

SOLVENCY II: QIS5 FOR NORWEGIAN LIFE AND PENSION INSURANCE

BY

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Abstract

This thesis describes, analyzes and applies the Solvency II on life and pension insurance by using the standard formulas in the Quantitative Impact Study 5 (QIS5) to calculate the Solvency Capital Requirement (SCR). We specifically examine the consequences for the Norwegian occupational defined benefit schemes, primarily for the private sector. The standard formulas in QIS5 to some extent specify stress scenarios without giving explicit formulas as they should be exact for the application. We therefore outline exact formulas for the Norwegian occupational defined benefit schemes. We do this both for the net expected cash flows and for the stressed cash flow. The latter we do by giving a method for calculating the stressed survival and hazard rate functions. We also price the embedded interest rate guarantee using market consistent prices from the Norwegian swaption market. We discuss bonds specifically and redistribution of cash flows generally to improve the precision. Using the contract boundary principle in Solvency II we base our calculations on that all policies are converted to paid up policies. This may primarily be relevant for pension schemes in the private sector. However, formulas for active policies are also given. Additionally one would only need the future risk premiums for market risk. At the end we perform a full QIS5 consequence study for a Norwegian pension fund, using the outlined formulas and discussing all relevant steps. To support this part we have developed algorithms in Mathematica to perform the necessary calculations.

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1 Introduction

1.1 Solvency II and QIS5

The Solvency Capital Requirement (SCR) is the regulatory amount of capital which insurance companies are required to hold in order to withstand unforeseen events. Under the current regulation, Solvency I, this is known as the solvency margin. Solvency I was adopted by the European Parliament and the Council in 2002 and was a limited reform of the earlier EU Solvency directives. During the Solvency I process it was acknowledged that a more profound reform was required to incorporate all aspects of an undertaking. A project now known as Solvency II was started to advance the shortcomings.

In December 2005 the first consequence study of the proposed solvency regulation was completed, known as the Quantitative Impact Study 1 (QIS1). This was followed up by QIS2 in June 2006, QIS3 in October 2007, QIS4 in July 2008, and finally QIS5 in November 2010, along with the developing progress of the Solvency II proposal. Implementation of Solvency II follows the Lamfalussy process where a level 1 framework directive sets out the general principles. The Solvency II framework directive, 2009/138/EU, was approved by the EU-Parliament and Council on April 22nd 2009 (The European Parliament and of the Council, 2009).

Detailed implementing measures are introduced at level 2 in the Lamfalussy process, with EIOPA¹ giving advice to the commission on implementing measures (e.g. EIOPA has developed a draft of the QIS5 technical documentation). QIS5 should according to the schedule be the last quantitative impact study before Solvency II is implemented on January 1st 2013. However, the technical specifications in QIS5 are not necessarily the final proposal for the level 2 implementing measures in Solvency II. Amendments will be likely adopted based on the results from QIS5 reporting.

Solvency II has three pillars similar to the Basel II banking regulation. Pillar 1 covers the quantitative requirements for calculating the technical provisions and solvency capital

¹ European Insurance and Occupational Pensions Authority (transformed from CEIOPS January 1st 2011)

requirements. Pillar 2 covers the internal risk management subject to supervisory review. Pillar 3 entails market discipline setting disclosure requirements.

1.2 General principles (QIS5)

The technical provisions and solvency capital requirements are based on the net expected cash flows of undertakings covering all assets, liabilities and financial instruments. This is a broad term covering both cash-in and cash-out. The technical provisions discount all cash flows on a market consistent basis. Additionally a risk margin is included to cover the cost of holding solvency capital. This follows the principle of that the technical provision should cover the price an undertaking would have to pay another undertaking for assuming the liabilities.

The Solvency Capital Requirement (SCR) is calculated as the value-at-risk using a one year time horizon and a 99.5 percent confidence level. An undertaking will then need to hold assets covering the sum of the technical provisions and the SCR. There is also the Minimum Capital Requirement (MCR) and the Absolute Minimum Capital Requirement (AMCR) setting a floor on the MCR.

The SCR may be calculated in several ways, using (European Commission, 2010a); a) full internal model, b) standard formula and partial internal model, c) standard formula with undertaking-specific parameters, d) standard formula, and e) simplification. The QIS5 technical documentation details the standard formula and simplifications where allowed for. However, the standard formula is the principle rule. The standard formula uses a modular approach, specifying the details of a stress scenario for each risk. A standard formula may or may not include an explicit formula.

Simplifications may sometimes be used if the simplified formula is proportionate to the underlying risk, and it is an undue burden for the undertaking to perform the complete calculations. The steps for assessing the proportionality assumption are outlined. Internal models may be used if they are approved by the national regulatory authorities.

1.3 Outline of the thesis

The project assignment is to describe how the stress scenarios are designed in QIS5 for life and pension insurance, and analyze how these are used for calculation of the solvency capital

requirement. This includes; a) a verbal description, b) a mathematical description, and c) a consequence study of a pension fund.

The plethora of life insurance products is immense and we will therefore confine the discussion to the Norwegian occupational defined benefits schemes. This is relevant for the consequence study of the pension fund. We will only focus on the private sector schemes. With some alterations the discussion and formulas will also apply to the public sector schemes. Furthermore, some of the discussion may be relevant for other life insurance products, and may be computed if the net expected cash flows can be accounted for. We have however specifically left out unit linked and defined contribution schemes from the formulas. These products diverge since the investment risk is born by the policyholders. Assets under management in Norway for these products are also low compared to the defined benefit schemes. Lastly we will only consider the standard formula, except from one simplification.

In chapter 2 we start out with giving the formulas for calculating the net expected cash flows for the Norwegian defined benefit schemes based on the survival models. We will also need the stressed cash flows, and these may be computed by using the stressed survival functions outlined in appendix A. In chapter 3 we introduce the necessary concepts for calculating cash flows for bonds, redistributing of cash flows, discounting of cash flows, and handling the interest rate guarantee on a market consistent basis. Chapter 4 describes the QIS5 formulas for credit risk, interpreted broadly. In chapter 5 we address the Norwegian legislation covering relevant aspects of the Norwegian defined benefit schemes. These chapters outline the necessary actuarial and financial theory for calculating the SCR. We proceed by describing the structure of QIS5 in chapter 6. We will need to calculate the technical provisions, since the assets in the insurance funds may not cover the technical provisions, and reduce the available amount of own funds for covering the SCR. However, this difference is technically not part of the SCR. We continue by describing both the modular and equivalent scenario approach. In chapter 7 we describe the financial stress scenarios and use the financial theory from previous chapters to calculate the capital charges. Analogously in chapter 8, we describe the life underwriting shocks and use the actuarial formulas (and discounting formula) for calculating the capital charges. Having explained and defined the necessary tools, we proceed with the practical assignment in chapter 9. We explain the algorithms, and discuss relevant issues and results in relation to the consequence study. At last in chapter 10, we conclude by giving some reflections of the project.

2 Life Insurance

Life insurance has existed for centuries and can be traced back to Roman times in the form of annuities when marine insurance was available. A contract called an annua consisted of a stream of payment either for a fixed term or for life and was offered by those who sold marine insurance. The earliest known guide to pricing of such contracts is dated back to 220 AD (Cannon & Tonks, 2008).

More recently in relative terms, the seventeenth and eighteenth centuries proved to be an influential period for the advancement of life insurance schemes. The Scottish Ministers' Widows' Fund², which effectively started March 25th 1744³, is often cited as the first successful life insurance fund. The Church of Scotland had earlier also made attempts to organize financial provision for the widows and orphans of its ministers in various schemes, but had failed due to inadequate support or poor organization (Dow, 1971-1973).

The Scottish Ministers' Widows' Fund scheme let the ministers choose premiums from four different levels. The premiums were initially invested in loans to member ministers at a fixed 4 percent interest rate level⁴. This simplified return calculations and enabled the fund to pay annuities to new widows in the amount £10-25 and the orphans were able to receive stock capital. The actuarial calculations were based on the world's first life table constructed by Sir Edmund Halley's in 1693 (Gerber, 1997), using the Breslau statistics. Interestingly these contracts are typically still in place with some modifications in the pension benefits scheme discussed in this thesis.

The first section in this chapter introduces the general basic survival time model followed by a description of the Gompertz-Makeham model which presumably is the most widely used model among actuaries. The second section puts the model in the context of life insurance contracts by defining the net single premiums (and equivalently the reserves) for each type of insurance contract considered in this thesis. As described in the onset of the thesis the primary focus will be on the Norwegian private occupational defined benefit schemes. The third section introduces tables of expected cash flows for the remaining lifetime of a given life.

² Church of Scotland Ministers' and Scottish University Professors' Widows' Fund

³ Approved by the Assembly in May 1743

⁴ Compulsory loans to members was ended in 1778, having proved to be "hurtful" to members and the Fund

This is an alternative approximation to the continuously discounted cash flows. However, we defer elaborating on the mapping and discounting mechanism for this approach until chapter 3. Finally in section four we will comment on underwriting risk and briefly discuss recent research that model biometric risk explicitly to allow for an actual Value-at-Risk model in contrast to the scenario based approach in standard formula in QIS5.

2.1 Survival times

In life insurance the underwriter agrees to make a single payment, or a stream of payments, contingent on predetermined “life” events unfolding (or not) in relation to the insured person. Survival time models are used to model probabilities of these events occurring over time and therefore have a natural representation as a stochastic process, see e.g. (Aalen, Borgan, & Gjessing, 2010). We will mostly refrain from this approach although section four makes some reference to this approach. In this section “survival” is used as a generic term which can have two meanings, a) that a life has not deceased, or b) that a life is not disabled.

2.1.1 The basic model

We look at a life of age x (years) as a starting point from a subpopulation (i.e. the population is subdivided into male and female). Let T represent the future lifetime of the individual, so that $x + T$ represents the time of death (or disability). T is unknown in advance and we assume that it is a stochastic variable with a cumulative density function (2.1) which gives the probability of not surviving until time $t \geq 0$. The actuarial notation for this is (2.2), while the probability of surviving past time $t \geq 0$ is denoted by (2.3). The latter is known as the survival function.

$$G(t) = P(T \leq t), \quad t \geq 0 \quad (2.1)$$

$${}_t q_x = G(t) \quad (2.2)$$

$${}_t p_x = 1 - G(t) \quad (2.3)$$

In the case of mortality rates, the survival function will tend to go to zero as t approaches infinity (or approximately zero for values greater than ω below). In contrast, a survival

function for another type of event may converge to a value within the unit interval as the full population may not experience the event under consideration (e.g. becoming disabled).

Implicitly $G(t)$ is conditional on the individual having survived past age x since $G(0) = 0$ by construction. The one year death and survival probability, ${}_1q_x$ and ${}_1p_x$, are often denoted by q_x and p_x respectively. A life table, which is also called a mortality table, is a sequence of one-year death probabilities $q_0, q_1, \dots, q_\omega$, where ω is an old age that is almost unattainable, e.g. 120 years not being futuristic about advances in medicine.

The complete distribution of ${}_tp_x$ and ${}_tq_x$ for $t \in \{1, 2, 3, \dots\}$ can be calculated recursively by the formulas below using an arbitrary life table as a basis:

$${}_tP_x = P_x P_{x+1} P_{x+2} \cdots P_{x+t-1}, \quad t = 1, 2, \dots$$

$${}_tq_x = \begin{cases} q_x & \text{if } t = 1 \\ q_x + \sum_{s=1}^{t-1} P_{x+s-1} \cdot q_{x+s} & \text{if } t \in \{2, 3, \dots\} \end{cases}$$

Identities (2.4) – (2.6) will also be useful, see (Gerber, 1997) for details:

$${}_tP_{x+s} = P(T > s+t | T > s) = \frac{1-G(s+t)}{1-G(s)} = \frac{{}_{s+t}P_x}{{}_sP_x} \quad (2.4)$$

$${}_tq_{x+s} = P(T \leq s+t | T > s) = \frac{G(s+t) - G(s)}{1-G(s)} = \frac{{}_{s+t}q_x - {}_sq_x}{{}_sP_x} \quad (2.5)$$

$${}_s|tq_x = P(s < T \leq s+t) = [1-G(s)] \frac{G(s+t) - G(s)}{1-G(s)} = {}_sP_x {}_tq_{x+s} \quad (2.6)$$

2.1.2 The hazard rate

The survival function gives the unconditional probabilities for the event occurring after time $t \geq 0$. A related function is the hazard rate⁵ implicitly assuming that T has a probability density function (i.e. $G'(t) \geq 0$ exists for all $x \geq 0$ and $t \geq 0$). In contrast to the survival function, the hazard rate is conditional on the event not happening before time $t \geq 0$. The product of hazard

⁵ (Aalen, Borgan, & Gjessing, 2010) use the term the hazard rate for (2.4) and cumulative hazard rate for the integral expression in (2.6).

rate and an infinitesimal time interval, yields the instantaneous probability of an event happening over the next infinitesimal period. The hazard rate is formally defined in (2.4).

$$\mu_{x+t} \equiv \lim_{\Delta t \rightarrow 0} \frac{1}{\Delta t} P(t \leq T < t + \Delta t \mid T > t) = \lim_{\Delta t \rightarrow 0} \frac{[1 - G(t)] - [1 - G(t + \Delta t)]}{\Delta t} \frac{1}{1 - G(t)} = \frac{G'(t)}{1 - G(t)} \quad (2.4)$$

The relationship between the two functions becomes more apparent after rewriting the right hand expression of (2.4) as the derivative of the integrated expression, which is shown below.

$$\mu_{x+t} = \frac{G'(t)}{1 - G(t)} = -\frac{d}{dt} \ln[1 - G(t)] = -\frac{d}{dt} \ln {}_t p_x \quad (2.5)$$

Finally solving for the survival function in (2.5) yields (2.6) which establish the well-known relationship between the survival and the hazard rate.

$${}_t p_x = \exp\left(-\int_0^t \mu_{x+s} ds\right) \quad (2.6)$$

At first glance it may seem only complicating introducing the hazard rate, but in survival analysis it is more common to estimate the hazard rate (or more often the cumulative hazard rate) rather the survival function directly from empirical data. A comprehensive description of models and statistical methods can be found in (Aalen, Borgan, & Gjessing, 2010). We will briefly consider the Gompertz-Makeham hazard rate function which will be used later in the quantitative part.

2.1.3 The Gompertz-Makeham hazard rate model

The model was in part developed by Gompertz who postulated that the hazard rate would grow exponentially as a function of age, illustrated by the second part of the right hand side in (2.7). In a mortality model this has the intuitive interpretation of an average ageing factor. Makeham later generalized the model by adding a constant to the hazard rate which is the first part of the right hand in (2.7). The constant tries to capture factors independent of age, e.g. like accidents. Put together this yields the Gompertz-Makeham hazard rate model.

$$\mu_{x+t} = \alpha + \beta \cdot c^{x+t} \quad (2.7)$$

The cumulative hazard rate of Gompertz-Makeham can easily be calculated as below.

$$\int_0^t \mu_{x+t} dt = \int_0^t \alpha + \beta \cdot c^{x+t} dt = \alpha \cdot t + \beta \cdot c^x \cdot (c^t - 1) / \ln(c)$$

Finally, the survival function can be found by inserting the cumulative hazard rate into (2.6).

$${}_t p_x = \exp(-\alpha t - \beta \cdot c^x (c^t - 1) / \ln(c)) \quad (2.8)$$

The Gompertz-Makeham model usually gives a reasonable approximation to mortality rates, which presumably is why it is quite popular considering its simplicity. However, it normally gives an inadequate description of mortality rates for older ages, yielding too high mortality rates (Bølviken & Moe, 2008). In spite of this, the net calculation basis for the Norwegian collective defined benefit scheme (K2005), is based on this model amongst others.

2.2 Benefits and net single premiums

A net premium for an insurance policy is calculated in such a way that the difference between expected present value of the benefits and the expected present value of the net premiums are zero. This is known as the equivalence principle. Generally, the premiums paid by the policy holders have loadings compensating for underwriting risk, operating expenses and profits, which to some degree are discretionary factors to individual insurance companies. The net premium is a pure notion and excludes all loadings and should therefore be relevant for all undertakings on a net basis.

A policyholder typically agrees to pay premiums on a periodic basis set forward in the insurance policy terms, commonly on an annually, quarterly or monthly basis. If the terms on the other hand set forward a single premium, the premium is fully paid up front. Thus, when entering the insurance policy, the net single premium and the present value of the net single premium is equal and furthermore also equal to the expected value of the benefits. This relation is useful, and we will use the net single premium approach to value the contractual obligations of an undertaking. In a fully funded pension scheme the technical reserves needs as a minimum to cover the net single premiums for the earned benefits at all times.

In this section we aim to describe basic elements of the workers benefits and lay out the formulas for calculating the net single premiums in the current Norwegian system for occupational collective defined benefit scheme primarily for the private sector. However, we

note that the system will undergo significant changes over the next couple of years in response to the Norwegian state pension reform which was implemented on the 1st of January 2011. The state pension reform is in principle an adaption to a defined contribution scheme where 18.1 percent of a taxpayer's salary up to a limit is accumulated into a taxpayer's "premium reserve" each year. The retirement age is flexible within a preset range and it is also possible to retire/work part time. The biggest change is, however, the adjustment factor for life expectancy which transfers a large part of the longevity risks from the taxpayers onto the beneficiaries.

The Government had earlier noticed that a significant part of the workers in the private sector was only covered by the state pension scheme, and therefore enacted a law which came into force in 2006 requiring all employers to run a pension scheme (OTP⁶) as part of the employee's compensation. The minimum required level is quite modest requiring only a 2% contribution of the salary into a defined contribution scheme. Although there is a tendency of employers shifting towards defined contribution schemes, the collective defined benefit schemes are still the most widespread pension scheme in Norway. The majority of the employers with pension schemes established before OTP, have a defined benefit scheme and all workers in the public sector are in principle covered by a defined benefit scheme.

There are basically four different components in the Norwegian collective defined benefit model: retirement pension after turning 67 years until death, disability pension until retirement, widows' pension and orphan's pension until the age of 18 and/or 21 years which are all life annuities paying a certain fixed amount on a regular basis (labeled net benefit in the next paragraph). As an additional component, Widow's pension may also extend to registered partners. It is also quite common to include a whole life insurance policy. We disregard this also since only life insurance companies can be licensed to offer these contracts.

Gross benefits in a pension scheme are defined as a percentage of the salary and are equal for all members within a pension scheme. The gross benefits include expected social security benefits under the prevailing social security system. This is, however, uncertain due to the risk of possible legislative reforms or possible variations of an assumed income path. The net benefits are simply the difference between these terms and constitute the actual obligations of

⁶ Obligatorisk tjenestepensjon

an undertaking for the private sector since it is illegal to guarantee gross benefits⁷. We will not discuss the formulas defining the social security benefits. These are somewhat tedious and don't involve actuarial calculations. Secondly the gross and net benefits have not been fully compatible after the state pension reform came into effect, and in practice awaiting legislative changes for the private collective pension schemes. Thirdly as already discussed insurance policies for the private sector only cover net benefits.

In the continuation we assume that the net benefits readily available and that these are paid out as a fixed continuous payment stream. Consequently, we simply need to consider the case of paying one unit continuously per year and scale these appropriately since the net single premiums will be proportional to the net benefits. We also informally introduce the discounting function ${}_s d_n$ which we define as the forward price of a zero coupon bond from time n and maturing at time $(n + s)$. We will discuss this concept more in chapter 3. In the meantime we assume a flat interest rate term structure representing the the technical reserve rate, i , and the usual discounting function ${}_s d_n = (1 + i)^{-s}$.

Below we follow the notation used in (Lillevold & Partners AS, 2010) with some slight modifications in addition to introducing the discounting function above. E_y is the net single premium for the relevant benefit with a continuous payment stream of one unit. We define n as the number of years until retirement and let it equal zero for the retired members. ω is introduced in section (2.1.1), ${}_s p_y$ is the mortality survival function and y represents the age of the insured individual. Now, using the equivalence principle and inserting the discounting function which leads to (2.9), the net single premium for a retirement benefit can be calculated straightforward by and multiplying the net retirement benefits by (2.9) (the latter is not shown). The mortality function is implicitly also dependent on the gender of the insured.

$$E_y = {}_n d_0 \cdot {}_n p_y \cdot \int_0^{\omega-y} {}_s d_n \cdot {}_s p_{n+y} \cdot ds = \int_0^{\omega-y} {}_{s+n} d_0 \cdot {}_{s+n} p_y \cdot ds \quad (2.9)$$

We now shift attention to the widows' pension rights which is paid out in the event of the insured passing away for the remainder of the widow's life time. The net single premium for this benefit is shown in (2.10) and may not be fully intuitive. The first expression is fairly straightforward noting that ${}_s p_y \cdot \mu_{y+s}$ is the marginal probability for the insured passing away

⁷ In contrast to this the public sector pension scheme is based on gross benefits.

in s years⁸, and simultaneously defining K_y^s as the forward present value of widow's benefits in s years conditional on the insured passing away exactly in s years.

$$\begin{aligned}
 E_y &= \int_0^{\omega-y} d_0 \cdot {}_s p_y \cdot \mu_{y+s} \cdot K_y^s \cdot ds \\
 K_y^s &= g(y+s) \int_0^{\omega-((y+s)-f(y+s))} d_s \cdot {}_\tau p_{(y+s)-f(y+s),2} \cdot d\tau
 \end{aligned} \tag{2.10}$$

The second expression in (2.10) however requires some more explanation. The Norwegian archetype for the collective defined benefit scheme for widows' pension is based on population demographics. Orphan's pension is treated similarly below. No information about marital status is actually required. The function $g(y + s)$ is simply the probability that an individual of age $y + s$ is married, and is conditional on the gender. Furthermore, $f(y + s)$ represents the average age difference for a married couple where the insured is $y + s$ years old, also conditional on the gender. Consequently, the age of the insured's spouse is on average $y + s - f(y + s)$ years old. Finally, we need the spouse's survival function. This is simply the survival function for the insured's opposite sex conditional on the spouse being alive when the insured passes away. The last part follows since $g(y + s)$ already accounts for the possibility that the spouse may have passed away earlier. If the benefits extend to a partner this is treated similarly replacing $g(y + s)$ in (2.10) with an appropriate analogous function. We omit further details to avoid repetition.

Calculating the net single premium for orphan's pension rights should be straightforward having worked through (2.10). The first expression in (2.11) below is perfectly identical although the function K is actually redefined. Now, shifting to the second expression, $k(y + s)$, which is the average number of children, while $z(y + s)$ is the children's average age when the insured is $(y + s)$ years. The payment stream ends when the orphans turn S_{BP} years, usually 18 or 21 years or some combination. This is early in life and the survival function is therefore approximated by the constant 1.

⁸ Which is easily obtained by multiplying (2.3) by (2.4)

$$E_y = \int_0^{\omega-y} d_0 \cdot {}_s p_y \cdot \mu_{y+s} \cdot K_y^s \cdot ds \quad (2.11)$$

$$K_y^s = k(y+s) \int_0^{s_{BP}-z(y+s)} d_s \cdot d\tau$$

In (2.10) and (2.11) we have defined the net single premiums for the widows and orphans pension rights. This is dependent on the insured passing away. We also need to consider the situation where the insured in fact has passed away and the widow and orphans are receiving benefits. This is represented by expression (2.12) and (2.13) respectively where the payment stream no longer is dependent on originally insured individual having passed away. We now regard the insured person having shifted focus from the originally insured to the widow or the orphan represented by (2.12) and (2.13).

$$E_y = \int_0^{\omega-y} d_0 \cdot {}_s p_y \cdot ds \quad (2.12)$$

$$E_y = \int_0^{s_{BP}} d_0 \cdot ds \quad (2.13)$$

Finally we need to consider disability pension benefits. So far we have only considered the insured having been able to enter two states, either being alive or having deceased. In order to calculate the disability benefits we need to introduce a third state, which is being alive but in a disabled condition. As a simplification we don't allow an individual to recover from this, i.e. being disabled until death. We assume that mortality survival function is independent of the disability state, which leave expressions (2.9) – (2.13) unchanged. Likewise, using the assumption of independence between mortality and disability, the probability of receiving disability pension at a certain time is simply the product of the mortality survival function and the probability of being disabled, the latter being $(1-{}_s p_y^{dis})$. A disabled individual will receive benefits until retirement in n years. This leads to expression (2.14).

$$E_y = \int_0^n d_0 \cdot {}_s p_y \cdot (1-{}_s p_y^{dis}) \cdot ds \quad (2.14)$$

When an individual is in a disabled state, the probability of being disabled is always one using the assumption that the recovery rate from the disability state is zero. Now, replacing $(1 - {}_s p_y^{dis})$ (2.14) by 1 immediately gives (2.15).

$$E_y = \int_0^n {}_s d_0 \cdot {}_s p_y \cdot ds \quad (2.15)$$

Expressions (2.9) – (2.15) are in principle the necessary formulas to calculate the technical reserves and solvency margin capital in subsequent chapters. It may however be more convenient to consider the net expected cash flows which we introduce below.

2.3 Net expected cash flows

A cash flow is a nominal received or paid quantity at a certain time or period of time. The QIS5 discounting helper tab uses the calendar year as a basis. We will take this as a starting point and define ${}_t E_y$ as the expected cash flow in year $t = 1, \dots, \omega$ for each benefit under consideration, when the insured is of age y old. We continue suppressing the implicit conditionality on sex. We will also use an indicator function $I(\text{logical expression})$ which is equal to one if a logical expression is true and zero otherwise.

The expected cash flows in a given year, t , can then easily be found by inserting an indicator function with an appropriate logical condition into (2.9) – (2.15) and removing the discounting function everywhere. This is shown in (2.16) – (2.22), respectively. Note that expression (2.16), (2.17) and (2.18) may involve forward starting benefits, consequently t is shifted appropriately.

$$\text{Retirement benefits: } {}_t E_y = \int_0^{\omega-y} {}_{s+n} p_y \cdot I(t-n-1 \leq s \leq t-n) \cdot ds \quad (2.16)$$

$$\begin{aligned} {}_t E_y &= \int_0^{\omega-y} {}_s p_y \cdot \mu_{y+s} \cdot K_y^s \cdot ds \\ \text{Widow's benefits: } K_y^s &= g(y+s) \int_0^{\omega-(y+s)-f(y+s)} {}_{\tau} P_{(y+s)-f(y+s),2} \cdot I(t-s-1 \leq \tau \leq t-s) \cdot d\tau \end{aligned} \quad (2.17)$$

Orphan's benefits:

$${}_tE_y = \int_0^{\omega-y} {}_sP_y \cdot \mu_{y+s} \cdot K_y^s \cdot ds \quad (2.18)$$

$$K_y^s = k(y+s) \int_0^{s_{BP}-z(y+s)} I(t-s-1 \leq \tau \leq t-s) \cdot d\tau$$

Widows receiving benefits:

$${}_tE_y = \int_0^{\omega-y} {}_sP_y \cdot I(t-1 \leq s \leq t) \cdot ds \quad (2.19)$$

Orphans receiving benefits:

$${}_tE_y = \int_0^{s_{BP}} I(t-1 \leq s \leq t) \cdot ds \quad (2.20)$$

Disability benefits:

$${}_tE_y = \int_0^n {}_sP_x \cdot (1 - {}_sP_x^{dis}) \cdot I(t-1 \leq s \leq t) \cdot ds \quad (2.21)$$

Disabled receiving benefits:

$${}_tE_y = \int_0^n {}_sP_y \cdot I(t-1 \leq s \leq t) \cdot ds \quad (2.22)$$

The indicator functions are computationally inefficient. It is, however, straightforward to factor them out by implementing them as in the listed Mathematic code in appendix C.

2.4 Biometric risk

Survival time analysis aims to identify important covariates that may affect survival times. An example is demographical factors for mortality models like age, sex, marital status, smoking habits, critical diseases that run in families, occupation, exercising habits. This is, however, beyond the scope of the thesis.

We saw in the latter section that the marital status, number of children, and age were treated as biometric averages conditionally on the insured's age and sex. It's quite likely this approach wouldn't work in a voluntary insurance scheme at the individual level since pronounced selection effects could evolve over time. In the long run this could ruin an insurance scheme.

3 Interest Rates

Albert Einstein is sometimes quoted having said that “the most powerful force in the universe is compound interest”⁹. If so, this can work two ways depending on being a creditor or a borrower. Life insurance companies and pension funds assume both roles, so it is obviously important to have a good understanding of the interest rate risks in order to make necessary adjustments to changing interest rate regimes. Attention to this has increased dramatically the last decade along with interest rates falling to very low levels compared to the recent historical standard. We will only cover the basic parts and briefly discuss yield curves, bonds and forward prices, modified duration and mapping of cash flows into interest rate vertices.

3.1 Yield curves

A yield curve, or interest rate term structure, is essentially a set of quotations for bonds or interest rate swaps of similar credit quality in the same currency, but sorted by increasing maturities. The prices may be quoted as yields. Otherwise a price may be converted to a yield by calculating the internal rate of return over the term to maturity. Zero-coupon bond prices are of particular interest since the discounted value of a cash flow can be calculated by simply taking the product of the cash flow and the zero coupon bond price having the same term to maturity.

The zero-coupon bond should be of similar credit quality as the cash flow when the discounted value is used for valuation purposes. On the other hand, if the purpose is to quantify the general interest rate risk it may be meaningful to use a benchmark zero-coupon yield curve, shock the yield curve appropriately and calculate the price change of the zero-coupon bond with relevant time to maturity. The change in value resulting from general interest rate risk may then be calculated as the product of the latter and the cash flow. However, this assumes that the relevant credit spreads are unchanged.

Yield curves are a theoretical concept since these cannot actually be observed directly in the market without making some assumptions. This is even the case when drawing a curve through

⁹ See e.g. <http://seekingalpha.com/article/263090-compound-interest-the-most-powerful-force-in-the-universe?source=feed>

a set of points representing yields for bonds across different maturities. Fortunately, term structure models have been developed and researched extensively and different models may have distinctive virtues that are suited for specific purposes.

An important objective in the Solvency II directive is to calculate the market value of assets and liabilities in undertakings simultaneously. Life insurance involves cash flows with very long terms to maturity which may well extend beyond the longest maturities quoted in the market. This introduces some issues. Firstly, more assumptions are needed to construct and extrapolate the yield curves. Secondly, there won't be a market where the interest rate risk can be immunized perfectly. In Solvency II extrapolation of yield curves is done by assuming that there exists a long term real rate and a long term inflation rate for each currency. This yields the long term nominal rate. The portions of the yield curves that extend beyond the maturities in the markets are then assumed to (mean) revert to the long term nominal rates. These are pre-calculated yield curves and will be taken as given going forward.

3.2 Zero-coupon bonds and forward prices

We informally introduced the discounting function ${}_s d_n$ in section (2.2) where we defined it as the forward price of a zero coupon bond from time n with maturity at time $(n + s)$. We should also add that the principal of the zero-coupon bond is implicitly assumed to be 1 currency unit which is received by the bond bearer at maturity. Thus, letting n equal zero we can find the spot value or present value of a cash flow of one currency unit received at maturity. When n is positive, ${}_s d_n$ is a forward price. Letting r_s be a zero-coupon yield curve described above, we can calculate ${}_s d_n$ as (3.1).

$${}_s d_n = \frac{(1 + r_n)^n}{(1 + r_{n+s})^{n+s}} \quad (3.1)$$

This follows from the definition of the zero-coupon yield curve, i.e. the internal rate of return of a zero-coupon bond as calculated in (3.2), and the usual arbitrage arguments. If the forward price differs from (3.1) there exists an arbitrage opportunity in the market which is exploited by buying the inexpensive bond and selling the expensive bond.

$$r_n = \left(\frac{1}{{}_n d_0} \right)^{\frac{1}{n}} - 1 \quad \text{and} \quad r_{n+s} = \left(\frac{1}{{}_{n+s} d_0} \right)^{\frac{1}{n+s}} - 1 \quad (3.2)$$

When the yield curve is flat, i.e. a constant, the spot price equals the forward prices and is independent of n as already assumed in section (2.2). This can be derived by replacing r_n and r_{n+s} by the constant, i , in (3.1).

Now, having described the discounting function properly we can calculate the net single premiums to market value in section (2.2) by inserting (3.1) into (2.9) – (2.15). We also introduced the expected cash flows, ${}_tE_y$, in section (2.3) and may calculate the net single premiums to market value by using (3.3).

$$V_y \approx \sum_{t=1}^{\omega} {}_t d_0 \cdot {}_t E_y \quad (3.3)$$

This is however an approximation that will slightly undervalue the benefits persistently (assuming that all forward interest rates are positive). The reason for this is that the stream of expected payments during a period is allocated to end of a period. It is, however, straightforward to improve this approximation which we will discuss in (3.4).

3.3 Interest rate sensitivity

In the financial literature, modified duration is the standard tool for managing interest rate risk. We shall in addition use this quantity to approximate the cash flow of a bond if we only have information about the market value and the duration. This will be covered below. We will again restrict the discussion to zero-coupon bonds since a coupon or interest paying bond computationally can be broken down into a series of zero-coupon bonds (which actually does trade in some markets, and are in the US known as STRIPS¹⁰).

The modified duration of a bond is derived by differentiating the (spot) bond price with respect to the yield and subsequently dividing by the (spot) bond price as shown in (3.4). In the derivation we have assumed a yearly compounding as set out in the QIS5 discounting cash flow helper tab. Thus, the bond price sensitivity to the interest rate can be calculated as a product of its modified duration and market value.

$$\frac{1}{{}_s d_0} \frac{d({}_s d_0)}{dr_s} = \frac{1}{{}_s d_0} \frac{d}{dr_s} \left(\frac{1}{(1+r_s)^s} \right) = \frac{1}{{}_s d_0} \frac{s}{(1+r_s)^{s+1}} = \frac{{}_s d_0}{{}_s d_0} \frac{s}{(1+r_s)} = \frac{s}{(1+r_s)} \quad (3.4)$$

¹⁰ Separate Trading of Registered Interest and Principal Securities

3.4 Redistribution of cash flows

JPMorgan introduced the RiskMetrics¹¹ methodology in 1994 which was a Value-at-Risk methodology for measuring market risk (J.P. Morgan, 1996) and included a dataset covering fixed income instruments, equities, foreign exchange and commodities. The interesting part in section is the simplification technique used for handling fixed income instruments. The RiskMetrics methodology mapped cash flows from fixed income instruments onto fourteen different vertices each representing a certain maturity on a yield curve ranging from 1 month to 30 year term to maturity.

The principles used for redistribution cash flows were that; a) market value should be preserved, but ignoring credit spreads, (b) market risk should be preserved and (c) cash flow sign should be unchanged. In relation to (3.3) only criteria (c) is satisfied. We will therefore suggest a mapping which approximately satisfies a) and b). The idea is simply to split the cash flow between the two nearest vertices.

Once again we confine the discussion to a zero-coupon bond maturing in $t > 1$ years. The nearest vertices are $\lfloor t \rfloor$ and $\lceil t \rceil$ assuming there are vertices for every year. In order to satisfy a) and b) simultaneously we actually need to use three vertices in the mapping algorithm. To keep things simple we are, however, satisfied if both conditions are approximately true. Equation (3.5) uses principle a), while equation (3.6) uses principle b) and yields identical approximate mapping rules.

$$\begin{aligned} \lfloor t \rfloor d_0 \cdot (1 - \alpha) + \lceil t \rceil d_0 \cdot \alpha = {}_t d_0 \Rightarrow \alpha = \frac{{}_t d_0 - \lfloor t \rfloor d_0}{\lceil t \rceil d_0 - \lfloor t \rfloor d_0} = \frac{{}_t d_{\lfloor t \rfloor} - 1}{\lceil t \rceil d_{\lfloor t \rfloor} - 1} \approx \\ \approx \frac{\left(1 + (t - \lfloor t \rfloor) \cdot \left(\frac{1}{\lceil t \rceil d_{\lfloor t \rfloor}} - 1 \right) \right)^{-1} - 1}{\left(1 + \left(\frac{1}{\lceil t \rceil d_{\lfloor t \rfloor}} - 1 \right) \right)^{-1} - 1} \approx t - \lfloor t \rfloor \end{aligned} \quad (3.5)$$

$$\frac{\lfloor t \rfloor}{1 + r_{\lfloor t \rfloor}} \cdot (1 - \alpha) + \frac{\lceil t \rceil}{1 + r_{\lceil t \rceil}} \cdot \alpha = \frac{t}{1 + r_t} \Rightarrow \alpha = \frac{\frac{t}{1 + r_t} - \frac{\lfloor t \rfloor}{1 + r_{\lfloor t \rfloor}}}{\frac{\lceil t \rceil}{1 + r_{\lceil t \rceil}} - \frac{\lfloor t \rfloor}{1 + r_{\lfloor t \rfloor}}} \approx t - \lfloor t \rfloor \quad (3.6)$$

¹¹ RiskMetrics is today commercialized and merged with MSCI in 2010.

In (3.6) we have assumed that $r_{\lfloor t \rfloor} = r_{\lceil t \rceil} = r_t$. This should be a fairly close approximation except potentially in the shorter end of the yield curve in periods where high or low policy rates lead to inverted or steep yield curves, respectively. In (3.5) we make use of two approximations. Firstly, $(1+r)^{-\alpha} \approx (1+\alpha \cdot r)^{-1}$ for $\alpha \in [0,1]$, secondly $(1-\alpha \cdot r)^{-1} \approx 1+\alpha \cdot r$ which is a first order Maclaurin series. Thus, we have established the following simple mapping rules:

- A. Allocate $t - \lfloor t \rfloor$ of the cash flow to vertex $\lceil t \rceil$
 - B. Allocate $\lceil t \rceil - t$ of the cash flow to vertex $\lfloor t \rfloor$
 - C. If $\lfloor t \rfloor = \lceil t \rceil$: Allocate the cash flow to vertex t
- (3.7)

Conveniently, t is the duration of the zero-coupon bond. We may approximate a bond's cash flow (not necessarily a zero-coupon bond) for risk measuring purposes by using only the information about duration, t years, and the market value by formula (3.8). The cash flow may then be redistributed to vertices by using the mapping rules in (3.7).

$$\text{Cash flow} \approx \frac{\text{Market value}}{{}_t d_0} \quad (3.8)$$

We discussed life annuity products in chapter 2 assuming a continuous payment stream. In section (2.3) we defined the expected cash flow of these products and formula (3.3) gives the net single premium market value. We can improve the approximation (3.3) by applying (3.7). For simplicity, we assume that the survival function is constant between each vertex and that initiation or termination of a life annuity only happens at a vertex. We can then simply split the cash flow equally between the two nearest vertices as defined by mapping algorithm (3.9). This follows from (3.7) since the duration is equal to the midpoint between two neighboring vertices. The approximation is less exact for the higher ages. But the midpoint approximation may anyway be a significant improvement since the duration for higher age are shortened relative to the midpoint.

$$\begin{aligned} \text{A. } E_y^1 &= E_y^1 + \frac{E_y^2}{2} \\ \text{B. } E_y^t &= \frac{E_y^t + E_y^{t+1}}{2}, \quad t = 2, \dots, \omega - 1 \\ \text{C. } E_y^\omega &= \frac{E_y^\omega}{2} \end{aligned} \quad (3.9)$$

We round off the discussion about cash flows by referring to the technical documentation of RiskMetrics (J.P. Morgan, 1996). This document contains methods to represent many common financial instruments as synthetical cash flows which can be useful for representing interest rate exposure as cash flows.

3.5 Swaptions

We will briefly describe swaptions as they are useful for pricing embedded interest rate guarantees in insurance policies. We will specifically address the option premium of a receiver swaption. The owner has the right to enter an interest rate swap, receiving the fixed leg and paying the floating leg. The interest rate on the fixed leg is determined by the strike of the swaption, while the interest rate on floating leg will be determined by the spot interest rate as the swaption expires. Thus a policyholder having a yearly interest guaranty may be thought of as owning a series of receiver swaptions with one year tenors ensuring that the investment returns each year are not less than the strike when the portfolio (premium reserves) is invested similarly to the floating leg.

$$\begin{aligned}
 & L e^{-r(t+1)} (i N(-d2) - F_t N(-d1)) \\
 & d1 = \frac{\ln(F_t / i) + \sigma^2 t / 2}{\sigma \sqrt{t}} \\
 & d2 = d1 - \sigma \sqrt{t} \\
 & F_t = \frac{1}{{}_1d_t} - 1
 \end{aligned}
 \tag{3.10}$$

The upper formula in (3.10) gives the premium of a receiver swaption, having; strike level i (the technical rate), one year tenor, principal L , expiring in t years, the risk free interest rate r , the forward one year interest rate F_t , volatility σ , and N is the standard cumulative normal distribution. This is the Black model applied to swaptions (Hull, 1993). If we assume that r equals F_t , all parameters except the volatility are known. If the price of a swaption is quoted in the market we may calculate the implied volatility yielding by finding the volatility that yields a theoretical premium equal to the quoted premium. Furthermore, if we do this for all expiries and strikes (holding the one year tenor constant), we will end up with a volatility surface (e.g.

see figure 9.5). Essentially, the volatility surface expresses the quoted prices as volatilities, using the Black formula as a translation rule.

We are then able to calculate the price of the embedded interest rate guarantee, using the relevant one year forward interest rates and volatility surface from the market. The premiums are calculated for each year using the Black formula. Hence, the theoretical swaption premiums are market consistent since these equal the quoted prices.

Using this approach we don't need to assume that the Black model is correct since it is only used to translate the quotations into implied volatilities. In fact, the market prices are often quoted in this way. We therefore refrain from discussing the underlying assumptions or deriving the Black model which may be found in most standard finance text books, see for instance (Hull, 1993). However, we will need to make some assumptions when using the Black model in chapter 9, in the stress scenarios. The implied volatility surface is readily available for the present yield curve. This is not the case for the stressed yield curves. We will therefore assume that the volatility surface is unchanged. Other possibilities exist, e.g. we may scale the volatilities to preserve the absolute volatilities (relative to interest rate level). This has also its weaknesses. Presumably there are more realistic models than the Black model which may improve this issue. Internal value-at-risk model may also be more appropriate enabling stochastic volatility models.

4 Counterparty risk

For simplicity, we interpret counterparty risk generally for the purpose exposition in this chapter. In QIS5 the term is limited to the counterparty risk module. The reason for the general interpretation is that both the market risk module and the counterparty default module include some elements of credit risk. This chapter seeks to give an intuitive background covering the risk calculations which are based explicitly or implicitly on ratings. We therefore compile the spread risk sub module, the concentration risk sub module, and the counterparty (default) risk module in this chapter.

The two previous chapters have discussed applicable theory and methodology, although not directly contained in the QIS5 technical documentation. This chapter differs in this respect, since all formulas are explicitly stated in the technical documentation, and therefore the standard formulas in these cases are explicit formulas. This is not the case for all (sub) modules. Ratings for each security or counterparty may be found by using one of three methods; a) official credit ratings from rating agencies, b) implicitly by solvency capital ratios using a QIS5 translation table, and c) unrated, which results in the lowest credit rating.

4.1 Spread risk

We confine the discussion to fixed income securities. This includes amongst others corporate bonds, subordinated debt, hybrid debt, mortgage backed securities, municipal bonds, and government bonds. In QIS5 the spread risk only captures the widening of the spreads. The spreads are calculated relative to their respective risk free yield curves. The standard formula (4.1) uses the relationship in expression (3.4) and a specified increase in the spreads for each rating class in QIS5.

$$\sum_i MV_i \cdot \text{duration}_i \cdot F^{up}(\text{rating}_i) \quad (4.1)$$

Spreads may also narrow. This is not accounted for in (4.1). Furthermore, rating migration is non-existent. This would typically be accounted for in a credit risk model which is not solely confined to spreads, e.g. see CreditMetrics (J.P. Morgan, 1997).

4.2 Concentration risk

Concentration risk may take several forms. On the aggregate level it could for instance apply to country and sector exposure. Another possibility is correlated investment themes. For instance, exposure to energy, materials, industrials, agriculture and emerging markets, which have been driven by the same economic super-cycle the last decade. However, the concentration risk in QIS5 is defined as the concentration of exposure against the same counterparty.

$$\left(\sum_i E_i\right) \cdot \sqrt{\sum_i \left(g_i(\text{rating}_i) \cdot \text{Max} \left[\frac{E_i}{\sum_i E_i} - \text{CT}(\text{rating}_i), 0 \right] \right)^2} \quad (4.2)$$

The proportion above a threshold level depending on each counterparty's rating is scaled by a concentration factor also depending on the rating. This is squared to yield a variance-like property. Using an assumption of being non-correlated, the formula aggregates all terms and takes the square root. The concentration risk is then found by multiplying previous result by the assets applicable to the concentration risk sub module. Thus, exposure to several counterparties above the threshold levels will still gain diversification.

4.3 Counterparty default risk

There are two models used in the QIS5 counterparty default risk module, based on Type 1 or Type 2 exposure. Type 1 is mainly credit default risk from risk mitigating contracts, either from reinsurance or finance. The exposure is often undiversified but the counterparties are usually rated. Type 2 is the remaining exposure captured by the counterparty risk module (e.g. mortgage loans and deposits if sufficiently diversified). Type 2 is simple to calculate using formula (4.3), taking a 15 per cent charge of the exposure that haven't been due for more than three months, and 90 per cent of the exposure which have been due for more than three months.

$$15\% \cdot E + 90\% \cdot E_{\text{post-due}} \quad (4.3)$$

The counterparty default risk of Type 1 is associated with more standard credit risk models, for instance CreditMetrics (J.P. Morgan, 1997). However, it relies on more complicated

assumptions. We will therefore not discuss these in detail and will only give the necessary background for calculating the risk. For an explanation we refer to the consulted advice for the counterparty risk module (EIOPA, 2009).

Formula (4.4) calculates the variance of the losses. The variance formula may at first look similar to a normal variance formula. However, the last term is non-standard. The j and k indexes are used for indexing rating categories, while the i index is used for accumulating exposures within a rating category. Formula (4.5) is calculated by setting γ equal to 0.25. The probability of default for rating category j is p_j .

$$\sigma^2 = \sum_j \sum_k u_{jk} \cdot y_j \cdot y_k + \sum_j v_j \cdot z_j \quad (4.4)$$

$$u_{jk} = \frac{p_j(1-p_j)p_k(1-p_k)}{(1+\gamma)(p_j+p_k)-p_jp_k} \quad \text{and} \quad v_j = \frac{(1+2\gamma)p_j(1-p_j)}{(2+2\gamma)-p_j} \quad (4.5)$$

Formulas (4.6) accumulate the loss given default (LGD) within each rating category and the squared LGDs.

$$y_j = \sum_i LGD_i \quad \text{and} \quad z_j = \sum_i (LGD_i)^2 \quad (4.6)$$

Formula (4.7) calculates the loss given default for each counterpart. The loss given default is calculated as a share, x, of the best estimate and the risk mitigation that is not covered by collateral. The risk mitigation of a contract is calculated according to the contracts risk mitigation effect in the QIS5 stress scenarios. The share x is either 50 percent or 90 percent.

$$LGD_i = \text{Max}[x \cdot (\text{Recoverables}_i + RM_i - \text{Collateral}_i), 0] \quad (4.7)$$

To compute the capital charge for Type 1 risk in section (7.8), the standard deviation is scaled to roughly a 99.5 percent confidence level with some additional safety adjustments, depending on the relative size of the standard deviation to the accumulated LGD as shown in formula (4.8).

$$3\sigma \text{ if } \sigma \leq 5\% \sum_i LGD_i, \text{ or otherwise } \min \left[5\sigma, \sum_i LGD_i \right] \quad (4.8)$$

5 The Norwegian legislation

In this chapter we will give a short overview of the most relevant parts of Norwegian legislation for Solvency II. The discussion will be limited to occupational defined benefit schemes managed by life insurance companies or pension funds. In the exposition we will assume that the current Norwegian legislation will continue in the present form. However, Finanstilsynet¹² has proposed to modify existing buffer funds into a single buffer fund with increased flexibility in light of Solvency II (Finanstilsynet, 2011a), which may impose severe constraints for the undertakings under the current framework. We base the discussion on (Banklovkomisjonen, 2010).

The Norwegian system is in general characterized by; 1) fully funded schemes, 2) linearly earned benefits by employees, 3) the right to transfer paid up policies¹³ 4) well defined profit sharing principles, 5) yearly capitalization guarantee, and 6) pricing of risk. We will discuss this in more detail below.

5.1 Assets and liabilities

The *premium reserves* are provisions covering the actuarial value of the benefits capitalized at a fixed rate, which is called the technical rate. The authorities define the maximum technical rate, which normally has a significant margin of safety, to the actual bond yields for longer times to maturities. The technical rate may not be a single rate at any given point in time, since lower rates sometimes are phased in gradually for existing contracts. An undertaking may however choose to use a lower technical rate than the maximum rate and can even operate with several technical rates. The *additional reserve* is a buffer fund which can be used to cover part or all of the yearly required accrual of the premium reserves by the undertaking's chosen technical rate(s). For this reason the technical rate is also known as an interest rate guarantee and is issued by the insurance company or pension fund to the insured. Both of these funds are broken down to the individual level of each insured and may be transferred to another undertaking if a policy has been converted to a paid up policy.

¹² The Norwegian regulatory authority for banking and insurance

¹³ Fripolis

The *premium fund* can be used for provisions into the premium reserves. A sponsor may use the fund to cover premiums during a year (e.g. the linearly earned benefits or due to wage increases). Another possible option is to revalue the premium reserves by a change of tariff, e.g. reducing the technical rate or increasing life expectancy or account for possible increasing trends in disablement. In this respect the fund belongs to the insured, but the sponsor decides how the provisions are used. The *retirees' surplus fund* on the other hand belongs to the pensioners and is used to regulate benefits which otherwise will stay unchanged. The interest rate guarantee is applicable to all four funds discussed in this section hitherto.

The *price adjustment fund* is a buffer fund which constitutes the unrealized profits for some parts of the liquid assets (usually, listed equities, and bonds that are not classified as hold-to-maturity). It is simply the difference between the market value and the book value of these assets, and one may tap from the fund just by realizing the profits. Thus, the price adjustment fund may be used to cover negative returns, yearly lawful accrual and returns above the technical rate. Another frequently used fund is the *risk adjustment fund* which works similarly to a buffer fund, but is classified as equity. It may be used to cover losses in the actuarial profits in the technical account.

Turning to the assets side, regulation requires that the assets are split into a collective portfolio(s) and a company portfolio. The collective portfolio(s) constitute eligible assets that must amount to the sum of premium reserves, additional reserves, premium fund, retirees' surplus fund and the price adjustment fund (not necessarily a complete list). The book return on the collective portfolio(s) constitutes the financial profit(s). The difference between actual return based on market values, except for bonds classified as "hold to maturity", and the book return equals the change in the price adjustment fund. Bonds classified as "hold to maturity" are amortized at book value over the time to maturity. The accounting effect is similar to a buffer if interest rates rises, but is then in reality a negative reserve in contrast to the other buffer funds which act as "true" risk mitigation. The sponsor(s) decide on the strategic asset allocation, directly or indirectly, and accordingly pays a return guarantee premium which we will touch on in section (5.3).

In contrast to the collective portfolio(s), the undertaking has full control of the strategic asset allocation in the company portfolio and the investment returns are directly linked to the undertaking's equity. For pension funds this division may be less clear as the sponsor(s) may

define the investment guidelines for both portfolios although not necessarily being equal, also representing the owner.

5.2 Profit sharing

Until 2008 the interest rate guarantee was paid for implicitly by the paid premiums and a profit sharing model yielding 35 percent to the undertaking and 65 percent to the client. From 2008 this was changed by new legislation to make risk pricing more transparent. Risk is now priced explicitly based on three types of profit centers; 1) financial profit, 2) actuarial profit, and 3) management profit. Any negative profits in a given year are covered fully by the undertaking's own funds, while any profits are credited in full to the clients (normally the premium fund, retirees' surplus fund¹⁴). Thus, the clients receive any profits while the undertaking bears any losses. This risk is priced individually and charged each client.

The book return on a given collective portfolio constitutes the financial earnings, while financial expenses amount to the yearly accrual by the technical rate for the applicable funds. The actuarial result is the difference between the expectations and the actual outcome in a given year with respect to the defining biometric risk categories, e.g. mortality, disability or longevity for the defined benefit scheme considered here. Over time premium loadings (safety margins) will materialize as actuarial profits if the actuarial expectations come true. Lastly, the management profit is the difference between charged management expenses for the insurance scheme subtracted by actual management cost.

The discussion above assumes that the policies are not paid up policies, and therefore have a sponsor backing the scheme to which the undertaking can charge insurance and risk premiums. This is obviously not an option for paid up policies since there is no sponsor backing the scheme. Paid up policies are therefore on a modified profit sharing model. For these contracts 80 percent of the combined profit is credited the clients while the undertaking receives the remaining 20 percent of the combined profit to compensate for the undertaken risks.

¹⁴ Provisions for the additional reserve are charged as an expense in the financial profit center.

5.3 Risk pricing

Since 2008 undertakings have been obliged to price and charge the risk associated with the clients as discussed in the previous section, assuming that a policy is not a paid up policy. The risk premiums are priced at the beginning of each year, while on the other hand any compensation for undertaking risk on paid up policy, potentially only will materialize after the accounts for the year are settled.

This seemingly difference in regimes is however not the full story. Sponsors in the private sector may terminate the defined benefit scheme¹⁵. In this case the insurance policies will be converted to paid up policies and the modified profit sharing model will apply to these contracts as well. Thus, if the pension schemes become too expensive for sponsors (or for other reasons) this may be a probable outcome leaving the undertaking bearing the risk. We will follow this line of reasoning in chapter 6.

Generally, the price of risk depends on the available buffers which can act as risk mitigation, in particular dampening the effect of short to medium term fluctuations in market values. In the long run trends in interest rates, equity returns, mortality rates and disability rates may in any case have a significant effect on the undertaking's solvency position, if risk is not priced accordingly or additional provisions are set aside to cover the changing environment.

This becomes especially critical if insurance schemes are terminated, in these events. The price of risk should reflect an undertaking's risk of covering losses from its own funds. This is a complex matter and will depend on future actions. For instance if the additional reserve is emptied in a single year, this will not be available the following year presumably increasing the risk of needing to cover subsequent losses with own funds. Even worse, if the yield curve falls below the technical rate the interest rate guarantee can only be met by taking market risk, realizing unrealized profits from the price adjustment fund, or using own funds. In the latter scenario an undertaking may be inclined to increase the bond duration in the company portfolio to hedge against lower interest rates.

¹⁵ And at a minimum establish a defined benefit scheme in accordance with OTP discussed in chapter 2.

6 Solvency II: The structure of QIS5

Hitherto we have been concerned with explaining the analytical tools from life insurance mathematics and financial theory that are necessary assist the calculations of SCR and MCR. In the next three chapters we will describe the formal computations using the standard formulas in the QIS5 technical documentation (European Commission, 2010a) and the annexes (European Commission, 2010b). The documentation covers both life and non-life insurance. Here we will only discuss the relevant parts for Norwegian life and pensions insurance, focusing on the Norwegian occupational defined benefit schemes. It may also be helpful to consult the manual for completion of the spreadsheets for solo undertakings (EIOPA, 2010a) and the QIS5 questions and answers (EIOPA, 2010b). The two latter documents are, however, not part of the formal documentation and are published by EIOPA. The interpretation of the QIS5 specification is not unique, in the sense that additional assumptions will be necessary which we will take in due course. Consequently, the standard formula does not yield a unique solvency capital requirement for a given undertaking.

This chapter outlines the calculations at the aggregate level, while the two next chapters describe the calculations for each risk category using the standard formula. Each risk category may be interpreted as a partial solvency capital requirement, which subsequently is aggregated to a total solvency requirement. This is generally lower due to diversification effects (in the modular approach).

6.1 Introduction

Figure 6.1 illustrates the general approach in Solvency II requirements, which is borrowed from (Finanstilsynet, 2010). The left side represents an undertaking's total assets at market value excluding intangibles, unless intangibles have a documented transaction value. The right hand side shows the technical provisions for the insurance liabilities, which is based on the following Solvency II principle (European Commission, 2010a):

“Solvency 2 requires undertakings to set up technical provisions which correspond to the current amount undertakings would have to pay if they were to transfer their (re)insurance obligations immediately to another undertaking”.

The technical provisions consist of the Best Estimate and the Risk Margin. In addition the undertaking needs assets to cover the Solvency Capital Requirement (SCR). The Minimum Capital Requirement (MCR) is the absolute minimal level of solvency capital for avoiding pervasive regulatory actions.

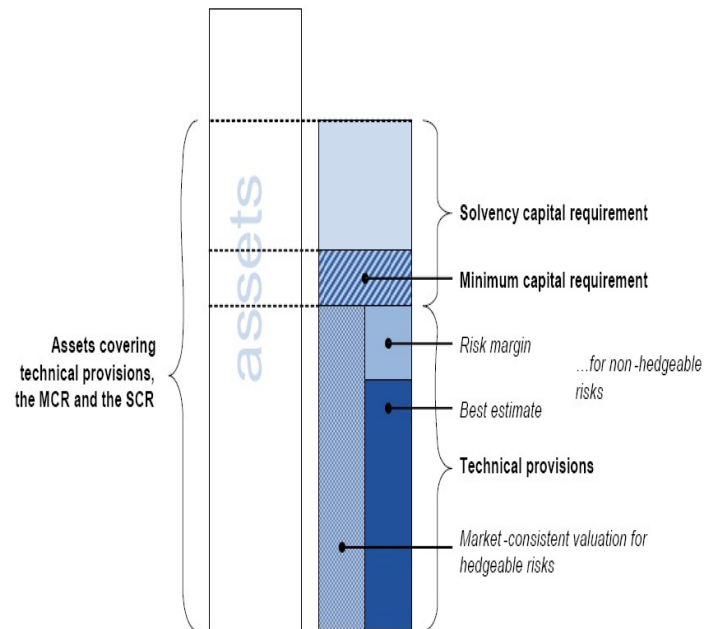


Figure 6.1: Solvency and minimum capital requirement

The Solvency II balance sheet above doesn't take account of which liabilities the assets belong to. The assets may not float as freely on the undertaking's actual balance sheet and Solvency II therefore requires that SCR and MCR must be covered by eligible own funds. This is briefly described in section (6.6).

The stress scenarios for the standard SCR formulas are explicitly given, but in theory they should correspond to a 99.5 percent confidence level over a year. In accordance with Solvency II principles they may follow a counter cyclical approach dampening the effect of the business cycle which often results in large fluctuations in asset values. Correspondingly, QIS5 is somewhat adapted to the state of business cycle at the end of 2009 with asset values having recovered only partially after the downturn in 2008. Undertakings may choose to use internal models, fully or partially, based on a Value-at-Risk approach with a 99.5 percent confidence level over a one year horizon. However, the internal model(s) must be approved by the national authorities. We note that an undertaking relying on dynamic risk management

may only account for this in internal models, which may encourage some undertakings to develop such models.

6.2 The Best Estimate

The contract boundary is a concept determining the extent of the liabilities that may be incurred from existing contracts. The contract boundary, fully or partly, is defined as the line where the undertaking unilaterally can terminate the contract or regulate premiums without limitations. In relation to the Norwegian defined benefit schemes, one must consider to which extent it is possible to change the tariffs or terminate the contracts with sponsors, which in this event will result in paid up policies. This should define the line of the contract boundary.

The best estimate concept corresponds to the discounted value of future expected cash flows until the contract boundary. Obviously one needs to take into account expected cash flows from the insurance liabilities, but the best estimate also includes future servicing expenses, management expenses, and taxes. On the other hand the undertaking has the benefit of including the expected future premiums and certain receivables, but the policyholder's behavior needs to be accounted for.

The expected cash flows should be calculated on a net expected basis excluding any safety loadings. The risk margin covers this part and standardizes the safety loadings so they are comparable. In addition the cash flows should be calculated gross of reinsurance and SPV's in order to calculate the solvency capital requirement for default risk, which is also assumed to carry over to the reference undertaking when considering the risk margin. The cash flow calculations should also be computed at the policyholder level and should cover the full lifetime. Furthermore, undertakings should take into account inflation appropriately, but not investment returns.

Simplifications are allowed for, but the technical document stresses the proportionality assumptions repeatedly. To ensure homogeneity and realistic simplifications, the best estimate should be calculated by each line of business. For life insurance there are sixteen lines of businesses resulting from two levels of segmentation; life insurance with profit participation, index-linked and unit-linked life insurance, other life insurance, and accepted reinsurance. The second level of segmentation divides this further into primary risk drivers; death, survival, disability and morbidity, and savings contracts (the risk is born by the policyholder).

The defined benefit schemes discussed in this thesis relates only to life insurance contracts with profit participation having longevity risk as the main risk driver. Unbundling of policies between sub segments is not necessary.

Using the above guidelines life and pension undertakings need to assess at least seven cash flow elements. I.e. cash flows resulting from; a) insurance liabilities, b) reinsurance contracts, c) premiums, d) servicing expenses, e) interest rate guarantees, f) future discretionary benefits, and g) taxes. In discussion we will assume that policies are paid up so we can disregard future premiums. Neither will we consider reinsurance and taxes, as our main focus is the Norwegian occupational defined benefit schemes.

We will therefore in this section calculate the best estimate as the sum of the discounted values of insurance liabilities (guaranteed benefits), the yearly interest rate guarantee, future discretionary benefits and servicing expenses. We discuss each element below.

6.2.1 Insurance liabilities

We defined the net single premiums and the net expected cash flows in chapter 2. Thus, for benefits related to the Norwegian occupational defined benefit schemes we may compute the discounted value of the expected cash flows straightforward. We do this by extending formula (3.3). The standardized cash flows are scaled by the policyholder's yearly benefits for each liability, i , from the set of available liabilities, P . Let S be the stock of policyholders, then (6.1) is the discounted value of the guaranteed benefits.

$$V|(yc, p) = \sum_{y \in S} \sum_{i \in P} Y_y^{(i)} \cdot \sum_{t=1}^{\omega} {}_t d_0(yc) \cdot {}_t E_y^{(i)}(p) \quad (6.1)$$

The formula obviously applies to other life and pension insurance contracts also, but the expected cash flows need to be estimated using methods satisfying the guidelines discussed above. The function V depends on several factors, specifically the yield curve (yc) and the survival functions (p) used to calculate the expected cash flows. We condition on these two explicitly since they capture the stress scenarios related to the liabilities.

We may also add the premium fund to the liabilities following a presentation by Finanstilsynet (Finanstilsynet, 2010). However, the premium fund may also be used to change the schemes' tariffs, which may increase the premium reserve fund without regulating the

benefits. We therefore choose to consider the premium fund as something between premium reserves and the additional reserve fund.

The QIS5 package includes the necessary yield curves, which should be used to discount the cash flows. There are several yield curves for each currency taking different levels of illiquidity premium into account. The Norwegian yield curve including the 75 percent liquidity premiums is presumably the most relevant yield curve for most life insurance contracts with profit sharing in Norway. The criteria for using a 100 percent illiquidity premium is; a) underwriting risk is only longevity and expense risk, b) no risk of surrender, and c) is a paid up policy. Criteria c) may qualify, but b) conflicts with the right to transfer the liabilities, and c) conflicts that with the occupational defined benefit schemes typically include disability insurance and insurance related to death.

6.2.2 The interest rate guarantee

There are potentially a wide range of different policyholder options that should be assessed when calculating the best estimate. However, we will only consider the so-called interest rate guarantee discussed in chapter 5. The guarantee relates to the book return to the policyholders each year. We will only consider the financial risk in this case. Net income from actuarial and management accounts will also affect the book return. Therefore, we will assume that they are independent from financial returns.

Undertakings may value the options using three methods; a) stochastic approach, either closed form or by simulation, b) scenario based with assigned probabilities, and c) a deterministic valuation based on the expected cash flows if it is market consistent. We will use method c), although it may also be interpreted as method a).

We will price the risk inherently from the yield curve. Deviating asset allocations strategies may be priced separately or viewed as the owner's risk. In some sense, this is parallel to the methodology used for calculating the risk margin (where undertakings only should include unavoidable market risk).

Assuming the book returns equal the one year forward yield curve for each year, one may price the interest guarantee as a receiver swaptions as discussed in chapter 3. These are quoted as over-the-counter contracts, but prices are available for maturities up to 30 years. Prices

may be quoted as implicit volatilities, e.g. by using the Black formula, yielding a volatility surface across strikes and expiries for one year tenors. We may therefore find the relevant volatility (i.e. price) for a certain combination of expiries and strikes. This method is market consistent and corresponds to method c). However, it may also be viewed as a stochastic model.

Having obtained the prices in chapter 3, we only need to calculate the principals. We do this by using formula (6.1) and a flat discount rate equal to the technical rate, i . We also scale the expected cash flows accordingly to the safety loading used for calculating the premium reserves. We assume that all benefits are covered by the interest rate guarantee.

$$L_n = (1 + \text{loading}) \cdot \sum_{y \in si \in P} Y_y^{(i)} \cdot \sum_{t=n}^{\omega} {}_t d_0(\text{technical rate}) \cdot {}_t E_y^{(i)}(p) \quad (6.2)$$

Now, using formula (3.10) we can calculate the value of the interest rate guarantee as in (6.3), where d_1 , d_2 , r , F_t are given as in (3.10).

$$IRG | (yc, p) = \sum_{t=0}^{\omega-1} L_t \cdot \exp(-r \cdot (t+1)) \cdot (i \cdot N(-d_2) - F_t \cdot N(-d_1)) \quad (6.3)$$

We also use the conditional denotation for IRG as the forward yield curve is implied from the yield curve and the cash flows depend on the survival functions for the stress scenarios. Formula (6.3) takes into account of the shape of the yield curve which is relevant for undertakings considering matching assets with liabilities. One should also note that realistically there are no market consistent prices for long maturities. But, in reality this is also the case for the extrapolated yield curves. As long as the technical rate is sufficiently low compared to the long term macroeconomic assumptions used in the extrapolation, this should be a minor issue.

We also note that in the case of active policies (which we have assumed do not exist) the sponsors are charged yearly risk premiums. The risk represented by (6.3) should be reflected in risk pricing charged the sponsors. However, as discussed earlier, the undertaking may end up bearing the whole risk if the risk premiums become expensive. Eventually, risking that all policies may be converted to paid up policies. In this context, our assumption that all policies are paid up, may not be completely unrealistic in a stress scenario.

In the next sub section we discuss future discretionary benefits (FDB). The value of the interest rate guarantee for the policyholders will depend on these, as the management may draw from buffer funds when needed to yield the guaranteed interest rate. In this respect the specification chosen above for modeling the interest rate guarantee is consistent with the principle that the future discretionary benefits should be calculated separately. For instance if we price the option using simulation and take into account future management actions of drawing from buffer funds, this would be inconsistent with adjusting for FDB separately. In this case it would also be necessary perform simulations holding FDB unchanged, which we intuitively believe should yield similar results to (6.3).

6.2.3 Future discretionary benefits

Future discretionary benefits are potential benefits belonging to the policyholders, and depend on being realized. This may happen by management action, e.g. by realizing profits, or indirectly through changes in the interest rate level. We will assume that the FDB is always positive in the discussion. This is not the case for under-funded pension schemes relative to the market rates. For the Norwegian occupational defined benefit scheme, the FDB is given by expression (6.4). There are, however, other possible buffers which we haven't taken account. Expression (6.4) may then be appropriately modified¹⁶.

$$FDB = br \cdot (PR - V | (yc, p) + \text{Max}[PAF + ARF - IRG | (yc, p), 0]) + (1 - br) \cdot ARF \quad (6.4)$$

The first part is the difference between the premium reserves and the discounted value of the guaranteed benefits. This is the excess value from the bond market when yields are higher than the technical rate, and the value of safety loadings. Over time this should accumulate as profits if the net expected cash flows are reasonably correct. The second part is the unrealized profits on the collective portfolio, and the additional reserves which may be used to cover the yearly accumulation by the technical rate. We subtract IRG, since the buffers may be used to cover the technical rate in periods where interest rates are too low. However, if the buffers are too low, this mitigation is not possible. The final term makes sure that the policyholder receives the whole additional reserve fund.

¹⁶ PR = premium reserves, PAF = price adjustment fund, ARF = additional reserve fund, and br = bonus rate.

6.2.4 Expenses

We won't discuss expenses, but give a simplified formula for calculating the discounted value based on the simplification for expense risk in (8.5). In this formula, following the notation in the technical documentation, i is the expected inflation rate, n is the average number of years the risk runs off weighted by renewal expenses, and E is a representative expense figure.

$$E \cdot \left(\sum_t^{\lfloor n \rfloor} (1+i)^t \cdot {}_t d_0 + (n - \lfloor n \rfloor) \cdot (1+i)^n \cdot {}_n d_0 \right) \quad (6.5)$$

This ends our discussion of the calculation of the best estimate for life and pension insurance. The risk premium will later be discussed briefly in order to calculate the full technical provisions. However, for this we will need the solvency capital requirements.

6.3 The Solvency Capital Requirement

This is the main objective of the thesis. In this part we describe the aggregate level, and the overlaying structure. This is necessary to aggregate the capital charges from each sub module using the standard formula. We will also only consider the standard formula. Simplifications may be possible conditional on qualifying assumptions.

The standard formula is based on a modular stress scenario approach. This is illustrated in figure 6.2 borrowed from the QIS5 technical documentation (European Commission, 2010a). The approach taken is to calculate the change in the undertaking's net asset value for each sub module resulting from the stress scenario. The net asset value is the difference between the assets and liabilities, which is denoted by SCR_i for (sub) module i ¹⁷. These calculations will be addressed thoroughly in chapter 7 and chapter 8, covering respectively financial risk and life underwriting risk. For all SCR_i calculations (and $nSCR_i$ calculations below) the risk margin in the technical provision are left out to avoid circularity since the SCR is used to calculate the risk margin. In addition the future discretionary benefits are kept unchanged when calculating each SCR_i , i.e. the loss absorbing capacity of the technical provisions is not accounted for. Furthermore, a positive change in net asset value is defined as a loss of net asset value.

¹⁷ The technical documentation uses different notation for capital requirements at the sub module level. In this chapter we simplify for improving the clarity in the exposition.

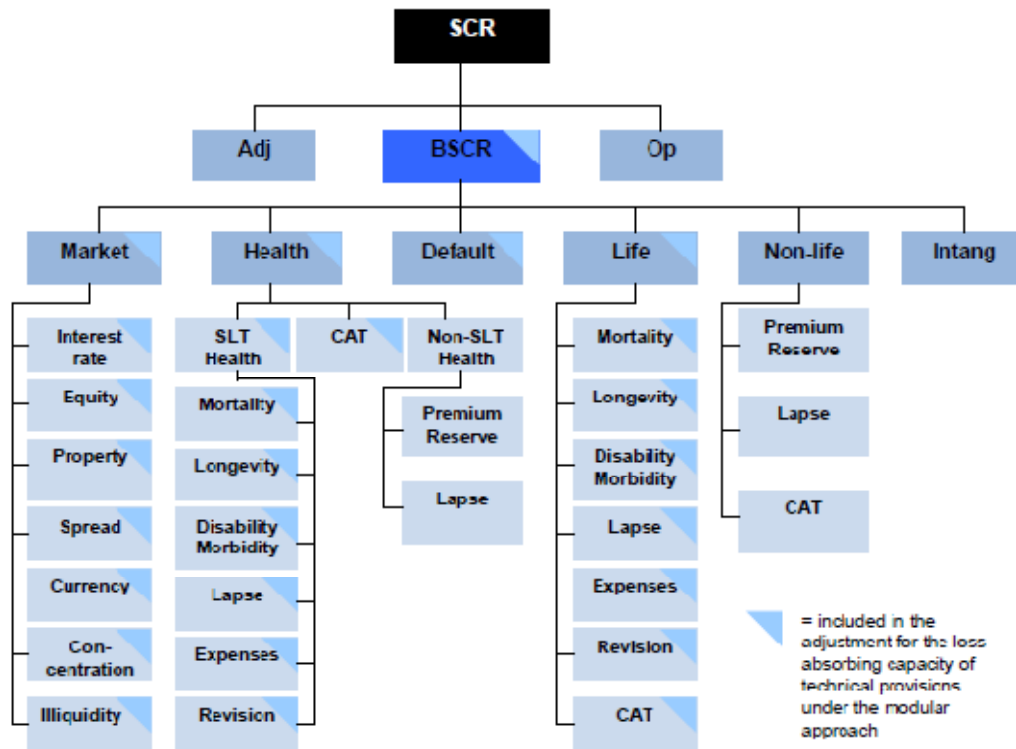


Figure 6.2: Risk modules in the standard formula

Assuming that each individual SCR_i is calculated, one may then aggregate these to the module level. The module level is depicted by level three in figure 6.2. The aggregation uses the QIS5 correlation matrices and the formula for calculating a portfolio's standard deviation with all weights equal to one, as shown in (6.6). The aggregation is performed for each module resulting in a SCR_i for each module i . For life and pension insurance, primarily market, default and life risk modules will be relevant. In addition, the disability and morbidity (SLT) sub module of the Health module may be necessary, if contracts having this risk as the primary risk driver, are unbundled.

$$\sqrt{\sum_i \sum_j SCR_i \cdot SCR_j \cdot Corr_{ij}} \quad (6.6)$$

The Basic Solvency Capital Requirement (BSCR) is similarly computed by aggregating from module to the top level. For illustrative purposes we have included the top level correlation matrix in figure 6.3. We note that there are two interest rates scenarios (higher and lower interest rates). The correlation matrix associated with the market risk module depends on which scenario is chosen, i.e. the scenario resulting in the largest loss.

i, j	Market	Default	Life	Health	Non-Life
Market	1.00	0.25	0.25	0.25	0.25
Default	0.25	1.00	0.25	0.25	0.50
Life	0.25	0.25	1.00	0.25	0.00
Health	0.25	0.25	0.25	1.00	0.00
Non-Life	0.25	0.50	0.00	0.00	1.00

Figure 6.3: Top level correlation matrix

Formula (6.7) defines the SCR. We have so far calculated the BSCR, and need in addition the adjustment term and the capital charge for operational risk.

$$SCR = BSCR + Adj + SCR_{Op} \quad (6.7)$$

SCR_{Op} is the solvency capital requirement for operational risk. We only consider defined benefit schemes and state the formula for these below:

$$SCR_{Op} = \min(0.3 \cdot BSCR, \max(0.04 \cdot (Earn + \max(Earn - pEarn, 0)), 0.0045 \cdot TP)) \quad (6.8)$$

Earn = earned premiums the past 12 months

pEarn = earned premiums over the prior 12 month period

TP = technical provisions

The Adj(ustment) term is the loss absorbency of the technical provision and deferred taxes. These should always be negative. We will assume that taxes have a minimal loss absorbing effect on the discussed insurance schemes and disregard deferred taxes in the exposition.

The calculation of the adjustment term for loss absorbency of the technical provisions must follow two methods in QIS5. These are explained in sub section (6.3.1) and (6.3.2). The reason for using two methods is that the political level has not yet decided which method will be adopted in Solvency II. However, the equivalent scenario approach should be used for calculations which depend on the SCR (e.g. possibly when calculating the risk margin).

Continuing below we will (or may) need the $nSCR_i$ for each sub module. This is the net SCR_i taking account of the loss absorbing of the technical provisions in the stress scenario. The difference between SCR_i and $nSCR_i$ at the sub module level is simply the reduced value of future discretionary benefits resulting from the stress scenario (if both terms are strictly positive). Undertakings must in each scenario calculate the loss absorbing effect cautiously to ensure that the loss absorbing capacity is not used in other scenarios. This is to avoid potential double counting.

6.3.1 The equivalent scenario approach

This approach requires the undertakings to compute a single stress scenario where all risks are accounted for simultaneously. This scenario is called the equivalent scenario and the reduction in net asset value for this scenario is denoted by $nBSCR$ (although this also being the case for the modular approach). In the calculation of the change of technical provisions one should take account of the relevant management actions that would be applied. Having calculated $BSCR$ above, the adjustment term for technical provisions is calculated by the formula (6.9). FDB is the total value of future discretionary benefits from the best estimate. The formula ensures that the assumed loss absorbing in the scenario is not larger than the maximum possible loss absorbing, which is equal to FDB .

$$Adj_{TP} = -\min(BSCR - nBSCR, FDB) \quad (6.9)$$

The method for calculating the equivalent scenario is based on the Component VaR partition which indicates the change in VaR if a component is taken away (i.e. the risk is immunized). We shall therefore label this the Component BSCR where component i , denoted by ΔSCR_i , approximate the change in the BSCR if the risk resulting from sub module i is immunized. The principle rule is to use the SCR_i s in the calculations, but there is an option to use the $nSCR_i$ s if they are more appropriate for the business and don't yield significantly different results. The Component BSCR partition can be computed directly by taking the scalar product between the vector of the SCR_i s and the gradient of the BSCR. The i th partition of the Component BSCR is calculated in (6.10). The featuring property of the Component BSCR partition is that the sum of the partitions equals the BSCR as shown in (6.11).

$$\Delta SCR_i = SCR_i \cdot \frac{\partial BSCR}{\partial SCR_i} = SCR_i \cdot \frac{\sum_j Corr_{ij} \cdot SCR_j}{BSCR} \quad (6.10)$$

$$\sum_i \Delta SCR_i = \frac{\sum_i SCR_i \cdot \sum_j Corr_{ij} \cdot SCR_j}{BSCR} = \frac{BSCR^2}{BSCR} = BSCR \quad (6.11)$$

Thus, one may calculate the SCR_i 's net of portfolio diversification ratio by dividing ΔSCR_i by $BSCR$ (but will only be true for marginal changes and using a linear model). The equivalent scenario approach uses these ratios to scale the original stress scenarios in QIS5 as shown in expression (6.12).

$$\text{Equivalent scenario}_i = \frac{\Delta \text{SCR}_i}{\text{BSCR}} \cdot \text{stress scenario}_i \quad (6.12)$$

For instance if sub module i's ratio is 0.8, the equivalent scenario reduces the risk for sub module i to 80% of the original stress scenario. The Component BSCR assumes a linear relationship which is only true for some of the sub modules. However, we may view this as a first order approximation for the non-linear cases. Chapter 9 gives a numerical example using the case study.

The advantage of the equivalent scenario is that the shocks are calculated simultaneously. Thus, it is more straightforward to compute the loss absorbing effect of the technical provision (and deferred taxes) and appropriate management actions, while avoiding double counting the loss absorbing capacity. The disadvantage is, however, that the equivalent scenario is specific for an undertaking and depends on the partial solvency capital requirements resulting from each sub module. An undertaking may therefore “optimize” the equivalent scenario by juggling the risk on the balance sheet, and ultimately also “optimize” the employed method in each sub module since there are openings for using simplifications.

6.3.2 The modular approach

The modular approach also equates the adjustment term by using formula (6.9). However, for this approach the nBSCR is calculated by aggregating up the nSCR_is from the sub module level by using the correlation matrices in the same way as when calculating the BSCR, as explained above. If an undertaking wishes to simplify the calculations it may choose to substitute one or several nSCR_is by the respective SCR_is which may result in a higher SCR.

The advantage of the modular approach is that it uses the original stress scenarios without modification. The disadvantage is however that it may be complicated to calculate the loss absorbing capacity of the technical provisions (and the deferred taxes) since the shocks are calculated one-by-one and not simultaneously. If this is not carefully calculated, this could lead to double counting of loss absorbing capacity of the technical provisions. This is not allowed for since the standard formula, using the modular approach, is essentially a capital charge for each stress scenario, while accounting for some risk reduction due to diversification. To a lesser extent it may be possible to “optimize” the solvency capital requirement by being selective where the future discretionary benefits are used.

6.4 The Minimum Capital Requirement

The Minimum Capital Requirement (MCR) for life insurance undertakings is calculated as a combination of a linear formula depending on components of the technical provisions and a minimum floor of the guaranteed benefits. Possibly overruling the linear formula, MCR must be at a minimum 25 percent of the SCR but not more than 45 percent of SCR, if the Absolute Minimum Capital Requirement (AMCR) is satisfied. The AMCR is expressed as an absolute amount in Euros. For life insurance undertakings the AMCR is EUR 3 200 000. The linear MCR formula for life insurance undertakings is given in (6.13). However, we have excluded contracts where the policyholders bear the investment risk and contracts without profit participation.

$$\text{MCR}_L = \text{Max}[5\% \cdot \text{TP}_1 - 8.8\% \cdot \text{TP}_2, 1.6\% \cdot \text{TP}_1]$$

(6.13)

TP_1 = Technical provisions for guaranteed benefits
 TP_2 = Technical provisions for future discretionary benefits

6.5 The Risk Margin

The risk margin is part of the technical provision to ensure that the technical provisions are equivalent to the amount that another undertaking would require to assume the liabilities. If the undertaking only receives the best estimate, it can only expect to earn the risk free rate on the risk capital and should therefore not be able to attract risk capital, being unable to meet the solvency capital requirements. Since risk capital is expensive, corporations have a natural incentive to optimize the use of risk capital. The required risk capital for an insurance undertaking is the SCR. The technical provisions should therefore include a loading for servicing the appropriate level of SCR when undertaking the liabilities. This is the risk margin. In QIS5 the Cost-of-Capital is 6% above the risk free rate¹⁸.

We will only outline the principles for calculating the risk margin briefly. Firstly, it is assumed the undertaking at all times only holds the required solvency capital as the risk runs off the balance sheet. Secondly, the risk margin only accounts for the necessary SCR to assume the liabilities. Thus, the risk margin only covers the unavoidable market risk, life underwriting risk, the default risk due to reinsurance contracts or SPV's and operational risk. Thirdly, the future discretionary benefits carry over to the new undertaking. Finally, the SCRs

¹⁸ Undertakings replicating the liabilities should only be charged for operational risk.

should be discounted with the risk free rate, and not include any illiquidity premiums. The risk margin is 6% of the discounted SCR_s (in QIS5).

As discussed in section (6.3) the risk margin should be based on the equivalent scenario. This is convenient since one may simply add the capital charges resulting from the relevant stresses. However, one must account for unavoidable market risk which is usually not part of the calculation, unless an undertaking seeks to minimize risk. Unavoidable market risk is assumed to come from the cash flows where the maturities are longer than the available maturities from the risk-free financial instruments.

In order to calculate the risk margin we need to calculate the appropriate SCR for every year as the risk runs off. This may be cumbersome for some sub modules. For instance if the stress scenario the first year is different from the subsequent years, undertakings will need to run the SCR calculations for all relevant future years (e.g. the disability shock). To facilitate this, there are several possible simplifications. The simplest formula for life undertakings is the level four of the hierarchy, where one roughly takes the product of the appropriate current SCR, the modified duration of the obligations, and the Cost-of-Capital discounted one year. In order to use this simplification the undertaking also needs to estimate the unavoidable market risk. There is also a simplified formula for this. It is the product of the following terms; the part of modified duration of obligations extending beyond the market, the interest rate shock for the maturity of the longest financial instrument, the average number of years the unavoidable market risk exists, and the best estimate.

6.6 Own funds

Solvency II defines the own funds which may be used to cover the solvency capital requirements. Own funds are divided into three categories depending on the characteristics of the capital. The characteristics are described along three dimensions; availability, durability, and the ability to absorb losses during normal business activity. We will only give a simplified description. Tier 1 capital comprises of equity with full loss absorbing capability and certain hybrid capital within a maximum limit. The equity is in the context of Solvency II roughly defined as the difference between the assets and liabilities. Tier 2 capital consists of sub ordinate debt of a certain quality and equity with limited ability to absorb losses. Tier 3 capital includes essentially capital elements not qualifying for Tier 2 capital.

The SCR must be covered by minimum 50 percent Tier 1 capital and maximum 15 percent Tier 3 capital. The MCR normally being less than SCR, has stronger requirements. At least 80 percent must be covered by Tier 1 capital, and the remaining may only be covered by Tier 2 capital.

7 Financial Risk

Life and pension undertakings are generally exposed to financial risk in two ways. Either through investments and counterparties, or indirectly through the time value of money used for discounting the expected cash flows of the insurance liabilities. The investment and counterparty risk results in actual losses on the financial accounts when being realized. Losses from the latter often play out over time being realized through lower income on the financial accounts. But, when income doesn't cover the charge by the technical rate, undertakings will need to use own funds to cover the difference.

In more extreme cases undertakings may also need to use own funds to cover increases in the premium reserves. This especially applies when there are no sponsors backing the scheme. If long term interest rates are close to or below the technical rate, the maximum allowed technical rate may be lowered by the authorities. This will increase the premium reserves by the discounting mechanism. Similarly for financial risk, if the market value of the assets falls below the technical reserves, undertakings will need to use own funds to cover the difference.

This chapter covers the market risk module and default risk module, which we discuss at the end. We describe each sub module which may be aggregated to BSCR level using the relevant correlation matrices and formula (6.6). We note that the default module is not a sub module and therefore should be used directly as input at the top level. Furthermore, there are two methods to aggregate to the nBSCR level. Both are discussed in chapter 6.

Before discussing each risk we will introduce the necessary formulas for calculating the gross and net solvency capital charges resulting from the stresses. We denote the total market value of the collective portfolio and company portfolios respectively as AK and AS . We may then compute and undertakings loss or profit from a change in market values using expression (7.1). The bonus rates (br) in the profit sharing models discussed in chapter 5, are either 80 percent or 100 percent of the profits received by the clients on the collective portfolio, while any losses are covered by the undertaking. The company portfolio is the undertaking's portfolio and consequently receives all profits and bears all losses.

$$\Delta A = (1 - br) \cdot \text{Max}[\Delta AK, 0] + \text{Min}[\Delta AK, 0] + \Delta AS \quad (7.1)$$

Equation (7.2) and (7.3) will only be affected by interest rates in this chapter. Hence, we may define them equal to zero when the yield curve is not stressed. This is done in order to generalize expression (7.4) – (7.7) below to all (sub) modules. Expression (7.2) gives the change in the discounted value of the guaranteed benefits by using (6.1) for both the stressed and the normal yield curve. Similarly, (7.3) yields the change in market value of the embedded interest rate guarantee using (6.3).

$$\Delta L = V | (yc_{stress}, p) - V | (yc, p) \quad (7.2)$$

$$\Delta IRG = IRG | (yc_{stress}, p) - IRG | (yc, p) \quad (7.3)$$

The gross capital charge may then be computed by formula (7.4). When assets and liabilities are matched, the expression inside the brackets should yield values close to zero¹⁹. Capital charges may not be negative, so we ensure that it is positive by using the Max operator.

$$Mkt_i = \text{Max}[\Delta L + \Delta IRG - \Delta A, 0] \quad (7.4)$$

We calculate the loss absorbing of the technical provisions in (7.5) and (7.6). The first expression shows the loss absorbing of the premium reserves. This cannot be larger than the capacity, which is the difference the premium reserves and the discounted value of the obligations before the stress scenarios has occurred²⁰. The expression otherwise follows the same reasoning as in sub section (6.2.3).

$$\Delta FDB_1 = -br \cdot \text{Min}[\Delta L, PR - V | (yc, p)] \quad (7.5)$$

Expression (7.6) is slightly more complicated. An undertaking may draw from the price adjustment fund (PAF) and/or if qualified from the additional reserve fund (ARF) to cover losses in the collective portfolio and/or an increase in value of the embedded interest guarantee. The last part of the equation takes into account that the additional reserve fund absorbs the covered loss completely independent of the bonus rate.

$$\Delta FDB_2 = -br \cdot (\text{Min}[-\Delta PAF_i - \Delta ARF_i, \Delta IRG - \Delta AK]) - (1 - br) \cdot \text{Min}[-\Delta ARF_i, \Delta IRG - \Delta AK] \quad (7.6)$$

¹⁹ In this case we assume that the company portfolio is also used to match liabilities.

²⁰ The illiquidity scenario turns out to be insignificant for Norwegian yield curves (see chapter 9), so we have simplified the equation by assuming that we in practice only need to bound the loss absorbing in the interest rate stress scenarios.

We can finally calculate the net capital charge after accounting for the loss absorbing of the technical provisions by using (7.7). The net capital charge should not be larger than the gross charge since the gross charge is bounded by zero, i.e. negative capital charges are not allowed for in profitable scenarios. We therefore ensure that the net figure is equal or less than the gross figure.

$$nMkt_i = \text{Min}[Mkt_i + \Delta FDB_1 + \Delta FDB_2, Mkt_i] \quad (7.7)$$

Undertakings must also ensure that the accumulated use of future discretionary benefits in each scenario is less than the capacity in the modular approach. We therefore impose restrictions (7.8) and (7.9) to guarantee this.

$$\sum_i -\Delta PAF_i \leq PAF \quad (7.8)$$

$$\sum_i -\Delta ARF_i \leq ARF \quad (7.9)$$

Having outlined the necessary equations we now describe each (sub) module. We have explicitly addressed the change in values of guaranteed benefits and the embedded interest rate guarantee in expression by using formulas in (7.2) and (7.3). However, we have not addressed the formulas for calculating the change in asset values. This will depend on each (sub) module, and we will address this in each sub section below.

7.1 Interest rate risk

Purpose: To account for interest rate risk from all assets and liabilities sensitive to yield curves and/or interest rate volatilities. Both nominal and real yield curves should be accounted for. The sub module does not extend to assets indirectly sensitive to interest rates (e.g. equity and property).

Definition: There are two (instantaneous) scenarios that should be evaluated; a) higher interest rates, and b) lower interest rates. Undertakings exposed to yield curves in several currencies should calculate the capital charge resulting from all yield curves in a combined scenario. The shocked yield curves are constructed by scaling the appropriate yield curves by factors depending on the time to maturity. In scenario a), the factor ranges from 1.70 in the

short end to 1.25 for 30 years and longer maturities. In scenario b), the factors range from 0.25 to 0.70. The absolute change in interest rates should at a minimum be one percentage point. Nominal interest rates are however bounded below by zero (this does not apply to real interest rates). We refer to the QIS5 technical documentation of a complete specification of the yield curve shift (European Commission, 2010a).

Calculation: The expected cash flows from the insurance obligations are already accounted for by using (7.2) and (7.3). We therefore only need to estimate the cash flows from the interest rate sensitive assets. We will only consider bonds and refer to chapter 3 for a discussion. If the cash flows are readily available, one may use the mapping algorithm (3.7) to redistribute the cash flows to the nearest interest rate vertices. Otherwise one may use (3.8) to approximate cash flows and proceed with (3.7) thereafter. Continuing one may use (3.3) to calculate (7.1), and the loss absorbing of the technical provisions is found by using (7.5) and (7.6). Finally, the gross and net capital charge may be estimated using (7.4) and (7.7). We have not explicitly addressed the multi-currency case. This may simply be performed by iterating the steps above for one currency at a time, while accumulating. The liability equations however only reflect the Norwegian pension system covering only liabilities in local currency, so (7.2) and (7.3) should be zero for all other currencies.

The steps above need to be calculated for both interest rates scenarios. The interest rate scenario that yields the highest net capital charge is chosen for calculating the solvency capital requirement. This will also define which correlation should for the market risk module.

7.2 Equity risk

Purpose: To account for equity risk resulting from the price level or price volatility in the equity markets. The sub module extends to all assets and liabilities which are exposed to equity risk.

Definition: Equities are categorized into two groups; a) “global”, and b) “other”. The QIS5 stress scenarios are an immediate fall of 30 percent in the “global” category, and 40 percent in the “other” category. All listed equities on stock exchanges in EEA and/or OECD member countries belong to the “global” category. So for the purpose of calculating the capital charge, Norwegian listed equities are “global”.

However, there are some exceptions. Equity participations in financial and credit institutions get a capital charge of zero, while the same rate for other strategic participations is 22 percent. The latter is not confined to EEA and/or OECD countries.

Calculation: For each category we take the product of the price fall and the market value or the equity exposure. We may then proceed by using formula (7.1) and (7.4) to calculate the gross capital charge, and continuing with (7.5) and (7.7) to find the net capital charge. The equity categories may then be aggregated using the equity correlation matrix and formula (6.6). We have implicitly assumed that there is no equity exposure in the liabilities. Furthermore, we have also assumed that the price sensitivity of the equity instruments is constant. This may easily be refined using appropriate pricing models, which we will not cover in the thesis. Finally, we note that undertakings may not take into account dynamic hedging strategies when using the standard formula in QIS5, i.e. undertakings can only take into account positions that are in place at the valuation date.

7.3 Property risk

Purpose: To account for property risk resulting from the price level or the price volatility in the property markets. The sub module extends to all assets and liabilities which are exposed to property risk.

Definition: The change in the net asset values from an instantaneous fall of 25 percent in the property prices.

Calculation: We may calculate the net and gross capital charge using the same steps as above. In addition, listed real estate companies are covered by the equity risk sub module and should not be included. Furthermore, property funds are typically leveraged. Undertakings will in these cases need to calculate the property exposure instead of using the market value of the property funds.

7.4 Spread risk

Purpose: To capture the risk from widening spreads or volatility of the spreads relative to the risk free yield curves in both assets and liabilities. This risk is for instance applicable to

government bonds issued by non-EEA members, corporate bonds, covered bonds, subordinated debt, hybrid capital, and credit derivatives.

Definition: The change in net asset value using formula (4.1) for bonds and the QIS5 defined spread risk factor for each category. An additional matrix of spread risk factors is provided for non-EEA government bonds. There are also specific duration floors and caps which should be accounted for in the formula. Formulas for structured products and credit derivatives depending on spread risk are given in the QIS5 technical documentation.

Calculation: We may calculate (7.1) by using (4.1). We have assumed that (7.2) and (7.3) do not depend on spread risk. So, we may carry on by using (7.4), (7.6) and (7.7) to calculate the net and gross capital charge.

7.5 Currency risk

Purpose: To account for foreign exchange risk from all sources within an undertaking resulting from the changes in price levels or the volatility of the prices. Both assets and liabilities are included in the sub module.

Definition: The change in net asset value, resulting from two instantaneous scenarios using the undertakings local currency as basis²¹; a) a 25 percent appreciation of each currency, and b) a 25 percent depreciation of each currency.

Calculation: We may then calculate the net and gross charge similarly to the calculations for the equity risk, for each currency risk scenario separately. The scenario with the highest net capital charge is used for calculating the solvency capital requirement. We don't address foreign exchange risk on the insurance liabilities for the same reasons as in section (7.1).

7.6 Concentration risk

Purpose: To capture concentration risk arising from having exposure to the same counterparty. The sub module only accounts for exposure considered in the equity, property and spread risk sub modules. In particular, the exposure accounted for in the credit default module should not be included.

²¹ The local currency is defined as the currency the undertaking prepares its financial statement in.

Definition: Formula (4.2) defines the concentration risk. Thresholds levels and concentration factors depending the rating is specified in the QIS5 technical documentation.

Calculation: We may calculate (7.1) using (4.2). Otherwise the calculation is identical as for the spread risk in section (7.4).

7.7 Illiquidity risk

Purpose: The illiquidity premium is incorporated in the QIS5 yield curves and results in lower technical provision. When the illiquidity premium falls the technical provisions will rise. This sub module therefore accounts for a fall in the illiquidity premium, but not a rise. The technical documentation (European Commission, 2010a) states that: *“The effect of an increase of the illiquidity premium is captured in the calibration of the spread risk module²²”*.

Definition: The change in the net asset values resulting from an instantaneous 65 percent fall in the illiquidity premium.

Calculation: The capital charge may be calculated by following the steps for calculating the interest rate risk, while using the stressed yield curves from the illiquidity premium shock. The multi-currency may also be relevant.

7.8 Counterparty risk (module)

Purpose: This module captures counterparty default risk not accounted for in the spread risk sub module. Risk mitigating contracts, such as derivatives and reinsurance arrangements, deposits, receivables and loans belong to this module. The counterparty default risk is defined as possible losses due to unexpected default or deteriorating credit standings.

Definition: There are two types of counterparty risk exposures, Type 1 and Type 2. These are discussed in section (4.3).

Calculation: Formula (4.3) defines the capital charge for Type 2 risk. The capital charge for Type 1 risk may be calculated using formula (4.8) and the specified default probabilities in the QIS5 technical documentation. The capital charge for the counterparty default module

²² We are however not convinced about this argument since the spread risk module is primarily related to bonds, subordinated debt and credit derivatives. Insurance obligations are usually unaffected by higher spreads in QIS5.

may then be calculated by using formula (6.6) on both Type 1 and Type 2 risk, using a correlation coefficient of 0.75. We can then proceed by calculating (7.1). The steps thereafter are identical to the spread risk in section (7.4).

8 Life Underwriting Risk

In this chapter we will describe the standard formula for the life underwriting module. All sub modules are relevant for life and pension insurance underwriting risk. We end this chapter by briefly referring to the disability and morbidity sub module in the SLT²³ Health Module for the purpose of completeness. This may be relevant for life insurance, but not for pension funds in Norway which only cover disability benefits as part of an occupational defined benefit scheme. Unbundling is in this case unnecessary and the risk is represented by the disability and morbidity sub module of the life underwriting module. Finally, we note that the sub modules in this chapter may be relevant for non-life insurance contracts which can give rise to life annuities.

Unbundling of insurance policies involves splitting contracts into its basic parts or groups of basic parts. Let P denote the set of available insurance products. A policy is a subset $U_y \subseteq P$ with coverages $Y_y^{(i)} > 0$ for $i \in U_y$. This is a slight abuse of notation since we use y to represent both the age and the life itself. We assume that an insurance policy only contains one principally insured. Otherwise, each insurance policy needs to be split into contracts of single principal lives.

We may unbundle the insurance products sharing one or several common characteristic, e.g. a primary risk driver, by forming the subset $U \subseteq P$ of insurance products sharing the common characteristic(s) and taking the disjunction $U_y \cap U$. The discounted expected value of the unbundled part of a life insurance policy can then be computed by formula (8.1).

$$V_y(U) | (yc, p) = \sum_{i \in U_y \cap U} Y_y^{(i)} \cdot \sum_{t=1}^{\omega} {}_t d_0(yc) \cdot {}_t E_y^{(i)}(p) \quad (8.1)$$

Furthermore, let S represent the stock of insurance policies. We may then calculate the change in the discounted expected value of the liabilities resulting from a stress scenario by using (8.2). $p\Delta$ indicates that the shocked survival function(s) should be used for calculating the stressed cash flows, while p indicates using the expected survival function(s) as before.

²³ Similar to Life Techniques

$$\Delta L(U) | (yc, p, p\Delta) = \sum_{y \in S} \text{Max}[V_y(U) | (yc, p\Delta) - V_y(U) | (yc, p), 0] \quad (8.2)$$

Note that (8.2) aggregates on a policy-by-policy level. This allows for no diversification between different principally insured lives since only increases in liabilities are accounted for. On the other hand, if $|U| > 1$, (8.2) allows for diversification effects for risk depending on the same life. This representation follows the principles set forward by the QIS5 technical documentation.

Let U be the relevant level of unbundling for Life sub module i . (8.3) is then the gross solvency capital requirement for this risk without accounting for loss absorbency of the technical provisions. If the profit sharing is on the actuarial accounts level, then this is also the net requirement (8.4). This is the case for active policies in the Norwegian defined benefit schemes as discussed in chapter 5.

$$Life_i = \Delta L(U) | (yc, p, p\Delta) \quad (8.3)$$

$$nLife_i = \Delta L(U) | (yc, p, p\Delta) \quad (8.4)$$

In the Norwegian system paid up policies have profit sharing on the combined profit center as also discussed in chapter 5. Assuming that the combined profit is non-positive, an undertaking may draw on the price adjustment fund (PAF_i) to cover the loss fully or partially, depending on the bonus rate. This equates to the net solvency charge in (8.5), and may be viewed as a management action in line with the following formulation in the QIS5 technical document (European Commission, 2010a):

“Additionally, the result of the scenario should be determined under the condition that the value of future discretionary benefits can change and the undertaking is able to vary its assumptions in future bonus rates in response to the shock being tested”.

Below the bonus rate is not allowed to change, but the undertaking may realize unrealized profits from the price adjustment fund appropriately.

$$nLife_i = (1 - br) \cdot \Delta L(U) | (yc, p, p\Delta) + br \cdot \text{Max}[\Delta L(U) | (yc, p, p\Delta) - PAF_i, 0] \quad (8.5)$$

Additionally the undertaking needs to check that the sum of the draws for all sub modules, not limited to the sub modules considered in this chapter, is less than the capacity resulting in the

restriction represented by (8.6). An undertaking may also consider using the additional reserve fund (ADR), but the fund is significantly more restricted as discussed in chapter 5.

$$\sum_i PAF_i \leq PAF \quad (8.6)$$

We now turn to discuss each relevant sub module, having discussed some principles and defined the necessary formulas for the three first considered sub modules. We describe each sub module based on purpose, definition of the standard formula, unbundling, if simplification is allowed and discuss the Norwegian occupational defined benefit scheme (NODBS) outlined in chapter 2 in the context of each sub module. The definitions are enclosed in quotation marks being taken directly from the QIS5 technical documentation (European Commission, 2010a). The capital requirement for each sub module is the change in net asset value resulting from the applied method.

8.1 Mortality risk

Purpose: To account for mortality risk associated with (re)insurance obligations resulting from death of a policyholder.

Definition: “A permanent 15 percent increase in mortality rates for each age and each policy where the benefits are contingent on mortality risk.”

Unbundling: Insurance policies covering benefits in the events of both death and survival of the same life don’t need not to be unbundled. The natural diversification of mortality and longevity risk associated with the same person is allowed for.

Simplification: Is possible at the aggregate level if the standard method is an undue burden for the undertaking and the proportionality criteria is qualified.

NODBS: Mortality risk does apply since widow’s and orphan’s benefits depend on death of the policyholder. There is however no need for unbundling the contracts since all terms, represented by (2.9) – (2.15), depend on the life status of the policyholder, i.e. $U = P$. Furthermore, widow’s benefits and orphan’s benefits are typically sufficiently low compared to the policyholder benefits to mitigate the mortality shock completely for paid up policies. However, this may not be the case for active policies. Expression (8.3) and (8.4) or (8.5) may

be used to calculate the capital charge for mortality risk according to the standard formula. The stressed mortality survival function(s), $p\Delta$, may be computed by using the method in appendix A and setting $F = +15$ percent.

8.2 Longevity risk

Purpose: To account for longevity (or survival) risk associated with (re)insurance obligations resulting from a decrease in mortality rates, typically stemming from life annuities or term insurance.

Definition: *“A 20 percent (permanent) decrease in mortality rates for each age and each policy where the benefits are contingent on longevity risk.”*

Unbundling: Same issues as for mortality risk.

Simplification: Is possible at the aggregate level if the standard method is an undue burden for the undertaking and the proportionality criteria is qualified.

NODBS: Longevity risk is the primary risk driver and does apply since the benefits to the policyholder depends on survival. No need for unbundling equivalently to the case for mortality risk, i.e. $U = P$. We use (8.3) and (8.4) or (8.5) to calculate the gross and net capital charge longevity risk by the standard formula. We compute the stressed mortality survival function(s), $p\Delta$, by using the method in appendix A and setting $F = -20$ percent.

8.3 Disability and Morbidity risk

Purpose: To account for the disability or morbidity risk resulting from (re)insurance obligations contingent on a definition of disability. The risk is characterized by changes in level, trend or volatility of disability rates.

Definition: *“An increase of 35 percent in disability rates for the next year, together with a (permanent) 25 percent increase in disability rates at each age in following years. Plus, where applicable, a permanent decrease of 20 percent in morbidity/disability recovery rates”.*

Unbundling: This sub module is only applicable where it is not appropriate to unbundle contracts. Otherwise the risk should be handled in the Health SLT disability and morbidity sub module.

Simplification: Is possible at the aggregate level, if the standard method is an undue burden for the undertaking and the proportionality criteria is qualified.

NODBS: The sub module is applicable when NODBS includes disability pension with earned rights. Otherwise the risk, if existent, should be handled by the Health SLT underwriting module. In the calculation we set $U = \{\text{disability insurance}^{24}\}$ since this is the only part depending on disability rates. However, in a model where mortality rates depend on disability rates there would likely be some diversification and we would use $U = P$. Now, continuing as above, we use (8.3) and (8.4) or (8.5) to calculate the gross and net capital charge for disability and morbidity risk by using the standard formula. We compute the disability survival function(s), $p\Delta$, by using the method in appendix A and setting $F_1 = 35$ percent and $F_2 = 25$ percent. We implicitly use a recovery rate of 0 percent, so a permanent decrease of 20 percent in recovery rates is not feasible and therefore not applicable.

8.4 Lapse risk

Purpose: Account for lapse risk which is defined as: “*the risk of loss or change in liabilities due to a change in the expected exercise rates of policyholder options*”. The assessed policyholder options are the legal and/or contractual policyholder options such as, terminations, renewals, extensions, that would significantly alter the future cash flows.

Definition: The definition is comprehensive. In broad terms, it is basically the maximum change in net asset value resulting from one of three possible scenarios; 1) exercise rates increase by 50 percent, but not to a higher absolute level than 100 percent, 2) exercise rates decrease by 50 percent, but not by more than 20 percentage points reduction in absolute level, and 3) a mass lapse of 30 percent of policies with (positive) surrender strain for retail business and 70 percent for non-retail business.

Unbundling: Necessary, following the policyholders’ legal and contractual options.

²⁴ Disability insurance is understood as not receiving (fully) disability pension at the valuation date.

Simplification: Is possible, must be but calculated at an appropriate granularity, and only if the standard method is an undue burden for the undertaking and the proportionality criteria is qualified.

NODBS: This can be applicable, but we have no empirical data covering this issue. If an employee leaves a company the employee has an option to continue in NODBS by paying premiums privately. In this case married employees with children may have bigger incentive to extend since NODBS discussed in chapter 2 is based on population means. Likewise could there potentially be a selection effect occurring from individuals having personal health information. We will however neglect this risk since the tax incentives is on the employers' hands which should moderate the incentive. This is not relevant in regards to pension funds, since this type of insurance must be contracted through a life insurance company.

8.5 Expense risk

Purpose: To account for the risk of increases in servicing expenses for (re)insurance contracts in the future. Expense payments that are fixed at the valuation date should be excluded from the analysis, and one should also take into consideration realistic management actions for policies with adjustable expense loadings.

Definition: *“Increase of 10 percent in future expenses compared to best estimate anticipations, and increase by 1 percent per annum of the expense inflation rate compared to anticipations.”*

Unbundling: Conditional on expense loading structure.

Simplification: Possible conditional that the standard method is an undue burden for the undertaking and the proportionality criteria qualifies. We will use the proposed simplification in chapter 9 and state the formula in (8.7). The input variables are; 1) E = servicing expenses during the last year for life insurance contracts, 2) n = average number of years the risk runs off weighted by renewal expenses, and 3) i = the expected inflation rate. The expected inflation rate used in the extrapolation of the Norwegian risk free yield curve is 2 percent.

$$Life_{exp} = 10\% \cdot n \cdot E + \left(\frac{(1+i+0.01)^n - 1}{i+0.01} + \frac{(1+i)^n - 1}{i} \right) \cdot E \quad (8.7)$$

NODBS: Applicable. The formula above yields a gross capital charge. We may arrive at a net figure by using (8.4) or (8.5), where the gross change in liabilities is defined as in this section rather than (8.3).

8.6 Revisions risk

Purpose: To account for the revision risk associated with the state of health of a policyholder or possible changes in legal environment affecting an annuity.

Definition: *“Increase of 3 percent in the annual amount payable for annuities exposed to revision risk. The impact should be assessed considering the remaining run-off period of the annuities.”*

Unbundling: Yes, the sub module should only be applied to annuities where the payable benefits may increase as a result of revision risk.

Simplification: Not possible.

NODBS: Applicable for earned disability pension benefits not called for. Thus we take $U = \{\text{disability insurance}\}$. In addition we need to modify (8.1). Instead of using the expected cash flows relevant for this benefit, we will calculate the discounted value as if the policyholders are disabled. This is done by shifting the expected cash flow index to $\{\text{receiving disability pension}\}$ in (8.1). The capital charge is 3 percent of the total discounted value in (8.1). Note that the technical documentation doesn't state a net figure, so for calculation purposes in the continuation we will use $\text{net capital requirement} = \text{gross capital requirement}$.

8.7 Catastrophe risk (CAT)

Purpose: To account for the risk of major irregular events resulting in the death of policyholders, e.g. pandemic events, nuclear explosions and earth quakes.

Definition: *“Absolute increase in the rate of policyholders dying over the following year of 0.15 percentage points (only applicable to policies which are contingent on mortality).”*

Unbundling: Yes, the sub module is restricted to (re)insurance obligations contingent of mortality. The technical documentation doesn't explicitly allow for the natural diversification

from benefits depending on both death and longevity of the policyholder in the description of the sub module. We will therefore assume that this diversification is not allowed for.

Simplification: Is possible at the aggregate level if the standard method is an undue burden for the undertaking and the proportionality criteria is qualified.

NODBS: Applicable for earned widow's and orphan's benefits. Thus we take $U = \{\text{widow's insurance, orphan's insurance}\}^{25}$. Additionally we also need to modify (8.1) analogously to the modification in section (8.6), but this time for widow's and orphan's benefits. The gross capital charge is 0.15 percent of the total discounted value. We may arrive at a net figure by using (8.4) or (8.5), where the gross change in liabilities is defined as in section rather than (8.3). We note we may also calculate this figure by using (8.3) and shocking the mortality survival function appropriately. However, we have not designed the algorithm in appendix A with respect to absolute changes in mortality rates, so we refrain from this since it essentially will yield the same result.

8.8 SLT Health Risk: Disability and morbidity

Purpose: To cover risk of disability or morbidity risk from (re)insurance liabilities resulting from shifts and variability in claim frequencies or claim amounts from disability, sickness and morbidity rates, and medical inflation. This is notably more comprehensive than the disability and morbidity sub module of the life underwriting module in section (8.3). The sub module distinguishes between medical expense insurance and income protection insurance obligations.

Definition: We will skip a general definition and refer to the QIS5 technical documentation for more information. The purpose of this section is only to bring attention to this sub module. We also note that the income protection insurance obligations are defined similarly to the disability and morbidity sub module in section (8.3).

NODBS: Not applicable if NODBS is of the form discussed in chapter 2. However, it should anyway not yield significantly different results if the medical expense insurance component mentioned above is not covered. On the other hand, there may be small diversification

²⁵ Widow's and orphan's insurance are understood as not receiving widow's and orphan's benefits at the valuation date.

differences since the SLT Health module uses a slightly different correlation matrix. Furthermore, undertakings would also lose the diversification in combination with the Life underwriting module.

9 Case study: QIS5 for a pension fund

We have so far described and outlined relevant theory, methods, principles, regulation and legislation in relation to the forthcoming Solvency II directive and QIS5. Ultimately, the objective of the assignment is to calculate the solvency capital requirement for a real life insurance or pension undertaking. This is the subject in this chapter where we will use the methods discussed in the previous chapters. Pål Lillevold has provided data for a Norwegian pension fund. We will refer to this pension funds as PF in this chapter.

9.1 Overview

We will base our calculations on the pension funds status at the end of 2010. We note that the formal QIS5 reporting assumed undertakings filing the status at the end of 2009. In this respect the calculations may not be completely representative in a formal Solvency II filing, since Solvency II uses a counter cyclical approach. However, the potential tightening for a supposed 2010 Solvency II requirement is rather uncertain since the economic recovery after 2008 has been slow for the developed countries. The distressed situation among some indebted countries in Europe presumably may delay or dampen the counter cyclical tightening. Furthermore, government bonds issued by distressed EU countries are rated as prime quality in QIS5 even if yields on some of these bonds trade above 10 percent. The details of QIS5 were finalized as the distressed situation was emerging. One may assume that regulators wish to avoid escalating these issues further as some financial institutions are heavily exposed and may need government aid if reconsidered.

The QIS5 filing forms are comprehensive Excel spreadsheets, but with simplifications for solo undertakings. Nevertheless, they are unsuitable for the exposition in this thesis. We have therefore prepared our own spreadsheets performing the necessary calculations from a sub module level similar to the formal spread sheets. Computations comprising or depending on (expected) cash flows for the pensions fund are calculated in Mathematica. The code is listed in appendix C. We will briefly sketch the logic behind the algorithms in sub section (9.3.1), but will refer to the previous chapters for a theoretical and principle discussion.

The Norwegian legislation for life and pension insurance protects the policyholder holder from fluctuations in the discounted value of the guarantee benefits. This may motivate undertakings to choose cautious strategic asset allocation strategies trying to preserve the shareholders equity in the short run. Solvency II may amplify this effect since it explicitly reveals the inherent risk of the yearly interest rate guarantee.

Contrary to this, PF is a well-managed and well positioned pension fund with very comfortable buffer and own funds. This enables the pension fund to run a long term focused investment strategy which is more comparable to international counterparts operating without short term return guarantees. As we will see, PF is sufficiently capitalized to pursue its long term investment strategy, even under Solvency II using the standard formulas in QIS5.

Table (9.1) depicts PF balance sheet. It holds 1.85 billion (NOK) in own funds and 2.41 billion (NOK) in buffer funds. This amounts to 4.27 billion (NOK) in risk mitigating capital, and compares to 6.97 billion (NOK) in funds that are covered by the embedded interest rate guarantee. PF uses a comfortably low technical rate of 2.6 percent for all policyholders.

ASSETS	31.12.2010	EQUITY AND LIABILITIES	31.12.2010
Equities	1 522 768 000	Unrealized profits	644 374 000
Fixed Income	283 962 000	Retained earnings	1 203 492 000
Other	53 958 000	Total Equity	1 847 866 000
Company portfolio	1 860 688 000	Premium reserve fund	6 480 844 000
		Additional reserve fund	160 283 000
Equities	4 256 044 000	Price adjustment fund	2 259 562 000
Fixed Income	4 769 227 000	Other insurance funds	324 887 000
Other	200 305 000	Liabilities	9 225 576 000
Collective portfolio	9 225 576 000	Other liabilities	12 822 000
		Total liabilities and equity	11 086 264 000
Total assets	11 086 264 000		

Table 9.1: Assets and liabilities

The financial information we have used throughout the chapter is the annual report and accounts for 2010. The statement contains the most important data for completing QIS5. However, some details are not disclosed and we will therefore make appropriate assumptions when necessary. We believe the undisclosed details won't have a major impact on the conclusions since the main risk stems from equity risk and interest risk from guaranteed benefits (while not accounting for changes in future discretionary benefits). To assess the

liabilities we are provided with a complete dataset covering benefits and premium reserves for each individual policyholder. Technically, the additional reserve fund is also allocated to each policyholder, but we will treat it at an aggregate level since this information is not disclosed.

The pension fund covers defined benefits for retirement and disability, widow or partner, and orphans. Retirement benefits principally start when the policyholder turns 67 years and lasts for the remaining lifetime. We will assume that all policyholder receive retirement benefits when turning 67 years, although there are some cases where the policyholder must wait until turning 68 years. Disability insurance lasts until retirement in the event of disablement, while widow's and partner's benefits lasts for the whole lifetime in the event of policyholder's death. In the latter case any orphans will receive benefits until turning 18 and/or 21 years. Quite commonly the orphan's benefits double in size after turning 18 years, while lasting until turning 21 years. These are essentially the benefits discussed in detail in chapter 2.

9.2 Technical provisions

As discussed previously the technical provisions in the Solvency II for insurance liabilities should be equal to the (market) value counterparts are willing to undertake the insurance liabilities. The valuation is split into two parts; a) calculating the expected value of the liabilities using no safety loadings, and b) calculating the risk premium to cover the cost of capital for the necessary solvency capital requirements. We will pursue part a) in this section.

We will use the K2005 parameters for calculating the expected cash flow. K2005 gives Gompertz-Makeham parameters for survival functions and additional functions describing population means and age differences necessary to calculate net expected cash flows for all benefits covered by PF, except for disability benefits. Lillevold & Partners AS has additionally provided relevant disability Gompertz-Makeham parameters. Parameters for calculating net expected cash flows are therefore readily available. However, it is important to be critical since these are based on population mean and the insurance liabilities of the undertaking may not be representative for the entire population. We shall, however, assume that this is satisfactory. PF has more than 15.500 members which are geographically dispersed over various parts of Norway.

The contract boundary is central for determining a pension fund's liabilities as discussed in chapter 6. About 35 percent of the policyholders are employed and acquire benefits linearly

by seniority. The remaining stock are paid up policies and/or receiving benefits. The sponsors, also representing the owners' stake in this case, may decide to change the pension scheme or entirely close it. In this case all policies will be converted to paid up policies. This is ultimately the contract boundary for PF, and we will use this as a basis for calculating the solvency capital requirement.

With this assumption we can treat all contracts equally and part of the same modified profit sharing model discussed in chapter 5. We will include earned benefits over the next year for increased seniority, but not potential wage increases. We will assume that the premium for the earned benefits is charged to the premium fund and credited the premium reserve fund. The one year timeframe is chosen accordingly to the VaR specification of the solvency capital requirement (i.e. one year time horizon with 99.5 percent probability).

To discount the expected benefits we use the Norwegian yield curve from the QIS5 discounting helper tab, including the 75 percent illiquidity premium which is relevant for these contracts. In addition we will use the implied volatility surface calculated from Norwegian swaptions market (as of 31st of December 2009) which is illustrated in figure 9.5. We discuss both in more detail below. Using these assumptions we are able to compute the technical provisions excluding the risk margin as shown in table 9.2.

TECHNICAL PROVISIONS ex. RM	
Guaranteed benefits	4 185 830 000
Interest rate guarantee (IRG)	140 690 000
Other insurance funds	205 239 600
Expenses	254 157 000
FDB: 80% (PRF - guaranteed benefits)	1 931 853 920
FDB: 100% (Additional reserve fund -IRG)	19 593 000
FDB: 80% Price adjustment fund	1 807 649 600
TP ex. Risk Margin	8 545 013 120

Table 9.2: Technical provisions ex. risk margin

The value of the guaranteed benefits, interest rate guarantee and expenses in table 9.2 is calculated using the Mathematica algorithms listed in appendix C. The value of the embedded interest rate guarantee reflects only time value since the intrinsic value is zero. The expenses are calculated using formula (6.5) based on servicing costs (total management expenses was 22.77 million NOK last year), the expected inflation rate (2%) and the duration of the expected cash flows. The “other insurance funds” may be obtained from table 9.1 while

deducting the estimated gross premium for the earned benefits for next year (about 120 million NOK). The premium reserves are credited with the same amount. The future discretionary benefits in the table are then straightforward to calculate.

Before proceeding to the solvency capital requirement, we will briefly look at the Norwegian yield curves and volatility surface used in the calculations. The zero-coupon yield curves are displayed in figure 9.3 up to maturities of 40 years, while figure 9.4 shows the implied forward (one) year yield curves.

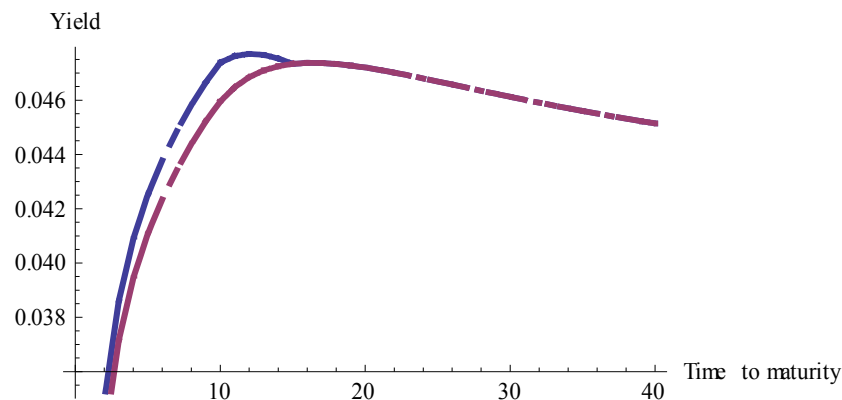


Figure 9.3: Zero-coupon yield curves (NOK)

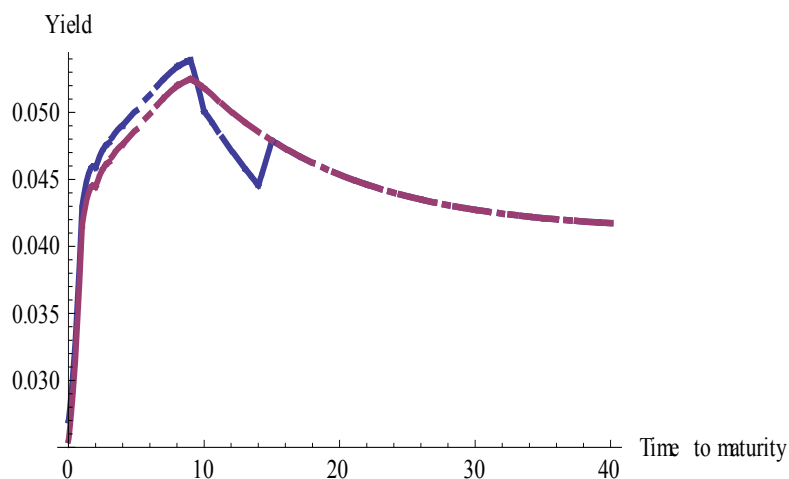


Figure 9.4: Forward one year yield curves (NOK)

The smoother curve in each graph depicts the risk free rates, while the other curve in each graph includes the relevant illiquidity premium (75 percent of 20 basis points). As can be seen from figure 9.4, the effect of the illiquidity premium runs completely off from 10 to 15 year maturities having a negative spread to the risk free yield curve. This is somewhat irregular and effectively means that (expected) cash flows with long maturities are not exposed to and

neither are discounted by the illiquidity premium obtainable in the shorter end of the curve. Consequently, the illiquidity premium is irrelevant for maturities longer than 15 years. When the illiquidity premium narrows in the stress scenario, forward interest rates will rise for maturities between 10 and 15 years, and fall for maturities less than 10 years. Thus, the effect of the illiquidity premium is not obvious for an undertaking.

Figure 9.5 depicts the Norwegian volatility surface for swaptions with one year tenors based on implied volatilities using the Black model and market prices as of 31st of December 2009. The time scale follows from right to left to improve visualization. The strikes in the graph vary from -200 to +200 basis points on top of the relevant implied forward curve which should be approximately similar to the forward curves in figure 9.4. Prices are only available for expiries up to 30 years. We have therefore extrapolated the volatility surface using 30 year volatilities also for longer maturities. The volatility surface converges to about 11 - 12 percent volatility for strikes above the forward yield curve. This is however fairly low compared to the interest rate level. At the end of last year, the same part of the surface was 2 - 3 percentage points higher than this. To be consistent with dates we shall, however, use the volatility surface below as input. The shape of the volatility surface will actually result in larger absolute price changes of the embedded options in the interest rates down scenario.

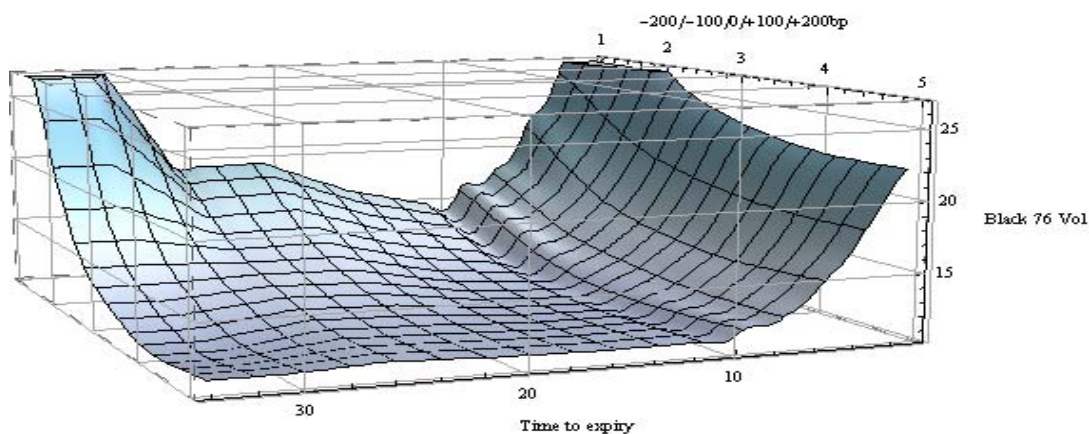


Figure 9.5: Volatility surface for Norwegian swaptions with one year tenor

9.3 Solvency capital requirement

Calculating the solvency capital requirement is the main objective in this thesis. We start first by briefly outlining the algorithms in appendix C below. The algorithms compute the life underwriting risks in the first sub section, and the interest rate risk and the illiquidity risk in the second sub section. The remaining part of the market risk and the default risk is also covered in the second sub section. We keep the future discretionary benefits unchanged, so we can calculate the basic solvency capital requirement in (9.3.3). Changes in the future discretionary benefits are accounted for in the two next sub sections. Finally sub section (9.3.6) calculates the operational charge and the adjustment factor obtained from the two previous sub sections. This yields the Solvency Capital Requirement (SCR).

9.3.1 Life underwriting risk

The algorithms consist of three parts. The first part calculates vectors of net expected cash flows for standardized benefits (100 NOK per year) for each standardized age between 0 and 120 years and each gender, looking 120 years into the future. The algorithms produce both the net expected cash flows and the stressed cash flows taking the relevant stress parameters as input. For ages where certain benefits are not applicable, the net expected cash flow is simply zero. For instance we assume that employees may enter the defined benefit scheme at age 20. Thus, the net expected cash flows are zero for employees under 20 years old. On the other hand, orphans receiving benefits lose these when turning 18 or 21, and so the net expected cash flows for orphans receiving benefits are zero for ages above 18 or 21 respectively. The net expected cash flows for each gender and benefit are saved into tables ranging from 0 to 120 years, and likewise for the stressed cash flows. Widow's and partner's benefits depend on two lives, and we "stress" both lives simultaneously to be consistent²⁶.

The second part computes the expected cash flows and the stressed cash flows for each policy using the tables from part one. For non-integer lives we use linear interpolation to approximate the relevant cash flows. This is a fair approximation speeding up the algorithm considerably. This is done for each covered benefit and scaled appropriately according to the terms in each policy. All expected and stressed cash flows for each policyholder are

²⁶ And effectively reduces the diversification when considered in combination with benefits having longevity as the primary risk driver.

accumulated²⁷. Additionally expected and stressed cash flows relating to disability benefits (but not currently receiving disability benefits) are also accumulated. Similarly, expected cash flows assuming that all policyholders have deceased, are accumulated for the purpose of calculating CAT risk as outlined in section 8.7. This yields all the necessary cash flows from the liabilities. But, we will need to rerun these algorithms when computing the stresses in the equivalent scenario.

Part three computes the risk sub modules. We don't give any risk figures here as they are displayed in table 9.10 and discussed in sub section (9.3.3). The relevant yield curves are loaded. We also apply the redistribution algorithm to the cash flows as discussed in chapter 3. Mortality risk is considered equal to zero as discussed in section (8.1). Longevity risk is computed by calculating the discounted value of the difference between the stressed and net expected cash flows. Similarly, the disability risk is calculated as the discounted value of the difference between the stressed and net expected cash flows. These were accumulated additionally. We assume that lapse risk is zero according to the discussion in section (8.4). Expense risk is computed by formula (8.7) needing three inputs. The last year's servicing expenses, the inflation rate and the average number of years the risk runs off weighted by the renewal expenses. Here we have used the duration of the expected cash flows. Revision risk is 3 percent of the discounted value of the disability benefits (not including those that are currently receiving disability benefits). Finally, CAT risk is 0.15 percent of the discounted value of the CAT cash flows.

9.3.2 Financial risk

In the discussion we follow the same order as in chapter 7, starting with interest rate sub module. We continue using the Mathematica algorithms from the previous section, and additionally load the appropriate stressed yield curves, usually denoted by up and down, and the volatility surface. We will assume that latter also applies in stressed situations. Realistically however it is likely that the implied volatilities will rise somewhat in the event of lower interest rates. We also compute the cash flows for the fixed income portfolios, and keep the collective and company portfolio separately.

²⁷ This is a slight simplification since we should separate cash flows by longevity or mortality as the primary risk driver. However, we assume that longevity risk is the primary risk driver for all policy holders since this is the natural risk driver for this defined benefit scheme. This is even more appropriate when all policies are considered paid up as here.

Part three of the algorithms yields the changes in values resulting from stressing the yield curve, upwards or downwards. We calculate the change in values individually for the guaranteed benefits, the embedded interest rate guarantee (with strike equal to the technical rate of 2.6 percent), price change of the collective portfolio and the company portfolio.

The financial statement contains information about the market values and durations across risk categories for the bond portfolios. We use formula (3.8) to estimate cash flows and mapping algorithm (3.7) to distribute cash flows to the appropriate vertices. The average duration in both portfolios is fairly short (2.05 in total). Higher interest rates will therefore only have a moderate impact on the market values of the bond portfolios. The discounted value of the guaranteed benefits and the embedded interest rate guarantee will fall. Thus, it is highly likely that the scenario with lower interest rates will be relevant for PF (which is verified by the calculations in appendix C). We will assume that all bonds are issued in NOK, having no information that contradicts this.

We also estimate the interest income for the next 12 months in table 9.6. The effective yield is 6.7 percent according to the financial statement. This is rather high compared to short term Norwegian government bonds, so we suspect there are some floating rate notes in the portfolio. We have subtracted 0.5 percentage points to take account of the moderate yield curve steepness. In table 9.6 we have charged the interest income account for the collective portfolio for the yearly interest rate guarantee by the technical rate. Thus, the interest income should cover the interest rate guarantee. However, no financial returns should be included, and we therefore assume that the net income is zero. There are no bonds classified as hold-to-maturity.

INTEREST PAY/REC	COLL. PORTFOLIO	COM. PORTFOLIO
Market value bonds	4 769 227 000	283 962 000
Effective yield	6.70 %	6.70 %
Yield curve steepness	0.50 %	0.50 %
Implied yield 2011	6.20 %	6.20 %
Capital IRG	6 966 014 000	
IRG	2.6 %	
Implied yield return	295 692 074	17 605 644
IRG 2011	181 116 364	
Net Income	114 575 710	17 605 644

Table 9.6: Interest payment account

The financial statement gives all equity holdings. 87.4 percent are listed on the Oslo Stock Exchange, 1.3 percent on Oslo Axess, 2.8 percent on foreign stock exchanges, while 8.5 percent are unlisted equities. Having looked through the holdings, we find that most of the unlisted equities are in the company portfolio. For simplicity we will assume that the collective portfolio holds only listed equities. All stock exchanges are in EEA and/or OECD countries. All listed equities therefore qualify for the “global” category. Using the market values we categorize the equities as in the upper part of table 9.7. The “other equities” are the unlisted part which is 8.5 percent of the total market value. We have also added 7.9 million NOK of call options to the company portfolio. When doing this we need to assume that the option is very deep in-the-money so that the option essentially behaves as a stock (i.e. delta equal to one). The lower part of the table shows the stress scenario.

EQUITIES	COLLECTIVE PORTFOLIO	COMPANY PORTFOLIO
Global	4 256 044 000	1 039 449 065
Other	-	491 198 935
Sum	4 256 044 000	1 530 648 000
STRESS SCENARIO	COLLECTIVE PORTFOLIO	COMPANY PORTFOLIO
Global: -30%	1 276 813 200	311 834 720
Other: -40%	-	196 479 574

Table 9.7: Equity exposure and risk

We may skip the property sub module since PF holds no property. When considering foreign exchange risk, the financial statement states that PF seek to reduce foreign exchange risk, i.e. there are no active bets but there may be passive bets. The market value of the currency hedge in the collective portfolio is 2 million NOK, while zero for the company portfolio. The latter may indicate that the company portfolio is unhedged. Having assumed that all bonds are issued in NOK, FX risk can only stem from equities. Table 9.8 shows the equity exposure for each currency using the holdings in the financial statement. We will assume that none of these currencies are hedged since the company portfolio holds the larger risk and presumably is unhedged. We therefore use a 25 percent capital charge on the exposure for all currencies.

	COLLECTIVE PORTFOLIO		COMPANY PORTFOLIO	
Currency	Exposure	Risk	Exposure	Risk
SEK	55 666 956	13 916 739	9 429 710	2 357 427
DKK	34 806 000	8 701 500	-	-
EUR	-	-	3 191 531	797 883
GBP	-	-	32 492 719	8 123 180
USD	1 339 000	334 750	83 022 031	20 755 508
CAD	11 851 200	2 962 800	13 746 438	3 436 609
ISK	-	-	546 000	136 500
Sum	103 663 156	25 915 789	142 428 429	35 607 107

Table 9.8: FX exposure from the equities and FX risk

The capital charge from the spread risk module is difficult to calculate without more information about the fixed income portfolios. We will therefore use the following assumptions; a) financial institutions are rated A, b) securities with the highest collateral level are rated A, c) mortgage backed securities are rated AAA, and d) the rest is unrated. Table 9.9 shows the capital charge, which is copied from the QIS5 spread risk helper tab.

COLLECTIVE PORTFOLIO					COMPANY PORTFOLIO				
Mktspbonds				244 600 089	Mktspbonds				12 693 101
Bonds	ΣM_{ζ_i}	weighted average duration	sum per rating of $MV_i \cdot m(\text{duri})$	capital charge per rating	Bonds	ΣM_{ζ_i}	weighted average duration	sum per rating of $MV_i \cdot m(\text{duri})$	capital charge per rating
AAA	0	-	0	0	AAA	0	-	0	0
AA	0	-	0	0	AA	0	-	0	0
A	637 138 000	3.5	2 228 521 180	31 199 297	A	0	-	0	0
BBB	0	-	0	0	BBB	0	-	0	0
BB	0	-	0	0	BB	0	-	0	0
B	0	-	0	0	B	0	-	0	0
CCC or lower	0	-	0	0	CCC or lower	0	-	0	0
Unrated	3 469 173 000	2.0	7 105 014 920	213 150 448	Unrated	283 962 000	1.5	423 103 380	12 693 101
Covered Bonds	ΣM_{ζ_i}	weighted average duration	sum per rating of $MV_i \cdot m(\text{duri})$	capital charge per rating	Covered Bonds	ΣM_{ζ_i}	weighted average duration	sum per rating of $MV_i \cdot m(\text{duri})$	capital charge per rating
AAA	10 726 000	3.9	41 724 140	250 345	AAA	0	0.0	0	0

Table 9.9: Helper tab spreadsheet for spread risk

In order to calculate concentration risk we will need to assume that the bond portfolios are well diversified and do not hold issuers from the four largest equity stakes. From the listed equity holdings we have identified the four largest equity holdings as of 5.82, 2.92, 2.16 and 1.66 percent of the total assets. These are unrated and therefore have a concentration risk threshold of only 1.5 percent. We may compute the concentration risk by using the QIS5 concentration risk helper tab. Another option is simply to multiply the portion exceeding the threshold by the relevant concentration risk factor (which is 0.73), squaring and adding all terms, taking the square root of the sum, and finally multiply by the total asset values excluding the loan portfolio considered below (about 11 billion NOK).

The Mathematica code in appendix C calculates the illiquidity risk using the same method as for the interest rate down scenario, but using the relevant stressed illiquidity premium yield curve. We have obtained this from the QIS5 discounting helper tab.

Finally, we will consider the counterparty default risk. There are three potential sources we need to assess. The counterparty risk related to derivatives. We will disregard this since we have assumed that all equities are unhedged. The same applies to the reinsurance contract (which is related to catastrophe insurance) since we implicitly have assumed yields no risk

mitigating effects in the stress scenarios. Furthermore, PF has a portfolio of loans collateralized by property (of about 96 million NOK). The loans are primarily mortgage loans to policyholders. We therefore categorize these as Type 2 exposure. According to the financial statement none of these are at risk, so we will assume that none of the down-payments are overdue. The capital charge is therefore simply 15 percent of the loans.

9.3.3 The Basic Solvency Capital Requirement (BSCR)

Having worked through all relevant parts of the risk sub modules, we can calculate the basic solvency requirement. Table 9.10 shows the relevant information from the risk sub modules (and sub-sub module in the case of equities) holding the value of future discretionary benefits unchanged. We use the following column headings; ΔL = change in market value of guaranteed benefits, ΔIRG = change in market value of embedded options, ΔAK = change in asset values on the collective portfolio, and ΔAS = change in asset value on the company portfolio.

STANDARD SCENARIO		ΔNAV , HOLDING $\Delta FDB=0$			
	SCR _i	ΔL	ΔIRG	ΔAK	ΔAS
Market	2 752 984 450	-	-	-	-
Default	14 400 750	-	-	-14 400 750	-
Life	243 709 309	-	-	-	-
BSCR	2 827 643 238				
Mktint	1 139 239 580	955 930 000	234 843 000	211 697 000	9 194 020
Mkteq	1 740 865 226	-	-	-	-
Mktprop	-	-	-	-	-
Mktsp	257 293 190	-	-	-244 600 089	-12 693 101
Mktfx	61 522 896	-	-	-25 915 789	-35 607 107
Mktconc	371 795 087	-	-	-371 795 087	-
Mktip	10 252 664	12 374 300	199 498	9 466 750	427 784
SCRmarket	2 752 984 450				
Global	1 588 647 920	-	-	-1 276 813 200	-311 834 720
Other	196 479 574	-	-	-	-196 479 574
Mrkeq	1 740 865 226				
Lifemort	-	-	-	-	-
Lifelong	195 063 000	195 063 000	-	-	-
Lifedis	80 946 000	80 946 000	-	-	-
Lifelapse	-	-	-	-	-
Lifeexp	53 103 000	53 103 000	-	-	-
Liferev	11 652 500	11 652 500	-	-	-
LifeCAT	7 043 040	7 043 040	-	-	-
SCRlife	243 709 309				

Table 9.10: BSCR calculation table

The row in the SCR_i column may now be calculated by simply adding the ΔL and ΔIRG columns, subtracting the ΔAS column, and subtracting 20 percent of the ΔAK column if

positive and 100 percent if negative. The latter follows from the modified profit sharing model which shares the profits (20/80) but not the losses (100/0).

The losses may then be aggregated to a higher level using formula (6.6) and the relevant correlation matrices (e.g. using the correlation matrices for the scenario where interest rates fall). We have listed the correlation matrices in appendix B. They may also be found in the QIS5 technical documentation. This adds up to a BSCR of 2.83 billion NOK. To arrive at an SCR we need to calculate the adjustment factors and operational risk capital charge. We calculate the adjustment according to the two prescribed methods in the next two sub sections. We shall assume that loss absorbing of deferred taxes is insignificant.

As a note, in the discussion above we have implicitly assumed that the net income from the actuarial and management accounts is nil before the stress scenarios appear.

9.3.4 nBSCR: The modular approach

In this approach we calculate a net Basic Solvency Capital Requirement (nBSCR) by accounting for the loss absorbing of technical provisions for each scenario separately. There are three possible sources as shown in table 9.11 under the “source” heading; the change in the difference between the premium reserves and the value of the guaranteed benefits, potential use of the adjustment reserve fund, and the (possibly potential) use of the price adjustment fund. Potential in this context means (possibly future) management actions.

The net change in FDB takes account of the profit sharing, while the change in gross FDB simply adds the three columns under the “source” heading. For each sub module we calculate the net capital charge under the column headed “nSCR_i”, found by adding the sub modules amount from the “SCR_i” column with the change in net FDB. The “SCR_i” column follows from the previous section.

Although the modular approach calculates each scenario individually, we should be careful not to double count the loss absorbing capacity. We have therefore tracked the utilization in the last row of table 9.11. As can be seen both the price adjustment fund and additional reserve fund are depleted. The difference between the premium reserves and the value of the benefits is still large, but utilization will mostly depend on the interest rate level. We have chosen to use the adjustment reserve fund in the equity stress scenario since this yields the

highest effect being the most important risk driver (the additional reserve fund can only be used to cover the yearly capitalization by the technical rate). We have also assumed that the management appropriately draws from the price adjustment fund in each case of the life stress scenarios. This is possible since we have assumed that all policies are converted to paid up policies, enabling profit sharing on the combined net income of the financial, actuarial and management account.

MODULAR APPROACH		SOURCE			ΔFDB		
	SCRi	Δ(PR-VL)	ΔARF	ΔPAF	ΔgrossFDB	ΔnetFDB	nSCRi
nMarket	2 752 984 450	-	-	-	-	-	877 808 597
nDefault	14 400 750	-	-	-14 400 750	-14 400 750	-11 520 600	2 880 150
nLife	243 709 309	-	-	-	-	-	48 741 862
nBSCR							891 997 372
nMktint	1 139 239 580	-955 930 000	-	-234 843 000	-1 190 773 000	-952 618 400	186 621 180
nMkteq	1 740 865 226	-	-	-	-	-	694 763 412
nMktprop		-	-	-	-	-	-
nMktsp	257 293 190	-	-	-244 600 089	-244 600 089	-195 680 071	61 613 119
nMktfx	61 522 896	-	-	-25 915 789	-25 915 789	-20 732 631	40 790 265
nMktconc	371 795 087	-	-	-275 263 134	-275 263 134	-220 210 507	151 584 580
nMktip	10 252 664	-12 374 300	-	-199 498	-12 573 798	-10 059 038	193 626
nSCRmarket							877 808 597
nGlobal	1 588 647 920	-	-160 283 000	-1 116 530 200	-1 276 813 200	-1 053 507 160	535 140 760
nOther	196 479 574	-	-	-	-	-	196 479 574
nMkteq							694 763 412
nLifemort	-	-	-	-	-	-	-
nLifelong	195 063 000	-	-	-195 063 000	-195 063 000	-156 050 400	39 012 600
nLifedis	80 946 000	-	-	-80 946 000	-80 946 000	-64 756 800	16 189 200
nLifelapse	-	-	-	-	-	-	-
nLifeexp	53 103 000	-	-	-53 103 000	-53 103 000	-42 482 400	10 620 600
nLiferev	11 652 500	-	-	-11 652 500	-11 652 500	-9 322 000	2 330 500
nLifeCAT	7 043 040	-	-	-7 043 040	-7 043 040	-5 634 432	1 408 608
nSCRLife							48 741 862
Capacity		2 359 760 000	160 283 000	2 259 560 000	4 779 603 000	3 855 739 000	
Deployed		-968 304 300	-160 283 000	-2 259 560 000	-3 388 147 300	-2 742 574 440	
Unused		1 391 455 700	-	-	1 391 455 700	1 113 164 560	

Table 9.11: nBSCR calculation modular approach

Finally, the nBSCR calculated by the modular approach may be found by using formula (6.6) appropriately in combination with the same correlation matrices used above. As the table shows, this yields an nBSCR of 892.0 million NOK. The adjustment term may then be computed as $BSCR - nBSCR = -1935.6$ million NOK, since the sources are not depleted using formula (6.9). We will make use this nBSCR in sub section 9.3.6 and will now turn to calculating the adjustment term using the equivalent approach.

9.3.5 nBSCR: The equivalent scenario

The equivalent scenario is specific for each undertaking as discussed in chapter 6. We therefore need to calculate this first, by using the method described in chapter 6. Table 9.12 outlines the calculations. We note that this is done at three levels²⁸ starting at the bottom level and working upwards to the top level. The principle rule is to use the SCRI's and not the nSCRI's. The "SCRI" column can be recognized from the standard scenarios in sub section 9.3.3. The multiplication of SCRI by Y and division by Capital corresponds to formula (6.10). We have chosen this setup following the example in Annex J in (European Commission, 2010b). We note that formula (6.10) assumes only one level, so BSCR in the formula (6.10) corresponds with Capital in the table, while pΔSCRI is the "ΔSCRI" at the sub-sub or sub module level. Y represents the remaining part of formula (6.10).

STANDARD SCENARIO		COMPONENT BSCR PARTITION			DIVERSIFICATION FACTOR RATIO			ΔSCRI	
	SCRI	Y	Capital	pΔSCRI	% sub sub	% sub	% Top		
Market	2 752 984 450	*	2 817 511 965 /	2 827 643 238 =	2 743 120 675			100 %	3 888 765
Default	14 400 750	*	763 574 190 /	2 827 643 238 =	3 888 765			27 %	
Life	243 709 309	*	935 555 609 /	2 827 643 238 =	80 633 797			33 %	
SCR	2 827 643 238			2 827 643 238					
Mktint	1 139 239 580	*	2 153 699 512 /	2 752 984 450 =	891 243 584		78 %	78 %	888 050 313
Mkteq	1 740 865 226	*	2 518 835 633 /	2 752 984 450 =	1 592 799 902		91 %	91 %	-
Mktprop	-	*	2 019 296 029 /	2 752 984 450 =	-		0 %	0 %	-
Mktsp	257 293 190	*	2 142 816 292 /	2 752 984 450 =	200 267 037		78 %	78 %	199 549 493
Mktfx	61 522 896	*	845 872 395 /	2 752 984 450 =	18 903 310		31 %	31 %	18 835 581
Mktconc	371 795 087	*	371 795 087 /	2 752 984 450 =	50 211 539		14 %	13 %	50 031 634
Mktip	10 252 664	*	-118 393 931 /	2 752 984 450 =	-440 923		-4 %	-4 %	-439 343
SCRmarket	2 752 984 450			2 752 984 450					
Global	1 588 647 920	*	1 736 007 600 /	1 740 865 226 =	1 584 215 033	100 %	91 %	91 %	1 444 279 861
Other	196 479 574	*	1 387 965 514 /	1 740 865 226 =	156 650 193	80 %	73 %	73 %	142 813 137
Mrkeq	1 740 865 226			1 740 865 226					
Lifemort	-	*	-13 492 740 /	243 709 309 =	-		0 %	0 %	-
Lifelong	195 063 000	*	211 251 875 /	243 709 309 =	169 084 327		87 %	29 %	55 943 334
Lifedis	80 946 000	*	109 258 260 /	243 709 309 =	36 289 213		45 %	15 %	12 006 669
Lifelapse	-	*	77 078 010 /	243 709 309 =	-		0 %	0 %	-
Lifeexp	53 103 000	*	149 928 760 /	243 709 309 =	32 668 703		62 %	20 %	10 808 785
Liferev	11 652 500	*	94 012 790 /	243 709 309 =	4 495 044		39 %	13 %	1 487 233
LifeCAT	7 043 040	*	40 555 290 /	243 709 309 =	1 172 021		17 %	6 %	387 776
SCRlife	243 709 309			243 709 309					2 827 643 238

Table 9.12: Calculating the equivalent stress scenario

The diversification factor ratio in the "% Top" column yields the undertaking specific equivalent scenario. Each scenario is quoted as a percentage of the original stress. For instance the global equity shock is only 91 percent of the original scenario where market values fall 30 percent. Thus, equities fall 27.3 percent in the equivalent scenario. All gross

²⁸ Effectively this means that correlations between sub modules from different modules can change since the correlations at the module level are constant, depending on each specific undertaking's positions.

charges are in fact calculated by table 9.12 if the risk sub modules are linear. This is not the case for the interest rate, illiquidity premium, longevity, disability and expense risk sub modules. We therefore need to recalculate these sub modules using the new parameters and the modified yield curves. To do this, we reuse part three of the Mathematica code, adjusting parameters and importing appropriately scaled stressed yield curves for the interest rate down scenario and the illiquidity premium scenario. The yield curves may be appropriately scaled using the QIS5 discounting helper tab. The other parameters follow the same line of reasoning as described for equities. For instance the longevity shock may be found by multiplying the original stress (-20 percent) by the diversification factor ratio for the longevity sub module from the top level (29 percent), which yield a reduced stress scenario of -5.8 percent. We proceed in a similar way for the other risk sub modules.

Having done this, table 9.13 shows the equivalent scenario using a setup similar to table 9.10 for the modular approach. However, in this case we don't use correlation matrices to aggregate the individual capital charges to the top level. The equivalent scenario is a single scenario where all stresses happen simultaneously, so we simply add the scenarios. The gross charge is 2.65 billion NOK, assuming future discretionary benefits are unchanged.

EQUIVALENT SCENARIO		ΔNAV, HOLDING ΔFDB=0			
	Gross charge	ΔL	ΔIRG	ΔAK	ΔAS
Market	2 567 618 880	-	-	-	-
Default	3 888 765	-	-	-3 888 765	-
Life	74 757 408	-	-	-	-
Top level	2 646 265 053				
Mktint	712 133 470	728 557 000	156 736 000	165 946 000	7 213 530
Mkteq	1 587 092 997	-	-	-	-
Mktprop	-	-	-	-	-
Mktsp	199 549 493	-	-	-189 705 074	-9 844 419
Mktfx	18 835 581	-	-	-7 934 265	-10 901 316
Mktconc	50 031 634	-	-	-50 031 634	-
Mktip	-24 296	-123 297	223	-94 507	-4 271
Market	2 567 618 880				
Global	1 444 279 861	-	-	-1 160 783 058	-283 496 802
Other	142 813 137	-	-	-	-142 813 137
Equity	1 587 092 997				
Lifemort	-	-	-	-	-
Lifelong	48 513 200	48 513 200	-	-	-
Lifedis	12 744 200	12 744 200	-	-	-
Lifelapse	-	-	-	-	-
Lifeexp	11 625 000	11 625 000	-	-	-
Liferev	1 487 233	1 487 233	-	-	-
LifeCAT	387 776	387 776	-	-	-
Life	74 757 408				
Sum	2 646 265 053	803 191 111	156 736 223	-1 246 491 304	-439 846 415

Table 9.13: The gross change in net asset value under the equivalent scenario

One may compare the non-linear sub modules in 9.13 with the linear approximation in table 9.12 resulting from the Component BSCR partition. We see that interest rate risk is about 20 percent off the exact calculations, while longevity risk is about 13 percent lower. Disability risk is on the other hand roughly 6 percent higher.

We can now calculate the changes in future discretionary benefits. This is illustrated in table 9.14 which is similarly structured as table 9.11. However, we now categorize the rows by liabilities, collective portfolio and company portfolio which are the relevant levels for profit sharing when considering a combined simultaneous stress scenario. We use the same assumptions as in the modular case regarding possible future management actions.

EQUIVALENT SCENARIO		SOURCE			ΔFDB	
	Δ	Δ(PR-VL)	ΔARF	ΔPAF	ΔgrossFDB	ΔnetFDB
Liabilities						
ΔVL	803 191 111	-728 433 703	-	-74 757 408	-803 191 111	-642 552 889
ΔIRG	156 736 223	-	-	-156 736 223	-156 736 223	-125 388 978
Sum	959 927 334	-	-	-	-	-
Collective portfolio						
Price change	-1 246 491 304	-	-	-	-	-
Sum	-1 246 491 304	-	-160 283 000	-1 086 208 304	-1 246 491 304	-1 029 249 643
Company portfolio						
Price change	-439 846 415	-	-	-	-	-
Sum	-439 846 415	-	-	-	-	-
Deployed FDB						
		-728 433 703	-160 283 000	-1 317 701 935	-2 206 418 638	-1 797 191 510
Gross loss						
	2 646 265 053					
ΔnetFDB						
	-1 797 191 510					
nBSCR						
	849 073 543					
Capacity FDB						
		2 359 760 000	160 283 000	2 259 560 000	4 779 603 000	3 855 739 000
Unused FDB						
		1 631 326 297	-	941 858 065	2 573 184 362	2 058 547 490

Table 9.14: nBSCR under the equivalent scenario

This results in an nBSCR of 849.1 million NOK. The adjustment term is again found by formula (6.9) which yields -1978.6 million NOK. We note that this is different from the change in net FDB in table 9.14 which is only -1797.2 million NOK. We have calculated the latter to keep track of the sources. When using formula (6.9) we must ensure that the loss absorbing is not greater than the capacity. The loss absorbing capacity is 3.86 billion NOK, and 1.98 billion is therefore well within the limits.

9.3.6 The solvency capital requirement

Having worked through the BSCR and nBSCR calculations, we are almost able to calculate the solvency requirement. We will also need the capital charge for operational risk. This is straightforward using the expression (6.8). We note that (6.8) is incomplete since it disregards some aspects, but, it is exact for PF. The necessary input is either previously calculated or can be found in the financial statement. Table 9.15 shows the calculation. It is simply the maximum of the charge for either premiums or technical provisions, but bounded at above by 30 percent of the BSCR. We see that the 0.45 percent charge for the technical provisions is binding. Technical provisions in this case don't include the risk margin to avoid circularity.

SCRop	INPUT	CHARGE	PREMIUM	TP	BSCR
BSCR	2 827 643 238	30 %	-	-	848 292 971
Earnlife 2010	427 359 000	4 %	17 094 360	-	-
Earnlife 2009	333 786 000	(diff) 4%	2 407 776	-	-
TP EX. RM	8 545 013 120	0.45 %	-	38 452 559	-
			19 502 136	38 452 559	848 292 971
SCRop			-	38 452 559	-

Table 9.15: Solvency capital charge for operational risk

We are finally able to compute the solvency capital requirement which is shown in table 9.16. We see that the equivalent scenario approach requires approximately 5 percent lower SCR than the modular approach. In addition the equivalent scenario approach has significantly more loss absorbing capacity left in the technical provisions. The difference is about 764 million NOK when comparing the BSCR – nBSCR for each approach.

SCR	MODULAR	EQUIVALENT
BSCR	2 827 643 238	2 827 643 238
Adj	-1 935 645 865	-1 978 569 695
SCRop	38 452 559	38 452 559
SCR	930 449 931	887 526 102

Table 9.16: Solvency capital requirement

The pension fund is well capitalized and has sufficient funds to cover the required solvency requirement using both methods. For completeness we will additionally compute the minimum capital requirement in table 9.17, using formula (6.13).

MCR	BASE	RATE	CHARGE
TP for guaranteed benefits	4 185 830 000	5.0 %	209 291 500
TP for FDB	3 759 096 520	-8.8 %	-330 800 494
Sum TP			-121 508 994
TP Floor	4 185 830 000	1.6 %	66 973 280
MCRIlinear			66 973 280
SCR Modular approach: Floor	930 449 931	25.0 %	232 612 483
SCR Modular approach: Ceiling	930 449 931	45.0 %	418 702 469
SCR Equivalent scenario: Floor	887 526 102	25.0 %	221 881 525
SCR Equivalent scenario: Ceiling	887 526 102	45.0 %	399 386 746

Table 9.17: Minimum capital requirement

The solvency capital requirement under the current regulation is 283 million NOK according to the financial statement. The MCR is just 18 and 22 percent lower, while the SCR is 229 and 214 percent higher than under the current regime, under the modular and equivalent scenario approach respectively. This may involve a significant tightening which it may be difficult to operate under for undertakings with weaker balance sheets. One should also take into consideration that Solvency II uses a counter cyclical approach. This may result in even more demanding solvency capital requirements when the developed economies have recovered sufficiently. For instance, the base level equity stresses are 39 percent and 49 percent for the “global” and the “other” category, compared to 30 percent and 40 percent in QIS5 respectively.

9.4 The pension fund’s solvency II balance sheet

We have worked through an extensive number of calculations, and round up by showing the pensions’ fund liability side of the Solvency II balance sheet. In order to do so we also need to calculate the risk margin. This is shown in table 9.18. We have used the crudest approximation that is allowed for (i.e. level 4). This may lead to an unnecessary high risk margin. However, the loss absorbing capacity of PF is abundant and we may not gain too much using a more refined method. In addition, the life stress scenarios were drastically reduced in the equivalent scenario which benefits the calculation of the risk margin. This follows since we only need to use the capital charges from the life module, the operational risk and a capital charge for unavoidable market risk from the equivalent scenario approach. The gross capital charge for the unavoidable market risk is, however, large as can be seen from table 9.18. We can probably improve this significantly if needed, e.g. if the pension fund

experiences a large loss of absorbing capacity. But, in that case the unavoidable market risk may also decrease some since the future discretionary benefits are included in the best estimate.

Risk Margin (level 4)	
Duration NGB 10y	8.30
Mod. duration ECF	13.34
Interest rate shift	1.22 %
Interest rate first year	2.59 %
Life Risk	74 757 408
Unavoidable market risk	1 579 114 012
Loss absorbing of TP	-1 323 097 136
SCRop	38 452 559
SCRru(0)	369 226 843
CoCM	288 007 519

Table 9.18: Calculation of risk margin

Finally, in table 9.19 we see that the technical provisions (8.83 billion NOK) are 4.3 percent lower than the insurance liabilities (9.23 billion NOK) from table 9.1. The technical provisions are therefore fully covered by the insurance liabilities.

Solvency II balance sheet	
Guaranteed benefits	4 185 830 000
Interest rate guarantee (IRG)	140 690 000
Other insurance funds	205 239 600
Expenses	254 157 000
FDB: 80% (PRF - guaranteed benefits)	1 931 853 920
FDB: 100% (Additional reserve fund -IRG)	19 593 000
FDB: 80% Price adjustment fund	1 807 649 600
TP ex. Risk Margin	8 545 013 120
Risk Margin	288 007 519
Technical provisions	8 833 020 639
MCR - Modular approach	232 612 483
MCR - Equivalent scenario	221 881 525
SCR - Modular approach	930 449 931
SCR - Equivalent scenario	887 526 102
Excess assets over liabilities	2 227 599 361
Solvency ratio MCR - Modular approach	957.6 %
Solvency ratio MCR - Equivalent approach	1004.0 %
Solvency ratio SCR - Modular approach	239.4 %
Solvency ratio SCR - Equivalent scenario	251.0 %

Table 9.19: Solvency II balance sheet

We have assumed that the pension fund's own funds are Tier 1 so that all excess assets over liabilities have Tier 1 characteristics. The solvency capital is significantly higher than under the current regime. This is partly due to the liabilities having a lower value, and that the equity is also including the unrealized earnings. This counteracts a significant part of the higher solvency capital requirement. The solvency ratio only falls from 324 percent under the current regulation to 251 percent or 239 percent in QIS5 depending on which approach is adopted when Solvency II is finalized.

The Norwegian regulatory authority has published the results from the Norwegian QIS5 reporting (Finanstilsynet, 2011b). The presentation does not cover pension funds, but we may compare with the ten life companies that have reported. The average best estimate and risk margin is about 1.6 percent lower than the gross reserve funds. This compares with 4.3 percent lower for PF. Only three have solvency ratios above 150 percent, while four are below 100 percent. PF is considerably better capitalized than the average, while probably also running a much higher investment risk.

We may also compare the risk decomposition. The life module represents on average about 40 percent of the BSCR, while 75.6 percent of the BSCR comes from the market risk module. The risk figures are stated in undiversified terms in the report, and therefore accumulate to more than 100 percent. PF is significantly more skewed towards investment risk with only 9 percent of the BSCR coming from the life module, and 97 percent from the market risk module. We end this chapter by concluding that the pension fund is extremely well capitalized and is well prepared to handle the forthcoming solvency requirements.

10 Conclusions

The Solvency II directive is a fundamental review of the capital adequacy requirements for the European insurance industry scheduled to be implemented January 1st 2013. It will have a major impact on life and pension insurance undertakings. This is in particular true for the Norwegian industry illustrated by the results from the Norwegian QIS5 reporting (Finanstilsynet, 2011b) and in the previous chapter. The Norwegian QIS5 reporting indicates that the situation is challenging. The average solvency capital requirement is 240 percent of the solvency margin under the current regime, while own funds are increased by only 5 percent. The pension fund, analyzed in chapter 9, is on the other hand well capitalized. QIS5 is no real concern for this pension fund. The solvency capital requirement is increased dramatically, however, being offset to a large degree by the increase in own funds. We will in this chapter summarize some of our findings and reflections throughout the project.

The calculations and assumptions behind QIS5 are non-trivial. It assumes thorough insight into an undertaking's insurance business as well as financial positions. This sets a high standard for all participants needing qualified expertise combining actuarial science and finance. This applies both to undertakings and regulatory authorities. Furthermore, the technical documentation outlines the calculations for the solvency capital requirements based on the so-called standard formula. The standard formula is however in several cases only a description of a stress scenario and does not necessarily state an explicit formula. In some of these cases simplifications are given, but stresses repeatedly that the proportionality assumption must be reasonable and that a full calculation must be an undue burden. Furthermore, when undertakings opt to use internal models, an even higher degree of complexity is introduced.

The complexity of QIS5 is also illustrated by the fact the participants in the QIS5 reporting have used varying assumptions leading to that the result may not be completely comparable. This reveals that the QIS5 methodology probably needs additional tuning before implemented in Solvency II. The issue of the contract boundary is non-trivial for Norwegian life and pension insurance. We have used the assumption that all policies are converted to paid up policies. This is a real possibility for defined benefits schemes in the private sector, but may

not be relevant for the public sector. Under the current stress test for life insurance undertakings, Finanstilsynet balances this outcome for the private sector, but not for the public sector (Finanstilsynet, 2009). Furthermore, it is unclear if one rather should use the lapse sub module to account for the possibility of mass conversion to paid up policies. This may be relevant for the life insurance undertakings that have many sponsors backing the defined benefit schemes. Pension funds usually have only one or a few sponsors, and differ from life insurance companies since the sponsors also typically represent the owners.

Continuing this reasoning one may also question how large risk premiums the sponsors are willing to pay for the interest rate guarantee, if long term interest rates falls significantly below the technical rate. We note that it's only favourable for undertakings to assume business as usual in the QIS5 reporting, if the assumed risk premiums paid by the sponsors covers this mismatch sufficiently in the subsequent years after the stress scenario occurs. A reasonable assumption may however be that lower interest rates may amplify the trend towards replacing defined benefit schemes with defined contribution schemes, already being under pressure from the ongoing "pension reform". Thus, undertakings may in the end bear the risk of lower interest rates anyway.

Classical value investing strategies essentially boils down to buying cheap cash flows and selling expensive cash flows having a long term view, while avoiding momentum strategies. Solvency II combined with the Norwegian interest rate guarantee may motivate the reverse. If the buffer between the premium reserves and the discounted value of guaranteed benefits becomes low, undertakings will be threatened by the risk of bearing the cost of lower interest rates, and may buy bonds with long durations to hedge this risk. Short term this may lead to a self-reinforcing effect of even lower interest rates due to the increased demand for bonds with long durations. Momentum strategies are already strongly motivated by the yearly interest rate guarantee²⁹. Solvency II may amplify this additionally and can result in lower investment returns to the clients. However, we note that Solvency II otherwise to some degree have adopted a counter-cyclical approach in the calibration of the stress tests.

We also observed that the exploited capacity of the buffer funds were significantly lower in the equivalent scenario approach compared to the modular approach, although the respective solvency capital requirements were similar. Which method that finally will be adopted in

²⁹ Dynamic risk management and constant proportion portfolio insurance (CPPI) are frequently used techniques in financial markets.

Solvency II, may be a concern for the Norwegian undertakings. The buffer funds are necessary for enduring market fluctuations when operating under the yearly interest rate guarantee. Furthermore, more flexible buffer funds may ease the burden somewhat. In the equivalent scenario where all stresses occurs simultaneously the additional reserve fund can only cover capitalization by the technical rate. Thus, additional reserve funds beyond this will stay unused in this approach. We note that Finanstilsynet has suggested amendments to Norwegian legislation in relation to Solvency II, amongst others proposing a single more flexible buffer fund (Finanstilsynet, 2011a).

The results from the Norwegian QIS5, and in particular for the pension fund analyzed in chapter 9, show that market risk constitutes the larger part of the solvency capital requirement. This will presumably bring attention to developing internal market risk models in order to reduce the solvency requirement. Furthermore, it is only possible to take account of dynamic hedging programs in internal models, displaying the behavior of an undertaking in turbulent markets. The standard formula can only include existing positions (except for rolling hedging programs, which will only account for the rolling of existing contracts into new contracts as they expire).

The illiquidity premium used for Norwegian QIS5 yield curves is small (maximum 20 basis points). However, we observed irregular effects on the yield curve as the illiquidity premium is phased out between 10 and 15 year maturities in the Norwegian QIS5 yield curves. This may be improved in the calibration (by EIOPA) using the forward yield curves instead of the zero-coupon yield curves.

As a final point, we may also question the so-called macroeconomic method for extrapolating yield curves beyond the longest bond maturities in the market³⁰. The argument is based on some of the same principles of the inflation targeting adopted by many central banks, which became a central banking paradigm in the 1990'ies. The globalization in the following years put strong downwards pressure on consumer goods resulting in low imported inflation in developed economies. The aftermath of the Great Recession in 2008 may change the inflation targeting paradigm, as the dysfunctional global trade model has been uncovered. It is therefore reassuring to observe that Finanstilsynet is uncomfortable with increasing the incentives to hold long bond durations at low interest rate levels.

³⁰ Although, probably not very different from the prevailing ultra long bond yields for the time being.

Appendix A – Stressing survival functions

In this appendix we describe how we compute the stressed survival functions. We assume a Gompertz-Makeham hazard rate function defined in sub section (2.1.3), although not needed before (B.8). The shocks in (European Commission, 2010a) are defined with respect to non-survival rates (i.e. mortality or disability rates) for each age. They are typically formulated as; a) a permanent F percent increase/decrease for each age, b) F_1 percent increase/decrease next year, and c) a F_2 percent increase/decrease permanently in the following years after the first year.

In each case we respectively define; a) $F_t = F$, b) $F_1 = F_1$ and $F_t = 0$ for $t > 1$, and c) $F_1 = 0$ and $F_t = F_2$ for $t > 1$. We may combine b) and c) to obtain an individual shock rate for the first year and equal shock rates in the subsequent years (but leaving out the zero assignments in b) and c)). Each shock may then be expressed by (B.1) for a given age x .

$$q_{x+t}^{shock} = (1 + F_{t+1}) \cdot q_{x+t}, \quad t = 0, 1, \dots, \lceil \omega - x \rceil \quad (B.1)$$

By using relation (2.3) we find the shocked survival function in (B.2).

$$p_{x+t}^{shock} = 1 - (1 + F_{t+1}) \cdot q_{x+t} = 1 - (1 + F_{t+1}) \cdot (1 - p_{x+t}), \quad t = 0, 1, \dots, \lceil \omega - x \rceil \quad (B.2)$$

The idea is now to find coefficients c_1, \dots, c_t which satisfies expression (B.3) so that we can describe the shocked survival function as a product the ordinary survival function and the exponential of the accumulated c_t 's. Note that the coefficients depend on x when specified this way. This allows for a general shock specification of F_t .

$${}_t p_x^{shock} = \exp \left(\sum_{j=1}^t c_j \right) \cdot {}_t p_x, \quad t = 1, 2, \dots, \lceil \omega - x \rceil \quad (B.3)$$

We continue by finding an analytical solution for c_t by inserting the shocked survival function into (2.4) and letting $s = t - 1$ and $t = 1$. This results in (B.4) which is solved for c_t in (B.5).

$$p_{x+(t-1)}^{shock} = \frac{t p_x^{shock}}{(t-1) p_x^{shock}} = \exp(c_t) \cdot \frac{t p_x}{(t-1) p_x} = \exp(c_t) \cdot p_{x+(t-1)} \quad (B.4)$$

$$c_t = \ln \left(\frac{p_{x+(t-1)}^{shock}}{p_{x+(t-1)}} \right) = \ln \left(\frac{1 - (1 + F_t) \cdot (1 - p_{x+(t-1)})}{p_{x+(t-1)}} \right) \quad (B.5)$$

In order to use the formulas in chapter 2 we need an appropriate continuous shocked survival function. We therefore define a continuous function f_{x+t} in (B.6). The shocked survival function (B.7) satisfies (B.1) exactly for $t = 1, \dots, \lceil \omega - x \rceil$.

$$f_{x+t} \equiv \sum_{j=1}^{\lfloor t \rfloor} c_j + (t - \lfloor t \rfloor) \cdot c_{\lceil t \rceil}, \quad t \in [0, \lceil \omega - x \rceil] \quad (B.6)$$

$$p_{x+t}^{shock} = \exp(f_{x+t}) \cdot p_{x+t}, \quad t \in [0, \omega - x] \quad (B.7)$$

The derivative of f_{x+t} is stepwise continuous. If the coefficients are sufficiently bounded the integral of (B.7) does exist. Numerically, we may need to be cautious when choosing ω to ensure that the coefficients, c_t , don't change too rapidly as the survival function approaches zero. For instance, a mortality shock is more challenging since it is difficult to decrease the survival function further when the survival function is close to zero. Conversely, a longevity shock results in a survival probability of approximately F_t for the next year, assuming the ordinary survival function is close to zero. Thus, the shocked survival function runs off similar to a geometric series which is numerically stable.

Finally, combining (2.6) – (2.7) and (B.7) yields the shocked hazard rate function in (B.8).

$$\mu_{x+t} = \alpha + \beta \cdot c^{x+t} - c_{\lceil t \rceil} \quad (B.8)$$

Appendix B – QIS5 correlation matrices

We have included the relevant correlation matrices from the QIS5 Technical Specification for completeness. These are part of the formal QIS5 documentation issued by the European Commission (European Commission, 2010a).

Top level

i, j	Market	Default	Life	Health	Non-Life
Market	1.00	0.25	0.25	0.25	0.25
Default	0.25	1.00	0.25	0.25	0.50
Life	0.25	0.25	1.00	0.25	0.00
Health	0.25	0.25	0.25	1.00	0.00
Non-Life	0.25	0.50	0.00	0.00	1.00

Figure B.1: Top level correlation matrix

MarketDown

i, j	Mktint	Mkteq	Mktprop	Mktsp	Mktfx	Mktfconc	Mktip
Mktint	1.00	0.50	0.50	0.50	0.25	0.00	0.00
Mkteq	0.50	1.00	0.75	0.75	0.25	0.00	0.00
Mktprop	0.50	0.75	1.00	0.50	0.25	0.00	0.00
Mktsp	0.50	0.75	0.50	1.00	0.25	0.00	-0.50
Mktfx	0.25	0.25	0.25	0.25	1.00	0.00	0.00
Mktfconc	0.00	0.00	0.00	0.00	0.00	1.00	0.00
Mktip	0.00	0.00	0.00	-0.50	0.00	0.00	1.00

Figure B.2: MarketDown correlation matrix (lower interest rates scenario)

MarketUp

i, j	Mktint	Mkteq	Mktprop	Mktsp	Mktfx	Mktfconc	Mktip
Mktint	1.00	0.00	0.00	0.00	0.25	0.00	0.00
Mkteq	0.00	1.00	0.75	0.75	0.25	0.00	0.00
Mktprop	0.00	0.75	1.00	0.50	0.25	0.00	0.00
Mktsp	0.00	0.75	0.50	1.00	0.25	0.00	-0.50
Mktfx	0.25	0.25	0.25	0.25	1.00	0.00	0.00
Mktfconc	0.00	0.00	0.00	0.00	0.00	1.00	0.00
Mktip	0.00	0.00	0.00	-0.50	0.00	0.00	1.00

Figure B.3: MarketUp correlation matrix (higher interest rates scenario)

Equity		
i, j	Global	Other
Global	1.00	0.75
Other	0.75	1.00

Figure B.4: Equity sub module correlation matrix

Life							
i, j	Lifemort	Lifelong	Lifedis	Lifelapse	Lifeexp	Liferev	LifeCAT
Lifemort	1.00	-0.25	0.25	0.00	0.25	0.00	0.25
Lifelong	-0.25	1.00	0.00	0.25	0.25	0.25	0.00
Lifedis	0.25	0.00	1.00	0.00	0.50	0.00	0.25
Lifelapse	0.00	0.25	0.00	1.00	0.50	0.00	0.25
Lifeexp	0.25	0.25	0.50	0.50	1.00	0.50	0.25
Liferev	0.00	0.25	0.00	0.00	0.50	1.00	1.00
LifeCAT	0.25	0.00	0.25	0.25	0.25	0.00	1.00

Figure B.5: Life module correlation matrix

Appendix C – Mathematica code

Part I - See sub section 9.3.1:

```
(* 2020 survival function *)
p2020[s_, y_] := Exp[-s × α - β × cy × (cs - 1) / Log[c]]
μ2020[y_] := α + β × cy;
(* Choose parameters dependent on gender *)
par2020[male_] := If[male, α = αm; β = βm; c = cm, α = αf; β = βf; c = cf];
(* Calculates stress coefficients *)
Setδ2020[ΔM_, y_] := Module[{t}, δ2020 = Table[Log[ $\frac{1 - (1 + ΔM) × (1 - p2020[1, t])}{p2020[1, t]}$ ], {t, y, 131, 1}]];
(* Stressed survival function: 2020 *)
p2020Δ = Compile[{{s, _Real}, {y, _Real}, {δ2020, _Real, 1}},
  Exp[-s × α - β × cy × (cs - 1) / Log[c]] × Exp[Sum[δ2020[[j]], {j, 1, IntegerPart[s]}] ×
  Exp[FractionalPart[s] × δ2020[[IntegerPart[s] + 1]]]];
Off[Part::"pspec"]

(* 2005 survival function *)
p2005[s_, y_] := Exp[-s × α5 - β5 × c5y × (c5s - 1) / Log[c5]];
μ2005[y_] := α5 + β5 × c5y;
(* Choose parameters dependent on gender *)
par2005[male_] := If[male, α5 = αm5; β5 = βm5; c5 = cm5, α5 = αf5; β5 = βf5; c5 = cf5];
(* Calculates stress coefficients *)
Setδ2005[ΔM_, y_] := Module[{t}, δ2005 = Table[Log[ $\frac{1 - (1 + ΔM) × (1 - p2005[1, t])}{p2005[1, t]}$ ], {t, y, 131, 1}]];
(* Stressed survival function: 2005 *)
p2005Δ = Compile[{{s, _Real}, {y, _Real}, {δ2005, _Real, 1}},
  Exp[-s × α5 - β5 × c5y × (c5s - 1) / Log[c5]] × Exp[Sum[δ2005[[j]], {j, 1, IntegerPart[s]}] ×
  Exp[FractionalPart[s] × δ2005[[IntegerPart[s] + 1]]]];

(* Disability survival function *)
pUp[s_, y_] := Exp[-s × θ0 - θ1 × θ2y × (θ2s - 1) / Log[θ2]];
(* Choose parameters dependent on gender *)
parUp[male_] := If[male, θ0 = θm0; θ1 = θm1; θ2 = θm2, θ0 = θf0; θ1 = θf1; θ2 = θf2];
(* Calculates stress coefficients *)
SetδUp[ΔU1_, ΔU2_, y_] :=
  Module[{t}, δUp = Prepend[Table[Log[ $\frac{1 - (1 + ΔU2) × (1 - pUp[1, t])}{pUp[1, t]}$ ], {t, y + 1, 68, 1}], Log[ $\frac{1 - (1 + ΔU1) × (1 - pUp[1, y])}{pUp[1, y]}$ ]]];
(* Stressed survival function: disability *)
pUpΔ = Compile[{{s, _Real}, {y, _Real}, {δUp, _Real, 1}},
  Exp[-s × θ0 - θ1 × θ2y × (θ2s - 1) / Log[θ2]] × Exp[Sum[δUp[[j]], {j, 1, IntegerPart[s]}] ×
  Exp[FractionalPart[s] × δUp[[IntegerPart[s] + 1]]]];

(* Probability of having a spouse entitled to benefits as a function of age and gender *)
g[x_, male_]
(* Probability of having a partner entitled to benefits as a function of age and gender *)
s[x_, male_]
(* Average age difference of spouse or partner as a function of age and gender *)
f[x_, male_]
(* Probability of having a children (under 18 and 21 years) entitled to benefits as a function of age and gender *)
k18[x_, male_]
k21[x_, male_]
(* Average age difference between insured and children as a function of age and gender *)
z18[x_, male_]
z21[x_, male_]
```

```

(* Retirement benefits - ECF or SCF*)
ApEcf[x_?NumberQ, male_, ΔM_] := Module[{ecf, n, t, i},
  ecf = Table[0, {120}]; par2020[male];
  n = Max[67 - x, 0]; (* n age to retirement *)
  If[ΔM == 0,
    If[FractionalPart[n] > 0, ecf[[Floor[n] + 1]] = NIntegrate[p2020[t, x], {t, n, Ceiling[n]}];
    For[i = Ceiling[n], i ≤ 119 - x, i++, ecf[[i + 1]] = NIntegrate[p2020[t, x], {t, i, i + 1}];
    n = FractionalPart[119 + 1 - x]; If[n > 0, ecf[[i + 1]] = ecf[[i + 1]] + NIntegrate[p2020[t, x], {t, i, i + n}];,
    Setδ2020[ΔM, x];
    If[FractionalPart[n] > 0, ecf[[Floor[n] + 1]] = NIntegrate[p2020Δ[t, x, δ2020], {t, n, Ceiling[n]}];
    For[i = Ceiling[n], i ≤ 119 - x, i++, ecf[[i + 1]] = NIntegrate[p2020Δ[t, x, δ2020], {t, i, i + 1}];
    n = FractionalPart[119 + 1 - x]; If[n > 0, ecf[[i + 1]] = ecf[[i + 1]] + NIntegrate[p2020Δ[t, x, δ2020], {t, i, i + n}];,
    ]; ecf];

(* Disability benefits - ECF or SCF, not disabled *)
UpEcf[x_?NumberQ, male_, ΔM_, ΔU1_, ΔU2_] := Module[{ecf, n, i, t},
  ecf = Table[0, {120}]; parUp[male]; par2005[male];
  If[ΔU1 == 0 && ΔU2 == 0 && ΔM == 0,
    For[i = 0, i ≤ 66 - x, i++, ecf[[i + 1]] = NIntegrate[p2005[t, x] × (1 - pUp[t, x]), {t, i, i + 1}];
    n = FractionalPart[66 + 1 - x];
    If[n > 0, ecf[[i + 1]] = NIntegrate[p2005[t, x] × (1 - pUp[t, x]), {t, i, i + n}];,
    If[ΔM == 0,
      SetδUp[ΔU1, ΔU2, x];
      For[i = 0, i ≤ 66 - x, i++, ecf[[i + 1]] = NIntegrate[p2005[t, x] × (1 - pUpΔ[t, x, δUp]), {t, i, i + 1}];
      n = FractionalPart[66 + 1 - x];
      If[n > 0, ecf[[i + 1]] = NIntegrate[p2005[t, x] × (1 - pUpΔ[t, x, δUp]), {t, i, i + n}];,
      Setδ2005[ΔM, x];
      For[i = 0, i ≤ 66 - x, i++, ecf[[i + 1]] = NIntegrate[p2005Δ[t, x, δ2005] × (1 - pUp[t, x]), {t, i, i + 1}];
      n = FractionalPart[66 + 1 - x];
      If[n > 0, ecf[[i + 1]] = NIntegrate[p2005Δ[t, x, δ2005] × (1 - pUp[t, x]), {t, i, i + n}];,
      ];
    ]; ecf];

(* Disability benefits - ECF or SCF, disabled *)
ULpEcf[x_?NumberQ, male_, ΔM_] := Module[{ecf, n, i, t},
  ecf = Table[0, {120}]; par2005[male];
  If[ΔM == 0,
    For[i = 0, i ≤ 66 - x, i++, ecf[[i + 1]] = NIntegrate[p2005[t, x], {t, i, i + 1}];
    n = FractionalPart[66 + 1 - x];
    If[n > 0, ecf[[i + 1]] = NIntegrate[p2005[t, x], {t, i, i + n}];,
    Setδ2005[ΔM, x];
    For[i = 0, i ≤ 66 - x, i++, ecf[[i + 1]] = NIntegrate[p2005Δ[t, x, δ2005], {t, i, i + 1}];
    n = FractionalPart[66 + 1 - x];
    If[n > 0, ecf[[i + 1]] = NIntegrate[p2005Δ[t, x, δ2005], {t, i, i + n}];,
    ]; ecf];

(* Spouse benefits - ECF or SCF *)
EpEcf[x_?NumberQ, male_, ΔM_] := Module[{ecf, h, t, i, p},
  ecf = Table[0, {120}]; par2005[male]; par2020[! male]; p = 4;
  If[ΔM == 0,
    For[i = 0, i ≤ 119 - x, i++,
      ecf[[i + 1]] = Sum[((1 - p2005[t/p, x]) - (1 - p2005[(t - 1)/p, x])) × g[x + t/p, male] ×
        Kep[x + t/p - f[x + t/p, male], t/p, i], {t, 1, p × (i + 1)}];,
    Setδ2005[ΔM, x]; Setδ2020[ΔM, x - f[x, male]];
    For[i = 0, i ≤ 119 - x, i++,
      ecf[[i + 1]] = Sum[((1 - p2005Δ[t/p, x, δ2005]) - (1 - p2005Δ[(t - 1)/p, x, δ2005])) × g[x + t/p, male] ×
        KepΔ[x + t/p - f[x + t/p, male], t/p, i], {t, 1, p × (i + 1)}];,
    ]; ecf];

```

```

(* Partner benefits - ECF or SCF *)
SpEcF[x_?NumberQ, male_, dM_] := Module[{ecf, h, t, i, p},
  ecf = Table[0, {120}]; par2005[male]; par2020[! male]; p = 4;
  If[dM == 0,
    For[i = 0, i ≤ 119 - x, i++,
      ecf[[i + 1]] = Sum[{(1 - p2005[t/p, x]) - (1 - p2005[(t - 1)/p, x])} × s[x + t/p, male] × Kep[x + t/p - f[x + t/p, male], t/p, i],
        {t, 1, p × (i + 1)}}];,
    Setd2005[dM, x]; Setd2020[dM, x - f[x, male]];
    For[i = 0, i ≤ 119 - x, i++,
      ecf[[i + 1]] = Sum[{(1 - p2005Δ[t/p, x, d2005]) - (1 - p2005Δ[(t - 1)/p, x, d2005])} × s[x + t/p, male] ×
        KepΔ[x + t/p - f[x + t/p, male], t/p, i], {t, 1, p × (i + 1)}}];
  ]; ecf];

(* Axillary functions for spouse and partnerbenefits *)
Kep[y_?NumberQ, t_?NumberQ, i_?NumberQ] := NIntegrate[p2020[h, y], {h, Max[Min[100 - y, i - t], 0], Max[Min[100 - y, i - t + 1], 0]}];
KepΔ[y_?NumberQ, t_?NumberQ, i_?NumberQ] := NIntegrate[p2020Δ[h, y, d2020], {h, Max[Min[100 - y, i - t], 0], Max[Min[100 - y, i - t + 1], 0]}];

(* Widows receiving benefits - ECF or SCF *)
ElpEcF[x_?NumberQ, male_, dM_] := Module[{ecf, n, t, i},
  ecf = Table[0, {120}]; par2020[male];
  n = 0; (* n age to retirement *)
  If[dM == 0,
    For[i = n, i ≤ 119 - x, i++, ecf[[i + 1]] = NIntegrate[p2020[t, x], {t, i, i + 1}];
    n = FractionalPart[119 + 1 - x]; If[n > 0, ecf[[i + 1]] = ecf[[i + 1]] + NIntegrate[p2020[t, x], {t, i, i + n}]]];,
    Setd2020[dM, x];
    For[i = n, i ≤ 119 - x, i++, ecf[[i + 1]] = NIntegrate[p2020Δ[t, x, d2020], {t, i, i + 1}];
    n = FractionalPart[119 + 1 - x]; If[n > 0, ecf[[i + 1]] = NIntegrate[p2020Δ[t, x, d2020], {t, i, i + n}]]];
  ]; ecf];

(* Benefits to children < 18 - ECF or SCF *)
Bp18EcF[x_?NumberQ, male_, dM_] := Module[{ecf, h, t, i, p},
  ecf = Table[0, {120}]; par2005[male]; p = 4;
  If[dM == 0,
    For[i = 0, i ≤ 75 - x, i++, (* unnecessary to calculate beyond 76*)
      ecf[[i + 1]] = Sum[{(1 - p2005[t/p, x]) - (1 - p2005[(t - 1)/p, x])} × 0.9 × k18[x + t/p, male] × Kbp18[z18[x + t/p, male], t/p, i, male],
        {t, 1, p × (i + 1)}}];,
    Setd2005[dM, x];
    For[i = 0, i ≤ 75 - x, i++, (* unnecessary to calculate beyond 76*)
      ecf[[i + 1]] = Sum[{(1 - p2005Δ[t/p, x, d2005]) - (1 - p2005Δ[(t - 1)/p, x, d2005])} × 0.9 × k18[x + t/p, male] ×
        Kbp18[z18[x + t/p, male], t/p, i, male], {t, 1, p × (i + 1)}}];
  ]; ecf];

(* Benefits to children < 21 - ECF or SCF *)
Bp21EcF[x_?NumberQ, male_, dM_] := Module[{ecf, h, t, i, p},
  ecf = Table[0, {120}]; par2005[male]; p = 4;
  If[dM == 0,
    For[i = 0, i ≤ 75 - x, i++, (* unnecessary to calculate beyond 76 *)
      ecf[[i + 1]] = Sum[{(1 - p2005[t/p, x]) - (1 - p2005[(t - 1)/p, x])} × 0.9 × k21[x + t/p, male] × Kbp21[z21[x + t/p, male], t/p, i, male],
        {t, 1, p × (i + 1)}}];,
    Setd2005[dM, x];
    For[i = 0, i ≤ 75 - x, i++, (* unnecessary to calculate beyond 76*)
      ecf[[i + 1]] = Sum[{(1 - p2005Δ[t/p, x, d2005]) - (1 - p2005Δ[(t - 1)/p, x, d2005])} × 0.9 × k21[x + t/p, male] ×
        Kbp21[z21[x + t/p, male], t/p, i, male], {t, 1, p × (i + 1)}}];
  ]; ecf];

(* Axillary functions *)
Kbp18[y_?NumberQ, t_?NumberQ, i_?NumberQ, male_] := NIntegrate[1, {h, Max[Min[18 - y, i - t], 0], Max[Min[18 - y, i - t + 1], 0]}];
Kbp21[y_?NumberQ, t_?NumberQ, i_?NumberQ, male_] := NIntegrate[1, {h, Max[Min[21 - y, i - t], 0], Max[Min[21 - y, i - t + 1], 0]}];

(* Orphans receiving benefits < 18 - ECF *)
Blp18EcF[x_?NumberQ] := Module[{ecf, n, i},
  ecf = Table[0, {120}];
  For[i = 0, i ≤ 17 - x, i++, ecf[[i + 1]] = NIntegrate[1, {t, i, i + 1}];
  n = FractionalPart[17 + 1 - x];
  If[n > 0, ecf[[i + 1]] = NIntegrate[1, {t, i, i + n}]]];
  ecf];

(* Orphans receiving benefits < 21 - ECF *)
Blp21EcF[x_?NumberQ] := Module[{ecf, n, i},
  ecf = Table[0, {120}];
  For[i = 0, i ≤ 20 - x, i++, ecf[[i + 1]] = NIntegrate[1, {t, i, i + 1}];
  n = FractionalPart[21 + 1 - x];
  If[n > 0, ecf[[i + 1]] = NIntegrate[1, {t, i, i + n}]]];
  ecf];

```

```

(* Parameters using standard stress scenario *)
(* Generate ECF and SCF tables: Retirement benefits *)
Folder = "C:\\Users\\Bull\\Documents\\Studier\\1Masteroppgave\\Datafiles\\ECFy\\Expt120\\";
ks = 20; kk = 120; ΔM = -0.2;
ecfVM = Table[0, {kk}, {120}]; ecfVF = Table[0, {kk}, {120}]; ecfSM = Table[0, {kk}, {120}]; ecfSF = Table[0, {kk}, {120}];
For[k = ks, k ≤ kk, k++,
  ecfVM[[k]] = 100×ApEcf[k, True, 0];
  ecfVF[[k]] = 100×ApEcf[k, False, 0];
  ecfSM[[k]] = 100×ApEcf[k, True, ΔM];
  ecfSF[[k]] = 100×ApEcf[k, False, ΔM];
];
Export[StringInsert["AP_M.txt", Folder, 1], ecfVM, "Table"];
Export[StringInsert["AP_F.txt", Folder, 1], ecfVF, "Table"];
Export[StringInsert["AP_M_S.txt", Folder, 1], ecfSM, "Table"];
Export[StringInsert["AP_F_S.txt", Folder, 1], ecfSF, "Table"];
(* Generate ECF and SCF tables: Disability benefits, not disabled *)
Folder = "C:\\Users\\Bull\\Documents\\Studier\\1Masteroppgave\\Datafiles\\ECFy\\Expt120\\";
ks = 20; kk = 67; kkk = 120; ΔM = -0.2; ΔU1 = 0.35; ΔU2 = 0.25;
ecfVM = Table[0, {kkk}, {120}]; ecfVF = Table[0, {kkk}, {120}]; ecfSM = Table[0, {kkk}, {120}]; ecfSF = Table[0, {kkk}, {120}];
ecfUM = Table[0, {kkk}, {120}]; ecfUF = Table[0, {kkk}, {120}];
For[k = ks, k ≤ kk, k++,
  ecfVM[[k]] = 100×UpEcf[k, True, 0, 0, 0];
  ecfVF[[k]] = 100×UpEcf[k, False, 0, 0, 0];
  ecfSM[[k]] = 100×UpEcf[k, True, ΔM, 0, 0];
  ecfSF[[k]] = 100×UpEcf[k, False, ΔM, 0, 0];
  ecfUM[[k]] = 100×UpEcf[k, True, 0, ΔU1, ΔU2];
  ecfUF[[k]] = 100×UpEcf[k, False, 0, ΔU1, ΔU2];
];
Export[StringInsert["UP_M.txt", Folder, 1], ecfVM, "Table"];
Export[StringInsert["UP_F.txt", Folder, 1], ecfVF, "Table"];
Export[StringInsert["UP_M_SM.txt", Folder, 1], ecfSM, "Table"];
Export[StringInsert["UP_F_SM.txt", Folder, 1], ecfSF, "Table"];
Export[StringInsert["UP_M_SU.txt", Folder, 1], ecfUM, "Table"];
Export[StringInsert["UP_F_SU.txt", Folder, 1], ecfUF, "Table"];
(* Generate ECF and SCF tables: Disability benefits, disabled *)
Folder = "C:\\Users\\Bull\\Documents\\Studier\\1Masteroppgave\\Datafiles\\ECFy\\Expt120\\";
ks = 20; kk = 67; kkk = 120; ΔM = -0.2;
ecfVM = Table[0, {kkk}, {120}]; ecfVF = Table[0, {kkk}, {120}]; ecfSM = Table[0, {kkk}, {120}]; ecfSF = Table[0, {kkk}, {120}];
For[k = ks, k ≤ kk, k++,
  ecfVM[[k]] = 100×ULpEcf[k, True, 0];
  ecfVF[[k]] = 100×ULpEcf[k, False, 0];
  ecfSM[[k]] = 100×ULpEcf[k, True, ΔM];
  ecfSF[[k]] = 100×ULpEcf[k, False, ΔM];
];
Export[StringInsert["ULP_M.txt", Folder, 1], ecfVM, "Table"];
Export[StringInsert["ULP_F.txt", Folder, 1], ecfVF, "Table"];
Export[StringInsert["ULP_M_S.txt", Folder, 1], ecfSM, "Table"];
Export[StringInsert["ULP_F_S.txt", Folder, 1], ecfSF, "Table"];
(* Generate ECF and SCF tables: Spouse benefits *)
Folder = "C:\\Users\\Bull\\Documents\\Studier\\1Masteroppgave\\Datafiles\\ECFy\\Expt120\\";
ks = 20; kk = 120; ΔM = -0.2;
ecfVM = Table[0, {kk}, {120}]; ecfVF = Table[0, {kk}, {120}]; ecfSM = Table[0, {kk}, {120}]; ecfSF = Table[0, {kk}, {120}];
For[k = ks, k ≤ kk, k++,
  ecfVM[[k]] = 100×EpEcf[k, True, 0];
  ecfVF[[k]] = 100×EpEcf[k, False, 0];
  ecfSM[[k]] = 100×EpEcf[k, True, ΔM];
  ecfSF[[k]] = 100×EpEcf[k, False, ΔM];
];
Export[StringInsert["EP_M.txt", Folder, 1], ecfVM, "Table"];
Export[StringInsert["EP_F.txt", Folder, 1], ecfVF, "Table"];
Export[StringInsert["EP_M_S.txt", Folder, 1], ecfSM, "Table"];
Export[StringInsert["EP_F_S.txt", Folder, 1], ecfSF, "Table"];

```



```

(* Generate ECF and SCF tables: Partner benefits *)
ks = 20; kk = 120; ΔM = -0.2;
ecfVM = Table[0, {kk}, {120}]; ecfVF = Table[0, {kk}, {120}]; ecfSM = Table[0, {kk}, {120}]; ecfSF = Table[0, {kk}, {120}];
For[k = ks, k ≤ kk, k++,
  ecfVM[[k]] = 100×SpEcF[k, True, 0];
  ecfVF[[k]] = 100×SpEcF[k, False, 0];
  ecfSM[[k]] = 100×SpEcF[k, True, ΔM];
  ecfSF[[k]] = 100×SpEcF[k, False, ΔM];
];
Export[StringInsert["SP_M.txt", Folder, 1], ecfVM, "Table"];
Export[StringInsert["SP_F.txt", Folder, 1], ecfVF, "Table"];
Export[StringInsert["SP_M_S.txt", Folder, 1], ecfSM, "Table"];
Export[StringInsert["SP_F_S.txt", Folder, 1], ecfSF, "Table"];

(* Generate ECF and SCF tables: Widows receiving benefits *)
ks = 20; kk = 120; ΔM = -0.2;
ecfVM = Table[0, {kk}, {120}]; ecfVF = Table[0, {kk}, {120}]; ecfSM = Table[0, {kk}, {120}]; ecfSF = Table[0, {kk}, {120}];
For[k = ks, k ≤ kk, k++,
  ecfVM[[k]] = 100×ELpEcF[k, True, 0];
  ecfVF[[k]] = 100×ELpEcF[k, False, 0];
  ecfSM[[k]] = 100×ELpEcF[k, True, ΔM];
  ecfSF[[k]] = 100×ELpEcF[k, False, ΔM];
];
Export[StringInsert["ELP_M.txt", Folder, 1], ecfVM, "Table"];
Export[StringInsert["ELP_F.txt", Folder, 1], ecfVF, "Table"];
Export[StringInsert["ELP_M_S.txt", Folder, 1], ecfSM, "Table"];
Export[StringInsert["ELP_F_S.txt", Folder, 1], ecfSF, "Table"];

(* Generate ECF and SCF tables: Children benefits < 18 *)
Folder = "C:\\Users\\Bull\\Documents\\Studier\\1Masteroppgave\\Datafiles\\ECF\\Expt120\\";
ks = 20; kk = 75; kkk = 120; ΔM = -0.2;
ecfVM = Table[0, {kkk}, {120}]; ecfVF = Table[0, {kkk}, {120}]; ecfSM = Table[0, {kkk}, {120}]; ecfSF = Table[0, {kkk}, {120}];
For[k = ks, k ≤ kk, k++,
  ecfVM[[k]] = 100×Bp18EcF[k, True, 0];
  ecfVF[[k]] = 100×Bp18EcF[k, False, 0];
  ecfSM[[k]] = 100×Bp18EcF[k, True, ΔM];
  ecfSF[[k]] = 100×Bp18EcF[k, False, ΔM];
];
Export[StringInsert["BP18_M.txt", Folder, 1], ecfVM, "Table"];
Export[StringInsert["BP18_F.txt", Folder, 1], ecfVF, "Table"];
Export[StringInsert["BP18_M_S.txt", Folder, 1], ecfSM, "Table"];
Export[StringInsert["BP18_F_S.txt", Folder, 1], ecfSF, "Table"];

```

```

(* Generate ECF and SCF tables: Orphan receiving benefits < 18 *)
ks = 1; kk = 18; kkk = 120; ΔM = -0.2;
ecf = Table[0, {kkk}, {120}];
For[k = ks, k ≤ kk, k++,
  ecf[[k]] = 100×BLp18Ecf[k];
];
Export[StringInsert["BLP18.txt", Folder, 1], ecf, "Table"];

(* Generate ECF and SCF tables: Children benefits < 21 *)
ks = 20; kk = 75; kkk = 120; ΔM = -0.2;
ecfVM = Table[0, {kkk}, {120}]; ecfVF = Table[0, {kkk}, {120}]; ecfSM = Table[0, {kkk}, {120}]; ecfSF = Table[0, {kkk}, {120}];
For[k = ks, k ≤ kk, k++,
  ecfVM[[k]] = 100×Bp21Ecf[k, True, 0];
  ecfVF[[k]] = 100×Bp21Ecf[k, False, 0];
  ecfSM[[k]] = 100×Bp21Ecf[k, True, ΔM];
  ecfSF[[k]] = 100×Bp21Ecf[k, False, ΔM];
];
Export[StringInsert["BP21_M.txt", Folder, 1], ecfVM, "Table"];
Export[StringInsert["BP21_F.txt", Folder, 1], ecfVF, "Table"];
Export[StringInsert["BP21_M_S.txt", Folder, 1], ecfSM, "Table"];
Export[StringInsert["BP21_F_S.txt", Folder, 1], ecfSF, "Table"];

(* Generate ECF and SCF tables: Orphan receiving benefits < 21 *)
ks = 1; kk = 21; kkk = 120; ΔM = -0.2;
ecf = Table[0, {kkk}, {120}];
For[k = ks, k ≤ kk, k++,
  ecf[[k]] = 100×BLp21Ecf[k];
];
Export[StringInsert["BLP21.txt", Folder, 1], ecf, "Table"];

```

Part II – See sub section 9.3.1:

```
(* Path to folder *)
Folder = "C:\\Users\\Bull\\Documents\\Studier\\1Masteroppgave\\Datafiles\\ECFy\\Eqsc120\\";
(* Retirement ECF *)
APM = Import[StringInsert["AP_M.txt", Folder, 1], "Table"];
APF = Import[StringInsert["AP_F.txt", Folder, 1], "Table"];
APMS = Import[StringInsert["AP_M_S.txt", Folder, 1], "Table"];
APFS = Import[StringInsert["AP_F_S.txt", Folder, 1], "Table"];
(* Disability pension rights *)
UPM = Import[StringInsert["UP_M.txt", Folder, 1], "Table"];
UPF = Import[StringInsert["UP_F.txt", Folder, 1], "Table"];
UPMSM = Import[StringInsert["UP_M_SM.txt", Folder, 1], "Table"];
UPFSM = Import[StringInsert["UP_F_SM.txt", Folder, 1], "Table"];
UPMSU = Import[StringInsert["UP_M_SU.txt", Folder, 1], "Table"];
UPFSU = Import[StringInsert["UP_F_SU.txt", Folder, 1], "Table"];
(* Disability pension *)
ULPM = Import[StringInsert["ULP_M.txt", Folder, 1], "Table"];
ULPF = Import[StringInsert["ULP_F.txt", Folder, 1], "Table"];
ULPMS = Import[StringInsert["ULP_M_S.txt", Folder, 1], "Table"];
ULPFS = Import[StringInsert["ULP_F_S.txt", Folder, 1], "Table"];
(* Widows pension rights - spouse *)
EPM = Import[StringInsert["EP_M.txt", Folder, 1], "Table"];
EPF = Import[StringInsert["EP_F.txt", Folder, 1], "Table"];
EPMS = Import[StringInsert["EP_M_S.txt", Folder, 1], "Table"];
EPFS = Import[StringInsert["EP_F_S.txt", Folder, 1], "Table"];
(* Widows pension rights - partner *)
SPM = Import[StringInsert["SP_M.txt", Folder, 1], "Table"];
SPF = Import[StringInsert["SP_F.txt", Folder, 1], "Table"];
SPMS = Import[StringInsert["SP_M_S.txt", Folder, 1], "Table"];
SPFS = Import[StringInsert["SP_F_S.txt", Folder, 1], "Table"];
(* Widows pension *)
ELPM = Import[StringInsert["ELP_M.txt", Folder, 1], "Table"];
ELPF = Import[StringInsert["ELP_F.txt", Folder, 1], "Table"];
ELPMS = Import[StringInsert["ELP_M_S.txt", Folder, 1], "Table"];
ELPFS = Import[StringInsert["ELP_F_S.txt", Folder, 1], "Table"];
(* Orphans pension rights - 18 year *)
BP18M = Import[StringInsert["BP18_M.txt", Folder, 1], "Table"];
BP18F = Import[StringInsert["BP18_F.txt", Folder, 1], "Table"];
BP18MS = Import[StringInsert["BP18_M_S.txt", Folder, 1], "Table"];
BP18FS = Import[StringInsert["BP18_F_S.txt", Folder, 1], "Table"];
(* Orphans pension rights - 21 year *)
BP21M = Import[StringInsert["BP21_M.txt", Folder, 1], "Table"];
BP21F = Import[StringInsert["BP21_F.txt", Folder, 1], "Table"];
BP21MS = Import[StringInsert["BP21_M_S.txt", Folder, 1], "Table"];
BP21FS = Import[StringInsert["BP21_F_S.txt", Folder, 1], "Table"];
(* Orphans pension 18 year *)
BLP18 = Import[StringInsert["BLP18.txt", Folder, 1], "Table"];
(* Orphans pension 21 year *)
BLP21 = Import[StringInsert["BLP21.txt", Folder, 1], "Table"];
```

```

(* Function calculating ECF and SCF at policyholder level *)
IpEcF[x_, male_, YAp_, YEp_, YSp_, YELp_, YUp_, YULp_, YBp18_, YBp21_, YBLp18_, YBLp21_] := Module[{f, ks, kk},
  f = x - Floor[x]; kk = Floor[x]; ks = Ceiling[x]; If[kk == 0, kk = 1]; If[ks > 120, ks = 120; kk = 120]
  If[male,
    ecfV = ecfV + YAp / 100 × (f × APM[[ks]] + (1 - f) × APM[[kk]]);
    ecfS = ecfS + YAp / 100 × (f × APMS[[ks]] + (1 - f) × APMS[[kk]]);
    ecfV = ecfV + YEp / 100 × (f × EPM[[ks]] + (1 - f) × EPM[[kk]]);
    ecfS = ecfS + YEp / 100 × (f × EPMS[[ks]] + (1 - f) × EPMS[[kk]]);
    ecfV = ecfV + YSp / 100 × (f × SPM[[ks]] + (1 - f) × SPM[[kk]]);
    ecfS = ecfS + YSp / 100 × (f × SPMS[[ks]] + (1 - f) × SPMS[[kk]]);
    ecfV = ecfV + YELp / 100 × (f × ELPMS[[ks]] + (1 - f) × ELPMS[[kk]]);
    ecfS = ecfS + YELp / 100 × (f × ELPMS[[ks]] + (1 - f) × ELPMS[[kk]]);
    ecfV = ecfV + YUp / 100 × (f × UPM[[ks]] + (1 - f) × UPM[[kk]]);
    ecfS = ecfS + YUp / 100 × (f × UPMSM[[ks]] + (1 - f) × UPMSM[[kk]]);
    UecfV = UecfV + YUp / 100 × (f × UPM[[ks]] + (1 - f) × UPM[[kk]]);
    UecfS = UecfS + YUp / 100 × (f × UPMSU[[ks]] + (1 - f) × UPMSU[[kk]]);
    ecfV = ecfV + YULp / 100 × (f × ULPM[[ks]] + (1 - f) × ULPM[[kk]]);
    ecfS = ecfS + YULp / 100 × (f × ULPMS[[ks]] + (1 - f) × ULPMS[[kk]]);
    ecfV = ecfV + YBp18 / 100 × (f × BP18M[[ks]] + (1 - f) × BP18M[[kk]]);
    ecfS = ecfS + YBp18 / 100 × (f × BP18MS[[ks]] + (1 - f) × BP18MS[[kk]]);
    ecfV = ecfV + YBp21 / 100 × (f × BP21M[[ks]] + (1 - f) × BP21M[[kk]]);
    ecfS = ecfS + YBp21 / 100 × (f × BP21MS[[ks]] + (1 - f) × BP21MS[[kk]]);
    ,
    ecfV = ecfV + YAp / 100 × (f × APF[[ks]] + (1 - f) × APF[[kk]]);
    ecfS = ecfS + YAp / 100 × (f × APFS[[ks]] + (1 - f) × APFS[[kk]]);
    ecfV = ecfV + YEp / 100 × (f × EPF[[ks]] + (1 - f) × EPF[[kk]]);
    ecfS = ecfS + YEp / 100 × (f × EPFS[[ks]] + (1 - f) × EPFS[[kk]]);
    ecfV = ecfV + YSp / 100 × (f × SPF[[ks]] + (1 - f) × SPF[[kk]]);
    ecfS = ecfS + YSp / 100 × (f × SPFS[[ks]] + (1 - f) × SPFS[[kk]]);
    ecfV = ecfV + YELp / 100 × (f × ELPF[[ks]] + (1 - f) × ELPF[[kk]]);
    ecfS = ecfS + YELp / 100 × (f × ELPFS[[ks]] + (1 - f) × ELPFS[[kk]]);
    ecfV = ecfV + YUp / 100 × (f × UPF[[ks]] + (1 - f) × UPF[[kk]]);
    ecfS = ecfS + YUp / 100 × (f × UPFSM[[ks]] + (1 - f) × UPFSM[[kk]]);
    UecfV = UecfV + YUp / 100 × (f × UPF[[ks]] + (1 - f) × UPF[[kk]]);
    UecfS = UecfS + YUp / 100 × (f × UPFSU[[ks]] + (1 - f) × UPFSU[[kk]]);
    ecfV = ecfV + YULp / 100 × (f × ULPF[[ks]] + (1 - f) × ULPF[[kk]]);
    ecfS = ecfS + YULp / 100 × (f × ULPFS[[ks]] + (1 - f) × ULPFS[[kk]]);
    ecfV = ecfV + YBp18 / 100 × (f × BP18F[[ks]] + (1 - f) × BP18F[[kk]]);
    ecfS = ecfS + YBp18 / 100 × (f × BP18FS[[ks]] + (1 - f) × BP18FS[[kk]]);
    ecfV = ecfV + YBp21 / 100 × (f × BP21F[[ks]] + (1 - f) × BP21F[[kk]]);
    ecfS = ecfS + YBp21 / 100 × (f × BP21FS[[ks]] + (1 - f) × BP21FS[[kk]]);
  ];
  ecfV = ecfV + YBLp18 / 100 × (f × BLP18[[ks]] + (1 - f) × BLP18[[kk]]);
  ecfV = ecfV + YBLp21 / 100 × (f × BLP21[[ks]] + (1 - f) × BLP21[[kk]]);
];

(* Function calculating CAT-ECF relating to mortality at policyholder level *)
CATEcF[x_, male_, YELp_, YBLp18_, YBLp21_] := Module[{f, ks, kk},
  f = x - Floor[x]; kk = Floor[x]; ks = Ceiling[x]; If[kk == 0, kk = 1]; If[ks > 100, ks = 100; kk = 100]
  If[male,
    CATecfV = CATecfV + YELp / 100 × (f × ELPMS[[ks]] + (1 - f) × ELPMS[[kk]]);
    ,
    CATecfV = CATecfV + YELp / 100 × (f × ELPF[[ks]] + (1 - f) × ELPF[[kk]]);
  ];
  CATecfV = CATecfV + YBLp18 / 100 × (f × BLP18[[ks]] + (1 - f) × BLP18[[kk]]);
  CATecfV = CATecfV + YBLp21 / 100 × (f × BLP21[[ks]] + (1 - f) × BLP21[[kk]]);
];

```

```
(* Aggregates policy ECF and SCF, files depend on policyholders stats.
Files may be merged with some modification *)
ImportFolder = "C:\\Users\\Bull\\Documents\\Studier\\1Masteroppgave\\Datafiles\\Member Data\\";
ExportFolder = "C:\\Users\\Bull\\Documents\\Studier\\1Masteroppgave\\Datafiles\\Member ECF\\Expt\\";
CATEcfV = Table[0, {120}];

F = Import[StringInsert["AK.txt", ImportFolder, 1], "Table"];
kk = Dimensions[F][[1]];
ecfV = Table[0, {120}]; ecfS = Table[0, {120}]; UecfV = Table[0, {120}]; UecfS = Table[0, {120}];
For[k = 2, k ≤ kk, k++,
  x = F[[k, 1]];
  male = If[F[[k, 2]] == 1, True, False];
  Optj = F[[k, 3]] / (F[[k, 3]] + F[[k, 4]]);
  YAp = Optj × F[[k, 6]];
  YEp = Optj × F[[k, 9]];
  YSp = Optj × F[[k, 10]];
  YBp18 = Optj × (F[[k, 7]] - F[[k, 8]]);
  YBp21 = Optj × F[[k, 8]];
  YUp = Optj × F[[k, 11]];
  IpEcf[x, male, YAp, YEp, YSp, 0, YUp, 0, YBp18, YBp21, 0, 0];
  CATEcf[x, male, YEp, YBp18, YBp21];
];
Export[StringInsert["cf-AK.txt", ExportFolder, 1], {ecfV, ecfS, UecfV, UecfS}, "Table"];

F = Import[StringInsert["AKOPTJ.txt", ImportFolder, 1], "Table"];
kk = Dimensions[F][[1]];
ecfV = Table[0, {120}]; ecfS = Table[0, {120}]; UecfV = Table[0, {120}]; UecfS = Table[0, {120}];
For[k = 2, k ≤ kk, k++,
  x = F[[k, 1]];
  male = If[F[[k, 2]] == 1, True, False];
  Optj = F[[k, 5]] / 100;
  YAp = Optj × F[[k, 6]];
  YEp = Optj × F[[k, 9]];
  YSp = Optj × F[[k, 10]];
  YBp18 = Optj × (F[[k, 7]] - F[[k, 8]]);
  YBp21 = Optj × F[[k, 8]];
  YUp = Optj × F[[k, 11]];
  IpEcf[x, male, YAp, YEp, YSp, 0, YUp, 0, YBp18, YBp21, 0, 0];
  CATEcf[x, male, YEp, YBp18, YBp21];
];
Export[StringInsert["cf-AKOPTJ.txt", ExportFolder, 1], {ecfV, ecfS, UecfV, UecfS}, "Table"];

F = Import[StringInsert["FAK.txt", ImportFolder, 1], "Table"];
kk = Dimensions[F][[1]];
Optj = 1; ecfV = Table[0, {120}]; ecfS = Table[0, {120}]; UecfV = Table[0, {120}]; UecfS = Table[0, {120}];
For[k = 2, k ≤ kk, k++,
  x = F[[k, 1]];
  male = If[F[[k, 2]] == 1, True, False];
  YAp = Optj × F[[k, 4]];
  YEp = Optj × F[[k, 7]];
  YSp = Optj × F[[k, 8]];
  YBp18 = Optj × (F[[k, 5]] - F[[k, 6]]);
  YBp21 = Optj × F[[k, 6]];
  YUp = Optj × F[[k, 9]];
  IpEcf[x, male, YAp, YEp, YSp, 0, YUp, 0, YBp18, YBp21, 0, 0];
  CATEcf[x, male, YEp, YBp18, YBp21];
];
Export[StringInsert["cf-FAK.txt", ExportFolder, 1], {ecfV, ecfS, UecfV, UecfS}, "Table"];
```

```

F = Import[StringInsert["PA.txt", ImportFolder, 1], "Table"];
kk = Dimensions[F][[1]];
ecfV = Table[0, {120}]; ecfS = Table[0, {120}];
For[k = 2, k ≤ kk, k++,
  x = F[[k, 1]];
  male = If[F[[k, 2]] == 1, True, False];
  Optj = 1;
  YAp = Optj × F[[k, 7]];
  YEp = Optj × F[[k, 10]];
  YSp = Optj × F[[k, 11]];
  YBp18 = Optj × (F[[k, 8]] - F[[k, 9]]);
  YBp21 = Optj × F[[k, 9]];
  IpEcf[x, male, YAp, YEp, YSp, 0, 0, 0, YBp18, YBp21, 0, 0];
  CATEcf[x, male, YEp, YBp18, YBp21];
];
Export[StringInsert["cf-PA.txt", ExportFolder, 1], {ecfV, ecfS}, "Table"];

F = Import[StringInsert["FPA.txt", ImportFolder, 1], "Table"];
kk = Dimensions[F][[1]];
Optj = 1; ecfV = Table[0, {120}]; ecfS = Table[0, {120}];
For[k = 2, k ≤ kk, k++,
  x = F[[k, 1]];
  male = If[F[[k, 2]] == 1, True, False];
  YAp = Optj × F[[k, 7]];
  YEp = Optj × F[[k, 10]];
  YSp = Optj × F[[k, 11]];
  YBp18 = Optj × F[[k, 8]];
  YBp21 = Optj × F[[k, 9]];
  IpEcf[x, male, YAp, YEp, YSp, 0, 0, 0, YBp18, YBp21, 0, 0];
  CATEcf[x, male, YEp, YBp18, YBp21];
];
Export[StringInsert["cf-FPA.txt", ExportFolder, 1], {ecfV, ecfS}, "Table"];

F = Import[StringInsert["PE.txt", ImportFolder, 1], "Table"];
kk = Dimensions[F][[1]];
Optj = 1; ecfV = Table[0, {120}]; ecfS = Table[0, {120}];
For[k = 2, k ≤ kk, k++,
  x = F[[k, 1]];
  male = If[F[[k, 2]] == 1, True, False];
  YELp = Optj × F[[k, 7]];
  IpEcf[x, male, 0, 0, 0, YELp, 0, 0, 0, 0, 0, 0];
];
Export[StringInsert["cf-PE.txt", ExportFolder, 1], {ecfV, ecfS}, "Table"];

F = Import[StringInsert["FPE.txt", ImportFolder, 1], "Table"];
kk = Dimensions[F][[1]];
Optj = 1; ecfV = Table[0, {120}]; ecfS = Table[0, {120}];
For[k = 2, k ≤ kk, k++,
  x = F[[k, 1]];
  male = If[F[[k, 2]] == 1, True, False];
  YELp = Optj × F[[k, 5]];
  IpEcf[x, male, 0, 0, 0, YELp, 0, 0, 0, 0, 0, 0];
];
Export[StringInsert["cf-FPE.txt", ExportFolder, 1], {ecfV, ecfS}, "Table"];

```

```

F = Import[StringInsert["PB.txt", ImportFolder, 1], "Table"];
kk = Dimensions[F][[1]];
Optj = 1; ecfV = Table[0, {120}]; ecfS = Table[0, {120}];
For[k = 2, k ≤ kk, k++,
  x = F[[k, 1]];
  male = If[F[[k, 2]] == 1, True, False];
  YBLp18 = Optj × F[[k, 7]];
  YBLp21 = Optj × F[[k, 8]];
  IpEcf[x, male, 0, 0, 0, 0, 0, 0, 0, 0, YBLp18, YBLp21];
];
Export[StringInsert["cf-PB.txt", ExportFolder, 1], {ecfV, ecfS}, "Table"];

F = Import[StringInsert["FPB.txt", ImportFolder, 1], "Table"];
kk = Dimensions[F][[1]];
AM = -0.2; Optj = 1; ecfV = Table[0, {120}]; ecfS = Table[0, {120}];
For[k = 2, k ≤ kk, k++,
  x = F[[k, 1]];
  male = If[F[[k, 2]] == 1, True, False];
  YBLp18 = Optj × F[[k, 5]];
  YBLp21 = Optj × F[[k, 6]];
  IpEcf[x, male, 0, 0, 0, 0, 0, 0, 0, 0, YBLp18, YBLp21];
];
Export[StringInsert["cf-FPB.txt", ExportFolder, 1], {ecfV, ecfS}, "Table"];

F = Import[StringInsert["PUOGPUT.txt", ImportFolder, 1], "Table"];
kk = Dimensions[F][[1]];
ecfV = Table[0, {120}]; ecfS = Table[0, {120}];
For[k = 2, k ≤ kk, k++,
  x = F[[k, 1]];
  male = If[F[[k, 2]] == 1, True, False];
  Optj = 1;
  YAp = Optj × F[[k, 9]];
  YULp = Optj × F[[k, 9]] × F[[k, 16]] / 100;
  YEp = Optj × F[[k, 12]];
  YSp = Optj × F[[k, 13]];
  YBp18 = Optj × F[[k, 10]];
  YBp21 = Optj × F[[k, 11]];
  IpEcf[x, male, YAp, YEp, YSp, 0, 0, YULp, YBp18, YBp21, 0, 0];
  CATEcf[x, male, YEp, YBp18, YBp21];
];
Export[StringInsert["cf-PU.txt", ExportFolder, 1], {ecfV, ecfS}, "Table"];

F = Import[StringInsert["FPU.txt", ImportFolder, 1], "Table"];
kk = Dimensions[F][[1]];
ecfV = Table[0, {120}]; ecfS = Table[0, {120}];
For[k = 2, k ≤ kk, k++,
  x = F[[k, 1]];
  male = If[F[[k, 2]] == 1, True, False];
  Optj = 1;
  YAp = Optj × F[[k, 7]];
  YULp = Optj × F[[k, 13]];
  YEp = Optj × F[[k, 10]];
  YSp = Optj × F[[k, 11]];
  YBp18 = Optj × F[[k, 8]];
  YBp21 = Optj × F[[k, 9]];
  IpEcf[x, male, YAp, YEp, YSp, 0, 0, YULp, YBp18, YBp21, 0, 0];
  CATEcf[x, male, YEp, YBp18, YBp21];
];
Export[StringInsert["cf-FPU.txt", ExportFolder, 1], {ecfV, ecfS}, "Table"];

Export[StringInsert["cf-CAT.txt", ExportFolder, 1], {CATEcfV}, "Table"];

```


Part III – see sub section 9.3.1 and 9.3.2:

The Calculations at the end is based on the standard formulas. The results from the equivalent scenario is not shown.

```
Folder = "C:\\Users\\Bull\\Documents\\Studier\\1Masteroppgave\\Datafiles\\Yield Curves\\";
iMRf = Log[1 + Flatten[Import[StringInsert["NOKSpot.csv", Folder, 1]]]];
iM = Log[1 + Flatten[Import[StringInsert["NOKSpot75.csv", Folder, 1]]]];
iMip = Log[1 + Flatten[Import[StringInsert["NOKSpotIp.csv", Folder, 1]]]];
iMup = Log[1 + Flatten[Import[StringInsert["NOKSpot75Up.csv", Folder, 1]]]];
iMdown = Log[1 + Flatten[Import[StringInsert["NOKSpot75Down.csv", Folder, 1]]]];
iMdownEqsc = Log[1 + Flatten[Import[StringInsert["NOKSpot75DownEqsc.csv", Folder, 1]]]];
iMupEqsc = Log[1 + Flatten[Import[StringInsert["NOKSpot75UpEqsc.csv", Folder, 1]]]];
iMipEqsc = Log[1 + Flatten[Import[StringInsert["NOKSpotIpEqsc.csv", Folder, 1]]]];

iMRf = Prepend[iMRf, First[iMRf]];
iM = Prepend[iM, First[iM]];
iMip = Prepend[iMip, First[iMip]];
iMup = Prepend[iMup, First[iMup]];
iMdown = Prepend[iMdown, First[iMdown]];
iMdownEqsc = Prepend[iMdownEqsc, First[iMdownEqsc]];
iMupEqsc = Prepend[iMupEqsc, First[iMupEqsc]];
iMipEqsc = Prepend[iMipEqsc, First[iMipEqsc]];
iG26 = Table[Log[1.026], {j, 1, 136}];

Folder = "C:\\Users\\Bull\\Documents\\Studier\\1Masteroppgave\\Datafiles\\Volatility Cubes\\";
VolCube = Import[StringInsert["Vol2009.txt", Folder, 1], "Table"];

(* Primary yield curve based on bond rates with 75% liquidity premium *)
(* May be used for actual curve and shocks *)
yc[t_] := yb[[IntegerPart[t] + 1]] + FractionalPart[t] × (yb[[IntegerPart[t] + 2]] - yb[[IntegerPart[t] + 1]]); (* yb = yield buckets *)
Fw[s_, t_] := Exp[-(s + t) × yc[s + t]] / Exp[-t × yc[t]];

(* Risk free yield curve based only calculation of risk margin *)
ybRf = iMRf; (* Initialization immediately *)
ycRf[t_] := ybRf[[IntegerPart[t] + 1]] + FractionalPart[t] × (ybRf[[IntegerPart[t] + 2]] - ybRf[[IntegerPart[t] + 1]]);
(* yb = yield buckets *)
FwRf[s_, t_] := Exp[-(s + t) × ycRf[s + t]] / Exp[-t × ycRf[t]];
(* Fw = Forward bond rates based on risk free yield curve (RM calculations) *)

(* Auxiliary yield curves *)
yc1[t_] := yb1[[IntegerPart[t] + 1]] + FractionalPart[t] × (yb1[[IntegerPart[t] + 2]] - yb1[[IntegerPart[t] + 1]]);
(* yb = yield buckets *)
Fw1[s_, t_] := Exp[-(s + t) × yc1[s + t]] / Exp[-t × yc1[t]];

(* Modified Duration of a cash flow vector (equal to duration since we have used continuous compounding) *)
MDur[ecf_] := Module[{n, t, D, BE},
  D = 0; n = Dimensions[ecf][[1]]; BE = Decf[ecf, ecf][[1]];
  For[t = 1, t ≤ n, t++, D = D + (t × Exp[-yc[t]] × (ecf[[t]] × Fw[t, 0] / BE));
  D];

(* Mapping of bonds, based on market value and duration *)
BondMapp[Bdata_] := Module[{i, n, MV, D, cf, ecf},
  n = Dimensions[Bdata][[1]]; ecf = Table[0, {120}];
  For[i = 2, i ≤ n, i++,
    MV = Bdata[[i, 2]]; D = Bdata[[i, 3]]; If[D < 1, D = 1];
    cf = MV / Fw[D, 0];
    ecf[[Floor[D]]] = ecf[[Floor[D]]] + (Ceiling[D] - D) × cf;
    ecf[[Ceiling[D]]] = ecf[[Ceiling[D]]] + (D - Floor[D]) × cf;
    If[Ceiling[D] == Floor[D], ecf[[Ceiling[D]]] = ecf[[Ceiling[D]]] + cf];
  ];
  ecf
];
```



```

(* Redistribution of life annuity cash flows *)
LifeMapp[ecf_] := Module[{i, recf, n},
  recf = Table[0, {120}]; n = Dimensions[ecf][[1]];
  For[i = 2, i ≤ n - 1, i++, recf[[i]] = 0.5 × ecf[[i]] + 0.5 × ecf[[i + 1]]];
  recf[[1]] = ecf[[1]] + 0.5 × ecf[[2]];
  recf[[n]] = 0.5 × ecf[[n]];
  recf
];

(* Returns discounted value, capital charge for cash flow differences *)
Decf[ecf_, ecfS_] := Module[{be, diff, i, p, rm, scr},
  rm = 0; be = 0; diff = ecfS - ecf; p = Dimensions[diff][[1]]; scr = Table[0, {p}];
  scr[[p]] = diff[[p]] × Fw[1, p - 1];
  For[i = p - 1, i ≥ 1, i = i - 1, scr[[i]] = (diff[[i]] + scr[[i + 1]]) × Fw[1, i - 1];
  For[i = 1, i ≤ p, i++, be = be + ecf[[i]] × Fw[i, 0]];
  {be, scr[[1]]};

(* Calculates the interest rate guarantee, based on Black-76 *)
OV[ecf_, l_, r_, VolCube_] := Module[{a, b, i, k, t, n, nV, hVol, Vol, IRG, f, σ, d1, d2, TR},
  (* Calculates interest rate guarantee cash flows from TR(t) *)
  n = Dimensions[ecf][[1]]; TR = 0; IRG = 0;
  hVol = VolCube[[1]] / 10000; nV = Dimensions[hVol][[1]];
  Vol = VolCube[[2 ;; Dimensions[VolCube][[1]]]] / 100;
  yb1 = Table[Log[1 + r], {136}];
  For[t = 1, t ≤ n, t++, TR = TR + (1 + l) × ecf[[t]] × Fw1[t, 0]];
  For[t = 1, t ≤ n, t++,
    f = (1 / Fw[1, t - 1] - 1);
    If[t == 1,
      IRG = TR × Fw[t, 0] × Max[r - f, 0];
      For[i = 1, i ≤ nV, i++, If[r - f ≥ hVol[[i]], k = i, If[i == 1, k = 1]]];
      If[k == 1 || k == nV, σ = Vol[[t - 1, k]], a = hVol[[k]]; b = hVol[[k + 1]];
      σ = Vol[[t - 1, k]] × (1 - (r - f - a) / (b - a)) + Vol[[t - 1, k + 1]] × (r - f - a) / (b - a);
      d1 = (Log[f / r] + σ^2 × (t - 1) / 2) / (σ × (t - 1)^0.5);
      d2 = d1 - σ × (t - 1)^0.5;
      IRG = IRG + TR × Fw[t, 0] × (-f × CDF[NormalDistribution[0, 1], -d1] + r × CDF[NormalDistribution[0, 1], -d2]);
    ];
    TR = TR × (1 + r) - (1 + l) × ecf[[t]];
  ];
  IRG];

(* Load net expected cash flows *)
Folder = "C:\\Users\\Bull\\Documents\\Studier\\1Masteroppgave\\Datafiles\\Member ECF\\Expt\\";
{AKecfV, AKecfS, AKUecfV, AKUecfS} = Import[StringInsert["cf-AK.txt", Folder, 1], "Table"];
{AKOPTJecfV, AKOPTJecfS, AKOPTJUecfV, AKOPTJUecfS} = Import[StringInsert["cf-AKOPTJ.txt", Folder, 1], "Table"];
{FAKecfV, FAKecfS, FAKUecfV, FAKUecfS} = Import[StringInsert["cf-FAK.txt", Folder, 1], "Table"];
{PAecfV, PAecfS} = Import[StringInsert["cf-PA.txt", Folder, 1], "Table"];
{FPAecfV, FPAecfS} = Import[StringInsert["cf-FPA.txt", Folder, 1], "Table"];
{PEecfV, PEecfS} = Import[StringInsert["cf-PE.txt", Folder, 1], "Table"];
{FPEecfV, FPEecfS} = Import[StringInsert["cf-FPE.txt", Folder, 1], "Table"];
{PBecfV, PBecfS} = Import[StringInsert["cf-PB.txt", Folder, 1], "Table"];
{FPBecfV, FPBecfS} = Import[StringInsert["cf-FPB.txt", Folder, 1], "Table"];
{PUecfV, PUecfS} = Import[StringInsert["cf-PU.txt", Folder, 1], "Table"];
{FPUecfV, FPUecfS} = Import[StringInsert["cf-FPU.txt", Folder, 1], "Table"];
{CATecfV} = Import[StringInsert["cf-CAT.txt", Folder, 1], "Table"];

```

```

In[46]:= (* Load bond data *)
Folder = "C:\\Users\\Bull\\Documents\\Studier\\1Masteroppgave\\Datafiles\\Bond Data\\";
BondDataColl = Import[StringInsert["BondDataCollPort.txt", Folder, 1], "Table"];
BondDataComp = Import[StringInsert["BondDataCompPort.txt", Folder, 1], "Table"];

(* Redistribute net expected cash flows - liabilities *)
ecfV = LifeMapp[AKDPTJecfV] + LifeMapp[FAKecfV] + LifeMapp[PAecfV] + LifeMapp[FPAecfV] + LifeMapp[PEecfV] +
  LifeMapp[FPEecfV] + LifeMapp[PBecfV] + LifeMapp[FPBecfV] + LifeMapp[PUecfV] + LifeMapp[FPUecfV];
ecfS = LifeMapp[AKDPTJecfS] + LifeMapp[FAKecfS] + LifeMapp[PAecfS] + LifeMapp[FPAecfS] + LifeMapp[PEecfS] +
  LifeMapp[FPEecfS] + LifeMapp[PBecfS] + LifeMapp[FPBecfS] + LifeMapp[PUecfS] + LifeMapp[FPUecfS];
UecfV = LifeMapp[AKDPTJUecfV] + LifeMapp[FAKUecfV];
UecfS = LifeMapp[AKDPTJUecfS] + LifeMapp[FAKUecfS];
CATecfV = LifeMapp[CATecfV];
dummy = ecfV;

(* Mapping of bond cash flows *)
yb = iM;
BondCollCf = BondMapp[BondDataColl];
BondCompCf = BondMapp[BondDataComp];

(* Calculating technical provisions ex. risk margin *)
PR = 6481000000 + 119647400; (* Premium reserves *)
PAF = 2259562000; (* Price adjustment fund *)
ARF = 160283000; (* Additional reserve fund*)
OIF = 205239600; (* Other insurance funds *)
IGR = 0.026; (* Technical rate *)
l = 0.07; (* Loading - to calculate value of embedded interest rate guarantee *)

Print[""]
dummy = Table[0, {Dimensions[ecfV][[1]]}];
yb = iM; GB = Decf[ecfV, dummy][[1]]; Print["Guaranteed benefits: ", GB]
yb = iM; AG = OV[ecfV, l, IGR, VolCube]; Print["Interest rate guarantee: ", AG]
yb = iM; n = MDur[ecfV]; EE = 22770000; i = 0.02;
EX = EE * (Sum[Fw[t, 0] * (1 + i)^t, {t, 1, IntegerPart[n]}] + (n - IntegerPart[n]) * Fw[n, 0] * (1 + i)^n); Print["Expenses: ", EX]
FDB = 0.8 * Max[PR - GB - AG + PAF + ARF, 0] + 0.2 * Max[(ARF - AG), 0]; Print["Profit sharing: ", FDB]
TP = GB + AG + OIF + EX + FDB; Print["Technical Provisions ex. RM: ", TP]
Print[""]

Guaranteed benefits:  $4.18583 \times 10^9$ 
Interest rate guarantee:  $1.4069 \times 10^8$ 
Expenses:  $2.54157 \times 10^8$ 
Profit sharing:  $3.7591 \times 10^8$ 
Technical Provisions ex. RM:  $8.54501 \times 10^9$ 

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```

ln[79]= (* Interest and illiquidity risk *)
(* Interest rates as is *)
yb = iM; VL = Decf[ecfV, dummy][[1]]; AK = Decf[BondCollCf, dummy][[1]]; AS = Decf[BondCompCf, dummy][[1]];
yb = iM; AG = OV[ecfV, 1, IGR, VolCube];
(* Interest rates down stress scenario *)
yb = iMdown; VLdown = Decf[ecfV, dummy][[1]]; AKdown = Decf[BondCollCf, dummy][[1]]; ASdown = Decf[BondCompCf, dummy][[1]];
yb = iMdown; AGdown = OV[ecfV, 1, IGR, VolCube];
(* Interest rates up stress scenario *)
yb = iMup; VLup = Decf[ecfV, dummy][[1]]; AKup = Decf[BondCollCf, dummy][[1]]; ASup = Decf[BondCompCf, dummy][[1]];
yb = iMup; AGup = OV[ecfV, 1, IGR, VolCube];
(* Illiquidity stress scenario *)
yb = iMip; VLip = Decf[ecfV, dummy][[1]]; AKip = Decf[BondCollCf, dummy][[1]]; ASip = Decf[BondCompCf, dummy][[1]];
yb = iMip; AGip = OV[ecfV, 1, IGR, VolCube];

Print["ΔL down: ", VLdown - VL]
Print["ΔIRG down: ", AGdown - AG]
Print["ΔAK down: ", AKdown - AK]
Print["ΔAS down: ", ASdown - AS]
Print[""]
Print["ΔL up: ", VLup - VL]
Print["ΔIRG up: ", AGup - AG]
Print["ΔAK up: ", AKup - AK]
Print["ΔAS up: ", ASup - AS]
Print[""]
Print["ΔL ip: ", VLip - VL]
Print["ΔIRG ip: ", AGip - AG]
Print["ΔAK ip: ", AKip - AK]
Print["ΔAS ip: ", ASip - AS]

ΔL down:  $9.5593 \times 10^8$ 
ΔIRG down:  $2.34843 \times 10^8$ 
ΔAK down:  $2.11697 \times 10^8$ 
ΔAS down:  $9.19402 \times 10^6$ 

ΔL up:  $-7.19432 \times 10^8$ 
ΔIRG up:  $-6.51459 \times 10^7$ 
ΔAK up:  $-2.14758 \times 10^8$ 
ΔAS up:  $-9.05803 \times 10^6$ 

ΔL ip:  $1.23743 \times 10^7$ 
ΔIRG ip: 199498.
ΔAK ip:  $9.46675 \times 10^6$ 
ΔAS ip: 427784.

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```

ln[72]:= (* Gross longevity risk *)
yb = iM; SCR = Decf[ecfV, ecfS][[2]]; Print["Longevity risk: ", SCR]
(* Gross disability risk *)
yb = iM; SCR = Decf[UecfV, UecfS][[2]]; Print["Disability/morbidity risk: ", SCR]
(* Gross expense risk *)
yb = iM; n = MDur[ecfV]; EE = 22 770 000; i = 0.02; k = 0.03;
SCR = 0.1×n×EE + ((1+k)^n-1)/k - ((1+i)^n-1)/i ×EE; Print["Expense risk: ", SCR];
Print["Modified duration: ", n]
(* Gross revision risk *)
yb = iM; SCR = Decf[UecfV, dummy][[1]]×0.03; Print["Revision risk: ", SCR]
(* Gross CAT risk *)
yb = iM; SCR = Decf[CATecfV, dummy][[1]]×0.0015; Print["CAT risk: ", SCR]

Longevity risk:  $1.95063 \times 10^8$ 
Disability/morbidity risk:  $8.0946 \times 10^7$ 
Expense risk:  $5.30103 \times 10^7$ 
Modified duration: 13.3368
Revision risk:  $1.16525 \times 10^7$ 
CAT risk:  $7.04304 \times 10^6$ 

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References

- Aalen, O. O., Borgan, Ø., & Gjessing, H. K. (2010). *Survival and Event History Analysis*. Springer.
- Banklovkommissjonen. (2010). *Pensjonslovene og folketrygdreformen I*. Norges offentlige utredninger.
- Bølviken, E., & Moe, A. (2008). *Gablers nye beregningsgrunnlag GAP07*.
- Cannon, E., & Tonks, I. (2008). A short history of annuities. *Annuity Markets* , 43-70.
- Dow, J. B. (1971-1973). Early actuarial work in eighteenth-century Scotland. *Transactions of the Faculty of Actuaries* , 33: 193-229.
- EIOPA. (2009). *CEIOPS' Advice for Level 2 Implementing Measures on Solvency II: SCR standard formula - Counterparty default risk module*.
- EIOPA. (2010a). *Manual for the completion of the QIS5 spreadsheets for solo undertakings*.
- EIOPA. (2010b). *Quantitative Impact Study 5 - Questions & Answers*.
- European Commission. (2010b). *Annexes to the QIS5 Technical Specifications*.
- European Commission. (2010a). *QIS5 Technical Specification*.
- Finanstilsynet. (2011b). *Erfaringer fra QIS5 - og veien videre*.
- Finanstilsynet. (2010). *QIS5 Informasjonsmøte for livsforsikringsselskap - 25. august 2010*.
- Finanstilsynet. (2009). *Veiledning for stresstest for livsforsikring - versjon 6c*.
- Finanstilsynet. (2011a). *Virksomhetsregler i livsforsikring. Letter to the Ministry of Finance, March 8th 2011* .
- Gerber, H. U. (1997). *Life Insurance Mathematics*. Springer-Verlag.
- Hull, J. C. (1993). *Options, futures, and other derivative securities*. Prentice Hall.
- J.P. Morgan. (1997). *CreditMetrics - Technical Document*.
- J.P. Morgan. (1996). *RiskMetrics - Technical Document*.
- Lillevold & Partners AS. (2010). *Referanseplan*.
- The European Parliament and of the Council. (2009). Directive 2009/138/EC. *Official Journal of the European Union* , 1-335.