A Dynamic Market Model

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Preface

This paper is the result of a combination of drifting thoughts during syllabus reading, the author being forced to make a spontaneous decision, and periods of hard thinking. I would like to give special thanks to Professor Kjell Arne Brekke for giving me excellent guidance during the early stage of writing, and to Professor Aanund Hylland for being a superb supervisor. My deepest gratitude also goes to two good friends, Tori Løge and Marit Solum, for having the patience of proof-reading my work. In addition, I would like to thank my friends and colleagues at the University of Oslo for creating a pleasant environment.

In my heart are Jon Anders Grønvold, for his reliability; my family, for their support; and Tineke de Chavonnes Vrugt, for reminding me of the fragility of life and the depth of eternal adventure and exploration.

Abstract

This paper looks at a dynamic process in a partial market where there are lags in the adjustment of the consumers as a group, and where the firms are not perfectly equal. What I model is the movements of price and realized quantum of a market good given the allocation of the demand and supply curves. A major conclusion is that given the circumstances, the system will always converge to the equilibrium point in the long run, but if shocks occur, the market allocation of price and realized quantum of the good may very well remain in disequilibrium. Unless one studies a particular market, it is not possible to make a precise general description of how the movements in disequilibrium behave quantitatively and describe the speed of convergence to equilibrium. However, qualitative conclusions are more easily drawn on a general basis.

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1. Introduction

This paper looks at a dynamic process (Tâtonement process) in a partial market where there are lags in the adjustment of the consumers as a group, and where there are some differences, although not large, among the firms. What I mean by "lags in adjustment" is that the group of consumers does not adapt instantaneously to a price change, but needs some time before settling in a steady state where the consumers' preferences are optimized. The purpose is to create a foundation for how dynamic processes may occur in reality, and see how partial markets potentially behave when not residing in equilibrium. What I model is the movements of price and realized quantum of a market good given the allocation of the demand and supply curves. Current location of these curves, as well as shocks changing them, are exogenous factors not explained. Central questions are which side of the market dominates in deciding the driving forces' movements, whether or not the system moves towards equilibrium and how movements may develop and at what speed.

Although the theme of this paper is related to microeconomics, the way of analyzing is similar to what is more often used in macroeconomics. Figures similar to phase diagrams are introduced and split into sets, which are used for explaining how the system behaves in different market circumstances. Different scenarios are discussed.

It is important to underline that the model in this paper in general is of speculative nature, as I cannot claim with confidence that it is well applied in reality. The model is mostly a result of my own philosophizing, and an interesting next step would be to challenge it through empirical research.

2. Framework

Three assumptions are central to this model, these works as a framework for the following dynamic analysis. In this section each of these assumptions will be given a detailed explanation, regarding why I choose them and what implications they have.

2.1 Assumption 1

Assumption 1: The market of interest is not affected by other markets. That is, I look at a partial market model.

The reason I use a partial market is mostly because it is convenient, not because it is a better choice than using a general equilibrium framework. Franklin M. Fisher (1983) argues in his book "*Disequilibrium foundations of equilibrium economics*" that when it comes to market dynamics and questions about stability, it is not sufficient to use a partial market in the analysis. He argues that since partial market approaches only observe the behavior of the given market, assuming that all other prices are constant, they will miss out the extended effects of price changes in other markets. Thus, even if one concludes that a partial market will obtain stability over time given constant prices in all other markets, this will be a waste of knowledge if the same is not true for the general market case where all markets are endogenous. And vice versa: if stability is not obtained in the partial market, but is so when all markets are under consideration, the latter should be of interest since the partial case is not close to reality and will therefore never happen.

I have no intentions of arguing against Fisher. A partial market is used in this paper merely because it is easier than the general case, as the latter demands a large set of difference equations to describe all changes in every market. It is also plausible to expect mutual dependence in many of these equations. As a consequence, the model in this paper is probably best applied as either an approximation of reality, which is better the smaller or more isolated the market of interest is, or just as a foundation for strengthening the intuition of the reader.

2.2 Assumption 2

Assumption 2: There is lag in adjustment of the consumers as a group, because of habit formation.

The behavior aspect of consumers and firms in this partial market is used as a tool for connecting the theoretical market aspect to reality, and the reason I want to model a lag in the adjustment of the consumers is two-sided. Firstly, it is intrinsically interesting to see how a market may behave when the consumer group has this attribute, as it can be argued to be a way of better adapting the model to reality. Secondly, it is a way of creating a driving force for the dynamics of the model. The last point will be a major focus of this paper and will be returned to in section 3.2, "The driving forces of the consumers", while the first point will be explained in more detail just below.

There are several arguments for why it is plausible for the consumer group to have adjustment lags:

- As a result of habit formation, each consumer may individually have lags in his or her adjustment of consumption as a response to a price change. If a consumer is used to a certain consumption level and to some degree disfavors change of habits, the person in question will not react instantaneously to a change of prices, and it takes some time to adjust to any new ideal quantum of demand. Thus, there are differences in short and long term preferences.
- The argument above can be strengthened by giving room for differences among consumers, and at the same time let consumers influence each other. This is the case with trends, where some start consuming a good, and others follow gradually, until a steady state eventually is reached (if the trend is not just a passing fashion).
- Furthermore, consumers may exhibit some lack of self-knowledge, in the sense that they actually do not have a clear idea of how much of the good they are willing to buy for each price level. When a change of price actually occurs the consumers change their consumption only gradually, until reaching the real preferred quantum given this new price level. An interpretation of this is that even though individual demand curves exist, consumers are not perfectly aware of their demand curves' shape or location.

A simple way to summarize this assumption is to consider an optimization behavior of consumers with a utility function where the utility of time t depends on both consumption today and on the change of consumption from the previous period:

$$u(c_t, |\Delta c_t|, |\Delta p_t|) \equiv u_t,$$

where

$$\frac{\partial u_t}{\partial c_t} > 0, \qquad \frac{\partial^2 u_t}{\partial {c_t}^2} < 0,$$

and

$$\frac{\partial u_t}{\partial |\Delta c_t|} < 0$$

for all t, and $|\Delta c_t|$ and $|\Delta p_t|$ can be interpreted as the absolute value of the change of respectively consumption and price from time $t-1$ to time t, that is $\Delta x_t = x_t - x_{t-1}$. For simplicity, the partial effect of these changes on utility is assumed to be independent of whether the changes are positive or negative. The reason for including $|\Delta p_t|$ in the utility function will not be explained until section 4.2; for now it is sufficient to think of this argument as a way of including price uncertainty and instability caused by price changes as factors influencing the utility of the consumers.

2.3 Assumption 3

Assumption 3: The firms are not perfectly equal.

The main reason for this third assumption is also two-sided. Firstly, I believe that firms are likely to have some differences in their cost functions due to different technologies, even within the same market. The consequence is that they may not show the exact same behavior, and their products may not be perfectly equal. In a partial equilibrium context, this means that the goods different firms produce are not perfect substitutes, although quite close (or else a partial approach would not be appropriate). This again means that there is room for small deviations in the price each firm sets for the market good, even though they do not possess any true market power, and the environment is otherwise competitive.

This leads to the second aspect: that it is a practical tool for solving a problem occurring when considering dynamic market processes, namely the problem of how market price actually changes. For instance, Fisher describes the difficulty of using the theory of individual behavior to explain price movement:

The equilibrium theory of individual competitive behavior on which we shall build is one in which prices are taken as given and quantities optimally set. This can readily be extended to *take price expectations as given. But, as Koopmans (1957) among others has remarked, in a world in which all prices are taken as given, how do prices ever change?*

- Franklin M. Fisher, *Disequilibrium foundations of equilibrium economics, 1983*, page 12.

My argument is that although market price often is considered as something both the consumers and firms take for granted, it is still possible to look at prices as endogenous for the firms in the short run. A real life example illustrates the point. Norway has two major tabloid daily newspapers, *VG* and *Dagbladet*, which are very similar products both in design, size, content et cetera. Yet the price of *VG*, which is the best selling one, is today 12 NOK on weekdays, while the price of *Dagbladet* is 15 NOK. Since they are more or less always put right next to each other when sold in stores, the reason for different prices can not be because of transportation costs (for consumers) or lack of information about the other good's allocation, but is more likely caused by differentiations between the two products. If the publisher of *VG* decided to raise the price to 13 NOK given everything else unchanged, they could loose readers, but I believe they would still keep enough to sustain a major share of the daily tabloid newspaper market. If they were to raise the price to 25 NOK, the story might be quite different. The point is that with small differences between the firms and their products, small differences and changes in price are possible without facing severe losses of sale. The firms thus have some range of prices they can choose from without falling out of the market due to severe losses, and from this range they will choose the price maximizing their profit given their cost function. This range gets narrower the more equal the firms and their products are, and with perfect equality they can only choose one point which is the market price. However, I cannot think of a single market that behaves in such a perfect way.

The implication this assumption has for my model, is that firms to some extent are able to change the price of their products, and this in turn can work as a key to how market price actually changes over time. If one firm increases its price by a small amount, and then other firms do the same, this can be a repeated process where the result is a substantial change of market price. Such a process is one way of answering the complex questions about which economic agents are in charge of prices (for instance asked in *Microeconomic Theory* by Mas-Colell, Whinston and Green), and how it is possible that prices ever change. I will present a more in-depth discussion of this last topic and Assumption 3 in general in the appendix at the end of the paper.

For now, it is important to ascertain that the small amount of market power each firm possesses does not imply that we are considering a monopolistic model. For instance, if a single firm fails to supply enough of its product relative to consumer demand, the products of the competitors are sufficiently close substitutes to work as adequate replacements for the consumers. And the model still works with perfect competition in the sense that the firms do not have the power to set the price in any way they want, they always have to keep it close to a certain level.

3. The separate driving forces

In this part of the paper I will separately explain the driving forces for the dynamics of realized price of the market good and its quantum, and how these two elements change separately, taking the other as given. I start by explaining what is meant by the market equilibrium in this model and defining eight different sets outside the equilibrium point. These sets are then used in the explanation of separate movements in price and quantum.

To keep the analysis simple, I have chosen to use discrete time in this paper. The notations Δx and Δp define changes of quantum and price with respect to time t.

3.1 Equilibrium and sets of disequilibrium

The equilibrium is the point where $\Delta x = 0$ and $\Delta p = 0$ simultaneous, and is therefore a steady state of this model, where neither price nor quantum are going to change. There is one unique such equilibrium, which is the point where the demand and supply curves intersect. In other words, the steady state equilibrium is the same equilibrium as the standard market equilibrium. The intuition behind this is that at this point both suppliers and demanders are satisfied with the allocation of price and realized quantum of the good, leading to no interest in change.

If I denote demand by $D(p)$ and supply by $S(p)$, the stady state equilibrium is the unique point (given that it exist) where $D(p) = S(p)$ and $x > 0, p > 0$. Further I can define four different two-dimensional sets in relation to this, all in the area of strictly positive price and quantum.

Set I: All allocations where $x > D(p)\ and\ x > S(p)$.

Set II: All allocations where $x > D(p)\ and\ x < S(p)$.

Set III: All allocations where $x < D(p)\ and\ x < S(p)$.

Set IV: All allocations where $x < D(p)\ and\ x > S(p)$.

In addition to these four sets and the equilibrium, there are four different one-dimensional sets:

Set S^+: All allocations where $x = S(p)\ and\ x > D(p)$.

Set S^-: All allocations where $x = S(p)\ and\ x < D(p)$.

Set D^+: All allocations where $x = D(p)\ and\ x > S(p)$.

Set D^-: All allocations where $x = D(p)\ and\ x < S(p)$.

All the sets just defined will be used as references from now on. Figure 1 gives a graphical illustration of the sets.

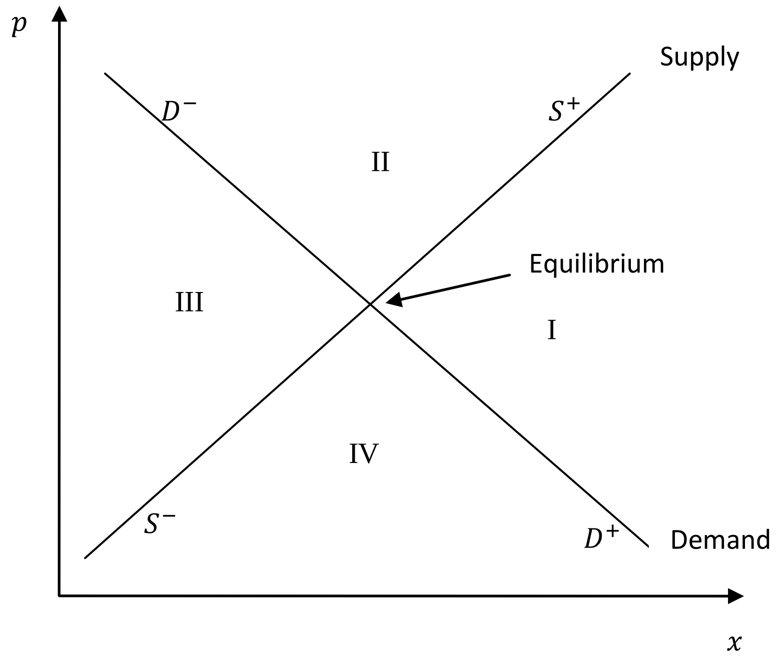

Figure 1

In order to explain the mechanics of this model, I will look at the perspectives of the consumers and the producers separately, before combining them using Figure 1.

3.2 The driving forces of the consumers

Along the demand curve, the consumers are satisfied with the allocation of price and quantum of the good, and it is thus possible to state that

$$\Delta x_D = 0$$

$$\Delta p_D = 0$$

along this curve. The notation implies that the consumers are satisfied with what they consume, and have no incentives to change this quantum given price or to bargain for any other price given quantum. The difference equations equal zero to express this, and "D" is used to underline that it is from the demanders' perspective.

If the allocation for some reason is to the left of the demand curve, the consumers want to either consume more, or they are willing to pay a higher price for the amount they already consume. I mark this by stating that $\Delta x_D > 0$ and $\Delta p_D > 0$ in this case, to show the direction of movement as a consequence of the driving forces of market allocation resulting from the consumers' behavior.

Here it is important to refer to Assumption 2. Because of the lag in the consumers' adjustment, we will not experience an immediate jump from an allocation outside the demand curve directly to the curve, because the consumers adapt gradually. To clarify: if the allocation already resides along the demand curve, and a sudden change of economic environment happens causing the demand curve to make a rightward shift, the allocation will not immediately shift to somewhere along the new demand curve were it only for the consumers to decide. Instead it would move gradually towards it over time. This is illustrated in Figure 2.

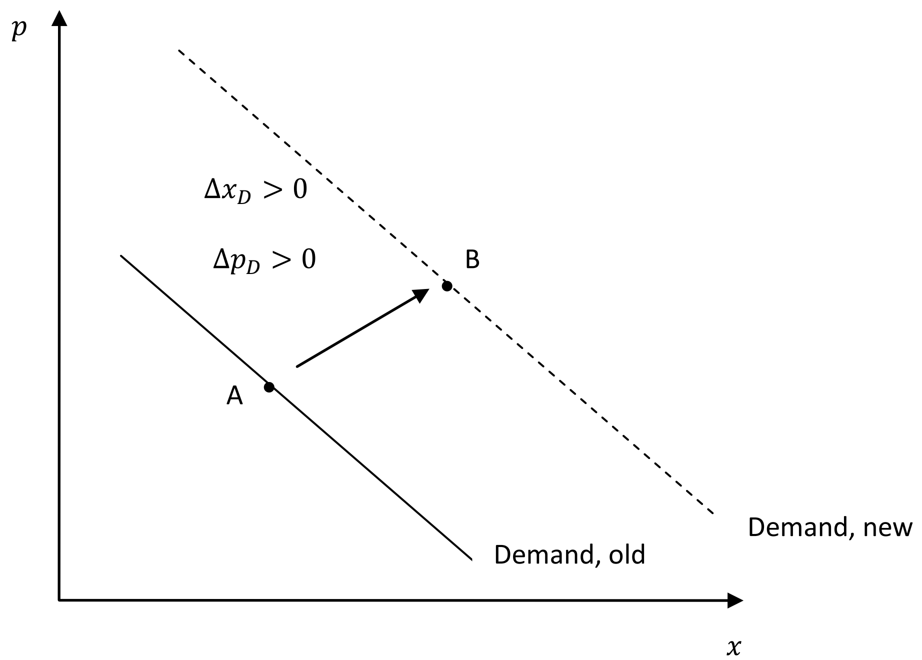

Figure 2

Let us say that the good of interest is a certain kind of ecological food, and that the rightward shift of the demand curve is caused by a period of positive media attention towards such products. Because the consumers get more interested in this kind of product, they are willing to pay more for what they already consume of the good, and they want more of the good. Some people are what could be called "trend-setters", meaning they are easily willing to change their consumption towards this ecological food, implying that for them the utility loss of change, $\partial u_t / \partial |\Delta c_t|$, is relatively low. Others need more persuasion to consider any change, meaning that it takes some time before the true effect of this demand curve change is realized. The result is that over time, aggregate consumption gradually rises, as well as the possibility of a higher price level due to increased willingness to pay.

At some point however, the effects of the shock will settle. This happens when both price and consumption have risen so much that the new demand curve is reached, at point B in the figure above. Thus, I can state that

$$\lim_{t \to \infty} x_D(t) = k_D^x, and$$

$$\lim_{t \to \infty} p_D(t) = k_D^p,$$

where k_D^x and k_D^p are constants. This illustrates that when only the consumers are taken into consideration, the allocation will eventually settle.

The opposite is true to the right of the demand curve. Here $\Delta x_D < 0$ and $\Delta p_D < 0$ will be the case, as consumers want less of the good of interest, or are not willing to pay the current price for continuing to consume the same amount. Again the change will occur gradually because of habit formation, until the demand curve is reached.

To conclude:

- $\Delta x_D = 0$ and $\Delta p_D = 0$ at the sets D^- and D^+ and at the equilibrium point.
- $\Delta x_D < 0$ and $\Delta p_D < 0$ in Set I, Set II and Set S^+.
- $\Delta x_D > 0$ and $\Delta p_D > 0$ in Set III, Set IV and Set S^-.

The same information is summarized in Figure 3, with arrows showing the directions of the driving forces.

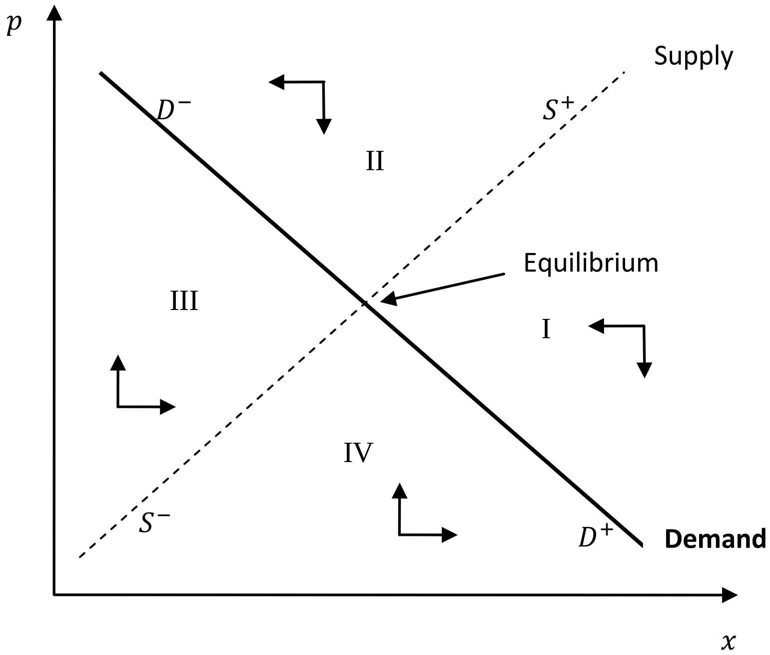

Figure 3

3.3 The driving forces of the firms

Now I will do a similar analysis as the one done for consumers above, this time regarding the behavior of the firms or producers. All combinations of price and quantum of the market good along the supply curve are allocations where the firms are satisfied with the amount they supply given the price level, from a profit maximizing point of view. This also means that given the current quantum of supply, the firms do not prefer any marginal change of price. This can be formulated by stating that

$$\Delta x_S = 0,$$

$$\Delta p_S = 0$$

along the supply curve. The interpretation of these equations is that there are no driving forces for changing price given supplied level and vice versa from the suppliers' point of view.

Notice, however, that if the firms had the theoretical choice of freely changing both price and quantum of the good, there would probably exist a point among the profit maximizing points on the supply curve where profit was the largest possible. In practice this is irrelevant since the firms cannot choose freely when also consumers are taken into consideration.

Also notice that although $\Delta p_S = 0$ may give the impression that there exists a specific price p_S, this is only partly true. The expression $\Delta p_S = 0$ means that the firms have no need to change their own price level, however it is not the same as stating that they all settle at the same price level p_S. As explained earlier, the firms are free to make certain deviations of price within a small range, so the intuition behind a stable market price is either that the firms by coincidence end up with exactly the same price level in steady state, or that the market price can be regarded as an average price among the firms. The important point is the dynamic perspective, which is that when $\Delta p_S = 0$, no individual firm wants to change the price level set, given the small range of possibilities they have.

At all allocations to the left of the supply curve, the driving forces from the firms' perspective will be such that given the amount they already supply, $\Delta x_S > 0$, and given the price of their products $\Delta p_S < 0$. The inequality $\Delta x_S > 0$ simply means that at the given market price, the firms want to produce and sell more goods as this will result in increased profits, while $\Delta p_S < 0$ means that the firms are in a position where they can cut the price of their products, and will probably do so because of competition. This is illustrated in Figure 4.

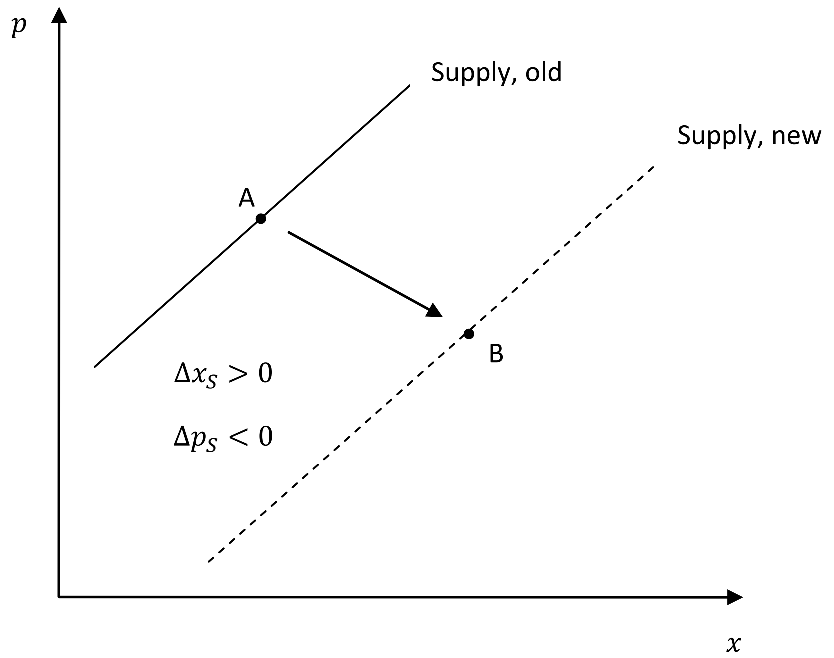

Figure 4

The market of interest could for instance be that of filter coffee, and the rightward shift of the supply curve caused by an exceptional good year of harvest of coffee beans, reducing the cost of producing a bag of coffee. There are several firms supplying this product, but the nature of the different firms' products differ through small differences in their quality, taste, design et cetera allowing for small differences in price, but not too large because competition still is prevalent. Some firms will start both increasing their supply and lower their price, to steal market shares from their competitors and in general make consumers buy more of their product. Other firms more or less voluntary do the same as a response, and this process is repeated until point B is reached. How long time the movement from A to B takes, will depend on how aggressively the firms cut their prices and how fast they are able to increase their production. When only the firms are taken into consideration the allocation may in theory move directly from A to B in a single jump. A stepwise transition is also possible, depending on behavior and competition.

Independent of the transition being immediate or stepwise, at some point the new supply curve will be reached, and thus also from the point of view of the suppliers it can be stated that

$$\lim_{t\to\infty} x_S(t) = k_S^x, and$$

$$\lim_{t\to\infty} p_S(t) = k_S^p,$$

with k_S^x and k_S^p being constants, meaning that the driving forces for change will settle when only the firms are taken into consideration.

To the right of the supply curve the opposite will be the case, meaning that the firms want to supply less than what they do given the price, or that they want the price to be higher given the amount they already supply. Thus, in this area $\Delta x_S < 0$ and $\Delta p_S > 0$. This will be the case until the supply curve is reached, where the change in price and supplied quantum will settle.

To conclude:

- $\Delta x_S = 0$ and $\Delta p_S = 0$ at the sets S^- and S^+ and at the equilibrium point.
- $\Delta x_S < 0$ and $\Delta p_S > 0$ in Set I, Set IV and Set D^+.
- $\Delta x_S > 0$ and $\Delta p_S < 0$ in Set II, Set III and Set D^-.

The same information is summarized in Figure 5, with arrows showing the direction of the driving forces.

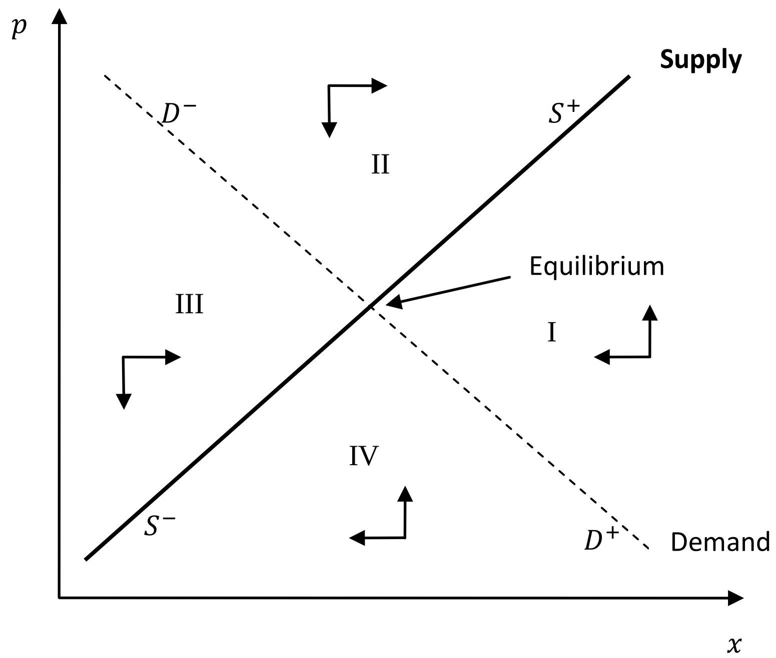

Figure 5

4. The combined driving forces

Until now I have described separately how the dynamics of the consumers and firms work. In this section I will attempt to see what happens when both driving forces explained in the previous section work at the same time, making the system more complex. This will be done through a stepwise analysis of what happens in each of the possible sets explained. A combination of logical reasoning, intuition and situational scenarios will be used to describe how the combined driving forces will interact in this system. I start with the equilibrium point as an anchor, before turning to Set I and go counter-clockwise through every set until finally Set D^+ completes the circle. As a helpful tool I will include an unrefined figure (Figure 6) showing the combination of the individual driving forces of the consumers and firms, unrefined in the sense that what happens when both groups interact is not yet considered. The letters "S" and "D" next to an arrow illustrate that the direction of movement is part of the driving force of the suppliers and demanders respectively, and when no letter is added it is because the direction of movement is unambiguous. As can be seen from Figure 6, the direction of movement in each set is always definite in one factor, either price or quantum. In the ambiguous cases, I will discuss which of the demand and supply side of the market will be the dominating part in deciding the outcome of realized direction of movement.

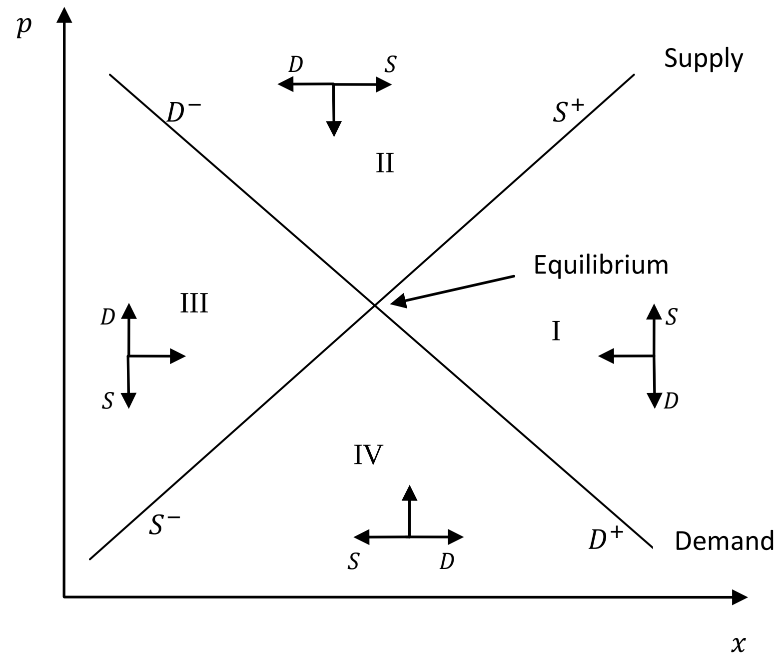

Figure 6

Before going through the different sets, I will give a short explanation of what decides the direction of price movement, as this will be important in some of the coming sets and situations. No deviation from the major literature is done here, and in this paper the principle of price movements is according to a differential equation put forward by Samuelson (1947),

$$\dot{p} = cz(p),$$

where $z(p)$ is the excess demand function, and $c > 0$ is a constant affecting adjustment speed (from *Microeconomic theory*).To fit this with the model in this paper, I specify a discrete version leading to a similar price difference equation:

$$p_{t+1} = p_t + cz(p_t).$$

This means that in cases of excess demand, the price level will grow, while in cases of excess supply, the price level will fall. I will elaborate on the mechanics in all the different set descriptions below.

4.1 The equilibrium point

The equilibrium point is the only point among all possible allocations where the system is in a steady state as long as no exogenous shocks appear. This is clear, as it is the only point where $\Delta x_D = 0$, $\Delta x_S = 0$, $\Delta p_D = 0$ and $\Delta p_S = 0$, which can be summarized by simply stating $\Delta x = 0$ and $\Delta p = 0$, and I define x^* and p^* as the equilibrium levels of quantum and price that are realized in the steady state point. At this point, neither the consumers nor the firms want any change of allocation given their possibilities. This makes sense, as in static models equilibrium is the point where the allocation always dwells, making a sort of agreement between the dynamic model in this paper and the static ones.

Yet, the existence of a steady state point is no guarantee that the system of the dynamic model is stable in the sense that when it is outside of equilibrium it will always return to this stable point. The question of stability will be discussed in section 5, as I in this section only look at each set separately and disregard the long term movements between sets.

4.2 Set I

At this set the situation is as follows:

- $\Delta x_D < 0$

- $\Delta x_S < 0$
- $\Delta p_D < 0$
- $\Delta p_S > 0$

Since both $\Delta x_D < 0$ and $\Delta x_S < 0$, the direction of movement of realized quantum of the good is unambiguous, as both the demanders and suppliers would like the quantum to be smaller given the price level. An important question here, however, is who has the most power in influencing the scope of reduction in quantum of the good. At all allocations (except those sufficiently close to the supply curve) the firms are initially wanting a larger reduction of market quantum compared to the consumers because of the lag in adjustment of the demander group, meaning that $|\Delta x_S| > |\Delta x_D|$, and a number of scenarios are possible. Here are three examples:

- Scenario 1: All firms may simply do cuts of supply so large that they more or less reach their individual supply curves, meaning that one at the aggregate level almost immediately reaches the aggregate supply curve, with no consideration taken to the welfare of the consumers. When below the line of p^*, the firms may implement this strategy to a weaker extent, because they are aware that reaching their individual long run equilibrium point will cause them to first cut and then raise their production. To prevent having to do too much readjustment of supply, each firm averts cutting too much from the start. This strategy, however, requires that the firms have a certain level of information about the aggregate market structure and about their individual long run supply curves.
- Scenario 2: Doing adjustments of supply may be costly, making the firms unwilling to do extensive cuts in the short run, also causing the firms to adapt gradually to shocks. The more costly adjustments are, the slower the supplied amount will change (no costs just lead to Scenario 1). Costs in this context may be of a direct kind, expressed through the cost function of the firm, or of a more indirect character, such as consumers regarding firms that fail to satisfy their demand as "unprofessional" and thus making the firm lose customers in the long run.
- Scenario 3: There may exist an institution that to some degree emphasizes the welfare of the consumers, and thus through laws, regulations and other kinds of market interventions regulates the market in cases when the firms do not sufficiently satisfy the consumers' demand, forcing the change of supply to also be gradually. The stronger this market intervention is, the smaller the change of supply is compared to the adjustment of the consumers.

Regardless of scenario, it can be concluded that in Set I, the total effect will be a reduction in realized quantum of the good, which can be expressed by stating that $\Delta x = \Delta x_S < 0$. In general when the direction of movement of quantum or price is unambiguous, I will drop the small “D” and “S” in the corner of the difference expressions like I have done here.

When it comes to price, the driving forces move in both direction as $\Delta p_D < 0$ and $\Delta p_S > 0$. However, it is plausible to assume that $\Delta p_S > 0$ will dominate because even though the consumers prefer to bargain the price level downward, they cannot force the firms to set lower prices, and the firms will set their prices higher despite the wishes of the consumers. This may either be because the firms gain negative profits because of too high costs compared to income, or simply because profit rises as a result of marginally higher prices.

To conclude that $\Delta p > 0$ in Set I can also be done by using the assumption that $\Delta p = cz(p)$, because of the fact that $|\Delta x| = |\Delta x_S| > |\Delta x_D|$. Since the reduction in realized quantum is larger than what the demanders prefer, current demand will always be higher than supply. I use the term “current demand” to underline that it is what the consumers at a certain time demand due to a lag of adjustment. This must not be confused with any point along the long term demand curve.

One small caveat here is allocations sufficiently close to the supply curve where the situation may be that $|\Delta x| = |\Delta x_D| > |\Delta x_S|$, and thus make $\Delta p < 0$. Intuitively this will happen when the consumers' lower willingness to pay (in combination with competition among firms) starts to dominate the price movement, and can only happen in cases where firms have positive profits, which may happen in this model. Profit can be positive at allocations close to the supply curve (and on the supply curve) because firms are not equal, and thus competition is not totally perfect. I expect this situation can only occur so close to the supply curve that it will not matter at all, because the next movement of market allocation will then cause the system to leave Set I before the price has time to get consequently lower.

Two things are important to note concerning the change of price here. The first is that the increase in price will have to be gradual, because no firm can at one instance raise prices too much. At any point in time the firms are restricted by a ”scope of price action” (SPA, an above bounded set), and to raise prices beyond this will cause a firm to lose enough customers to force it out of the market due to competition from other firms. The SPA will be wider the more different the firms and their products are, and more narrow the fiercer the competition is.

Secondly, if the SPA is wide enough to make it possible for firms to increase prices extensively, the consumers' lag of adjustment will not make them "stupid" in the sense that they do not react to severe price changes. The expression $\partial u_t/\partial|\Delta c_t|$ will be more negative the larger the change of price is, which means that the utility functions of the consumers have the property that

$$\frac{\partial^2 u_t}{\partial|\Delta c_t|\partial|\Delta p_t|} < 0,$$

where $|\Delta p_t|$ represent degree of price change at time t. Thus, the firms cannot use temporarily higher price levels as a tool to get short run profit boosts through fooling the consumers even if they had the possibility to do so because of a wide SPA. This is why I in section 2.2 included $|\Delta p_t|$ in the utility function.

To summarize:

- In Set I, $\Delta x_t < 0$ and $\Delta p_t > 0$.

4.3 Set S^+

Along this upper part of the supply curve, we have the following driving forces:

- $\Delta x_D < 0$
- $\Delta x_S = 0$
- $\Delta p_D < 0$
- $\Delta p_S = 0$

Only the consumers cause any change in price or quantum; if it is up to the firms to decide, no change will occur.

Although the firms do not want any change of realized quantum x, they cannot prevent the consumers from buying less of the good, and thus $\Delta x_D < 0$ dominates, meaning that $\Delta x < 0$ will be the case.

When it comes to price, consumers cannot force the firms to reduce the price of their good, and at the set of S^+ the firms will not do so since they are at a profit maximizing allocation. The consequence is that $\Delta p_S = 0$ will be the dominating part for price movement, which means that $\Delta p = 0$ will be the case.

Thus:

- In Set S^+, $\Delta x < 0$ and $\Delta p = 0$.

4.4 Set II

At this set the situation is as follows:

- $\Delta x_D < 0$
- $\Delta x_S > 0$
- $\Delta p_D < 0$
- $\Delta p_S < 0$

Each allocation in Set II has the property that at the given price level the consumers want to reduce their demand over time, while the producers actually prefer to supply more. As in the previous case, the firms have no power to enforce a higher demand level, the consumers are free to reduce the amount of the market good they buy. $\Delta x < 0$ will thus be the case also in this set.

Both driving forces will cause the price to be reduced, as both $\Delta p_D < 0$ and $\Delta p_S < 0$. The explanation is that the consumers get a lower willingness to pay over time, in combination with the price level being at levels where firms' income far exceeds costs, meaning that the competitive pressure will cause the firms to cut the price level.

There is an important difference between the behavior of the firms when cutting and when raising their price level. Raising the price level has to be done stepwise because of consideration of the SPA, as discussed in section 4.2. When it comes to lowering the price however, the firms may make severe reductions instantaneously as an aggressive attempt to steal market shares. It is likely that if one firm does so, others will have to follow quickly to avoid losing too many market shares, and knowing this, all firms will prefer to also make price reductions in a stepwise manner. Whether an instantaneous or a stepwise price reduction is most likely to occur in reality, depends on how firms behave in this game theoretic context, and will not be discussed in more detail here.

The main point is to summarize:

- In Set II, $\Delta x < 0$ and $\Delta p < 0$.

4.5 Set D^-

This set is the part of the demand curve above p^*, and the situation here is as follows:

- $\Delta x_D = 0$
- $\Delta x_S > 0$
- $\Delta p_D = 0$
- $\Delta p_S < 0$

The argument of what happens to changes in realized quantum in this set is simply a repetition of the previous case, because the suppliers cannot force the consumers to buy more than they prefer at any given price level, and thus $\Delta x = 0$ will be the case.

With regards to price, even though the consumers are happy with the current price level given their consumption, competition will still drive the price level down. Thus, Set D^- can be seen as a pure "consumers' market", letting them consume the same amount to a reduced price level.

To summarize:

- In Set D^-, $\Delta x = 0$ and $\Delta p < 0$.

4.6 Set III

At this set, we have the following relation:

- $\Delta x_D > 0$
- $\Delta x_S > 0$
- $\Delta p_D > 0$
- $\Delta p_S < 0$

At any price level in this set, the consumers will gradually want to increase their demand, and the firms are happy to increase their supply in accordance with this gradual change of consumer preferences. Therefore, it can unambiguously be concluded that $\Delta x > 0$ in this set, and it is likely that $\Delta x = \Delta x_D$, as it is plausible that Δx_D takes a lower value than Δx_S, unless factors in some way restrict the firms to satisfy the consumers' demand or that there is costs of supply adjustment (see the discussion in section 4.2).

Price movement is different however. Just as in the case of Set I, the driving forces for price change go in both directions. Over time, the consumers are willing to pay more for the amount of goods they consume, while the producers are in such a position that they for the given quantum supplied are able to reduce price without bearing negative profits. Even though the firms prefer to set their prices at a higher level and still sell at least the same amount, the competition will force them to do the opposite. As long as at least one firm starts a price war, the others will have to follow, and even if no one did, the market would be so lucrative for the supply side that new firms can easily enter the market and steal market shares by setting a considerably lower price. The more fierce competition is, the faster the price level will fall.

This suggestion of letting $\Delta p < 0$ is strengthened by concidering the price change equation $\Delta p = cz(p)$, because in Set III, $z(p)$ is likely to be negative since Δx_D is likely to be smaller than Δx_S, unless there are very high costs of increasing supply in the short run.

To conclude:

- In Set III, $\Delta x > 0$ and $\Delta p < 0$.

4.7 Set S^-

This set makes up the part of the supply curve below p^*, and here the following is in force:

- $\Delta x_D > 0$
- $\Delta x_S = 0$
- $\Delta p_D > 0$
- $\Delta p_S = 0$

Regarding the question of whether $\Delta x_D > 0$ or $\Delta x_S = 0$ will dominate, the discussion from Set I can again be referred to. As long as the firms are free to let supply take any level they prefer when the quantum related to the supply curve is lower than what the consumers demand, $\Delta x_S = 0$ will dominate so that $\Delta x = 0$ will be the case. Even if the firms in some way are restricted to satisfy the demand of the consumers, it is not obvious that they have to commit to any forced increases in supply before leaving the set, since any such action of forcing requires a third part with unrealistically precise monitoring abilities.

The movement of price in this set is interesting, and not immediately intuitive. At first, one may expect that $\Delta p_S = 0$ will dominate and thus no price movement occurs, because this is a natural continuation of the argumentation from Set III, where competition drives the price level down. Note however that at the previous set, Δx_D was assumed to take lower values than Δx_S due to lag in the consumers' adjustments, while the firms had incentives to raise their supply at a faster pace, leading to a situation of excess supply. In Set S^-, on the other hand, the firms are satisfied with their current level of supply for the given price level, not wanting to adjust, causing a situation of excess demand since $\Delta x_D > 0$. This induces a pressure for higher prices, allowing the firms to raise their price level in spite of the competitive environment. Therefore, $\Delta p_D > 0$ will be the dominating driving force.

It can thus be summarized:

- In Set S^-, $\Delta x = 0$ and $\Delta p > 0$.

4.8 Set IV

For this set, which is below both the demand and supply curve, the following relations are true:

- $\Delta x_D > 0$
- $\Delta x_S < 0$
- $\Delta p_D > 0$
- $\Delta p_S > 0$

The situation regarding movements of quantum in this set is similar to that of Set S^-, in the sense that the consumers want a higher level of quantum than the producers, for any given price level. What really happens in this set is to a high degree dependent on the cost level the firms face when changing their supply, again I will refer to the discussion on the scenarios portrayed in section 4.2.

If the firms face no costs in altering their level of supply (Scenario 1), they will simply prefer an allocation along the supply curve, and thus immediately cut the quantum of supply in accordance with this, taking no regard to the consumers' demand. Then, it is obvious that $\Delta x < 0$ will be the case.

On the other hand, if the firms face some costs in adjusting their supply, the case may be quite different. I will explain this through the use of Figure 7.

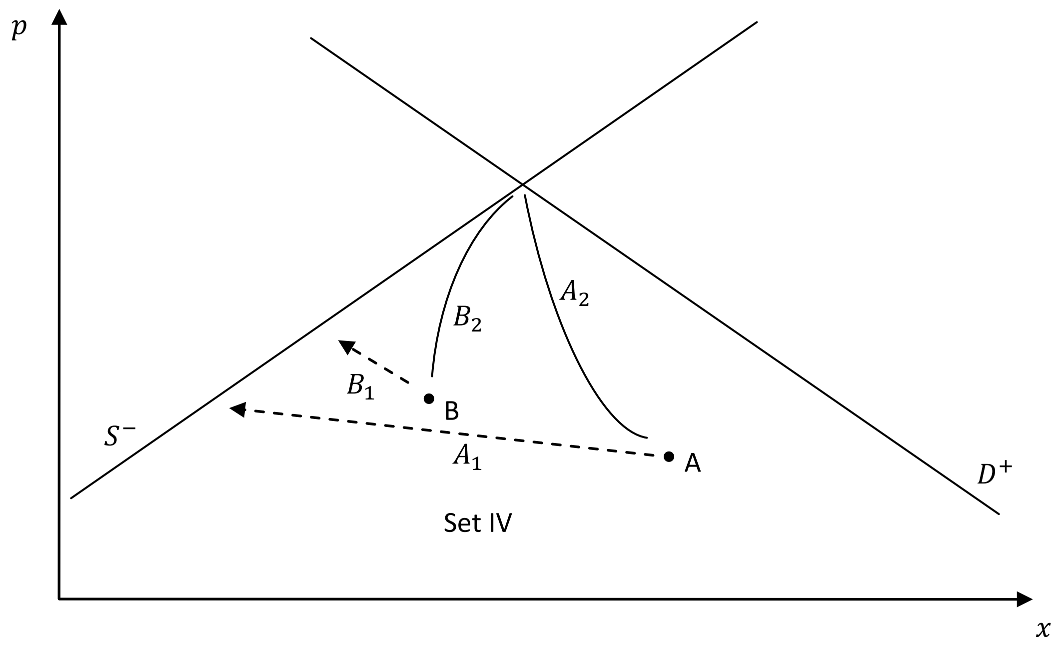

Figure 7

Two arrows are drawn from the allocation point A. The arrow A_1 illustrates what typically can be the case if the firms do not face any costs at all in their supply adjustment, and the allocation will immediately converge to Set S^-. If the firms on the other hand face costs of adjustment, they have incentives to make more cautious reductions of supply, to avoid a long run overshooting of supply reduction, and instead choose a path like the one marked with A_2, reaching the equilibrium point without detours. There is no obvious reason that the firms should have knowledge of the allocation of their individual long run demand curve, so it is plausible that the movements' path converges to Set S^- or Set D^+ before reaching the equilibrium point. In any case, the movements from point A both have the property that $\Delta x < 0$.

From point B, two similar arrows are drawn. The arrow B_1 simply illustrates the case of no adjustment costs similar to arrow A_1, only from allocation B this time, and again reaching S^- immediately. Arrow B_2 is more interesting because it illustrates that if the firms face adjustment costs of changing their supply level, an ambiguous relation in the movement of x occurs. In the short run, the firms prefer to reach their individual supply curve. They may however understand that in the long run their production will increase from the level corresponding to this curve because of the fact that the current quantum for many firms is likely to be lower than their individual equilibrium levels, due to the fact that the aggregate supply level is lower than the equilibrium level x^*. If for each firm the adjustment costs of first decreasing and then again increasing their supply is higher than the profit loss of choosing a path leading directly to their individual equilibrium point, a path like B_2 is likely to arise on the aggregate level, and the result will be increased supply already in Set IV. Thus, depending on allocation, there is a possibility that $\Delta x > 0$ in this set.

To summarize: if the firms face no adjustment costs, $\Delta x < 0$ and convergence towards Set S^- is likely (unless price is already close to p^*, then the equilibrium point may be reached at once), but if the firms face adjustment costs, they may to a certain degree prefer taking a more direct path towards the equilibrium point, meaning that $\Delta x < 0$ if the current quantum is higher than x^*, and $\Delta x > 0$ if the opposite is true.

The direction of price movements in this set is unambiguous, as the consumers over time will have a higher willingness to pay for the amount they consume, and the firms are in such a position that they need and want to raise their price level because of high cost levels compared to income at the current allocation. It is therefore clear that $\Delta p > 0$ in this set.

To conclude:

- In Set IV, $\Delta p > 0$ always, while $\Delta x < 0$ if firms face no adjustment costs or the allocation is to the right of x^*. If firms face adjustment costs, $\Delta x > 0$ for allocations to the left of x^*.

4.9 Set D^+

This is the last set, and here the following is the case:

- $\Delta x_D = 0$
- $\Delta x_S < 0$
- $\Delta p_D = 0$
- $\Delta p_S > 0$

The consumers are satisfied with the level they consume at the current price, while the firms want to cut their supply. Adjustment costs or not, $\Delta x < 0$ will be the case as long as the firms are free to decide the amount they supply, but the speed of reduction depends on the level of adjustment costs.

Since this set is along the demand curve, the consumers are also satisfied with the price level, while the firms want to set higher prices, similar to the situation in the previous set. Although there is a possibility that the firms immediately raise their price levels, it may be more satisfying to assume that the increase in price level will not happen until after the reduction in supplied quantum, to be in accordance with the price change difference equation $\Delta p = cz(p)$, meaning that $\Delta p = 0$ will be the case. It seems plausible that the reduction of supply will dominate any potential price raise (due to the fact that firms cannot raise their price too much at any point in time) in such a way that Set IV is reached before Set I, so that applying $\Delta p = cz(p)$ will not cause any critical losses of realism.

To summarize:

- In Set D^+, $\Delta x < 0$ and $\Delta p = 0$.

5. Stability

Until now I have mostly described the isolated driving forces within each set. Now, the attention will be turned to review the whole picture and see how the total dynamic movements of market allocation may act. In this part the focus will be on whether or not we can expect the system to be stable, while section 6 will concentrate on speed of adjustment.

Section 4 already made it clear that if equilibrium is reached, the whole system will be in a steady state since both $\Delta x = 0$ and $\Delta p = 0$ at this point. The question of stability is thus a question of whether or not the system will ever reach equilibrium when first in disequilibrium. To study this, I will include a diagram, Figure 8, illustrating all the updated information about the combined driving forces in each set, and use this figure as an anchor for the coming discussion.

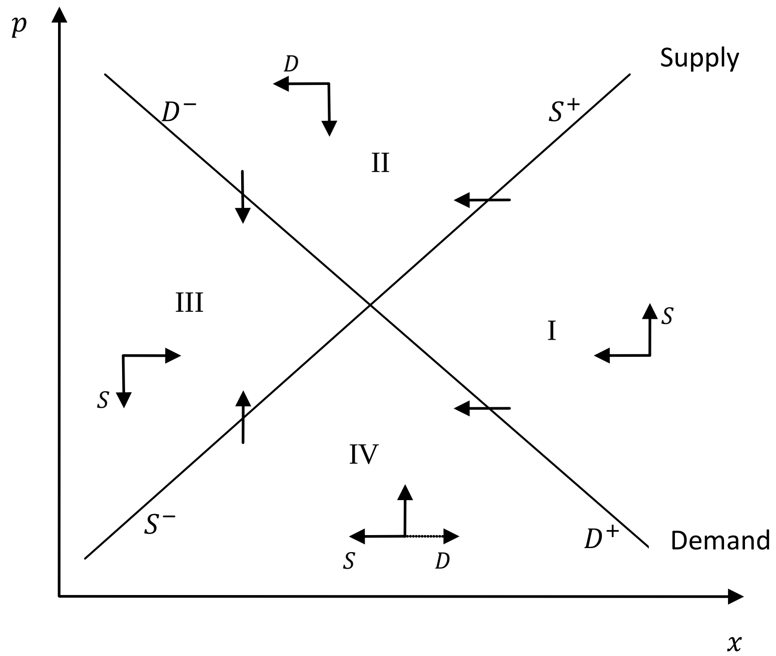

Figure 8

What is obvious from Figure 8 is that the system will not be unstable in the sense that when outside of the equilibrium point, price and quantum will converge to either infinity or zero. All combinations of arrows point towards the steady state allocation, so it seems plausible that the movements will either move towards this point, or at least circle around it. In section 5.1 I will explain all the possible ways of leaving every set by discussing each one of them paragraph by paragraph. I will sum up by drawing a tree in section 5.2.

5.1 Possible movements out from each set

Before going through each set, note that some of the possibilities that will be described are mostly of theoretical interest, as the probability that an allocation in reality exactly reaches the one-dimensional sets is equal to zero. They are still included, both to make the analysis theoretically watertight, and because convergence towards the one-dimensional sets often will occur. In such cases of convergence, I assume that movements of the market allocation will be according to the descriptions of these sets. Thus, in cases where I state that the one-dimensional sets are *reached*, this should in reality be interpreted as *convergence* towards these sets.

In Set I, there are six possibilities of sets to arrive at when leaving the area. They will be listed:

- If at or below p^*, the allocation may move directly to the equilibrium point, either stepwise or in an instant if the firms have the possibility to raise price to p^* immidiately, and cut supply so the supply curve is reached at once.
- If below p^*, and the costs of adjustment are non-existing, the firms may cut supply extensively so that the Set S^- is reached instantaneously. Remember to interpret this as convergence towards the set, as it is unlikely that each individual firm knows exactly the location of their supply curve, and the probability of reaching an exact point on the curve is thus zero.
- If below p^* and there are costs of adjustment, the level of supply may be cut to such an extent that Set IV is reached.
- If below p^* and there are costs of adjustment, the level of supply may be cut to such an extent that Set D^+ is reached.
- If the allocation is above p^*, the movements will either stepwise or immediately reach Set S^+. This set may also in general be reached from any point in Set I as long as the firms' adjustment costs are sufficiently high and they miss the equilibrium point.
- When the allocation is close to the supply curve, the reduced demand of the consumers may be sufficiently high to lead the allocation directly to Set II.

In Set S^+ the allocation is almost certain to reach Set II. The only exception is if is close to the equilibrium point: then the reduction in the consumers' demand may in one instance move fast enough to reach D^- even though there are lags in their adjustment.

From Set II, the movements may either go gradually towards the equilibrium point, or instead reach Set D^-. Both cases are possible when the original consumption level is above x^*, while only Set D^- can be reached if the consumption level already is below x^*. I ignore the theoretical possibility of prices being cut so fast relative to consumption that the allocation returns to Set S^+, because this implies firms cutting prices more than needed, as it means they react to the excess supply level the instant before it actually occurs, and the firms have no incentives to act in such a way.

In the case of reaching Set D^-, the next movement will in general lead to Set III. Again there is an exception at allocations close to the equilibrium point, since there is a possibility that prices may be cut so much that Set S^- is reached before Set III.

When in Set III, there are two possibilities for leaving the area. The first, which only occurs when the original price level is above p^*, is that the path step by step goes to the steady state point. The second, which may occur for any price level, is that the allocation reaches Set S^-. There is also a theoretical possibility that demand grows so fast relative to price reduction that Set D^- is again reached, but this will be ignored because it means consumers react instantaneously to the price reduction, which contradicts the assumption of lags in their adjustment.

In Set S^-, the price is the only factor that will increase, meaning that in general the allocation returns to Set III. When close to the equilibrium point, the price can increase sufficiently to jump directly to Set D^-. If this happens we may experience a system where price forever jumps up and down between the supply and demand curve without quantum of realized consumption ever changing, because along the demand curve the consumers will not increase their demand, and along the supply curve the firms will not increase their supply. However, I do not regard this scenario as plausible, since it requires the firms to display very short-term thinking, and in addition it requires a relatively strong increase of excess demand, which is likely to be weak close to the equilibrium point. Also, remember that it is unlikely to exactly reach any one-dimensional set in reality. I thus rule out this theoretical case of infinite price jumps by assuming that only Set III will be reached from Set S^-.

There are two general possibilities of leaving Set IV. As discussed in section 4.8, the movements in this set depend on the degree of adjustment costs the firms face, and the higher these are, the higher is the likelihood that the movements will take a path directly towards the equilibrium point, which is the first possibility. The second possibility, which is especially likely with low or no adjustment costs (then $\Delta x < 0$ to all allocations at this set), is that the movements instead reach Set S^-, which in theory can happen in an instant in the extreme case of no adjustment costs. In the first case, the firms may miss equilibrium (because they don´t have perfect information of their individual demand curve's allocation) in such a way that Set D^+ is actually reached, but I ignore this scenario, as it will not have any major consequence for the question of stability.

In Set D^+, I simplified by stating that only realized quantum will be reduced, and thus there are two possible ways of leaving this set. The first is that firms cut supply so much that the allocation goes directly to Set S^-, which is the case related to low or no costs of adjustment for the firms. The second occurs with a more modest reduction so that the allocation instead reaches Set IV, which is more likely as long as there is some degree of adjustment costs facing the firms when they change their level of supply.

5.2 Overview of movements

Now that all of the possible sets are discussed, I will in Figure 9 draw a tree summing up all the information, thus making it easier to see the whole picture. In the figure, Set I is used as a starting point, and each arrow leaving the box corresponding to this set leads to one of the theoretically possible sets from Set I. A similar procedure is then used for each of the next sets, causing a branched shape. What I mean by the stippled boxes titled "Already covered" is that the branching out from the corresponding set is already covered in another part of the tree, and repetition is not necessary. All sets will be covered in the tree, and the boxes corresponding to the equilibrium point is the only ones where no further branching occurs.

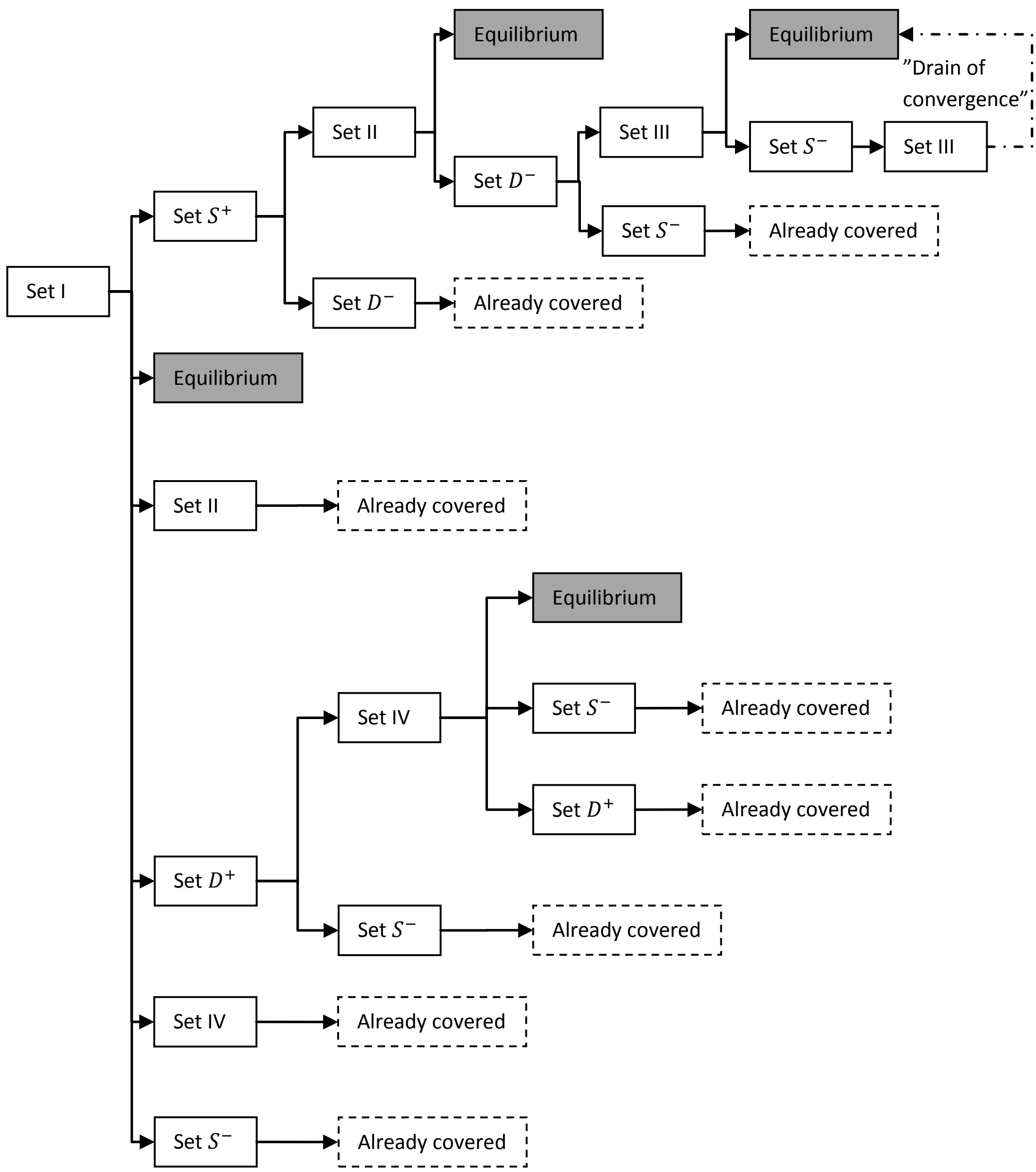

Figure 9

From Figure 9, one can see that no matter where the starting point is, the system will either find a path that moves directly to the equilibrium point, or it will inevitable reach or converge to Set S^- . This set (or the area close to this set) works as a "drain of convergence" where the system converges towards the equilibrium. What happens is that price will raise, reaching Set III, but then quantum of realized consumption will increase combined with a price reduction, leading either to the equilibrium or again to a convergence to Set S^-, but this time closer to the steady state point than what originally was the case. This process will be repeated in a stepwise manner until steady state is reached. Thus, it is possible to conclude that the whole system is stable in the sense that the equilibrium point inevitably will be reached no matter where the starting point of the original allocation is.

Figure 10 and 11 illustrate different scenarios of how the movements towards equilibrium may take place. Figure 10 uses Set I as the starting point, while Figure 11 uses Set D^+ as the starting point. The solid curve shows the longest path to equilibrium among the paths drawn, while the stippled curves show alternative shorter routes.

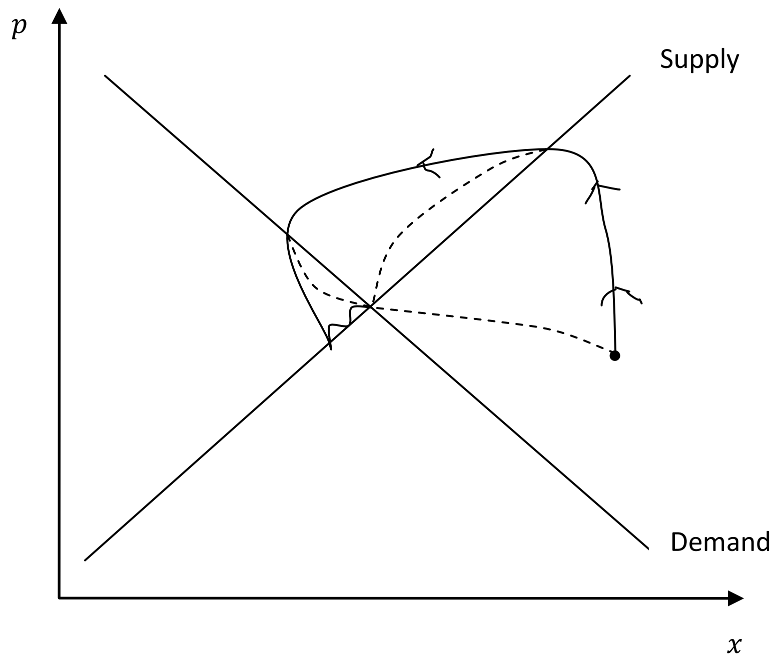

Figure 10

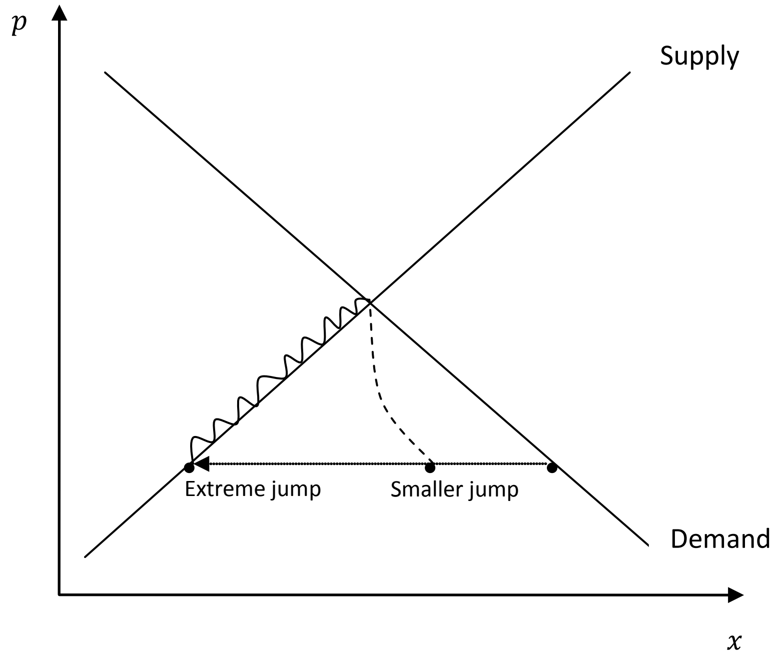

Figure 11

<u>Conclusion</u>:

As time goes to infinity, x goes towards x^* and p goes towards p^*. The system is thus stable, and the equilibrium point will be reached. This is conditional on a constant environment, and that the firms and consumers do not miss their preferred target consequently.

6. Speed of adjustment in disequilibrium

So far I have mostly used an intuitive approach in describing the model, and this has been sufficient to argue for the result of stability. In order to discuss the speed of adjustment, especially the speed of convergence towards the equilibrium point when in disequilibrium, assumptions about the behavior of consumers have to be made, and decisions made as to what degree of adjustment costs the firms face. These properties will in reality depend on what kind of partial market is studied, the technology of the firms and the subjective preferences of the consumers participating in the market, all of which are factors depending on both time and location.

6.1 The general case

First I will be very general. The price movement equation has already been specified, and takes the following form:

$$\Delta p = cz(p).$$

This equation is however not adapted to the environment of this model, for instance because it does not include the possibility that prices are cut faster downward than upward, as the firms only have competitive restrictions when increasing price. A more flexible model can be made by specifying c as a function $c(A)$ of the current allocation and time, $A = (x, p, t)$. The size of $c(A)$ depends on facors such as how equal the firms' goods are, or how fierce competition is.

It is also important to understand that $z(p)$ is not the expression $D(p) - S(p)$, because these only consider the long term demand and supply curves, while it is the current demand and supply that matters. Thus, it is better to state that $z(p) = x_D - x_S$, and one can in addition note that $x = \min(x_D, x_S)$.

The movements of realized quantum will be according to the following relation:

$$\Delta x = \min(\Delta x_D, \Delta x_S) = \min[f(D(p) - x), g(S(p) - x)],$$

where $f(*)$ and $g(*)$ have the same sign as their arguments. It is not possible to make general statements about how strongly the functions react to changes in their arguments, since this depends on the circumstances of the market and its participants. For instance, if one considers it as plausible that consumers first are relative quick to react to price changes, but then adapts more slowly as time goes by, then $f(*)$ could be of the form $f(*) = (D(p) - x)\lambda$, where $0 < \lambda < 1$.The function form of $f(*)$ depends on the behaviour of the consumers. In the special case $\lambda = 0$, the consumers are perfect creatures of habit, never wanting to leave the current consumed quantum, while if $\lambda = 1$, the whole adjustment lag of the consumers disappear.

In a similar manner, $g(*)$ captures the behavior of the firms, and the function form depends on what degree of adjustment costs the firms face. In the extreme case of no adjustment costs at all, the function will take the simple form

$$g(*) = S(p) - x,$$

meaning that the allocation will simply converge to the supply curve in cases when $\Delta x_S < \Delta x_D$. With adjustment costs, the function could for instance take a similar form of the example given for $f(*)$:

$$g(*) = (S(p) - x)\mu.$$

By altering the functions, many interesting properties can be included. The simple examples I have given assumes the actors to lack memory; they only care about the previous levels of realized quantum. The functions could of course be extended, for instance in such a way that the consumers' speed of adjustment depends on the original level of consumption, and not just on what they recently consumed.

The point is that many properties can be included in the equations, and it is not possible to make a universal statement about how the system will behave quantitatively. This is the reason why I had a non-technical approach when arguing for the stability of the system, and it makes it difficult to say much about speed of adjustment without studying a specific case. In the following sections, I will therefore provide different examples and scenarios of how speed of adjustment towards stability may occur.

6.2 A case of no adjustment costs for firms, and no lags of adjustment for consumers

Now I will see what happens in a market where Assumption 2 does not apply, combined with letting firms have no costs in cutting or increasing their supply. The following relation will apply:

$$\Delta x = \min(\Delta x_D, \Delta x_S) = \min[D(p) - x, S(p) - x]$$

$$\Delta p = c(A)z(p)$$

The price relation is complex because Assumption 3 still applies, and I assume $c(A)$ to take smaller values when $z(p) > 0$ compared to when $z(p) < 0$, because competition is quite fierce.

In this special case, both D^- and S^- will work as drains of convergence, and the speed of the convergence towards equilibrium will only depend on the functional form of $c(A)$ combined with the original allocation A. If A is such that $p > p^*$, then D^- will be reached in one step, if $p < p^*$, then S^- will be reached in one step, and if $p = p^*$, the allocation will reach equilibrium at once. When S^- or D^- is reached, the firms will set price higher or lower respectively, the latter case going faster by the assumption about $c(A)$. Demand or supply will react instantaneously to the price change, and the allocation thus jumps right back to the set without touching Set III, so the result is a stepwise movement along the supply or demand curve towards equilibrium, where the size of each jump depends on $c(A)$.

Below I include three figures, Figure 12-14, illustrating examples of how movements may be with the assumptions made. Figure 12 is similar to Figure 10-11, while Figure 13-14 show the same relationship, only with focus on quantum and price movements respectively, with time along the horizontal axis. All three figures includes one case where $p > p^*$ and one where $p < p^*$ marked with the label "Path A" and "Path B", and in both cases $x > x^*$ originally. To understand how the original allocation occurs, think of it as first being an equilibrium, but then a shock happens in the economy causing both the supply and demand curves to jump so that the allocation suddenly is in Set I.

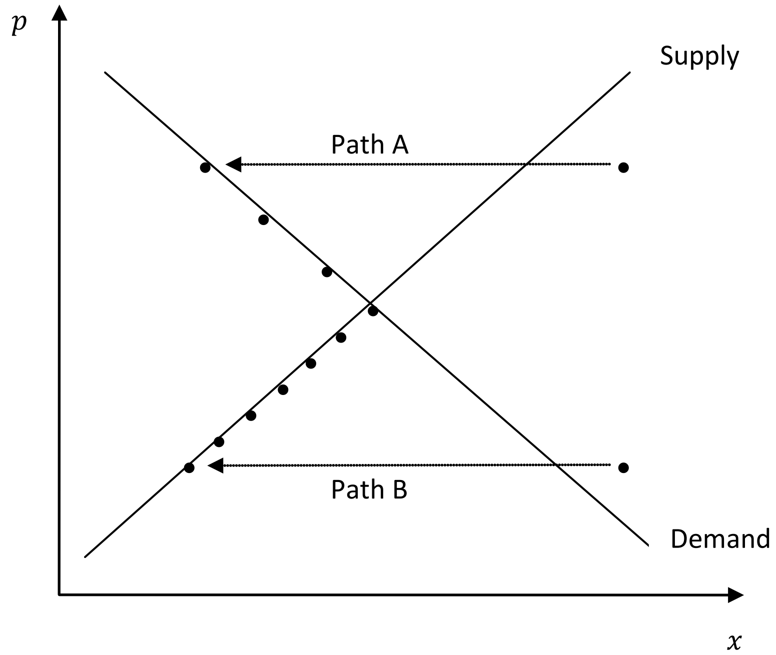

Figure 12

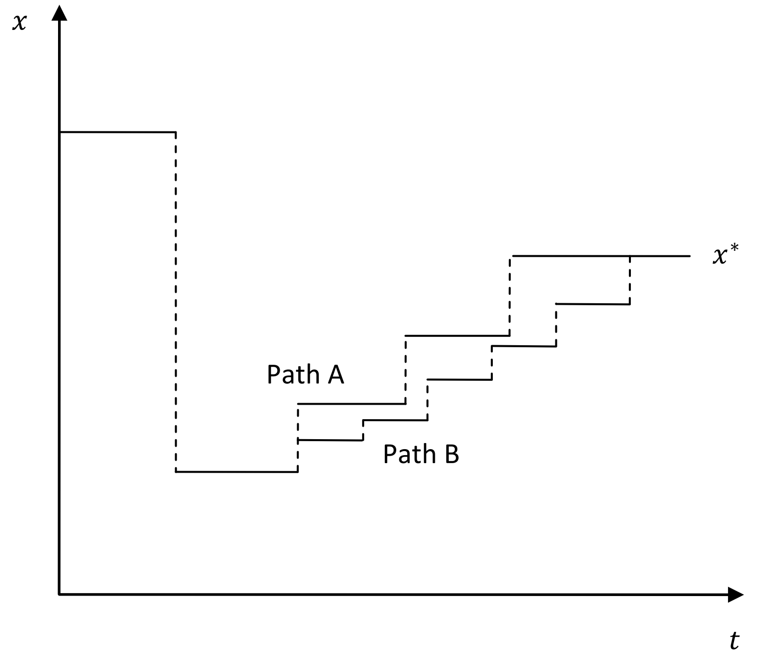

Figure 13

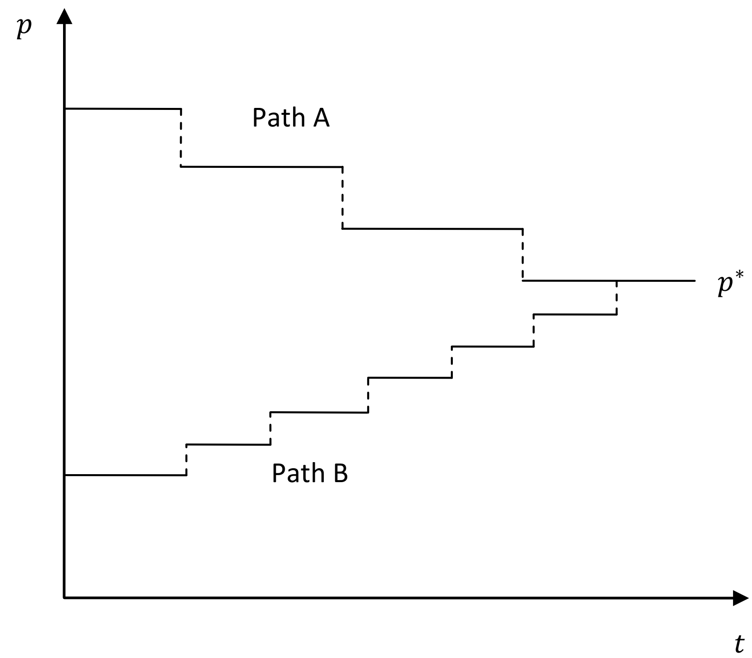

Figure 14

Note that through the assumption on $c(A)$, it takes longer time to reach equilibrium when $p > p^*$.

6.3 A case of no adjustment costs of the firms, but lags in the consumers' adjustment

In this case, Assumption 2 applies, but the firms have no adjustment costs in changing their supply level. I choose to use the case of $f(*) = (D(p) - x)\lambda$, $0 < \lambda < 1$, meaning that the consumers' adjustment will go faster far away from the demand curve relative to close to the demand curve. The difference equations will take the following form:

$$\Delta x = \min(\Delta x_D, \Delta x_S) = \min[(D(p) - x)\lambda, S(p) - x]$$

$$\Delta p = c(A)z(p)$$

How fast the system will move towards equilibrium depends on the size of λ and $c(A)$. The consumers will have more lags in their adjustment to low values of λ relative to high values, and the price movements are slower to low values of $c(A)$.

Just as with the case of section 6.2, I will use three figures (Figure 15-17) to illustrate movements based on different kind of circumstances. This time, the starting allocation is in

Set I above p^*, and I will draw three different movements where the follwing relations will be implied:

- Relation 1: High $c(A)$ relative to λ.
- Relation 2: Low $c(A)$ relative to λ.
- Relation 3: Low $c(A)$ relative to λ in Set II, and opposite in Set III.

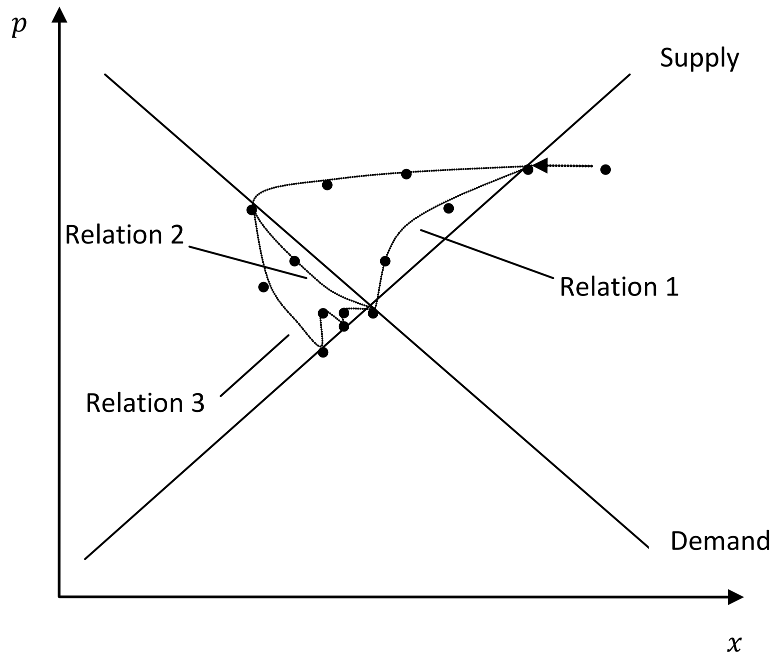

Figure 15

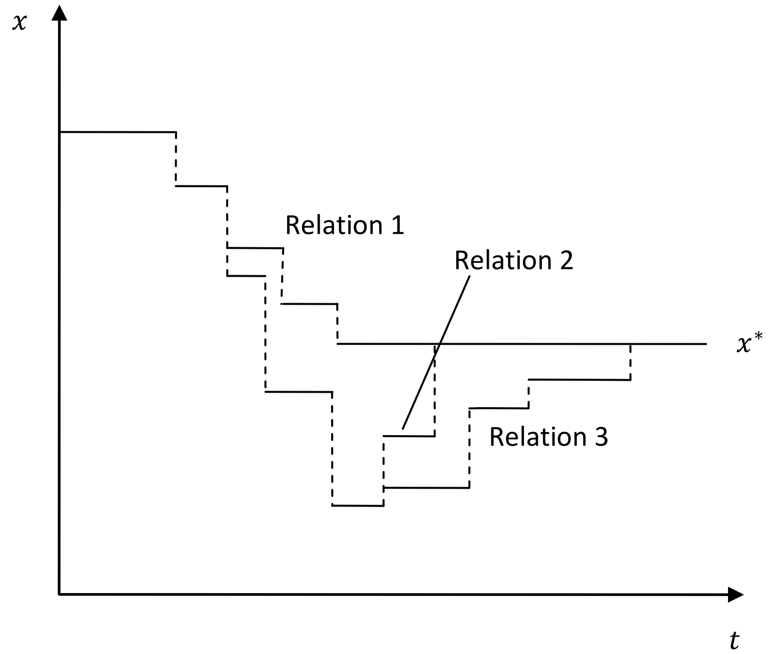

Figure 16

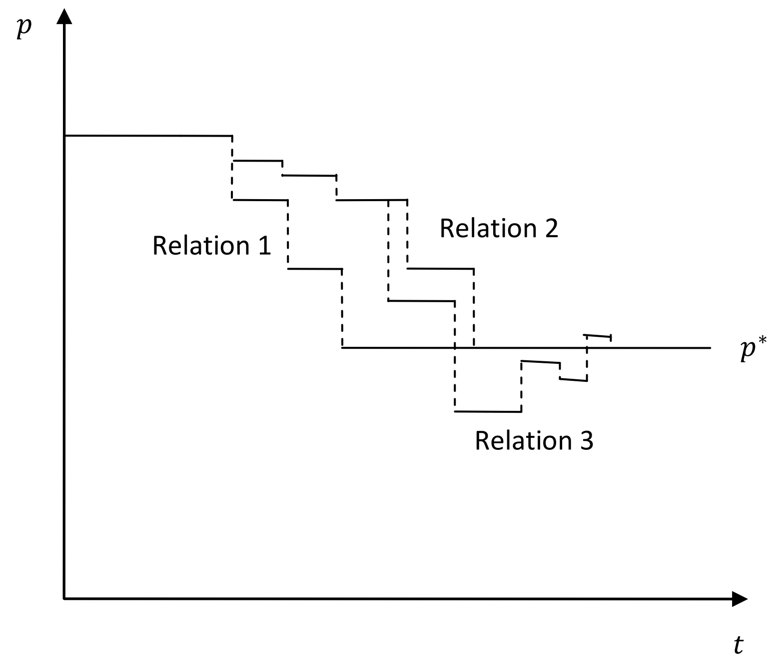

Figure 17

It can be seen that in the first relation the combination of λ and $c(A)$ works in such a way that the path will move directly towards equilibrium. Even within this path however, the speed of movement may vary depending on how the combination is, which makes it difficult to say something about how long time it takes until equilibrium is reached even when the path is known. The other two relations show different ways the system may behave when Set III is reached. Relation 3 is probably the path that will take the longest time to reach the steady state point.

Also, it is plausible to assume that the movements of Relation 2 and 3 will be slower relative to the case in section 6.2, where the consumers had no lags of adjustment at all.

6.4 A case of firms having relatively high adjustment costs

I will include one last case, this time with the firms having such high costs when adjusting their supply level that they aim for a path leading directly to equilibrium if they can. This implies a low value of μ, and the difference equations will take the following form:

$$\Delta x = \min(\Delta x_D, \Delta x_S) = \min\,[(D(p) - x)\lambda, (S(p) - x)\mu]$$

$$\Delta p = c(A)z(p)$$

The starting point will be in Set I bellow the p^* level, and two paths will be used as examples, one going directly to equilibrium, and the other having relatively higher levels of λ and/or μ such that Set IV is reached first. See Figure 18-20 for possible movements.

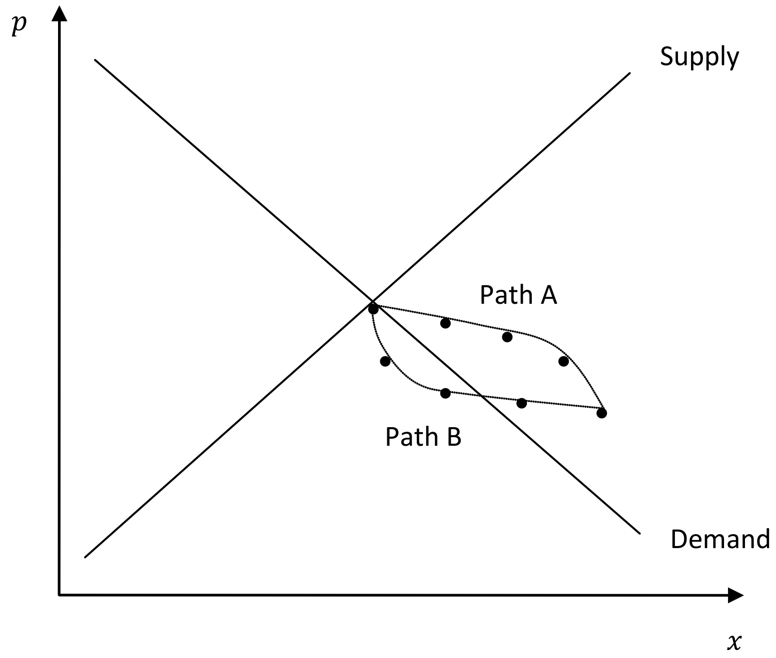

Figure 18

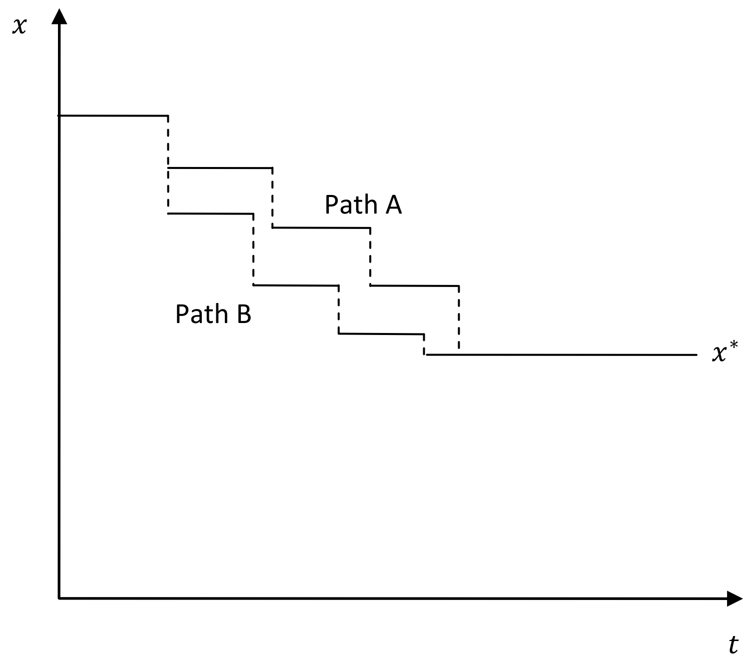

Figure 19

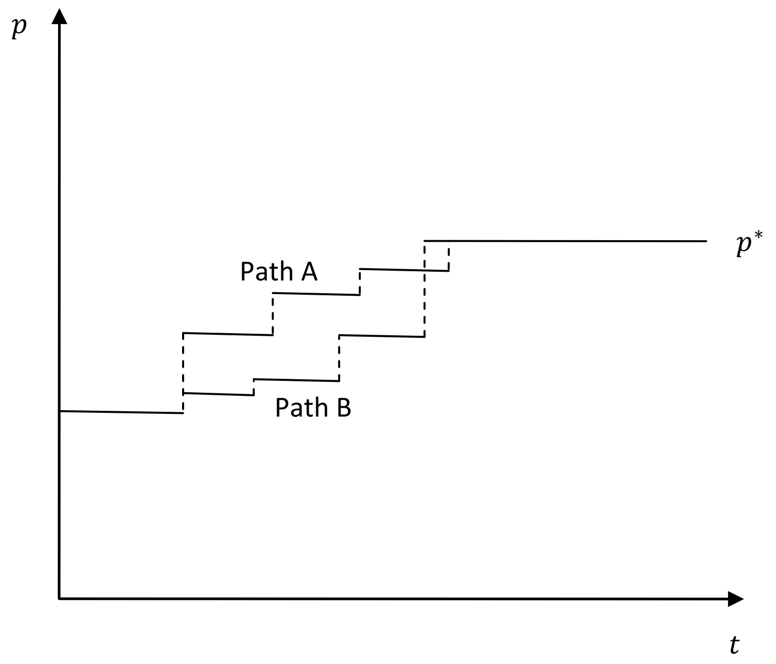

Figure 20

6.5 Concluding remarks

In this section I have included many different paths of how movements towards the equilibrium may occur. The major point has been to underline the fact that there exists no general path the system will take, and that how the movements go and at what speed depend on the relative values of λ, μ and $c(A)$, or even more genereal, depend on the forms of the functions $f(*)$, $g(*)$ and $c(A)$. All these functions depend on the nature of the market, the firms and the consumers in consideration.

7. Interesting implications in an expanded context

So far the focus has been on different ways the system may behave when moving from one equilibrium point to another after an exogenous shock, and the approach has thus been to look at long term behavior in a short term context. What I mean by this is that the environment is assumed to be static when studying movements over time, and the analysis ignores the possibility of the economic environment being exposed to new exogenous shocks in the process of movement towards equilibrium. In reality shocks may occur any time, often or rare, depending on how stable the market of interest is, and it is highly plausible that new changes of allocation of the demand and supply curves happen before the system reaches the steady state point. If this is true, it can be very difficult to identify the allocation of equilibrium, at least for markets experiencing frequent shocks, and although there is a theoretical short term stable point the system moves towards, the market can still be subject to long term instability. In the following part of the paper, I will discuss some implications of exogenous shocks under transition.

In a market with frequent shocks, it may be very difficult to identify the equilibrium point in the context of this model. There is for instance no reason that anybody will be aware of the demand curve's allocation, as it is likely that not even the consumers themselves are aware of their individual demand curve, they only recognize what amount of consumption they want of a certain good for the current price. A person trying to analyze the market can only observe the realized amount consumed, the price level, and maybe the amount the firms supply. In cases of excess demand the true value of $z(p)$ is not observable, but indicators of its level may excist. In Figure 21-23 below, I show an example of movements of market allocations where the equilibrium point is in different random positions due to frequent shocks. I use a case of relatively high adjustment costs of supply change facing the firms, and consumers also adjusting slowly. The stipulated line is what I call the "equilibrium path", showing the unobservable movements of the steady state point towards which the system will move in the long run, but maybe never reach before it changes allocation, and the solid curves show the true movements of realized market quantum and price. In this example with slow adjustments, the solid curve is close to being continuous, and will be so drawn to simplify. Along the curve I include information about which set the allocation resides in, and mark the places where new shocks occur by notation such as "A→B" (change from equilibrium A to B). The long run demand and supply curves are excluded, as the figures would become too messy if these curves were included.

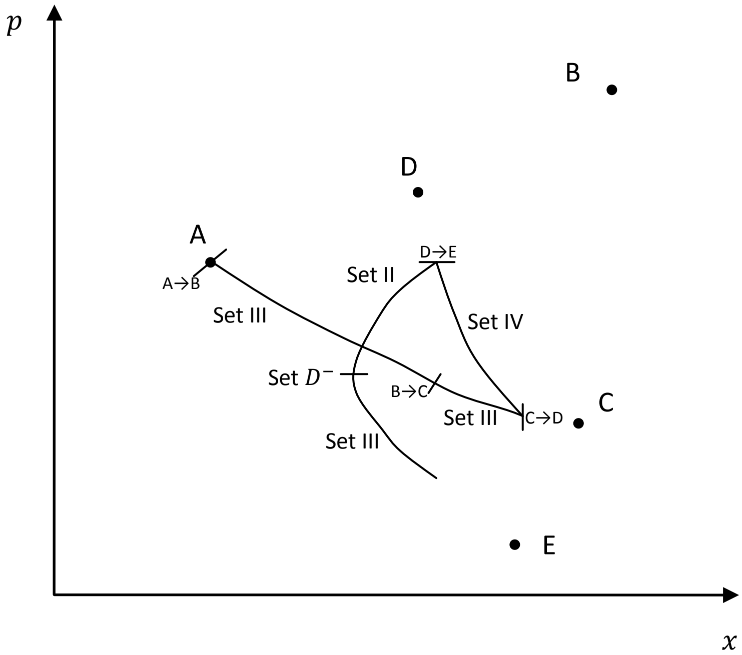

Figure 21

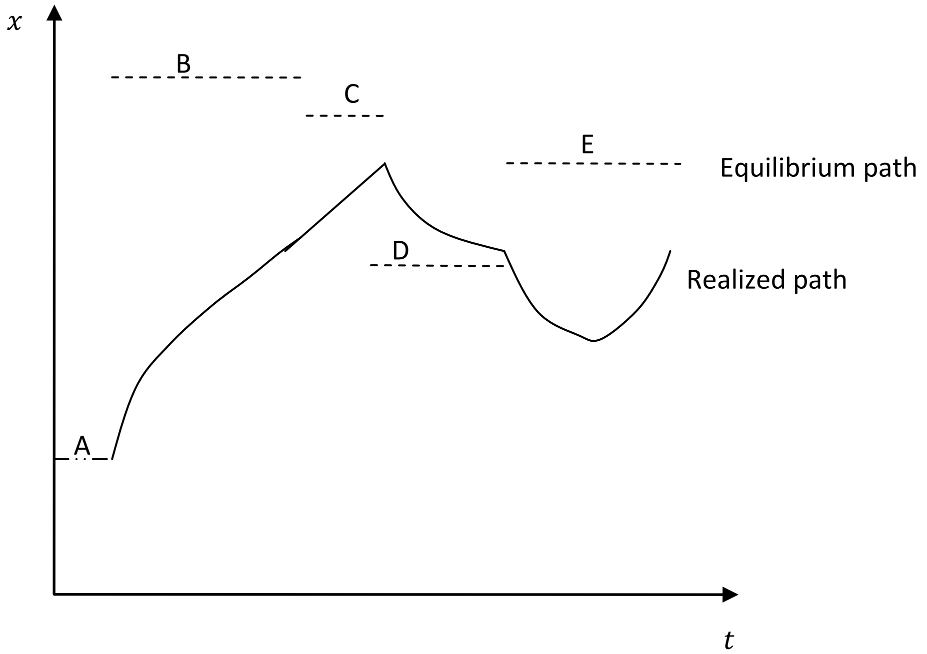

Figure 22

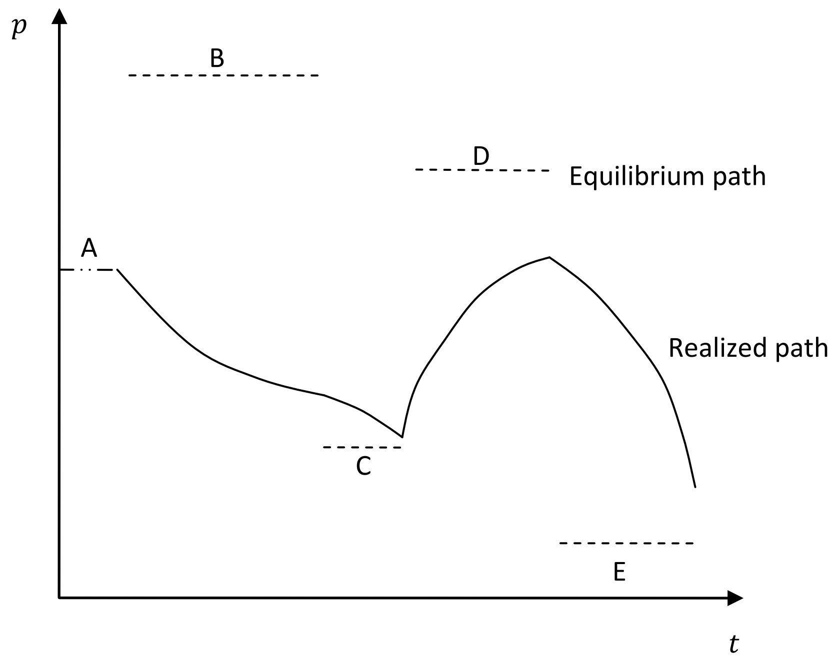

Figure 23

From this example a number of interesting elements can be noticed:

- At some time intervals, the distance between the current allocation and the equilibrium point may grow larger, even though the long run behavior will be movements towards this point.
- Due to the ”overshooting” property of the system, in the sense that price or quantum may move away from equilibrium before getting closer, an analyzer risks loosing parts of the whole picture. For instance, after the shift of equilibrium from point A to point B, price level will be higher, but the movements of price start going in the opposite direction. Then a new shock occurs bringing the steady state to point C, and the movements keep being downward towards this point. In this process, the analyzer may totally miss out the fact that equilibrium B existed, as the price movement towards this point will not be revealed unless Set D^- is reached. A similar case with quantum can also occur, for example if a shock happens in the movements from point D towards point E before Set D^- is reached, and the next equilibrium causes the quantum to continue being reduced.
- Note however, that in the framework of this model, it is always possible to identify when being in Set III, as it is the only set where price and quantum go down at the same time. The same holds for Set II, which is the only set where quantum goes down as price also goes down. More uncertainty is surrounding Set I and Set IV (but not in the case of Set IV where quantum grows due to firms facing adjustment costs). The special ”drain of convergence” around Set S^- and Set III is also easy to identify when it occurs.

- If shocks happen frequently enough, the system may never be in equilibrium.

8. Conclusion

A major conclusion of this paper is that given the circumstances, the system will always converge to the equilibrium point in the long run, but if shocks occur, the market allocation of price and realized quantum of the good may very well remain in disequilibrium. Furthermore, as is clear from section 6, it is not possible to make a precise general description of how the movements in disequilibrium behave quantitatively and related to speed of convergence to equilibrium, but qualitative conclusions are more easily drawn. In order to do a more precise quantitative analysis, a certain market must be chosen, and estimation of the difference equations describing movements has to be done through empirical data.

Extensions of the model can also be made. For example it could be interesting to see how the system changes in the context of a monopoly market, or explore what happens with a market where the supply or demand curve is either horizontal or vertical.

As noted in the introduction, the model in this paper is of a speculative nature, and empirical research would be an interesting next step to see if it has any reliability. Such research could be challenging, however, for instance because it can be difficult to separate movements related to the current location of the demand and supply curves from movements due to changes of these curves' location. Additionally, the fact that this paper studies only a partial market may cause problems. If the model nevertheless turns out to have a value in explaining some real world markets, it would be a good tool for understanding and predicting short term movements of market allocation.

9. Appendix on Assumption 3

In this section I will discuss Assumption 3, which states that the firms are not perfectly equal. First of all, one has to make clear what the meaning of “equal firms” really is, here are some suggestions of what the interpretation of equality may be:

- Equality of technology, or internal equality, which means that Firm A and Firm B face the same cost function, but the consumers may still regard their products x_A and x_B as different due to image etc.
- Equality of products, or external equality, which means that Firm A and Firm B face different cost functions, but that the consumers regard their products x_A and x_B as perfectly equal.
- Perfect equality, which means that both internal and external equality operate at the same time.
- Ultimate equality, which means that Firm A and Firm B is the same firm, and thus that x_A and x_B is the same product.

To be more pedantic, one can also distinguish between how goods can be equal. Here, it is also possible to distinguish between external equality, meaning differences in how the goods are *perceived* by the consumers, and internal equality, which would somehow mean that the goods are equal in function or *nature*, but are still perceived as unequal due to “vain” consumers. One important point of the model in this paper is that firms can have small differences in their price level due to differences among them, and in the end the only relevant property for the possibility of setting unequal price levels is that the consumers perceive the goods as different. Because of this, I will only look at different kinds of external equality of goods. Examples are:

- Dependent equality, meaning that if goods x_A and x_B are sold under the exact same circumstances (place, time, amount of announcement etc.), the consumers are perfectly indifferent to them.
- Independent equality, meaning that the consumers are perfectly indifferent to goods x_A and x_B no matter under which cirmumstance they are sold.
- Ultimate equality, meaning that the goods x_A and x_B are perceived as equal, and that the circumstance under which they are sold are perfectly equal. For most goods this simply means that x_A and x_B is the same good, because two objects cannot be at the exact same location.

These are only extreme cases, and in reality approximations are sufficient. Two goods may have small differences in circumstances, but not large enough for the consumers to distinguish between them.

There are many different ways that the circumstances of goods differ, but in this paper I assume that consumers are enough influenced by the marketing of firms to make them perceive otherwise similar goods of firms as unequal. Thus,

Assumption 3 ⇒ Good perceived as unequal,

in the context of dependent equality. Furthermore, regarding the differences of firms, it does not matter for this paper whether this counts as internal equality or not, only external equality is relevant. The relation thus becomes:

Firms in the market do not have external equality ⇒Assumption 3.

Assumption 3 ⇒ Goods perceived as unequal.

Goods perceived as unequal ⇒ Room for small differences in price level.

If we neglect Assumption 3 an intriguing situation occurs concerning the price movement equation $\Delta p = c(A)z(p)$ (this equation is introduced in section 6). Since all firms are equal, so are their supplied goods, which means that consumers only will buy the goods from the firm or firms setting the lowest price level (assuming the consumers to be rational). Thus, in a situation such as in Set S^{-} where $\Delta p > 0$ is the case, the mechanisms behind these driving forces collapse, because there is no way for the firms to raise their price level. If some firms want to marginally increase their price level, they will lose all customers, and thus no firm can ever raise their price level. An exception would be if they cooperate, but that is not a part of this model.

With this argumentation I will give a comment on what is written on page 621 in "Microeconomic theory", where a statement about a relation similar to $\Delta p = c(A)z(p)$ is made:

"(…) is best thought of not as modeling the actual evolution of a demand-and-supply driven economy, but rather as a tentative trial-and-error process taking place in fictional time and run by an abstract market agent bent on finding the equilibrium level of prices (…)"

This sentence is related to the question of which agent is in charge of prices, and it is further stated that the time in the differential equation (they use continuous time) cannot be real time since a disequilibrium p cannot be compatible with feasibility. My point of view is that what really is fictional is a frictionless world, and that it is *because* of friction (for instance through differences in firms' products) that $\Delta p = c(A)z(p)$, or its continuous time equivalence $\dot{p} = c(A)z(p)$, may apply also in the real world.

10. References

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