as in the reference case.

Second, I will consider a different evolution path for the real interest rate. In the reference forecasts, I have chosen to rely on forecasts for the nominal interest rate based on Norges Bank (2004c). Now, I will investigate how the different forecasts changes, when relying on some older forecasts for the nominal interest rate presented in Norges Bank (2003).[59] Again,

[58] In the AR(8)-model for consumption, all insignificant variables are excluded.
[59] In Norges Bank (2003) there are only forecasts for the interest rate until 2006.4.

all other exogenous variables are assumed to follow the same path as in the reference case. The forecasts for the interest rate presented in Norges Bank (2003) and (2004c) differ in the way that Norges Bank (2004c) assumes both a lower level of the interest rate and a slower increase in the interest rate over the specific forecasting period. Results from the forecasts obtained by the different assumptions of the exogenous variables are given in Appendix VII. The forecasts are produced both with and without intercept correction.

Comparing the results, one can clearly see that the reference-forecasts and the forecasts obtained by the alternative consumption path, gives quite similar forecasts. On the other hand, forecasts obtained by the alternative interest rate path differ. They differ in the way that they give a lower demand for cash. This is also reasonable since the alterative interest rate path, suggests a higher interest rate than in the reference case. There may be different reasons explaining why the forecasts obtained when using an alternative interest rate path differ more from the reference-forecasts than the ones obtained when using an alternative consumption path. However, the main reason is probably that the alternative path for the interest rate differs more than the alternative path for consumption, compared to the interest and consumptions paths in the reference case. On the other hand, there is reason to believe that it is harder to forecast the interest rate than consumption. The reason for this is that consumption is likely to have minor fluctuations around a smooth trend while the interest rate is likely to be more volatile.

7. Conclusions

Despite major innovations in transactions technology, the demand for cash increased yearly until 1999, and has been reduced slowly since then. This paper has studied the determinants of cash demand in Norway during the period 1980-2004. A thorough discussion of the different determinants for cash demand has been carried through. As a starting point for the discussion, the concept of money and its role, the evolution of the payment system and theoretical models of money demand were considered. Since most of the theoretical models of money demand focus on a quite narrow concept of money, it is believed that some of these models are also relevant when investigating the demand for cash. Especially models considering private households demand for money are believed to be relevant. Based on this discussion, empirical models of demand for real cash have been developed. Rather than estimating the theoretical models themselves, they are rather used to determine which explanatory variables that should be considered when an empirical model is specified.

In the empirical analysis, quarterly data and a general-to-specific approach were used. Due to problems with autocorrelation in the residuals and lack of data, a VAR model was not deemed appropriate to use for estimating the demand for cash. Instead a single equation equilibrium correction model was used. Considering different initial models, eliminating insignificant variables and applying different statistical tests, suggest that there are two competitive, but also quite similar, models of relevance. Both models have only private point-of-sale consumption and real deposit interest rate as significant explanatory variables in addition to a linear trend, seasonal variables and impulse dummies. The rapid evolution in the payment system seems to have a negative effect on the demand for cash. This effect is represented in the models by a negative linear trend.

The estimation results show that the two models short-run effects differ slightly, while the long-run effects are quite similar. The long-run elasticity for consumption was found to be approximately 0.63 and the semi-elasticity for the interest rate was found to be approximately 0.02 in both models. These results differ compared to the results obtained by Fischer, Köhler and Seitz (2004) for the Euro area.

The explanatory variables were found to be weakly exogenous with respect to all parameters in the structural equation for real cash, validating a single equation approach. In addition to this, the highly significant equilibrium correction term suggests that an equilibrium correction model is appropriate.

In order to choose between one of the two competitive models, parameter stability and ex post forecasting properties were investigated. However, this did not lead us to a clear conclusion of which model to choose. Both models seem to have parameters that are reasonable stable. On the other side, both models have some problems when it comes to forecasting in the sample period 2002.1-2004.2. An explanation for this may be that this forecasting period was a rather turbulent period for the Norwegian economy. The main reason for this was probably that the economy needed some time to adjust to the change in the monetary policy regime in March 2001. Since the model selection analysis, gave no clear suggestion of which model to choose, I treated both models on an equal basis.

For given evolution paths of the exogenous variables, ex ante forecasts for the period 2004.3-2007.4 were obtained by the two models. In addition to this forecasts produced by a simple AR(8) model were also considered. The forecasts produced by the two competitive models that are developed, suggest that the demand for real cash will rise for the next couple of years, and then slowly decrease from the end of 2006. In contrast the simple AR(8) model suggests that the demand for cash will increase throughout the whole forecast period. In order to make the forecasts more robust for potential breaks, forecasts with intercept correction were also conducted. Finally, forecasts were obtained under alternative assumptions of the evolution paths for the exogenous variables.

Finally, the work can be extended in different ways. First, it is believed that the illegal economy has a great impact on cash demand. An investigation of how large the illegal economy is and what determines the size of it, would be of interest in a further investigation of the demand for cash. Second, a further investigation of how consumers respond to the different prices related to alternative payment instruments would be of great interest. Finally, an investigation of the demand for different denominations of notes and coins would be of interest. Alternatively, this could be done by dividing the denominations into different groups, for example large, medium and small banknotes and coins. It is then likely that especially the demand for large banknotes is strongly affected by the illegal economy.

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Appendices

Appendix I. Data used in the empirical analysis

Table I.1: Data, real values

Period	CU	C	r	(t+p)/y	ATM	EFTPOS	P_{alt}
1980.1	51722	63382	3.36	0.47	38	0	0.00
1980.2	51376	66592	3.60	0.45	75	0	0.00
1980.3	52615	70175	4.13	0.44	113	0	0.00
1980.4	50975	76050	4.87	0.48	150	0	0.00
1981.1	46751	60568	2.52	0.45	175	0	0.00
1981.2	47234	65764	4.65	0.44	200	0	0.00
1981.3	47582	68025	4.95	0.44	225	0	0.00
1981.4	47415	74707	6.65	0.46	250	0	0.00
1982.1	44243	60030	3.58	0.44	270	50	0.00
1982.2	44769	65334	5.83	0.44	290	100	0.00
1982.3	45693	67283	5.35	0.43	310	150	0.00
1982.4	44353	71232	6.63	0.46	330	200	0.00
1983.1	42569	61427	5.58	0.44	334	250	0.00
1983.2	42927	64873	6.80	0.44	338	300	0.00
1983.3	44125	68205	6.96	0.43	341	350	0.00
1983.4	43505	73294	7.25	0.44	345	400	0.00
1984.1	41411	63032	6.57	0.42	355	475	0.00
1984.2	42541	67491	7.02	0.44	365	550	0.00
1984.3	44256	70139	7.85	0.42	375	625	0.00
1984.4	44118	76315	7.68	0.45	385	700	0.00
1985.1	43518	68171	7.33	0.45	406	750	0.00
1985.2	43404	73019	7.21	0.46	427	800	0.00
1985.3	43955	76034	7.96	0.45	448	850	0.00
1985.4	45532	82851	7.96	0.46	469	900	0.00
1986.1	44614	71317	7.43	0.45	552	1050	1.17
1986.2	44588	78690	7.31	0.45	636	1200	1.15
1986.3	44108	79271	6.57	0.42	719	1350	1.11
1986.4	45154	87151	8.46	0.43	802	1500	1.06

Period	CU	C	r	(t+p)/y	ATM	EFTPOS	P_{alt}
1987.1	41530	73336	8.03	0.45	922	1813	2.15
1987.2	42673	78937	9.18	0.45	1041	2125	2.15
1987.3	43808	81598	9.68	0.43	1161	2438	2.12
1987.4	45355	86908	9.89	0.45	1280	2750	2.03
1988.1	41809	75823	8.98	0.44	1371	3580	2.18
1988.2	42230	78857	9.42	0.44	1463	4409	2.17
1988.3	42365	81596	10.15	0.43	1554	5239	2.17
1988.4	43990	83906	9.79	0.43	1645	6068	2.10
1989.1	40618	73253	8.42	0.42	1669	6215	2.56
1989.2	41141	78580	7.46	0.42	1694	6362	2.53
1989.3	42665	81283	8.03	0.41	1718	6509	2.55
1989.4	43125	84107	8.21	0.43	1742	6656	2.45
1990.1	39049	73311	7.61	0.43	1750	7295	3.30
1990.2	40529	79493	7.91	0.41	1759	7935	3.31
1990.3	42074	82818	8.29	0.41	1767	8574	3.28
1990.4	42800	85346	7.62	0.43	1775	9213	3.17
1991.1	39430	76289	8.17	0.41	1780	9988	3.53
1991.2	39223	81526	7.53	0.41	1785	10762	3.50
1991.3	41625	87264	8.13	0.41	1789	11537	3.48
1991.4	43237	89653	8.18	0.42	1794	12311	3.41
1992.1	39382	77970	8.19	0.41	1779	13101	3.29
1992.2	40550	84873	7.25	0.42	1765	13891	3.28
1992.3	42050	89074	8.49	0.41	1750	14681	3.22
1992.4	41870	91656	8.67	0.42	1735	15471	3.14
1993.1	39609	79093	7.02	0.42	1728	16219	3.58
1993.2	41303	85939	5.14	0.42	1721	16966	3.54
1993.3	42220	93531	5.01	0.41	1714	17714	3.51
1993.4	45523	95389	4.24	0.4	1707	18461	3.46
1994.1	43746	83772	3.89	0.43	1716	21176	3.03
1994.2	46197	89360	3.11	0.43	1724	23891	3.04
1994.3	46764	95170	3.44	0.41	1733	26606	3.00
1994.4	47526	98596	3.88	0.42	1741	29321	2.94

Period	CU	C	r	(t+p)/y	ATM	EFTPOS	P_{alt}
1995.1	44235	85523	3.00	0.43	1754	30667	2.93
1995.2	46936	92630	3.29	0.43	1767	32014	2.89
1995.3	46671	99462	3.75	0.42	1779	33360	2.86
1995.4	48013	100794	3.76	0.43	1792	34706	2.80
1996.1	45783	91057	4.24	0.44	1809	35788	2.56
1996.2	47304	95481	2.78	0.44	1827	36870	2.53
1996.3	47147	103701	3.04	0.42	1844	37951	2.50
1996.4	48992	106956	2.69	0.43	1861	39033	2.46
1997.1	46118	91429	1.59	0.42	1870	40923	2.43
1997.2	46347	100565	2.13	0.43	1879	42813	2.41
1997.3	46208	108127	2.66	0.43	1887	44702	2.41
1997.4	51047	112270	2.27	0.42	1896	46592	2.38
1998.1	45713	93888	1.91	0.42	1908	48003	2.21
1998.2	46661	102649	2.50	0.43	1920	49414	2.19
1998.3	45905	111504	5.53	0.42	1932	50824	2.17
1998.4	49026	112559	5.75	0.43	1944	52235	2.14
1999.1	45898	99149	4.67	0.44	1960	53862	2.06
1999.2	45476	103787	4.14	0.44	1976	55489	2.02
1999.3	44228	114919	4.68	0.42	1991	57115	2.01
1999.4	49434	116773	3.15	0.42	2007	58742	1.98
2000.1	43613	102410	3.40	0.44	2035	60388	2.12
2000.2	44700	109886	3.82	0.44	2063	62034	2.08
2000.3	42996	116144	5.08	0.42	2091	63680	2.08
2000.4	47340	119637	4.89	0.43	2119	65326	2.07
2001.1	41925	103510	4.35	0.43	2125	66917	2.10
2001.2	43002	110993	4.60	0.44	2132	68507	2.08
2001.3	41943	116240	6.88	0.43	2138	70098	2.12
2001.4	46957	122641	5.37	0.43	2144	71688	2.12
2002.1	42336	106739	4.85	0.43	2155	73793	1.96
2002.2	41754	115383	4.74	0.44	2166	75897	1.94
2002.3	40108	119268	5.83	0.43	2177	78002	1.94
2002.4	44829	127012	4.63	0.43	2188	80106	1.94

Period	CU	C	r	(t+p)/y	ATM	EFTPOS	P_{alt}
2003.1	39391	108221	2.08	0.44	2150	82889	1.91
2003.2	40836	118964	5.53	0.44	2112	85673	1.90
2003.3	40062	124887	2.63	0.43	2073	88456	1.91
2003.4	46258	132934	1.36	0.44	2035	91239	1.92
2004.1	41531	114781	1.27				
2004.2	42847	123651	0.66				

Notes:

Capital letters indicate that the variables are in levels.

Exceptions: *r* and the variables in the ration *(t+p)/y* are both in levels, see specifications of variables at p. 50.

Real currency (CU) and real consumption (C) are given in millions of 2001 NOK.

Real currency (CU), real consumption (C) and real price of using alternative payment instruments are all obtained by deflating the nominal values by the (POS) consumption price index (CPIC).

The real interest rate is obtained by subtracting the CPI inflation rate from the nominal interest rate.

The values for EFTPOS in the period 1980.1-1981.4, and the values for P_{alt} in the period 1980.1-1985.4 were set equal to 1 when log-transforming the data. The reason for this is that otherwise it is not possible to calculate the logarithm of these variables.

Table I.2: Data, nominal values

Period	Cash in circulation	Point-of-sale (POS) consumption	Nominal interest rate	CPI*	CPIC*	Inflation (CPI)
1980.1	17627	21600	6.97	0.35	0.34	3.61
1980.2	18110	23474	7.08	0.36	0.35	3.48
1980.3	19336	25789	7.15	0.38	0.37	3.02
1980.4	19814	29561	7.33	0.38	0.39	2.46
1981.1	18635	24143	7.48	0.40	0.40	4.96
1981.2	19300	26871	7.59	0.42	0.41	2.94
1981.3	20346	29087	7.67	0.43	0.43	2.72
1981.4	21133	33297	7.88	0.43	0.45	1.23
1982.1	19989	27121	7.99	0.45	0.45	4.41
1982.2	20455	29851	8.06	0.46	0.46	2.23
1982.3	21608	31818	8.07	0.47	0.47	2.72
1982.4	22008	35345	8.38	0.48	0.50	1.75
1983.1	20944	30222	8.32	0.50	0.49	2.74
1983.2	21279	32158	8.40	0.50	0.50	1.60
1983.3	22376	34587	8.36	0.51	0.51	1.40
1983.4	23014	38772	8.52	0.52	0.53	1.27
1984.1	21716	33054	8.71	0.53	0.52	2.14
1984.2	22364	35480	8.58	0.54	0.53	1.56
1984.3	23655	37489	8.88	0.54	0.53	1.03
1984.4	24534	42439	8.87	0.55	0.56	1.19
1985.1	23552	36894	8.96	0.56	0.54	1.63
1985.2	24024	40416	8.91	0.57	0.55	1.70
1985.3	25160	43522	9.04	0.57	0.57	1.08
1985.4	27196	49487	9.14	0.58	0.60	1.18
1986.1	25729	41128	9.18	0.59	0.58	1.75
1986.2	26276	46372	9.22	0.60	0.59	1.91
1986.3	27003	48530	9.52	0.62	0.61	2.95

Period	Cash in circulation	Point-of-sale (POS) consumption	Nominal interest rate	CPI*	CPIC*	Inflation (CPI)
1986.4	28980	55933	10.36	0.63	0.64	1.90
1987.1	26596	46964	10.90	0.65	0.64	2.87
1987.2	27383	50654	10.79	0.66	0.64	1.61
1987.3	28449	52990	11.02	0.67	0.65	1.34
1987.4	30832	59080	11.32	0.68	0.68	1.43
1988.1	28263	51256	11.43	0.69	0.68	2.45
1988.2	28733	53654	11.18	0.71	0.68	1.76
1988.3	28821	55510	10.92	0.71	0.68	0.77
1988.4	30938	59011	10.70	0.72	0.70	0.91
1989.1	28522	51439	9.66	0.73	0.70	1.24
1989.2	29231	55831	9.10	0.74	0.71	1.64
1989.3	30083	57313	8.69	0.74	0.71	0.66
1989.4	31606	61642	8.83	0.75	0.73	0.62
1990.1	28646	53781	8.97	0.76	0.73	1.36
1990.2	29647	58149	9.07	0.77	0.73	1.16
1990.3	31084	61186	8.88	0.77	0.74	0.59
1990.4	32682	65170	8.94	0.78	0.76	1.32
1991.1	30255	58537	8.88	0.79	0.77	0.71
1991.2	30335	63052	8.58	0.80	0.77	1.05
1991.3	32376	67874	8.51	0.80	0.78	0.38
1991.4	34291	71104	8.68	0.80	0.79	0.50
1992.1	31057	61487	8.65	0.81	0.79	0.46
1992.2	32083	67152	8.31	0.82	0.79	1.06
1992.3	33888	71785	8.79	0.82	0.81	0.30
1992.4	34689	75937	9.05	0.82	0.83	0.38
1993.1	31889	63678	7.85	0.83	0.81	0.83
1993.2	33707	70135	6.10	0.84	0.82	0.96
1993.3	34751	76985	4.97	0.84	0.82	-0.04
1993.4	38003	79630	4.43	0.84	0.83	0.19
1994.1	36134	69196	4.00	0.84	0.83	0.11

Period	Cash in circulation	Point-of-sale (POS) consumption	Nominal interest rate	CPI*	CPIC*	Inflation (CPI)
1994.2	38029	73561	3.84	0.84	0.82	0.73
1994.3	39034	79439	3.98	0.85	0.83	0.54
1994.4	40454	83925	4.28	0.85	0.85	0.40
1995.1	37047	71626	3.98	0.86	0.84	0.98
1995.2	39853	78652	4.00	0.87	0.85	0.71
1995.3	40048	85348	3.96	0.87	0.86	0.21
1995.4	42069	88316	4.01	0.87	0.88	0.25
1996.1	38920	77407	3.93	0.87	0.85	-0.31
1996.2	40696	82142	3.63	0.88	0.86	0.85
1996.3	40980	90137	3.63	0.88	0.87	0.59
1996.4	43324	94582	3.32	0.89	0.88	0.63
1997.1	40727	80741	2.63	0.89	0.88	1.04
1997.2	41314	89644	2.54	0.90	0.89	0.41
1997.3	41264	96557	2.83	0.90	0.89	0.17
1997.4	46014	101200	2.85	0.91	0.90	0.58
1998.1	41704	85654	2.86	0.91	0.91	0.95
1998.2	42947	94478	3.03	0.92	0.92	0.53
1998.3	42485	103197	5.79	0.92	0.93	0.26
1998.4	46070	105772	6.32	0.93	0.94	0.57
1999.1	42832	92526	5.54	0.93	0.93	0.87
1999.2	43152	98484	4.86	0.94	0.95	0.72
1999.3	42233	109736	4.48	0.94	0.95	-0.20
1999.4	48020	113433	4.43	0.95	0.97	1.28
2000.1	42178	99040	4.43	0.96	0.97	1.03
2000.2	43936	108007	4.62	0.97	0.98	0.80
2000.3	42390	114506	5.39	0.97	0.99	0.31
2000.4	46952	118656	5.81	0.98	0.99	0.92
2001.1	42034	103779	5.85	1.00	1.00	1.50
2001.2	43608	112558	5.78	1.01	1.01	1.18
2001.3	41591	115263	5.84	1.00	0.99	-1.04

Period	Cash in circulation	Point-of-sale (POS) consumption	Nominal interest rate	CPI*	CPIC*	Inflation (CPI)*
2001.4	46633	121795	5.74	1.00	0.99	0.37
2002.1	42002	105896	5.40	1.01	0.99	0.55
2002.2	41900	115787	5.29	1.01	1.00	0.55
2002.3	40188	119506	5.80	1.01	1.00	-0.03
2002.4	44955	127368	5.79	1.02	1.00	1.16
2003.1	39718	109119	4.89	1.05	1.01	2.81
2003.2	41253	120178	3.88	1.03	1.01	-1.65
2003.3	40262	125511	2.27	1.03	1.01	-0.36
2003.4	46249	132907	1.84	1.04	1.00	0.48
2004.1	41872	115723	1.42	1.04	1.01	0.15
2004.2	43704	126124	1.25	1.04	1.02	0.59

* Base year 2001

Notes:

Cash in circulation is calculated at end of quarter.

Cash in circulation and POS-consumption is denoted in millions of NOK.

CPI denotes consumer price index.

CPIC denotes the specific price index, which only includes prices of the consumption goods included in the POS variable. This is the price index which is used to construct real values of currency in circulation and POS consumption.

Sources:

Cash in circulation and nominal interest rate: Norges Bank.

POS-consumption and CPI: Statistics Norway.

CPIC: Terje Skjerpen.

Appendix II. Construction of point-of-sale consumption variable

The categories of private consumption

Code	Description
62A1	Groceries
62A2	Non-alcoholic beverage
62B1	Alcoholic beverage
62B2	Tobacco
62C1	Clothes and shoes
62C2	Repairs of clothes and shoes
62D2	Rent (house)
62D3	Materials for repairs in rented house
62D4	Services connected with house
62D5	Electricity
62D6	Fuel and remote heating
62E1	Repairs of household equipment and paid housework
62E3	Furniture and white goods
62E6	Miscellaneous household articles and equipments
62F1	Medicine, glasses and orthopaedic equipment
62F3	Health services
62G1	Purchase of private means of transport
62G2	Spare parts
62G3	Gas and oil
62G4	Local and long-distance transport
62G5	Other services in connection with transport
62H2	Telecommunication equipment
62H3	Post and telecommunication services
62I1	Photo and information technology equipment
62I2	Durable leisure equipment
62I3	Books, leisure commodities
62I4	Leisure service
62J0	Education
62K0	Hotel and restaurant services
62L3	Private commodities
62L4	Private services

62L5 Insurance

62L6 Financial and legal services

62L8 Norwegians consumptions abroad

62L9 Foreigners consumptions in Norway

Calculation of private point-of-sale (POS) consumption:

All the private consumption items that are shaded in blue are included in the private point-of-sale (POS) consumption variable. The others are taken out, because it is believed that cash is not considered as an alternative mean of payment when purchasing these items. They are typically paid with giros.

Calculation of CPIC (POS price index):

This is a price index related to POS consumption. It is calculated as the ratio between POS consumption in values and POS consumption in constant prices. Implicitly this means that the individual price indexes are weighted by their value shares in relation to the value of POS consumption.

Appendix III. Calculation of the variable P_{alt} (price alternative payment instruments)

Table III.1. Annual data series, which are of relevance for the construction of P_{alt}

Period	Transactions Cards (millions)	Transactions Bank card (millions)	Price Bank card (NOK)	Transactions Cheque (millions)	Price Cheque (NOK)
1980	0.00	Missing	0.00	92.00	0.00
1981	0.00	Missing	0.00	92.00	0.00
1982	1.00	Missing	0.00	92.00	0.00
1983	2.00	Missing	0.00	94.00	0.00
1984	3.50	Missing	0.00	101.00	0.00
1985	4.50	Missing	0.00	86.00	0.00
1986	7.50	Missing	0.00	70.00	0.75
1987	13.75	9.64	1.00	76.00	1.50
1988	30.34	14.20	1.00	72.20	1.90
1989	40.81	19.00	1.00	60.30	2.70
1990	53.51	24.60	1.44	50.00	4.31
1991	58.90	33.30	1.11	46.20	5.36
1992	76.60	46.00	1.16	38.10	6.42
1993	97.40	63.04	1.87	31.60	8.05
1994	123.10	83.90	1.88	26.30	8.22
1995	143.40	103.80	2.02	22.40	8.82
1996	182.50	137.60	2.05	17.10	8.89
1997	234.70	184.20	2.20	12.90	9.81
1998	288.60	234.80	2.13	9.40	10.59
1999	338.80	282.70	2.07	6.30	12.31
2000	391.60	342.50	2.19	4.00	15.07
2001	446.10	394.70	2.24	2.90	21.06
2002	500.80	451.70	2.07	2.00	21.75
2003	575.50	519.40	2.07	1.50	22.54
Weight:					
1. quarter	0.213	0.213		0.213	
2. quarter	0.253	0.253		0.253	
3. quarter	0.253	0.253		0.253	
4. quarter	0.281	0.281		0.281	

Notes, Weight:
These weights are used when transforming the annual data series into quarterly data series. There exist monthly data on the total number of transactions with card for 2000-2003 (BBS). I have used these numbers to derive the seasonal weights given above. Then I have assumed that the same seasonal pattern is also present in the rest of the sample, and I have derived quarterly data for the whole data sample by applying this assumption.

Explanation of the data:
Total transactions with cards (in million of transactions):
In other words, total use of EFTPOS terminals. This includes use of bank cards, domestic credit cards, payment issued by international credit card companies and oil companies' cards.
1980-1988: Estimates done by Gresvik and Kaloudis (2001).
1989-1990: Numbers are from Norges Bank (1990, 1991).
1991-2003: Numbers are from Norges Bank (2001, 2004a).

Transactions with bank cards (in million of transactions):
1987-1990: Numbers taken from Norges Banks annual report for the respective years.
1991-1994: Numbers from Norges Bank (2001). I have estimated bank cards used in the banks payment terminals as 96.4% of the total use of cards in the banks terminals.
1994-2003: Numbers from Norges Bank (2004a)

Price per transaction when using bank card:
There are only prices in connection with using bank cards. To use the other types of payment cards is free.
1982-1986: Price assumed to be equal to 0.
1987-1989: Assumption, price equal to 1.
1980-1989: There are no prices registered for use of cards. However we know that prices for using card existed at least for the last couple of years. EFTPOS terminals were introduced in 1982.
1990-2003: Actual prices, numbers from Norges Banks annual reports on payment system.

Total transactions when using cheques (in millions of transactions):
1980-1981: Estimates done by Gresvik and Kaloudis (2001).
1982-2003: Actual numbers, taken from Norges Banks annual reports.

Price per transaction when using cheque:
The price is an average of the prices per transaction when using personal cheques and business cheques.
1980-1985: No pricing on use of cheque.
1986: Estimate, average price of the years 1985 and 1987.
1987-1988: Numbers from appendix in Norges Banks (1990). The numbers are calculated in a different way than in the other years, they represent average fee per transaction.
1989: Estimate, average price of the years 1988 and 1990.
1990-2003: Actual numbers from Norges Bank (2001, 2004a).

Table III.2. Quarterly data series involved in the construction of the variable P_{alt}

Period	Transactions cards (millions)	Transactions Bank card (millions)	Price Bank Card (NOK)	Transactions Cheque (millions)	Price Cheque (NOK)	Total Transactions	Nominal Price
1980.1	0.00	Missing	0.00	19.60	0.00	19.60	0.00
1980.2	0.00	Missing	0.00	23.28	0.00	23.28	0.00
1980.3	0.00	Missing	0.00	23.28	0.00	23.28	0.00
1980.4	0.00	Missing	0.00	25.85	0.00	25.85	0.00
1981.1	0.00	Missing	0.00	19.60	0.00	19.60	0.00
1981.2	0.00	Missing	0.00	23.28	0.00	23.28	0.00
1981.3	0.00	Missing	0.00	23.28	0.00	23.28	0.00
1981.4	0.00	Missing	0.00	25.85	0.00	25.85	0.00
1982.1	0.21	Missing	0.00	19.60	0.00	19.81	0.00
1982.2	0.25	Missing	0.00	23.28	0.00	23.53	0.00
1982.3	0.25	Missing	0.00	23.28	0.00	23.53	0.00
1982.4	0.28	Missing	0.00	25.85	0.00	26.13	0.00
1983.1	0.43	Missing	0.00	20.02	0.00	20.45	0.00
1983.2	0.51	Missing	0.00	23.78	0.00	24.29	0.00
1983.3	0.51	Missing	0.00	23.78	0.00	24.29	0.00
1983.4	0.56	Missing	0.00	26.41	0.00	26.98	0.00
1984.1	0.75	Missing	0.00	21.51	0.00	22.26	0.00

Period	Transactions cards (millions)	Transactions Bank card (millions)	Price Bank Card (NOK)	Transactions Cheque (millions)	Price Cheque (NOK)	Total Transactions	Nominal Price
1984.2	0.89	Missing	0.00	25.55	0.00	26.44	0.00
1984.3	0.89	Missing	0.00	25.55	0.00	26.44	0.00
1984.4	0.98	Missing	0.00	28.38	0.00	29.36	0.00
1985.1	0.96	Missing	0.00	18.32	0.00	19.28	0.00
1985.2	1.14	Missing	0.00	21.76	0.00	22.90	0.00
1985.3	1.14	Missing	0.00	21.76	0.00	22.90	0.00
1985.4	1.26	Missing	0.00	24.17	0.00	25.43	0.00
1986.1	1.60	Missing	0.00	14.91	0.75	16.51	0.68
1986.2	1.90	Missing	0.00	17.71	0.75	19.61	0.68
1986.3	1.90	Missing	0.00	17.71	0.75	19.61	0.68
1986.4	2.11	Missing	0.00	19.67	0.75	21.78	0.68
1987.1	2.93	2.05	1.00	16.19	1.50	19.12	1.38
1987.2	3.48	2.44	1.00	19.23	1.50	22.71	1.38
1987.3	3.48	2.44	1.00	19.23	1.50	22.71	1.38
1987.4	3.86	2.71	1.00	21.36	1.50	25.22	1.38
1988.1	6.46	3.02	1.00	15.38	1.90	21.84	1.48
1988.2	7.68	3.59	1.00	18.27	1.90	25.94	1.48
1988.3	7.68	3.59	1.00	18.27	1.90	25.94	1.48
1988.4	8.53	3.99	1.00	20.29	1.90	28.81	1.48
1989.1	8.69	4.05	1.00	12.84	2.70	21.54	1.80
1989.2	10.33	4.81	1.00	15.26	2.70	25.58	1.80
1989.3	10.33	4.81	1.00	15.26	2.70	25.58	1.80
1989.4	11.47	5.34	1.00	16.94	2.70	28.41	1.80
1990.1	11.40	5.24	1.44	10.65	4.31	22.05	2.42
1990.2	13.54	6.22	1.44	12.65	4.31	26.19	2.42
1990.3	13.54	6.22	1.44	12.65	4.31	26.19	2.42
1990.4	15.04	6.91	1.44	14.05	4.31	29.09	2.42
1991.1	12.55	7.09	1.11	9.84	5.36	22.39	2.71
1991.2	14.90	8.43	1.11	11.69	5.36	26.59	2.71

Period	Transactions cards (millions)	Transactions Bank card (millions)	Price Bank Card (NOK)	Transactions Cheque (millions)	Price Cheque (NOK)	Total Transactions	Nominal Price
1991.3	14.90	8.43	1.11	11.69	5.36	26.59	2.71
1991.4	16.55	9.36	1.11	12.98	5.36	29.53	2.71
1992.1	16.32	9.80	1.16	8.12	6.42	24.43	2.60
1992.2	19.38	11.64	1.16	9.64	6.42	29.02	2.60
1992.3	19.38	11.64	1.16	9.64	6.42	29.02	2.60
1992.4	21.52	12.93	1.16	10.71	6.42	32.23	2.60
1993.1	20.75	13.43	1.87	6.73	8.05	27.48	2.89
1993.2	24.64	15.95	1.87	7.99	8.05	32.64	2.89
1993.3	24.64	15.95	1.87	7.99	8.05	32.64	2.89
1993.4	27.37	17.71	1.87	8.88	8.05	36.25	2.89
1994.1	26.22	17.87	1.88	5.60	8.22	31.82	2.50
1994.2	31.14	21.23	1.88	6.65	8.22	37.80	2.50
1994.3	31.14	21.23	1.88	6.65	8.22	37.80	2.50
1994.4	34.59	23.58	1.88	7.39	8.22	41.98	2.50
1995.1	30.54	22.11	2.02	4.77	8.82	35.32	2.46
1995.2	36.28	26.26	2.02	5.67	8.82	41.95	2.46
1995.3	36.28	26.26	2.02	5.67	8.82	41.95	2.46
1995.4	40.30	29.17	2.02	6.29	8.82	46.59	2.46
1996.1	38.87	29.31	2.05	3.64	8.89	42.51	2.17
1996.2	46.17	34.81	2.05	4.33	8.89	50.50	2.17
1996.3	46.17	34.81	2.05	4.33	8.89	50.50	2.17
1996.4	51.28	38.67	2.05	4.81	8.89	56.09	2.17
1997.1	49.99	39.23	2.20	2.75	9.81	52.74	2.15
1997.2	59.38	46.60	2.20	3.26	9.81	62.64	2.15
1997.3	59.38	46.60	2.20	3.26	9.81	62.64	2.15
1997.4	65.95	51.76	2.20	3.62	9.81	69.58	2.15
1998.1	61.47	50.01	2.13	2.00	10.59	63.47	2.01
1998.2	73.02	59.40	2.13	2.38	10.59	75.39	2.01
1998.3	73.02	59.40	2.13	2.38	10.59	75.39	2.01

Period	Transactions cards (millions)	Transactions Bank card (millions)	Price Bank Card (NOK)	Transactions Cheque (millions)	Price Cheque (NOK)	Total Transactions	Nominal Price
1998.4	81.10	65.98	2.13	2.64	10.59	83.74	2.01
1999.1	72.16	60.22	2.07	1.34	12.31	73.51	1.92
1999.2	85.72	71.52	2.07	1.59	12.31	87.31	1.92
1999.3	85.72	71.52	2.07	1.59	12.31	87.31	1.92
1999.4	95.20	79.44	2.07	1.77	12.31	96.97	1.92
2000.1	83.41	72.95	2.19	0.85	15.07	84.26	2.05
2000.2	99.07	86.65	2.19	1.01	15.07	100.09	2.05
2000.3	99.07	86.65	2.19	1.01	15.07	100.09	2.05
2000.4	110.04	96.24	2.19	1.12	15.07	111.16	2.05
2001.1	95.02	84.07	2.24	0.62	21.06	95.64	2.11
2001.2	112.86	99.86	2.24	0.73	21.06	113.60	2.11
2001.3	112.86	99.86	2.24	0.73	21.06	113.60	2.11
2001.4	125.35	110.91	2.24	0.81	21.06	126.17	2.11
2002.1	106.67	96.21	2.07	0.43	21.75	107.10	1.95
2002.2	126.70	114.28	2.07	0.51	21.75	127.21	1.95
2002.3	126.70	114.28	2.07	0.51	21.75	127.21	1.95
2002.4	140.72	126.93	2.07	0.56	21.75	141.29	1.95
2003.1	122.58	110.63	2.07	0.32	22.54	122.90	1.92
2003.2	145.60	131.41	2.07	0.38	22.54	145.98	1.92
2003.3	145.60	131.41	2.07	0.38	22.54	145.98	1.92
2003.4	161.72	145.95	2.07	0.42	22.54	162.14	1.92

Notes:
Total transactions: Denotes the sum of transactions (in millions) where either cards or cheques are used as the payment instrument.
Nominal Price: Denotes the average nominal price per transaction, when an alternative payment instrument to cash is used.
The average real price per transaction for the use of an alternative payment instrument to cash is obtained by deflating the nominal price by the price index for (POS) consumption.

Appendix IV. Calculation t-values on long-run

Assume that the empirical equilibrium correction term is given by:

$$\hat{\alpha} cu_{t-1} + \hat{\beta}_1 c_{t-1} + \hat{\beta}_2 r_{t-1} = \hat{\alpha}\left(cu_{t-1} - \hat{\rho}_1 c_{t-1} - \hat{\rho}_2 r_{t-1}\right), \text{ where } \hat{\rho}_i = -\frac{\hat{\beta}_i}{\hat{\alpha}} \quad \textit{for} \quad i = 1,2$$

Then the large-sample variance of $\hat{\rho}_i$ can be approximated as, see Kmenta (1997, p. 486) for a more detailed discussion:

$$\hat{\text{var}}(\hat{\rho}_i) = \hat{\delta}_{1,i}\, \hat{\text{var}}(\hat{\beta}_i) + \hat{\delta}_{2,i}\, \hat{\text{var}}(\hat{\alpha}) + 2\hat{\delta}_{3,i}\, \hat{\text{cov}}(\hat{\beta}_i, \hat{\alpha})$$

$$\textit{where} \quad \hat{\delta}_{1,i} = \left(\frac{\partial \hat{\rho}_i}{\partial \hat{\beta}_i}\right)^2 = \left(\frac{1}{\hat{\alpha}}\right)^2, \quad \hat{\delta}_{2,i} = \left(\frac{\partial \hat{\rho}_i}{\partial \hat{\alpha}}\right)^2 = \left(\frac{\hat{\beta}_i}{\hat{\alpha}^2}\right)^2 \quad \textit{and} \quad \hat{\delta}_{3,i} = \left(\frac{\partial \hat{\rho}_i}{\partial \hat{\beta}_i}\right)\left(\frac{\partial \hat{\rho}_i}{\partial \hat{\alpha}}\right) = \left(\frac{1}{\hat{\alpha}}\right)\left(\frac{\hat{\beta}_i}{\hat{\alpha}^2}\right) = \frac{\hat{\beta}_i}{\hat{\alpha}^3}$$

Table IV.1: Components involved in the calculation of he long-run t-values

Values:	Model 1	Model 2
$V\hat{a}r(\hat{\beta}_1)$	0.0106	0.0056
$V\hat{a}r(\hat{\beta}_2)$	0.0000053	0.0000051
$V\hat{a}r(\hat{\alpha})$	0.0087	0.0076
$\hat{\text{cov}}(\hat{\beta}_1, \hat{\alpha})$	- 0.0064	- 0.0045
$\hat{\text{cov}}(\hat{\beta}_2, \hat{\alpha})$	0.00019	0.00016
$\hat{\delta}_{1,c}$	4.02	3.75
$\hat{\delta}_{2,c}$	1.64	1.47
$\hat{\delta}_{3,c}$	- 2.57	- 2.35
$\hat{\delta}_{1,r}$	4.02	3.75
$\hat{\delta}_{2,r}$	0.0016	0.0014
$\hat{\delta}_{3,r}$	0.0812	0.0733
$\hat{\text{var}}(\hat{\rho}_c)$	0.0899	0.0531
$s\hat{d}(\hat{\rho}_c)$	0.30	0.23
$\hat{\text{var}}(\hat{\rho}_r)$	0.000067	0.000054
$s\hat{d}(\hat{\rho}_r)$	0.0082	0.0073

t-values	Model 1	Model 2
$\hat{\rho}_c$	- 2.13	- 2.71
$\hat{\rho}_r$	2.48	2.66

Appendix V. Output for models in Chapter 6

Table V.1: Empirical measures and diagnostics

	Model 1	**Model 2**	**Model 3**
Sigma	0.019	0.020	0.023
RSS	0.029	0.030	0.044
R^2	0.901	0.893	0.845
R^2-adj	0.899	0.891	0.841
DW	2.14	2.04	2.01
Mean(Δcu_t)	- 0.000937	-0.000360	-0.000360
Var(Δcu_t)	0.003106	0.003191	0.003191

Table V.2: Residual misspecification tests (p-values in parentheses)

	AR 1-5 **F(5,65)**	**ARCH 1-4** **F(4,62)**	**Norm** $\chi^2(2)$	**Hetero** **F(21,48)**	**RESET** **F(1,69)**
Model 1	0.9523	0.1019	0.1440	0.8275	0.8192
	[0.4529]	[0.9815]	[0.9305]	[0.6877]	[0.3683]
Model 2	0.6078	0.3225	2.7058	1.2795	2.2077
	[0.6942]	[0.8620]	[0.2585]	[0.2331]	[0.1415]
Model 3	0.3745	0.7086	2.2267	0.5898	0.3256
	[0.8647]	[0.5885]	[0.3285]	[0.8172]	[0.5698]

Appendix VI. Forecasts for the exogenous variables

Table VI.1. Forecasts for the exogenous variables

Forecast Period	C reference	r reference	CPIC	C Case A	r Case B
2004.3	128607	1.28	1.020	130508	2.58
2004.4	134276	0.66	1.025	137479	2.36
2005.1	122237	1.05	1.027	119117	2.85
2005.2	127953	0.91	1.031	128327	2.91
2005.3	134254	1.68	1.030	134829	3.68
2005.4	139212	0.65	1.041	142104	2.65
2006.1	126042	1.73	1.044	123468	3.63
2006.2	131344	1.67	1.051	132575	3.47
2006.3	137344	2.50	1.051	139539	4.20
2006.4	142590	1.52	1.064	146277	3.12
2007.1	129010	2.51	1.069	127946	
2007.2	134407	2.39	1.077	136822	
2007.3	140145	3.20	1.078	144094	
2007.4	145136	2.27	1.091	150681	

Notes:

C reference: are the forecast for real (POS) consumption used when producing reference forecasts for the demand for real cash. It is based on forecasts produced by Statistics Norway for private consumption. I have here assumed that the change in nominal POS consumption follows the same change as the forecasted nominal private consumption. The real POS consumption forecasts are then obtained by deflating it by the forecasts for the POS consumption price index (CPIC).

r reference: are the forecasts for real deposit interest rate used when producing the reference forecasts for the demand for real cash. It is based on forecasts produced by Norges Bank for the folio rate. I have assumed that the change in the nominal deposit rate follows the exact same path as the forecasted change in the nominal folio rate. The forecasts for the real interest rate are then calculated by using the forecast for the nominal deposit rate produced by Norges Bank and the forecast for the consumer price index (CPI) produced by Statistics Norway.

CPIC: are the forecasts for the POS price index. I have here assumed that the forecasts for the change in this price index follow the same path as the forecasts for the change in the consumer price index (CPI) produced by Statistics Norway.

C Case A: are forecasts for POS consumption used when producing Case A forecasts for real cash. These are the forecasts produced by an univariate AR(8) model for POS consumption.

The AR(8)-model for consumption, which is used to construct the alternative evolution path of consumption, is given below (t-values are in parentheses).

$$\underset{}{\Delta c_t} = \underset{(2.32)}{0.006} - \underset{(-7.27)}{0.609\Delta c_{t-1}} + \underset{(6.20)}{0.569\Delta c_{t-4}} + \underset{(3.95)}{0.405\Delta c_{t-5}} - \underset{(-2.44)}{0.199\Delta c_{t-6}} - \underset{(-2.25)}{0.184\Delta c_{t-7}} + \underset{(2.34)}{0.204\Delta c_{t-8}} + \hat{\varepsilon}_t,$$

where $\hat{\varepsilon}_t$ is the residual from the OLS estimation of the AR(8) model.

r Case B: are forecasts for real deposit interest rate used when producing Case B forecasts for real cash. These forecasts are based on forecast produced by Norges Bank (2003) for the folio rate.

Appendix VII. Forecast results

The tables below present the different forecast results that are obtained by Model 1 and Model 2 under different assumptions about the evolution of the exogenous variables.

Reference-forecasts refer to the forecasts obtained when the initial assumptions of the evolution of the exogenous variables are used. In other words the ones used in Section 6.3 and Section 6.4.

Case A refers to simulation experiment no. 1, where forecasts are obtained using an alternative consumption path compared to the one used in the reference case.

Case B refers to simulation experiment no. 2, where forecasts are obtained using an alternative interest rate path compared to the one used in the reference case.

Table VII.1: Ex ante forecasts for demand for real cash (in billions of 2001-NOK), **Model 1**

Forecast period	Reference-case	Case A	Case B	Reference case	Case A	Case B
	Intercept correction					
	No	No	No	Yes	Yes	Yes
2004.3	43.978	44.163	43.635	43.165	43.346	42.828
2004.4	49.406	49.793	48.613	47.911	48.287	47.142
2005.1	45.277	45.054	44.751	43.566	43.352	43.060
2005.2	45.914	46.174	45.118	43.957	44.206	43.195
2005.3	47.082	47.561	45.761	44.440	44.892	43.194
2005.4	51.925	51.755	50.139	48.753	48.594	47.076
2006.1	46.652	46.297	45.148	43.762	43.429	42.351
2006.2	46.002	46.311	44.345	43.197	43.486	41.640
2006.3	46.898	47.661	44.952	43.831	44.544	42.012
2006.4	50.790	50.470	48.661	47.572	47.272	45.577
2007.1	45.079	45.020		42.390	42.334	
2007.2	43.924	44.625		41.457	42.118	
2007.3	45.008	46.153		42.439	43.519	
2007.4	48.498	48.442		45.885	45.833	

Table VII.2: Ex ante forecasts for demand for real cash (in billions of 2001-NOK), **Model 2**

Forecast period	Reference case	Case A	Case B	Reference case	Case A	Case B
	Intercept correction					
	No	No	No	Yes	Yes	Yes
2004.3	43.147	43.234	42.821	42.609	42.695	42.287
2004.4	49.315	49.586	48.548	48.304	48.569	47.552
2005.1	44.019	44.082	43.070	42.888	42.949	41.963
2005.2	45.859	45.848	44.637	44.526	44.515	43.339
2005.3	45.850	45.913	44.344	44.207	44.268	42.755
2005.4	51.772	52.047	49.854	49.772	50.037	47.928
2006.1	45.507	45.521	43.756	43.714	43.728	42.031
2006.2	46.656	46.638	44.860	44.825	44.808	43.100
2006.3	46.359	46.540	44.457	44.274	44.447	42.457
2006.4	51.305	51.812	49.174	48.964	49.447	46.929
2007.1	44.552	44.702		42.579	42.723	
2007.2	45.589	45.738		43.658	43.801	
2007.3	45.109	45.530		43.096	43.498	
2007.4	49.391	50.220		47.282	48.076	

Table VII.3: Ex ante forecasts for demand for nominal cash (in billions of NOK), **Model 1**

Forecast period	Reference case	Case A	Case B	Reference case	Case A	Case B
	Intercept correction					
	No	No	No	Yes	Yes	Yes
2004.3	44.843	45.031	44.493	44.014	44.198	43.670
2004.4	50.624	51.021	49.812	49.092	49.477	48.305
2005.1	46.486	46.257	45.946	44.730	44.509	44.210
2005.2	47.346	47.614	46.525	45.328	45.585	44.542
2005.3	48.486	48.979	47.126	45.766	46.231	44.482
2005.4	54.061	53.884	52.201	50.758	50.592	49.013
2006.1	48.726	48.355	47.156	45.708	45.359	44.234
2006.2	48.328	48.652	46.587	45.380	45.684	43.745
2006.3	49.294	50.096	47.248	46.070	46.820	44.158
2006.4	54.041	53.699	51.775	50.616	50.297	48.494
2007.1	48.174	48.110		45.300	45.240	
2007.2	47.298	48.052		44.642	45.353	
2007.3	48.539	49.774		45.769	46.933	
2007.4	52.919	52.858		50.068	50.011	

Table VII.4: Ex ante forecasts for demand for nominal cash (in billions of NOK), **Model 2**

Forecast period	Reference case	Case A	Case B	Reference case	Case A	Case B
	Intercept correction					
	No	No	No	Yes	Yes	Yes
2004.3	43.995	44.085	43.663	43.447	43.535	43.118
2004.4	50.531	50.808	49.745	49.495	49.767	48.725
2005.1	45.195	45.259	44.220	44.033	44.096	43.084
2005.2	47.289	47.277	46.029	45.914	45.903	44.691
2005.3	47.218	47.283	45.667	45.525	45.588	44.030
2005.4	53.901	54.188	51.904	51.819	52.095	49.899
2006.1	47.531	47.546	45.701	45.658	45.672	43.900
2006.2	49.014	48.995	47.128	47.091	47.073	45.279
2006.3	48.727	48.917	46.727	46.535	46.717	44.625
2006.4	54.589	55.127	52.321	52.097	52.611	49.933
2007.1	47.610	47.771		45.502	45.656	
2007.2	49.091	49.251		47.011	47.165	
2007.3	48.649	49.102		46.477	46.911	
2007.4	53.893	54.798		51.592	52.458	