Models of Conflicts

by

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Preface

This work has taken many turns since I first started thinking about it in the early Autumn 2009. For good advice and stimulating discussions at especially the later of those turns I thank my supervisor Kalle Moene.

I am grateful to the Consortium for Research on Terrorism and International Crime for providing me with a monthly stipend and the opportunity to participate in their interesting seminars, and the Norwegian Institute of International Affairs for giving me access to their resources and my own desk. At NUPI, I thank John Kristen Skogan for his frankfulness in discussions, and Nina Græger for her efforts to make me feel welcome in the Department of International Politics and at the Institute in general. It is surely my own responsibility that I did not exploit more all the expertise that is gathered there.

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Summary

What determines the relative power of opposed groups? Power, in the sense of the ability to impose one’s own preferred outcome, is often treated as an exogenous property that actors simply “have” or “do not have”, but it may also be viewed as determined by underlying variables. Formal models of conflict take the latter approach: The power of an actor is seen as the equilibrium probability that the group will win a prize for which it is competing, with this win probability being a function of the resources that are spent by the groups in the fight.

In this thesis I survey the literature on formal models of conflict and their determination of power. I go through various important dimensions of the models and highlight central findings. I also look at some applications, and I make use of the violent political situation in Somalia to illustrate the explanatory potential of the models.

Chapter 2 is a survey of formal models of conflict. I start out with a discussion of models of contests and conflict in general. I then present a basic conflict model to illustrate some important concepts and fix ideas about what a formal contest is. Since this part establishes a baseline for a more advanced understanding, it will be quite thorough and technical material will be explained in some detail. I further discuss how to incorporate the widely employed feature of production into the basic model, before I move on to provide an overview of various extensions to this simple set-up that have been proposed in the literature.

Chapter 3 is a closer look at applications of these models and challenges they face. I use the violent political situation in present-day Somalia as an illustration of an application of a model. The model does capture some very broad traits of the actual conflict, but more careful empirical work is needed for offering policy recommendations.

Chapter 4 suggests several ideas for future work.
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1 Introduction

What determines the relative power of opposed groups? Power, in the sense of the ability to impose one’s own preferred outcome, is often treated as an exogenous property that actors simply “have” or “do not have”, but it may also be viewed as determined by underlying variables. Formal models of conflict take the latter approach: The power of an actor is seen as the equilibrium probability that the group will win a prize for which it is competing, with this win probability being a function of the resources that are spent by the groups in the fight.

In this thesis I survey the literature on formal models of conflict and their determination of power. I go through various important dimensions of the models and highlight central findings. I also look at some applications, and I make use of the violent political situation in Somalia to illustrate the explanatory potential of the models.

A better understanding of the elements of power struggles, which in their violent forms ravage countries and destroy lives, is of obvious value. Further, the resources spent in the struggles typically have other uses, making competition a source of a public bad rather than good. One recent report estimates the level of military spending in 2008 at approximately 2.4 % of global GDP, with a much higher percentage of GDP in many countries involved in open conflict (SIPRI, 2009). That is without including the destructive and growth-reducing effects of violent conflict and other conflict-related spending, nor, on the other hand, the benefits of the spending. Studies of the costs of specific conflicts have also been conducted: In a critical review of existing methods for estimating the economic costs of conflict, De Groot et al. (2009) present a number of studies of the cost of the long-lasting conflict in Sri Lanka, the most comprehensive of which estimates that the conflict from 1983 up to 1996 may have cost up to the double of Sri Lanka’s 1996 GDP (Arumatilake et al., 2001). Although measuring the direct and especially the indirect costs of war is fraught with methodological difficulties (Bozzoli et al., 2008), the figures are vast.

In a tradition going back to Haavelmo (1954), economists and political scientists have modeled conflicts and the decisions therein analytically. This literature explores the implications of not taking the institutions of secure property rights
and the rule of law as given. Without enforced property rights, it becomes im-
possible to write contracts, and cooperation is not easily sustained. In short,
the problem the agents face is the decision to allocate resources to production of
regular utility increasing goods, e.g. consumption goods, or appropriative goods,
which are used for grabbing from someone else or defending what one have. All
agents are assumed to maximize their expected utility taking factors such as
their own and the opponent’s resources, the relative numbers in the groups and
the technology of conflict into account. These factors then constitute the deep
parameters on which power rests.

After being dormant for a long time after Haavelmo, the field was revived from
the late 1980’s and onwards. Hirshleifer (1988) is the earliest modern contribu-
tion, and sets out the basic elements of production and contest success functions
(‘combat power functions’ in Hirshleifer’s terminology) and explores two different
contest success functions and three classes of solution concepts/equilibria. The
contest success functions discussed are of the ratio form, where the win probabil-
ities depend on relative appropriative effort, and the difference form, where the
win probabilities depend on absolute difference in effort. The class of difference
contest success functions is exemplified by the logit or logistic specification, and
may be thought of as an auction where all bidders pay their bid (Konrad, 2009).
This article contains many of the topics that Hirshleifer and others later treated
more rigorously, such as allocation of resources between productive and appro-
priative uses, the role and properties of contest success and production functions,
and relative gains from conflict. Hirshleifer (1989) is a further elaboration on the
differences between ratio and difference contest success functions, emphasizing
the advantage of the difference-form specification in allowing for two-sided peace
and one-sided submission. The ratio-form specification has nevertheless turned
out to be the most widely used, according to a recent book on contests in general
(Konrad, 2009) and an overview of the conflict literature in economics (Garfinkel
and Skaperdas, 2007).

Hirshleifer (1991) takes up a thread from Hirshleifer (1989) about how a less
resourceful party may gain from conflict through spending a larger share of its
resources on fighting, in some cases until the parties are rewarded equally, and so
may end up improving its position relative to the stronger party. Skaperdas (1992)
employs more general assumptions about the technologies and emphasizes the equilibrium win probability as a definition of power. An important difference from Hirshleifer (1991) is the possibility of asymmetrical marginal productivities in production of consumption goods. This results in an inverse relationship between equilibrium win probability and marginal productivity since the party with the lower marginal productivity has a lower opportunity cost for fighting.

Garfinkel (1990) analyzes a repeated two-period game where two countries receive a stochastic income in the first period that they must allocate between present consumption and consumption and arms in the next period, with punishment strategies allowed. The key variable under scrutiny is arms investment in the first period.

The above contributions model a contest between two unitary actors. Grossman (1991) considers a contest between a ruler and a population, the former setting taxes and employing soldiers, the latter producing, soldiering or participating in an insurrection. I will discuss many of these contributions at greater length in chapter 2.

It should be noted that the conflict literature constitutes an exploration of the classical theoretical assumption of pure selfishness - as one of the main contributors to the field has put it: “[A] genuine Homo Economicus would not be restrained from using force if using force were to enhance his material interests” Skaperdas (2006, p. 882). In a conflict, the world is anarchic in the sense that there is no higher authority to appeal to - every agent is on his own.

In general, analytical models make arguments transparent and provide greater scope for empirical testing. Although their abstract forms may seem detached from reality, the models improve our understanding through identifying causal mechanisms and helping give a consistent picture of the world. In addition, they allow for precise definitions of concepts that may be vague in other contexts, like “power”. An even though structural estimation of the parameters of the conflict models may not be feasible, one can obtain valuable insights through comparative statics analysis, and one should not shy away from more direct applications when suitable assumptions are found and the level of aggregation is right. Bates et al. (1998) is a small collection of attempts to do this for some specific historical events, although not employing the contest framework
explicitly. Critiquing this work, Elster (2000) claims that such ‘deductive history’ is close to impossible because the historical record will not be able to provide sufficient detail for application of the actual, fine-grained micromechanisms, but at least when it comes to events that have not already occurred, that is a question to be settled by the best studies.

Formal models are often seen as providing explanations based solely on the material incentives of their actors, but conflict models transcend the greed vs. grievance distinction by being able to incorporate many types of goals. However, the interpretation in terms of material gain does provide an accessible and credible metric in a wide range of circumstances, and going beyond it demands a deeper understanding of the situation. I will take up some of the difficulties with this in the discussion on Somalia in chapter 3.

The conflict framework has often been employed with long-lasting civil conflict in mind. Hirshleifer (1995) discusses how military decisiveness largely has moved towards greater decisiveness through history. A natural extension to that discussion would be that weapons development also has increased the scope for terrorism in asymmetrical conflict, terrorist attacks being a cost-effective way to hurt the opponent for a weak party. Consequently, much fighting effort in the models to be considered in the following may be expected to take the form of terrorism, especially when one of the parties approaches its resource constraint for fighting effort.

The real world conflicts which I take these models to illuminate are cases where two parties with a certain degree of permanence face each other repeatedly. This kind of protracted conflict typically takes place inside states - between a weak state and rebel movements, or in the form of an outright civil war. A case in point is Somalia, where after the fall of the dictator Barre in 1991, a feeble government, various warlords and communities have been consumed in continuous fighting for territorial control in the central and southern parts of the country. I consider this conflict in detail in chapter 3. Another contemporary example is the conflict in Israel/Palestine, while historical cases include the battles for control of cities and territories that took place in much of continental Europe preceding the formation of the modern national states. A historical case further away in time but closer geographically is the fighting between various factions, most notably
“Birkebeinerne” and “Baglerne”, over the throne of Norway in the 12th and 14th century.

There is a large body of quite recent statistical work on various aspects of conflict on the aggregate level that I will not consider in the following. That literature falls outside the scope of this thesis, but it should be noted that it establishes several empirical facts that conflict models should be able to reproduce.

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Chapter 4 concludes and suggests several ideas for future work.
2 Models of Conflicts

2.1 Contests

Formal models of conflict are closely related to contest theory, a subfield of game theory. A contest is a situation where agents spend resources to try to win over others in some way. The competition is formalized by means of a “contest success function”, which gives each participant’s winning probability as a function of the resources allocated to contest effort by all the players. The total amount of resources spent on contest effort, a key variable, is referred to as “dissipation”. The dissipation is often interpreted as waste or the intensity of the conflict. The agents may be individuals, groups, countries, or other units, resources may for instance be constituted by money, time, or natural resources, and prizes and the concept of winning may likewise take a multiplicity of forms.

Accordingly, the contest approach is very general, and it has been used to model such seemingly diverse phenomena as rent-seeking (Tullock, 1980), advertising (Schmalensee, 1983), elections (Congleton, 1986) and athletic competition (Szymanski, 2003). The last two decades have seen an increase in applications to political or violent conflict, which is what I am focusing on.

2.2 Conflicts

In the generic conflict situation, the agents involved each exert effort on appropriative activities that influence their probability of winning. The relationship between effort and winning probabilities - the “technology of conflict” - is represented by the contest success function. The probabilities produced by the contest success function usually denote either the probability of winning the whole prize (winner-takes-all contests), or the share of the prize received. It is common to let the probabilities range freely on the interval [0, 1], but restraining them to take only discrete values, for example only 0 or 1, is possible. In any case, the equilibrium values of the probabilities provide a precise definition of relative power.

The cost of effort is the value of the resources spent. This opportunity cost of fighting is either modeled explicitly by specifying the cost structure of effort, like in the baseline model below, or as emerging endogenously as in models with
production, which I will consider in section 2.4. Spending on appropriation may include both offensive and defensive measures. Moreover, what is spent on appropriative activities may be interpreted as the amount of fighting, or violence, that takes place. Although most models emphasize the consumption of quite standard goods as the alternative, this need not take such a narrow form.

Although I emphasize conflict, it should not be forgotten that there are good reasons for adversaries to reach a settlement, for instance that they then do not have to bear the additional destructive costs of open conflict or suffer from lowered utility by having to face the risky outcome of battle (if being risk-averse). Fearon (1995) particularly stresses the incentives for settlement and the demands they place on any rationalist explanations of war.

A general word of caution is in order before embarking on the models: As noted in connection with Hirshleifer (1989)’s discussion of contest technologies in the introduction, some results may hinge on the functional forms assumed for utility, contest success, revenue collection, etc.

2.3 The baseline model

To introduce the basic framework and the most important concepts of conflict models, this section presents a simple model of a conflict between two parties that are both after a prize that only one can obtain. I show how the optimizing behavior of the parties lead to an equilibrium where the values of the key variables depend on the parties’ valuations of the prize and the parameters of the conflict technology.

One can interpret the prize as control of a specific geographic area that may be a source of income or be valuable for other reasons. Both parties are supposed to be able to use resources to increase their chances of winning the prize. Interpreting the agents as two groups with military resources and the resources spent as the number of soldiers employed in a battle over the area, we see that this set-up can be seen as a very simple representation of a military conflict.

Even though there is only one time-period in the model, one may think of the battle as taking place over a somewhat extended period of time. The players in any case do not consider what happens after the battle is over. The players
make their decisions on how much to fight simultaneously, i.e. they have no way
to observe how many soldiers the adversary plans to take to the battle. The
solution concept in this section will be that of Nash equilibrium.

Whenever the expressions for the two players are symmetric in an obvious
way, I often show the case for player 1 only.

The most common functional form for the contest success function is the “ratio
form” (or “power form”) - an intuitive and analytically tractable specification. In
the two-player case, the standard ratio form specification gives agent 1’s winning
probability as

\[
p_1(F_1, F_2) = \begin{cases} \frac{F_1}{F_1 + F_2} & \text{if } F_1 + F_2 > 0 \\ 1/2 & \text{otherwise} \end{cases} \tag{1}
\]

The probability is a function of the two fighting choices \(F_1\) and \(F_2\). We see that
a group’s power, the probability that that group gets its way, depends on the
relative level of force employed. However, the amounts of force actually employed
are not given, but depend on the attributes of the groups and the environment in
which they operate. We shall see below how they are optimal choices on behalf of
each group. The parameter \(m\), often referred to as the “effectiveness” parameter,
plays an important role in describing the conflict technology. Higher values of
\(m\) imply stronger effects of differences in fighting effort. \(m = 1\) is an often used
benchmark case in which the winning probability of an agent is simply equal to
the agent’s share of total fighting effort.

A cost function \(C\) describes the cost of fighting:

\[
C_i = C^i(F_i), \quad i = 1, 2. \tag{2}
\]

An assumption usually made about the cost function is that it is simply a linear
function of fighting with unit cost equal to or normalized to 1:

\[
C^i(F_i) \equiv F_i, \quad i = 1, 2, \tag{3}
\]
a convention I will follow. In that case, the terms “the fighting” and “the re-
sources spent on fighting” become interchangeable. A natural extension would
be to require the cost function to be convex. Another could be to include inter-
dependencies between the contestants’ costs.
Assuming that the players are risk neutral, player $i$ chooses $F_i$ to maximize the payoff
\[ \pi_i = p_i v_i - F_i, \quad i = 1, 2, \]  
where $v_i$ is the valuation of the contested prize of player $i$. When each player solves this maximization problem taking the other player’s choice as given, and assuming positive efforts, we obtain the optimal effort choices as implicit solutions to the first order conditions
\[ \frac{v_i m F_i^{m-1} F_j^m}{(F_i^m + F_j^m)^2} - 1 = 0, \quad i = 1, 2, \quad i \neq j. \]  
These equations define each player’s best response to the other player’s choice. Combining them implies
\[ F_j^* = \frac{v_j}{v_i} F_i^*, \quad i = 1, 2, \quad i \neq j. \]  
Using this in the first order condition yields the optimal fighting effort for player 1:
\[ F_1^* = m \frac{v_1^{m+1} v_2^m}{(v_1^m + v_2^m)^2}, \]  
and symmetrically for player 2. We can see that the player with the higher valuation exerts most fighting effort. By differentiating with respect to the valuations, it can be shown that the equilibrium fighting efforts are increasing in both $v_1$ and $v_2$ as long as the valuations are positive.

By inserting the optimal fighting choice of player 1 into the contest success function, we obtain the equilibrium win probability for player 1
\[ p_1^* = \frac{(F^*_1)^m}{(F^*_1)^m + (F^*_2)^m} \]  
\[ = \left( \frac{v_1^{m+1} v_2^m}{(v_1^m + v_2^m)^2} \right)^m \left[ \left( \frac{v_1^{m+1} v_2^m}{(v_1^m + v_2^m)^2} \right)^m + \left( \frac{v_1^m v_2^{m+1}}{(v_1^m + v_2^m)^2} \right)^m \right] \]  
\[ = \frac{v_1^m}{v_1^m + v_2^m}. \]
The player with the highest valuation is the most powerful, as he has the highest probability of winning. In this case this is because the higher valuation of the prize leads him to spend more in the fight. We also see that the higher is $m$, i.e. the more decisive is the conflict technology, the the more powerful is the player.
with the highest valuation. If we can compare the valuations and express one as a multiple of the other, we can also get an expression for the difference in power. Assuming \( v_1 = tv_2 \), a constant describing how many times higher player 1’s valuation is that of player 2, relative power can be written as a fraction of the equilibrium win probabilities:

\[
\frac{p_1^*}{p_2^*} = t^m. \quad (11)
\]

If player 1 values the prize most highly, we will have \( t > 1 \), and player 1 will be \( t^m \) times more powerful than player 2, i.e. \( t^m \) times more likely to win.

The equilibrium expected payoff for player 1 is

\[
\pi_1^* = \frac{v_1^{m+1}(v_1^m + v_2^m(1 - m))}{(v_1^m + v_2^m)^2}. \quad (12)
\]

Continuing with \( v_1 = tv_2 \), \( t \geq 1 \) from above, we have

\[
\frac{\pi_1^*}{\pi_2^*} = \frac{v_1^{m+1}(v_1 + v_2(1 - m))}{v_2^{m+1}(v_1(1 - m) + v_2)}
= t^{m+1} \left( \frac{t - m + 1}{t - tm + 1} \right). \quad (13)
\]

We have seen that the player with the higher valuation brings more soldiers to the battle and thus is the more powerful. Since the expression above is strictly greater than 1, we see that this pays off: The payoff of player 1 is higher than that of player 2, even if player 1 also spends more.

It is instructive to write the equilibrium payoff in (12) as a product of player 1’s valuation and another factor:

\[
\pi_1^* = v_1 \left[ \frac{v_1^m}{v_1^m + v_2^m} - \frac{v_1^m v_2^m}{(v_1^m + v_2^m)^2} \right]. \quad (15)
\]

With positive valuations, we see that the term in brackets, a number between 0 and 1 (actually, the equilibrium win probability) from which is subtracted a positive number, must be between 0 and 1, and thus that the equilibrium payoff is lower than the valuation. Since resources are spent on unproductive activities, this comes as no surprise, but here we have a precise expression for how the payoffs are affected.
We can also look explicitly at the resources spent in the battle: Adding the optimal fighting efforts of the two players gives the dissipation, the total level of fighting or how much is “wasted”:

\[ F = F_1^* + F_2^* \]
\[ = m \frac{v_1^{m+1} v_2^m}{(v_1^m + v_2^m)^2} + m \frac{v_1^m v_2^{m+1}}{(v_1^m + v_2^m)^2} \]
\[ = m \frac{v_1^m v_2^m (v_1 + v_2)}{(v_1^m + v_2^m)^2}. \]

Rewriting this as

\[ F = m \frac{(v_1 v_2)^m}{v_1^{2m} + (v_1 v_2)^m + v_2^{2m}} (v_1 + v_2), \]

we see that the total level of fighting does not exceed the total valuation of the prize \((v_1 + v_2)\) as long as \(m \leq 1\).

In general, increasing the valuation often effectively increases the violence. It of course increases the payoff as well, but if there are externalities in war, such as when the true costs are not taken into account by the decision makers, this may have negative average effects.

Above, I assumed positive levels of effort. It is easy to see that a situation with \(F_1 = F_2 = 0\) cannot be an equilibrium, since both players would then have an incentive to exert an infinitesimal amount of effort and capture the whole prize with certainty in stead of being allocated it for free with a 50 % probability.

What about the case where one of the players, player 1, say, expends zero effort? Player 1 would then get a payoff of zero, and so will choose a strictly positive effort level if the expected equilibrium payoff derived above is strictly positive:

\[ \pi_1^* = m \frac{v_1^{m+1} + v_2^m (1 - m)}{(v_1^m + v_2^m)^2} > 0. \]

This leads to the following condition for positive effort for player 1:

\[ v_1^m + v_2^m (1 - m) > 0. \]

Since both valuations are positive, this condition will at least be satisfied unless \(m\) is higher than 1. In the following, I will assume that the combinations of the
valuations and the parameter $m$ are such that this condition is satisfied, or in other words that it pays to take part in the fight.

Another issue, however, is that positive effort requires positive resources, and more specifically that an optimal amount of fighting requires a sufficient amount of available resources. What happens when one of the players cannot afford his desired level of spending? The player that is not constrained will play according to his first order condition in any case. The above assumption that it pays to take part in the fight implies that the marginal utility of fighting exceeds the marginal cost. This has to hold all the way up to the optimal level of fighting, hence the constrained player will spend all his resources on fighting. An illustration of such a situation is provided as the last special case below.

After summarizing the equilibrium solutions for the general case for ease of comparison, I below show the solutions for some special cases.

The general case

\begin{align}
F_1^* &= m \frac{v_1^{m+1} v_2^m}{(v_1^m + v_2^m)^2} \\
p_1^* &= \frac{v_1^m}{v_1^m + v_2^m} \\
\pi_1^* &= \frac{v_1^{m+1}(v_1^m + v_2^m(1 - m))}{(v_1^m + v_2^m)^2} \\
F &= m \frac{v_1^m v_2^m(v_1 + v_2)}{(v_1^m + v_2^m)^2}.
\end{align}

**Special case: $v_1 = v_2$** The case with equal valuations is effectively the basic rent-seeking model (Tullock, 1980). If there exists a monetary unit of measurement for fighting, equal valuations seems especially plausible when the prize is also monetary, such as in Tullock’s original application, or for instance represents a revenue stream from a natural resource. In the special case where $v_1 = v_2 = v$, the equilibrium values for player 1’s fighting effort, win probability and payoff
and the total dissipation reduce to:

\[ F^*_1 = \frac{mv}{4} \]  
\[ p^*_1 = \frac{1}{2} \]  
\[ \pi^*_1 = \frac{v(2 - m)}{4} \]  
\[ F = \frac{mv}{2}. \] (27) (28) (29) (30)

The players are in perfectly symmetric positions, and end up spending the same amount and being equally powerful. With equal valuations, each individual fighting effort, and hence the total level of fighting, is unambiguously increasing in \( m \).

**Special case:** \( m = 1 \)

\[ F^*_1 = \frac{v_1^2 v_2}{(v_1 + v_2)^2} \]  
\[ p^*_1 = \frac{v}{v_1 + v_2} \]  
\[ \pi^*_1 = \frac{v_1^3}{v_1 + v_2} \]  
\[ F = \frac{v_1 v_2}{v_1 + v_2}. \] (31) (32) (33) (34)

\( m = 1 \) is an often used simplification, and I will indeed use it myself later on.

**Special case:** \( v_1 = v_2 \) and \( m = 1 \) Combining the two previous cases gives:

\[ F^*_1 = \frac{v}{4} \]  
\[ p^*_1 = \frac{1}{2} \]  
\[ \pi^*_1 = \frac{v}{4} \]  
\[ F = \frac{v}{2}. \] (35) (36) (37) (38)

The reaction functions for this last case, i.e. the situation where \( v_1 = v_2, m = 1 \), are shown in Figure 1. The Nash equilibrium is at the intersection of the curves. In this particular case, both players spend 1/4 of the value of the prize on fighting.
and have a 50-50 chance of winning the prize. The resources spent on fighting is half the value of the prize. It is obvious that both parties would be better off by reducing this and instead simply split the prize in two, but that is not a viable option.

![Graph showing reaction functions](image)

**Figure 1**: Reaction functions when $v_1 = v_2 = v$ and $m = 1$

**Special case**: One player resource constrained and $m = 1$. Because of its relevance for the application in chapter 3, I here include the solution of a case where one player have insufficient resources to fight at his optimal level. As seen above, the constrained player will use all he has in the battle, while the unconstrained simply optimizes taking this into account. I assume $m = 1$ for expositional clarity. This yields the following equilibrium, where player 1 is
assumed to be the constrained player:

\[
F_1^* = R_1 \\
F_2^* = \sqrt{v_2 R_1} - R_1 \\
p_1^* = \sqrt{\frac{R_1}{v_2}} \\
p_2^* = 1 - \sqrt{\frac{R_1}{v_2}} \\
\pi_1^* = \sqrt{\frac{v_1}{v_2} R_1} - R_1 \\
\pi_2^* = (\sqrt{v_2} - \sqrt{R_2})^2 \\
F = \sqrt{v_2 R_1}
\]

A situation where one player hits his resource constraint such as this seems relevant in many war situations, where the value placed on winning is so high that one is willing to give everything, and one of the equilibria in the application in chapter 3 will in fact conform exactly to this special case.

2.4 Production

One important topic in the economics of conflict literature has been the trade-off between spending resources on production and spending them on appropriative activities. Production decisions are standard in most models, and was in fact part of Haavelmo’s formulation. Whereas in the model above the agents did not have a way to derive utility other than winning the prize, now production of an alternative consumption good is assumed. Immediately below, I retain the aspect of pure rent-seeking in that there is an exogenous prize for which the players compete, but each player may also produce something on his own. In many conflicts, such as those that are over pieces of land or when there are sharp divides between the parties, a contested object that is independent of production seems like a reasonable assumption.

In the literature, however, a more common way of incorporating production has been to assume that the non-appropriative efforts of the agents are joint inputs in the production of a final good, which in turn is what is in contention. This makes for a general equilibrium case and will be treated in the subsequent
section.

2.4.1 Exogenous prize and production of fighting

The baseline model could have been specified with each agent being endowed with an initial resource $R_i$ to be allocated between production of a consumption good $X_i$ and fighting effort $F_i$. The payoff to be maximized by player $i$ would then be given by

$$\pi_i = p_i v_i + X_i, \quad i = 1, 2.$$  \hfill (46)

If the initial resource could be consumed directly or transformed into fighting effort on a one-to-one basis, we would have

$$\pi_i = p_i v_i + X_i, \quad i = 1, 2$$  \hfill (47)

$$= p_i v_i + (R_i - F_i), \quad i = 1, 2.$$  \hfill (48)

Provided an internal solution, i.e. none of the agents would choose not to fight or spend all available resources on fighting, all the equilibrium relations derived above would continue hold, except that the initial resource would also enter in each player’s payoff.

For a more general case, one could assume that the amounts of fighting effort $F_i$ and consumption good $X_i$ were functions of the initial resource allocated to each purpose:

$$F_i = f^i(R_{F,i}), \quad i = 1, 2, \hfill (49)$$

$$X_i = g^i(R_{X,i}), \quad i = 1, 2. \hfill (50)$$

A natural further assumption would be requiring the functions to be increasing and concave.

2.4.2 Endogenous prize

Most of the conflict literature assumes that the prize is some function of all the players’ productive inputs. This allows for different links between production and appropriation and may be a reasonable assumption in some circumstances, for instance with a mobile prize or very intertwined groups. In these models, valuation of the prize is nearly always equal.
A typical example of this type of specification is Hirshleifer (1991), where each player $i$ allocates his exogenously given resource $R_i$ between productive use $E_i$ and fighting effort $F_i$:

$$E_i + F_i = R_i. \tag{51}$$

The joint production process is given by

$$I = A \left( E_1^{1/s} + E_2^{1/s} \right)^s, \quad i = 1, 2, i \neq j, \tag{52}$$

where $I$ denotes income and $A, s$ are exogenously given parameters. Since the players fight over the whole income, each maximizes

$$I_i = p_i I \tag{53}$$

subject to their resource constraint. For simplicity, he treats the case with $m = 1$. Also, the major part of the analysis deals with the case where there is no complementarity between the productive efforts, i.e. mixing the inputs or not makes no difference. All the income goes to the winner.

With equal amounts of resources, two risk-neutral agents will spend exactly equal amounts in the contest, be equally powerful, and get equal shares of the prize:

$$F_1 = F_2 = E_1 = E_2 = (R_1 + R_2)/4, \quad p_1 = p_2 = 1/2. \tag{54}$$

This is not very surprising, but the same results obtain also in the case with unequal amounts of initial resources, a finding that will be discussed in the following section.

### 2.5 Resource inequality

Warring parties are often very unequal when it comes to the resources that they are able to spend. Many civil wars, for instance, are fought between a government benefitting from a state apparatus and support from abroad and a relatively poor insurgency.

To introduce the topic of resource inequality, I will continue with Hirshleifer (1991). As we have seen, two parties allocate their resources between productive effort and fighting effort, and the income generated by the productive efforts is what is in contention, so we have the case of an endogenous, common prize.
side is assumed to have more initial resources than the other. When the resource disparity is not too large, securing an internal solution, the optimal fighting efforts, and hence the winning probabilities and the payoffs, remain equal. One way to view this result, which Hirshleifer terms the “strong” form of “the paradox of power”, is that since both sides are risk-neutral, they allocate their resources such that their marginal benefit of fighting is equal. Since the prize in any case is the total production, and it does not matter who produces, the fighting efforts will be equal. Hirshleifer’s interpretation is that is pays the poorer side “to fight harder” (Hirshleifer, 1991, p. 43). As both agents put in the same fighting effort, one should be aware that “harder” is relative to endowment.

The “weak” version of the paradox obtains when the poorer side would want to devote more than its total resources to fighting effort. We then get a corner solution where the poorer side fights with all its endowment. The resulting payoffs are not equalized, as above, but they are less unequal than the initial resources. Since the richer side does not have to consider reactions to fighting levels higher than the other side’s total resources, its need to fight is reduced, and so corner solutions involve lower dissipation of resources than internal equilibria for a given total of initial resources.

When there is complementarity between the productive efforts, meaning that it is beneficial to have productive effort from both, the results change somewhat. With the type of complementarity considered by Hirshleifer, only internal equilibria obtain. More income is actually produced. The richer side fights more, but the poorer side more than weighs up this by fighting less, so total fighting effort is reduced. Both gain absolutely, with the richer side gaining the most, but the weak paradox still applies - resulting incomes are less unequal than initial resources. The preceding analysis has assumed the decisiveness parameter $m = 1$. Improvements in fighting technology, which amounts to an increase in the $m$, as usual lead to greater dissipation of resources, but the richer side benefit absolutely as long as the disparity is large enough. As $m$ continues to increase, the paradox of power gradually vanishes.

Skaperdas and Syropoulos (1997) consider also differences in productive efficiency, and expands on the counterintuitive result that the agent with the lower marginal productivity actually receives a higher equilibrium payoff share than
the more productive agent, which is due to each agent exploiting his comparative advantage. The less productive agent produces more guns and therefore in effect is the more powerful. The more productive agent is aware of this, but his optimal response is nevertheless to produce somewhat fewer guns.

Esteban and Ray (1999) generalize the aspect of inequality by assuming a distribution of individual characteristics in the society. It is important to note that the characteristics stand in a one-to-one relationship to preferences, and need not have any connection to individual resources, although they may be interpreted as pertaining to income, as some of the authors’ examples suggest. For all individuals there is one outcome that is strictly preferred to all other outcomes, but in contrast to the standard specification, they also have a preference ordering over which of these other outcomes obtains. Individuals with the same strictly preferred outcome are assumed to form an interest group and expend contest effort together. The model yields few general results, but a larger difference between groups and the presence of two opposed groups of similar size both tend to increase conflict. Groups of equal size are associated with local maxima of conflict (note: not necessarily global), and fewer, more similar-sized groups tend to increase conflict.

2.6 Fighting asymmetries

2.6.1 Ruler and ruled

The parties in a conflict may differ in other important respects than initial resources. In the preceding section I referred to as resource inequality as a possible consequence of a conflict between a government and rebels. Grossman (1991) goes beyond this and models the two sides specifically as a ruler and a population of ruled that have different institutional roles, and very different options available to them.

The actors are one ruler and many peasant families, essentially a representative household. The ruler taxes the peasants and employs some of them as soldiers to deter insurrection, while peasants choose between producing, soldiering, and participating in an insurrection. An equilibrium in a one-shot game consist of an allocation of resources (time), a probability distribution of income for the ruler
and the peasant families, and a probability for a successful insurrection. Grossman utilizes a contest success function that is somewhat non-standard, and also specifies that the ruler moves first.

An important result is that relatively inefficient soldiering technology increases soldiering and insurrection activity and decreases production, but nevertheless may make peasants better off (absolutely) than in the case with more efficient soldiering technology and higher production. Political instability, in the sense of relatively much time allocated to insurrection and a relatively high probability of a successful insurrection, is here bad for the ruler, but to the advantage of the peasants.

2.6.2 Offense vs. defense

In most models, “fighting” describes all activities related to violent conflict. Grossman and Kim (1995) introduce a distinction between offensive weapons and defense/fortification. By distinguishing between offense and defense, the model admits analysis of nonaggressive equilibria - i.e. equilibria where spending on defense deters predation. The authors remark that even though few people spend resources on predation, most spend at least some on defensive measures. The contest success function for player $i$ is given by

$$p_i = \frac{h_i}{h_i + \theta g_j}, \quad i \neq j,$$

where $h_i$ and $g_i$ denote defensive and offensive measures, respectively. $p_i$ is here the fraction of player $i$’s endowment, which may be used for production for direct consumption, that is retained after the battle. We see that the more player $i$ spends on defensive measures, the more he is able to retain, while, on the other hand, the more the other player, $j$, spends on offensive measures, the more that player is able to take away. The entire endowments, which is now allocated to three uses, consumption, defense and offense, are in contention. The players do not simply maximize the amount they retain, but trades it off against the possibility of appropriating the property of the other - in effect, the model has an endogenous determination of property rights, whose strength comes in degrees.
Nonaggressive equilibria, i.e.

\[ g_i = g_j = 0 \]  
\[ p_i = p_j = 1, \]

are feasible when offensive weapons are sufficiently ineffective (\( \theta \) is sufficiently low), or predation have large destructive effects (the amount that is actually received by the appropriator is much less than the amount that is lost by the defendant). The strength of property rights depends positively on the effectiveness of defense relative to offense, and the destructive effects of predation, with nonaggressive equilibria being feasible at sufficiently high levels of these parameters. While secure property rights are associated with higher welfare for the richer agent, the poorer may benefit from weak property rights, since he may then get his hands on more productive capital.

In equilibria involving predation, the poorer agent has a higher ratio of expected income to endowment than the richer, reminiscent of the paradox of power in Hirshleifer (1991).

### 2.7 Risk aversion

With risk-neutral agents, the distinction between the values of the contest success function being probabilities of winning the whole prize and shares of the prize in many cases does not matter. Risk aversion is a well supported empirical fact, however, and there have been several attempts to incorporate it into contests. Risk aversion has been proposed as a solution to the “Tullock paradox”, the prediction, counter to experience, that the full prize is dissipated in the limit with no barriers to entry.

There is a natural inclination to think that aversion to risk should lead to less fighting, but the effects of introducing risk aversion depend on the model set-up. Skaperdas and Gan (1995) see contest effort as a form of insurance, and find conditions under which the more risk averse player in a two-player game expend most effort and consequently receives the prize with the highest probability, or in other words where risk aversion leads to power.

A problem with risk aversion in contests is that for contests with multiple risk-averse players, the best-response functions quickly become intractable. Cornes
and Hartley (2003b) avoid such complications by considering share functions, which give the optimal win probability for each individual as a function of aggregate effort. The authors manage to show that risk aversion is not enough to achieve less than full dissipation in the limit when the number of players become large - one needs also some conditions on the contest technology.

2.8 The Future

Most conflicts take place over long stretches of time and involve repeated clashes between adversaries, yet time does not enter in the basic model. Hirshleifer (1995) changes this by demanding that the resources of the players themselves be in contention and determined as equilibrium outcomes. The actual one-shot game that is analyzed is seen as one in a continuing series of battles. For a solution to be stable, the decisiveness of conflict should not be too high (‘dynamic stability’), and the income must be high enough to support survival of both parties.

Another way to consider the future is of course to specify a dynamic game explicitly. Axelrod (1984) popularized the result that a strategy of cooperation may be sustainable and even evolutionary advantageous in a quite general game with repeated interactions. In that formulation, placing a higher value on the future increases the gain from cooperation because of the waste resulting from war.

In a related analysis, Garfinkel (1990) considers a situation where two countries follow ‘trigger’ strategies (cooperate if the other has always cooperated, do not cooperate if the other has ever deviated) and their resources are stochastic. Whether cooperation takes place depends on the actual draw of the resources and the distribution from which they are (identically) drawn. Empirical implications are that, for a particular realization of the resources, if countries are acting cooperatively, military spending should be negatively correlated with the expected value of the endowment and positively correlated with the variance of the endowment, whereas there should be no such relation if the countries are acting ‘opportunistically’.

However, in a multi-period setting, conflict may shift the initial conditions of subsequent periods, a feature not present in Axelrod’s stationary game or the
related game of Garfinkel. In Skaperdas and Syropoulos (1996) the players have to make their choice of arms in two subsequent periods. The initial resources of the players are exogenously given, but in the second period they depend on the outcome in the first period. In the case of war in the first period, the output is divided according to win probabilities, and the resources available to a player in the second period are specified as a fraction of the player’s share of the output in the first period. Such a set-up increases the importance of the outcome in the first period, strengthening the motive to arm then, and an increasing value of the future may accordingly make cooperation harder to obtain. Credible commitments to abstain from arming are ruled out.

Garfinkel and Skaperdas (2000) extend this analysis by considering the possibility of bargaining, which make it possible to distinguish between settlement with arms and open warfare. A negotiated settlement avoids the destructive costs of war, but still contains arms, which influence the terms of the settlement. In the case of war, the loser from the first period is assumed to start out with no resources in the second period, he is effectively eliminated, so the winner will then not need to worry about fighting. The bargaining rule is assumed to be equal division of the surplus between a situation with settlement and one with conflict.

Key parameters are the destructive effects of war and the share of resources that the winner from the first period retains in the second period. For certain combinations of these, war is preferred (in the first period), while settlement is preferred for others.

Mehlum and Moene (2008) also treat the issue of incumbency advantage in a two-player game. In their infinite game, the party in power collects an incumbency rent and enjoys a cost advantage in the fight, while the challenger fights to become the incumbent and be able to reap these two benefits. Introducing an incumbency advantage to one of the contestants will make the other contestant disadvantaged when in position as a challenger, and thereby increase the value of the prize to both players, in many cases with more fighting effort as a result. This contrasts with the case in a static setting, where the party not getting a cost advantage would simply adjust by fighting less. Although the contestant who receives the advantage will increase his power, in the sense of equilibrium win probability, the ‘positional dynamics’ considered may actually have the perverse
effect of reducing the payoffs for both players. The incumbent and the challenger are in asymmetrical positions, and will typically have different win probabilities.

2.9 Multiple parties, Alliances

Conflict may take place between multiple parties, and alliances are a key ingredient in many real world wars.

The basic models allows for quite straight forward generalization to multiple parties (Tullock, 1980; Haavelmo, 1954). However, a complication arising if individuals are members of groups is that within-group conflict may be an issue. This has been treated in the literature on collective rent-seeking: Nitzan (1991) models the two levels of rent-seeking, but assumes a connection between the effort in the two contests, in the sense that fighting effort within a group equals the group’s fighting effort.

More intuitively, however, the individuals within a group could be engaged in the same type of distributional conflict as the group taken as a whole. Wärneryd (1998) specifies two such levels of rent-seeking contests in a model of how (the inhabitants of) countries or regions may be better off as part of a federal system. Garfinkel (2004) employs a similiar model with production to explore the endogenous formation of the number and size of alliances.

Hausken (2005) models both within- and between-group conflict with a general ratio form contest success function, and systematically compares the comparative statics resulting from adding individuals and groups in pure rent seeking models and models with endogenous production.

2.10 Summing up

What determines the relative power of opposed groups? We have seen a number of features that may be relevant. It is often the case that a feature interacts with the rest of the model set-up and through that has an ambiguous effect. Here I summarize I few that are more clear cut, and that are of particular interest for the application next chapter.

i) Valuations. A high valuation of the prize makes a player willing to spend much in the fight. This may seem tautological, but it does draw attention to
the fact that relative intensity of preferences matters and points to the importance of factors that influence the valuation, either directly or indirectly.

ii) Resources. Having a high valuation is not sufficient for fighting much, however, one must also afford doing so. On the other hand, when inequalities are not too large and the players contest a joint prize, resources often have less to say.

iii) Conflict technology. With unequal valuations and not too great inequality, the decisiveness parameter $m$ magnifies power differences.

iv) Comparative advantage in fighting. Being a better producer is not necessarily a big advantage, being a better fighter, on the other hand, is likely to pay off.
3 Applications

3.1 On the application of conflict models

The formal approach to conflicts is quite abstract and aims at identifying causal mechanisms and relations between variables rather than at accurately representing the messy reality that is found on the ground. However, being relevant for real-world conflicts is also a goal, and most authors at least suggest empirical applications for their work, such as the continuing struggle between the city-states of ancient Greece (Hirshleifer, 1995), the civil war in Angola (Mehlum and Moene, 2002), and mafia activities in Eastern Europe (Konrad and Skaperdas, 1998).

Some studies go beyond this and focus on real-world applications more specifically, addressing such topics as competition between warlords (Skaperdas, 2002), political transitions and the conflict between elites and the poor in Western Europe and Latin America (Acemoglu and Robinson, 2001), the persistence of elites even if political transitions take place (Acemoglu and Robinson, 2008), modern guerilla warfare (Fearon, 2008), ethnic/religious versus class-based conflict (Esteban and Ray, 2008), and how the level of violence in a country may be affected by an aid agency’s decision on how to deliver food aid (Blouin and Pallage, 2008).

However, most of also these more applied studies use empirical data only to a limited extent. Some exceptions are: Bates et al. (2002) do not use numerical data, but keep close attention to case studies in their model of the negative and positive effects of violence on political order in post-colonial Africa (A less formal, but more comprehensive version of this analysis is Bates (2008)); Mehlum et al. (2006) use data on resource dependence and the quality of institutions of a sample of 42 countries to test their hypothesis that institutions that encourage rent-seeking have a negative effect on growth while producer friendly institutions have a positive effect; Banerjee et al. (2001) develop a model of conflict between rich and poor members of Indian sugar cooperatives and confront the model’s predictions with data on prices, capacity levels and participation rates.

In general, it is fair to say that most studies address questions from a quite theoretical standpoint. As noted, conflict models are very flexible, but with such flexibility comes the challenge of finding the right operationalizations and
relevant data, a task complicated by the participants’ incentives to misrepresent information. Indeed this challenge may help account for the fact that most of the literature has tended toward theoretical exploration rather than empirical application. It should be noted that although I undertake an application in this chapter, I do not see this as in any significant way playing a role in filling this hole in the literature that I am suggesting exists.

Predictions from the models are typically obtained from comparative statics analysis, i.e. looking at the effects of changes in the parameters or exogenous variables on the equilibrium values of the variables of interest. One can then for instance examine how the level of fighting is affected by a new weapons technology, which influence the parameter $m$, or an income shock.

3.2 Somalia

To what extent does the simplest model that I have set out above admit an interpretation of considerable traits of the violent political situation in Somalia? In this section I consider the conflict between the “Government” and the “Islamists” in the southern and central parts of Somalia within the baseline model framework to illustrate explanatory potential and empirical challenges. I begin with a brief recent history of the country.

3.2.1 Introduction: Somalia after state collapse

A very short history of Somalia’s two last decades: Conditions were steadily deteriorating during the last years of the dictatorship of Siad Barre. After Barre was deposed in 1991, civil war continued. Responding to the famine brought about by the civil war, the United Nations created the United Nations Operations in Somalia (UNOSOM I), which failed to have much effect, and then the US-led Unified Task Force (UNITAF), that intervened militarily in late 1992. Humanitarian assistance delivery and security were initially improved, but the following operation UNOSOM II became engaged in heavy fighting with the troops of the warlord Mohamed Farah Aideed. The United States withdrew in late 1993 after 18 US soldiers were killed in a failed hunt for Aideed in Mogadishu in October of that year. The fighting decreased, but UNOSOM II withdrew in 1995. Shifting
alliances between warlords, businessmen, clans, sharia courts and local militias in the southern and central parts of the country and high variation in the degree of violence have characterized the situation since then. There have been many attempts to revive a central government, but all have failed. The situation in Somaliland to the North and Puntland to the North-East have been relatively stable.

My main sources on Somalia are Boutros-Ghali (1996), Little (2003), Menkhaus (2007a,b), reports of the Independent Expert appointed by the Secretary-General on the situation of human rights in Somalia (UNHRC, 2009, 2010), and various articles and reports from the International Crisis Group.

3.2.2 Conflict: Government against Islamists

A faction consisting of several anti-Ethiopian, Mogadishu-based clans controlled the Transitional National Government (TNG) from 2000, while their rivals, the Somali Reconciliation and Rehabilitation Council (SRRC), gained the upper hand in the Transitional Federal Government (TFG), which was formed in 2004. The president subsequently elected by the transitional parliament in 2004, Abdullahi Yusuf, and his prime minister were close allies of Ethiopia and strongly anti-Islamist. With the dominance of the TFG by Ethiopia-friendly anti-Islamists, the struggling Islamist movement in the capital Mogadishu gained support as an alternative force, and formed the Supreme Council of Islamic Courts (SCIC) in 2005. The transitional parliament broke down in 2005. Concerned about Somalia as a safe haven for terrorists, the United States managed to forge an alliance between several militiamen in Mogadishu. However, this alliance was not founded on the best of terms, and after a series of battles against a loose alliance of militiamen throughout the first half of 2006, the Islamists emerged decisively as winners and in full control of Mogadishu. It is from this point that my analysis starts.

The players: Government and Islamists

The alliance of various factions and clans controlling the government at the time of parliamentary breakdown I from now on denote “the Government”. This identification is somewhat accidental, as there are other parties that have controlled the government before, but it
is this group that has been recognized by the international community in recent years. Because of the security situation in the Mogadishu, parliamentary session were held in Nairobi, Kenya, and the Government was never able to govern from the capital. The members of the Government traditionally had strong ties to Ethiopia. When using variables from the model, the Government will be referred to by the subscript $G$.

The Government’s adversary will be denoted “the Islamists”, and will have $I$ as its subscript. This group gets its name from comprising the SCIC, but includes also the Mogadishu-based clans referred to above. The Islamists had more tense relations with Ethiopia.

Each group is assumed to behave like one, unitary actor when it comes to dealings with the other group. It is clear that both groups are constituted by a wide range of actors with possibly very different goals, so this is obviously a big assumption. However, the interests may be well aligned in this particular conflict, or the leaders may have ways of making the members of its group yield to their way.

**The prize: Mogadishu** Being in charge of the capital is of importance in itself. In addition, Mogadishu is a commercial center and an important port, thus controlling it offers a variety of economic benefits. In general, control of a country’s capital often opens many sources of profits, such as access to the revenue of the state apparatus, proceeds from corruption, extortion of regular businesses, control of aid, or in general taxation on economic activity. These types of rent should at least in principle be possible to estimate.

Another, more elusive factor is nonmonetary goals. If we believe the word of the Islamists, much of their valuation stems from the desire to implement a particular brand of religion and legislation. Even if one is sceptical of such claims, the relevancy of hard-to-quantify sources of valuation must be recognized. In some situations it may be possible to difference out these variables. An analysis aiming to engage with empirical data would have to take a standpoint on these issues.

To the extent that there is some production to be seized or players may invest in the part of city they control, the prize may be some function of both players’
productive effort. However, I will assume that this is insignificant compared to pure rent that may be extracted.

Variables: $v_G, v_I$

**The fighting: Soldiers**  The decision made by the players is how many soldiers to employ in the fight in the present period. Men with combat experience and low opportunity cost abound, so it does not seem unreasonable to assume that at least on the margin it is possible to hire the amount of soldiers one needs and that the ability to do so is fairly equal, a notion supported by reports of foot soldiers shifting sides. I assume $m = 1$ in the contest success function for simplicity.

Variables: $F_G, F_I$

**The win probability: Degree of control**  I take the win probability, the outcome of the contest success function, to be the share of Mogadishu that is controlled. This assumption seems plausible, as Mogadishu has been divided by various frontlines in recent years. It appears from newspaper reports that these frontlines are well defined and quite observable for someone in the field, but data are scarce, probably for both practical and tactical reasons. A further complication is created by terrorist attacks employed by insurgents that often make for the non-effective control of either party. I will not pursue this topic further, only note that a way of incorporating this into the model would be to specify an option for terrorism effort in the contest success function that decreased the win probability for all contestants.

I begin by considering the case where the control may be exercised in “this” period only, and where in the “next” period, the decisions have to be made over again, with no trace of the preceding period. The game is thus essentially a one-shot, one-period game. A period may be thought of as lasting for about the time it takes to regroup and organize after a battle. The perks enjoyed by the one in charge of the capital may make for a definite incumbency advantage, but I will only consider the stationary case in the following.

Variables: $p_j = \frac{F_j}{F_G + F_I}$, $j = G, I$
The Payoff  As in the baseline model, the payoff of a player $i$ is equal to valuation times win probability minus the costs of fighting:

$$\pi_i = p_i v_i - F_i,$$

We can think of the payoff as accruing jointly to the leaders of the groups. War decisions are made by the leaders, and it is their valuations that matter. The costs of war are simply the expenses on soldiers and military material. The general population is with this interpretation assumed to respond to local incentives and not have any say in decisions at the aggregate level, or simply be coerced in some way by the leaders. An interpretation where the prize is shared within the groups and the true costs of lives lost are incorporated into the opportunity cost of $F_i$ is possible, but I believe the former interpretation is both the more plausible and the more quantifiable one. A middle way could be to include a parameter measuring the leaders’ internalization of the society-wide costs of war.

Variables: $\pi_G, \pi_I$

The initial situation  At my starting point, midway through 2006, the Islamists are in charge of Mogadishu, and the Government confined to the provisional capital Baidoa. The Government’s absence at this point is taken to mean that they had hit their resource constraint and were spending all they could, which was quite little, in the fight, even though their valuation was high. It may be argued that the Government enjoyed funding from abroad and should have been able to procure the military services they needed. However, at the time the official government had been ineffectual for quite some time, and was not heavily supported, even though recognized legally internationally. In addition, funding may not have been available for war purposes. The Government did have backing by Ethiopia, but at nowhere near the level that it would enjoy a few months later.

We thus have:

$$F^*_G = R_G$$

$$F^*_I = \sqrt{v_I R_G} - R_G$$

$$p^*_G = \sqrt{\frac{R_G}{v_I}}$$

$$p^*_I = 1 - \sqrt{\frac{R_G}{v_I}}$$
As \( R_G \) is quite small, the power of the Government, \( p_G^* \), will be close to 0, while the Islamists be in almost full charge, with \( p_I^* \) close to 1. The reason here is very simple: The very few available resources for one party allows for the dominance of the other. However, fighting does not necessarily take place on a large scale.

**Intervention and a new equilibrium**  Islamists in power were always feared as a destabilizing element by neighboring Ethiopia, and the radicalization of the Islamic Courts during 2006 and the unruly militants in the shabaab militia did not help to quell these fears. Ethiopia intervened in support of the Government in late 2006 with the approval and support of the United States. This had the dual effect of providing resources to the Government itself, i.e. increasing \( R_G \), and adding to any Government fighting effort actually provided. We can express this with a modified contest success function for each side that includes the Ethiopian forces as a fixed, exogenous quantity \( F_E \):

\[
p_G = \frac{F_G + F_E}{F_G + F_E + F_I}, \quad p_I = \frac{F_I}{F_G + F_E + F_I}.
\]  

(63)

Both sides take the Ethiopian impact as given. The Government now has sufficient resources to do what it likes, but of course welcomes the Ethiopian support. However, its objective function remains the same, so the Government does not welcome the support in any other way than treating it as a gift, which can be seen from the equilibrium values of fighting and power:

\[
F_G^* = \frac{v_G^2 v_I}{(v_G + v_I)^2} - F_E
\]

(64)

\[
F_I^* = \frac{v_G v_I^2}{(v_G + v_I)^2}
\]

(65)

\[
p_G^* = \frac{v_G}{v_G + v_I}
\]

(66)

\[
p_I^* = \frac{v_I}{v_G + v_I}
\]

(67)

These solutions are almost exactly those of the baseline model with \( m = 1 \) (equations (31) and (32)), which were considered in chapter 2. The only difference is in the solution for the Government’s fighting (64), where the value of the Ethiopian forces is subtracted. The interpretation of this is that the Government adjusts its own use of soldiers since there is an exogenous army rolling in anyway. The
consequences, or lack of thereof, of the Ethiopian army rolling out will be seen below.

It was assumed in the initial situation that the Government spent the whole of its $R_G$ in the fight and that it wanted to spend more. Now since it is able to adjust the spending, the effect on $F_G^*$ may be negative, but the important point is that the combination of Government and Ethiopian soldiers greatly exceeds the previous number of Government soldiers alone. For an internal solution, the first order condition for the Islamists for any given value $\bar{F}$ of opposition fighting in any case implies

$$F_I = F \sqrt{v_I \bar{F} - \bar{F}},$$

so the Islamist also increase their number of soldiers.

The heightened level of fighting affects the distribution of power: The Islamists loose their hegemony over the capital, while the Government gets a share.

An issue with the potential to complicate the analysis is changes in valuations. We have seen above that the Islamists fought more because they had to. Another reason may have had to do with a shift in their valuation. Due to historical feelings of animosity between segments of the Somali population and Ethiopia, the actual involvement of Ethiopian forces in Mogadishu may have served to increase the stakes in the conflict, thereby effectively strengthening the Islamists’ desire to remain in power, and hence effectively increasing their valuation $v_I$. That the Ethiopian forces were backed by the US may have increased tensions even further. A simple way to include this formally is by defining an ‘antagonism parameter’ $\theta$:

$$\pi_i = p_i v_i - F_i - \theta p_j, \quad i = G, I, i \neq j.$$  \hspace{1cm} (69)

The power of one player here has a direct negative impact on the payoff of the other. The model above can be seen as the special case of $\theta = 0$, or alternatively as including the antagonism directly in the valuations. Choosing this latter interpretation of the initial situation, I now want to consider the case of the Ethiopian support of the Government leading to increased antagonism on behalf of the Islamists and their supporters. In other words, the payoffs after the intervention are given by (69), with $\theta_I = \theta > 0$, $\theta_G = 0$. The equilibrium values of fighting

33
and power would then be given by:

\[
F^*_G = \frac{v_G^2 (1 + \theta)v_I}{(v_G + (1 + \theta)v_I)^2} - F_E
\]  

(70)

\[
F^*_I = \frac{v_G((1 + \theta)v_I)^2}{(v_G + (1 + \theta)v_I)^2}
\]  

(71)

\[
p^*_G = \frac{v_G}{v_G + (1 + \theta)v_I}
\]  

(72)

\[
p^*_I = \frac{(1 + \theta)v_I}{v_G + (1 + \theta)v_I}
\]  

(73)

Both players spend more on fighting than without antagonism. The Islamists increase their spending more and somewhat paradoxically increase their power at the same time that their payoff decreases.

Assuming that such feelings of antagonism are relevant, important questions are how they are to be operationalized and obtained data on? Even if one could obtain data on antagonism on a point in time through surveys, preference-revealing behavior or the study of the cultural environment of the conflict, there are additional questions of the feelings’ stability. This is a point where any explanatory account faces a serious challenge, not least when it comes to testing.

**Withdrawal and the same equilibrium**  
Ethiopian soldiers withdrew at the turn of the year 2008, but the fighting has continued. A reason for this can be seen in our simple model: Provided that the Government now had sufficient resources for soldiers, they would increase their fighting effort one-for-one as the Ethiopians pulled out. I am here assuming that the constraint on the resources of the Government continues not to bind. The new levels of fighting and power would be characterized by:

\[
F^*_G = \frac{v_G^2 v_I}{(v_G + v_I)^2}
\]  

(74)

\[
F^*_I = \frac{v_G v_I^2}{(v_G + v_I)^2}
\]  

(75)

\[
p^*_G = \frac{v_G}{v_G + v_I}
\]  

(76)

\[
p^*_I = \frac{v_I}{v_G + v_I}
\]  

(77)

The only difference from the expressions characterizing the previous equilibrium is in equation (74), which says that now the Government is supplying all its
soldiers itself. The new level of Government forces, however, is exactly equal to the previous sum of Government and Ethiopian forces. The Government’s payoff, on the other hand, is markedly lower, since it is now not receiving the free gift. In effect, the Islamists face the same level of opposition as before, and employ the same response.

The distribution of power does not change, as the effective amounts of fighting on each side remain identical. Ethiopia achieved its goal of restraining the power of the Islamists, but the level of violence, presumably also a destabilizing factor, increased from the initial situation.

One may hypothesize that antagonism is more easily increased than decreased. In that case, the antagonism parameter $\theta$ from the intervention situation would still be relevant, and the corresponding equilibrium values of fighting and power would be the ones obtaining also here. Thus the lasting effect of the Ethiopian intervention may have been to increase the level of violence on a permanent basis.

**Discussion** The model does capture some very broad traits of the contest to control Mogadishu, but what have we learned from the application? Some lessons are the following: It is often thought that one resourceful actor will inevitably repress a weaker one severely, but in moving from the first to the second equilibrium, we saw how making the Government and the Islamists more equal resulted in a more equal distribution of power, but also more violence. In moving from the second equilibrium to the third, it became clear that in the model this did not have to do with the extra troops provided by the external power, but rather with the Government’s ability to procure soldiers. In other words, without the binding resource constraint, there was no difference in fighting whether exogenous forces were present or not, only one optimal level. We also examined a way in which the intervention in itself did have an effect on the level of violence, namely through increasing the antagonism present in the conflict, perhaps even on a permanent basis.

Contributing policy advice is a goal of most applications - what does this simple model have to offer in that respect? Is there for instance something that can be done to reduce the fighting? The equilibrium equations suggest reducing the valuation of the prize. This will also reduce the payoffs, so a better solution would
be to distinguish the part of the prize that may be captured through fighting. Power sharing agreements and negotiations can be interpreted as playing this role.

Developing the economy is often seen as promoting peace. In a more elaborate model, one could have analyzed the role of production of non-military goods, for instance through production functions as suggested in section 2.4.1. Production may have various complicated effects, but in a model such as the one above without production that is available for appropriation, better productive capacities will increase the opportunity cost of fighting and lead to less violence, more production and higher payoff. However, if the leaders depend on capturing the rent for which there is military competition, measures enhancing productive ability, like jobs training and civilian business opportunities, is not unlikely to be resisted, paradoxically in particular if offered to one’s own side.

Another dimension in the case of Somalia is that resources have kept flowing on each side - the Islamists have been able to raise money and at least some fighting resources abroad, while the government side have also received official financial and military support internationally. The end result has been enabling both sides to spend more resources on fighting. The recent development of the Islamists claiming allegiance to Al-Qaeda can be seen as an attempt at securing this support also in the future. It may indeed succeed in this, but such moves are likely to induce foreign support of the government’s side as well. In addition, more open collaboration with terrorist groups may now increase the Government’s antagonism parameter, fueling a spiral of violence. However, the Islamists may be in badly need of the money - charging the World Food Program payment for being allowed to deliver food to such an extent that the WFP had to suspend their activities suggests desperation, and contrary to many popular claims, there are no clear indications of ransom money from piracy benefiting rebels onshore (Rollins et al., 2010).

More relevant may be another topic related to terrorist financing that has received much attention in the media: popular funding from abroad through Hawala, a remittance system based outside the formal banking sector. Given that huge sums are transmitted from the large Somali diaspora, it is probably inevitable that it will play a role in supporting violent actions. Hawala offers
nearly full anonymity, its transactions are difficult to trace as no written records are usually kept. These features have made the system attractive for money-laundering purposes and for people wanting to hide their financial transactions, which in turn have made many argue for regulation of the network. Because of its huge social importance and decentralized organization, however, few developments have been seen, and few are perhaps desirable, in endeavours to regulate Hawala (Reuter and Truman, 2004). Large diasporas have in fact been found to be associated with conflict, but the point about social importance is a general one: Remittances and support from the diaspora serves too many other functions to be cut off.

It is important to have in mind the empirical challenges connected with applying a model, especially when it comes to policy advice. I have emphasized the important valuation variable. Different parties may view a prize differently; an impact on a player’s effective valuation may come from a multitude of sources, of which only antagonism toward the adversary is formalized here, and hard data on these issues are difficult to obtain. Other assumptions have been made more specifically, the most tenuous of which is perhaps that the two groups behave like perfectly coherent units. This may be a valid approximation, but one should not forget the many reasons for such a relationship to be unstable or not to hold at all, and the possible links to these reasons from the conflict situation itself.
4 Conclusion

With historical cases, we have seen how the models may help throw some light on specific questions like why support of the government have had certain effects. A more ambitious goal is to analyze future developments and draw policy implications. For this one must deal with serious challenges relating to assumptions about the suitable level of aggregation and operationalization and measurement of important variables. The models that have been presented are far from being near the level of sophistication and empirical accuracy that would be required for such an endeavor, but any analysis need to start with a thorough understanding of simple cases, and these models are parts of such a first step. When that step is complete, one should go further and build larger models. However, there remains more foundational work, one strand of which is emphasized by a recent review of the literature on civil war: “[psychological factors and misperception] have yet to be applied to formal models of civil war” (Blattman and Miguel, 2009, p. 30). I end by considering some ideas for future research that take up this challenge.

Modern behavioral economics have revealed many aspects of human behavior that only to a limited extent have been incorporated into formal models of conflict. Two such aspects are the well-established phenomena of loss aversion and the endowment effect. Loss aversion refers to the finding that people often attach a higher disutility to a loss than to a corresponding gain (Kahneman and Tversky, 1979). The endowment effect is displayed when the value that is placed on an object is higher when one possesses the object than when one does not (Thaler, 1980). Cornes and Hartley (2003a) analyze loss aversion in a static rent-seeking contest and show that it reduces dissipation. However, loss aversion could have the opposite effect in a dynamic setting, by increasing an incumbency’s fear of losing and reducing the desirability of settlement. A warlord would be loth to have to give up his controlled area, and spend inordinate amount of resources on defending it. This could cement factional divisions and further hinder a peaceful solution.

With the endowment effect, where one’s valuation of an object varies before and after getting to own it, time is an essential element. Although the starting point is that property rights are non-enforced, the endowment effect should be
readily incorporated by having valuation increasing with time of possession. The endowment effect seems especially relevant in situations where the contested object is not primarily about money, such as in fights over territories with strong religious or cultural connotations.

Other findings from behavioral economics may also prove illuminating in the analysis of conflicts. Even if the stakes are very high in conflict situations, the participants may exhibit overconfidence and common biases. It should be noted that a bias need not be irrational; it may for instance take the form of favoring one’s own group. That could simply be a natural tendency of humans, but it may also be interpreted as allowing for personal connections that are not modeled explicitly.

“Misperceptions” sounds more unambiguous, but what constitutes rational beliefs is not so clear-cut. If agents to some extent base their beliefs on past experience, e.g. of fighting success, this could take the form of distorting the contest success function. This could provide a powerful source of misperceptions, as actors with mostly unsuccessful experiences would be perceive themselves to be weaker than they really are, while the opposite would be the case with successful actors. In a multi-group setting, it seems plausible that increased volatility of fighting could result.

One of the appealing features of the contest framework is its great versatility. In Haavelmo (1954)’s canonical formulation, regions or countries are the decision-making units. Bowles and Choi (2003) scale this down to the individual level and combine it with a game-theoretic model where agents are of different types. They effectively microfound the modern institution of individual property rights by showing how a shift from a hunter-gatherer social order to an agriculturalist social order may have occurred with the advent of agriculture around 11 000 years ago. Leaving the question of the historical accuracy of their story aside, the approach of disaggregation is promising with regards to clarifying the foundations of institutions and tackling the collective action problem directly. Bowles (2004) stresses that preferences and institutions evolve, and that they do so together. One simple way of incorporating this insight into contest models would be by specifying an action-specific utility as a function of earlier patterns of behavior, letting such a pattern represent an institution. One could then study the effects
of different initial configurations and stochastic shocks.
References


