Dept. of Math. University of Oslo
Pure Mathematics No. 34
ISSN 0806–2439 December 2003

Explicit strong solutions of stochastic differential equations on Hilbert spaces

Alberto Lanconelli[1),2)] and Frank Proske[1)]

December 18, 2003

1) Department of Mathematics , University of Oslo
Box 1053 Blindern , N-0316 Oslo, Norway
2) Department of Mathematics, University of Pavia,
via Ferrata,1 , 27100 Pavia, Italy

e-mails: albertol@math.uio.no, proske@math.uio.no

Abstract

We give an explicit representation of strong solutions of Itô-SDE's in Hilbert spaces in terms of a non linear operation on a stochastic distribution space. This formula can be potentially used to obtain solutions of SDE's and SPDE's with non Lipschitzian coefficients.

Key words and phrases: infinite dimensional Itô-diffusions, strong solutions, SPDE, white noise analysis
AMS 1991 classification: 60H10; 60H15; 60H40

1 Introduction

The main objective of this paper is to utilize methods and concepts from white noise theory to construct explicit solutions of fully nonlinear stochastic (partial) differential equations. The results of this paper are based on ideas developed in [17]. In the latter paper the authors demonstrate how white noise concepts can be applied to give an explicit representation of global strong solutions of one dimensional Itô-SDE's. We intend to extend this result to SDE's in Hilbert spaces driven by Gaussian noise. The solutions of such equations can be potentially used to derive solutions of certain classes of SPDE's. Employing connections between infinite dimensional SDE's and SPDE's it would be interesting for instance to study stochastic Navier Stokes equations ([2]). The current literature about explicit solutions of stochastic (partial) differential equations is dominated by the idea to fill up the "gap" between ordinary (or partial) differential equations and S(P)DE's (see [21],[4],[22],[10]). The latter authors' idea is to reduce the problem of finding explicit solutions to that of determining solutions of ODE's and PDE's pathwisely. Our methodology to solve S(P)DE's is novel

since we are not in need to resort to solutions of deterministic differential equations. Our point of view is even converse and it could possibly serve as a starting point to investigate solution of ODE's and PDE's.
The approach can be for instance applied to finite dimensional SDE's when the drift coefficient is not necessarily Lipschitz. We point out that in this case the above mentioned reduction method fails . Our explicit formula can be invoked to gain information on the structure of solutions of SDE's from a different point of view. This closed form expression can be for instance used to study path properties and long time behaviour of solutions. We think that it is also possible to verify this formula as a strong solution of an S(P)DE directly. This would open the possibility of constructing strong solutions of S(P)DE's involving coefficients under regularity assumptions (e.g. non-Lipschitzian diffusion coefficients) not covered by general existence theorems.
We propose a method which can be possibly adapted to inquire into the case of S(P)DE's driven by Levy processes ([16]).

2 Preliminaries from White Noise theory

In this Section we pass in review some concepts from Gaussian white noise analysis. For more information about white noise theory we recommend the reader to consult for instance the books [6], [14], [20], [7]. In the following we consider the space of temperated distributions $S'(\mathbb{R})$. By the Bochner theorem there exists a unique probability measure on $(S'(\mathbb{R}), \mathcal{B}(S'(\mathbb{R})))$ such that

$$\int_{S'(\mathbb{R})} e^{i\langle\omega,\varphi\rangle} d\mu(\omega) = e^{-\frac{1}{2}\|\varphi\|^2_{\mathcal{L}^2(\mathbb{R})}}$$

holds, where $\langle\omega, \varphi\rangle$ denotes the action of $\omega \in S'(\mathbb{R})$ on $\varphi \in S(\mathbb{R})$. The triple $(S'(\mathbb{R}), \mathcal{B}(S'(\mathbb{R})), \mu)$ is referred to as *white noise probability space*. We denote by H a real separable Hilbert space. Let us choose an orthonormal basis $\{e_i\}_{i=1}^{\infty}$ of H. As shorthand notation we write $\mathcal{L}^2(H)$ for $\mathcal{L}^2(S'(\mathbb{R}), \mu; H)$.
We recall the Wiener-Itô chaos expansion in terms of Hermite polynomials (see for instance [9]):

$$h_n(x) = (-1)^n e^{\frac{x^2}{2}} \frac{d^n}{dx^n}(e^{-\frac{x^2}{2}}), \quad n = 0, 1, 2, ...$$

Consider the orthonormal basis of $\mathcal{L}^2(\mathbb{R})$ consisting of the Hermite functions

$$\xi_n(x) = \pi^{-\frac{1}{4}}((n-1)!)^{-\frac{1}{2}} e^{-\frac{x^2}{2}} h_{n-1}(x), \quad n = 1, 2, ...$$

Then one constructs an orthogonal $\mathcal{L}^2(\mu)$ basis $\{\mathcal{H}_\alpha(\omega)\}_{\alpha\in\mathcal{I}}$ given by

$$\mathcal{H}_\alpha(\omega) := \prod_{i=1}^{\infty} h_{\alpha_i}(\langle\omega, \xi_i\rangle).$$

The multiindex set $\mathcal{I}$ stands for the space of sequences $\alpha = (\alpha_1, \alpha_2, ...)$ with components $\alpha_i \in \mathbb{N}_0$ and with compact support. Thus every $F \in \mathcal{L}^2(\mu)$ can be uniquely represented as

$$F(\omega) = \sum_{\alpha\in\mathcal{I}} c_\alpha \mathcal{H}_\alpha(\omega), \quad c_\alpha \in \mathbb{R}$$

where each $\mathcal{H}_\alpha(\omega)$ has the norm expression

$$\|\mathcal{H}_\alpha\|^2_{\mathcal{L}^2(\mu)} = \alpha! := \alpha_1!\alpha_2! \cdots .$$

For convenience we introduce the notation

$$Index\alpha = \sup\{k|\alpha_k \neq 0\}.$$

Using the family $\{\mathcal{H}_\alpha(\omega)\}_{\alpha\in\mathcal{I}}$ one obtains the $\mathcal{L}^2(H)$ orthogonal basis $\{\mathcal{H}_\alpha(\omega)e_i\}_{\alpha\in\mathcal{I},i\in\mathbb{N}}$. The Hida stochastic test function space (S) can be defined as the space of

$$f(\omega) = \sum_{\alpha\in\mathcal{I}} a_\alpha \mathcal{H}_\alpha(\omega) \in \mathcal{L}^2(\mu)$$

such that the growth condition

$$\|f\|^2_{0,k} := \sum_{\alpha\in\mathcal{I}} \alpha! a_\alpha^2 (2\mathbb{N})^{k\alpha} < \infty \text{ for all } k \in \mathbb{N}$$

with weights

$$(2\mathbb{N})^\beta = 2^{\beta_1}4^{\beta_2} \cdot\cdot (2k)^{\beta_k} \cdot\cdot \quad \text{if } \beta = (\beta_1, \beta_2, ...) \in \mathcal{I}$$

is satisfied. The norms $\|\cdot\|_{0,k}, k \in \mathbb{N}$ induce the projective topology on (S). The Hida stochastic distribution space $(S)^*$ is defined as the topological dual of (S). So we get the following Gel'fand triplet

$$(S) \hookrightarrow \mathcal{L}^2(\mu) \hookrightarrow (S)^*.$$

This concept of distribution spaces can be extended to the Hilbert space setting (see [5]). Similarly to the one dimensional case the Hida stochastic test function space on H, indicated by $S(H)$, can be described by the set of all

$$f(\omega) = \sum_{\alpha\in\mathcal{I}} \sum_{i=1}^{\infty} a_{i\alpha} \mathcal{H}_\alpha(\omega)e_i, \quad a_{i\alpha} \in \mathbb{R}$$

in $\mathcal{L}^2(H)$ such that for all $k \in \mathbb{N}$

$$\|f\|^2_{0,k} := \sum_{\alpha\in\mathcal{I}} \sum_{i=1}^{\infty} \alpha! a_{i\alpha}^2 (2\mathbb{N})^{k\alpha} < \infty.$$

$S^*(H)$ is the dual of $S(H)$ and we obtain the chain of inclusions

$$S(H) \hookrightarrow \mathcal{L}^2(H) \hookrightarrow S^*(H).$$

The dual paring between $F(\omega) = \sum_{\alpha\in\mathcal{I}} b_\alpha \mathcal{H}_\alpha(\omega) \in S^*(H)$ and $f(\omega) = \sum_{\alpha\in\mathcal{I}} a_\alpha \mathcal{H}_\alpha(\omega) \in S(H)$ can be represented as

$$\langle F, f\rangle = \sum_{\alpha\in\mathcal{I}} \alpha! \langle a_\alpha, b_\alpha\rangle_H.$$

We are going to establish the definition of the singular noise of a Q-Wiener process on H. For this purpose take the bijective map

$$\tau : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$$

given by

$$(i,j) \mapsto j + \frac{(i+j)(i+j+1)}{2}.$$

We consider a positive symmetric trace class operator on H denoted by Q with eigenvalues $\lambda_i > 0$. The Q-Wiener process on H can be written as

$$B^Q(t) := \sum_{i=1}^{\infty} \sqrt{\lambda_i}\beta_i(t)e_i, \tag{2.1}$$

where $\{\beta_i(t)\}_{i\geq 1}$ are independent Brownian Motions given by

$$\beta_i(t) = \sum_{k=1}^{\infty} \delta_{\tau(i,j),k} \int_0^t \xi_j(s)ds\mathcal{H}_{\epsilon_k}(\omega)$$

where $\delta_{l,m}$ denotes the Christoffel symbol.
Equation (2.1) can be reformulated as

$$B^Q(t) := \sum_{k=1}^{\infty} \theta_k(t)\mathcal{H}_{\epsilon_k}(\omega), \quad t \geq 0$$

where

$$\theta_k(t) = \delta_{\tau(i,j),k}\sqrt{\lambda_i}(\int_0^t \xi_j(s)ds)e_i.$$

The Hida stochastic distribution space $S^*(H)$ exhibits the nice feature to contain the singular white noise $W^Q(t)$ of the Q-Wiener process $B^Q(t)$. The *singular noise* of the Q-Wiener process $B^Q(t)$ can be defined by the formal expansion

$$W^Q(t) := \sum_{k=1}^{\infty} \delta_{\tau(i,j),k}\sqrt{\lambda_i}\xi_j(t)e_i\mathcal{H}_{\epsilon_k}(\omega). \tag{2.2}$$

It can be checked that $W^Q(t)$ is an element in $S^*(H)$ for all t.
We introduce a multiplication of distribution in $S^*(H)$ via the Wick product

$$\diamond : S^*(H) \times S^*(H) \to S^*(H)$$

given by

$$(F,G) \mapsto \sum_{\gamma \in \mathcal{I}} g_\gamma \mathcal{H}_\gamma(\omega)$$

with

$$g_\gamma = \sum_{i=1}^{\infty} \sum_{\alpha+\beta=\gamma} c_{i\alpha} d_{i\beta} e_i$$

for

$$F(\omega) = \sum_{\alpha \in \mathcal{I}} \sum_{i=1}^{\infty} c_{i\alpha}\mathcal{H}_\alpha(\omega)e_i$$

$$G(\omega) = \sum_{\beta \in \mathcal{I}} \sum_{i=1}^{\infty} d_{i\beta}\mathcal{H}_\beta(\omega)e_i.$$

As an example the Wick version of the exponential function, denoted by $exp^{\diamond}$, can be defined by

$$exp^{\diamond}(F) = \sum_{n \geq 0} \frac{1}{n!} F^{\diamond n} \in S^*(H) \tag{2.3}$$

where $F^{\diamond n}$ is the n-th Wick power

$$F^{\diamond n} := F \diamond \cdots \diamond F \text{ (n times)} .$$

One of our main tools to derive the explicit solution of an SDE on H is the $\mathcal{H}$-transform. As in the finite dimensional case its definition is based on the expansion along the basis $\{\mathcal{H}_\alpha(\omega)\}_{\alpha \in \mathcal{I}}$. We define the Hermite transform of $F(\omega) = \sum_{\alpha \in \mathcal{I}} c_\alpha \mathcal{H}_\alpha(\omega) \in S^*(H)$ as

$$\mathcal{H}F(z) := \tilde{F}(z) := \sum_{\alpha \in \mathcal{I}} c_\alpha z^\alpha \tag{2.4}$$

for $z \in \mathbb{C}^{\mathbb{N}}$ such that the series converges in $H_{\mathbb{C}}$ (the complexification of H). It can be shown that $\mathcal{H}F(z)$ converges absolutely in the infinite dimensional neighborhood of 0 in $\mathbb{C}^{\mathbb{N}}$ given by

$$\mathbb{K}_q := \{z \in \mathbb{C}^{\mathbb{N}} : |z_i| < (2i)^{-q}, i \in \mathbb{N}\}.$$

Based on the $\mathcal{H}$-transform one can give necessary and sufficient conditions whether the power series (2.4) is the $\mathcal{H}$-transform of an element in $S^*(H)$. For instance, if

$$X(z) = \sum_{\alpha \in \mathcal{I}} c_\alpha z^\alpha, c_\alpha \in H$$

is bounded by some $M < \infty$, then there exists a unique element F in $S^*(H)$ such that $\mathcal{H}F(z) = X(z)$. Moreover F can be choosen as

$$F(\omega) = \sum_{\alpha \in \mathcal{I}} c_\alpha \mathcal{H}_\alpha(\omega).$$

Consider $F, G \in S^*(H)$. Then for all z such that $\mathcal{H}F_i(z) = \langle \mathcal{H}F(z), e_i \rangle$ and $\mathcal{H}G_i(z) = \langle \mathcal{H}G(z), e_i \rangle$ exist we get the relation

$$\mathcal{H}(F \diamond G)(z) = \sum_{i=0}^{\infty} \mathcal{H}F_i(z) \mathcal{H}G_i(z) e_i.$$

The latter relation shows that in the case of $(S)^* = S^*(\mathbb{R})$, $\mathcal{H}$ is an algebra homomorphism between $(S)^*$ (w.r.t.$\diamond$) and the algebra of the power series in infinitely many complex variables, that is we have

$$\mathcal{H}(F \diamond G)(z) = \mathcal{H}F(z) \mathcal{H}G(z).$$

3 Explicit Solution

In this section we aim at determining an explicit representation for global strong solutions of the Itô-SDE

$$dX_t(\omega) = b(X_t(\omega))dt + \sigma(X_t(\omega))dB_t^Q(\omega), \quad X_0 = x \in H \tag{3.1}$$

where $b : H \to H$ and $\sigma : H \to L(H,H)$ are continuous mappings.
$\{X_t\}_{t\geq 0}$ is called a *global strong solution* of (3.1) if the following integrability conditions are fulfilled

$$E[\int_0^t \|b(X_s)\|ds] < \infty,$$

$$E[\int_0^t \|\sigma(X_s)\|^2_{L(H,H)}ds] < \infty$$

and if X_t solves the equation

$$X_t(\omega) = x + \int_0^t b(X_s(\omega))ds + \int_0^t \sigma(X_s(\omega))dB_s^Q(\omega)$$

where it is assumed that

$$\sigma(X_s)Q^{\frac{1}{2}} \in L_{(2)}(H,H)$$

($L_{(2)}(H,H)$ is the space of Hilbert-Schmidt operators from H into itself).
Now suppose that b and σ satisfy a Lipschitz condition on bounded sets that means for all $n \in \mathbb{N}$ there exists a constant L_n such that for all $x, y \in H$, $\|x\| \leq n$, $\|y\| \leq n$ we have

$$\|b(x) - b(y)\| + \|\sigma(x) - \sigma(y)\|_{L(H,H)} \leq L_n\|x - y\|.$$

Further we require that b and σ fulfill the growth condition

$$\langle x, b(x)\rangle \leq K(1 + \|x\|^2)$$

and

$$\|\sigma(x)\|^2_{L(H,H)} \leq K(1 + \|x\|^2)$$

for all $x \in H$ and for some constant K. It follows from a result of Leha-Ritter [18] that there exists a unique global strong solution $\{X_t\}_{t\geq 0}$.
Let us assume that there exists $\Lambda : H \to H$ such that the following conditions hold:
i) There exists $\Lambda^{-1} : H \to H$ such that $\Lambda(\Lambda^{-1}(x)) = \Lambda^{-1}(\Lambda(x)) = x, \forall x \in H$;
ii) $\Lambda^{'}(x)\sigma(x) = Id_H, \forall x \in H$;
iii) $E[\|\Lambda^{-1}(B_t^Q)\|^2] < \infty$;
iv) $\|\Lambda^{'}(x)b(x) + \frac{1}{2}\sum_{i=1}^{\infty} \Lambda^{''}(x)[\sigma(x)\sqrt{\lambda_i}e_i, \sigma(x)\sqrt{\lambda_i}e_i]\|_0 < \infty, \forall x \in H$
where $\{\lambda_i\}_{i\geq 1}$ are the eigenvalues of Q and $\{e_i\}_{i\geq 1}$ are the corresponding eigenvectors which form an orthogonal system of H. The norm $\|\cdot\|_0$ is defined by $\|x\|_0 := \|Q^{-\frac{1}{2}}(x)\|$, $x \in H_0 := Q^{\frac{1}{2}}(H)$ and $\langle\cdot,\cdot\rangle_0$ is the inner product associated with $\|\cdot\|_0$.
Using the infinite dimensional Itô formula (see [1],[4]) we apply the transformation Λ to the strong solution X_t of (3.1) and obtain

$$d\Lambda(X_t) = \Lambda^{'}(X_t)(b(X_t)dt + \sigma(X_t)dB_t^Q) + \frac{1}{2}\sum_{i=1}^{\infty} \Lambda^{''}(X_t)[\sigma(X_t)\sqrt{\lambda_i}e_i, \sigma(X_t)\sqrt{\lambda_i}e_i]dt$$

Then condition ii) yields:

$$d\Lambda(X_t) = (\Lambda^{'}(X_t)b(X_t) + \frac{1}{2}\sum_{i=1}^{\infty} \Lambda^{''}(X_t)[\sigma(X_t)\sqrt{\lambda_i}e_i, \sigma(X_t)\sqrt{\lambda_i}e_i])dt + dB_t^Q. \tag{3.2}$$

Employing the proof of Theorem 2.7.10 in [7] we see that

$$\mathcal{H}X_t(z) = E[X_t exp\{\int_0^t \langle h(s,z), dB_s^Q\rangle_0 - \frac{1}{2}\int_0^t \|h(s,z)\|_0^2 ds\}]$$

where $h(s,z) = \mathcal{H}W_s^Q(z)$. Then by invoking the infinite dimensional Girsanov's theorem (see [3], page 290) we get that

$$\mathcal{H}X_t(z) = E[\tilde{X}_t]$$

where $\{\tilde{X}_t\}_{t\geq 0}$ is the strong solution of

$$d\tilde{X}_t = (b(\tilde{X}_t) + \sigma(\tilde{X}_t)h(t,z))dt + \sigma(\tilde{X}_t)dB_t^Q, \quad \tilde{X}_0 = x.$$

(3.2) applied to $\tilde{X}_t$ gives

$$d\Lambda(\tilde{X}_t) = (\Lambda'(\Lambda^{-1}(\Lambda(\tilde{X}_t)))b(\Lambda^{-1}(\Lambda(\tilde{X}_t))) + h(t,z) + \frac{1}{2}\Gamma(\Lambda^{-1}(\Lambda(\tilde{X}_t))))dt + dB_t^Q.$$

where

$$\Gamma(\tilde{X}_t) := \sum_{i=1}^{\infty} \Lambda''(\tilde{X}_t)[\sigma(\tilde{X}_t)\sqrt{\lambda_i}e_i, \sigma(\tilde{X}_t)\sqrt{\lambda_i}e_i].$$

We put $Z_t := \Lambda(\tilde{X}_t)$. By making use of Girsanov's theorem again we find that

$$\mathcal{H}X_t(z) = E[\Lambda^{-1}(\Lambda(\tilde{X}_t))] = E[\Lambda^{-1}(Z_t)] =$$

$$= E[\Lambda^{-1}(B_t^Q) exp\{\int_0^t \langle h(s,z) + \Lambda'(\Lambda^{-1}(B_s^Q))b(\Lambda^{-1}(B_s^Q)) + \frac{1}{2}\Gamma(\Lambda^{-1}(B_s^Q)), dB_s^Q\rangle_0 -$$

$$-\frac{1}{2}\int_0^t \|h(s,z) + \Lambda'(\Lambda^{-1}(B_s^Q))b(\Lambda^{-1}(B_s^Q)) + \frac{1}{2}\Gamma(\Lambda^{-1}(B_s^Q))\|_0^2 ds\}].$$

We are coming to our main result.

Theorem 3.1 *Suppose that the conditions (i)-(iv) hold. Further let us denote by* $\dot{\beta}_i(t) \in (S)^* = S^*(\mathbb{R})$ *the singular noise of the independent standard Brownian motions* $\beta_i(t)$ *in (2.1). Then the explicit strong solution* $X_t(\omega)$ *of (3.1) takes the form*

$$X_t(\omega) = \sum_{i\geq 1} \alpha_i(\omega)e_i$$

with

$$\alpha_i(\omega) = E_{\hat{\mu}}[\langle \Lambda^{-1}(\hat{B}_t^Q), e_i\rangle M_t^{\diamond}(\omega)]$$

where the quantity $M_t^{\diamond}(\omega)$ *is given by*

$$M_t^{\diamond}(\omega) = exp^{\diamond}\{\sum_{i\geq 1}\int_0^t (\dot{\beta}_i(s,\omega) + \sqrt{\lambda_i}\langle \Lambda'(\Lambda^{-1}(\hat{B}_s^Q))b(\Lambda^{-1}(\hat{B}_s^Q)) + \frac{1}{2}\Gamma(\Lambda^{-1}(\hat{B}_s^Q)), e_i\rangle_0)d\hat{\beta}_i(t) -$$

$$-\frac{1}{2}\int_0^t (\dot{\beta}_i(s,\omega) + \sqrt{\lambda_i}\langle \Lambda'(\Lambda^{-1}(\hat{B}_s^Q))b(\Lambda^{-1}(\hat{B}_s^Q)) + \frac{1}{2}\Gamma(\Lambda^{-1}(\hat{B}_s^Q)), e_i\rangle_0)^{\diamond 2} ds\}.$$

$E_{\hat{\mu}}$ *denotes a Bochner expectation on an auxiliary probability space* $(\hat{\Omega}, \hat{\mathcal{F}}, \hat{\mu})$, $\hat{B}_s^Q := B_s^Q(\hat{\omega})$ *and* $\hat{\beta}_i(t) := \beta_i(t, \hat{\omega})$.

The integrals occuring in the Wick exponential of $M_t^{\diamond}$ *are stochastic integrals (w.r.t.* $\hat{\beta}_i(t)$*) and Bochner integrals on* $S^*(\mathbb{R})$ *(see for instance [17] for details).*

Proof. Taking into account that the $\mathcal{H}$-transform of W_s^Q in (2.2) can be written as

$$\mathcal{H}W_s^Q(z) = \sum_{k\geq 1} \delta_{\tau(i^*,j^*),k}\sqrt{\lambda_{i^*}}\xi_{j^*}(t)e_{i^*}z_k$$

which converges in a certain neighborhood $\mathbb{K}_q$ of 0 in $\mathbb{C}^{\mathbb{N}}$, this implies that

$$\int_0^t \langle \mathcal{H}W_s^Q(z) + \Lambda'(\Lambda^{-1}(\hat{B}_s^Q))b(\Lambda^{-1}(\hat{B}_s^Q)) + \frac{1}{2}\Gamma(\Lambda^{-1}(\hat{B}_s^Q)), d\hat{B}_s^Q\rangle_0 =$$

$$= \sum_{i\geq 1}\int_0^t (\mathcal{H}(\dot{\beta}_i(t))(z) + \sqrt{\lambda_i}\langle \Lambda'(\Lambda^{-1}(\hat{B}_s^Q))b(\Lambda^{-1}(\hat{B}_s^Q)) + \frac{1}{2}\Gamma(\Lambda^{-1}(\hat{B}_s^Q)), e_i\rangle_0)d\hat{\beta}_i(t).$$

Using (2.3) the statement of the theorem will follow if we are allowed to extract the $\mathcal{H}$-transform in the latter relation. But this can be done as consequence of

Lemma 3.2 *Adopting the above notation the series*

$$\sum_{i\geq 1}\int_0^t \dot{\beta}_i(s,\omega)d\beta_i(s,\hat{\omega})$$

converges in $(S)^* = S^*(\mathbb{R})$. *Moreover* $M_t^\diamond(\omega)$ *is Bochner integrable w.r.t.* $E_{\hat{\mu}}$ *on* $(S)^*$.

Proof. The proof is based on the use of Fernique's theorem as in Lemma 3.1 in [17] applied to the $(S)^*$-valued Gassian element

$$\sum_{i\geq 1}\int_0^t \dot{\beta}_i(s,\omega)d\beta_i(s,\hat{\omega}).$$

□ □

Acknowledgments

We thank B.Øksendal for fruitful discussions.

References

[1] Curtain,R.F. and Falb,P.L.:"Itô's lemma in infinite dimensions", J.Math.Anal,Appl., 31, 1970, pp. 434-448.

[2] Capinski,M. and Gatarek,D.:"Stochastic equations in Hilbert spaces with applications to Navier-Stokes equations in any dimensions", J.Funct.Anal., 126, 1994, pp. 26-35.

[3] DaPrato,G. and Zabczyk,J.:"Stochastic equations in infinite dimensions", Cambridge University Press, Cambridge, 1992

[4] Doss,H.:"Liens entre equations differentielles stochastiques et ordinaires", Ann.Inst.H.Poincare' Sect.B(N.S.), 13, 1977, pp. 99-125.

[5] Filinkov,A. and Sorensen,J.:"Differential equations in spaces of abstract stochastic distributions", Stoch.Stoch.Rep., 72, 2002, pp. 129-173.

[6] Hida,T., Kuo,H.-H., Potthoff,J. and Streit,L.: "White noise analysis". Kluwer 1993.

[7] Holden,H.,Øksendal,B.,Ubøe,J. and Zhang,T.-S.:"Stochastic Partial Differential Equations- A Modeling, White Noise Functional Approach" Birkhäuser, Boston 1996.

[8] Isobe,E. and Sato,Sh.:"Wiener-Hermite expansion of a process generated by an Itô stochastic differential equation", J.Appl.Prob, 20, 1993, pp. 754-765.

[9] Ito,K.: "Multiple Wiener Integral", J.Math.Soc.Japan, 3, 1951, pp. 157-169.

[10] Kouritzin,M.A. and Remillard,B.:"Explicit strong solutions of multidimensional stochastic differential equations", Preprint, University of Alberta, 2002.

[11] Kunita,H.:"Stochastic differential equations and stochastic flows of diffeomorphisms", Ecole d'ete de Probabilites de Saint-Flour,XII,1982,P.L.Hennenquin ed.,LNM 1097, Springer, Berlin, 1984,pp. 143-303.

[12] Kondratiev,Y., Da Silva,J.L, and Streit,L.:"Generalized Appell systems", Methods Funct. Analysis Topology, 3 (3), 1997, pp. 28-61.

[13] Karatzas,I. and Shreve,S.E.:"Brownian motion and stochastic calculus", second edition, Springer-Verlag, 1991.

[14] Kuo,H.H.:"White noise distribution theory", Prob and Stoch, Series, Boca Raton, FL: CRC Press, 1996.

[15] Kallianpur,G. and Xiong,J.:"Stochastic differential equations in infinite dimensional spaces", IMS Lecture Notes-Monograph series, Vol 26, 1995.

[16] Løkka,A.,Øksendal,B. and Proske,F.:"Stochastic Partial Differential Equations driven by Lévy Space-Time White Noise", to appear in Annals of Applied Prob., 2002.

[17] Lanconelli,A. and Proske,F.:"On explicit strong solution of Itô-SDE's and the Donsker delta function of a diffusion", Preprint series in Pure Mathematics, University of Oslo, 27, 2003.

[18] Leha,G. and Ritter,G.: "On diffusion processes and their semigroups in Hilbert spaces with an application to interacting stochastic systems", Ann. Probab., 12, 1984, pp. 1077-1112.

[19] Mikulevicius,R. and Rozovskii,B.:"Linear parabolic Stochastic PDE's and Wiener Chaos", SIAM J.Math.Anal. 29, 2, 1998 pp. 452-480.

[20] Obata,N.:"White noise SPDE'scalculus and Fock space",LNM 1577, Berlin, Springer-Verlag, 1994.

[21] Sussmann,H.J.:"On the gap between deterministic and stochastic ordinary differential equations", Ann. Prob., 6, 1978, pp. 19-41.

[22] Veretennikov,A.Yu. and Krylov,N.V.:"On explicit formulas for solutions of stochastic differential equations", Math.USSR Sbornik, 29, 1976, pp. 229-256.