

SPECIAL ISSUE ARTICLE

Précis

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Funding information

Norges Forskningsråd, Grant/Award Number: 314435

Abstract

Thin Objects has two overarching ambitions. The first is to clarify and defend the idea that some objects are ‘thin’, in the sense that their existence does not make a substantive demand on reality. The second is to develop a systematic and well-motivated account of permissible abstraction, thereby solving the so-called ‘bad company problem’. Here I synthesise the book by briefly commenting on what I regard as its central themes.

KEYWORDS

abstract, abstract objects, criteria of identity, Frege, modality, ontology

Thin Objects (Linnebo, 2018, henceforth, *TO*) has two overarching ambitions. The first is to clarify and defend the enticing, though initially obscure, idea that some objects are ‘thin’, in the sense that their existence does not make a substantive demand on reality. I pursue this ambition by developing a broadly Fregean conception of abstraction. Consider (a plural version of) the so-called ‘Hume’s Principle’:

$$\#xx = \#yy \leftrightarrow xx \approx yy, \quad (\text{HP})$$

which says that the number of some objects xx (in symbols: $\#xx$) is identical with the number of some objects yy just in case xx and yy are *equinumerous* (in symbols: $xx \approx yy$, defined as there being a one-to-one matching of xx with yy). This promises an account of thin objects. The idea is that the existence of the cardinal numbers that figure on the left-hand side requires no more than the truth of the corresponding right-hand side. The existence of the number 2, say, would require no more than the existence (and thus self-equinumerosity) of some pair. Indeed, any two objects will do, irrespective of their nature and location, if any.

Regrettably, other Fregean abstraction principles are inconsistent or otherwise problematic. A famous example is (a plural and simplified version of) Frege’s Basic Law V:

$$\{xx\} = \{yy\} \leftrightarrow \forall u(u < xx \leftrightarrow u < yy), \quad (\text{V})$$

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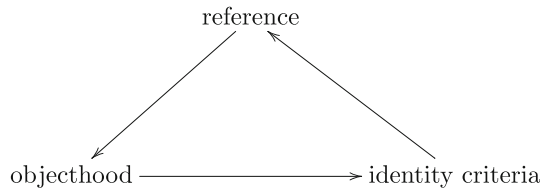
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where ‘ $u < xx$ ’ means that u is one of xx . Absent further restrictions, (V) falls prey to Russell’s paradox. This is worrisome. What separates the permissible forms of abstraction from the impermissible ones? Without a convincing answer, it is natural to worry that the bad cases somehow ‘corrupt’ the good. My second overarching ambition is therefore to respond to this ‘bad company problem’ by developing a systematic and well-motivated account of permissible abstraction.

I now wish to synthesise the book by briefly commenting on what I regard as its central themes.

1 | THE FREGEAN TRIANGLE

The general shape of my project can be explained in terms of the following Fregean triangle:



The idea is to connect three fundamental logico-philosophical notions. Let me start on the right and follow the arrows around the triangle. First, it suffices for a singular term to refer that it has been associated with a specification of the would-be referent, which figures in an appropriate criterion of identity. In this way, I argue, we can explain how various forms of reference are constituted, including to abstract objects such as numbers.¹ Second, to be an object is to be a possible referent of a singular term. This is a logical conception of objecthood, inspired by Frege.

These two steps encapsulate my approach to thin objects – and thus to my first overarching ambition. There are criteria of identity that do not involve F s but that nonetheless suffice to constitute reference to F s. Here is an example from Frege. We can specify a direction by producing an appropriately oriented line. And two such specifications determine the same direction just in case they are parallel.² Thus, given just lines and parallelism, we can constitute reference to ‘new’ objects, namely directions. Directions are therefore thin, requiring for their existence (and indeed status as possible referents of singular terms) nothing more than the existence of appropriately oriented lines, which could be located anywhere.

I turn now to the third step, along the triangle’s bottom edge. The first two steps are, as observed, capable of introducing objects that are ‘new’ in the sense that they are neither referred to nor quantified over in the criterion of identity with which we began. The third step is to use this expanded domain to formulate criteria of identity for yet further objects. Consider the plural version of Basic Law V. Suppose we start with two concrete objects a and b . We can use these objects to constitute reference to four ‘new’ sets, namely \emptyset , $\{a\}$, $\{b\}$ and $\{a,b\}$.³ The ensuing expanded domain enables us to constitute reference to yet more sets. Thus, by looping around the triangle, we can successively account for ever larger domains of sets, as well as other abstract objects.

These looping explanations encapsulate my approach to the bad company problem – and thus to my second overarching ambition. The central idea is that abstraction is permissible provided that we only ever presuppose objects already accounted for – and thus added to our domain. ‘New’ objects can be introduced by abstraction if, *but only if*, they can be fully

¹The obtaining of these relations is not a primitive fact. So we want to know in virtue of what the relations obtain.

²As usual, I set aside the wrinkle that we need not only parallelism but also sameness of orientation.

³To obtain \emptyset we can either (unlike English and other natural languages) accept an empty plurality or use second-order logic restricted to properties in extension (i.e., properties defined by listing their instances, if any).

specified on the basis solely of the ‘old’ objects already introduced. As just illustrated, when this requirement is enforced, even Basic Law V – a bad companion *par excellence* – is rehabilitated.

This provides one safe route to thin objects, namely, via this constrained form of Fregean abstraction. I hasten to add that I make no attempt to shut down other possible routes. More work remains for the curious!

2 | THE NOTION OF SUFFICIENCY

I have talked about the existence of thin objects not making any substantive demand on reality. To make this more precise, I introduce a notion of *sufficiency*, intended to represent the mentioned demands: ‘ $\varphi \Rightarrow \psi$ ’ means that φ suffices for ψ . In particular, we want $xx \approx yy \Rightarrow \#xx = \#yy$ and various generalisations thereof.

A central task of Chapter 1 is to articulate a ‘job description’ for the (so far entirely programmatic) notion of sufficiency. This description consists of four philosophical constraints.⁴ The first two are needed to sustain the promised advance from ‘old’ to ‘new’ objects.

Ontological expansiveness constraint: There are true sufficiency statements $\varphi \Rightarrow \psi$ where the ontological commitments of ψ exceed those of φ .

Face value constraint: The formulas involved in a sufficiency statement $\varphi \Rightarrow \psi$ can be taken at face value in our semantic analysis. In particular, abstraction terms such as ‘ $\#xx$ ’ function semantically as singular terms, that is, as terms standing for objects.

The last two constraints are needed to reap the desired epistemic and explanatory benefits:

Epistemic constraint: If $\varphi \Rightarrow \psi$, then it is possible to know $\varphi \rightarrow \psi$.⁵

Explanatory constraint: If $\varphi \Rightarrow \psi$, then $\varphi \rightarrow \psi$ admits of an acceptable metaphysical explanation.

Is there a candidate that satisfies our job description? I argue that $\varphi \Rightarrow \psi$ cannot be defined in terms of the corresponding material conditional being either analytic or metaphysically necessary. A better option, I suggest, would be to understand sufficiency as ‘a species of metaphysical grounding’ (p. 18).⁶ Still, ‘I would resist any *identification* of the notion of sufficiency with that of grounding, for two reasons’ (ibid.). First, grounding does not in general satisfy the Epistemic constraint. Second, merely to claim that abstraction gives rise to relations of grounding would be insufficiently explanatory. Any such claims would have to be defended and integrated with the metasemantics and epistemology of abstraction. So, I decided simply to develop my own candidate for the job. This is the task of Chapters 2 and 8.

I have since warmed to the idea of understanding sufficiency as a ‘grounding potential’ (p. 43 n. 41) found in a large and diverse class of cases of Fregean abstraction.⁷ The arguments of *TO* are still needed, though, to defend the existence of these potentials and to provide the needed integration with metasemantics and epistemology.

⁴There are also some technical constraints described in *TO*, p. 12.

⁵I simplify slightly. See *TO*, p. 16 for the official, slightly stronger formulation.

⁶See also pp. 43–44 n. 41, p. 56 n. 18 and p. 193.

⁷See my Replies, Sections 1–4.

3 | THE FLEXIBLE CONCEPTION OF REALITY

The hardest part of my journey around the Fregean triangle is probably the step where I attempt to use a criterion of identity, involving neither reference to nor quantification over *Fs*, to explain how reference to *Fs* is constituted. An example would be the constitution of reference to directions solely on the basis of lines and parallelism. Keenly aware of the puzzling nature of this step, I attempt two softening-up manoeuvres before embarking on a direct defense. The first such manoeuvre is to canvass (in Section 2.4) a *flexible conception of reality*, intended to ‘locat[e] the approach to be developed [...] within a broader philosophical landscape’ (p. 30). The second is to provide (in Section 2.3) an example of the desired step, involving concrete (and therefore less controversial) objects, namely *physical bodies*. Let me now describe these manoeuvres, trusting the reader not to conflate the softening-up with the defense proper.

According to the flexible conception,

reality is articulated into objects only through the concepts that we bring to bear. And we often have some choice in this matter. [...] This] conception of ontology gets its bite by adding the controversial claim that there is no unique, privileged set of concepts in terms of which to ‘carve up’ reality, namely the concepts that match some rigid concept-independent articulation of reality into objects. (p. 31)

The question, as I analyse it, concerns what it takes to apply the apparatus of first-order logic – and the associated logical concept of object – to reality. The rigid conception says there is a unique correct application – namely, that which matches some stock of objects, given independently of our language and concepts. The flexible conception disagrees, insisting that there can be different, equally legitimate applications.⁸

While I hope this is helpful as a way of locating the desired view, it is obviously highly schematic. We need to develop, in proper detail, an account of permissible applications of the apparatus of first-order logic to reality. This sets the agenda for the book as a whole.

4 | PHYSICAL BODIES

I believe physical bodies, such as sticks and stones, provide a good illustration of how criteria of identity can figure in the constitution of reference. To show this, I develop, in Section 2.3 of *TO*, a simple model of robots embedded in, and interacting with, a physical environment. My focus is on the senses of sight and touch.

What does it take for one of these robots to refer to a physical body? At the very least, the robot must receive ‘perceptual’ information from some part of the body, which thus serves as a *specification* of the referent. More interestingly, the robot needs some mechanism for determining when two such specifications are associated with one and the same body. I call this a *unity relation*. This unity relation will reflect fundamental features of physical bodies such as: bodies are three-dimensional, solid objects; bodies have natural and relatively well distinguished spatial boundaries; bodies are units of independent motion; bodies move along continuous paths; bodies have natural and relatively well-distinguished temporal boundaries. Write ‘ \sim ’ for this unity relation (which is a partial equivalence relation) and ‘*B*’ for the ‘body builder’ that maps a specification to the body, if any, that it determines. Then we obtain the following criterion of identity:

⁸I argue in my Replies, Section 3, that the flexible conception is compatible with realism. See also Linnebo (2023) for further discussion.

$$u \sim u \wedge v \sim v \rightarrow (B(u) = B(v) \leftrightarrow u \sim v). \tag{CI-B}$$

This criterion achieves something remarkable. It figures at the heart of an account of what it takes for the robots to refer to a physical body. Yet the ingredients of this account – the parcels of matter that serve as specifications and the unity relation – do not mention or involve any bodies. We have, in other words, an illustration of the advertised puzzling phenomenon, namely, an account of the constitution of reference to *Fs* that does not itself refer to or quantify over *Fs*.

This reveals an important structural similarity between the constitution of reference to physical objects and to abstract ones. The main difference is that the facts required to constitute reference to a physical body make more substantive demands on the world than those required to constitute reference to abstract objects such as directions, numbers or pure sets. In particular, only in the former case are there demands on a specific region of spacetime.

5 | REFERENCE BY ABSTRACTION

With the two softening-up manoeuvres in place, I turn to the general thesis of reference by abstraction, which I develop and defend in Sections 2.4–2.6 and Chapter 8.

To begin, we note that there is vast potential for generalising from the example of physical bodies. Here is the general form of type of criterion of identity exemplified by (CI-B):

$$\alpha \sim \alpha \wedge \beta \sim \beta \rightarrow (f(\alpha) = f(\beta) \leftrightarrow \alpha \sim \beta), \tag{CI}$$

(‘ α ’ and ‘ β ’ can be any type of variable: singular, plural or higher-order.) There are many promising examples of how a criterion might figure in the constitution of reference:

Referents	Specifications (α, β)	Unity relation (\sim)
Physical bodies	Parcels of matter	Connectedness
Directions	Lines	Parallelism
Linguistic types	Tokens	‘Co-typicality’
Cardinal numbers	Pluralities	Equinumerosity
Sets	Pluralities	Coextensionality

All these examples are subsumed by a general thesis of reference by abstraction, formulated on p. 37 of *TO*.⁹

Why believe these examples and the associated general thesis? The heaviest lifting takes place in Chapter 8. On pp. 150–151, I distill my argument into three steps. Let me illustrate using the case of direction. The intended full generalisation should be clear.

First: we start speaking *as if* there are directions. More precisely, we use the following sufficiency statements as assertibility conditions for an extension of our language in which we talk, not only about lines and parallelism, but also about directions:

⁹In a separate discussion of *TO*, Agustín Rayo (2023) asks why criteria of identity are needed at all. Why not achieve even greater generality by developing the flexible conception of reality without my insistence on criteria of identity? While I have no principled objection to this kind of generalisation, I argue in (Linnebo, 2023) that criteria of identity ensure a more robust objectivity of the resulting objects, by securing their independence from the particular perspective one may have on it – whether a spatial or temporal point of view, an occasion of representing the object, or the individual who represents it.

$$\begin{array}{ll}
 l_1 \parallel l_2 \Rightarrow d(l_1) = d(l_2) & l_1 \nparallel l_2 \Rightarrow d(l_1) \neq d(l_2) \\
 l_1 \perp l_2 \Rightarrow d(l_1) \perp^* d(l_2) & l_1 \not\perp l_2 \Rightarrow d(l_1) \not\perp^* d(l_2).
 \end{array}$$

For example, we take the orthogonality of two lines, $l_1 \perp l_2$, as a basis for asserting that their respective directions are orthogonal, $d(l_1) \perp^* d(l_2)$. This step is permissible because of the reductive (or ‘predicative’) character of the assertibility conditions: every statement in the extended language receives an assertibility condition formulated solely in terms of the antecedently accepted lines and which provably respects all inferential relations in the extended language.¹⁰

Second: if available, an interpretation of the extended language that takes the apparent reference to directions at face value would be preferable, for two reasons. One reason is based on considerations about compositionality, in the presence of generalised quantifiers such as ‘most’ (cf. *TO*, Section 8.4.2 and p. 171). In essence: it can be true that most directions point north even when most of the specifying lines do not. Another reason is that speakers often have little or no cognitive access to the relevant specifications and unity relation, as they would need to have if these were directly involved in the *semantics* of their language, as opposed to figuring in a *metasemantic* account of how this language obtains its semantic interpretation (cf. *TO*, Section 8.4.3).

Third: the preferred face-value (or ‘non-reductionist’) interpretation *is* available. I defend this claim by invoking a form of ‘internalism’ about reference, according to which the metalanguage in which a semantic interpretation is formulated can avail itself of the very form of reference under consideration (in our illustration, reference to directions).

I call the resulting view *metasemantic reductionism*. The *reductionism* is manifest in the requirement on the assertibility conditions in step one. The adjective ‘*metasemantic*’ signals that the reduction pertains to *the constitution of relations of reference*, not to semantic analysis. As concerns semantic analysis, I defend a face-value (or ‘non-reductionist’) interpretation based on reference to abstracta. But these relations of reference obtain in virtue of facts that do not involve these abstracta.

6 | DYNAMIC ABSTRACTION, MODALLY DEVELOPED

The first two steps around the Fregean triangle, summarised above, expand the domain by adding to it certain ‘new’ objects that are fully specified in terms of the ‘old’. The laws governing these expansions are codified by the relevant sufficiency statements.

How can these laws be harnessed to justify a formal theory of dynamic abstraction, describing the effects of successively looping around the Fregean triangle? Each of the expandable domains is a plurality of objects, available for reference and quantification, relative to some interpretation of the language. Given such a plurality of ‘old’ objects, it is possible to expand the interpretation so as to introduce various ‘new’ objects. One natural option is thus to represent this possibility using the resources of modal logic.¹¹

Consider the case of sets. Let ‘ $\text{Set}(xx, y)$ ’ express that y is the set obtained by abstraction on xx . Then my account licenses the following criterion of potential existence:

$$\Box \forall xx \diamond \exists y \text{Set}(xx, y). \quad (1)$$

¹⁰For this latter claim, see Corollary 8.1 on pp. 155–156 of *TO*.

¹¹See *TO*, Section 3.5. *TO* does not claim this is the only option. An attractive alternative has since emerged, namely to trade the modal resources for a more restrictive (‘critical’) form of plural logic (Florio & Linnebo, 2021).

We also have a modal criterion of identity:

$$\text{Set}(uu, x) \wedge \text{Set}(vv, y) \rightarrow (x = y \leftrightarrow \Box \forall z(z < uu \leftrightarrow z < vv)). \tag{2}$$

These two criteria can be obtained by first ‘factorising’ the plural version of Basic Law V into separate criteria of existence and identity, and then transposing these criteria to our modal-dynamic setting.¹²

By laying down further plausible principles, which explicate either the process of dynamic abstraction in general or its interaction with sets in particular, we obtain a modal theory of dynamic abstraction. This theory, it turns out, is strong enough to interpret ZF set theory. Since this is a story often told, I need not repeat it here.¹³

7 | EXTENSIONAL AND RELATIVE VERSUS INTENSIONAL AND ABSOLUTE GENERALITY

My modal theory of dynamic abstraction describes possible ways to advance from one domain to a larger one. Each of the domains in question corresponds to a plurality of objects. We may therefore say that each of these domains is *extensional*.

This extensionality is of critical importance (cf. *TO*, Section 3.3). It is their extensional character that allows each of these domains to be surpassed. To see this, consider the sufficiency statements for sets (where ‘ $xx \equiv yy$ ’ abbreviates $\forall u(u < xx \leftrightarrow u < yy)$):

$$\begin{array}{ll} xx \equiv yy \Rightarrow \{xx\} = \{yy\} & xx \not\equiv yy \Rightarrow \{xx\} \neq \{yy\} \\ x < xx \Rightarrow x \in \{xx\} & x \not< xx \Rightarrow x \notin \{xx\}. \end{array}$$

Once again, these statements achieve something remarkable. They enable us fully to characterise each of the ‘new’ sets that we seek to add solely in terms of the ‘old’ objects – as required by the first step of my argument for reference by abstraction (cf. Section 5). To understand how this works, we observe that the questions raised by left-hand sides – whether xx and yy are the very same objects and whether x is one of xx – are intrinsic to the objects in question, all of which are ‘old’; there is no need to consult any other objects. The resulting full characterisation of some ‘new’ objects in terms of the ‘old’ enables us to surpass the ‘old’ domain by adding the ‘new’ objects. As promised, this establishes that every quantifier with an *extensional* domain (namely, a plurality of ‘old’ objects) can be surpassed by a more inclusive such quantifier.

Does this mean that absolutely general quantification is impossible? Not quite (cf. *TO*, Section 3.6). In addition to the ordinary quantifiers with an extensional domain, we have stronger devices of generalisation in the form of the ‘modalised quantifiers’ $\Box \forall$ and $\Diamond \exists$, which express an *intensional* form of generality. We can even prove that these composite expressions behave logically precisely like genuine quantifiers – as far as first-order logic is concerned. I claim that the intensional generality expressed by the modalised quantifiers is absolute.

There is a worry, though (cf. *TO*, Section 3.7). Why cannot this modalised generality too be surpassed, much like we can surpass any quantifier with an extensional domain? To investigate, consider the intensional analogue of the extendability argument outlined above. The relevant sufficiency statements include:

$$\Box \forall x(\varphi(x) \leftrightarrow \psi(x)) \Rightarrow \{u : \varphi(u)\} = \{u : \psi(u)\},$$

¹²The two initial ‘factors’ of Basic Law V look like (1) and (2) with all modal operators deleted.

¹³See, e.g., (Linnebo, 2013) and *TO*, Chapter 12.

and its negated analogue. A crucial difference is apparent. In this intensional case, the left-hand sides are not intrinsic to any ‘old’ objects – contravening the requirement that the ‘new’ objects be fully characterised solely in terms of the ‘old’. Thus, the best extendability argument on offer does not get a grip on the intensional – and, I claim – *absolute* generality effected by the modalised quantifiers. (The possibility of other, more dangerous extendability arguments is discussed in my reply to James Studd.)

8 | ARISTOTELIAN REALISM

Our journey around the Fregean triangle yields an account of abstract objects and our ability to refer to them and know them. What is the metaphysical status of these objects? In their contribution, Sereni and Zanetti characterise my view as a form of Aristotelian realism. I wish to end by explaining why I find this characterisation apt.

I have emphasised that abstract objects, on my account, are thin: not very much is demanded for their existence. I can now be more specific. A physical body is thick, because its existence imposes demands on the specific region of spacetime where it is located, namely that this region contains matter appropriately organised. Directions or linguistic types are much thinner. For the lines or tokens that serve as specifications of such objects can be located anywhere. Thus, these abstract objects do not impose any demands on any specific regions of spacetime. Even so, these abstract objects are not entirely thin, since their existence requires there to be a space in the first place, in which appropriate specifications can be found. Thinnest of all are pure abstract objects, such as pure sets or cardinal numbers. The empty set, for example, can be abstracted from nothing whatsoever, and so, successively, can all other pure sets as well.

It is natural, then, to regard abstract objects as ontologically derivative: they owe their existence to the existence of appropriate specifications. The cardinal number 2, for example, owes its existence to the existence of some pair or other – any two objects will do, including ones that are themselves abstract. It is useful to think of this as an Aristotelian dependence profile – to be contrasted with a problematic and fundamentally different Kantian dependence profile, which would have the objects depend on us or other cognisers (cf. my Replies, Sections 2 and 3).

This broadly Aristotelian approach ensures that mathematical objects (and truths) are counterfactually independent of us. Had there been no intelligent life, there would still have been plenty of pairs around to ground the existence of (and truths about) the number 2. This is an important step towards a platonistic conception of mathematics. But I resist the assimilation of mathematical objects to concrete ones that one finds in more robust forms of platonism (cf. *TO*, Sections 11.1–11.3). On my view, there are profound differences between the abstract realm and the concrete. The denizens of the former are thin, not thick. And this thinness gives rise to a form of indefinite extensibility, whereby the plurality that makes up the domain of any extensional interpretation of the language can be surpassed by an even larger plurality.

ACKNOWLEDGEMENTS

Thanks to Neil Barton, Laura Crosilla, Louis deRosset, Salvatore Florio, Michael Glanzberg, Jon Litland, Eileen Nutting, Agustín Rayo, Andrea Sereni, Stewart Shapiro, James Studd, and the members of the Infinity & Intensionality seminar at the University of Oslo for valuable comments and discussion. This research was supported by the Research Council of Norway (project number 314435).

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How to cite this article: Linnebo, Ø. (2023) Précis. *Theoria*, 89(3), 247–255. Available from: <https://doi.org/10.1111/theo.12464>