Preface

This master thesis studies the method of interference cancellation applied on multiple access channels. The work has been conducted during the period August 2007 to December 2008. It has been done as a part of the program for Master of Electronics and Computer Technology at the Department of Physics, University of Oslo. The principal work has been done at the premises of the University Graduate Center at Kjeller (UNIK).

First of all, I would like to thank my supervisor Dr. Pål Orten and my colleagues Ph.D. candidate Hans Jørgen Bang and Post doc Ninoslav Marina who supported me by giving many useful advices connected to my research as well as suggesting the related literature.
Abstract

In present day wireless communication systems there is a need for high transmission data rates supporting large number of users. The available bandwidth is limited and expensive, therefore, the requirement of new technologies that utilize the available spectrum more efficiently. One possible way to improve the usage of the bandwidth is the method known as subtractive interference cancellation that can be either successive or parallel. It is low-complexity method that allows designing a system with more users within a given bandwidth. In a frequency division multiple access system, in practice there is an interference, which comes from adjacent channels and results in a degraded performance. In this case the system may in many cases be severely interference limited.

The main contribution of this master thesis is development of algorithms for simulation of adjacent channel interference cancellation, which offer an essential improvement in bit error rate performance.

Our results indicate that the parallel technique shows slightly better results than the successive one after the first stage of implementation, while after the second stage the successive technique performs better.

The proposed method could be used to increase the number of users in systems with fixed available bandwidth.
# Contents

1 Introduction 9

2 System model and problem formulation 13
   2.1 Model description
      2.1.1 Digital signal modulation techniques
      2.1.2 Baseband and bandpass transmission
      2.1.3 The channel model
      2.1.4 Practical approach through simulation
   2.2 Problem formulation
      2.2.1 Motivation
      2.2.2 Bit error rate
      2.2.3 Bandwidth

3 Simple algorithm for BPSK and QPSK simulations 19
   3.1 Simulated and theoretical BER for BPSK 19
   3.2 Simulated and theoretical BER for QPSK 20

4 Pulse shaping for QPSK 23
   4.1 Rectangular pulse shaping 23
   4.2 Square-root raised-cosine pulse shaping 27
   4.3 Spectra estimates in linear and log scales
      4.3.1 Plots of spectra 31
      4.3.2 The effect of the pulse shaping 34
      4.3.3 Spectra of transmitted signal and BER performance when varying the roll-off parameter of the square-root raised-cosine filter 34
   4.4 Conclusions 37

5 Adjacent channels interference 39
   5.1 Generation of adjacent channels 39
   5.2 Simulation programs for investigation of the BER performance characteristics
      5.2.1 ACI when varying the separation between the channels 43
      5.2.2 ACI when varying the roll-off factor 43
      5.2.2.1 Varying $\beta$ when $F_1 = 2$ 43
      5.2.2.2 Varying $\beta$ when $F_1 = 1$ 45
      5.2.3 Varying of several parameters and mapping the results at the same plot. 47
   5.3 Conclusions 50

6 Interference Cancellation 52
   6.1 The SIC technique
      6.1.1 SIC when varying the distance between the channels 54
      6.1.2 SIC when varying the roll-off factor 56
      6.1.3 Two stages of the SIC 57
6.2 The PIC technique
   6.2.1 PIC when varying the distance between the channels
   6.2.2 PIC when varying the roll-off factor
   6.2.3 Two stages of the PIC
6.3 Compare the SIC and PIC techniques

7 Conclusion and future work

Bibliography
**List of abbreviations**

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>AWGN</td>
<td>Additive white Gaussian noise</td>
</tr>
<tr>
<td>ACI</td>
<td>Adjacent channel interference</td>
</tr>
<tr>
<td>BER</td>
<td>Bit error rate</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
</tr>
<tr>
<td>CDMA</td>
<td>Code-division multiple access</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier transform</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital Signal Processing</td>
</tr>
<tr>
<td>DS-CDMA</td>
<td>Direct-sequence code-division multiple access</td>
</tr>
<tr>
<td>FFT</td>
<td>Fast Fourier transform</td>
</tr>
<tr>
<td>IC</td>
<td>Interference cancellation</td>
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<tr>
<td>IIR</td>
<td>Infinite impulse response</td>
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<td>ISI</td>
<td>Inter symbol interference</td>
</tr>
<tr>
<td>LTI</td>
<td>Linear time-invariant</td>
</tr>
<tr>
<td>MATLAB</td>
<td>Matrix Laboratory</td>
</tr>
<tr>
<td>MF</td>
<td>Matched filter</td>
</tr>
<tr>
<td>PIC</td>
<td>Parallel interference cancellation</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quaternary Phase Shift Keying</td>
</tr>
<tr>
<td>SIC</td>
<td>Successive interference cancellation</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-noise ratio</td>
</tr>
</tbody>
</table>
Chapter 1
Introduction

Wireless communication systems are currently becoming more and more significant. We are witnessing continuous improvements in the existing systems and devices, offering new services every day. As the demand for wireless mobile communication services is constantly growing, the need for mobile systems and devices that support much higher data rates is increasing. The available bandwidth is restricted and costly resource, and therefore, we need new technologies that utilize it more efficiently.

Types of interference

The problem of interference in communication systems is as old as communications itself. The obligatory coexistence of information and nosy signals (or useless electrical signals, noise) has been accommodated in the design of communication systems. Recent mathematical modeling and simulation techniques significantly eased this effort, and much of the interference or its unwanted effect can be removed through filtering, the choice of digital signal modulation, and the selection of an optimal receiver location.

There are two categories of the interfering signals in wireless communication systems: those that occur in environment and are not possible to eliminate, and those artificial signals created by humans that can generally be attenuated or controlled. The above-mentioned interference is subdivided into several types, such as: co-channel interference, adjacent channel interference (ACI), intermodulation interference and others.

Adjacent channel interference

This type of interference is characterized by undesirable signals from neighboring frequency channels which inject energy into the channel of interest. The closeness with which channels can be placed in frequency band is defined by the
spectral roll-off parameter of the filter and the width and shape of the main lobe. ACI is caused for the most part by insufficient filtering and nonlinearity of the amplifiers.

The effect of poor filtering is shown in Figure 1.1 below. The energy in the shaded regions corresponding to adjacent channels will cause interference.

![Power spectrum](image)

**Figure 1.1: Adjacent Channel Interference (shaded regions) [10].**

In most cases, it is enough to concern only the interference coming from the two channels on either side of the primary channel. The shaded regions represent the energy contributions to the channel of frequency $f_{c2}$ coming from the two adjacent channels, $f_{c1}$ and $f_{c3}$.

There will be signal interference from $f_{c4}$, $f_{c5}$, and other channels farther from the channel of interest, $f_{c3}$. However, the contributions from these channels decrease the farther these channels are from the channel of interest. A comparison of the ACI created by near neighbors and far neighbors due to insufficient filtering is illustrated in Figure 1.2.

One of the ways ACI can be attenuated is through an increase in the channel separation.
Figure 1.2: ACI from neighboring channels and from distant channels [10].

The effects of ACI can also be reduced by using advanced signal-processing techniques that, among other methods, apply successive interference cancellation (SIC) and parallel interference cancellation (PIC) techniques of the subtractive interference cancellation method.

**Main goals of the thesis**

The first goal of this thesis is to study the method of interference cancellation applied to multiuser wireless systems. The main goal is to build up a simulator of a communication system that allows its performance improvement when the method of subtractive interference cancellation is used.

**Outline and contributions of the thesis**

The thesis consists of seven chapters including this introduction. The thesis outline is in the following way.

In Chapter 2 we present the system model and formulate the main objectives for a successful interference cancellation. We also give an exact definition of bit error rate (BER), energy/bit-to-noise ratio and introduce the notion of bandwidth.
In Chapter 3 we propose an algorithm of simulation for Binary Phase Shift Keying (BPSK) and Quaternary Phase Shift Keying (QPSK) digital signal modulation techniques to investigate the BER performance. The simulation is performed on digits, but not on physical pulses, i.e. without the pulse shaping applied.

We develop the algorithm and MATLAB program that we described in Chapter 3, by introducing pulse shaping on our transmitted signal and including a matched filter at the receiver in Chapter 4. We study the effect of the rectangular and square-root raised-cosine pulse shaping together with the roll-off parameter of the filter, and present the simulation results of the proposed algorithm as the system performance in the terms of the BER characteristic.

In Chapter 5 we generate the adjacent channels in order to investigate the interference introduced by them and the BER performance of the main channel as we will vary the distance between channels and the filter’s roll-off parameter.

Chapter 6 constitutes the main contribution of this thesis. We consider the SIC and PIC interference cancellation techniques that have been applied in Direct-sequence code-division multiple access (DS-CDMA) systems and use them for ACI cancellation in our work as well. We also examine the improvement of the BER characteristics by applying the two stages of these techniques.

Chapter 7 contains conclusions of this work.
Chapter 2

System model and problem formulation

The main work of the thesis is related to adjacent channel interference cancellation in wireless and mobile communication systems. We shall make a number of assumptions to represent the simulation problem as simple and understandable as possible. At the same time we hope that the main results of our simulations will be used in practical communication systems. Further in this chapter in Section 2.1 the system model will be introduced together with some descriptions used in the subsequent chapters. The motivation and the main objectives of this thesis will be mentioned in Section 2.2. In the same section we also discuss how to define a performance of a digital communication system in terms of its bit error rate and introduce the notion of bandwidth.

2.1 Model description

We consider a frequency divided system which is shared by several users simultaneously. If it is possible to pack neighboring channels/users closer together, it results to a better exploitation of the frequency resources. The disadvantage is the interference that arises from the different channels. This interference can be removed with help of interference cancellation methods.

2.1.1 Digital signal modulation techniques

Digital symbols that we use in digital communications must be compatible with the characteristics of the channel which we send our signal through. The process of transforming digital symbols into waveforms is named digital modulation.

In this study we shall restrict our attention to the Phase-shift keying (PSK) digital modulation technique. This is one of the most spread digital modulation schemes used in both military and commercial communication systems. PSK uses a finite number of phases, each assigned a unique combination of binary digits. It is convenient to represent PSK schemes on a so-called constellation diagram by introducing the points equispaced around a circle in real and imaginary axes.
named, respectively, in-phase and quadrature axes. Since the points are placed on a circle, the symbols they represent are transmitted with the same energy.

Binary Phase Shift Keying (BPSK) modulation, the simplest and most robust of all techniques, uses two phases and the signal shifts the phase of the waveform to one of the two states, either zero or $\pi$. Its constellation diagram is shown in Figure 2.1(a) with in-phase and quadrature axes named as I and Q, respectively. It is only able to transmit 1 bit/symbol in this case and so this is considered to be a disadvantage when using high data-rate systems with limited bandwidth.

A four-level (4-ary) PSK is called Quaternary Phase Shift Keying (QPSK), and uses four points on the constellation diagram (Figure 2.1(b)). The signal shifts the phase to one of four states and so QPSK can transmit 2 bits/symbol as we see from the diagram as well. When applying Gray coding each adjacent symbol only differs by one bit.

![Figure 2.1: Constellation diagrams a) for BPSK, b) for QPSK [12].](image)

The QPSK scheme is unique among all PSK signal sets in the sense that the QPSK waveform set is represented by a combination of antipodal and orthogonal members. In comparison to BPSK the double data rate can be used with the same bandwidth, or the same data rate as for BPSK and twice less bandwidth. However, in order to achieve the same bit error probability as BPSK, QPSK has to use twice the power (since two bits are transmitted simultaneously).

### 2.1.2 Baseband and bandpass transmission

In this study we are going to deal with two types of signals/channels which are: baseband or low-pass signal, and passband signal. The same applies to channels.
The difference between them is in the frequency band that these signals (or channels) are occupying. The spectrum of the baseband signal is composed of frequencies that are equal to or very near zero and a baseband channel can transfer frequencies that are equal to or very close to zero. Usually baseband signal correspond to pulse-like waveforms. A passband signal is situated on a band between two higher frequencies.

When there is a need to translate the spectrum of a low-pass signal to a higher frequency, we can simply multiply the baseband signal with a carrier wave. Referring to Figure 2.1, the baseband signal can be represented as

\[ Z(t) = I(t) + jQ(t), \]

(2.1)

where \( I(t) \) is the in-phase signal, \( Q(t) \) the quadrature phase signal. Then, passband signal will correspond to

\[ I(t)\cos(2\pi F_c t) - Q(t)\sin(2\pi F_c t) = \text{Re}\{Z(t)e^{j2\pi F_c t}\}, \]

(2.2)

where \( F_c \) is carrier frequency which the baseband signal is translated on.

### 2.1.3 The channel model

We will assume that transmission characteristics of our main and adjacent channels are constant for any distance and time. However, since in any communication system there is a thermal noise (random motion of electrons that inevitable in receivers) that has additive, white, and Gaussian characteristics, we shall model and simulate the background noise of the channels as additive white Gaussian noise (AWGN). An AWGN channel adds white Gaussian noise to the signal that passes through it and, as noise samples are uncorrelated and also independent, affects each transmitted symbol independently. The received baseband signal for user \( i \) at time \( t \) is given by

\[ r_i(t) = s_i(t) + n_i(t), \]

(2.3)

where \( s_i(t) \) is a transmitted signal, and \( n_i(t) \) is a zero mean, complex white Gaussian noise process.

The model does not account for the phenomena of fading, frequency selectivity or nonlinearity.
2.1.4 Practical approach through simulation

In order to model a natural communication system for the purpose of teaching, research and development a computer simulation can be applied. Simulation is also used to show the eventual real effects and optimize the performance. As computer simulation is the imitation of some real processes, we need to substitute physical objects with their mathematical descriptions. In digital signal processing (DSP) we work with sequences of numbers, it is, therefore, useful to simulate a real-life system by using a computer simulation so that it can be studied to see how the system works and behaves when changing different variables and parameters.

For the purpose of ACI cancellation we develop our simulation in MATLAB. MATLAB (short for Matrix Laboratory) is used to implement numerical algorithms for a wide range of applications. It has the Signal Processing Toolbox inbuilt so it is considered to be convenient when employing MATLAB in the field of DSP. The above-mentioned toolbox contains the features that we further are going to apply, such as: digital filter design, analysis and implementation; access to the fast Fourier transform; tools for spectral analysis and statistical signal processing; and graphical user interfaces for designing, analyzing, and visualizing signals, filters, and windows.

One of the disadvantages of simulation in the field of DSP is the artificial aspect of assumptions introduced by simulation which can make the result of simulation different from the real-life communication system.

2.1 Problem formulation

The main objective of this study will be to build up a simulator that simulates the performance of the system with different degrees of overlap between the channels and how high improvement can be achieved using the interference cancellation techniques. Our design goal will be to jointly-optimize:

- the type of signal modulation,
- the type of pulse shaping,
- the roll-off parameter of the filter, and
- the distance between channels.

In addition we shall compare the successive and parallel interference cancellation techniques for single and two stages implementation.
In the next subsection we will try to motivate the importance of these optimization criteria when performing adjacent channel interference cancellation in a communication system.

2.1.1 Motivation

The significance of all criteria is apparent from a wireless system perspective. The available bandwidth for a wireless link is limited and we should therefore choose the signal modulation technique which utilizes bandwidth the most efficiently by transmitting more data at a given bandwidth. The choice of optimal pulse shaping is also essential since wireless networks function with the smallest possible bandwidth to provide a high number of users. Depending on the value of the pulse shaping filter’s roll-off parameter it becomes possible to further increase the bandwidth exploitation. Thus, the choice of digital modulation technique, pulse shaping and the roll-off parameter for shaping the pulses should absolutely be considered when designing a communication system. Concerning the distance between the interfering channels, we should first examine the system performance for different grades of channel overlap. Since the interference between the channels is undesirable, we should apply existing interference cancellation techniques to avoid it and observe how much improvement can be achieved.

2.1.2 Bit error rate

In order to compare different digital signal modulation techniques and measure the performance of a communication system, we need to define a bit error rate of this system. Bit error rate (BER) or error probability depends on the signal-to-noise ratio (SNR) or, more exactly for our case, on the energy/bit-to-noise ratio, $E_b/N_0$, where noise is assumed to be additive white and Gaussian, and $E_b$ is the energy per bit. $E_b/N_0$ is a normalized by bandwidth $W$ and bit rate $R$ version of the SNR parameter (2.4), and, unlike SNR, uses to measure the performance in digital systems, but not the analog ones.

$$
\frac{E_b}{N_0} = \frac{S}{N} \cdot \frac{W}{R}, \quad [11] \tag{2.4}
$$

where $N_0$ is the noise power spectral density, and $S/N$ is the average signal power to average noise power ratio.

Figure 2.2 illustrates a plot of the bit error probability $P_b$ versus $E_b/N_0$. The most of such curves has a “waterfall-like” shape.
When comparing the performances of different systems, we can conclude which system is more efficient by verifying the following: the smaller the required $E_b/N_0$ value for a given probability of error, the better the detection process.

2.1.3 Bandwidth

In the following text we will use the notion of bandwidth in two different contexts. In the first one bandwidth means a range of frequencies, while in the second one bandwidth means capacity of the system in terms of information bits per unit time.

In this thesis we improve the bandwidth efficiency of the system that we design in order to utilize the frequency bandwidth more efficiently since the latter is a limited and expensive resource.
Chapter 3

Simple algorithm for BPSK and QPSK simulations

In this section we are going to describe the algorithm of simulation for BPSK and QPSK digital signal modulation techniques with the view of investigation of the BER characteristics performance. The techniques themselves were described in some details in Subsection 2.1.1. The algorithm will be implemented, using the MATLAB.

3.1 Simulated and theoretical BER for BPSK

In order to generate transmitted symbols of unit energy for BPSK type of signal modulation, we first use a special MATLAB command RANDN to generate a vector of normally distributed random numbers. Then we apply the command SIGN, which returns 1 if the element is greater than zero and -1 if it is less than zero. Thus, the obtained sequence of symbols that consists of 1s and -1s, which is corresponding to two different states of phase change: either zero or π. Referring to Subsection 2.1.1 and Figure 2.1(a), we have to mention here that instead of generating the sequence of 0s and 1s, it is easier for MATLAB implementation to generate -1s as a substitute for 0s. When applying BPSK each symbol consists of one bit, that is why the number of symbols is equal to the number of bits in this case. Let us generate for the simulation not less than 100,000 bits to get a complete BER characteristic for the lowest value on the BER axis equal to $10^{-6}$ when plotting the BER curve. The digits are being transmitted through a baseband AWGN channel. Applying the MATLAB function RANDN again we generate AWGN noise samples, that is to say a vector of zero mean Gaussian distributed random variables of the same length as the transmitted signal vector. Furthermore, we put this noise on each bit of the signal. The noise is additive, thus, we employ the operation of addition for implementing the noise superimposition in MATLAB. As a result of addition, we obtain our received signal.
Next stage of the simulation process is estimating or in other words, detection of originally transmitted symbols. In order to achieve this we will apply the previously used MATLAB function SIGN to obtain the sequence of 1 and -1 values. Travelling through the noisy channel, a definite number of received symbols will become erroneous and different from the transmitted symbols. Having applied formula below, we estimate the simulated bit error rate $BER_s$ when simulating the BPSK modulation technique.

$$BER_s = \frac{N_e}{N_{tot}},$$

(3.1)

where $N_e$ is a number of errors and $N_{tot}$ is a total number of transmitted bits.

In order to compare this result with the theoretical, we obtain the theoretical bit error rate for BPSK $BER_t$, which is given by:

$$BER_t = Q\left(\sqrt{\frac{2E_b}{N_0}}\right),$$

(3.2)

where $Q(x)$ is the Q-function defined in [1] (or if to type HELP QFUNC in MATLAB for a definition).

Our algorithm contains a loop in order to repeat the simulation for several values of $E_b/N_0$ in the range from -10 to 11 dB.

Applying the SEMILOG MATLAB function we plot the bit error rate in dB as the function of $E_b/N_0$. The theoretical and simulated results for BPSK are shown in Figure 3.1.

### 3.2 Simulated and theoretical BER for QPSK

As with the simulation of BPSK technique, we start with generating transmitted symbols of unit energy for QPSK type of signal modulation. Since we deal with the four-level PSK, the modulating data signal shifts the phase of the waveform to one of four states, either $\pm\pi/4$ or $\pm3\pi/4$. So far as we use complex plane to represent QPSK as revealed in constellation diagram in Figure 2.1(b), the QPSK symbols in MATLAB are having the complex representation as well. If we generate 100 000 bits, the sequence of bits will contain 100 000/2=50 000 symbols, where each symbol consists of 2 bits. We first need to create a vector of normally distributed complex random numbers and apply the command SIGN on it to obtain the sequence of 1s and -1s, where -1s are generated instead of 0s (Figure 2.1(b)). The complex numbers are achievable by multiplication of second bit of a symbol by imaginary unit $j$, which amounts to $\sqrt{-1}$ and is written in MATLAB as SQRT(-1). The generated sequence is to be transmitted through a baseband AWGN channel.
Similarly as with BPSK example, we use the RANDN command to generate AWGN noise samples with only difference that we need complex noise samples, consisting of 2 bits each and every second bit is multiplied by imaginary unit $j$. Then we add the complex noise samples to the transmitted sequence of symbols and thereby obtain the received signal. By using the command SIGN we execute the symbol detection.

As far as we examine the bit error rate but not the symbol error rate, the BER for QPSK type of signal modulation will remain as in Equation (3.1) for BPSK technique. It concerns the theoretical BER as well. We obtain theoretical BER for QPSK, using Equation (3.2).

In the same manner as with BPSK the algorithm for the QPSK simulation contains a loop to obtain for values of $E_b/N_0$ in the range from $-10$ to $11$ dB.

For the comparison of BER characteristics of BPSK and QPSK techniques we plot the BER of QPSK in dB in the same figure. The theoretical and simulated results for QPSK are shown in Figure 3.1.

![Figure 3.1](image)

Figure 3.1: Simulated BER for BPSK and QPSK together with theoretical results.

From Figure 3.2 it can be seen that the simulated bit error rates of QPSK and BPSK are the same. Note, however, that the symbol error rate for QPSK is almost twice than its bit error rate. Our simulated results completely coincide with the theoretical ones, which indicates that our algorithm is accurate.
These simulations were made using binary digits which cannot be implemented in practical communication systems. For that reason we need to convert the binary digits into waveforms that could be sent through the channel.
Chapter 4
Pulse shaping for QPSK

In this section we are going to extend the algorithm and MATLAB program that we described in Chapter 3, by introducing pulse shaping on our transmitted signal and including a matched filter at the receiver. We implement two types of pulse shaping, such as: rectangular and square-root raised-cosine, and we will present the simulation results of the proposed algorithms as a performance of the BER characteristic. Since QPSK has greater bandwidth efficiency than BPSK so that we can transmit more data at a given bandwidth (2 bits/symbol), we will proceed with simulations employing the QPSK type of signal modulation.

4.1 Rectangular pulse shaping

In order to estimate BER in Chapter 3 we have executed the simulations on a sequence of binary digits, 1s and -1s. In spite of the fact that simulated result completely coincides with the theoretical, there is a big improbability when using such a representation of the signal. There is nothing “physical” about the digits resulting from this process. We use digits just as a way to describe the message information. Therefore, there is a need in something more physical that can represent or “carry” the digits.

When real digital transmission, the source information or binary digits have to be transformed to a form that is compatible with real digital communication system. There is no channel that could have transmitted the digits. In order to transmit the digits through a baseband channel, they first have to be transformed to some pulse-like waveforms or in other words, electrical pulses because pulses are compatible with such a channel. The probability that we detect pulses/symbols at the receiver correctly is a function of the received pulse energy. That’s why, it is always better to make the pulse width as wide as possible.

First we are going to represent the binary digits with rectangular and non-overlapping pulses. In order to achieve this we apply to our sequence such
MATLAB functions, as: UPSAMPLE and FILTER. Function UPSAMPLE upsamples input signal by inserting N-1 zeros between input samples, where N is the desired length of a pulse/symbol. We have chosen N=5. For example, if our input signal is \(-1 \ 1 \ 1\) then we get a new upsampled signal which is shown in Figure 4.1 by red stars.

![Upsampling of the signal -1 1 1 and rectangular pulse shaping applied on this signal.](image)

Then we create a filter or, so-called, rectangular window, consisting of 1s and with the length of 5, as the one upsampled symbol's length. Further we normalize the total energy of the rectangular window such that this energy will be equal to 1 and the SNR after matched filtering (that we will introduce at the receiver), sampling and detection, will stay the same as when we used the sequence of 1s and -1s in Chapter 3. Having denoted the rectangular pulse before energy normalizing as vector \(y\) with the length of \(n\) i.e. consisting of \(n\) samples, and after we have normalized the total energy - as vector \(h\) with the same length as \(y\), we can define the vector \(h\) by calculating each value/sample in \(h\) as

\[
h_i = \frac{y_i}{\sqrt{\sum_{i=1}^{n} y_i^2}},
\]

(4.1)
where \( i \) corresponds to a sample number in both \( y \) and \( h \), and \( \sum_{i=1}^{n} y_i^2 \) is the total energy of the pulse \( y \).

Function \text{FILTER} performs the convolution of the rectangular window and the upsampled signal. Thus, we achieve the rectangular pulse shaping on our sequence, which we transmit through an AWGN channel. Rectangular pulses are shown in Figure 4.1 by blue samples. After performing the pulse shaping we can say that the transmitted signal now is materially identical to a real transmitted signal and compatible with the baseband channel so that could be sent through the AWGN-channel when applying real digital communication system.

**Matched filter at the receiver**

Before starting the detection of received symbols, we apply matched filtering.

To describe in few words, a matched filter is a linear filter designed to provide the maximum signal-to-noise power ratio at its output for a specified transmitted symbol’s waveform [1]. That means that matched filter in our case contains a rectangular window of length 5 with its total energy equal to 1.

In real digital communication system a known signal \( s(t) \) plus AWGN \( n(t) \) is the input to a linear, time-invariant (receiving) filter followed by a sampler, as shown in Figure 4.2. The receiving filter RX is our matched filter.

![Figure 4.2: Typical baseband digital system [11].](image)

Next step of the simulation is to provide matching filtering by applying MATLAB function \text{FILTER} to convolve the rectangular window and the sequence of the received symbols. Maximum SNR, produced by the matched filter gives better detection result. Further we have to pick up the correct sample out of the convolved sequence. As we know, the basic characteristic of the matched filter is that its impulse response is a delayed version of the mirror image (rotated on the \( t=0 \) axis) of the input signal waveform. This delay equals to one symbol length (or symbol time duration). Without this delay, the response of the
mirror image is unrealizable because it describes a response as a function of negative time [1].

As long as the symbol length equals to 5 (or consists of 5 samples), we will pick up every 5th sample out of the matched filtered sequence. In order to achieve this we make a vector, starting from 5th sample and using a step of 5. Further we implement the detection by using function SIGN for making a decision on every sample for the purpose of getting 1 and -1 values.

Using (3.1), we estimate the simulated BER for QPSK when applying rectangular pulse shaping at the transmitter and matched filter at the receiver. In Figure 4.3 above-mentioned BER is plotted together with the previous estimated BER when simulation was applied to a signal consisting of binary digits.

![Figure 4.3: BER performance for QPSK when applying rectangular pulse shaping and without: the simulated and theoretical results.](image)

From Figure 4.3 it can be seen that the simulated BER with pulse shaping applied coincides with the simulated BER without pulse shaping applied and with the theoretical BER, which indicates the correctness of the proposed algorithm.
4.2 Square-root raised-cosine pulse shaping

In this section we will describe an algorithm of implementing the square-root raised-cosine pulse shaping on our transmitted signal and using the matched filtering at the receiver.

Referring to Figure 4.2 we can obtain the transfer function of the whole system, which includes the transmitting filter, the channel and the receiving (matched) filter, as

\[ H(f) = H_t(f)H_c(f)H_r(f) \]  \hspace{1cm} (4.2)

As far as we want to utilize the less available bandwidth in order to have place for most users in the multiuser system, we need to apply the ideal Nyquist filter which transfer function has to be the transfer function of the total system \( H(f) \). The Nyquist filter is used in order to overcome ISI, but is unrealizable since it applies an infinite impulse response (IIR) filter. In practice, for implementation of the Nyquist filter the truncated raised-cosine filter is usually applied. If assuming an ideal channel with its transfer function \( H_c(f) = 1 \), then the simple way to make the \( H(f) \) to become a raised-cosine filter is to chose

\[ H_t(f) = H_r(f) = \sqrt{H(f)}, \]  \hspace{1cm} (4.3)

where \( H(f) \) is a transfer function of the raised-cosine filter while the transmitting and the receiving filters are square-root raised-cosine filters [11].

Let us next introduce a special MATLAB function, which we are going to apply for generating the above-mentioned type of filter. The function, which represents squared-root raised-cosine filter is called SQR_RC and written by my supervisor Dr. Pål Orten in 1996. The function requires a user to specify three of the filter parameters that are \( L, \eta \) and \( \beta \). \( L \) is a pulse/symbol length which we chose to be equal to 5, \( \eta \) is oversampling (number of points/samples per symbol) or upsampling as we called it when generating rectangular pulse shape. Let \( \eta \) be equal to 5 (in subsequent chapters we will vary these parameters for some useful experiments). And \( \beta \) (further written as \( \beta \)) is a so-called roll-off factor of the square-root raised-cosine filter that characterizes the steepness of the filter roll off and varies from 0 to 1. In Figures 4.4 and 4.5 below the filter characteristics, such as: the impulse response and the transfer function are illustrated for three
different roll-off values. The blue plots are for $\beta = 0$, the red ones for $\beta = 0.5$ and the green plots are for $\beta = 1$.

![Graph showing impulse response of different roll-off values](image)

**Figure 4.4**: Square-root raised-cosine filter impulse response.

The impulse response of a square-root raised-cosine filter is given by

$$
 y(t) = \frac{4\beta}{\pi \sqrt{L}} \cos \left[ \frac{(1+\beta)\pi t}{L} \right] + \frac{\sin \left[ \frac{(1-\beta)\pi t}{L} \right]}{(4\beta t/L)^2},
$$

(4.4)

where $\beta$ is the roll-off factor of the filter, $t$ is the sampling instant, and $L$ is the sampling or the symbol period [7].

As we can see from Figure 4.4, impulse response of the filter is sinc-like, but truncated, and the larger the filter roll-off factor, the smaller the amplitude of the pulse tails closed to the main lobe.

Figure 4.5 illustrates the transfer function of the square-root raised-cosine filter. We note that the larger the roll-off factor $\beta$, the wider bandwidth is needed. We will discuss the advantages and drawbacks of increasing the roll-off parameter for square-root raised-cosine pulse shaping later in Subsection 4.3.3.
Going back to algorithm and MATLAB program description, we start with applying just depicted function \texttt{SQR\_RC} to create a squared-root raised cosine filter with roll-off parameter equal to 0.5. Further, as it was done in example with rectangular pulse shaping, we normalize the total energy of the square-root raised-cosine pulse such that this energy will be equal to 1. Having earlier denoted a squared-root raised cosine pulse before energy normalizing as vector \( y \) with the length of \( n \) samples, and after we have normalized the total energy - as vector \( h \) with the same length as \( y \), we can define the vector \( h \) by calculating each sample value in \( h \) applying (4.1).

By analogy with the rectangular pulse shaping, we generate a QPSK sequence of binary digits, which we upsample first and then with help of function \texttt{FILTER} perform the convolution of the square-root raised-cosine pulse and that upsampled signal. Thus, we have achieved the square-root raised-cosine pulse shaping on the initial sequence, which we then transmit through the AWGN channel.

![Figure 4.5: Square-root raised-cosine filter’s transfer function for various roll-off factors.](image)

At the receiver we repeat all those steps from the previous simulation described in Section 4.1. As before, the matched filter, which is now square-root raised-cosine pulse, gives one symbol length delay. That is why after matched filtering we start sampling with picking up the first sample just after the above-mentioned
symbol duration and continue to sample with the step of 5, since we have chosen the transmitted symbols to consist of 5 samples. Detection process remains the same as earlier.

Using (3.1), we estimate the simulated BER for QPSK when applying square-root raised-cosine pulse shaping at the transmitter and matched filter at the receiver. In Figure 4.6 this BER is plotted together with the previous estimated BER when simulation was applied to a signal consisting of binary digits.

![Figure 4.6: BER performance for QPSK when applying square-root raised-cosine pulse shaping ($\beta = 0.5$) and without: the simulated and theoretical results.](image)

### 4.3 Spectra estimates in linear and logarithmic scales

Electrical communication signals consist of time-varying voltage or certain waveforms, typically described in the time domain. It is also suitable to describe such signals in the frequency domain in terms of their frequency transfer functions. The frequency domain representation of a signal is called spectrum. The concept of spectrum is significant in analyzing and designing communication systems. A signal can be then described by its average power or energy content at various frequencies. A spectrum also illustrates how much of the bandwidth the signal occupies. All these aspects make us to be interested in spectra and Fourier techniques. The Discrete Fourier transform (DFT) is one of several mathematical
tools that is useful in the analysis and design of linear time-invariant (LTI) digital systems. A frequency-domain signal representation principally involves the decomposition of the signals in terms of sinusoidal (or complex exponential) components. With such decomposition, a signal is considered to be represented in the frequency domain.

In order to analyze advantages and drawbacks of rectangular and square-root raised-cosine pulse shaping and make a comparison of these techniques, we will examine their signal representations in the frequency domain, i.e., examine their spectra.

4.3.1 Plots of spectra

In the beginning of this section we would like to mention that the one of the most important and computationally efficient methods for evaluating the DFT and power spectra estimation is Fast Fourier transform (FFT). Principal advantages of this method are the reduction in number of computations and in the required core storage. The method involves sectioning the signal and based on time averaging over short, modified periodograms.

Applying the MATLAB function FFT, we first obtain the spectrum of the transmitted signal filtered by rectangular pulse with oversampling equal to 5. The spectrum estimate in linear scale is shown in Figure 4.7. We can clearly recognize the sinc shape of the spectrum, and the first zero is at the symbol rate. The frequency axis ranges from $-fs/2$ to $fs/2$, where $fs$ is the sampling frequency. Here $fs = 5*fsymbol$ ($fsymbol = symbol rate$). Before plotting we have used FFTSHIFT function in MATLAB which is useful for visualizing the Fourier transform with the zero-frequency component, shifted to the middle of the spectrum. Thereby we have arranged the spectrum from $-fs/2$ to $fs/2$ (instead of from 0 to $fs$).

Let us next obtain the spectrum of the transmitted signal filtered by a square-root raised-cosine pulse with oversampling equal to 5, pulse length equal to 5 and with roll-off factor equal to 0.5. The spectrum estimate in linear scale is shown in Figure 4.8.

Then we also would like to present plots of the spectra in a logarithmic scale which is common way of plotting the spectrum as it easier to analyze closer side lobes and function’s zeros. Figures 4.9 and 4.10 show the spectra of the transmitted signal in a logarithmic scale filtered by a rectangular pulse and square-root raised-cosine pulse, respectively. All these spectrum estimations serve the purpose of illustrating the bandwidth and shape of the spectra.
Figure 4.7: Spectrum estimate in linear scale, applying rectangular pulse shaping

Figure 4.8: Spectrum estimate, applying square-root raised-cosine pulse shaping
Figure 4.9: Spectrum estimate in log scale, applying rectangular pulse shaping

Figure 4.10: Spectrum estimate in log scale, applying sq.-root raised-cosine pulse
4.3.2 The effect of the pulse shaping

Let us now take a closer look on the spectra. From the spectra in linear scale it is obviously that when rectangular pulse shaping applied on the transmitted signal (Figure 4.7), the spectrum has large-amplitude side lobes to compare to when applying square-root raised-cosine pulse shaping (Figure 4.8). Such large side lobes are undesirable since they increase the bandwidth to the transmitted signal.

From the spectra in logarithmic scale we observe that when rectangular pulse shaping applied, zeros appear as values that go to minus infinity (Figure 4.9). The side lobes of the spectrum of the square-root raised-cosine pulse shaped signal (Figure 4.10) are significantly lower than the side lobes in Figure 4.9. This is considered to be a big advantage of square-root raised-cosine pulse shaping since compact spectrum provides optimum bandwidth utilization. Thereby in order to design successful and achieve an improvement in spectrum estimate we should rather have applied square-root raised-cosine pulse shaping with its smoothing effects to avoid large-amplitude side lobes (tails) instead of rectangular pulse shaping.

4.3.3 Spectra of transmitted signal and BER performance when varying the roll-off parameter of the square-root raised-cosine filter

In addition to implementation of the square-root raised-cosine pulse shaping on the transmitted signal with roll-off parameter $\beta$ equal to 0.5, let us vary $\beta$ and analyze the plots of spectra afterward.

To remind that $\beta$ is a roll-off factor of the square-root raised-cosine filter that characterizes the steepness of the filter and varies from $0 < \beta < 1$.

Choosing $\beta = 1$ we run our simulation program for QPSK type of signal modulation. The pulse length $L$ and number of samples per symbol $\eta$ (oversampling) remain the same as in previous simulation, respectively, 5 and 5.

The result is shown below in Figure 4.11 in a logarithmic scale.

Now we repeat the simulation experiment for $\beta = 0$ for the purpose of comparison the spectra for boundary values of $\beta$. The spectrum estimate in logarithmic scale is represented in Figure 4.12.

Using the spectrum estimate for $\beta = 0.5$ as well, we can now compare the plots. We observe that when $\beta = 1$ (Figure 4.11) the spectrum has the widest main lobe and the lowest side lobes. And contrariwise, when $\beta$ is equal to 0 (Figure 4.12) the spectrum has the narrowest main lobe and the highest side lobes. For $\beta = 0.5$ (Figure 4.10) the result appears to be medium for both main lobe and side lobes.
Figure 4.11: Spectrum estimate in logarithmic scale, applying square-root raised-cosine pulse shaping on the transmitted signal when $\beta=1$

Figure 4.12: Spectrum estimate in log scale, applying square-root raised-cosine pulse when $\beta=0$
So, in other words the parameter $\beta$ is directly proportional to the main lobe width, and inversely proportional to the height of the side lobes.

Since high side lobes increase the bandwidth of the transmitted signal, it is then clear that the higher the roll-off factor, the more the bandwidth of the transmitted signal gets decreased. In the case of mobile communications, for example, it is very positive and allows having a higher number of users who share the same bandwidth to communicate.

It is also useful to observe how the BER characteristics of the received signal change when varying the roll-off parameter. For this purpose we run the simulation program a few times for different values of roll-off parameter with the matched filter (which is a square-root raised-cosine filter in this case) included at the receiver, and after detection of the signal we calculate performance of the BER using (3.1). In Figure 4.13 the plots of the BER are illustrated for three different values of the roll-off factor of the square-root raised-cosine filter.

![Figure 4.13: BER performance for QPSK when applying square-root raised-cosine pulse shaping together with the theoretical result for non-shaped sequence. The simulated results are represented for $\beta=0; 0.3$ and 1.](image)

We can observe that when $\beta = 1$ or 0.5 (from Figure 4.6) the simulated BER performance completely coincides with the theoretical result. For $\beta = 0.3$ the
performance starts slightly to get worse, and finally when $\beta = 0$ the BER has become essentially worse.

The worsening of the BER when $\beta = 0$ can be explained by the filter’s low length when choosing $L$ to be equal to 5. What happens when we then decrease the roll-off factor is that the filter response becomes narrower in the frequency domain and the side lobes have higher amplitude in the time domain (filter’s impulse response). The total response of the transmitting and receiving filters when $L=5$ and $\beta$ varies from 0-0.1 does not result in a Nyquist pulse (Nyquist filter). Although 100% Nyquist filter is unrealizable since it applies an infinite impulse response (IIR) filter, in our case when $\beta$ is that low, we even cannot manage to implement the raised-cosine filter, which is realizable and uses in practice as an implementation of a low-pass Nyquist filter. The reason why the BER performance becomes worse is the inter symbol interference (ISI). From approximately $\beta = 0.3$ the ISI is negligible even when $L=5$.

Such a low roll-off as from 0-0.1 is not current in practice. But if to increase the length of the pulse in the time domain the filter becomes closer to Nyquist filter, no ISI occurs and the BER performance then does not depend on the value of the roll-off. In this study we represented the true result for the boundary values of $\beta$ when $L$ was low and equal to 5 since we will proceed with the simulations with the same value of $L$ to achieve the lower computational complexity.

### 4.4 Conclusions

**Pulse shaping**

- We need to send a physical signal instead of just digits. To this effect we represent a sequence of digits as specifically shaped electrical pulses.

- For most communication systems (with the exception of spread-spectrum systems), our goal is to reduce the required system bandwidth as much as possible to some rationally small bandwidth. We can achieve this by a proper choice of a pulse-shaping on the transmitted signal.

**Rectangular pulse shaping**

- Rectangular pulse shaping is simple to achieve and it has low complexity.

- It is necessary to normalize the total energy of the rectangular pulse in relation to one binary bit/symbol.
Square-root raised-cosine pulse shaping

- This technique for shaping the pulses means to round off the corners of the pulses so that side lobes of the spectra of the pulses fall off faster.

- A pulse-shaping filter should satisfy two requirements. It should provide the desired roll-off, and it should be realizable (the impulse response needs to be truncated to a finite length).

Matched filter

- Applying matched filtering at the receiver we provide the maximum SNR for more successful detection process.

Spectra

- Such large side lobes as we observe from the spectra with the rectangular pulse shaping applied on the transmitted signal are undesirable since they increase the bandwidth of the transmitted signal.

- The spectra of the signal with the square-root raised-cosine pulse shaping applied have significantly lower side lobes compared with the rectangular pulse shaping.

Varying the roll-off parameter

- When analyzing the spectra, we conclude that the parameter $\beta$ is directly proportional to the main lobe width, and inversely proportional to the height of the side lobes.

- Since high side lobes increase the bandwidth of the transmitted signal, the higher the roll-off factor, the more the bandwidth of the transmitted signal gets decreased. It allows a higher number of users who share the same bandwidth.

- When the square-root raised-cosine filter’s length and pulse length $L$ are high enough, then the BER performance of the channel does not depend on the value of the roll-off.

- When $L$ and $\beta$ take low values simultaneously then BER becomes worse because of the ISI phenomena since it is not possible to implement the Nyquist filter on the product of transmitting and receiving filters in frequency domain.
Chapter 5
Adjacent channel interference

So far we have dealt and examined only the one signal which we generated, using BPSK or QPSK modulation techniques and pulse shaping, then transmitted through the noisy channel and received, implementing a matched filtering, detection and calculation of the bit error rate of the received signal. This signal was considered a baseband signal that occupied a definite channel, which spectrum we were analyzing when applying specific digital signal modulation and pulse shaping techniques, and varying the roll-off parameter in the square-root raised-cosine pulse shaping case.

In this chapter we are going to generate neighboring or adjacent channels further to our existing baseband channel, let us call the latter by main channel, in order to investigate the interference, that these adjacent channels introduce, and the BER performance of the main channel while we will vary the distance between channels and the roll-off parameter.

Referring to discussion in Chapter 1, not only the nearest neighbors (two channels on either side of the main channel) contribute with the signal intrusions (Figure 1.1), but there will be interference coming from some far neighbors (distant channels) as well (Figure 1.2). Since farther channels contributions decrease the farther these channels are from the main channel, we will neglect their influence on the main channel and assume that exceptionally two nearest and two second-nearest neighboring channels introduce the interference.

5.1 Generation of adjacent channels

Let us proceed to built up our simulator by generating four signals/channels more in addition to the existing baseband signal. It is necessary to mention that all these transmitted signals will be generated as different sequences of binary digits first, and all will be applied the square-root raised-cosine pulse shaping on.
Similarly to the way we have generated the transmitted signal in Subsection 4.2, at first, four QPSK sequences of digits must be created by using the MATLAB functions RANDN and SIGN. Then we apply a square-root raised-cosine pulse shaping on these sequences, implementing upsampling of the sequences first and then convolving them with the square-root raised-cosine pulse. This pulse was described earlier as a filter with its impulse response, given by (4.2). Before convolution we have normalized the total energy of the square-root raised-cosine pulse, applying (4.1) such that this energy became equal to 1.

In order to transmit these four pulse-shaped signals through four different noisy channels (AWGN channels), what, in other words, means to distribute transmitted signals’ spectra in the frequency domain at different frequency bands, we need to transform these signals from baseband signals into passband signals. An easy way to translate the spectrum of a low-pass or baseband signal to a higher frequency is to multiply or heterodyne the baseband signal with a carrier frequency (discussed in Subsection 2.1.2). Applying (5.1) we obtain the new transmitted signals $a_{i(t)}$ modulated into specific frequencies:

$$a_{i(t)} = A_{i(t)} e^{j2\pi F_i t}, \quad (5.1)$$

where $A_{i(t)}$ is a baseband transmitted signal, $F_i$ is a frequency which signal $A_{i(t)}$ is carried forward on, $t$ is a vector of time, which consists of the same number of samples as the transmitted signal, and $i$ is a serial number of a signal.

After we have generated four passband transmitted signals, we obtain their spectra, applying MATLAB function FFT. Assuming that the main baseband channel is placed in the middle of the entire frequency band with its middle frequency equal to zero, we choose frequency of the right adjacent channel $F_1=2$ and frequency of the left adjacent channel $F_2=-F_1=-2$, since symmetry of the adjacent channels relative to the main channel. The next-nearest right and left channels are carried on frequencies $F_3=2F_1=4$ and $F_4=-F_3=-4$, respectively. Now we will run this simulation program for such parameters’ values as: pulse length $L=5$, upsampling $\text{eta}=5$ and roll-off factor $\beta=0.5$. The spectra estimates to all channels together are shown in Figure 5.1 in linear scale. The main channel is marked with red, the nearest channels are blue and the next-nearest channels are green.

We note that when upsampling is equal to 5 then it is not enough place to all 5 channels on the entire frequency band. The next-nearest channels instead of occupying bandwidths with middle frequencies in $F_3=4$ and $F_4=-4$, carry on onto $F=\pm 1$. The entire bandwidth ranges from $-2.5 < F < 2.5$ and a sampling rate $F_s = 5$ in the case of $\text{eta}=5$, that is why the next-nearest channels bandwidths (marked with green) fold up inwards the above-mentioned range relative to, so-called, folding frequency $F_s/2$, which in our experiment is equal to
2.5. It can lead to a significant worsening of the BER performance since channels strongly interfere with each other.

Figure 5.1: Spectra estimates for the main and four adjacent channels in linear scale, applying upsampling \( \text{eta}=5 \)

To avoid that neighboring channels fall out the range \(-F_s/2 < F < F_s/2\), we will increase upsampling or sampling rate and run the simulation for \( \text{eta} = 12 \). Frequencies \( F_1, F_2, F_3 \) and \( F_4 \) remain the same as in previous experiment. The spectra estimates in linear scale are illustrated in Figure 5.2 and in logarithmic scale in Figure 5.3 for better visual observation of channel overlap or as they say the adjacent channel interference.

Analyzing Figure 5.2, we note that when we have increased \( \text{eta} \) up to 12, then the folding frequency \( F_s/2 \) become equal to 6 and all five channels’ bandwidths appear in the range of \(-F_s/2 < F < F_s/2\). In addition to mention, the signals contain higher energy now and have more compact signaling spectra, although by increasing the \( \text{eta} \) parameter, it adds higher computational complexity that causes longer time to run the simulation experiment in this case. As a conclusion we affirm that the higher the oversampling of the transmitted signal, the more channels we can dispose on an available frequency range. Thereby, we will
Figure 5.2: Spectra estimates for the main and four adjacent channels in linear scale, applying upsampling $\eta=12$

Figure 5.3: Spectra estimates for the main and four adjacent channels in logarithmic scale, applying upsampling $\eta=12$
continue to run the simulation programs choosing *eta* = 12 in order to provide sufficient for five channels frequency range.

### 5.2 Simulation programs for investigation of the BER performance characteristics

The main objective of this section will be to proceed running some simulation programs when varying different parameters, and to decide for optimal parameters’ values by comparison of the characteristics.

#### 5.2.1 ACI when varying the separation between the channels

We first generate all signals/channels where the middle frequency of the right adjacent channel is $F_1=2$ and $F_2$, $F_3$ and $F_4$ are obtained according to the relations between them described above in Subsection 5.1. Then have computed their spectra, we add up the main signal with four neighboring signals to get a composite signal we are going to transmit through AWGN channel. Receiving this above-mentioned signal we perform matched filtering at the receiver (convolution with the square-root raised cosine pulse) and then detection of the main baseband signal out of the composite received signal. As previously we apply (3.1) to calculate the bit error rate of the simulated received signal and (3.2) to calculate the theoretical BER. After running this simulation several times for values of $F_1=0.5; 1; 1.2; 1.3; 1.5$, we plot the results at the same figure, placed below as Figure 5.4 (values of $F_1$ are small numbers on figure). The roll-off factor was constant and chosen to be equal to 0.5.

Studying the figure we note that BER performance is good or acceptable and coincides with the theoretical result until $F_1 = 1.3$, after what the performance takes a turn for the worse with decreasing the distance between the channels. The closer neighboring channels are placed towards the main channel, the more they disturb it, which increases the number of errors during the detection process and affects the performance.

#### 5.2.2 ACI when varying the roll-off factor

##### 5.2.2.1 Varying $\beta$ when $F_1 = 2$

In this subsection we are going to continue to run previous simulation program with only difference that we will vary the roll-off parameter of the square-root raised-cosine filter. BER characteristics are illustrated in Figure 5.5. The distance
Figure 5.4: BER performance for the main channel when varying distance between the channels $F_1; \beta = 0.5$. Simulated and theoretical results.

Figure 5.5: BER performance for the main channel when varying the roll-off factor (numbers on the figure); separation between the channels $F_1 = 2$. 
between the channels in a frequency domain $F_1$ is constant and equal to 2.

What we note and can conclude observing this figure is that the optimal BER characteristics are for the roll-off $\beta = 1; 0.3$ and for $\beta = 0.5$ (from Figure 5.4). When $\beta = 0.2$ performance is quite acceptable, but from the value of 0.2 the BER is getting worse. We explain this by the presence of the ISI phenomena when both $L$ and $\beta$ take the low values simultaneously (discussed in Subsection 4.3.3).

The second conclusion we make when comparing Figure 5.5 and Figure 4.13. In Figure 4.13 is shown the BER performance for different $\beta$ values for the only main channel without any adjacent channels on the frequency band. These characteristics are similar with the characteristics in Figure 5.5 where the adjacent channels are applied. Thus, it means that when the distance to the closest interfering channels $F_1 = 2$ and higher, then the neighboring channels interfere with the main channel intangibly and influence on neither detection nor BER performance of the main channel.

5.2.2.2 Varying $\beta$ when $F_1 = 1$

In this subsection we would like to pack the channels closer together on a frequency band and analyze the new result. Varying the roll-off parameter we run simulation several times while the distance between closest channels remains constant and equal to 1. The result of the simulations is shown in Figure 5.6.

First and foremost we note that the BER characteristics became much worse to compare to those in Figure 5.5 for the same values of $E_b/N_0$, which is caused by the decreasing the separation between the channels. The adjacent channels overlap the main channel more and interfere with it, introducing by this much many errors into detection process. We also note that starting from specific $E_b/N_0$ values, in the range when $\beta$ varies from $0.3 - 1$ characteristics experience so-called floor, which means that BER has already minimum value and will never get lower than that value even if to increase the energy per bit $E_b/N_0$. Thus, for $\beta = 1$ the floor is when $BER = 0.015$ and for $\beta = 0.5$ when $BER = 0.001$, which indicates bad performance. The only way to improve BER and to avoid the floor is to increase the distance between neighboring channels.

Secondly, we see that increasing the roll-off factor of the square-root raised-cosine filter the BER performance gets worse what did not happen in previous experiments. The worst performance is now for $\beta = 1$, while when distance between channels was $F_1 = 2$, the same characteristic was the best. We can explain this by using the spectra estimates for $\beta = 1$ and $\beta = 0$ in Figure 5.7. When $\beta = 1$ the spectra of the signals have the widest main lobes and the lowest side lobes, which is positive and decreases the signal’s bandwidth. But, on the
Figure 5.6: BER performance for the main channel when varying roll-off factor; distance between the channels $F_1 = 1$.

Figure 5.7: Spectra estimates of five transmitting channels for $\beta = 1$ (a) and for $\beta = 0$ (b); the distance between the closest channels $F_1 = 1$.
other part, in case when neighboring channels overlap the main channel tangibly, 
then their main lobes overlap first of all. Since the main lobes are wide, it causes 
many errors during the detection of the main channel. And on the contrary, when 
$\beta = 0$ and main lobes are narrowest, then the main lobes of the neighboring 
channels overlap the main lobe of the main channel much less. The BER 
characteristic in this case is still not optimal, which is caused by high-amplitude 
side lobes which interfere with the main signal. For $F_1 = 1$ the BER performance 
when $\beta = 0.1$ is slightly better than when $\beta = 0$.

5.2.3 Varying of several parameters and mapping the results at 
the same plot.

The goal of this subsection is to study the parameters’ dependences when we vary 
a few of them simultaneously.

We first would like to see the dependence of BER on the distance between the 
closest channels $F_1$ for three different values of the roll-off parameter. All BER 
values we will take for $E_b/N_0$ equal to 10 (dB) as we have noticed that around 10 
(dB) the characteristics start to have the own behavior each. In Figure 5.8 the 
curves are illustrated for $\beta = 0; 0.5; \text{ and } 1$ (corresponding to numbers under the 
curves on the figure).

![Figure 5.8: BER characteristics when varying the distance $F_1$ for three different roll-off values. $E_b/N_0 = 10$ (dB).](image)
From the figure we can notice that the most optimal BER is when the roll-off factor is equal to 0.5. Only when decreasing the distance to $F_1 = 1$, the BER turns to be better for $\beta = 0$.

Next figure (Figure 5.9) represents the dependence of BER on the roll-off parameter for different distances between the channels $F_1$ (small numbers on the figure on the right). We still reed the BER values for $E_b/N_0 = 10$ (dB). Figure indicates that BER performance is better for intermediate values of the roll-off factor (closer to low values, however), which varies in the range $0 < \beta < 1$. Generally, the performance becomes worse when decreasing the distance between the neighboring channels $F_1$ no matter value of the $\beta$ parameter.

![Figure 5.9](image)

Figure 5.9: BER characteristics when varying the roll-off factor for different distances $F_1$. $E_b/N_0 = 10$ (dB).

Let us next show the dependence $E_b/N_0$ on roll-off factor for different distances. It makes sense to use two different figures since in the case where, due to the neighboring channels overlap the main channel heavily ($F_1 = 1$) and the BER performance experiences floor, it needs high values of $E_b/N_0$ for much higher BER values than for increased separation between the channels. Figure 5.10 illustrates the above-mentioned dependence for constant BER that equal to $10^{-5}$ for three distances $F_1 = 1.2; 1.5$ and 2 (numbers on the right in figure). $E_b/N_0$ curve for $F_1 = 1$ is shown in Figure 5.11 for constant $BER = 10^{-3}$.

As it was expected we observe that the shorter the distance between the adjacent channels and the main channel, the higher energy per bit we need in order to
achieve the same bit error rate. In addition, it requires the least energy per bit when intermediate and closer to low values of $\beta$.

Figure 5.10: $E_b/N_0$ curves when varying the roll-off parameter for three different distances $F_1$. $BER = 10^{-5}$.

Figure 5.11: $E_b/N_0$ curve when varying the roll-off parameter for the distance between channels $F_1 = 1$. $BER = 10^{-3}$.

Looking closer on Figure 5.11 we note that there are no values for $\beta > 0.5$. Due to the floor in the BER performance when the distance as short as $F_1 = 1$, we will
never achieve the bit error rate better than $10^{-3}$ for $\beta > 0.5$, no matter if there are no limits in increasing the $E_b/N_0$.

5.3 Conclusions

Adjacent channels

- The interference is coming from two channels on either side of the main channel and from some distant channels as well.

- The main channel remains a baseband channel while we modulate neighboring channels into higher/lower frequencies.

- The upsampling parameter $\eta$ is increased to $12$. The more compact we make the signaling spectrum, the higher is the allowable data rate or the greater is the number of users that can simultaneously be served. This has important implications to communication service providers, since greater utilization of the available bandwidth translates into greater revenue.

Varying of the parameters

- There is a direct proportionality between channel distance and BER performance. Increasing the distance, we reduce the adjacent channel interference and BER becomes better. Until the distance $F_1=1.3$, we observe good performance (specifically when roll-off factor is equal to 0.5).

- When $F_1= 2$ and higher then no adjacent channel interference, influencing on BER is observed.

- Decreasing the distance between the channels from $F_1=1.5$ we conclude that to achieve the better result in BER performance, the roll-off factor is needed to be decreased simultaneously. Thus, when frequency $F_1$ is in range $1.3 < F_1 < 1.5$ we choose $\beta$ not higher than 0.7, but rather from the range $0.3 < \beta < 0.5$. When $F_1 = 1.2$, the performance is best and acceptable if $\beta$ varies from 0.2 - 0.3, and the best BER we can achieve when $F_1 = 1$ is for $\beta$ not higher than 0.1. That is due to the feature that the spectrum to a signal that was applied the square-root raised-cosine pulse shaping on with the roll-off factor $\beta = 1$, has the widest main lobe. Thus, when adjacent channels tangibly overlap the main channel, the overlap in their main lobes leads to the much worse performance than when $\beta$ is small.
• The BER performance characteristics can experience so-called floor when increasing of energy per bit does not help to achieve a better performance. Floor appears when the channels are packed on the frequency band as close as with $F_1 = 1$ and closer and having the roll-off parameter equal to 0.3 or higher.

• It is not reasonable to pack the channels even closer than when frequency to the nearest channel is $F_1 = 1$, because it causes large adjacent channel interference and much of information that the main channel contains, will be received with a large number of errors.

• One of the ways ACI can be attenuated is through an increase in the channel separation.
Chapter 6

Interference Cancellation

We have so far generated five signals/channels, placed on different frequencies on the specific frequency band where there are the main signal which we detect and calculate bit error rate for, and four adjacent channels, two on each side of the main channel. We also have experimented with the channels by varying the distance between them and the roll-off parameter of the square-root raised-cosine pulse, and have observed whither the adjacent channel interference influence on the BER performance of the received signal.

In order to improve the performance of the communication system which experiences ACI and design this system successfully, there are methods how to counteract the ACI and similar kinds of interference.

Referring at least to [4] and [8], it is clear that in recent years there has been done a good work on interference cancellation within wireless communications when applying the spread spectrum techniques in code-division multiple access (CDMA) systems, particularly in direct-sequence code-division multiple access (DS-CDMA) technology. Due to many advantages of this technology like, for example, a better utilization of bandwidth/frequency and increasing the number of users, it is useful to proceed researching in this range of CDMA.

In DS-CDMA communications there occurs mutual interference between the different users’ overlapping signals as they occupy the same frequency band simultaneously. The detection of every single user can be improved by using the information about the other users and detecting the users jointly instead of one by one. There is a method which is called subtractive interference cancellation and means that one or several users after being detected are subtracted from the received signal. There exist two main types of this method: successive- and parallel interference cancellation.

Similar interference cancellation techniques as in DS-CDMA can be applied for ACI cancellation in this work for our multiuser system too since that is about the overlapping channels placed on the same frequency band as well. In this chapter we will study successive interference cancellation (SIC) and parallel interference
cancellation (PIC) and improvement of the BER characteristics by applying the two stages of these techniques.

6.1 The SIC technique

Let us first describe the algorithm of implementing the successive interference cancellation technique on our received signal. The received signal is a composite signal which is the sum of five transmitted on different frequencies signals and white Gaussian noise. First step is the detection of the main signal/channel and calculating the BER when all four neighboring channels introduce ACI. The plot of this BER performance we are going to need when comparing characteristics before applying SIC and after that.

In second step we detect one of the adjacent channels. The optimal solution is first to detect the closest neighboring channel as it interferes with the main channel the most. Let us say we detect the channel №1 which was transmitted on frequency $F_1$ (on the right to the main channel). To find the transmitted low-pass equivalent to the channel №1, we transfer the channel back into baseband, applying

$$c_i(t) = \frac{S_k(t)}{e^{j2\pi F_i t}}$$

where $c_i(t)$ is a transferred to baseband received channel, $i=1..4$ is a serial number of a channel, $S_k(t)$ is a sum of received signals which is obtained over again on each step of IC process (in every step one of the channels is cancelled from this sum), $k=1..4$ is a step number, $F_i$ is a frequency to a received passband channel and $t$ is a vector of time, which consists of the same number of samples as the received signal.

Further we perform matched filtering (the filter is described by square-root raised cosine pulse), then sampling and detection of the received baseband signal №1. When the detected signal is upsampled over again, pulse-shaped and modulated to the previous higher frequency $F_1$ applying (5.1) (where $A_{i(t)}$ is now a baseband received signal and $a_{i(t)}$ is a obtained over again passband received signal), we subtract this signal from the sum of all channels and the noise, thereby eliminating the interference introduced by channel №1. The new sum value is then reduced by one channel. Not necessarily for the SIC technique but in order to see that system performance has improved already after cancellation of one adjacent channel, the BER of the main channel can be obtained again.

This method was called for successive because we detect and cancel the interfering signals’ contributions successively and one signal at a time.
So in next step we just repeat the second step with the whole procedure that has been performed on channel №1, but now on channel №2, which is the closest to the main channel to the left and placed on frequency $F_2 = -F_1$. Exception in the procedure is only that we subtract channel №2 not from the original sum but from the reduced by one channel sum that has been obtained in previous step.

To proceed the interference cancellation process it is necessary to repeat the second step on next-neighboring channels №3 ($F_3 = 2F_1$) and №4 ($F_4 = -F_3$) similarly.

After the fifth step is done when all interfering channels’ contributions are eliminated from the original composite received signal, we detect the main channel, performing first the matched filtering and sampling. The BER to the main channel is calculated as in (3.1).

This process can be repeated for multiple stages. Each stage takes as its input the data estimates of the previous stage and produces a new set of estimates as its output. In Subsection 6.1.3 we develop the algorithm and run the simulation for two stages.

In Figure 6.1 below the IC process is illustrated stepwise for the SIC technique in the terms of BER performance. The simulation program is run for $\beta = 0.5$ and the nearest channel to the main one is placed on frequency $F_1 = 1$.

Analyzing this figure we can conclude that due to IC technique applied on the received signal the BER performance is pretty much improved, although the adjacent channels overlap the main channel quite heavily. From figure 5.6 we note that without IC the BER performance for the same parameters’ values is rather bad and experiences floor when $BER = 10^{-3}$ for $E_b/N_0 = 29$ dB and higher. No increasing of energy per bit can help to avoid the floor in that case.

The second observation is that after have cancelled all interfering channels’ contributions the performance of the main channel stays practically the same as after have cancelled just two nearest adjacent channels. This can be explained by much higher interference from the closest overlapping channels than from the distant ones that worsens the performance of the main channel.

### 6.1.1 SIC when varying the distance between the channels

In this subsection we proceed with some simulations to investigate the performance of the one stage of SIC technique when varying the distance between the channels. Figure 6.2 shows the BER characteristics for frequencies $F_1 = 1$ (red plots); 1.2 (green plots) and 1.5 (blue plots). The roll-off parameter is the same for all three cases and equal to 1.
Figure 6.1: BER performance of the main channel for the SIC technique when $F_1 = 1$ and $\beta = 0.5$.

Figure 6.2: BER performance of the main channel for the SIC technique when $\beta = 1$ and $F_1 = 1; 1.2$ and $1.5$. The stars are for BER before IC and lines are for BER with IC applied. The red circles show the theoretical result for QPSK.
What we can say observing this figure is that there is an essential improvement in performance for all distances when the one stage of the SIC technique is applied on the received signal. Especially when $F_1 = 1.2$ the BER performance is improved by more than three orders of ten for value of $E_b/N_0 = 11$ (dB). For $F_1 = 1.5$ characteristic is improved and coincides with the theoretical result.

### 6.1.2 SIC when varying the roll-off factor

Let us now examine how the BER performance depends on varying the roll-off parameter of the square-root raised-cosine pulse when applying the one stage of the SIC technique on the received signal. Figure 6.3 shows the BER characteristics for $\beta = 0$ (yellow plots); $\beta = 0.5$ (blue plots); $\beta = 0.7$ (green plots) and $\beta = 1$ (red plots). The distance to the nearest adjacent channel $F_1$ is constant and chosen to be equal to 1.2.

![Figure 6.3: BER performance of the main channel for the SIC technique when $F_1 = 1.2$ and $\beta = 0; 0.5; 0.7; 1$. The stars are for BER before IC and lines are for BER with IC applied. The red circles show the theoretical result for QPSK.](image-url)
As have been expected a degree of the BER improvement is in direct proportion to the increase of the roll-off factor. The highest improvement is when $\beta = 1$ and, respectively, the less one when $\beta = 0$.

Referring to Figure 5.7 where spectra estimates for five channels with $\beta = 1$ and $\beta = 0$ are represented, it becomes clear that when $\beta = 1$, due to wide main lobes of interfering channels which heavily overlap the main lobe of the main channel, the performance of the received main channel before IC, therefore, will be the worst among all roll-off values. But, on the other hand, the same BER performance get improved the most when applying the SIC and cancel all adjacent channels’ contributions. As for the case when $\beta = 0$, since the main lobes of the channels are the narrowest among all roll-off values, they do not overlap the main channel’s main lobe that much, that is why the performance when using the one stage of SIC is just slightly improved to compare to the same performance without IC and to the performances for higher roll-off values.

### 6.1.3 Two stages of the SIC

In this subsection we develop the proposed algorithm of implementing the one stage of the SIC technique and run the simulation for two stages. Figure 6.4 illustrates the BER of the main channel when two stages SIC are applied.

![Figure 6.4: BER performance of the main channel for the two stages of the SIC technique when $F_1 = 1$ and $\beta = 0.5$.](image)
As we observe from Figure 6.4 the improvement in the performance is significant after applying one stage and is even better after two stages are complete, and BER characteristic completely coincides with the theoretical result. This is a proof of high effectivity of the SIC technique used when ACI degrades the performance of the multiuser system.

6.2 The PIC technique

We will now introduce the algorithm of implementing the parallel interference cancellation technique on our received signal. Unlike the SIC technique where we detect and cancel channels one at a time, in parallel IC method all interfering channels are detected at the same time (in parallel) and then eliminated from the received signal simultaneously.

The algorithm of simulation program for PIC technique we start in the same manner as for the SIC, described in Section 6.1. Having generated the composite received signal, we first detect all of the five signals. To detect the passband channels (all channels except the main one) we need to obtain their baseband equivalents by applying the formula

$$C_i(t) = \frac{S(t)}{e^{j2\pi F_i t}},$$  \hspace{1cm} (6.2)

where $c_i(t)$ is transferred to a baseband specific received channel, $i=1 \ldots 4$ is a number of a channel, $S(t)$ is a sum of five received channels, $F_i$ is a frequency to a received passband channel and $t$ is a vector of time, which consists of the same number of samples as the received signal.

Have performed matched filtering and sampling on our baseband channels, we detect them and modulate again to their original higher/lower frequencies that they were sent and received on, by using (5.1) with only difference in the names of variables. Thus, now $a_i(t)$ is obtained over again passband received signal and $A_i(t)$ is a baseband received signal.

The calculating of the BER of the main channel before starting IC is desirable in order to have a possibility to see improvement when the PIC is applied, so we fulfill this calculation.

Finally, we perform the IC by subtracting all adjacent channels from the sum of five received channels, thereby obtaining the received signal with all interfering signals’ contributions eliminated. The only thing left is detection of the main signal, performing the matched filtering and sampling first. Employing (3.1) we compute the BER of the main channel.
It is possible to generalize this cancellation procedure to more than one stage. We will perform the two stages of the PIC technique in Subsection 6.2.3.

Figure 6.5 illustrates the interference cancellation process for the one stage of the PIC technique in terms of BER performance. The roll-off factor $\beta = 0.5$ and the nearest adjacent channel to the main one is placed on frequency $F_1 = 1$.

![Figure 6.5: BER performance of the main channel for the PIC technique when $F_1 = 1$ and $\beta = 0.5$.](image)

Observing this figure we can conclude that due to PIC technique applied on the received signal the BER performance for the main channel is substantially improved, as it is in the case with the SIC technique as well. Before IC is applied on the received signal the BER performance when $F_1 = 1$ and $\beta = 0.5$ experiences floor when $BER = 10^{-3}$ (Figure 5.6) and does not improve if increase the energy per bit $E_b/N_0$.

### 6.2.1 PIC when varying the distance between the channels

In this section we will run some simulations with the PIC technique applied on the received signal, experimenting how the BER will change when varying the parameter of distance between the adjacent channels $F_1$. Figure 6.6 shows the
simulation results for $F_1 = 1$ (red plots); 1.2 (green plots) and 1.5 (blue plots). The roll-off parameter $\beta$ is constant for all cases and equal to 1.

![Graph of BER performance](image)

Figure 6.6: BER performance of the main channel for the one stage of the PIC technique when $\beta = 1$ and $F_1 = 1; 1.2$ and $1.5$. The stars are for BER before IC and lines are for BER with IC applied on the received signal. The red circles show the theoretical result for QPSK.

The characteristics above are almost identical to those ones on Figure 6.2. That is why the observations regarding Figure 6.6 are the same that we made in Section 6.1.1 when analyzing Figure 6.2.

### 6.2.2 PIC when varying the roll-off factor

To continue studying the performance of the main channel when employing the one stage of the PIC technique, we run more simulations while varying roll-off parameter of the filter. Figure 6.7 shows the BER characteristics for $\beta = 0$ (yellow plots); $\beta = 0.5$ (blue plots); $\beta = 0.7$ (green plots) and $\beta = 1$ (red plots). The distance to the nearest adjacent channel $F_1$ is constant and chosen to be equal to 1.2.
So far as characteristics are almost identical with the characteristics in Figure 6.3, we consider the observations made for Figure 6.3 as appropriate for Figure 6.7 when the one stage of the PIC technique is employed as well.

### 6.2.3 Two stages of the PIC

In this subsection we develop the proposed algorithm of implementing the one stage of the PIC technique and we will run the simulation for two stages. The BER characteristics of the main channel when two stages of the PIC technique are applied on our system are presented in Figure 6.8.

As we note from Figure 6.8 the improvement in the performance is significant after applying one stage and is even better after two stages are complete. It proves that the PIC technique can improve the performance of the system degraded by ACI when there is an overlap between the channels.
Figure 6.8: BER performance of the main channel for the two stages of the PIC technique when $F_1 = 1$ and $\beta = 0.5$.

6.3 Compare the SIC and PIC techniques

Finally, for the purpose of comparison the SIC and PIC techniques we now present the simulation results of one and two stages plotted on the same figure (Figure 6.9) in terms of the BER performance. The green curve shows the BER without the IC applied, red color (except circles) is for the SIC technique and blue color is for the PIC technique.

Figure 6.9 shows that after the one stage has applied, the PIC technique improves the performance just slightly better, while after the two stages applied on the system the SIC technique performs visibly better and coincides with the theoretical result. If there is a need in performance improvement, it makes sense to continue and repeat the process for more stages.

When implementation of these two techniques it is necessary to take into account that they have different complexity. The common for both techniques is that each stage takes as its input the data estimates of the previous stage and produces a new set of estimates as its output. The performance of these two techniques improves and becomes equal as the energy/bit-to-noise ratio increases.
Figure 6.9: BER performance of the main channel for the one and two stages of the SIC and PIC techniques when $F_1 = 1$ and $\beta = 0.5$. The stars indicate the BER for the one stage, and the lines are for the two stages applied. The red circles represent the theoretical result for QPSK.
Chapter 7

Conclusion and future work

In this thesis we studied method of subtractive interference cancellation applied to multiuser systems. We developed algorithms for simulation of a point-to-point communication system that uses the techniques of interference cancellation. Initially we made a choice of the optimal modulation technique, pulse shape and its corresponding roll-off factor, which improve the bandwidth efficiency. The next step was to generate adjacent channels to our channel of interest and study the degradation of the BER performance for different frequency separation between the channels. Further we explored the SIC and PIC techniques applied in one and two stages. For the one stage implementation our simulations indicate that the BER of the PIC technique is slightly better. The results for the two stages implementation scheme show a clear improvement over the single stage scheme. However, in this case the SIC technique performs visibly better. By using the IC techniques we can pack the channels much closer on a given frequency band such that there is a place for more channels, i.e., users. A possible direction for future work is to extend the algorithm with additional stages in order to improve the performance even more. In addition we can perform the interference cancellation techniques on all channels in the system the same way as we did for the main channel.
Bibliography


